DEVELOPMENT OF A FUZZY LOGIC-BASED ADAPTIVE KALMAN FILTER

P. J. Escamilla-Ambrosio, N. Mort

Department of Automatic Control and Systems Engineering
University of Sheffield
Mappin Street, Sheffield S1 3JD, United Kingdom
Tel: +44 (0) 114 222 5619, Fax: +44 (0) 114 222 5661
E-mail: COP99PJE@Sheffield.ac.uk

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Abstract

In this paper, after reviewing the traditional Kalman filter formulation, a development of a fuzzy logic-based adaptive Kalman filter is outlined. The adaptation is in the sense of adaptively tuning, on-line, the measurement noise covariance matrix *R* or the process noise covariance matrix *Q*. This improves the Kalman filter performance and prevents filter divergence when *R* or *Q* are uncertain. Based on the whiteness of the filter innovation sequence and employing the covariance-matching technique the tuning process is carried out by a fuzzy inference system. If a statistical analysis of the innovation sequence shows discrepancies with its expected statistics then a fuzzy inference system adjusts a factor through which the matrices *R* or *Q* are tuned on line. This fuzzy logic-based adaptive Kalman filter is tested on a numerical example. The results are compared with these obtained using a conventional Kalman filter and a traditionally adapted Kalman filter. The fuzzy logic-based adaptive Kalman filter showed better results than its traditional counterparts.

1 Introduction

1.1 Kalman Filtering

The Kalman filter is an optimal recursive data processing algorithm [6] that provides a linear, unbiased, and minimum error variance estimate of the unknown state vector $x_k \in \Re^n$ at each instant k = 1,2,…, (indexed by the subscripts) of a discrete-time controlled process described by the linear stochastic difference equations:

$$x_{k+1} = A_k x_k + B_k u_k + w_k \quad (1)$$

$$z_k = H_k x_k + v_k \quad (2)$$

where x_k is an $(n \times 1)$ system state vector, A_k is an $(n \times n)$ transition matrix, u_k is an $(l \times 1)$ vector of the input forcing function, B_k is an $(n \times l)$ matrix, w_k is an $(n \times 1)$ process noise vector, z_k is a $(m \times 1)$ measurement vector, H_k is a $(m \times n)$ measurement matrix, and v_k is a $(m \times 1)$ measurement noise vector.

Both w_k and v_k are assumed to be uncorrelated zero-mean Gaussian white noise sequences with covariance matrices:

$$E\{w_k w_i^T\} = \begin{cases} Q_k, & i = k \\ 0 & i \neq k \end{cases} \quad (3)$$

$$E\{v_k v_i^T\} = \begin{cases} R_k, & i = k \\ 0 & i \neq k \end{cases} \quad (4)$$

$$E\{w_k v_i^T\} = 0 \qquad \text{for all } k \text{ and } i \quad (5),$$

where $E\{\cdot\}$ is the statistical expectation, superscript T denotes transpose, Q_k is the process noise covariance matrix, and R_k is the measurement noise covariance matrix.

The Kalman filter algorithm is organised into two groups of equations [12],

i) Time update (or prediction) equations:

$$\hat{x}_{k+1}^- = A_k \hat{x}_k + B_k u_k \quad (6)$$

$$P_{k+1}^- = A_k P_k A_k^T + Q_k \quad (7).$$

These equations project, from time step k to step k+1, the current state and error covariance estimates to obtain the *a priori* (indicated by the super minus) estimates for the next time step.

ii) Measurement update (or correction) equations:

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \quad (8)$$

$$\hat{x}_k = \hat{x}_k^- + K_k [z_k - H_k \hat{x}_k^-] \quad (9)$$

$$P_k = [I - K_k H_k] P_k^- \quad (10).$$

These equations incorporate a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

In the above equations, $\hat{x}_k$ is an estimate of the system state vector x_k, and P_k is the covariance matrix corresponding to the state estimation error defined by,

$$P_k = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\} \quad (11).$$

The term $H_k \hat{x}_k^-$ is the one-stage predicted output $\hat{z}_k$, and $(z_k - H_k \hat{x}_k^-)$ is the one-stage prediction error sequence, also referred to as the innovation sequence or residual, generally denoted as *r* and defined as:

$$r_k = (z_k - H_k \hat{x}_k^-) \quad (12).$$

The innovation represents the additional information available to the filter as a consequence of the new observation z_k. The weighted innovation, $K_k[z_k - H_k \hat{x}_k^-]$, acts as a correction to the predicted estimate $\hat{x}_k^-$ to form the estimation $\hat{x}_k$; the weighting matrix K_k is commonly referred to as the filter gain or the Kalman gain matrix.

