# Development and Results of the Swedish Road Deflection Tester

by

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*"La déflexion d'une chaussée est un peu, pour l'ingénieur routier,
ce qu'est la température d'un malade pour un médecin."* [33]

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# Preface

The work presented in this licentiate thesis has taken far more time than originally planned. This is partly due to the rather unfortunate history of the project, and partly due to my, under certain circumstances, procrastinating nature and personality. For me, this research project started in February 1996 at the Department of Structural Engineering at The Royal Institute of Technology in Stockholm. From December 1999 to the present date, I've been working at the Swedish National Road and Transport Research Institute (VTI) in Linköping.

I thank my supervisor Professor Anders Eriksson for always being confident in me, and positive to my annual estimate, and sometimes promise, to have the work finished within the year.

Several people have been of great help in my work. First and foremost, my thanks go to Carl A. Lenngren at Swedish Road Administration Consulting Services for sharing his vast knowledge and experience in road research, collaboration on conference papers, and also, especially at the beginning of my career, for acting as a mentor to me.

At VTI I have especially enjoyed the company of the "RST group" — Stig Englundh, Inger Forsberg, Thomas Lundberg and Leif Sjögren. You have all made the time at work a constant source of joy and inspiration. Many thanks to the former VTI employee and "designated driver" Hans Velin. Not only for brilliant manoeuvring of the truck, but for very enjoyable company on the RDT trips in Sweden and abroad. Also, many thanks to the constantly very helpful and always friendly staff at the VTI library.

In September-October 2002 the RDT was taken to England and France for evaluation and comparative testing with the Deflectograph systems used in the respective countries. This was a most interesting trip, and it was very stimulating to see the big interest the RDT technique generated. Brian Ferne, David Gershkoff, Peter Watson, Patrick Werro, and Kim Adams at the Transport Research Laboratory in Crowethorne, thank you for making our stay in England so stimulating and pleasant. *Un grand merci á Jean-Michel Simonin et Denis Lievre à Laboratoire Central des Ponts et Chaussées pour votre générosité, bonne compagnie aux dîners, et pour les déjeuners — biens et longs.*

Linköping, May 2006.

Peter Andrén

# Abstract

A project to construct a high-speed road deflection tester was initiated in the 1991. A mid-sized truck was used as a carrier for the first prototype. The results were promising and it was decided to build a full-size truck system. The new vehicle, based on a Scania R143 ML, was completed in 1997.

The Road Deflection Tester (RDT) is equipped with two arrays of twenty non-contact laser sensors that collects transversal surface profiles at normal traffic speeds. One profile, placed between the wheel axles, constitutes an unloaded case. The other profile, just behind the rear axle of the vehicle, constitutes the loaded case. By subtracting the front cross profile from the corresponding rear one, the "deflection profile" is assessed. The deflection is assumed to vary with the stiffness of the road.

In order to produce a large load on the rear wheels the engine was mounted in the back of the vehicle, slightly behind the rear axle. In testing mode the rear axle force is approximately 112 kN, and the front axle force is about 30 kN. An incremental wheel pulse transducer, two force transducers and two accelerometers, an optical speedometer and a gyroscope are also mounted on the RDT.

The first test programme was carried out in 1998. Due to the careful choice of test sections, data from these sections still produce the best results. A smaller test programme was carried out in 2001, and a larger one in 2002 when the RDT was taken to England and France for demonstration. Promising results, both on an aggregated scale and for individual test sections, have been obtained. The RDT compares favourably with the Falling Weight Deflectometer.

Short histories of road construction and road research give some historical and cultural background to the more recent developments. A more comprehensive history of rolling deflectographs presents all devices found in the literature from the start in the mid-fifties when the California Traveling Deflectograph and Lacroix Deflectograph were constructed, to the latest laser based High-Speed Deflectograph. Many references are given for further reading.

The data acquisition hardware on the RDT system consist of sensors, signal converters, signal processing cards, an industrial computer for data communication, and an ordinary PC for operating the equipment and data storage.

The software used to evaluate the data is written entirely in Matlab. Many levels of pre-processing make evaluation relatively fast, and the modularised design makes it easy to implement new evaluation algorithms in a clean and efficient way.

A literature survey on the deformations of solids under static and moving load is presented in Appendix A. The static case started with Boussinesq in 1885, was much developed in the sixties, but since the eighties only a very limited amount of new results have been published. The moving load case, on the other hand, is still an field of active research and development.

# Sammanfattning

Ett projekt för att bygga en vägdeflektionsmätare för normal trafikhastighet påbörjades 1991, med en medelstor lastbil som bärfordon. Resultaten var lovande och det beslutades att ett större system skulle byggas. Det nya fordonet, baserat på en Scania R143 ML, färdigställdes 1997.

Road Deflection Tester (RDT) är utrustad med två uppsättningar à tjugo lasersensorer som mäter vägens tvärprofil i normal trafikhastighet. Den främre profilen, placerad mellan hjulaxlarna, utgör det obelastade fallet. Den bakre profilen, placerad strax bakom bakaxeln, utgör det belastade fallet. Deflektionsprofilen erhålls genom att subtrahera den främre profilen från motsvarande bakre profil. Denna deflektion antas variera med vägens styvhet.

För att skapa så stor last som möjligt på bakaxeln är motorn placerad i den bakre delen av lastbilen. I testläge är kraften på bakaxeln ungefär 112 kN, och kraften på framaxeln är cirka 30 kN. En hjulpulsgivare, två kraftgivare och två accelerometrar, en optisk hastighetsmätare och ett gyro är också monterade på RDT:n.

Det första testprogrammet utfördes 1998. Tack vare ett noggrant val av teststräckor erhålls fortfarande de bästa resultaten från dessa tester. Ett mindre testprogram utfördes 2001, och ett större i 2002 när RDT:n togs till England och Frankrike för demonstration. Lovande resultat har erhållits både på en aggregerad skala och för individuella teststräckor. Resultaten från RDT:n har jämförts med de från fallviktsmätaren, med gott resultat.

Korta historiebeskrivningar över vägbyggande och vägforskning ger lite historisk och kulturell bakgrund till den senare utvecklingen. En mer omfattande historiebeskrivning över fenomenet rullande deflektografer presenterar alla utrustningar från starten på mitten av femtiotalet när California Traveling Deflectograph och Lacroix Deflectograph konstruerades, till den senaste laserbaserade High-Speed Deflectograph. Åtskilliga referenser ges för ytterligare läsning.

Hårdvaran för datainsamlingen på RDT:n består av sensorer, signalomvandlare, signalbehandlingskort, en industridator för datakommunikation och en vanlig PC för att sköta utrustningen och lagring av data.

Mjukvaran som används för utvärdering är skriven uteslutande i Matlab. Flera nivåer av databehandling gör utvärderingen relativt snabb, och den modulariserade designen gör det lätt att implementera nya utvärderingsalgoritmer på ett snyggt och effektivt sätt.

En litteraturgenomgång rörande deformationer av solida kroppar under statiska och rörliga laster presenteras i Appendix A. Det statiska fallet startade med Boussinesq 1885, utvecklades betydligt på sextiotalet, men sedan åttiotalet har endast en begränsad mängd nya resultat publicerats. Fallet med rörliga laster, å andra sidan, är fortfarande ett aktivt forsknings- och utvecklingsområde.

# Contents

# Chapter 1

# Introduction

During the last one-hundred and fifty years a rapid development and expansion of global communication networks, such as landline and mobile telephones, radio and satellite communication and, more recently, the Internet have transformed the world we live in. One might think that this development would reduce the need for a more physical infrastructure — as we all can stay in contact with each other without travelling — but that certainly hasn't been the case. On the contrary, roads are still one of the most important part of the, both regional and global, infrastructure. Probably, albeit often indirectly, one form of communication stimulates other forms of communication, and we can expect to see an increase in both vehicular and Internet traffic in the future. But, even if roads are much older than the communication technologies of today, the knowledge about them is far from complete and, with the ever-increasing use of roads in modern society, the need for road research is as high as ever.

The Swedish annual budget for road maintenance has been about seven billion Swedish crowns over the last years [258]. Worldwide this figure would be enormous, which makes it easy to draw the conclusion that huge savings can be made if road construction and maintenance would be more cost effective. Not only would it mean good opportunities for the industrialised part of the world to economise on the road expenses, but it would also provide better possibilities for the underdeveloped regions, as transportation usually is one of the major barriers for industrial and agricultural development and growth.

Roads are a significant intrusion and interference in the environment. Poor roads will, in general, force vehicles to use more energy and thus pollute more. Of course, the major responsibilities for pollution caused by traffic is not in the hands of the road engineers, and no sustainable decrease of traffic pollution can be achieved by building better roads. However, the reduced pollution, due to better roads, comes hand in hand with higher riding comfort and safer driving, which makes the demand for improved road quality manifold.

Today a lot is known about how to build roads, but not so much is known on how to keep roads in a good condition, and very little is known about how to determine the structural condition of a road in some not too complicated and slow manner. Therefore much more effort must be put into the research on how to keep the existing road net in a permanently good condition. Any technique capable of doing this will be an immense assistance in any Pavement Management System (PMS).

The present work will present one such method, the Road Deflection Tester (RDT), aiming at assessing the structural condition of roads at normal traffic speeds. Road roughness, geometry and cracks can all be measured at normal traffic speeds today, but, on a road network level, the structural condition is normally assessed from road surface data. In all practicality this means that you get to know that the road is weak when it's already broken. With the RDT this information could be presented when there is still time to strengthen the road, and hence avoid more expensive repairs or even reconstruction.

## 1.1 Background

Before the RDT project no rolling deflectograph was ever used in Sweden. Both Norway and Finland purchased Lacroix deflectographs, and Denmark even developed their own deflectograph (see page 15). In 1976, the Swedish Road Administration (SRA) organised a comparative study of different bearing capacity meters [208]. The Lacroix from Norway, and the first generation Danish Deflectograph participated. Even though the deflectographs came out favourably in the tests, the SRA never purchased one of its own. With this background, Sweden might seem a strange place for the RDT project to originate. Nonetheless, with the excellent results from the Road Surface Tester (RST), developed in the early eighties, the idea came up that by using roughly the same technology it should be possible to measure the deflection caused by the wheel loads of a rolling vehicle.

A detailed feasibility study was conducted prior to the construction [12]. In this study the question whether the then existing sensors could actually measure the effect on the pavement surface of a moving wheel load was examined. The necessary resolution and accuracy of the system was determined, and the availability of sensors with the necessary performance were researched. The expected deflection was simulated with the Finite Element Method, indicating that the deflections would be large enough to detect. Some possible problems were identified but, all in all, it was considered to be possible to build a functioning high-speed deflection tester. However, in the conclusions to this feasibility study one can read that (in the author's translation) "Lastly, competence in behavioural sciences will be required to adapt this large and complicated system to the personnel — from the drivers to the decision-makers." In hindsight, this is something we definitely haven't done enough of (especially concerning the decision-makers).

A profitability analysis for the RDT system was prepared by Clas-Göran Rydén of SRA in 1994 [230]. The basic conditions for the analysis was (in the author's translation): "The revenue of the RDT system depends on the size of the profit made if you have access to the information the RDT system can provide, as compared to not having this information." The total "revenue" for all state-owned paved roads for one year was estimated at 97m SEK. The, by far, largest part of this sum was the 70m saved on the "ranking of maintenance objects", i.e. identifying the individual objects on the road network where an investment in maintenance would have the highest returns.

A first RDT prototype was constructed in the early nineties, and merited by the promising results from this, a second prototype was built in the mid-nineties (see

Chapter 3 for detailed information). At the time the construction of the second RDT was finished the SRA stopped funding the project. This was mainly because of a general overhaul of all research projects. The RDT project was, it's probably fair to say, guilty by association to the SweRoad and RST-Sweden companies. These two companies were involved in a somewhat complicated operation, set up by the former SRA general director Per Anders Örtendahl, were money from the SRA had been used to finance development and marketing of the RST and PAVUE crack detection systems in the USA. In January 1997 the newspaper *Dagens Nyheter* published articles dealing with the companies affiliated with the SRA. Two full page articles, where the headlines read 'SRA's USA-flop cost 50 millions' and 'Örtendalh's swindle fooled the government', are reproduced in Fig. 1.1(a) and 1.1(b).

DN SPECIAL

Vägverkets USA-flopp kostade 50 miljoner

DN SPECIAL

Örtendahls fiffel lurade regeringen

(a) Dagens Nyheter 19/1/1997

(b) Dagens Nyheter 20/1/1997

Figure 1.1: News paper articles from *Dagens Nyheter*. (Reproduced with kind permission of *Dagens Nyheter*.)

The SRA stopped funding the project just before serious testing of the system was about to start. The RDT was rushed from completion to production use. To a large extent this testing still remains to be done, even though some validation and system checks have been performed on test primarily for production use. So, since 1996 only very limited development has been conducted. Data from a test programme, consisting of about ten sites, in 1998 and from the demonstration in England and France in 2002 is the best working material so far. Also, a few months after completion of the second RDT the head development engineer left VTI. Much hands-on experience and undocumented knowledge for the system was lost.

Ten years ago, in 1996, a group of researchers headed by Peter P. Canisius were commissioned by the SRA and the Swedish Governmental Agency for Innovation

Systems (VINNOVA), to evaluate the research carried out at VTI and the Institution of Road Research at KTH. The quotation below is one of their conclusions, found under the "Road Surface Analysis" heading:

> "The road analysis group has done excellent research in developing very high quality devices for analysing road surfaces. The most advanced development is the high speed deflection tester (R.D.T.). The review team strongly suggests that sufficient funding is made available to complete the work on the prototype R.D.T..
>
> Furthermore it is suggested to fund a project on the necessary precision of the equipment. This means to define what is the needed accuracy and precision for the measurements in view of accuracy and precision of the various analysis and prediction models as well as the variability of the pavement structure itself." [54]

At present, the future of the RDT is uncertain. As of this writing, Urban Karlström, the director general of VTI has asked for a decommission plan to be made, due to the low "project portfolio". After many years of inattention from the people involved at the SRA, the attitude towards the RDT seems to improve, but as of 2006 there has been no funding.

## 1.2 Roads, a brief historical review

From about 4000 B.C. humans have had enough knowledge and proper tools to build roads. Ever since, roads have both existed and been an essential part of the human civilisation's further development and expansion, and it's almost impossible to underestimate the importance roads have had in over all evolution of human society. This importance is quite easily disregarded from, probably due to the fact that the road is so common and "simple" that it almost taken for granted. Nevertheless, roads has always been of the utmost importance to civilisation, and they are likely to hold that position for many decades to come. What will follow here is a very brief overview of the history of the road, in order to give some historical and cultural background to the more recent developments in this area. A more detailed survey can be found in *Ways of the world: a history of the world's roads and of the vehicles that used them* by Maxwell G. Lay [168].

For sure, no major human civilisation could have been built without a significant road system. Different civilisations might have had slightly different reasons to build their roads, but trade and transport were certainly the main influences. Transportation of soldiers and equipment in times of war has always been a strong incitement for building roads, with an emphasis on building, as the mostly already existing transport routes used in war needed to be much stronger to carry military vehicles, and other equipment.

More that one thousand years before the birth of Christ a quite well developed net of roads, used as caravan routes, existed in the Middle East. These roads, which weren't much more than beaten earth, linked not only the countries around the Mediterranean, but also large parts of northern Europe, China and India. The by far most known of these roads is the Silk Road dating from approximately 300 B.C.

when it merely consisted of many small caravan routes linked together. Two hundred years later it reached, in its own right, all the way from China to the Mediterranean, making it an active trade route for centuries to come (more or less until Vasco de Gama found a sea route to China in 1497).

In China nationwide road construction started at about 1000 B.C. under the reign of the Western Zhou dynasty. A stronger development phase occurred a few hundred years later during the rule of the Qin and the Han dynasties, but the most active period of road making in ancient China came with the first Emperor Shi, who ordered 15 metres wide post roads to be built all over his vast empire. As mighty an achievement their postal system might have been, the Shi emperors' main legacy to the world is the more lasting Great Wall of China.

The Incas in South America are certainly not as famous for their roads as for their art, religion and human sacrifices to their gods, but they also had a road net of the impressing 23,000 kilometres crossing the, from a road point of view, unfriendly terrain. When European explorers first saw these roads in the sixteenth century they were considered to be of much higher quality compared to the contemporary European roads. (Of course, the myths of Eldorado, with streets paved with gold might have contributed to this . . . )

With the exception of the Silk Road no roads would stay famous through history until the Romans started to build their extensive road system in order to rule their vast empire. Today, roads such as Via Appia, Via Nerva and Via Latina are still famous, and the Romans are almost as well-known for their roads as they are for their emperors, architecture and the Latin language. The Roman road system covered the, to them, entire civilised world, and the proverb "all roads lead to Rome" was actually more than a proverb.

With and after the fall of the Roman Empire not much happened with road development for many centuries, and if the Middle Ages can be said to represent a low water mark in technical evolution, this is certainly true for the art of road making. Not only were the Roman techniques for building roads forgotten, but the roads themselves were also allowed to deteriorate as the material from them often was used for other purposes. This situation remained more or less the same for more than a millennium, when the Industrial Revolution brought forth a revival in travel and transport, and with that a more systematic and scientific approach to road construction.

## 1.3 Road Research, a brief historical review

So, even if roads have existed since prehistoric times, road research and road technology are far more recent phenomena. Surely, the Romans had some kind of empirical road research, but all their roads were self-supporting structures relying on size rather than design, which made the need for labour far more important than the need for technical skill. This kind of rigid roads had the drawback of being very expensive, time consuming to build and not always comfortable to use. One the other hand, if properly built, they could last for centuries and even millennia, with the result that many of the old Roman roads are still in use in large parts of their old empire.

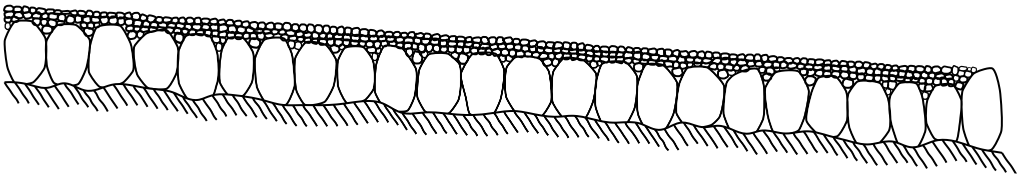


Figure 1.2: Illustration of Trésaguet's road construction. (Redrawn from [168].)

The French engineer Pierre-Marie Jérôme Trésaguet was the first to change the expensive Roman way of building roads. He argued that the natural formation should do the supporting, and the pavement should keep the natural formation dry and strong, and to protect it from pressures high enough to cause damage. Trésaguet's construction was based on a thick layer of large stones placed on a cambered surface, and an additional layer of smaller broken stones to make the surface smooth and easy to repair, as illustrated in Figure 1.2 above.

In England, Thomas Telford was the one who made road making a science. Telford was a multi-talented man who, apart from building roads, constructed bridges, harbours, canals and buildings. As Trésaguet, Telford used large stones on top of the natural foundation with the difference that the stones, and not the foundation, formed the cambered surface. Smaller stones were then used to form a good running surface for vehicles. The Trésaguet and Telford roads didn't differ very much, and they both required good drainage and, at least under heavy traffic, almost daily maintenance.

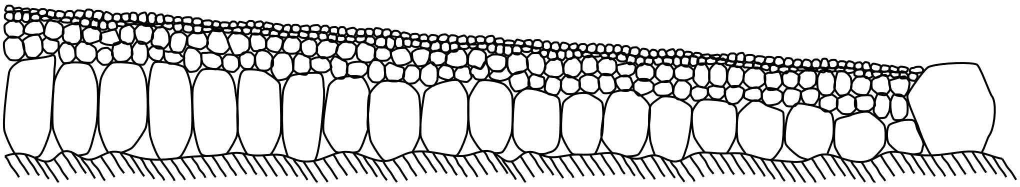


Figure 1.3: Illustration of Telford's road construction. (Redrawn from [168].)

A paradigm shift in road construction took place when John Loudon McAdam realised, in the beginning of the nineteenth century, that crushed rock (what we today, eponymously, call macadam) of the right size could be used instead of the costly and complicated use of larger hand crafted stones as the road base. The scientific explanation to the very good behaviour of macadam comes from the interlock between the individual pieces of broken stone, as opposed to the almost non-existing friction between the individual pieces in gravel. Now, macadam was not the sole solution to road paving difficulties — the roads tended to be very dusty in summer and little else than a series of mud holes for the other seasons. Though, the main reason for this poor quality was usually that organic material was used to repair the roads, strictly against McAdam's recommendations.

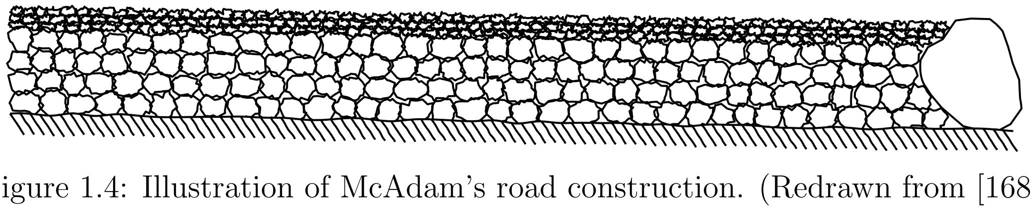


Figure 1.4: Illustration of McAdam's road construction. (Redrawn from [168].)

Numerous different paving materials, (e.g. rubber, wood, stone and gravel) was tested in the eighteenth and nineteenth centuries, resulting in two "winners", namely asphalt and concrete. The use of asphalt as a paving material has a long history that got going in the late eighteenth century when mastic (bitumen distilled from natural asphalt) was used to waterproof timber decks. As this turned out to be very slippery sand was added, and when the sand penetrated into the mastic a much stronger and stiffer material was born. The next step was to find the right mixture of stone and mastic, and to find a cheaper and better mastic than the quite rare natural bitumen. First, tar obtained as a byproduct from coal processing was used, but a better solution came with bitumen produced as a spin-off from oil refining. Today, bitumen mixed with rubber is a widely used technique for the binders, and the stone material must be chosen as a well-graded mixture, i.e. no piece of stone should float in the binding material, but always be in contact with other stones.

The probably only way to find out which paving material is best suited for some specific purpose is to perform a test. Many tests of this kind has been performed over the years, starting in 1838–1839 with a trial of different paving products on the Oxford Street in London. Later, many tests were carried out in Europe and especially in the United States.

How to make a full-scale test can be learnt from a 1962 paper by Lee and Croney [171]. "The principle followed in constructing full-scale experiments is simple. Pavement structures of different thicknesses and employing different base and surfacing materials are laid adjacent to each other on a subgrade, the properties of which are known, and their performance is assessed at regular intervals under traffic. The main criterion used to judge the performance of flexible pavements is the permanent deformations which takes place under the action of traffic." Today, test roads are equipped with lots of sensors, but the basic principle is the same.

The first American full-scale test was conducted at Bates, near Springfield in Illinois, between 1920 and 1923. These tests was primarily directed at design methods for concrete roads, and showed that the strength of the natural formation plays a key role. Between 1952 and 1954 the Western Association of State Highway Officials (WASHO) conducted a large test series in Idaho. During these tests the Benkelman Beam was developed by Alfred Benkelman [277]. But even larger tests were to come, and it would not be possible to deal with the history of road science without to mention the AASHO Road Tests, conducted in the USA from 1956 to 1961 [1]. In these tests 126 army trucks drove 27.500.000 kilometres on a specially built road constructed with many different techniques. The traffic rolled from November 1958 to November 1960. A very readable introduction to the AASHO project is published as Special Report 61A [2]. Many valuable results and techniques originates from these tests, and they are often used as reference for new tests.

However, the era of large road tests is not over. New traffic requirements call for new tests for a better traffic environment and more sustainable roads. The Mn/Road project in Minnesota is the latest and most promising road project of today, and this brief history section can suitably be ended with a quotation from the homepage of the Mn/Road project. *The Minnesota Road Research Project (Mn/ROAD) is the world's largest and most comprehensive outdoor pavement laboratory, distinctive for its electronic sensor network embedded within six miles of test pavements.*

## 1.4 Scope and Objective

The objective with the present work is to describe the history, development, results, and current status of the Swedish Road Deflection Tester (RDT). Furthermore, the history of rolling deflectographs and a literature survey on deformations of solids under static and moving loads are presented.

The main purpose of the RDT, and similar devices, is to provide data to a Pavement Management System. The actual use of the deflection data will, however, not be discussed at any length in the present report.

The history section is limited in scope to rolling deflectographs. Stationary and semi-stationary devices are not covered.

The original plan for the present thesis was to make a theoretical model of the deflection basin under a moving wheel using the theory of wave propagation in layered material. While at the KTH the work was done mainly in this direction, but no finished model existed when the author left KTH for VTI. At VTI, the limited time spent on the RDT project has solely been spent on measuring roads and evaluating the data. The literature survey for deflections under moving loads is nonetheless closely related to the project, and included in Appendix A. In the survey very little material on beam and plate theory is included, as these models are used primarily for concrete roads, which, in turn, are almost nonexistent in Sweden.

# Chapter 2

# Rolling Deflectometer History

During the nineteen-forties and fifties, with an increasing use of deflection measurements as a control method of road strength and structural condition, the methods available at that time were soon found to be too slow. In widespread use was the Benkelman Beam [277] and similar or derived devices, and stationary equipment as the General Electric travel gauge. In the fifties the Benkelman Beam, with about 300 deflection measurements per day for a skilled three-man crew [286], was thought to be a quick method. Soon, however, this was not enough and quicker and less labour intensive methods were being called for. The idea to mount the deflection measuring equipment on the truck causing the actual deflection was, apparently, obvious enough to present itself more or less simultaneously in the US and in France.

Many state-of-the-art reports on pavement deflection devices have been written in the past, e.g. [67, 99, 110, 154, 270]. Most of these reports have presented the then current status of all deflection devices, and not only the moving ones. In the present report, apart from giving the current picture, a more historical review has been attempted. The expression "rolling deflectograph" is used here as a generic term for all types of moving pavement deflection measuring devices, no matter of speed or implementation.

This history chapter is limited in scope to the moving deflectographs. However, many different kinds of deflection assessment equipment exists, or have existed. These can broadly be placed in three categories: stationary, semi-stationary, and moving. The first category includes equipment like the General Electric travel gauge [140], Linear Variable Differential Transformers and Multi-Depth Deflectographs [117, 261], light emitting diode systems [117], accelerometers and geophones [261], etc. The second category includes, e.g., devices like the Benkelman Beam [277], the different types of falling weight deflectometers [68], plate bearing tests (quite rare today but see [224] for a recent study) and the "Thumper" [202]. The third category consist of the moving mechanical or laser based deflectographs, covered here.

Further, the main focus is on the technical aspects of the devices. The actual use of the deflection data will not be discussed at any length. Interesting "starting-points" and further references on this can be found in following papers: Butler and Kennedy [51]; Leger and Autret [174]; du Mesnil-Adelee and Peybernard [81]; and Catt [55] on the use of deflections to characterise the pavement construction on a large scale. McCullough and Bailie [196]; Autret [16]; Hoyinck, van den Ban and Gerritsen [130]; and Kumar and Kennedy [164] all consider alternative ways to handle Deflectograph data (mainly by interpreting the shape of the deflection bowl to assess layer properties), and the benefits thereof on a road network. The methods used to process measurement data and database considerations are treated by Boulet and Gramsammer [42]. A paper by Lenngren [177] treats the possible strategies for the use of rolling deflectometer data in pavement management.

Deflections (or more generally deformations) is probably the most intuitive way to measure the strength and quality of something: apply a force at something and measure how much it yields. This is common practise in everyday life for most of us (checking the firmness of a bed, the pressure in a bicycle tyre, the ripeness of an avocado, etc.) and definitely common practise in engineering.

The purpose with any sort of deflectograph is, of course, to measure the deflection of a pavement under a given force and use this deflection either to calculate some strength or stiffness parameter (e.g. the elasticity modulus) or to use the deflection as a direct measure of the strength and stiffness. The definition of pavement deflection given by Hveem [140] will be used in the present report. Hveem states that deflection is: “A transient downward movement of the pavement when subjected to vehicle wheel loads. A deflected pavement rebounds shortly after the load is removed.” Pavement deflections under normal traffic loads are in the range from less than a tenth of a millimetre for Portland concrete pavements to one or a few millimetres for a weak asphalt concrete road.

In general, detailed descriptions on the different rolling deflectographs have not been widely published. Hard to find internal reports or no documentation at all seems to be the norm. One exception to this is the French Lacroix deflectograph, which is beautifully described and documented in a series of articles in *Bulletin de liaison des Laboratoires Routiers Ponts et Chaussées*.

## 2.1 Mechanical Systems

### 2.1.1 California Traveling Deflectograph

The data (ranging from 1938 to 1954) presented in Hveem’s 1955 paper “Pavement deflection and fatigue failures” [140] was collected with, first, the General Electric travel gauge and, later, with the Benkelman Beam. The G.E. gauge had to be installed in the pavement, and even if the results are of very high quality only the installation points are represented. The Benkelman Beam is more mobile but even a skilled three-man crew can only make about 300 measurements per day. To speed things up the California Division of Highways—Materials and Research Department developed and built the semi-automatic California Traveling Deflectograph during the years 1955–1960 [287]. (From the references available it’s not obvious what part of the measuring cycle that needs human involvement, making the process *semi*-automatic.)

This device is briefly described in another paper by Hveem [141] and shown in Figure 2.1. The operating principle is that of an automated Benkelman Beam. A truck plus trailer held a traversing frame that carried up to four Benkelman Beam type probe arms. The frame holding the probes was put to rest on the pavement while the steady-moving vehicle passed, and the frame was then moved

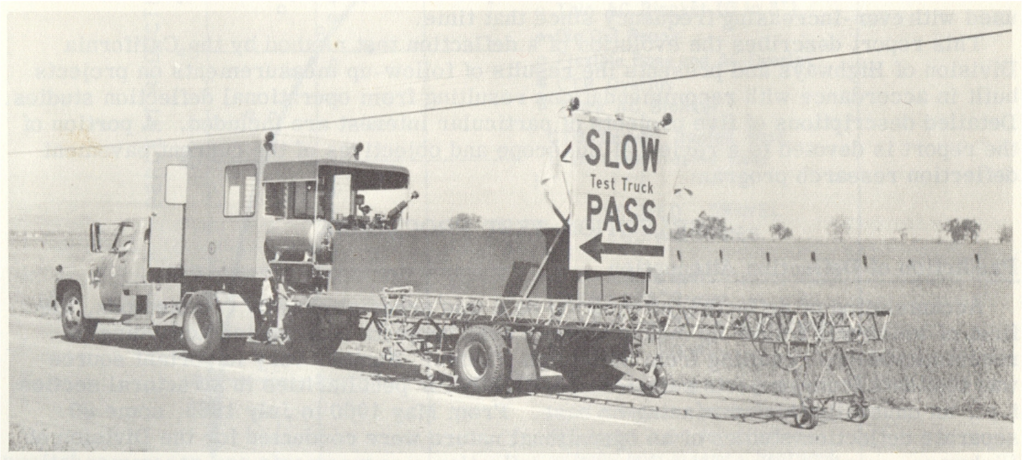


Figure 2.1: The California Traveling Deflectograph. (Reproduced from [287].)

automatically to the next point on the pavement. (It's probably safe to assume that two of the probes were always placed to measure the deflection between the dual tyres.) Data was originally registered on chart paper [140], and later recorded electronically on tape [286]. The operating speed was about 0.8–1.2 km/h, with one set of samples assessed every 3.8 metres. A three-man crew made 1500–2000 deflection measurements per day. The axle load could be varied, by means of a movable weight, from about 5 to 7 tonnes [140, 286, 287].

The California Traveling Deflectograph was used until 1969 for routine work and till 1980 in research [52, 53]. Only one device was manufactured, indicating that the project was not a total success. When the California Traveling Deflectograph was taken out of service the trailer part was retained to be used with Benkelman Beam test. This is now referred to as the California Deflectometer.

### 2.1.2 Lacroix Systems

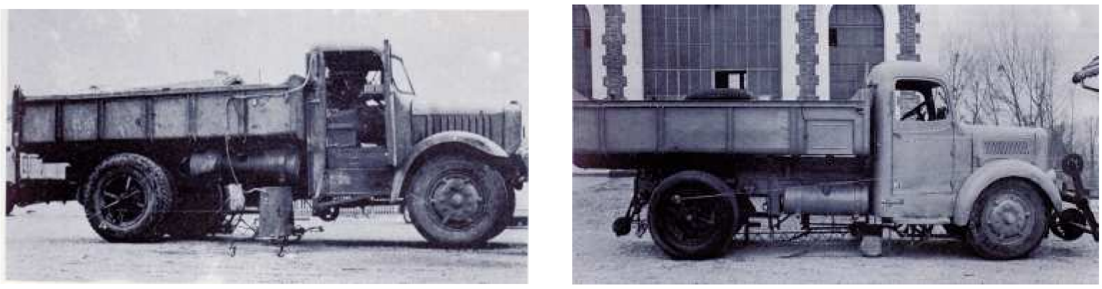
A, with the California Traveling Deflectograph, contemporary deflectograph project was the French Lacroix system. The Lacroix-style rolling deflectograph measures the deflection between a pair of double rear tyres, but the measuring probe arms and registration mechanism are a bit different from the Benkelman Beam.

The operating procedure is basically the same for all Lacroix deflectographs. First the frame holding the backward-pointing probe arms are placed on the surface of the pavement, on the wheel paths between the wheel axles. The lorry drives at a steady speed. The frame stays on the surface till the tip of the probe arm is positioned a short distance behind the rear axle. In this way the Lacroix records slightly more than a one-sided deflection basin for each test point. The wires and guidance system will then move the frame to a new position a given distance along the road, and the procedure is repeated. With this scheme deflection values, equally distanced at about four metres, will be assessed along the road.

The first Lacroix-style deflectograph was constructed in 1956 by M. J. Lacroix, at that time chief engineer at Ponts et Chaussées à Périgueux in Dordogne in the southwest of France. This first prototype had an operating speed at about 1.8 km/h, and could only measure the deflection in the right wheel path. The 0.8 metres L-shaped beam with one probe arm measured one point every 3.6 metres. The deflection measures were registered on paper. The article by Mr. Lacroix himself [165] from *Bulletin de liaison des Laboratoires des Ponts et Chaussées* describes the first version in detail.

The second prototype of the Lacroix was developed in 1961. With a 1.2 metres T-shaped dual probe arm this version assessed the deflections in both wheel paths, and also used an electro-optical photographic recording device beside the chart paper recorder.

The third version was developed in 1964 (and was also called version 1964) and is beautifully presented by Hubert, Noret, Donnat, Morin and Parey [135], and by Prandi in both French [227] and English [226]. The operational speed was increased to 2.0–2.7 km/h depending on the condition of the road surface. The length between two test points was 3.2 metres. The recording of data was still done with both a graph paper and an optical device. In the article by Hubert et al. the possibility to register the data electronically for use by a computer is mentioned, even if this would take a few years to realise on a larger scale. The third version (no longer called a prototype) was widely used in France (and other countries) and in 1965 1 300 km of road was measured in France alone [227].



(a) The first Lacroix prototype (1956). This version is built on a Willème S. 10 truck.

(b) The second Lacroix (1961), built on a Berliet GLM 10 M2 or M3 truck.

Figure 2.2: Early development of the Lacroix Deflectograph. (Both photographs are reproduced from [165].)

The different methods available to store and handle data by computers was discussed in a paper in 1969 by Ph. Léger [175]. Either data could be recorded directly to a machine readable media (experiments were done with an ordinary 1/4 inch tape recorder), or via a punched paper derived from the photographic film normally used.

With the 1972 paper by P. Autret [17] the Lacroix deflectographs were started to be called with version numbers 01, 02, 03 for the three different versions (starting with the second prototype from 1961). The paper presents, for the first time, the deflectograph with "inverse beam", i.e. the T-shaped beam is replaced with one with the middle arm pointing forwards instead of backwards, thus resembling a two-tinned fork more than a T. This modification allows for assessment of the whole deflection basin, giving additional information on the structural condition of the road. The operating speed is now 4.0 km/h. The third version was the first one with a longer chassis allowing for testing on stiffer pavements. Siffert [246] writes that in 1969 27 Lacroix' were used in France and seven trucks had been exported to Belgium, The Ivory Coast, Spain, Great-Britain, Roumania, Switzerland, and Czechoslovakia. The success continued, and Autret [17] writes that from 1969 to 1972 the Lacroix deflectograph had been sold to South-Africa, Finland, Holland, Turkey, and Venezuela.

(a) The third Lacroix (1964), built on a Berliet GLM 10 M2 truck. (Reproduced from [135].)

(b) The Lacroix Deflectograph 04. (Reproduced from [33].)

Figure 2.3: Later development of the Lacroix Deflectograph.

The version 04 of the Lacroix was introduced in 1980 and first presented by Boulet and Gramsammer [43]. A much more thorough description can be found a paper by Baucheron de Boissoudy, Gramsammer, Keryell and Paillard [33]. This model was, as model 03, intended for stiffer pavement. The main modification, according to Boulet and Gramsammer, was the introduction of an on-board mini-computer to process the data. Version 03 and 04 have a 6.75 wheelbase comparing to the 4.5 metres on version 01 and 02. Another big difference between versions 03 and 04 was that the deflection beam was about 2.9 metres longer on the 04 version. The longer beam used on the 04 version was also mounted on the existing 03s, then called 03.5.

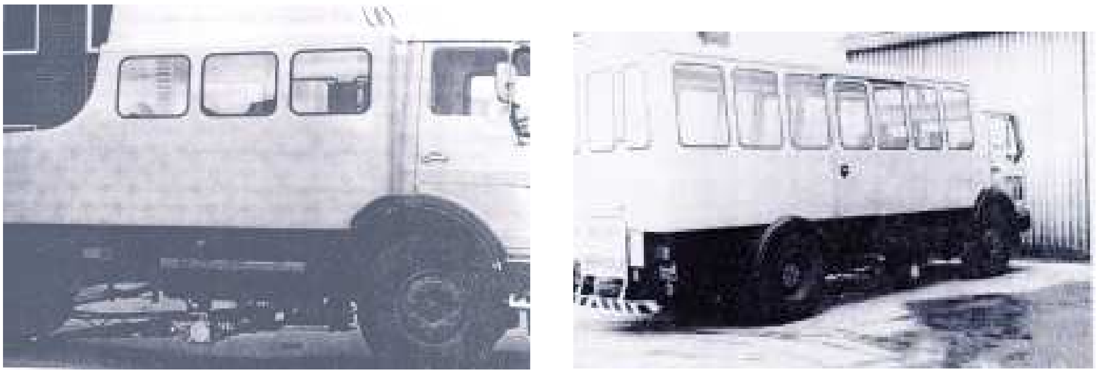
Starting from the late seventies the company MAP S.A. of Basel, Switzerland had the exclusive licence to manufacture and sell the Lacroix Deflectograph outside of France [106]. They also produced an intermediate version with a 5.5 metres wheelspan. In 1982 the deflectographs were still equipped with a graphic recording device, even though it's save to assume that most data at that time was processed with computers. In 1984 34 deflectographs were in service in France, 22 with the shorter chassis and twelve with the longer one. Abroad, 62 deflectographs were used in 30 countries [33]. In 1997, 12 countries of the 21 participating in the COST 325 [67] programme used Lacroix deflectographs.

A fifth version (not called 05, but the Flash deflectograph) was presented in 1997 [275] (and in English one year later [248]). The intention with the Flash deflectograph was to replace both types of the older versions (i.e. with long and short chassis). In order to achieve this most of the system was redesigned, even if the main concepts are the same. The beam, sensors, traction system, guidance, etc. was redesigned and the operating speed was pushed to about 7 km/h. Interestingly, the probe arms are now mounted on a T-shaped frame, as in the Lacroix 01.

Numerous articles have been presented on the evaluation and experiences with the Lacroix Deflectograph, but it's beyond the scope of this review to list them. The interested reader can consult the following references to begin with [80, 81, 129, 130, 161, 169, 179, 247, 254].

### 2.1.3 British Pavement Deflection Data Logging Equipment

The different models of the British Pavement Deflection Data Logging Equipment (PDDLE) [97, 151–153] are based on the French Lacroix deflectograph. The British Transport Research Laboratory purchased a Lacroix version 02 for evaluation in 1967. After a modified specification making the deflectograph more suitable for use in the United Kingdom six more Lacroix were bought in 1970 and the original was modified to comply with the new specification. The major changes from the original are given by Kennedy and Gardiner [152], but can in short be said to give a more sensitive system altogether. The technical data of the PDDLE is pretty much the same as for the Lacroix — an operating speed of 2.5 km/h and recording of the maximum deflection every 3.8 metres. The PDDLE 2000 series had an accuracy of 0.001 mm due to improved sensor technique [97]. The British (probably inspired by the French) also built a 6.5 metres wheel-base machine for stiffer pavements.



(a) British Pavement Deflection Data Logging Equipment. Mark I.

(b) British Pavement Deflection Data Logging Equipment. Mark II.

Figure 2.4: Development of the British Pavement Deflection Data Logging Equipment. (Both photographs reproduced from [152])

In the mid-seventies a private company, WDM Ltd, started to manufacture the PDDLE on a commercial scale, and had contracts to do the larger part of the routine surveys in the UK [97]. Papers on the use of the PDDLE system, rather than technical information, can be found in papers by Gardiner and Kennedy [98]; Catt [55]; Kumar and Kennedy [164]; and Butler and Kennedy [51]. A more general discussion on the use of pavement deflection data in pavement management in the UK can be found in a paper by Ferne and Roberts [89]. The authors conclude that "/.../ the Department has every confidence that the deflection approach will provide a reliable method of planning and designing structural maintenance into the future." One especially interesting paper on the use of deflectographs can be found in "The Deflectograph — A Practical Concept" by Hill and Thorpe [123]. It mainly deals with the types of evaluation made possible by the Lacroix. The paper also deals with the general "concept" of the deflectograph, and we can read that "This Paper attempts to promote lateral thinking with the hope that more people will consider the philosophy of the usage of the Deflectograph." The authors had had a couple years' experience of the Lacroix deflectograph and were convinced of the Deflectograph's qualities and "that there is no substitute for quantified assessments."

### 2.1.4 Danish Deflectographs

The first generation Danish Deflectograph was developed from 1972 and put in operation two years later [203, 204]. The construction seems to have been inspired more by the California Traveling Deflectograph than the Lacroix. A fifteen metres long trailer carries an eight metres long truss framework with the Benkelman Beam type probes. The measuring procedure is the same as for the Lacroix or California Traveling Deflectograph, where the probes are placed on the road surface to measure the deflection from the constantly moving lorry. The probes are then automatically moved to a new position for a new measurement cycle. One set of deflections was assessed every eleven metres, and the speed was 1.5 km/h. This deflectograph was called the "grasshopper", due to their similar movements while jumping along the ground/pavement. An interesting historical review on this device can be found in a paper by Jørgen Banke [24].

Figure 2.5: The Danish first generation Deflectograph. (Reproduced from [63].)

The second generation Danish Deflectograph [144] was completed and put in regular operation in 1988. With the need for only one deflectograph in Denmark the first was donated to the Danish Road Museum. Although the second generation was a complete rebuild from the first generation, the working principle, with minor modifications, is the same. The new deflectograph could operate in curves and the speed was raised to 7 km/h. The second generation Danish Deflectograph is no longer in use, but the trailer is used in the new Danish High Speed Deflectograph covered below. Both of the Danish deflectographs were one-of-a-kind and used only in Denmark.

### 2.1.5 Australian Systems

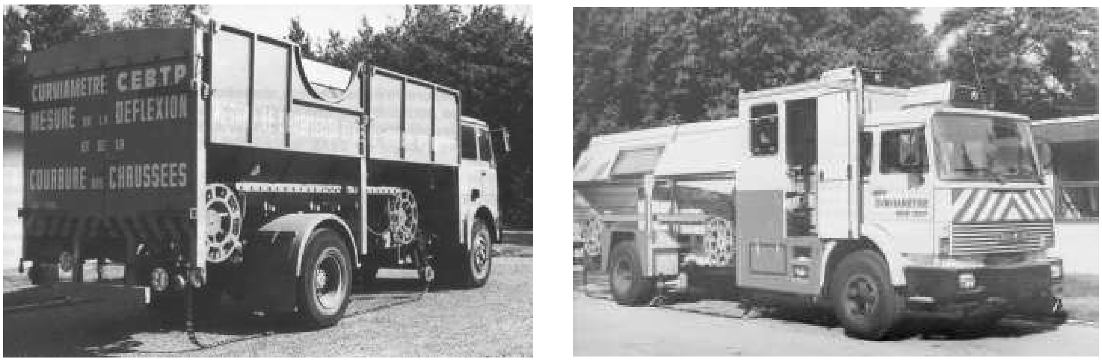
The Department of Main Roads, New South Wales, Australia purchased a Lacroix Deflectograph in 1975 and one more in 1978. Loosely based on the Lacroix concept the Deflectolab [124] was constructed in 1984–1987. Almost every detail on the Deflectolab project is given in the paper by Hill and his ten coauthors [124] — from the Scania P82M vehicle to the ASYST programming language used for the data processing. The Deflectolab is different to other deflectographs in that the Benkelman Beam type probe arm are mounted behind the rear axle. The measuring cycle then starts with with probes being positioned between the dual tyres and the unloading is recorded. The operating speed is 4 km/h, and samples are assessed variably every 4 to 20 metres.

The Country Roads Board of Australia were not so pleased with their Lacroix purchased in 1974. According to the paper by Veith [272] practically the complete system was redesigned by the Australian engineers. Veith writes "The Lacroix Deflectograph was found unreliable and extremely difficult and costly to maintain", and the paper includes a complete appendix with details in problems encountered with the Lacroix. Even though the Lacroix concept was held on to in the redesigned version, more or less all of the electronics, sensors and the recording equipment were replaced. In the late eighties Vicroad engineers fitted this new instrumentation on a new vehicle resulting in the Pavement Strength Evaluator (PASE) [270]. The operating speed is 4 km/h.

### 2.1.6 Curviamètre

The first deflectograph not based on the Benkelman Beam concept was the French Curviamètre. It was developed not by the LCPC but by the Centre Expérimental de Recherches et d'Etudes du Bâtiment et des Travaux Public (CEBTP). The first prototype, based on a Unic-Fiat 220 R with a 13 tonnes axle load, rolled in 1973 [215]. In 1977 the first unit suitable for production use was completed.

The basics of the Curviamètre's operating principle is similar to that of a caterpillar tank. Geophones or accelerometers are mounted on a continuous closed-loop



(a) The 1972 prototype. (Reproduced from [215].)

(b) The 1977 model. (Reproduced from presentation material.)

Figure 2.6: Development of the Curviamètre.

Figure 2.7: The Curviamètre MT-15. (Reproduced from presentation material.)

chain that travels on the pavement surface between the dual rear wheel. The acceleration or velocity of the surface during a passage of the rear wheel is recorded, starting two metres before the wheel passes and stops one metre after. The Curviamètre can assess both the deflection and the radius of curvature of the pavement deflection bowl at a speed of 18 km/h. The 1977 version had only one sensor on the chain which gave it a sampling distance of 12.45 metres, which was the length of the chain. A very ambitious comparison programme between the Curviamètre and the Benkelman Beam can be found in a paper by Liautaud and Bamba [180].

A new model, the MT 15, was produced in the early nineties. The operating speed was now one metre per second faster than before (6 m/s or 21.6 km/h), but the main improvement was that the now fifteen metres long closed-loop chain was equipped with three geophones generating a result every five metres [4, 71, 178]. A variable rear load made it possible to vary the rear axle load from 8 to 13 tonnes.

### 2.1.7 Russian УНК-systems

The Russian УНК-systems (UNK[1]) started being developed in 1975 by Сиденко (Sidenko) at ОНИЛ КАДИ (ONIL KADI) [245]. The УНК-1 then produced apparently suffered from construction defects and never met any real use. In 1977 the УНК-2 system was constructed. (Unfortunately, good information about these devices have proved hard to find. The information given here is solely based on the paper from Сиденко. However, that paper is quite short and the descriptions of the different systems are hard to interpret even for native Russian speakers (большое спасибо to Alexei Jolkin and Rune Karlsson for help with this)). The УНК-2 seems to work according to the same principle as the French Curviamètre, with the one difference that the УНК-2 uses a strain-gauge mounted on a steel plate on the chain, and not an accelerometer or geophone to assess the deflection (this seems really strange, and might be an error in the translation — a geophone would make more sense). The system was, at least, used for five years in Ukraine and Moldova, with satisfactory results. The operating speed was 5 km/h, and the test points were 8 metres apart. A similar, but trailer mounted, system, УНК-3, was developed to allow for measurements on a broader variety of roads.

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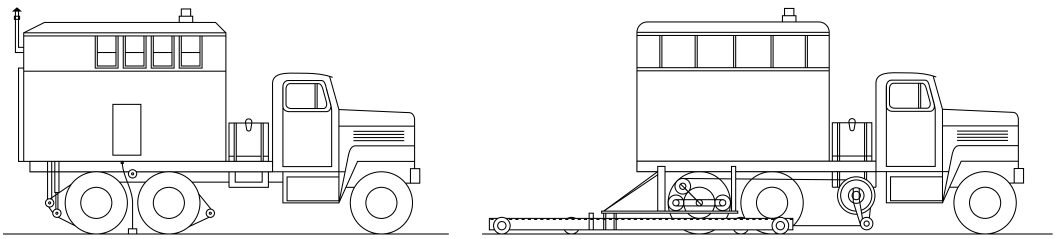
[1] English transcriptions of the Cyrillic text are given in parentheses.

A completely new concept was made with the УНК-4 in 1980. A four-metre beam was controlled with a mechanism that made the beam move periodically along the vehicle. The operating speed was 3 km/h and the sample distance 3 metres. The УНК-4 system was used for routine surveys on the Ukrainian road network, and according to the authors both the efficiency and accuracy of the УНК-4 was higher than that of the Lacroix system. In 1985 the УНК-4 system was valued to 3000 roubles.

It's unfortunate that this system never was brought to a comparative test with the Lacroix it is said to outdo, or the Curviamètre. No information has been found on the present status of the УНК-systems, or any other Russian rolling deflectograph.



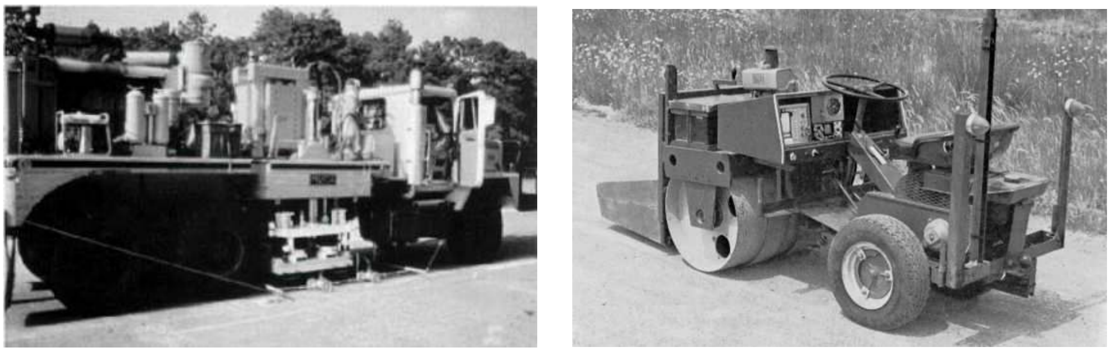
(a) УНК-2.

(b) УНК-4.

Figure 2.8: The Russian УНК-2 and УНК-4. (The pictures are redrawn from the paper "Непрерывные измерения прогиба нежестких дорожных одежд под подвижными нагрузками" [245] by the author due to poor copy quality of the original paper.)