1.2 Statement of the problem

As described previously, the traditional Kalman filter formulation assumes complete *a priori* knowledge of the process and measurement noise covariance matrices *Q* and *R* [9]. However, in most practical applications these matrices are initially estimated or, in fact, are unknown. The problem here is that the optimality of the Kalman filter estimates are closely connected to the quality of these a priori noise statistics [1, 7]. It has been shown how inadequate initial noise statistics may seriously degrade the precision of the state estimates or even provoke the divergence of the filter [3, 4]. From this point of view it can be expected that an adaptive formulation of the Kalman filter will result in a better performance or will prevent filter divergence.

Different traditional adaptive procedures for the Kalman filter have been devised [4, 8, 9, and 10] since its original formulation [5]. Recently, Mohamed and Schwarz [10] have classified these procedures into two main approaches: innovation-based adaptive estimation (IAE) and multiple-model-based adaptive estimation (MMAE). In the former the adaptation is made directly to the statistical information matrices *R* and/or *Q* based on the changes in the filter innovation sequence. In the second, a bank of Kalman filters runs in parallel with different models for the filter's statistical information. In both techniques the concept of utilising the new information available in the innovation (or residual) sequence is used but they differ in their implementation. In this work only the first approach will be addressed, for the second approach the reader is referred to [1].

The role of matrices *Q* and *R* in the Kalman filter setting is to adjust the Kalman gain in such a way that it controls the filter bandwidth as the state and the measurement errors vary [9]. Direct adaptation of *R* can be attainable via a curve fitting-like procedure while, in general, direct adaptation of *Q* is very hard or impossible to obtain.

In this work the adaptation procedure is concerned with the imposition of conditions under which the filter statistical information matrices *R* or *Q* are adaptively tuned via a FIS which uses the available new information given by the filter innovation sequence. We note that these matrices are considered as constants in the conventional Kalman filter.

The remainder of this paper is organised as follows. Section 2 describes the proposed fuzzy-logic based Kalman filtering approach. Here two cases are explained: when *Q* is known and *R* is adapted, and when *R* is known and *Q* is adapted. To show the effectiveness of this approach an illustrative example is outlined in section 3. The results obtained are compared with those obtained with a conventional Kalman filter and with those obtained with a traditionally adapted Kalman filter. Finally, the conclusions of this work are given in section 4.

2 Fuzzy logic-based adaptive Kalman filtering

In this section an on-line IAE algorithm employing the principles of fuzzy logic is presented. In particular, the technique known as covariance matching [8] is employed. The basic idea behind this technique is to make the actual value of the covariance of the residuals consistent with its theoretical value [10]. If a statistical analysis of the innovation sequence shows discrepancies between its theoretical covariance and its actual covariance then a fuzzy inference system (FIS) adjusts a factor through which the matrices *R* and/or *Q* are tuned, on line, in order to reduce this discrepancy. The adjusting factor is generated by a FIS based on the size of the discrepancy above mentioned.

The general idea explored here is to take advantage of the main characteristics that fuzzy systems have, like the simplicity of the approach, the capability to deal with imprecise information, and the possibility of including heuristic knowledge about the phenomenon under consideration. The next sections will describe the proposed fuzzy logic-based adaptive Kalman filtering approach (FL-AKF).

2.1 Adaptive adjustment of the measurement noise covariance matrix *R* with *Q* known

The measurement noise covariance matrix *R* represents the accuracy of the measurement instrument, meaning a larger *R* for measured data implies that we trust this data less and take more account of the prediction. Assuming that the noise covariance matrix *Q* is known, here an algorithm employing the principles of fuzzy logic has been derived to adaptively adjust the matrix *R*. This is done in two steps; first, having available the innovation sequence or residual r_k (defined by Equation 12), its theoretical covariance is,

$$S_k = H_k P_k^- H_k^T + R_k \quad (13),$$

obtained from the Kalman filter algorithm. Second, if it is found that the actual value of the covariance of r_k has a discrepancy with its theoretical value, then a FIS is used to derive adjustments for *R* based on knowledge of the size of this discrepancy. The objective of these adjustments is to correct this mismatch as well as possible.

Given the availability of the innovation sequence r_k, its actual covariance $\hat{C}_r$ is approximated by its sample covariance [10] through averaging inside a moving estimation window of size N,

$$\hat{C}_{rk} = \frac{1}{N} \sum_{i=i_0}^{k} r_i r_i^T \qquad (14),$$

where $i_0 = k - N + 1$ is the first sample inside the estimation window. This means that only the last N samples of r_k are used to estimate its covariance. The window size is chosen empirically to give some statistical smoothing.

Now, a new variable called the Degree of Matching (*DoM*), is defined to detect the size of the discrepancy between S and $\hat{C}_r$. This is:

$$DoM_k = S_k - \hat{C}_{rk} \qquad (15).$$

The basic idea of adaptation used by a FIS to derive adjustments for R is as follows. It can be noted from Equation 13 that an increment in R will increment S, and vice versa. Thus, R can be used to vary S in accordance with the value of *DoM* in order to reduce the discrepancies between S and $\hat{C}_r$. From here three general rules of adaptation are defined:

1. If $DoM \cong 0$ (this means S and $\hat{C}_r$ match almost perfectly) then maintain R unchanged.
2. If $DoM > 0$ (this means S is greater than its actual value $\hat{C}_r$) then decrease R.
3. If $DoM < 0$ (this means S is smaller than its actual value $\hat{C}_r$) then increase R.