## 2.1.8 Other systems

The Rolling Dynamic Deflectometer (RDD) was developed at The University of Texas at Austin. The RDD was constructed by modifying a Vibroseis truck. Particularly useful in oil prospecting, the Vibroseis trucks apply large dynamic forces to the ground in order to generate seismic waves. The hydraulic vibrator mounted on the RDD transmit sinusoidal forces in the 5–100 Hz range to the road surface, and rolling sensors to assess the deflections [35, 99]. The operating speed is about 2.5 km/h. Field results from the RDD can be found in papers by Bay with various coauthors [34, 36, 37] and by Kim, Röesset and Stokoe II [159].



(a) The Rolling Dynamic Deflectometer. (Reproduced from [35].)

(b) The Collograph. (Reproduced from [43].)

Figure 2.9: The RDD and the Collograph.

The French Collograph [43, 104, 105] can also be seen as a sort of rolling deflectograph. Derived from a small rolling and vibrating compactor it transmits a 50 Hz load with a peak of about 3 kN. The Collograph is primarily developed for detection of cracks, separated pavement layers, etc., but Boulet comments on the relationship between the output from the device and the structural capacity of the road that "[t]his is not the purpose of the Collograph, but a close correlation has nevertheless been noted /. . . /" [43].

Chiefly influenced of their Lacroix, a deflectograph was built in the late eighties in Czechoslovakia. The DEF 02 deflectograph was built on a LIAZ truck, used an inverse T beam, and measured every 6 to 9 metres in 2.88 km/h according to Kudrna [163]. No information has been found on the later history or present status of the DEF 02 deflectograph.

## 2.2 Laser-Based Systems

The mechanical deflectographs discussed in the previous section made it possible to make routine network level deflection measurements. With a top speed of about 20 km/h for the Curviamètre, they were, however, all far from normal traffic speed. With an ever increasing traffic volume during the nineteen-sixties, seventies and eighties their low speed started to be a problem. A method that could assess the deflection at normal traffic speeds would not only make it possible to test more, but the tests could be done in a much safer way — for both the deflectograph operators and other road users.

### 2.2.1 Purdue Deflectograph

The first practical solution to this was the Purdue Deflectograph. (This deflectograph never had an "official" name. It will be called the Purdue Deflectograph in the present report.) The Purdue Deflectograph system [85–87] did not only aim at measuring the deflection, but also the surface texture and longitudinal profile. The concept was based on the TRRL high-speed profilometer [78, 255] and thoroughly described in the PhD. thesis by Elton [85]. In short, at least four non-contact laser range finders are mounted in a line along the vehicle. A geometric relationship is then used to calculate the deflection (see e.g. the thesis by Elton [85] for details).

First tested with a loading truck in January 1982 alignment of the lasers turned out to be a big problem, causing the longitudinal profiles to drift. Even with adjustments to fix the end points to data from a manual survey the profiles differed as much as 37 cm over 150 metres, and many suggestions for improvements of the system are given by Elton. The wording in a paper published in 1988 [87] is much more positive, stating that "This method allows actual pavement profile to be measured /. . . / including every wavelength /. . . /". However, the results presented are the same as in 1982, suggesting that no significant development had taken place in the six year span. According to Harr [118] the speed during tests was only 16 km/h, even though, technically, the system should have been able to measure at normal traffic speeds. The system was patented by Elton and Harr in 1982 [86]. A paper describing the potential use on airfields was published in 1983 [50].

### 2.2.2 Ohio DoT and Surface Dynamics Inc.

In 1985 the Ohio Department of Transportation ordered a feasibility study from the company Surface Dynamics Inc. regarding high-speed measurement of highway pavement deflections under moving loads. With no explicit references to the work by Harr and Elton the TRRL walking beam reference system was chosen [253]. Six non-contact Selcom 2204-64 Optocator lasers was proposed to get some redundancy from the minimum four. In order to minimise the laser misalignment which caused problems for the Purdue Deflectograph a thermo-insulated and liquid cooled reference beam with a velocimeter correction unit was proposed. The deflection measuring devices should be mounted, with three vibration insulation mounting pads, on a suspended platform under the trailer. The feasibility study was positive, but no information has been found whether the Ohio Department of Transportation developed the project or not.

In any event, in the mid-nineties two American rolling deflectograph projects started. They had similar names (Rolling Wheel Deflectometer and Rolling Weight Deflectometer) and were both based on the work by Elton and Harr.

### 2.2.3 Rolling Weight Deflectometer

The Rolling Weight Deflectometer[2] (RWeD) of Quest Integrated, Inc. and Applied Research Associates [147–149, 229] was mainly aimed at airfield evaluation. It had the same setup as the Purdue Deflectograph, i.e. four equally distanced non-contact Selcom lasers. Designed for airfield evaluation, the load is transfered to ground through an F-15 wheel assembly.

To compensate for the misalignment problem a laser beam was aimed down the central cavity of the physical beam holding the lasers, and three optical position sensors were used. Instead of trying to make the beam infinite stiff, the idea was to allow the beam to bend from temperature and vibration and compensate for this. The compensation mechanism is thoroughly described in a patent application [146]. At the Road Profile User Group (RPUG) meeting in 1996 the concept of a highway version of a RWeD was presented [46], but no such unit has actually been built.

### 2.2.4 Rolling Wheel Deflectometer

The Rolling Wheel Deflectometer (RWhD) of, initially, Phoenix Scientific, Inc [110, 119, 120] also seems to be a descendant of the Purdue Deflectograph. At least, the long-time project manager Jim W. Hall, Jr. is acknowledged in Elton's thesis on the Purdue Deflectograph, mentioned above.

One major difference from the TRRL walking beam concept was that the RWhD made use of two scanning lasers (called the Control Area Scanner and the Loaded Area Scanner) instead of the four or more spot lasers used by the Purdue Deflectograph and the Rolling Weight Deflectometer. In this way, the complete longitudinal deflection basin would be assessed, possibly giving more details of the structural capacity of the road. However, problems with accuracy caused the RWhD researchers

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[2] Sometimes refereed to as the Rolling Wheel Deflectometer or Rolling Load Deflectometer.

to abandon the scanning laser technique for a more conventional four spot laser system [111].

The ERES Division of Applied Research Associates are now in charge of the RWhD project. The latest news on the project [107, 111] are quite promising, but only a very limited set of test have been performed so far.

### 2.2.5 Road Deflection Tester

See the next chapter for a thorough description of the Swedish Road Deflection Tester.

### 2.2.6 High Speed Deflectograph

The latest addition to the rolling deflectograph scene is the Danish High Speed Deflectograph (HSD) [121, 122]. Rather than the “standard” laser triangulation distance-meters, the HSD is using laser Doppler velocity-meters. These laser Doppler sensors assess the road surface deflection speed by measuring the shift in the outgoing and incoming laser light, i.e. the Doppler effect. (The basic idea, to measure the deflection velocity instead of the deflection, is the same as for the French Curviamètre (see Section 2.1.6) with the difference that the Curviamètre measured the deflection velocity in a large number of points and then could integrate this to a deflection.) By measuring the deflection speed, theoretically only one laser sensor is needed. As an absolute value is obtained no reference sensor is needed. This also does away with the problem of measuring in curves, which cause an alignment problems for more or less all other deflectographs.

How *quickly* the road surface deflects instantaneously at one point near a moving load is, however, not quite as interesting as how *much* it deflects. On the other hand, a relationship between deflection velocity and actual deflection should not be very hard to find, even though it’s likely that this relationship will vary with the road construction and, especially, the viscoelastic properties of the asphalt.

The Doppler sensor actually measures the relative speed of the sensor and the road surface, so it’s of utmost importance to filter out the movement of the sensor. On the HSD this is achieved with a combination an inertial three axle accelerometer and a three axle gyroscope. Data from the inertial units are used both in post-processing and as input to a servo system controlling the position of the sensor in real time.

To get the true deflection a large number of Doppler sensor would be needed. As of this writing only two laser Doppler meters are used. So far, very little results from the High Speed Deflectograph have been presented.

# Chapter 3

# The RDT System

The Road Deflection Tester (RDT) was built with the intention of providing a safe, fast, accurate and reliable way to assess the bearing capacity of roads, airport runways, and other pavement surfaces. Primarily, its use is intended for the Pavement Management System network level.

In the present chapter the RDT system is thoroughly documented — the configuration and technical solutions, the sensors, the data acquisition system etc. All from a technical point of view. For information on the RDT project per se, and not the results thereof, see the Background section on page 2.

## 3.1 History

As mentioned in Section 1.1 the idea with the RDT originated with the success of the laser based RST system. Whereas the RST used one array of non-contact laser sensors to measure the road surface, the RDT needed two — one for the undeflected state, and one for the deflected state. The difference between the two cross profiles would then be the deflection.

Before the construction of the first prototype got underway an operating environment analysis was conducted, which at that time meant, more or less, the Purdue Deflectograph. (A couple of patents by Gilbert Swift [259, 260] was also found. It's beyond the scope of this review to illustrate the very interesting concept developed by Swift, but a visit to the United States Patent and Trademark Office on-line database can be recommended.) A detailed feasibility study [12] was also conducted before construction started. All in all, it was a go-ahead.

A prototype RDT was built in the early nineties, Figure 3.1(a). The 1964 Volvo Titan truck proved to be a suitable carrier. The rear axle weight and the sensor locations could quite easily be altered, and many different sensor configurations were tested. However, the relatively short distance between the two wheel axles of the truck was assumed to limit the function of the system. A longer wheelbase would make the deflection reading more accurate, it was thought. Other problems were the low maximum speed of 70 km/h and the difficulty to keep an even speed while going uphill. Also, the facilities, comfort and working environment for driver and operator in the vehicle were very limited, making more than day-trips practically impossible. It served well as a research vehicle, but it was clearly unsuitable as a production unit.

(a) The prototype RDT. (b) The new RDT.

Figure 3.1: Evolution of the RDT vehicle. (Both photographs by VTI.)

Some of the results from the first tests with the prototype RDT are reported by Arnberg, Holen and Magnusson [13]. Further results can be found in a paper presented by Lenngren [176].

The first VTI project related to the RDT started in July 1985 and ran for one year. The budget was 75.000 SEK. A second project of about 4m SEK ran from 1989 to 1992. This project included construction and testing of the first prototype RDT. At this time the RDT technology was patented in Sweden, Switzerland, Germany, France, Great Britain and the USA. These patents all expired in 2005.

To address the problems mentioned above an ambitious project was initiated in the mid-nineties. The total budget for the new RDT was estimated to 76m SEK. This budget included a video system for surface crack detection and possibly even a ground penetrating radar for automatic assessment of the road layer thicknesses. The crack and layer detection systems were never implemented, and one can anyway argue that it's not always convenient to have all systems in one vehicle.

The new RDT was built on a modified Scania R143 ML truck, Figure 3.1(b). The major modification is that the engine is placed in the back of the truck in order to maximise the rear axle force on the road.

In Sweden, the maximum speed limit for trucks is 90 km/h on motorways and arterial major roads, and 80 km/h on other roads (but sometimes lower, of course). As higher speeds can be important for a detailed analysis the RDT is actually registered as a bus, and as a result it's permitted to do 90 km/h on arterial major roads too. (Even though the RDT formally is a bus the word truck will be used in the text throughout.) Even higher speeds are possible as the truck is not equipped with the speed limiter normally installed in Sweden. So, with a dispensation from the SRA the RDT can do almost 110 km/h. The extra seats needed for the bus registration provide for the possibility to carry an extra operator and driver on longer assignments, and they are also very handy for demonstrations.

A ambitious test programme was initiated in 1994. 100 test section were to be measured with both RDT prototypes and the FWD [177]. A correlation study and a method to translate results from the RDT to FWD domain was sought. Due to the sudden termination of the RDT project, mentioned in Section 1.1, the documentation from this test programme was never completed, nor published.

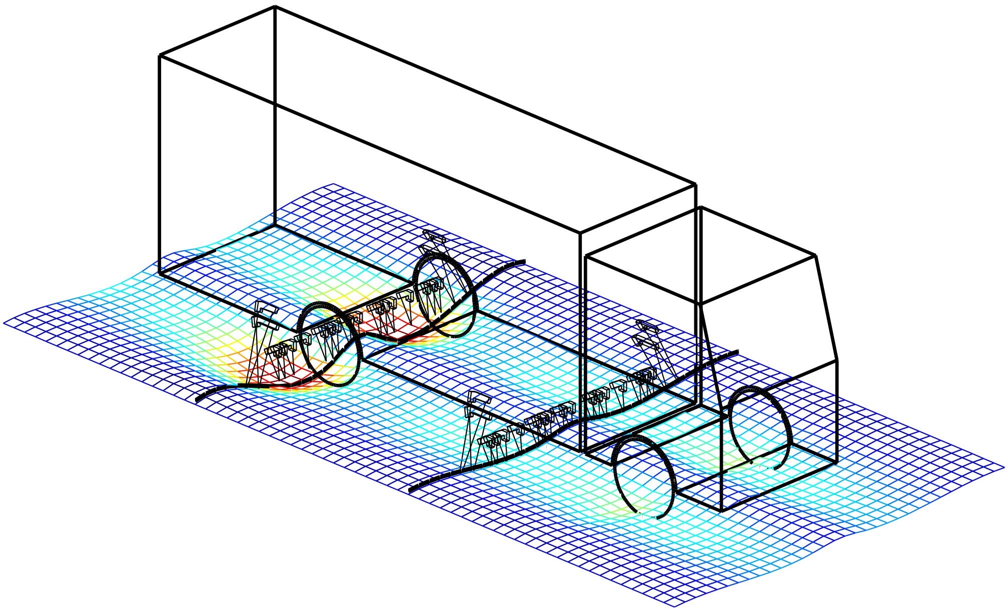


Figure 3.2: The working principle behind the RDT vehicle. Two arrays of non-contact laser sensors acquire the transversal deflection profile.

Even though all the development of the RDT was done by VTI, it was still owned by the SRA. In 1997, because of the strategic decision by the SRA not to develop new research equipment 'in-house', the RDT was handed over from the SRA to VTI.

## 3.2 Vehicle Configuration

To recapitulate, the RDT is equipped with two arrays of twenty non-contact laser sensors that collects transversal surface profiles at normal traffic speeds (up to 110 km/h). One array is mounted 2.5 metres behind the front wheel axle, where the road is considered to be in a non-deflected state. The other array measures the deflected state 0.5 metres behind the rear wheel axle. See Figures 3.2 and 3.3 for details on how the lasers, and laser arrays, are mounted.

The rear lasers need to be angled 35 degrees in order to collect the transversal profile 0.5 metres behind the centre of the wheel. (Where the maximum deflection actually occurs, for different types of road etc., is not known at present. An attempt to assess the longitudinal part of the deflection basin behind the right rear wheel was conducted on the prototype RDT, but no documentation on this has survived.). To keep the rear and front arrays identical, the front lasers are also angled.

An incremental wheel pulse transducer is mounted on the left front wheel for accurate travelled distance. Force transducers and accelerometers are mounted on the left and right sides of the rear axle. An optical speedometer for both longitudinal and transversal speed and a gyroscope are mounted near the front axle, right under the driver's cab. Taken together, these sensors give detailed data on how the truck behaves when operating. More details on the sensors are given on pages 25–31 below.

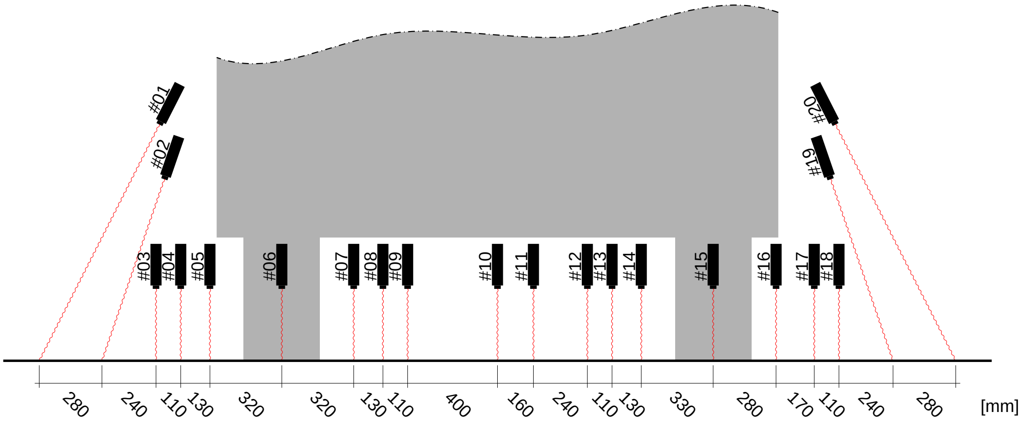


Figure 3.3: Configuration of the Laser Range Finders. The grey shadow illustrates the body and the rear wheels of the truck.

The engine in the RDT has been placed at the rear end of the vehicle in order to create as large difference as possible in load between the front and rear axles. In addition, two movable weights of 400 kilogrammes each have been installed in the truck. In transportation mode these weights are moved to a position close to the front wheel axle allowing a more even weight distribution. During tests, the loads are moved to their back position resulting in a higher rear axle force. In test mode the static rear and front axle loads are approximately 112 kN and 30 kN, respectively. The normal dual tyres have been replaced with Michelin super single. This makes the pressure distribution on the pavement surface somewhat higher and it reduces the complexity of the load.

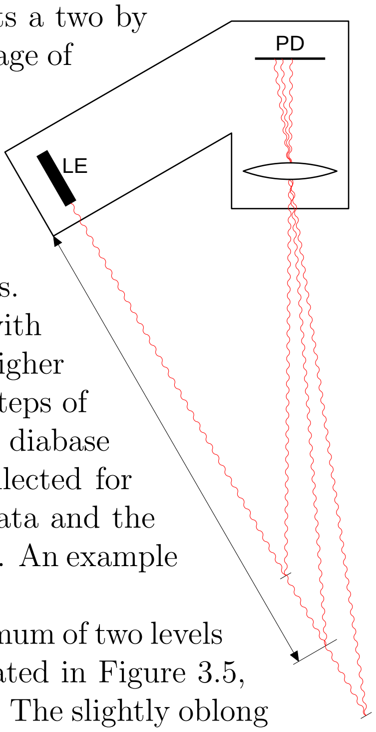
The RDT truck is 10.5 metres long and 2.5 metres wide. The aluminium beams holding the lasers are 3.1 metres wide. The extra width requires a dispensation issued by the Swedish Road Administration.

### 3.2.1 Sensors

In the present section the functions of all sensors on the RDT will be explained. As mentioned above forty non-contact laser range finders, an incremental wheel pulse transducer, two force transducers and two accelerometers, an optical speedometer and a gyroscope are mounted on the RDT. In the planning stage in the mid-eighties the intentions were that the RDT should also be equipped with a ground penetrating radar, video crack detection, and thermometers for both air and pavement surface temperature. These plans have not been realised, though.

#### Laser Range Finders

The laser range finders (LRFs) are of four different versions of the Selcom Optocator 2008, depending on their position on the truck. Lasers #01 and #20 have a 1178 mm stand-off and a 400 mm measuring range, 853/330 mm for #02 and #18, 390/180 mm for #03, #10 and #18, and 390/128 mm for the other. The stand-off is the distance from the aperture where the laser beam leaves the LRF to the centre of the measuring range. In general terms, the accuracy of the measurements is in inverse ratio to the range the cameras can handle. The resolution varies from 0.1 mm for the #01/#20 pair, to 0.032 mm for the lasers with the 128 mm measuring range. (It can be mentioned that the Selcom Optocator used for road surveying was developed by Selcom in collaboration with Ulf Sandberg at VTI. The first working Optocator, mounted on a sliding carriage on a steel beam, was presented in 1979. In 1982 an Optocator sensitive enough to allow mobile measurements was constructed.)



The laser range finders work according to the principle of optical triangulation, as illustrated to the right. A laser emitter (LE) projects a two by four millimetres beam onto the road surface, and the image of the light spot is focused on the position detector (PD). Analog processing will find the centre of gravity of the image, which in turn determines the actual position of the spot. An invalid error will be reported if the sensor is unable to find the spot.

The laser range finders are calibrated in two steps. First, laser range finders #04 and #17 are calibrated with ceramic gauge set blocks (more lasers can be added for higher accuracy). Gauge set blocks with heights from ten, in steps of ten, to one-hundred millimetres are placed on a levelled diabase surface plate. On each level data from the LRF is collected for about ten seconds. From the differences in raw laser data and the known difference in height scale factors can be calculated. An example is shown in Figure 3.4 below.

In the second step a liquid surface is placed on a minimum of two levels within the LRFs measuring range. A container, illustrated in Figure 3.5, holding the liquid has been constructed for this purpose. The slightly oblong "cups" allows the container to be lifted straight up and still keep the laser points from the angled lasers more than 50 millimetres from the edge (in order to avoid surface tension effects).

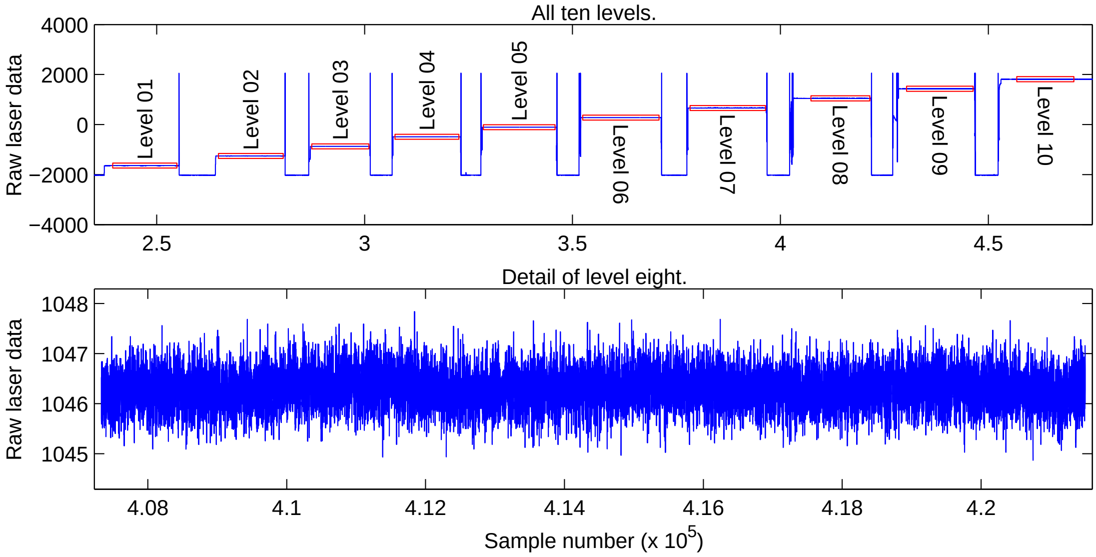


Figure 3.4: Example of a gauge block calibration.

The interconnected “cups” and the bent channels between the cups minimises the wave motion in the liquid more efficiently than holding the liquid in a single “trough”. Before measuring at a new level the liquid surface is given time to stabilise for about ten minutes. Screens are placed between #03/#04, #04/#05, #07/#08, #08/#09, #12/#13, #13/#14, #16/#17 and #17/#18 to prevent light scatter from one laser to interfere with the others. With the scale factors from LRF #04 and #17 the positions for the horizontal and plane liquid surfaces can be calculated, and, from this, scale factors for all LRFs are given. The laser cameras used are mounted on the racks as shown in Figure 3.3. Normal milk is used as the calibrating liquid.

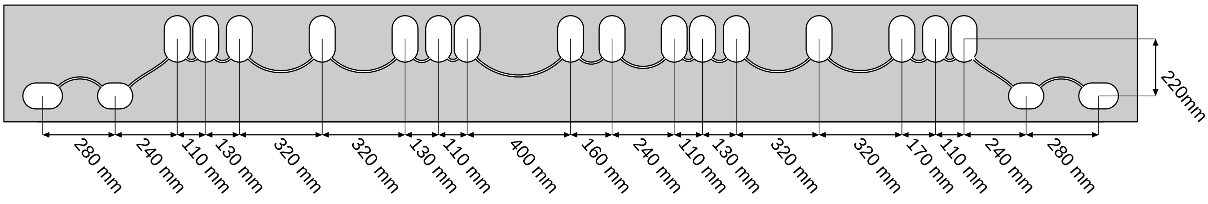


Figure 3.5: Container for liquid calibration.

Even if this calibration scheme has proved to be adequate for surface characteristics data samplers, it does not seem to be robust on the higher degree of accuracy needed for small deflection measurements. The deflections being measured are not far from the resolution of the lasers, and even small errors in calibration can have drastic results. Plans have been made to remount the laser. The advantage of having the lasers point straight down is probably larger than the possible disadvantage of assessing the rear profile a little bit further back. Anyhow, the extent of the deflection basin is not known at present, and the 0.5 metres distance isn’t necessarily the optimal. This could lessen the calibration uncertainty significantly.

### Speedometers

The DATRON speedometer gives both the longitudinal and transversal velocities. It is mounted in the front of the vehicle, under the driver’s cab. The working principle is, taken from the users manual

> ”The sensor DATRON V1 functions according to the principle of optical correlation. The image of a rough, illuminated surface is projected through an objective onto a grid of diode arrays arranged at equal intervals. The photocurrent of the diode arrays has a definite frequency directly proportional to the relative velocity of the diode array in relation to the surface. When the signal has been processed appropriately, the distance traversed can be calculated as to length and direction. Using two diode arrays, it is possible to record the distance in two directions /. . . /”

An example of recorded speeds (from the RD942 country road in France) is shown with blue colour in Figure 3.6 on the next page. In the figure is also the speed as calculated from the wheel pulse transducer plotted (in red), but the lines coincide for everything but sections with relatively high transversal speeds, when there is a difference between the general *speed* and the longitudinal *velocity*.

Notice how closely related the transversal speed and the signal from the gyroscope are. The main objective with having both the transversal speed and the gyroscope is to find out how large the transversal speed is when there is a zero output from the gyroscope, as this is a direct measure of how much the truck is "angled" while driving straight. This information could then be used as an indicator that the two laser arrays are out-of-line, and compensated for in the post-processing.

The problem with the two arrays being out-of-line was, at the planning stage of the first prototype, proposed to be solved by moving the rear laser array sideways with a servomechanism to compensate for the lateral movement — a construction that never left the drawing board.

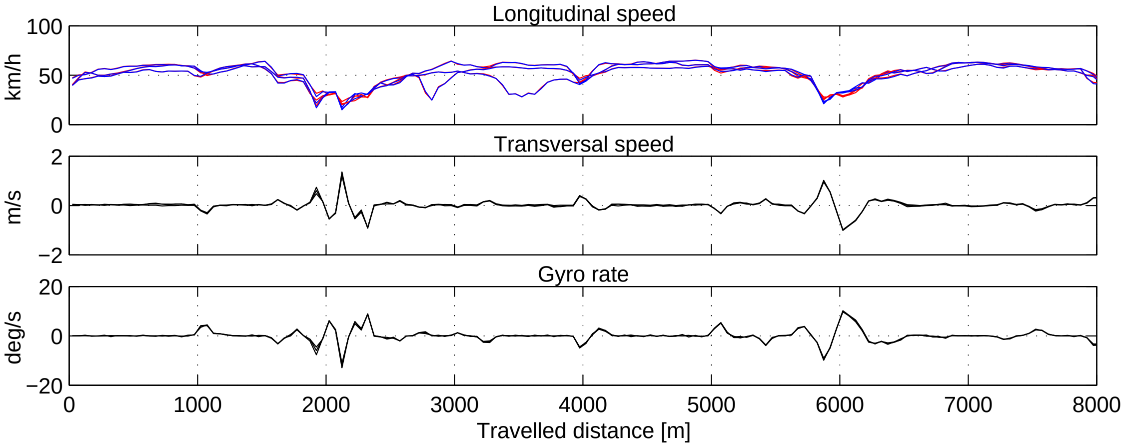
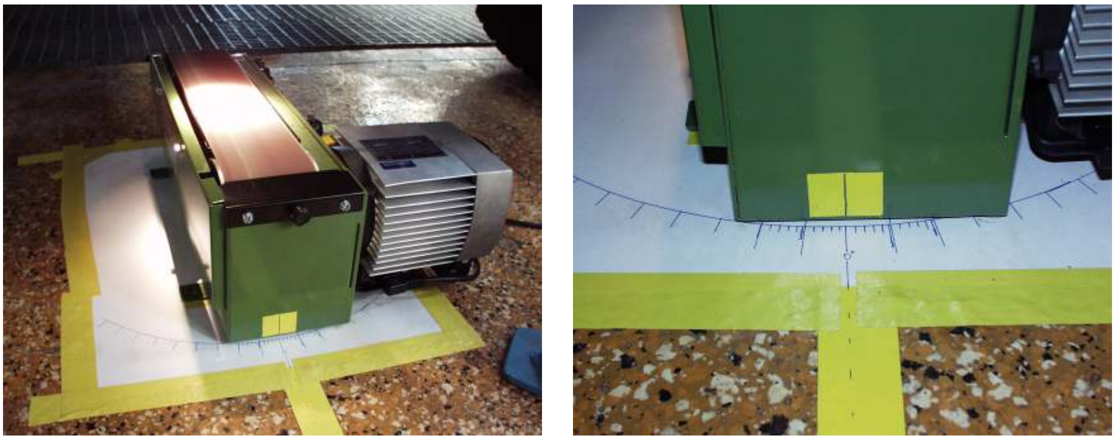


Figure 3.6: Example of speeds for three runs on the RD942.

The optical speedometer is calibrated with a belt sander with a known belt speed. The sander is placed on the floor below the speedometer with the paper moving in the direction of the longitudinal axis of the truck. The sander is then angled from minus seven to seven degrees in relation to the longitudinal axis. The scale factor is given as the quotient between known and measured speed.



(a) Overview

(b) Detail

Figure 3.7: Calibration of the optical speedometer.

### Accelerometers and Force Transducers

The accelerometers and (shear) force transducers are mounted close to the wheels on both sides of the rear axle. If the mass of the wheel assembly is known, these sensors make it possible to determine the wheel-to-ground force. The RDT have, for this purpose, been calibrated on a hydraulic shaker facility, where the known force input can be compared with the registered axle force. The result of the dynamic calibration is reported by Östergren and Magnusson [210], and general information on this type of calibration can be found in a paper by Leblanc, Woodrooffe and Papagiannakis [170].

The wheel force illustration in Figure 3.8 is taken from the circular TRL test road. The RDT was driven anticlockwise which puts a lot of load on the right side of the truck. At the very end of the loop a very steeply-banked curve made the truck lean on the left wheels at the lowest speed.

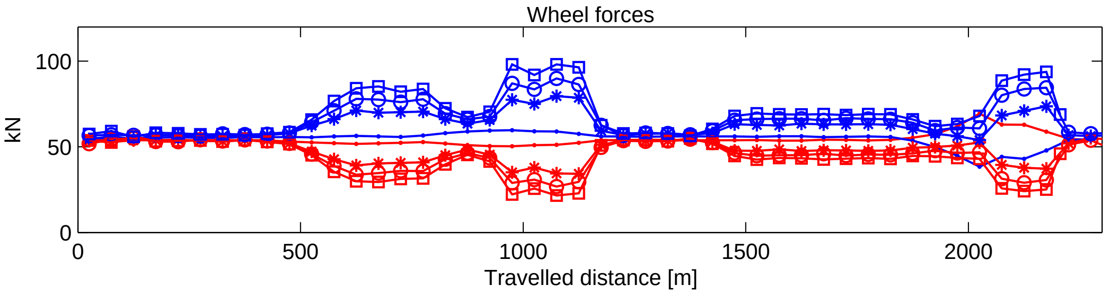


Figure 3.8: Force transducer. 10, 50 60, and 70 km/h with dots, stars, circles and squares, respectively. Left wheel force in red and right side in blue.

The force transducers are statically calibrated with a load cell placed under the rear wheels (one at the time). With a garage jack the load can be varied within the interesting range. The output from the load cell is recorded manually and compared to the signal from the rear axle force transducer on the truck. The wheels not being calibrated are placed on wooden blocks with the same height as the load cell.

(a) Overview (b) Detail

Figure 3.9: Calibration of the left wheel force transducer.

The accelerometers on the rear axle are calibrated by placing them in two extreme position. One position is the earth's gravity of 9.81 m/s$^2$, and the other case is the zero acceleration. The mount holding the accelerometer is designed to allow for easy calibration.

### Wheel Pulse Transducer

The incremental wheel pulse transducer is mounted on the front left wheel. This brings it as close to the centre of the road as possible. The transducer mounted on the RDT delivers 2500 pulses per rotation. The total number of pulses from the start is logged to the data file. Plans were made, but not realised, for the RDT to have wheel pulse transducers on both front wheels, in order to measure the turning rate, and compensate for the out-of-line problem mentioned on page 28.

The normal procedure for the RST system is to test measure a known length, and calibrate the wheel pulse transducer from this. For various reasons this method (or any other) has not been used with the RDT. Usually, the measured length has not been of crucial importance as the test sections normally have been short. Experience from the RST shows that this scale factor actually is very stable, and calibration is practically only needed when new tyres have been fitted. The RDT is still using its original tyres, and care is taken to keep the tyre pressure at specified levels when testing. When the RDT will be used for road network testing, the RST type of calibration will be used. In all the results presented below a fixed scale factor of $2500/(1.10\pi) = 723.4$ have been used, where $1.10\pi$ is the circumference of the wheel.

### Gyroscope

The gyroscope is placed in the front of the truck, right under the driver's cab. It's manufactured by the aviation branch of SAAB. The gyroscope is calibrated by slowly turning it 360 degrees. The scale factor is simply the measured rotation divided by 360. In Figure 3.10 below the raw data signal from the gyroscope is shown in the upper part, and the accumulated turn in the lower figure.

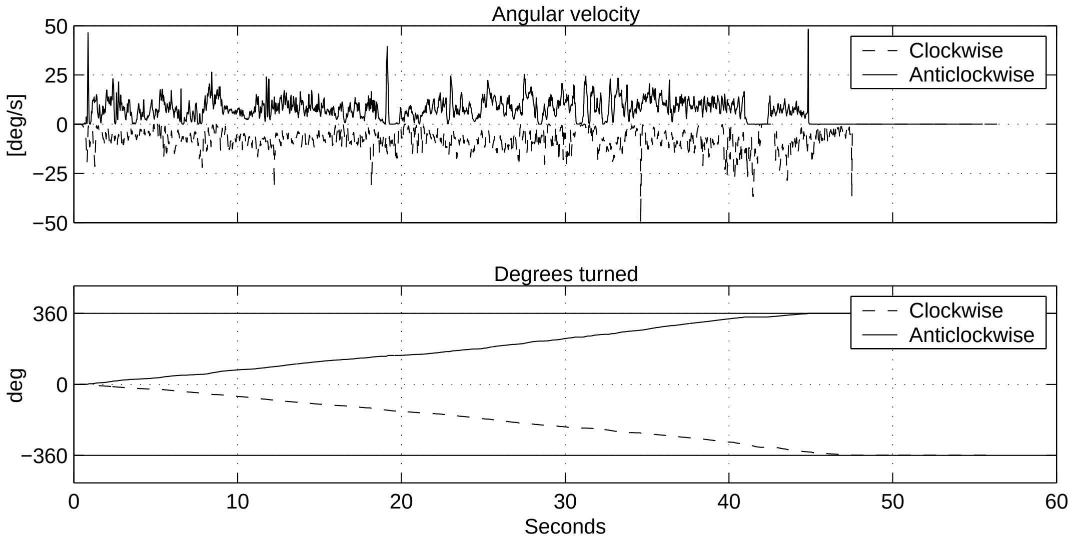


Figure 3.10: Calibration of gyroscope.

Illustrative "gyroscopic" results from the RD942 country road in France are shown in Figure 3.11. Three small roundabouts and some other sharp curves are easily seen in the data stream. Obviously, the RDT does not work very well under conditions like this, as the two laser arrays will read from different parts of the pavement surface.

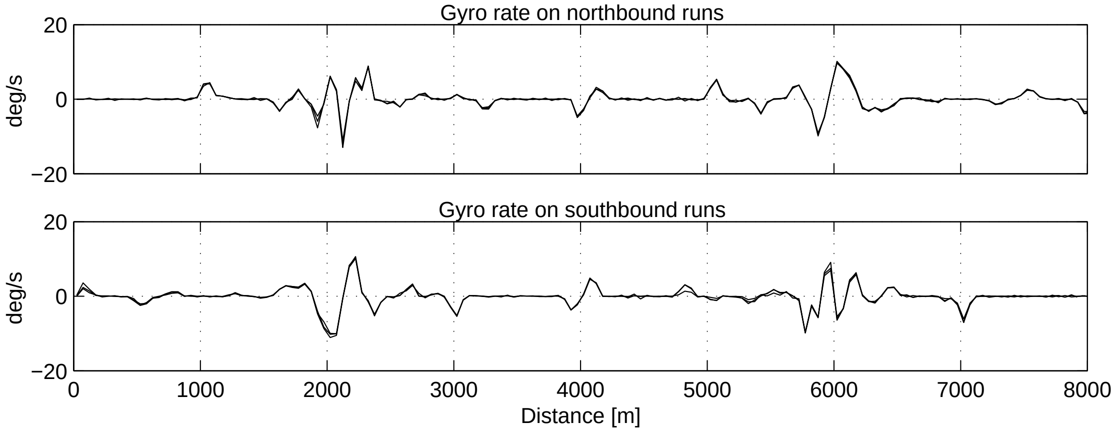


Figure 3.11: Example of gyroscope output from the RD942 test site in France.

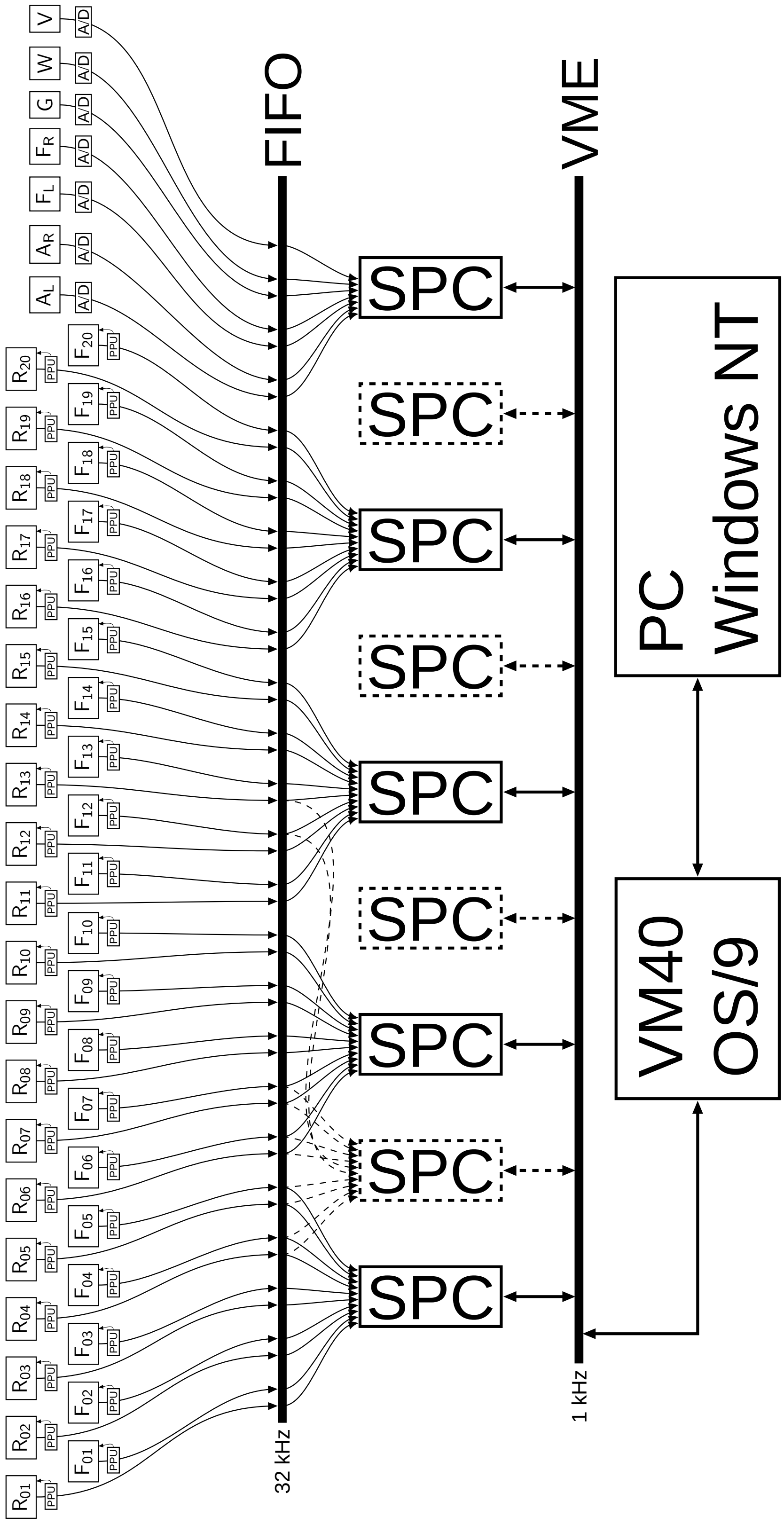


Figure 3.12: Data acquisition, and data flow.

## 3.3 Data Acquisition

### 3.3.1 Hardware

All the RDT sampling hardware work with a frequency of about 32 kHz. An averaging process over 32 samples at a time reduces this to 1 kHz. Invalid samples are, of course, not used in the averaging process (see page 26 for a brief explanation of how the lasers work). The number of valid samples actually used to get the mean sample value is stored as a quality index ranging from 0 to 1 in steps of $\frac{1}{32}$. (The averaging factor can be set to other values than 32. However, for the RDT tests to date the 1 kHz sampling rate has always been used.) With the normal sampling frequency of 1 kHz and a vehicle speed of 70 km/h one set of samples is stored about every 20 millimetres. It is also possible to save data in “raw mode”, i.e. without the averaging process activated. However, the raw mode only works for a few sensors, due to bandwidth limitations in the data collecting system.

When all sensors are active, 1 kHz data is stored to file at the rate of 11.3 MB per minute, or, in other words, about 9.5 MB per kilometre when driving at 70 km/h.

The data flow in the hardware is illustrated in Figure 3.12 on the opposite side. The rear and front lasers are labelled $\mathsf{R}_{\mathsf{xx}}$ and $\mathsf{F}_{\mathsf{xx}}$, respectively, where xx is the laser number ranging from 1 to 20. The accelerometers, left and right, are labelled with an $\mathsf{A}_{\mathsf{L/R}}$, and the same notation is used for the force transducers with an F. The gyroscope, wheel pulse transducer and speedometer are labelled with G, W and V, respectively. The A/D box represents an analog-to-digital converter. All laser range finders are connected to a Probe Processing Unit (PPU). The PPU processes the analog signal from the lasers, and delivers a scaled and linear digital signal. The PPU also controls the power used by the laser, as indicated with a small feedback arrow in the figure. The actual data flow is illustrated with arrows.

Much of the low-level data processing is performed by the signal processing cards (SPC). “Applications” are uploaded from the PC via the VM40 computer to the SPCs. An application can be thought of as a small computer program. In the RST system typical applications are rut depth and IRI. Applications for the RDT could be any, or all, of the deflection indices presented on pages 38–39. These could then be calculated and presented in real time. More SPCs can be added for more applications, as illustrated with the dashed SPCs, and dashed data flow arrows. With the current simple averaging application only three of eleven available SPC slots on the RDT are in use. The SPCs communicate with a VM40 computer running OS/9 over an industrial standard VME bus. The VM40 computer communicates with the PC over a 10Mb/s coaxial cable.

### 3.3.2 Software

The data is collected and stored, on an ordinary PC running Windows NT, with a computer program called WayWatch developed by OPQ Systems. From WayWatch, data is stored in a compact binary file called `*.mean`. A program called WWConvert is used to convert data from the `*.mean` format to an ordinary float representation. This file, typically called `*.data`, can be read by a simple `fread` command from C, Matlab or any similar environment. This, raw data, is essentially a large matrix with ninety-nine columns (forty-eight sensors with quality index plus three system channels) and any number of rows.

All code for pre- and post-processing of the raw data has been written in the Matlab environment. The structure of the code will be described briefly below. Due to the relatively large amount of data stored by the RDT data acquisition system, multiple levels of pre-processing is used.

As large batch-runs often have been of interest in the evaluation, all computations are controlled from just two files, called `rdt_pre_proc.m` and `rdt_post_proc.m` for pre- and post-processing, respectively. One segment of code in each file controls the analysis of a specific test section. A code segment, controlling the Arlanda test site, from the `rdt_pre_proc.m` is shown below for illustration. By changing the `true` to `false` the Arlanda test site would not be included in the next batch run. Lines beginning with the percent character are comments, and not executed.

```
if(true)
  %> Arlanda.
  RunTestVec                    = [01:18];
  MOD.FileDir                   = 'D:/';
  MOD.PresLength                = 50;
  MOD.Method                    = 'Defl_Eval';
  rdt_eval(RunTestVec, MOD, 'Arlanda');
end
```

The first step of pre-processing consists of a “synchronisation” of the individual runs. (With synchronisation it’s actually distance and not time that’s “coordinated”.) Next, the data is scaled and averaged over the chosen presentation length. Mean profiles from the front and rear laser arrays for all indices using all cleaning methods at all cleaning levels are stored at regular intervals of the chosen presentation length, as is data from all other sensors. (The different indices used are explained further on page 38, and more information on the data cleaning methods are given on page 36.) This, intermediate, data is stored on file in a Matlab structure called `TestRes`. In this way, a large batch run can produce intermediate results for many, or all, test sections. There is no need to repeat these calculations unless the basic evaluation methods change, or a different presentation length is needed.

The post-processing is controlled by another structure called `PostProc`, again exemplified with the Arlanda test section below. First, basic parameters controlling the presentation, printing method and what to print, cleaning methods, etc. are set for all tests in the batch run. Then, individual parameters can be set for specific tests. Usually this is the availability of falling weight deflectometer data, adjustments in which lasers to be used in the case of malfunctioning lasers, ranges in the plotting, statistical information on the test, etc.

```
if(true)
  %> Arlanda.
  PostProc.Test_Facts_File      = 'C:/1998/Arlanda/arlanda_test_facts.m';

  PostProc.RunTestVec           = [01:09];
  PostProc.LoadFWD              = 'fwd.txt';
  PostProc.FWD_Scale            = -0.001;
  PostProc.FWD_OffSet           = -408.0;
```

```
  [TestRes, TestFact]           = load_test_results(PostProc);
  PostProc                      = post_proc(PostProc, TestRes, TestFact);

  PostProc.RunTestVec           = [10:18];
  PostProc.LoadFWD              = 'ArlandaDataSouth.txt';
  PostProc.FWD_Scale            = -0.001;
  PostProc.FWD_OffSet           = -255.0;
  [TestRes, TestFact]           = load_test_results(PostProc);
  PostProc                      = post_proc(PostProc, TestRes, TestFact);
end
```

As can be seen in the code above a file called `arlanda_test_facts.m` is referred to. This file contains all information on the test, organised in a structure called `TEST`. The first member, `TEST{01}`, of the structure is included below. `TEST{02}`, the second member, will be defined for the second northbound run, etc. For the test at Arlanda an 18-membered structure holds all information on all runs.

```
  TEST{01}.Name                 = 'Arlanda';
  TEST{01}.Identifier           = 'arn';
  TEST{01}.Date                 = [1998, 06, 24];
  TEST{01}.Version              = 'OLD';
  TEST{01}.TargetSpeed          = 50;
  TEST{01}.Direction            = 'Northbound';
  TEST{01}.DirectionKey         = 'nb';
  TEST{01}.FileDir              = 'R:/PeterA/DATA/RDT/1998/Arlanda/';
  TEST{01}.ScaleFile            = 'scales1998-06-03.txt';
  TEST{01}.Sensors              = 98;
  TEST{01}.RunNumber            = 1;
  TEST{01}.File{01}.Name        = 'nb50_1.data';
  TEST{01}.ST{01}               = 1;
  TEST{01}.SP{01}               = 217659;
```

So, the `*_test_facts` file contains all the information about the test: location, test date, target speed, direction, data file name, run number, etc. Also included is information on "events". Typical events are start and stop of the test section (collection of data normally begins and ends before and after the section of interest), intersections, bridges, and other landmarks, but also information on other things that might influence the test: pause mode while avoiding cyclists, information on passing heavy vehicles, dirt on the road, etc. The WayWatch program implements the event functionality with the keyboard function keys `F1`-`F8`. The event is registered in the data stream for as long as the key is pressed, and two keys can be pressed at the same time, if, e.g., an intersection is crossed while holding the key for pause mode.

All the real post-processing work is done by the `post_proc` function mentioned above. Depending on the values set in the `PostProc` structure (and some information from the `TestRes` and `TestFact` structures) a number of small sub-functions return the final results. This, very modular, way of programming makes it possible to, in a clean way, add new functionalities and apply these on data from all tests in the database.

Figure 3.14 on the opposite side illustrates the RDT data stream. The circles represent samples (only laser samples are shown). Time flows from the top of the figure and down. The fifteen segments within braces represent equal length averaging sections, which contain different amounts of data due to a changing speed along the test run. In the example, the speed is lower in the beginning of the test and increases till segment nine where is stabilises. The first four metres of the rear data stream and the last four of the front can not be used for deflection analysis, as there is no corresponding data from the other array to compare with. (The illustration is, due to its schematic nature, imperfect. The braced segments for the rear laser array should actually be a little bit shorter in the acceleration phase as the speed is higher when the rear array passes, thus taking less time. The limited resolution made this hard to achieve.)

The quality of the averaged samples are indicated by the tone, white for perfect reading with all thirty-two samples valid, and black for all invalid.

The figure illustrates the so called `transversal` cleaning method. A threshold for the data quality is set, and if any laser in the transversal profile has lower quality the entire profile is discarded, as illustrated with the crosses. However, in order to ensure that only the same parts of the road surface is used for the deflection profile, the corresponding transversal profile is discarded from on the other array too, even if that data is of a high enough quality. This is marked with the thin arrows, pointing from the "bad" profile to the "good".

Other cleaning methods have been implemented. The `longitudinal` works the same way as the transversal, but discards the longitudinal profile if the threshold is met. A combination of the transversal and longitudinal methods have also been made. The `exact` method only discards the actual samples the are below the threshold. This will create mean transversal profiles with different amount of data on the different lasers. The `none` cleaning method will simply not remove any samples, no matter how low the quality is.

No major study to find the best cleaning method on an average has been done so far. However, on most test sites the quality is high enough to make the choice of cleaning method of little importance. In the present report the transversal method has been used.

The laser data quality is often lower on the rear array, and in the wheel paths. This can be seen in Figure 3.13 below, and it's also illustrated in Figure 3.14 to the right, where most of the "cancellation arrows" point from the rear stream to the front one. The lasers used on the RDT are very sensitive, and the data quality on a normal road surface is usually in the high 99 %.

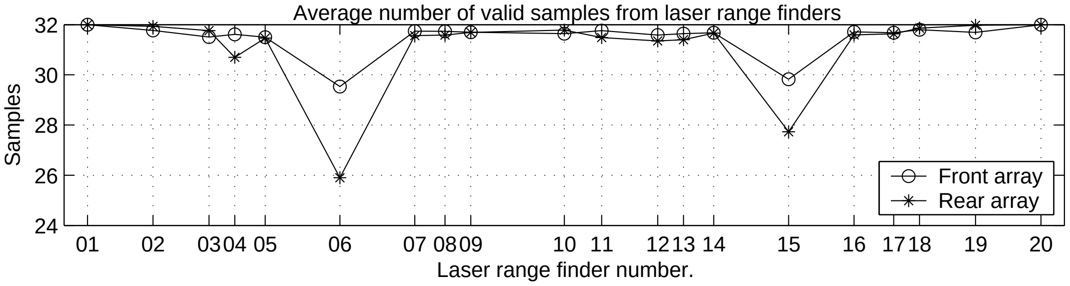


Figure 3.13: Laser quality on newly paved road.

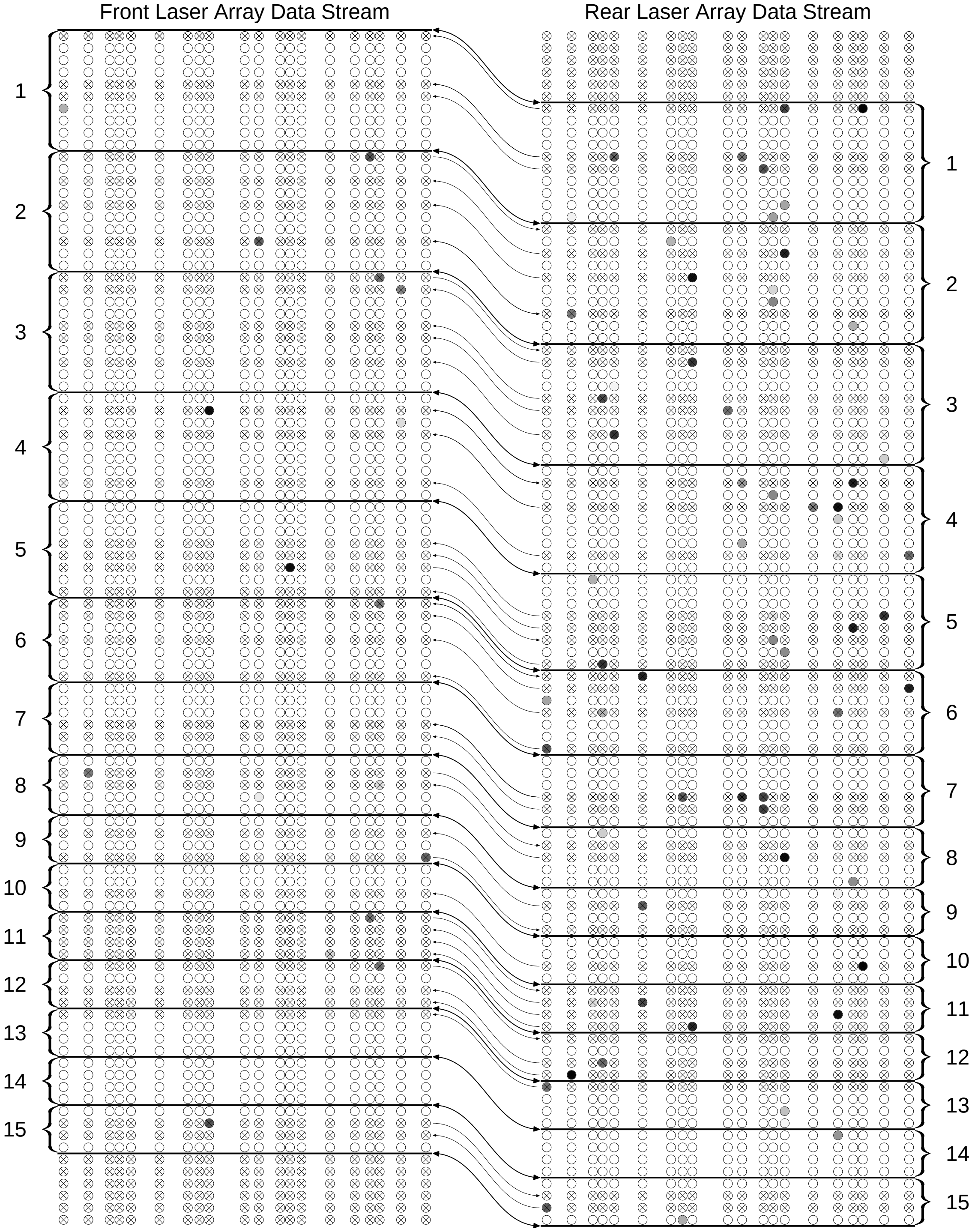
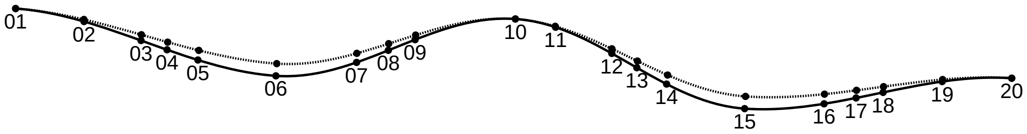
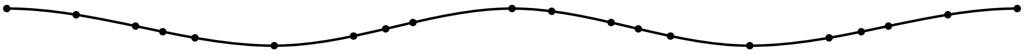


Figure 3.14: Schematic illustration of the RDT data stream.

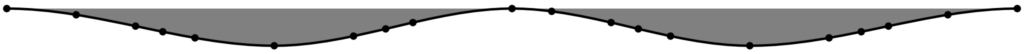
## 3.4 Deflection Indices



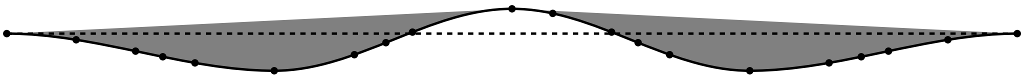
In the figure above, the front cross profile is illustrated with a dashed line, and the rear profile with a solid line. These two lines correspond to the two black lines in Figure 3.2. The black dots represent the points where the lasers hit the pavement surface. The deflection profile is produced by subtracting the rear profile from the front profile. As the rear profile is “deeper” the deflection profile will be sub-zero, as shown in the figure below. The endpoints, which are assumed to be uninfluenced by the load, are set to zero. The whole deflection profile is, in other words, rotated and translated until the endpoints end up on zero. From this deflection profile all indices described below are calculated, except the *rut depth difference*. The reason for having multiple indices is that different properties of the road can be assessed with different indices.



The *deflection area* is the area below a straight line between the endpoints. One advantage with the deflection area is that data from all lasers contribute to the index, making it robust to small measurement noise and external disturbances in individual lasers. A disadvantage is that the result is a surface, which can be hard to interpret and compare directly with other indices.



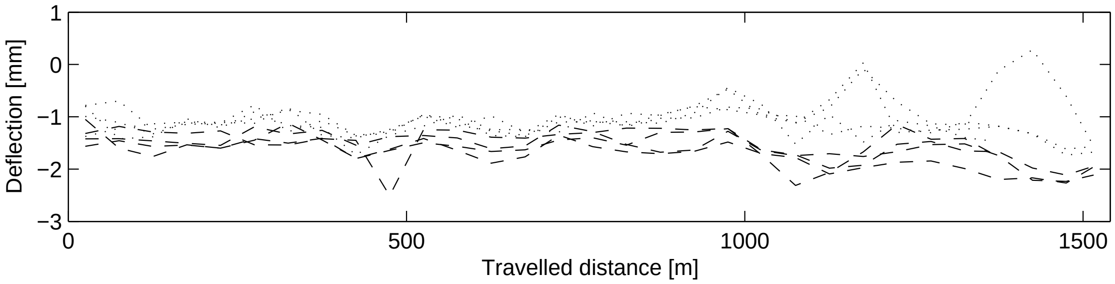
An index similar to the deflection area is the *wire deflection area*. The deflection area from above would be calculated from under the dashed line in the illustration below. The wire deflection area is defined as the surface between the deflection profile and a convex “wire” drawn between the endpoints. When the deflection profile is all negative the *deflection area* and the *wire deflection area* are identical.



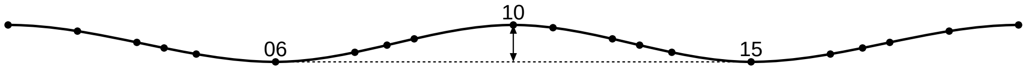
The index illustrated in the figure on the top of the next page is made for easy comparison with the Falling Weight Deflectometer, or some other deflection measuring equipment. The *maximum deflection* is calculated as the maximum distance between the deflection profile and the “wire” as described above, and illustrated by the thicker double arrow. The somewhat thinner double arrow illustrates the *baseline index*, which is defined as the deflection at a given point under a line connecting two symmetrically positioned lasers. The connecting line is illustrated with the thin

dashed line between laser #12 and #18. The idea with the baseline indices is to assess the properties in the upper layers of the road.

Under ideal conditions the index *rut depth difference* is the same as the maximum deflection. In reality they differ slightly as the convex "wire" covering the profiles are calculated *after* the rear and front profiles are subtracted for the maximum deflection, and *before* the rut depths are subtracted. The two indices will give the same result only if every point of the rear profile is more deflected than the corresponding point on the front profile, which, in reality, is uncommon. However, the differences are generally small, as illustrated in Figure 3.4 below, where the dotted lines are the maximum deflection, and the dashed lines the rut depth difference. This example is taken from the Gistad test site where the rut depth is in the 15–20 mm range.



An index with the strange-sounding name *bjelke height difference* was introduced recently in the evaluation process, but has shown much promise. The word "bjelke" is Norwegian for "beam", or in this case "straight edge". The bjelke height has been used on narrow roads in Norway as a complement to the rut depth. The bjelke hight difference is computed as illustrated in the figure below. Instead of using the endpoints as zero-points, the middle-point is assumed to be uninfluenced by the wheel-forces.



# Chapter 4

# Results

Some of the results presented in the present chapter have previously been published in conference proceedings. The paper *High-speed rolling deflectometer data evaluation* was presented at the conference "Nondestructive Evaluation of Aging Aircraft, Airports and Aerospace Hardware III" at Newport Beach, California in 1999. This paper is now somewhat dated. Since then, the understanding of data has improved a lot, and most of the evaluation algorithms have been totally rewritten. Much of the background material is valid and interesting, though. The following papers have all been made in collaboration with Carl A. Lenngren. In 2000 the paper *Evaluating pavement layer properties with a high-speed rolling deflectometer* was presented at the "Nondestructive Evaluation of Aging Aircraft, Airports and Aerospace Hardware IV" also in Newport Beach. This paper concentrates on the possibility to measure properties from the different layers of a road structure. A third, peer-reviewed, paper entitled *Evaluating subgrade properties with a high-speed rolling deflectometer* was presented at the "Pavement, Subgrade, Unbound Materials, and Nondestructive Testing" conference in Denver, Colorado in 2000. The focus in this paper was the subgrade layer only. Comparisons were made between the RDT and the subgrade E-moduli. The last paper so far was presented in Cassis in Portugal on the third Workshop at the "Sixth International Conference on the Bearing Capacity of Roads, Railways and Airfields". The title *Rolling wheel deflectometer/FWD correlation study* explains the content of the paper very accurately. Results from this paper are given in Section 4.6 on page 50.

The best results so far can be produced from the 1998 test programme, which most probably can be attributed to the fact that those test sites, with wide straight pavements, were expertly chosen to suit an evaluation of the RDT. Many different types of road constructions were included, making chances to find an appropriate test site higher. The smaller test programme of 2000 also included mainly wide and straight pavements, but they were also very homogeneous and stiff, complicating the analysis. Also, no FWD data is available for these test sites. The programme in England and France in 2002 was more of a test to see if the RDT was ready for production use. The test sites where chosen not to suit the RDT, but simply as interesting from a road technology perspective. The analysis of the 2000 and 2002 test programmes were interesting in many ways, resulting in better data cleaning methods and more flexible computer programs rather than good results concerning the deflections per se.

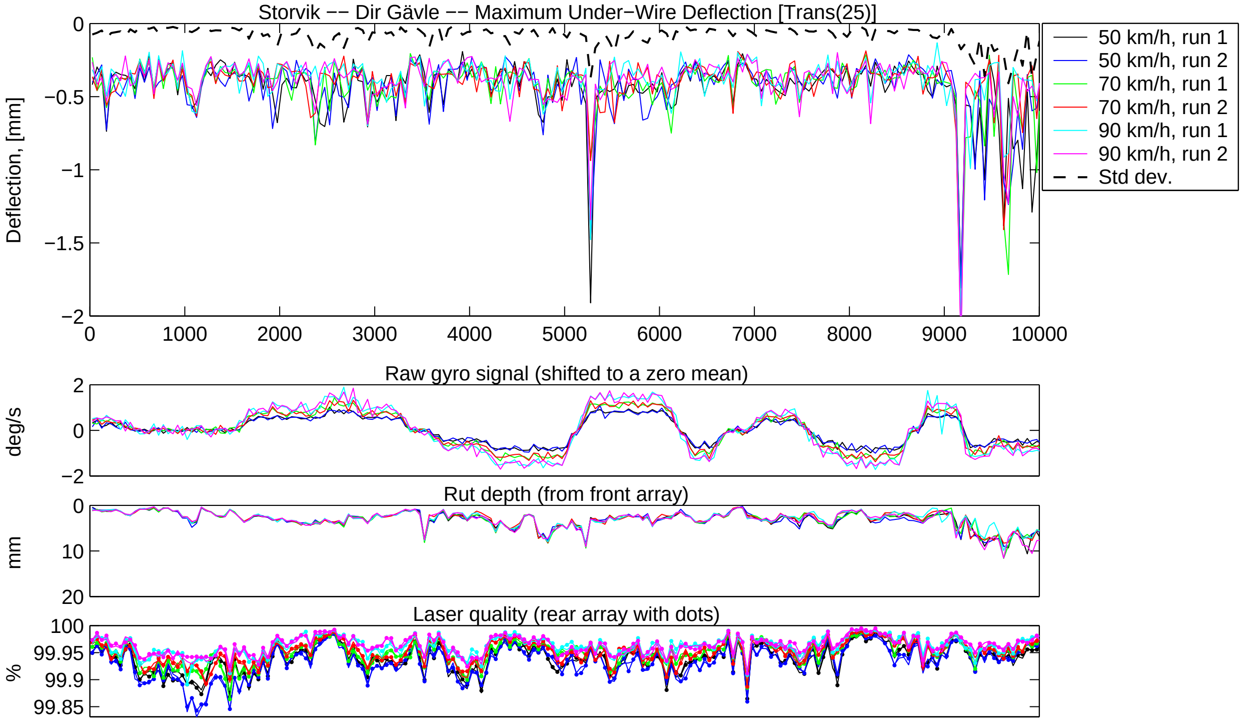


Figure 4.1: General repeatability of the RDT system.

Another boon with the tests in England was that the computer code in the evaluation programs as of 2002 were validated by Peter Watson at TRL. Peter wrote his own code that produced near identical results as the 'official' code at VTI.

The results presented in the present chapter illustrates some of the key issues one can expect to find from this type of device, namely, repeatability, speed dependency, and correlation with other deflection measuring devices. The latter can be seen as a replacement for the reproducibility test, which is impossible as the RDT is a one-of-its-kind device. Other important results are demonstrations of the ability to test long objects quickly, and the comparison with the Road Surface Tester for conventional surface characteristics such as IRI and rut depth.

## 4.1 Repeatability

Even during the worst conditions the RDT system has proved to be very repeatable. Sometimes the results have been difficult to interpret, but these results have also shown a high degree of repeatability. Figure 4.1 above illustrates the repeatability of six consecutive runs at a representative test site of Storvik. The standard deviation of the deflection index is marked as a dashed line. As can be seen, the standard deviation is very low, demonstrating good repeatability, which also is obvious by "visual inspection" of the graph. Not only the actual deflection index is repeatable, but the whole system works in a very repeatable way. After "synchronisation" the rut depth and gyroscope give near identical results, save for the change in magnitude of the gyroscope depending on the speed. Even the laser quality is highly repeatable, even though it is in the high 99 % range and should be very sensitive to small changes in each run.

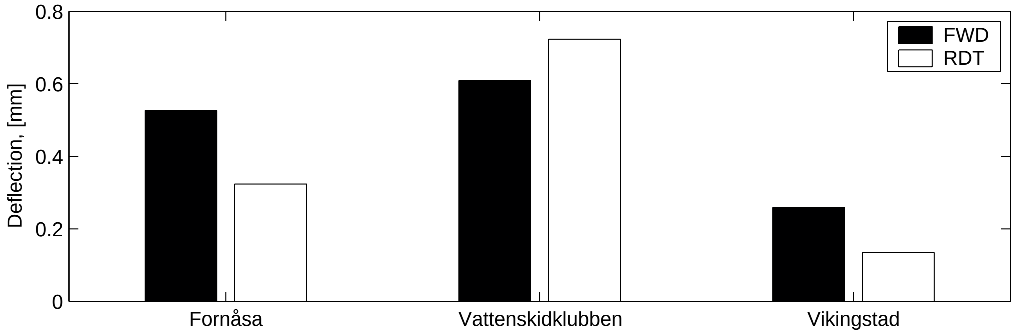


Figure 4.2: Overall deflections for the RDT and FWD on three targeted test sites.

Actually, low repeatability can only be found on very stiff roads where the signal to noise ratio is higher than normal. An example of this can be seen in Figure 4.8 on page 45. In this case a concrete road was tested, resulting in very low deflections, but also a very low repeatability. However, this type of road is less interesting to test for a structural condition, making the low repeatability less concerning.

## 4.2 Deflections

It's important to bear in mind that the RDT was designed primarily for road network use. The analysis presented below is directed more to individual road objects, with point-to-point comparisons with the FWD, which is a far more demanding task.

However, in 2001 three test sites in proximity to the VTI facility were measured with the sole objective of separating roads with different structural capacity. A weak road at the Linköping Water Ski Club, a medium strength road at Fornåsa, and a very strong road with a concrete base at Vikingstad were chosen as test objects. All three test sites are straight, similar in roughness and about one kilometre long. VTI's Falling Weight Deflectometer were used at all the sites shortly before the RDT. Using the *rut depth difference* a fair correlation between the RDT and the FWD deflections were found, as can be seen in Figure 4.2 above. (It's not shown here, but it can be mentioned that the *rut depth difference* gave only slightly better correlation than the other indices tested in the evaluation.) This test indicates that the RDT is capable to, at least, perform the task it was designed for.

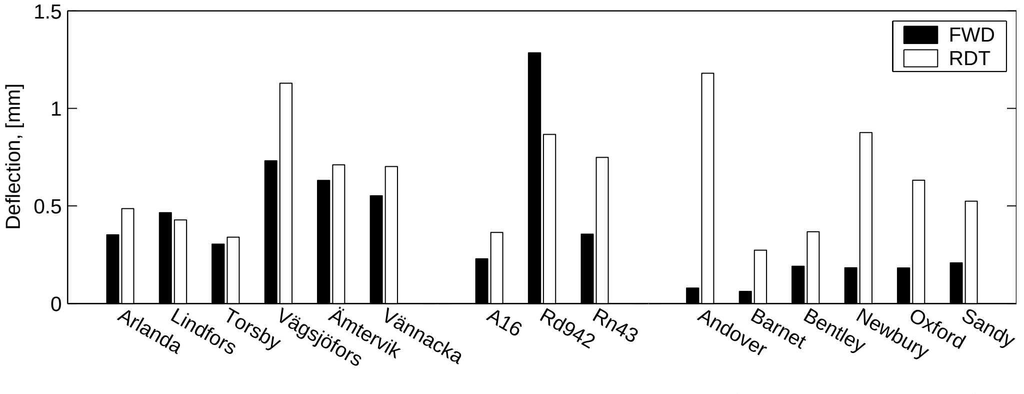


Figure 4.3: Overall deflections for the RDT and FWD (or Deflectograph) on all other test sites (cf. Figure 4.2).

(a) Test site at Fornåsa.	(b) Test site at the Water ski club.

Figure 4.4: Photographs from RDT test sites. Notice the pieces of sleeping pad used to indicate start and stop of the test section. (Both photographs by VTI.)

Analysing the other test sites where FWD or Deflectograph comparison data is available give the result as shown in Figure 4.3. Convincing and reliable results can be produced for the 1998 test programme, but the tests in England and France are clearly in need of a more thorough analysis. Results could in all probability be improved by choosing only straight sections, and removing parts where some laser range finder had meandered outside of the pavement surface. Such an algorithm would have to be based on a clear set of rules to avoid "cherry-picking" of data, but that remains to be done. The Andover test site, e.g., had severe ruts (almost 40 mm at some points) which clearly is a problem. Even though the Andover test site cast light on an important limitation in the RDT evaluation software, it can be discussed whether already deteriorated roads like this really needs to be measured for structural conditions.

Photographs from two test sites from the 2001 test programme are reproduced in Figure 4.4. Strips of ordinary sleeping pad are glued to the pavement surface and used to indicate start and stop of the test section. The 12 mm high foam layer is easily detectable in the data stream, and allows for very accurate lengthwise synchronisation.

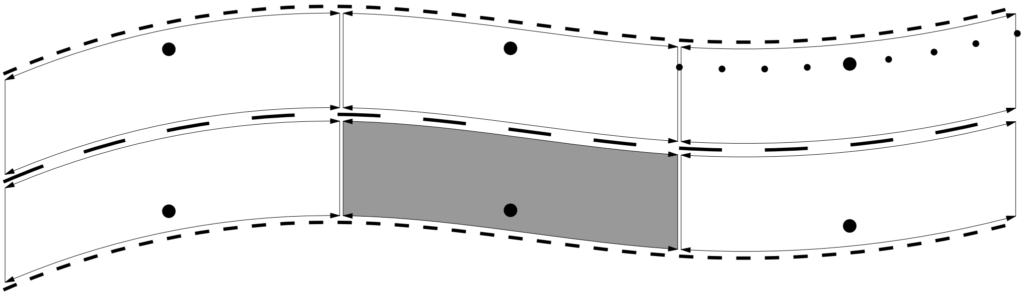


Figure 4.5: The dots represent the Falling Weight Deflectometer test stations, 40 metres apart. The grey part centred around the FWD spots represent the area over which the deflection index have been calculated.

Below, the more detailed analysis performed on individual objects is presented. Apart from being easier to present, this is what is currently asked for. The road network measurements are, according to the SRA representatives, a low priority due to the limited resources available. Relatively quick but detailed testing of object prior to rehabilitation, and as a control resource in “operational contracts” (swe. funktionsentrepenader) are higher priorities. For this type of tests speed is not of crucial importance, making is possible to measure the road in speeds from 10 to 90 km/h in steps of 10. This would give better resolutions, allow for a comprehensive analysis of speed effects, and give a high degree of redundancy in the data material.

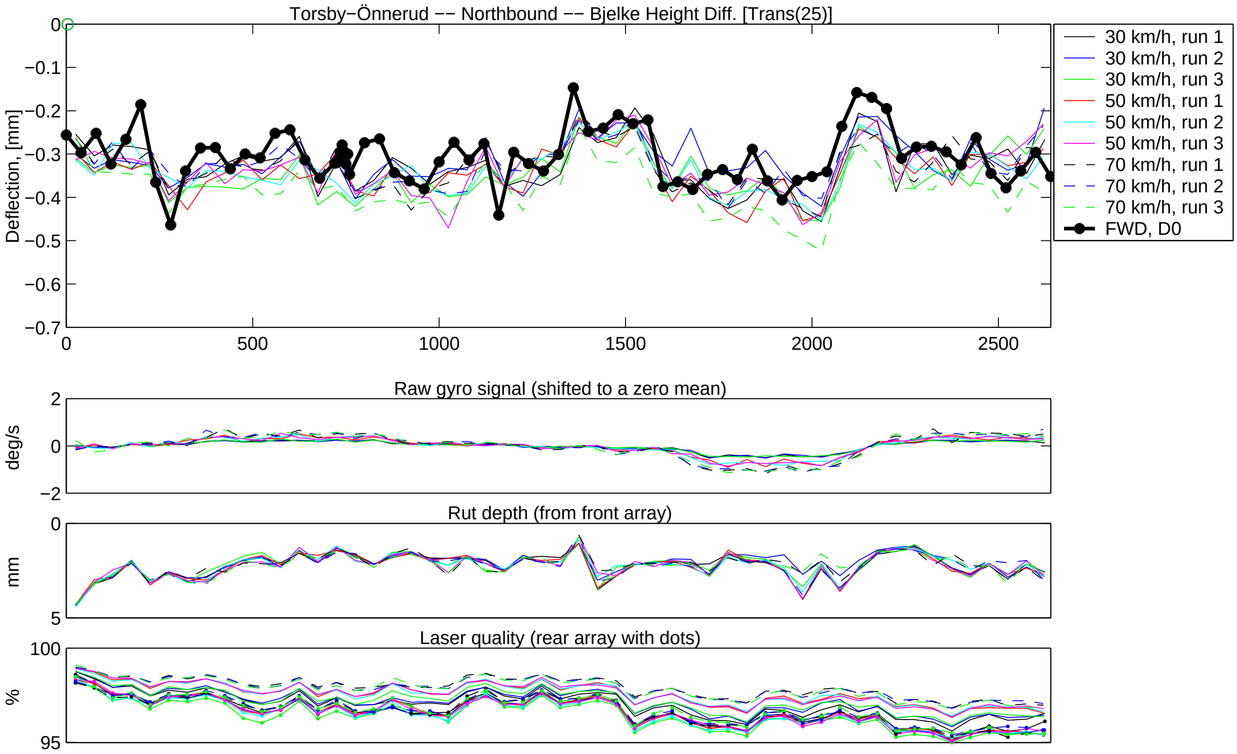


Figure 4.6: Results from the Torsby-Önnerud test section.

The best results so far, when compared to FWD data, can be produced from the Torsby-Önnerud test site measured at 1998. This test site was measured shortly after repair (which, with the black surface, explains the why the laser quality is below 99 %) on a varm and sunny day, making the asphalt about 40 °C warm and relatively soft. The road cuts through an esker and goes mostly through a wooded area. The annual daily traffic is about 3300. The width of the road is 8.0 m. The speed limit is 90 km/h.

As can be seen in Figure 4.6 there is an almost one-to-one correlation between the RDT and the FWD, both in trend and absolute deflection values. Especially, the parts at 1300–1600 and 2100–2300 with a lower deflection are clearly picked out by the RDT. The small differences occurring at the beginning of the section and at 800 metres can be explained with, e.g., the FWD testing a weak/strong spot in a strong/weak surrounding, as illustrated in Figure 4.5 on page 43. There should be little doubt that the good results presented here are incurred by anything else other than the bearing capacity of the road.

Promising results can also be seen at the Arlanda test site, Figures 4.7 and 4.8. This test site is interesting as is offers both asphalt and concrete overlay (AC northbound, PCC southbound). Although the deflections in the northbound direction are somewhat lower than those reported from the FWD, the trend with larger deflections to the end of the test sections, is also detected by the RDT. Very small changes in offset and scale factor of the laser range finders influence the deflections reported by the RDT. To some extent such an “post measure” calibration (or “dynamic calibration”) could be defendable as/if the FWD can be regarded as a true reference.

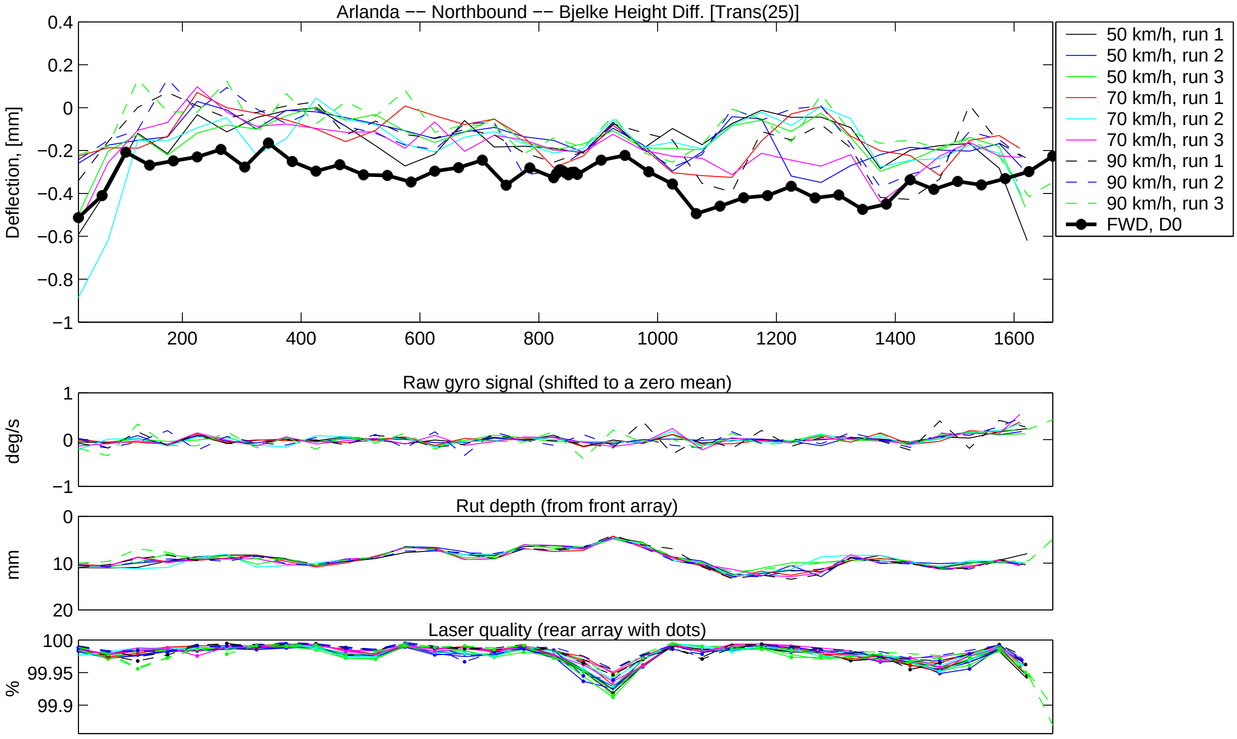


Figure 4.7: Deflections on the northbound asphalt road.

As mentioned above, in the southbound direction the road is made of concrete, which yields far lower deflections. It can also been seen that the signal to noise ratio is high, indicating that the measured deflection is close to the resolution of the system.

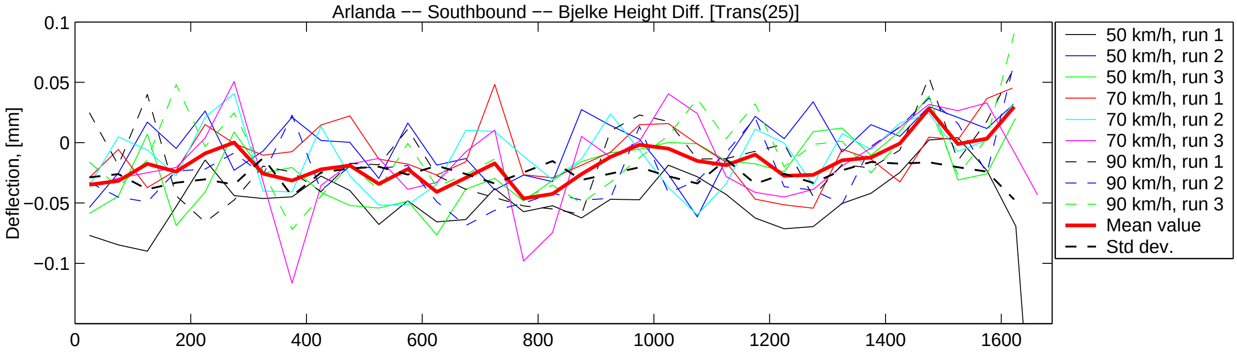


Figure 4.8: Deflections on the southbound concrete road.

## 4.3 Speed Dependency

The speed dependency can be very clearly seen at the Lindfors test site. Results from the FWD indicate that the road has very uniform material parameters, which is reflected in the uniform output from the RDT. Using the *deflection area* index a striking speed dependency can be noted. The higher the speed the lower the deflection is true for all but one sample at the beginning of the site. Why the speed dependency is so evident on this section, and not always so obvious on others (even though it is clearly discernible in, at least, parts of the most cases) has not been analysed yet. The asphalt concrete in this case did not differ from other test sections. A plausible explanation is that the subgrade is saturated, and at high speeds water pore pressure builds up, causing these dynamic effects.

The test site is a bypass of village Lindfors in a farmed area. The geometry of the road is straight. The annual daily traffic is about 3300. The width of the road is 8.0 m. The speed limit is 90 km/h.

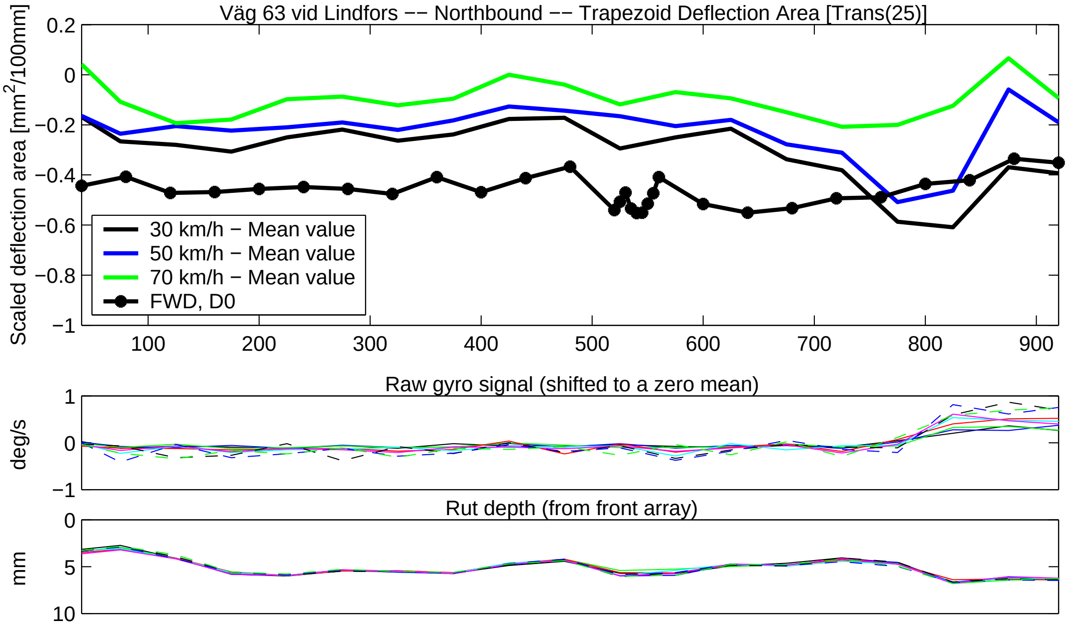


Figure 4.9: Speed dependency at the Fornåsa test site.

## 4.4 Long Tests

The very reason for building the RDT was the ability to measure large amounts of roads in a short time. Falling Weight Deflectometers are, comparatively, very slow and often still-standing during operation, which constitutes a safety risk both for operators and surrounding traffic.

Through the years a few long test sites have been measured, as shown in Table 4.1. With the storage capacity of hard disks available today there is no practical limit of the quantity of road that can be tested.

Table 4.1: All “network” test with the RDT. See Table 4.3 on pages 52–53 for explanations on the notation.

| Location | | Date | Length | Speeds (rep.) |
|---|---|---|---|---|
| Motorway E18 | ↔ | 1998-06-13 | 75000 | 70(1) |
| Highway 45 | → | 1998-04-14 | 75000 | 70(1) |
| Motorway E4 | ↔ | 2001-09-27 | 137000 | 80(1,2) |
| Svärdsjö | ↔ | 2002-04-09 | 25300 | 70(3), 90(2,3) |
| Svärdsjö | ↔ | 2002-06-11 | 25300 | 80(4) |
| Gistad | ↔ | 2004-09-24 | 20000 | 70(1) |
| Gistad | ↔ | 2004-11-15 | 20000 | 70(1) |

The longest test to date is the 137 kilometre long test run through the Östergötland County. This test took one hour and forty minutes, and resulted in a 1.2 GB data file, enough to make more than six million deflection profiles. The results from the evaluation can be seen in Figure 4.10. Due to the prohibitive cost no comparative FWD tests were carried out on this site.

The results, with deflections mainly in the 0.1–0.4 range seems realistic. One interesting aspect is the marked drop in laser quality on the newly repaired section at about 55–60 km. The laser quality drops as a results of the black, less reflecting, surface, the rut depth is close to nil, and the deflection index is, if not lower, more homogeneous than on the surrounding older pavement. (The quality drop at the very end of the test section is the result of a light rain, where the water on the road scatters the laser signal spot, making detection harder.)

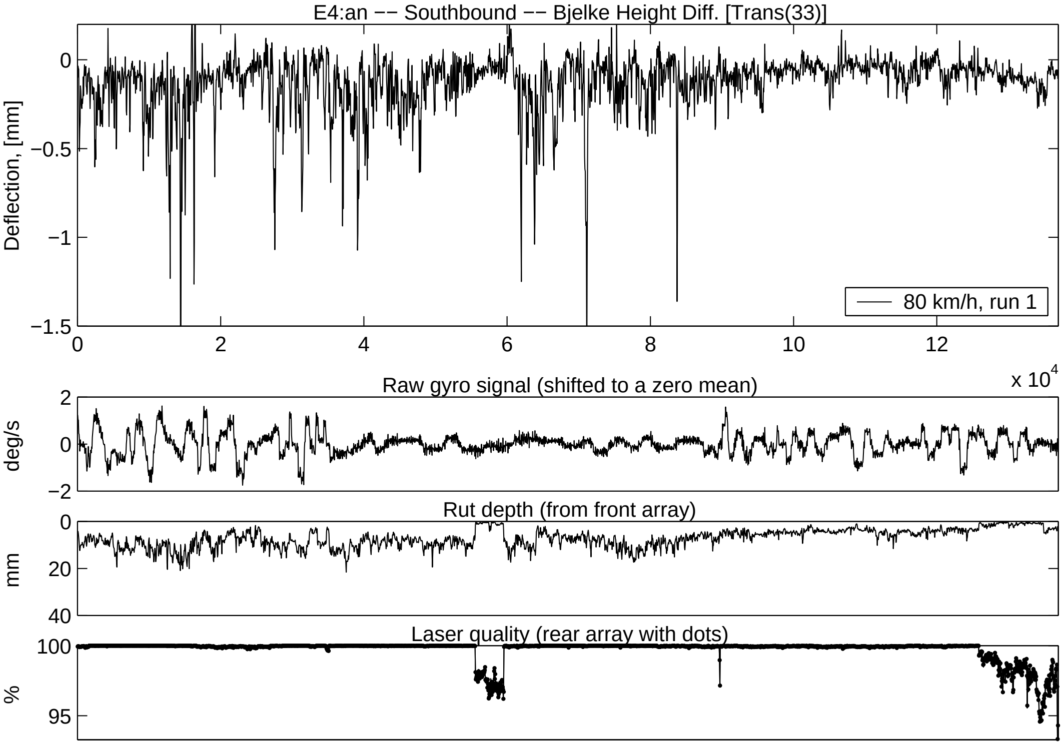


Figure 4.10: Southbound on motorway E4.

Another "network" test is illustrated with the motorway E18, tested as a part of the 1998 test programme. Complete information is given in Figure 4.11 for the eastbound run, and only the deflection index in Figure 4.12 for the westbound run. Again it can be seen that both the laser quality and rut depths drops slightly at sections newly paved sections, that also tend to be more homogeneous on the deflections. The increased homogeneity most likely depends on an increased signal to noise ration, which in turn could be a signal that a longer presentation length, or harder cleaning method should be used. The deflections, mostly placed in the 0.1–0.3 mm range, are very realistic, though. (The 0 km/h in the legend simply means that the normal traffic speed was held, and no special target speed was aimed for.)

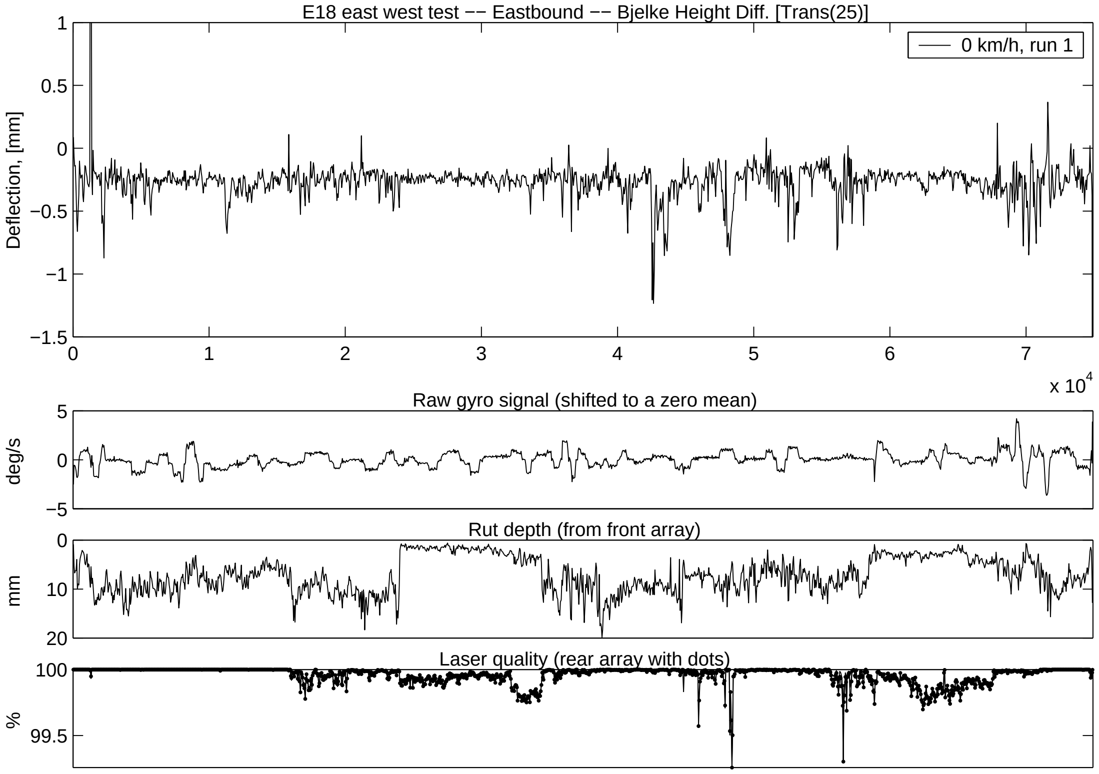


Figure 4.11: Eastbound on motorway E18.

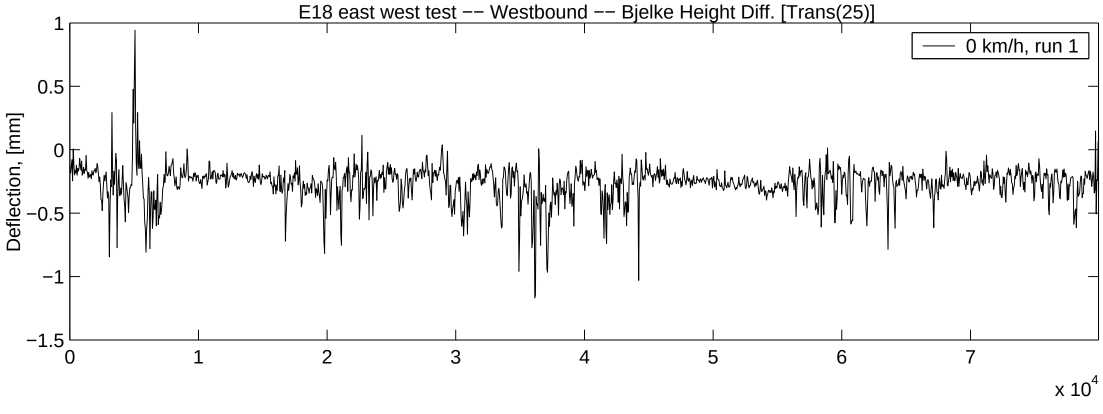


Figure 4.12: Westbound on motorway E18.

## 4.5 Comparisons Over Time

A couple of test sites have been measured at more than one occasion, allowing for comparisons over time. The Svärdsjö and Gistad test sites have also been measured more than once, but both with only about two months in between.

The most interesting site is obviously the Vikingstad, which has been measured four times, as listed in the table below.

Table 4.2: Site at Vikingstad measured at multiple occasions. See Table 4.3 on pages 52–53 for explanations on the notation.

| Location | | Date | Length | Speeds (rep.) |
|---|---|---|---|---|
| Vikingstad | ↔ | 1998-06-17 | 1350 | 30(2), 50(4,3), 70(3,4) |
| Vikingstad | ↔ | 2001-10-29 | 1060 | 30(2), 50(2), 70(2) |
| Vikingstad | ↔ | 2002-06-06 | 1080 | 30(2), 50(2), 70(2) |
| Vikingstad | ↔ | 2003-08-20 | 1070 | 30(2), 50(2,4), 70(2) |

The agreement between the four runs are somewhat lower than can be expected. This would be a good opportunity to investigate where these differences originate, and possible improve the calibration routines. An educated guess is that the laser offset values and scale factors play an important role, but driving behaviour or actual seasonal changed in the structural condition might influence the results as-well.

Also, these tests have not been synchronised with the same rigour as individual runs within one test session. These errors should be negligible, though.

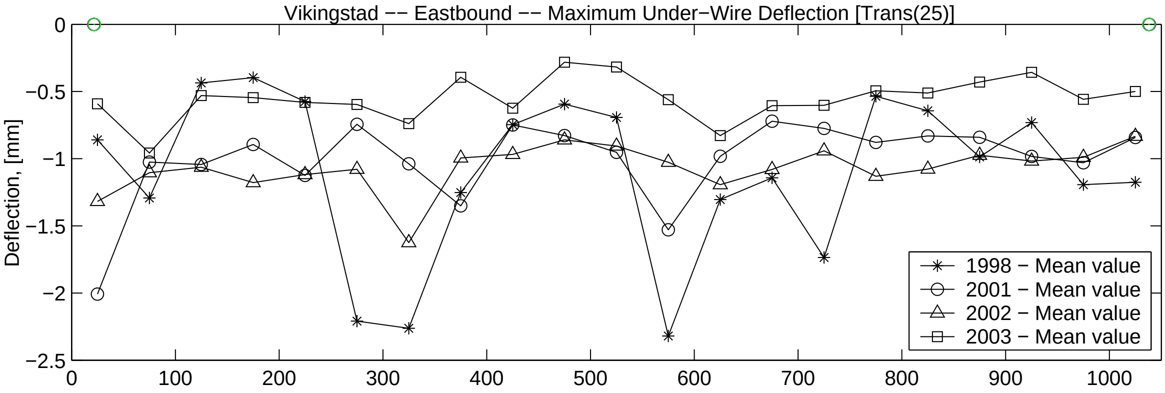


Figure 4.13: Four runs at Vikingstad.

The test of 2003 was carried out just to “air” the RDT after a long period of sill-standing in the garage, which could be one explanation to the lower deflection level in this test. No proper calibration was done for this test.

The results from annual surveys like this must be improved, if the RDT is to fill the purpose it was design for. The main point with doing road network surveys would be to find out when the structural condition is getting worse, but without any significant damage on the surface. Unfortunately, no test site producing promising results has been measured more than once.

## 4.6 Evaluating Subgrade Properties

The material in the present section have been previously published as a part of *Rolling wheel deflectometer/FWD correlation study* by the author and Carl A. Lenngren [9]. The author has no experience in using the FWD or backcalculating E-moduli from the deflection basins recorded. However, these results are relevant to the present study as they represent a closer link between the RDT and the FWD in the way it's normally used. All E-moduli presented below were backcalculated by Lenngren. Further, the text below, describing the FWD analysis is written by Lenngren, but slightly edited by the author to fit the present format.

Three objects chosen for tests during 2001 were also subjected to the VTI Falling Weight Deflectometer at a 50 kN load level. Each object was about 1 km long. A backcalculation with Clevercalc 3.8 of layer moduli was done so that the static deflection around the RDT wheels could be determined. This program is based on the Evercalc series described by Mahoney, Coetzee and Lee [190]. Resilient layer modulus linearity was assumed as the wheel load of about 55 kN is quite near 50 kN.

Vikingstad, the test section giving the least deflections is actually on the old Stockholm-Copenhagen road used fifty years ago. It is an asphalt concrete (AC) road over a cement treated base (CTB) resting on fine soils. A four-layer system clearly showed the CTB being much stiffer than then AC. As is common for unbound layers under relatively stiff layers the subbase exhibited a rather soft value of 80–120 MPa or about the same as the subgrade. The CTB was backcalculated to about 16000 MPa and the AC to 4500 MPa. At section 280 m the subgrade showed a very stiff response, likely due to bedrock near the surface.

The Fornåsa test section is a typical asphalt concrete road also being rather homogeneous throughout its entire length. The asphalt concrete was backcalculated to 11000 MPa at the time of testing. However it was about five degrees warmer at the time of the RDT visiting there so the AC modulus was adjusted down accordingly. The unbound base and subbase was determined to be 65 MPa on the average and the subgrade 135 MPa.

The Water Ski Club test section, a bituminous surface treatment (BST) was assumed to be of poor bearing capacity, but actually deflections on the average were just a tad higher than those for Fornåsa. Very often the unbound base gets compacted by traffic on such roads, so that they display a rather high modulus. It can not be considered homogeneous however as there were several sections influenced by bedrock.

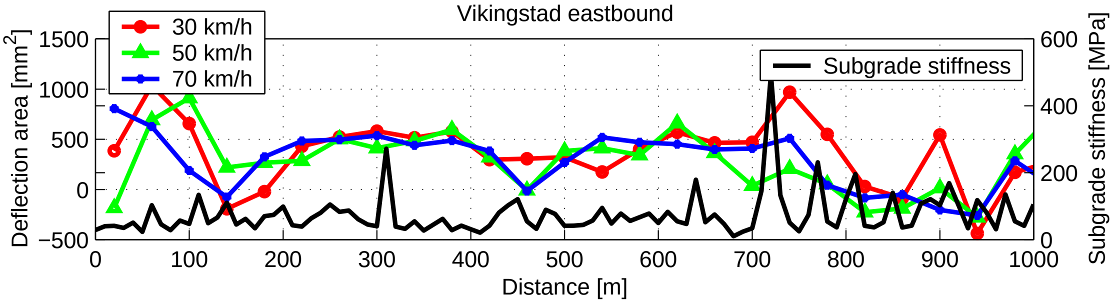


Figure 4.14: Comparison between the RDT deflection area and the subgrade stiffness.

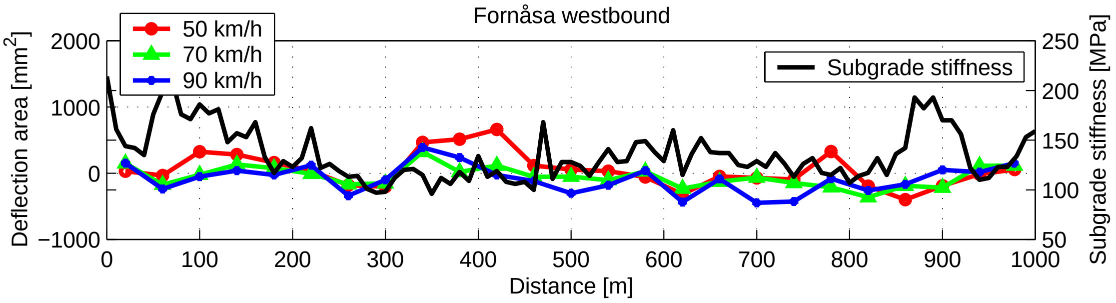


Figure 4.15: Comparison between the RDT deflection area and the subgrade stiffness.

The deflection profile areas are plotted against the left y-axis, and the subgrade E-modulus is plotted against the right y-axis. The plot legends refer to their respective y-axis. The RDT test results are all presented as the *deflection area* (given in mm$^2$) and are plotted in the travelled direction.

The RDT deflection area and the backcalculated subgrade stiffness at the Vikingstad test site (Figure 4.14) correlates fairly well. It can be seen that the homogeneous subgrade in section 100–700 results is reflected by the small variation in the RDT deflection area. The larger variation in subgrade stiffness in the 700–900 section is shown as a more varying deflection area. Note also that the cement treated base results in a very low speed dependency.

The Fornåsa test section is quite homogeneous regarding subgrade stiffness and RDT deflection area as well, Figure 4.15.

At the Water Ski Club test site a very clear correlation between the subgrade stiffness and the RDT deflection area was found. The RDT deflection area seems to be depend by the subgrade stiffness which is often seen on weaker constructions.

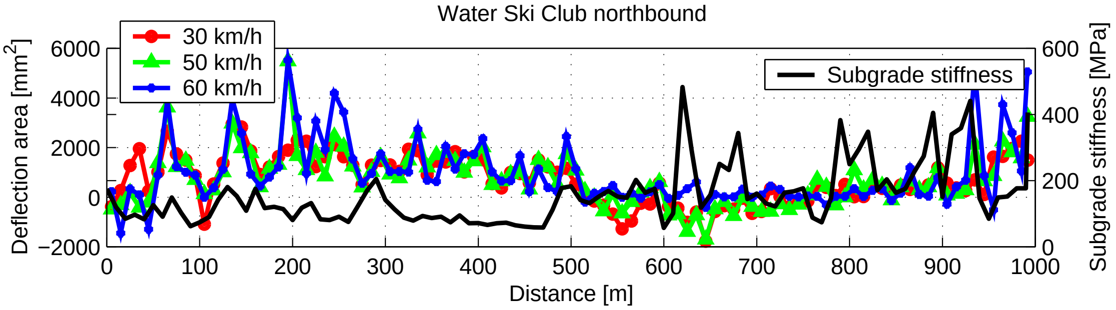


Figure 4.16: Comparison between the RDT deflection area and the subgrade stiffness.

## 4.7 List of Tests

For reference, the complete set of tests measured with the RDT is listed in Table 4.3 below. The ↔ symbol in the second column indicates that the test site was measured in both directions, and → that only one direction was tested. The speeds are given in km/h, with the number of repetitions for the speed in parentheses behind. When the number of repetitions in the two directions differ, this is noted with two numbers, as in 70(3,4). Speed ranges such as 50–80 mean 50, 60, 70, 80. A few test runs prior to the 1998 test programme have been omitted as they never have been used for any qualitative analysis.

Table 4.3: All test to date performed with the RDT.

| Location | | Date | Length | Speeds (rep.) |
|---|---|---|---|---|
| Arlanda | ↔ | 1998-06-24 | 1650 | 50(3), 70(3), 90(3) |
| Motorway E18 | ↔ | 1998-06-13 | 75000 | 70(1) |
| Flygrakan and Nykil | ↔ | 1998-06-22 | 1170 | 30(19) |
| Lindfors | ↔ | 1998-06-15 | 1000 | 30(3), 50(3), 70(3) |
| Ljungskile | ↔ | 1998-06-25 | 1050 | 50(4), 70(3), 90(3) |
| Stångån | → | 1998-06-26 | 1250 | 90(3) |
| Torsby-Önnerud | ↔ | 1998-06-14 | 2660 | 30(3), 50(3), 70(3) |
| Highway 45 | → | 1998-04-14 | 75000 | 70(1) |
| Vägsjöfors | ↔ | 1998-06-14 | 510 | 20(3), 30(3) |
| Västra Åmtervik | ↔ | 1998-06-15 | 2070 | 30(3), 50(3), 70(3) |
| Vännacka-Hajom | ↔ | 1998-06-12 | 2030 | 30(3), 50(3), 70(3) |
| Vikingstad | ↔ | 1998-06-17 | 1350 | 30(2), 50(4,3), 70(3,4) |
| Köping | ↔ | 2000-10-27 | 3950 | 50(4), 70(4), 90(4,5) |
| Litslena | ↔ | 2000-06-20 | 9650 | 50(3), 70(3), 90(3) |
| Sidensjövägen | ↔ | 2000-06-28 | 10050 | 30(3), 40(3), 50(3) |
| Storvik | ↔ | 2000-06-21 | 10930 | 50(2), 70(2), 90(2) |
| Fornåsa | ↔ | 2001-10-11 | 1090 | 50(2), 70(2), 90(2) |
| Uddevalla | ↔ | 2001-09-26 | 11300 | 50(2), 70(2), 90(2) |
| Highway 34 | ↔ | 2001-09-03 | 1160 | 50(1), 70(3,1), 90(1) |
| Vattenskidklubben | ↔ | 2001-10-15 | 1070 | 30(1), 50(3), 60(1) |
| Vikingstad | ↔ | 2001-10-29 | 1060 | 30(2), 50(2), 70(2) |
| Motorway E4 | ↔ | 2001-09-27 | 137000 | 80(1,2) |
| Fornåsa | ↔ | 2002-06-06 | 1050 | 50(2), 70(2), 90(2) |
| Svärdsjö | ↔ | 2002-04-09 | 25300 | 70(3), 90(2,3) |
| Svärdsjö | ↔ | 2002-06-11 | 25300 | 80(4) |
| Vikingstad | ↔ | 2002-06-06 | 1080 | 30(2), 50(2), 70(2) |
| Motorway A16 | → | 2002-10-07 | 11800 | 60(3) |
| —. 2 km test sect. | → | 2002-10-07 | 2090 | 60(5) |
| Highway RD942 | ↔ | 2002-10-08 | 8180 | 60(3) |
| —. 2 km test sect. | → | 2002-10-08 | 2150 | 30(3), 60(8) |
| Road RN43 | ↔ | 2002-10-09 | 14900 | 60(3) |
| —. 2 km test sect. | → | 2002-10-09 | 2500 | 30(2), 60(8), 80(1) |

*Continued on next page.*

*Continued from previous page.*

| Location | | Date | Length [m] | Speeds (rep.) |
|---|---|---|---|---|
| A303 Andover | → | 2002-10-03 | 1510 | 60(2), 70(2), 80(2) |
| M1 Barnet | → | 2002-10-02 | 2500 | 70(1), 90(2) |
| A31 Bentley | → | 2002-10-01 | 2320 | 50–80(1,2,2,2) |
| A339 Greenham | → | 2002-10-04 | 8340 | 60(3) |
| A420 Oxford | → | 2002-10-04 | 3700 | 60(1), 70(3), 80(2) |
| A1 Sandy | → | 2002-10-04 | 6700 | 60(1), 80(5) |
| TRL Test Road | → | 2002-09-30 | 2230 | 10(2), 50–70(6) |
| TRL Test Road | → | 2002-10-01 | 2230 | 10(4), 40–80(2,4,4,4,2) |
| TRL SRS | → | 2002-10-03 | 170 | 10–50(6,6,6,6,8) |
| Vikingstad | ↔ | 2003-08-20 | 1070 | 30(2), 50(2,4), 70(2) |
| Gistad | ↔ | 2004-09-24 | 20000 | 70(1) |
| Gistad | → | 2004-09-24 | 1800 | 50(1), 70(2), 90(1) |
| Gistad | ↔ | 2004-11-15 | 20000 | 70(1) |
| Gistad | → | 2004-11-15 | 1800 | 50(1), 70(2), 90(1) |

To summarise, the RDT has been used on thirty-four different test sections, in three countries. A little bit more than 3300 km of road has been measured, and analysed. 3300 km might sound like much, but this is less than a tenth of the road length measured every year for the SRA network monitoring for the conventional road surface characteristics in Sweden.

## 4.8 The RDT as an RST

Of course, the RDT can also be used to measure conventional road surface characteristics. The ability to measure transversal profiles are, as explained in the previous sections, the basic idea behind the RDT. Longitudinal profiles can be calculated from the laser signals in combination with the accelerometers on the rear wheel axle. True, this will not be as accurate as having the accelerometers mounted on the same beam that carries the lasers, but for practical purposes the errors seem to be negligible. An advantage with the RDT over the RST and similar systems is the wider, but still accurate, cross profile. The increased width is sometimes asked for, as data from the ordinary laser profilometers occasionally fail to assess the true condition of the road.

A longitudinal profile, the IRI calculated from it, and the rut depth from the front laser array is shown in Figure 4.17. The longitudinal profiles in the upper plot have been resampled to an 0.1 metre sample distance and filtered with a 100 metre highpass filter. The differences in the IRI are not significantly larger than the differences between two runs with the RST. The IRI is calculated with a 20 metre presentation length, which is the standard in Sweden. For the rut depth a special RDT data processing run with the 20 metre presentation length was done, to allow for point-to-point comparison with the RST. The RST and RDT both measured the test site in Svärdsjö in April 2002.

The ability to measure conventional surface characteristics simultaneously with the deflection measuring is a big advantage for the RDT, compared to other deflection measuring devices. Not only because of the reduced number of vehicles needed, but also for the absolute synchronisation between the surface characteristics and the information on the structural condition.

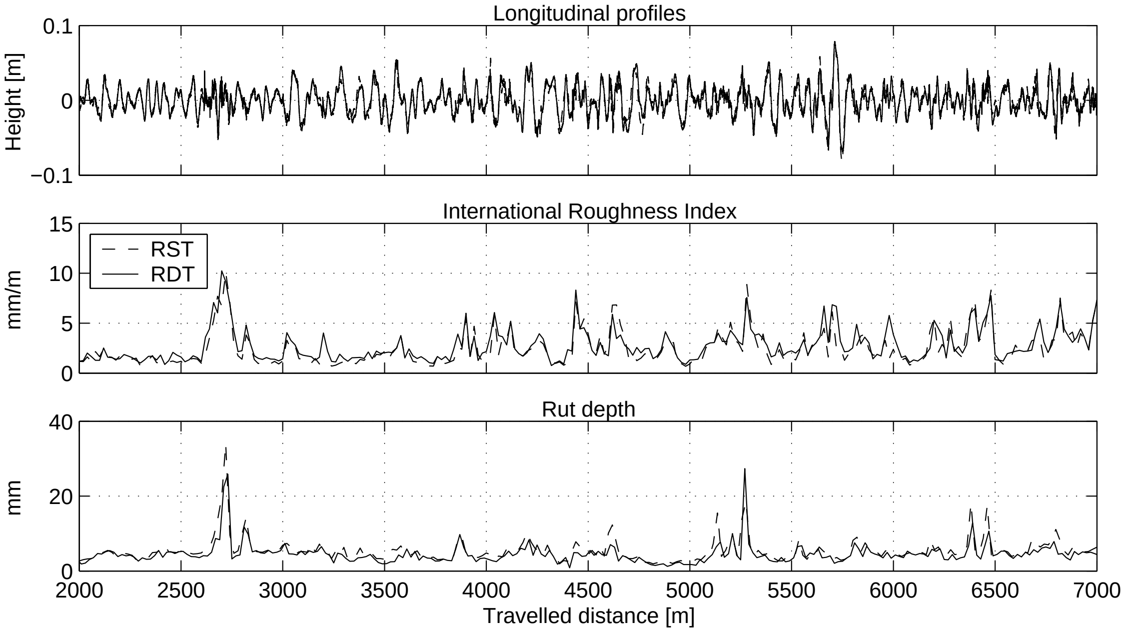


Figure 4.17: Rut depth and IRI calculated from the RDT data compared with the RST results.

The cross slope can, if needed, be estimated from the quotient of the wheel forces, but this is not very accurate. In Figure 4.18 the cross slope from the RST is compared with the right/left wheel force quotient from the RDT. The method works better for slopes on straight sections than in curves, where the very idea with the cross slope is to even the wheel forces. Of course, the needed accelerometers and gyroscopes needed to make the RDT truly function as an RST can easily be installed, at a relatively low cost, if wanted.

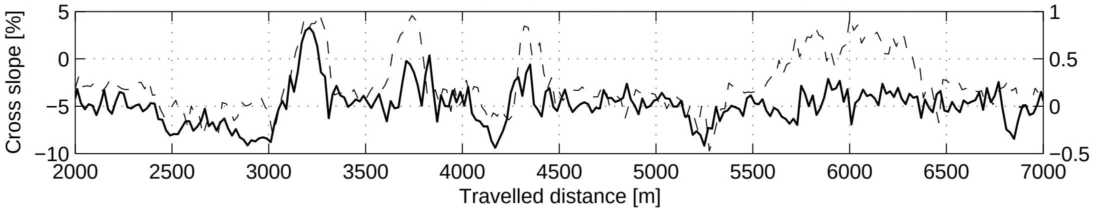


Figure 4.18: Estimation of the cross slope from the RDT wheel forces. (The same figure legend as in Figure 4.17.)

# Chapter 5

# Conclusions and Further Research

The high-speed RDT technique allows large amounts of road to be tested in a comparably short time. Two persons (one driver and one to operate the computer equipment) can test up to 500 kilometres of centreline in only one day's work. This generates large amounts of data but with efficient computer code and the ever increasing speed of computers this is no longer a problem. Presently, the RDT data are used as a relative indicator of the road bearing capacity. The results are not used as input for mechanistic design, but rather as a screening tool. Weak and strong pavements should be discernible and the present report shows that there is a good correlation between the RDT and the Falling Weight Deflectometer (FWD) as regards the ranking of a number of different strength pavements. Some test sections also show a good correlation when making point-to-point comparisons with the FWD. When comparing with the FWD it is important to understand the difference of the two methods, where the former represents a location and the latter a length along the road.

The repeatability of the RDT has been proved very good. Low repeatability is essentially only found on very stiff roads, where the signal to noise ratio in the data is higher. A reproducibility test is impossible as the RDT is a one-of-its-kind device. Conventional surface characteristics such as the IRI and rut depth can be measured with high accuracy.

Speed sensitivity of the deflection was noted in one particular case. As the asphalt concrete in this site did not differ from other test sections, it can be assumed that this behaviour can be attributed to high pore pressures in the subgrade soils. Considering that dynamics are involved in the evaluation process it is important to test at different speeds so that models can be tested coping with viscoelasticity.

The software for analysing the raw data is written entirely in Matlab. The analysis, post-processing and presentation of the results are controlled from only two Matlab data-structures, making it easy to quickly test many alternative evaluation techniques. Multiple levels of pre-processing is used in order to minimise the time needed for evaluations. The software is highly modularised, allowing new functions to be added in a clean and efficient way. The main limitations in the RDT technology as of this writing can, however, be found in the evaluation of data.

It should come as no surprise that some problems will be hit upon when trying to measure an 0.1–0.5 millimetres deflection on a rough surface from a moving platform at normal traffic speeds. All things considered, the RDT seems to be a very useful tool for determining the overall state of bearing capacity.

### Further Research

More accurate calibration methods for the laser range finders are needed. The deflections being measured are not far from the resolution of the lasers, and even small errors in calibration can have drastic results. However, this problem will probably be greatly reduced after the proposed reangling of the lasers to point straight down instead of the 35 degree angle they have now.

More tests must be done on roads of varying conditions, and these tests must be repeated annually. The RDT-FWD correlation can certainly be improved, and tests on instrumented test sections will be necessary some time in the future. A test on an instrumented road would help in determining any significant deviations as regarding speed, deflection delay and so forth.

The software needs to be improved. The basic structure of the code is probably adequate for the problems involved. The real challenge is to keep the data with actual deflection information, and remove the other. The cleaning methods implemented today do their job well, but more advanced methods are probably needed. One issue that must be addressed is the out-of-line complex of problem, i.e. when the two laser arrays measure transversal profiles shifted in relation to each other. Spline interpolation and subsequent matching and correction of the profiles have been tested, but not pursued at any length.

A numerical model of the sensors and the data flow would be of great help in pinpointing problems in calibration, assessing the effect of the out-of-line problem, etc. More theoretical simulations are also needed to get a more complete picture of the effects of the rolling wheel on the pavement. A literature survey, presented in Appendix A, on the relevant theories has been carried out.

Plans have been made to convert the RDT from a dedicated deflectograph to a more all-purpose research vehicle. The RDT will, e.g., be used this summer in a project evaluating the effects of dynamic loads on pavement surface roughness. It can also be used as a sort of “super RST”, measuring conventional surface characteristics very accurately.

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# Appendix A

# Deformations of Solids under Static and Moving Loads — A Literature Survey

This literature survey started and was mainly carried out when the project was still planned to deal with theoretical models of road deformations under moving loads. Had that actually been the case more than a literature survey would hopefully have been presented. Plans were made for a more thorough study of the theoretical models, with details of implementations, possibilities and limitations, results, comparisons between models, and comparisons with results from the Road Deflection Tester. Even though some work has been done on the different theoretical models for the steady-state and transient deformations, and their applicability to the RDT project none of that will be presented. Rather than presenting an incomplete review and analysis the "criticism" of Alexander Pope from three-hundred years ago was adopted. From Pope's "An Essay on Criticism" we learn that:

> A little Learning is a dang'rous Thing;
> Drink deep, or taste not the Pierian Spring:
> There shallow Draughts intoxicate the Brain,
> And drinking largely sobers us again.

which is very recommendable advice indeed. However, as just a literature survey it's relatively complete, and even though it only has a limited relation to the other parts of the project, the present report was found to be the proper place for its publication.

A more "down to earth" limitation in the survey is that only works dealing with conditions relevant for the Swedish state road network are presented. Very few roads in Sweden are made of Portland concrete cement, making models for this type of road less interesting. In a theoretical context this means that beam and plate theory have been excluded. Also, concrete road are, due to their stiffness, less likely candidates for the RDT. A few exceptions have to be made as these works are of fundamental importance and other, more relevant, works are based on them. The "plate on elastic foundation" problem was, at least for the highway engineering use, first solved by Westergaard [279] in 1926 for a simple Winkler foundation. Westergaard was using results from "recent investigations of stresses in railroad track" and the results could "be applied conveniently by highway engineers for the design of concrete road slabs". The theory was later developed by many researchers. See e.g. Marguerre [193] for a more complete theoretical analysis, and Hogg [125] for the thin plate case. Any elaboration on this would, however, be beyond the scope of the present survey.

All limitations that applies to the static load case applies to the moving loads too. Publications on beam or plate theory will not be reviewed, unless, again, they are very important to other works. See Kerr [157] for a thorough review on beam and plate theory for moving loads.

A moving load will generate fast compressional waves and slower shear waves in the solid on which it travels. Hence, loads can move with speeds higher than the compressional wave speed, in the range between the two wave speeds, or slower than the shear speed. These cases are called "superseismic", "transseismic", and "subseismic", respectively. (Some authors use the terms "supersonic", "transsonic" and "subsonic".) Papers dealing exclusively with the "superseismic" and/or "transseismic" load speeds will be excluded, as these speeds never, or at least extremely seldom, will be of interest for road vehicles.

The quite large research area of suddenly appearing and then radially expanding loads have also been excluded, as it is mainly used to assess the effects of pressure waves from detonations. See Miles [200], Papadopoulus [213, 214], Baron [28] or Gakenheimer (and Miklowitz) [95, 96] for information and further references in this field. Also, the connection between the theory of moving loads and crack propagation will not be covered here. An early example of such a study can be found in a paper by Atkinson [15].

Some general information on moving loads can be found in the vast literature on wave propagation. See e.g. Graff [103], Miklowitz [199], Achenbach [3], Bedford and Drumheller [39], Brekhovskikh [45], Davis [72], Ewing, Jardetzky and Press [88], Hudson [136], or Kennett [155].

## 5.1 Static Loads

The deformations of a semi-infinite elastic solid under the influence of a normal surface point load, and many special cases for distributed loads, was first solved by Joseph Boussinesq [44] in his seminal work "Application des potentiels [à l'étude de l'équilibre et du mouvement des solides élastiques]"[1]. As the title states Boussinesq used potential theory to find a solution to the problem. The original work by Boussinesq is only available in French, and entirely without illustrations on top of that. However, the main findings can be found in any standard book on elasticity or soil mechanics (see e.g. Timoshenko and Goodier [269] or Love [187]). (Boussinesq was a very productive researcher most remembered for his contributions in mathematics, mechanics, and thermodynamics, but his width can be exemplified with the book "Théorie de la bicyclette" published in 1899.)

In 1916 Kwan-ichi Terazawa of the Imperial University in Tokyo published a 64-page paper on the basic theory and many applications dealing "with the problem

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[1] The title, which almost serves as an abstract, actually continues with "principalement au calcul des déformations et des pressions que produisent, dans ces solides, des efforts quelconques exercés sur une petite partie de leur surface ou de leur intérieur".

in the case in which the boundary is subjected to any given normal pressure" [262]. This beautiful paper has, of some reason, not been referred to much at all in the later literature on the same subject. Even today this paper can be read with some interest, even though the material has been covered extensively in later publications.

To be of any real practical use the theory had to be developed from the fundamental semi-infinite elastic solid case to more realistic ones. The case for an elastic layer on a rough rigid base (i.e. with enough friction to prevent slipping between the two layers) was first solved, for plain strain, by Marguerre in 1931 [192].

Biot [40] published solutions, for both plain strain and three-dimensional case of axial symmetry, to the pressure distributions for rigid slippery base, rough rigid base (same results as Marguerre 1931), and a "flexible but inextensible thin layer embedded in the material". Biot's solutions were only concerned with the vertical pressure at the rigid base, and further developed by Pickett [223] in 1938, to "the complete stress analysis". In 1939 Holl [126] used superposition of the earlier solutions for the, in road construction relevant, special case of trapezoidal loads.

In 1943, Burmister presented the solution for stresses and displacements in a two-layer elastic system [48]. The method used is to solve the problem for a surface loading with a distribution $\sigma_z = -mJ_0(mr)$ where $J_n(x)$ is the $n$-th order Bessel function of the first kind. Using Bessel function theory the solution for any real world surface load can be obtained by superimposing many basic Bessel function solutions. For example

$$Y = -q\alpha \int_0^\infty \frac{Y^*}{m} J_1(m\alpha)dm$$

will give the solution for a circular distributed load, where $Y^*$ is the solution a single Bessel load distribution. Judging from the discussion published with the paper, this was very welcome research from the engineering community.

Two years later, in 1945, Burmister published a three part paper entitled "The General Theory of Stresses and Displacements in Layered Systems. I–III" [49]. These papers formalises the 1943 findings by dealing with two-layers systems with and without friction between the layers, in the I and II papers, respectively. The third paper deals with a three-layer system with full friction between layers, and a solution for the surface displacement is given. As Burmister derived closed form solutions the expressions grew very cumbersome in the three-layer case, but in theory this method could be expanded to any number of layers.

In 1948 Fox [91], from the British Road Research Laboratory, used Burmister's method and Southwell's relaxation method to calculate stresses in the lower layer of a two-layer system. One year later Acum and Fox [5, 6] extended this work further to include stresses in all layers in a three-layer system. The relaxation method was not used in the 1949 paper "since the labour of providing results of the required accuracy is prohibitive."

In 1949 the Swedish researcher Nils Odemark [209] published a much used way to simplify the calculations of layered systems. Apparently Odemark was inspired by the Russians Ivanov and Kriviskij who presented a similar method in as early as 1943. Odemark coined the method to *the theory of equivalent thicknesses*, but it's often referred to as the Odemark method. Broadly, the method is used to calculate the equivalent thicknesses of a number of layers of different E-moduli to one layer with a given E-modulus. Thus, the problem is reduced to a single layer problem.

A very thorough study, containing mainly tables and influence diagrams for the stresses and displacements, of the top layer in a two-layer rigid base system was published by Burmister in 1956 [47]. The closed form solutions from the 1945 paper were, in "1500 man-days of work," numerically evaluated by "special computational methods using I.B.M. computing machines and desk calculators in the Watson Scientific Computing Laboratory" [47] in what is likely to be one of the first uses of computers in civil engineering.

As mentioned above the method derived by Burmister could be used for any number of elastic layers with any combination of interface conditions. This is just what Mehta and Veletsos did in 1959 [197], by automating the procedure described by Burmister. A program was written for the ILLIAC computer of University of Illinois. Tabular results for stresses and displacements for up to a four-layer system are given.

Schiffman extended on Mehta's work [234] to allow for more general loading conditions. Apart from the concentrated and distributed load cases considered by Mehta, Schiffman also considered the rigid and tangential loads and the asymmetric "slightly inclined rigid loading".

In chapters 9 and 10 of Sneddon's influential book "Fourier Transforms" [250] from 1951 the Boussinesq point load problem, a few different stiff body indentation problems, and some basic crack analysis for axisymmetric systems are treated. It's clear from Sneddon's work that the Fourier transform is a very powerful tool for the solution of the otherwise very complicated problems of general elasticity, and, as will be seen later in this review, has been frequently employed by other researchers. The solution to the Boussinesq problem is also given by Eason, Fulton and Sneddon [82].

In "Fourier Transforms" Sneddon only briefly considered the problem of asymmetrical loading conditions. In 1960, however, Muki of Keio University in Tokyo generalised and extended the method of integral transforms to the asymmetric case [201]. Many examples of asymmetric cases, as for example "indentation by a slightly inclined flat-ended cylinder", are given in the paper.

Whiffin and Lister [282] made a review of many of the works referred to above for their applicability to pavement design and analysis. A lack of material parameters made evaluation difficult, though, and the authors conclude that "it is not yet possible to present a technique for designing roads from this information", and "[t]he elastic approach outlined in the paper may never give a complete design method /.../".

Using the "correspondence principle" proposed by Lee [172] the solution for the two-layer system with smooth interface from Burmister was extended to a quasistatic viscoelastic material model by Ishihara in 1962 [143]. The basic idea behind Lee's method is, by applying the Laplace transform, to convert the viscoelastic problem to an "associated" elastic problem, which can be solved with conventional theory of elasticity. The solution of the elastic problem (where time and elasticity are replaced with transformed quantities) is then transformed back to the viscoelastic domain. The solution of the two-layer problem was further complemented by Kraft [162] in 1965 to treat the rough interface, partly by using the numerical collocation method developed by Schapery [232] to perform the numerical Laplace transformations.

In 1967, the year of the Second International Conference on the Structural Design of Asphalt Pavements, many researchers presented new findings on viscoelastic models. Ishihara, with coauthor Kimura [142], extended his own work from 1962. The two-layer viscoelastic system was revisited and its applicability to pavement design was considered, using test data from AASHO to fit the theoretical models. The correspondence principle by Lee was used by Ashton and Moavenzadeh [14] for the detailed solution of a viscoelastic three-layer system under circular loads. But instead of using the more numerical method as outlined by Mehta and Veletsos the closed-form solution from Burmister was used, effectively restricting the authors to three layers. Huang [132] presented a very general solution for the stresses and displacements in multi-layered viscoelastic systems under circular loads. For two-layer systems the analytical solution with Laplace transforms was used, and for systems with three or more layers the Laplace transforms are made with the approximate collocation method originally proposed by Schapery, mentioned above. The collocation method was also used by Barksdale and Leonards [26] in 1967 to predict the performance of surface pavements. The theoretical model was used to assess the permanent long-term deformations after many repetitive loads.

Also in 1967, with the arrival of more powerful computers Sanborn and Yoder [231] calculated stresses and displacement under semi-ellipsoidal loads by simply integrating the Boussinesq solution for a point load. The semi-ellipsoidal shape was considered to be a more realistic tyre footprint than the circular shape, and the configuration of a Boeing 707-320 landing gear was used as an example.

In the nineteen-sixties many technical reports and PhD-theses were written on the subject of (surface) deformations in layered systems. These reports have probably been archived long ago, and the library personnel at VTI who usually manage to "dig things up" had no luck with the following reports. They are listed here for completeness, and as they might be of interest to a reader to whom the reports are more readily available.

"Viscoelastic and Thermoelastic Analysis of Layered Systems" (1962) by R. Westmann at University of California [280]. "Analysis of Stresses and Displacements in an N-layered Elastic System Under Uniformly Distributed on a Circular Area" (1963) by J. Michelow at the California Research Corporation [198]. "Numerical Computation of Stresses and Strains in a Multiple-layered Asphalt Pavement System" (1963) by H. Warren and W. L. Dieckmann also at the California Research Corporation [276]. "Theoretical Stress Distribution in an Elastic Multi-Layered Medium" (1964) by M. K. Charyulu at Iowa State University [57]. "Stresses and Displacements in Viscoelastic Layered Systems Under Circular Loaded Areas" (1966) by Yang Hsien Huang at University of Virginia [131]. "Elastic and Viscoelastic Analysis of Layered Pavement Systems" (1966) by R. D. Barksdale at Purdue University [27]. "The Response of Bituminous Mixtures to Dynamic and Static Loads Using Transfer Functions" (1969) by S. Swami at Purdue University [257].

Basic papers on stresses and displacements in layered systems with little or nothing novel continued to be presented for some years. In 1967 Verstraeten [274] published a paper titled "Stresses and Displacements in Elastic Layered Systems" with no references to Mehta, Huang, Schiffman or anyone else who had been working on the same problems. In 1968 Charyulu and Sheeler [58] published five sets of curves for a four-layer system under parabolic loads using the method of Mehta from 1959. Peutz, van Kempen and Jones [221] wrote a Fortran IV program in 1968 to calculate the stresses and displacements under any number of normal circular loads basically using the method given by Mehta ten years earlier. The authors claim that "we succeeded in simplifying the equations involved, which so far were thought to be too complicated to handle", even though Mehta [197], Huang [132] and Barksdale [26] all had published algorithms for multiple layered systems. Also in 1968, Ueshita and Meyerhof [271] published a paper on the surface displacements of an elastic layer on a rigid base, and for an elastic two-layer system under various normal loads. In 1968 and 1971 Thrower at the British Road Research Laboratory published two reports "Calculations of stresses and displacements in a layered elastic structure" [267] and "Calculations of stresses, strains and displacements in a layered elastic structure, part II" [268] with the same material all over again.

New research also continued to be presented, though. In 1968 Huang [133] used the general theory for elastic layered systems to calculate stresses and displacements in nonlinear soil media. Many layers with different elastic properties were used to approximate one layer with elastic properties varying with the depth. A Burroughs B5500 "high-speed computer" was used to get numerical results.

Duncan, Monismith and Wilson [79] introduced the application of the finite element method analysis for pavements. This is, of course, an altogether different approach than the more analytical methods discussed above. Even if the finite element method eventually would provide an almost limitless flexibility, this first test was an axisymmetric model with linear material properties. Some nonlinear material modelling was attempted, but the authors noted them as "preliminary". The calculated results were compared to the deflections measured with the California Traveling Deflectograph (see Section 2.1.1).

In the early nineteen-seventies computer code to calculate stresses and displacements were starting to be publicly available. Pichumani [222] compares three computer programs in 1971 and found the two ones based on the finite element methods (`WIL67` and `AFPAV`) to be both more efficient and economical than the `BISTRO` program based on the Burmister theory.

The finite layer method was introduced to pavement engineering by Cheung in 1979 [61]. In being semi-analytical, the finite layer method allows a full three-dimensional problem to be solved with computer storage requirements as a one-dimensional problem.

From this point and on little has been done on the theory of stratified media for static loads. A vast amount of computer programs for backcalculation of deflection basins from the falling weight deflectometer have been developed. These programs, however, usually use either the finite element method or Burmister's method (or, more recently, artificial neural networks or other more "modern" approaches). Papers published in this field from the mid-seventies are more concerned with the results than the underlying theory.

The basic Boussinesq problem still generates some attention, though. In 1990 Wolf [283] published a very theoretical report titled "The Viscoelastic Boussinesq's Problem", mainly aimed at planetary science. In 2001 Selvadurai [235] published a new way to solve the classic problem "through the use of a Lamé potential." The result was, of course, the same as with other methods.

Lastly, two years ago Becker and Bevis [38] published a paper in which they derived closed form solutions to the deformation under a "uniform pressure applied within a rectangular region", which is referred to as Love's problem.

However, the static load case is only the “forerunner” to the far more complicated moving load case, and again a strophe from Pope’s “An Essay on Criticism” is relevant.

> But more advanc’d, behold with strange Surprize
> New, distant Scenes of endless Science rise!
> So pleas’d at first, the towring Alps we try,
> Mount o’er the Vales, and seem to tread the Sky;

Truly, there seems to be an almost “endless Science” in the dynamics of moving loads.

## 5.2 Moving Loads

In the same way as the static load case started with the findings published by Boussinesq in 1885, the moving load case has it roots in a paper by Lord Rayleigh on wave propagation [228], also published in 1885. In that paper Rayleigh investigated waves propagating at the surface of an elastic solid — what is called Rayleigh waves today — and which is the subject of many papers discussed below. Of some reason, the paper of Lord Rayleigh is not referred to nearly as much as the book by Boussinesq.

A first step to solve the moving load problem was published by Lamb [166] in 1904. Actually, the moving load was not considered by Lamb, but the stationary point impulse load for both the two- and three-dimensional case was thoroughly investigated and solved. Pekeris extended the solution of the impulse load in two much referred to papers [218, 219].

Just as in the case with stresses and deformations under static loads Ian N. Sneddon used Fourier transforms to solve the moving load problem. The basic theory is outlined in section 50 (pp. 444–449) in “Fourier Transforms” [250]. The moving concentrated point force is mentioned as the simplest example of the theory, but “[t]he details of such a calculation are left to the reader.”

However, today we don’t necessarily have to do these calculations, as Sneddon himself produced a number of papers the following years with explicit formulae. Only one year after the publication of Fourier Transforms a more detailed description of the moving load problem was published [251]. Albeit not much for the practising engineer, more details are given, and expressions for the stresses and displacements under a moving stress pulse are given. Yet more details are given in the paper “Quelques solutions des équations du mouvement d’un solide élastique” [252] from 1954, and the problem is thoroughly examined in the 1956 paper by Eason, Fulton and Sneddon [82].

One other early, and very different, solution was given by Criner and McCann [70] in 1953. They solved the beam on elastic foundation problem by building an electric-analog-computer by transferring the mechanical problem to its electric analogy. The solution was very general and allowed for nonuniform beams, velocity variations, and nonlinear properties in the foundation. The major drawbacks of this technique are, of course, that a special purpose machine will have to be built for evaluation, and input/output can be tricky.

During the late nineteen fifties and early sixties a few very influential papers were written. All of these papers are concerned with the basic theory of the moving load. Again, the results are not much for the highway engineer to use directly. Kenney, Jr. [156] considered the constant-velocity moving load on a beam on elastic foundation, and added different types of damping in the elastic subgrade to the analysis. Cole and Huth [66] studied a concentrated vertical line load moving on the surface with constant velocity. Apart from the subseismic case, the transseismic and superseismic cases are considered. Mathews [194, 195] published two articles in 1958–1959 concerning the beam on undamped and damped elastic foundation problem. (Although, as Mathews adds in a postscript, his results were basically the same as those of the previously published Kenney, these papers have continued to be influential.) Ang [10] studied the interior stresses caused by a transient line load moving at a subseismic speed on an elastic half-space. A closed form solution was derived with the use of Fourier and Laplace transforms (and the Gagniard-de Hoop "trick"). Craggs [69] gave a solution for two-dimensional waves for the concentrated transient loads on an elastic half-space. The method of "dynamic similarity" is used, as the use of Fourier transforms for other that steady-state solutions would be, in Craggs' words "to invite mathematical complications." A basic model for deformation under both static and moving loads is presented by Schiffman [233]. Most of the paper deals with viscoelastic theory, and no results of computations are given. Bastiani [29] presented a mathematically complex paper in 1962 at the First International Conference on the Structural Design of Asphalt Pavements. The title "The Explicit Solution of the Equations of the Elastic Deformations for a Stratified Road Under Given Stresses in the Dynamic Case" summarises the content well. However, this paper, which should be of utmost interest for road engineers, has received very little attention, which is probably due to its complexity.

Milton E. Harr [116] wrote a far more accessible paper, also in 1962. Harr used a simple Voigt element to model the viscoelastic road, and used the results to make comparisons with test results from the AASHO Road Test. Harr would later be involved in the development of the first attempt to build a high-speed deflectograph (see Section 2.2.1). Pister and Westmann [225] made the theory of moving loads more accessible for road engineers by treating the beam on viscoelastic foundation problem. A brief analysis of the elastic bilinear case (material with different tensile and compressive moduli) was also presented. In 1963, Thompson [265] used the theory of plate on elastic foundation to analyse the behaviour of roads under loads moving with a constant velocity. A large number of examples with different velocities and damping ratios for the road deflection are given.

In 1965 Eason [83] extended his Fourier transform solution of the moving point load to that of a moving circular disk and rectangle of uniform pressure. Both normal and shear loads are considered. Eason notes that some of the results are "adequate for the type of situations encountered by highway engineers."

Mandel and Avramesco [191] investigated briefly the steady-state deformation under loads moving at low subseismic speeds on elastic half-spaces. A few years later, in 1966, Avramesco [18] treated many different loading conditions for an elastic stratified media. The work is quite theoretical but valuable for the presentation of the basic theory. The main ideas from the 1966 paper were presented (in English) at the Second International Conference on the Structural Design of Asphalt Pavements in Ann Arbour, Michigan, USA in 1967 [19]. If you prefer French a special edition of “Bulletin de liaison des Laboratoires des Ponts et Chaussées” (Decembre 1968) contains basically the same paper again [20].

Payton [217] used the dynamic Betti-Rayleigh reciprocal theorem to solve the problem with a transient point-body force moving on the surface. To give the reader an idea of the mathematical complexity involved (but without defining parameters and variables) a short quote from Payton: “Finally the normal surface displacement is found to be

$$\begin{aligned} w(x,y,t) = -\frac{Q_0}{16\mu}\Bigg\{ \Bigg[ & -2(3)^{1/2}\frac{r_{c_1}}{r_{c_R}^2} - 6\frac{r_{c_1}}{r_{2c_2}^2} + 2(3)^{1/2}\frac{r_{c_1}}{r_{c_2/\beta}^2} \\ & + \frac{(4+8(3)^{-1/2})^{1/2}}{r_{c_R}^2}\frac{N}{(r^2-c_R^2t^2)^{1/2}}H\left(\frac{r}{c_R}-t\right) + 2(3)^{1/2}\frac{N}{r_{2c_2}^2(4c_2^2t^2-r^2)^{1/2}} \\ & - \frac{(-4+8(3)^{-1/2})^{1/2}}{r_{c_2/\beta}^2}\frac{N}{(c_2^2t^2/\beta^2-r^2)^{1/2}}\Bigg] H\left(t-\frac{r}{c_1}\right) + \Bigg[-2(2+3^{1/2})\frac{r_{c_2}}{r_{c_R}^2} \\ & + 2\frac{r_{c_2}}{r_{2c_2}^2} - 2(2-3^{-1/2})\frac{r_{c_2}}{r_{c_2/\beta}^2} + \frac{(4+8(3)^{-1/2})}{r_{c_R}^2}\frac{N}{(r^2-c_R^2t^2)^{1/2}}H\left(\frac{r}{c_R}-t\right) \\ & -2(3)^{1/2}\frac{N}{r_{2c_2}^2(4c_2^2t^2-r^2)^{1/2}} + \frac{(-4+8(3)^{-1/2})^{1/2}}{r_{c_2/\beta}^2}\frac{N}{(c_2^2t^2/\beta^2-r^2)^{1/2}}\Bigg] H\left(t-\frac{r}{c_1}\right) \\ & + \left[-4(3)^{1/2}\frac{r_{c_1}}{r_{c_R}^2} - 12\frac{r_{c_1}}{r_{2c_2}^2} + 4(3)^{1/2}\frac{r_{c_1}}{r_{c_2/\beta}^2}\right] S(\Delta_1) \\ & + \left[-4(2+3^{1/2})\frac{r_{c_2}}{r_{c_R}^2} + 4\frac{r_{c_2}}{r_{2c_2}^2} - 4(2-3^{1/2})\frac{r_{c_2}}{r_{c_2/\beta}^2}\right] S(\Delta_2)\Bigg\}.\text{”} \end{aligned}$$

The general solution for the two horizontal surface displacements $u(x,y,t)$ and $v(x,y,t)$ are even more complicated and cover no less than an entire page each in the printed paper.

In 1966 Niwa and Kobayashi [207] solved and gave numerical examples for the problem of vertical arbitrarily distributed loads on an elastic half-space. Numerical results of some simple examples are presented in order to demonstrate the theory. Subseismic, transseismic and superseismic speeds were all considered in the paper.

The first in-depth engineering analysis of the moving concentrated load was published with Lansing’s NASA report from 1966 [167]. Steady-state solutions for the displacements are presented in a form suitable for numerical evaluation, and many numerical results are presented and discussed. Equations for transient effects were derived for the horizontal surface displacement in the special case of Poisson’s ratio equal to 1/4.

Transient effects of a suddenly applied load is analysed by Payton [216] in 1967. The paper is quite technical with an emphasis on the different techniques needed to solve the necessary Fourier and Laplace transforms.

Lister and Jones [183] presented a paper on the behaviour of flexible pavements under moving loads by treating the pavements as a two and three-layer elastic system. Approximate solutions for the deflection under the moving wheel is given, and the effect of different materials and speeds are investigated in detail. The formulae expressing the deformations are empirical approximations.

Stimulated by findings that the previous assumptions on time dependent material behaviour was "not even approximately correct" Perloff and Moavenzadeh [220] presented a paper on the deflections of a viscoelastic medium due to a moving load. Just as for viscoelastic solutions in the static case the principle of correspondance of Lee [172] is used to find the deflection of the surface. Another paper on the moving load on viscoelastic pavements was published in 1969 by Chou and Larew [62]. Using Burmister's formulae for the static case and the correspondence principle of Lee [172] solutions were derived for one and two-layer systems. The Laplace inversion for more complicated models were done with the collocation method originally proposed by Schapery [232] in 1962. The authors noted that it would be straightforward to use more layers, but that the needed computer time was excessive. A three-layer viscoelastic system was analysed by Elliott and Moavenzadeh [84]. The result for the normal deflection of the surface is presented in detail, and many different road structures are analysed. The authors believed that this kind of viscoelastic analysis was "a step in the right direction" as rate and accumulation effects, duration of loading etc. could be accounted for.

With the aim of assessing the remaining life of a pavement structure Westmann [281] looked into the problem of a moving concentrated load on a viscoelastic plate on an elastic foundation. With the methods presented the qualitative performance of layered systems could be predicted.

Singh and Kuo [249] studied the deformations of an elastic half space to moving circular loads, with "uniform" and "hemispherical" load distributions. Closed form solutions of the vertical deformation was given when the load speed was lower than the shear wave velocity of the medium.

A first step towards a theoretical design approach appeared with the 1972 paper by Ferrari [90]. Ferrari acknowledges that the viscoelastic semi-infinite model he uses is too simple to be used in a production environment. Also, the fatigue behaviour of the bituminous concrete must be further investigated. A method to measure material properties for bituminous roads is also presented.

In 1972 Freund [92] treated the problem with transient loads moving at nonuniform speed, albeit within an infinite elastic solid. A paper by the same author one year later [93] treated the semi-infinite case, for both a concentrated load and a line load. Both papers are quite theoretical, though.

A novel approach is demonstrated by Thrower, Lister and Potter [266] from the British Road Research Laboratory by combining their theoretical studies with experiments. The loading conditions and the road material and construction were carefully controlled by the "RRL Road Machine", and agreement between measured and experimental results were "on the whole good". Vertical deformations due to repeated moving loads were considered in 1976 by Verga, Battiato and Ronca [273]. As for Thrower et al. mentioned above Verga et al. compared their theoretical analysis with experiments. Residual deflections after 1 000, 5 000 and 10 000 load cycles are given in the paper, and they found the conventional elastic theory to underestimate deflection even for one single passage. The results presented in the Verga et al. paper was extended by the same trio of researches on year later [30]. This time most space is given to the laboratory methods, and comparisons between calculated and measured data. About the same results are also presented in [31] from the same year. Battiato and Verga [32] later extended their work to a three-layer system, where the top layer was considered to be viscoelastic and the lower two purely elastic. Both strains at a single passage and the permanent deformations was handled by their programs.

In 1973 Huang [134] extended his earlier work on static loads (see [132]) on multi-layered viscoelastic systems to that of moving loads. (From his papers we can learn that Huang had upgraded from the Burroughs B5500 computer to an IBM 360.) Numerical results are concentrated to compressive stresses and strains on the top of the bottom layer, and tensile strains on the bottom of the top asphalt layer.

A simple viscoelastic model was proposed Alpan and Baker [7] in 1977. They validated the model with experimental data from the AASHO Road Test and used it for road life expectancy evaluations.

Regarding the pavement system as "black box" Baladi [22] used time dependent transfer functions to relate the load input to the deflection output. This empirical/theoretical approach is, of course, limited to the types of roads were input and output data are available. Baladi notes in the PhD. thesis that "[c]hanges in parameters of the [transfer] function reflect changes in pavement performance and conditions." The results were also presented at TRB by Baladi and Herr [23].

In 1972 Ladislav Frýba published the first edition of "Vibration of Solids and Structures under Moving Loads" [94]. A slightly updated edition was published in 1999. The book is focused on problems relating to moving loads on beams and plates. The twenty page long chapter 18 presents a general solution to the elastic half-space problem.

Using theories previously used in seismology Apsel and Luco derived a method to calculate the dynamic response of a layered half-space. Their papers of 1983 [11,188] considered many different loading conditions, including the moving concentrated point force. An integral representation for the complete dynamic displacement field was presented in [188], and an efficient method of numerical evaluation of the integrals was presented in [11]. Some ten years later de Barros and Luco used the same theoretical frame-work to compute the steady-state response of a layered viscoelastic half-space subjected to a moving point load [76], and later for the moving line load [77]. Although the main focus still lies on seismology, the authors note that the model would be suitable for the study of, e.g., traffic induced ground vibrations.

Taking the work of de Barros et al. further, Jones, Le Houédec, Peplow and Petyt [150] and Lefeuve-Mesgouez, Le Houédec and Peplow [173] published papers on ground vibrations due to a harmonic rectangular load, and a harmonic strip load, respectively. A review paper on traffic induced ground vibrations was published by Hung and Yang in 2001 [138].

The Cambridge PhD. thesis by Michael Hardy [112] extends the work of Cebon [56] and Hunt [139], also from Cambridge. Hardy's main interest for the model was to estimate the road damage caused by heavy vehicles. Hence, a random varying load is considered, which was previously not covered in the literature, where the load input could be measured from moving vehicles, or generated from vehicle models. Hardy found the model to predict the strains and stresses in road from moving dynamic vehicles without significant errors. Hardy and Cebon [113] made a comparison of measured data to three theoretical models, namely beam on Winkler foundation, plate on Winkler foundation, and the layered elastic half-space. An impulse response function is used (with the convolution integral technique) to calculate the effect of a moving load. Hardy finds that the more simple beam on Winkler foundation is accurate enough for the "tyre on road" case. In 1993 Hardy and Cebon [114] presented a linear theory to predict the primary response of a road, and validated with experiments on the TRRL test road. One year later the importance of vehicle speed and load frequency on the pavement response was considered by the same authors [115].

To overcome the boundary condition problem in finite element models Pan, Okada and Atluri [211, 212] coupled a boundary element model (BEM) with the FEM. The elastic pavement is modelled with finite elements, while the underlying elastoplastic half-space is modelled with boundary elements. A pure finite element model was presented by Kirkner, Caulfield and McCann in 1994 [160]. The elastoplastic material model made it possible to study permanent deformations. A quasistatic steady-state solution for a load moving at constant velocity is presented. This work was later extended in a paper by Shen and Kirkner in 2001 [236].

More recently, many papers have been written on different aspects on FEM and BEM solutions for moving loads. Most interesting would probably be the work by Yang and Hung [284] on the use of 2.5D finite/infinite elements to get the steady-state response. The same result is produced by Andersen and Nielsen [8] with the use of BEM alone. Within the BEM framework González and Abascal [102] implements the correspondence principle to convert a viscoelastic problem to an elastic problem (as mentioned above).

Raj Siddharthan, with various coauthors, has been researching on moving loads on pavements for more than ten years. Starting in 1991 with Anooshehpoor and Epps [241] a model consisting of a three-layer rubber foam was investigated. No theory validation was performed, though. Two papers, with coauthors Zafir and Norris [243, 244], were published in 1993 and present a viscoelastic finite-layer model dealing with the response of a fluid saturated porous media (based on Biot's formulation) to a moving surface load. The theoretical model was compared to other theoretical solutions and validated with the foam model developed earlier. Results from the finite-layer model has been published in a number of interesting papers. These papers will be described briefly. In 1994 a paper on the longitudinal strain under moving traffic load was published [285]. A field verification with good agreement was published in 1996 [242]. In 1998 a three-dimensional version of the finite-layer program was presented [240]. Results from the 3D version has been presented extensively in recent years [237–239].

In order to evaluate laboratory tests of rut formations on asphalt roads Anders Björklund [41] derived formulae for stress and strain according to the theory of linear viscoelasticity. With the aim to make a computer program useful in everyday engineering Hopman [127, 128] wrote the VEROAD program. Using the results of Björklund and the work of Battiato et al. Fourier transformations are used to remove all time dependencies. The deformations, stresses and strains are solved in the frequency domain, and then transformed back to the time domain. Nilsson, Oost and Hopman [206] validated the VEROAD computer program with full-scale test pavements. Both transversal and longitudinal measured strains were found to agree well with those computed with VEROAD. In 2002 VEROAD was used by authors Nilsson, Hopman and Isacsson [205] to evaluate two different rheological models (Burgers and Huet-Sayegh).

The SAPSI-M computer program by Chatti and Yun [59, 60] is similar to the VEROAD program in functionality. The moving loads are modelled as a series of pulses. The problem is solved using Green's function in the frequency domain, and the final result is presented by an inverse Fourier transform to the time domain. The SAPSI-M have been validated to full-scale tests with "excellent agreement" [60].

Fourier transforms have been used since Sneddon published the first solution to the moving load problem. Analytical solutions can only be found for a few special cases, and numerical methods must be used for realistic cases. In 1997 Marcus Lieb improved the numerical evaluation substantially with the use of wavelet compression, which reduces the computational time about one tenth of the original time required [181, 182]. Using integral transform methods (ITM) further results and theory on the model developed by Lieb have been published more recently by Grundmann, Lieb and Trommer [108] and by Grundmann and Trommer [109].

Traffic induced vibrations in buildings have been investigated by a team of researchers centred at the university at Leuven in Belgium. A sophisticated mathematical vehicle model is used to compute road forces. These forces are then used in a mathematical model to assess the free-field vibrations. The models used are thoroughly described in the PhD. thesis of Geert Lombaert [184], and a series of scientific papers by one or more of Clouteau, Degrande and Lombaert [73–75, 185, 186] present results and validations of the model.

New theoretical results on the moving load problem recently been presented by a group of researchers from the Technical University of Athens. In the first paper by Georgiadis, Vamvatsikos and Vardoulakis from 1999 [101] the dynamic but stationary point load problem is considered. A mathematically very elegant method is presented with which deformations can be obtained, not only on the surface, but also in the sub-surface region. Two mathematically quite demanding papers by Georgiadis and Lykotrafitis explores the theory of moving loads on elastodynamic and thermo-elastodynamic half-spaces were published in 2001 and 2003 [100, 189]. Both papers use the same underlying theory and solution technique of "based on the use of the Radon transform and elements of distribution theory" [100]. Integral solutions for the steady-state are given for a number of problems.

During the last years the models for the deformation of poroelastic materials have been published. Theodorakopoulos [264]; Jin, Yue and Tham [145] and Theodorakopoulos, Chassiakos and Beskos [263] all deal with the dynamic response of poroelastic half-spaces to moving dynamic loads. The full dynamic poroelastic theory of Biot is employed by all authors. The poroelastic material model should be conveniently be applied to road models, where all unbound materials are poroelastic, or at least exhibit poroelastic behaviour.

The basic moving load problem is, more than the basic Boussinesq problem, still an active field of research. Inspired by the Russians Churilov and Sveklo, Barber [25, 64, 65, 256] used the Smirnov-Sobolev technique to derive closed form solutions for the vertical deformation of an elastic half-space for the moving load problem.

Bakker, Verweij, Kooij and Dieterman [21] gave a new and improved solution to the moving point load problem, earlier solved by Gakenheimer and Miklowitz [95]. The new solutions is "more straightforward, thus allowing more insight into this important canonical problem."

More recent contributions to the moving load problem are the work by Hung and Yang [137] on deformations and wave propagation in a viscoelastic half-space under different load conditions. Dynamic displacements and stress response on a viscous Winkler foundation has been studied by Kim and McCullough [158]. The influence of road surface roughness on the dynamic deflections is presented, with very interesting results. Focusing on "the treatment for highly oscillating wave number integration" Liao, Teng and Yeh [278] recently solved the very basic problem of a point load moving with subseismic speeds.

Much work remains to be done before a theoretical model of the deformations caused by the RDT will exist, but the theoretical work as surveyed above is clearly very developed. However, this is a complicated problem, and even if the deformations can be measured very accurately, there will always be only a limited knowledge of the materials in the pavement construction. A word of warning from Pope is, yet again, relevant. We will probably never have the full solution on how to best assess the structural condition of roads, so we will always have "Alps on Alps arise!"

Th' Eternal Snows appear already past,
And the first Clouds and Mountains seem the last:
But those attain'd, we tremble to survey
The growing Labours of the lengthen'd Way,
Th' increasing Prospect tires our wandering Eyes,
Hills peep o'er Hills, and Alps on Alps arise!