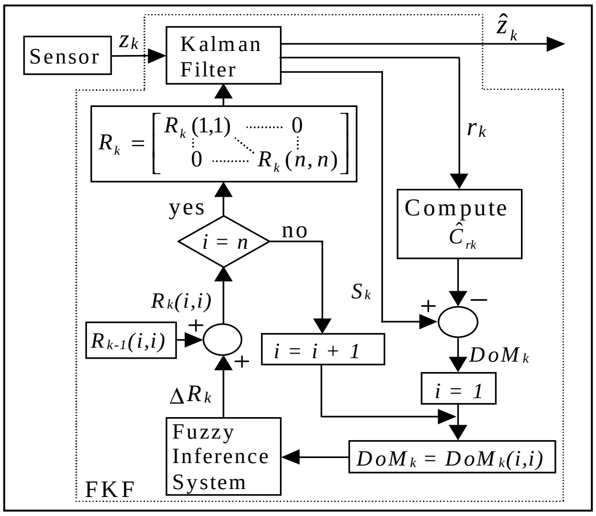

Fig. 1. Graphical representation of the adjusting process of R.

Because there is a direct relationship between the dimensions of the matrices S, R and *DoM*, the adaptation of the (i, i) element of R can be made in accordance with the (i, i) element of *DoM*. Thus, a single-input-single-output (SISO) FIS generates the tuning or correction factor for each element in the main diagonal of R, and the correction is made in this way:

$$R_k(i,i) = R_{k-1}(i,i) + \Delta R_k \qquad (16),$$

where ΔR_k is the tuning factor that is added or subtracted from the element (i, i) of R at each instant of time. ΔR_k is the FIS output and $DoM_k(i,i)$ is the FIS input. The FIS sequentially generates the correction factors for R until all the elements (n in total) in its main diagonal are adjusted. A graphical representation of this adjusting process is shown in Fig. 1.

2.2 Adaptive adjustment of the process noise covariance matrix *Q* with *R* known

The covariance matrix Q represents the uncertainty in the process model. An increase in the covariance matrix Q means that we have less trust in the process model and more in the measurement. Assuming that the noise covariance matrix R is known an algorithm to estimate the matrix Q can be derived.

The idea behind the process of adaptation of Q is as follows. Equation 13 can be rewritten as:

$$S_k = H_k (A_k P_k A_k^T + Q_k) H_k^T + R_k \qquad (17),$$

and from Equation 17 it can be deduced that a variation in Q will affect the value of S. If Q is increased, then S is increased, and vice versa. Thus, if a mismatch between S and $\hat{C}_{rk}$ is observed then a correction can be made through augmenting or diminishing the value of Q. Thus, similar to the previous case, three general adaptation rules are defined:

1. If $DoM \cong 0$ (this means $\hat{C}_{rk}$ and S are equal) then maintain Q unchanged.
2. If $DoM > 0$ (this means $\hat{C}_{rk}$ is smaller than S) then decrease Q.
3. If $DoM < 0$ (this means $\hat{C}_{rk}$ is greater than S) then increase Q.

If the dimensions of matrices S, R and *DoM*, are the same as that of matrix Q, then the same procedure used to adjust R can be used in this case. The adaptation of the (i, i) element of Q is made in accordance with the (i, i) element of *DoM*. Thus, a SISO FIS generates the tuning factor for each element in the main diagonal of Q, and the correction is made in this way:

$$Q_k(i,i) = Q_{k-1}(i,i) + \Delta Q_k \qquad (18),$$

where ΔQ is added or subtracted from each element of the main diagonal of Q at each instant of time. ΔQ is obtained from a FIS were $DoM(i,i)$ is the FIS input. Thus the FIS sequentially generates the correction factor for the (i, i) element of Q. The graphical representation of this adjusting process is the same as that shown in Fig. 1, but replace an R with a Q.

A more difficult task is that of adjusting Q when there is not a direct correspondence between the dimension of the matrices S, Q and *DoM*. However, as it will be shown in the illustrative example, some empirical considerations can be used by a FIS to successfully overcome this problem.

3 Illustrative example

To demonstrate the efficiency of the fuzzy logic-based adaptive Kalman filtering (FL-AKF) approach, a simple numerical example is presented. The results are compared with those obtained with a traditional Kalman filter without adaptation (TKF) and a traditional adaptive Kalman filter (TAKF).

Consider the following linear system, which is a modified version of a tracking model [2, 11],

$$\begin{bmatrix} x_{k+1}^1 \\ x_{k+1}^2 \\ x_{k+1}^3 \end{bmatrix} = \begin{bmatrix} 0.77 & 0.20 & 0.00 \\ 0.25 & 0.75 & 0.25 \\ 0.05 & 0.00 & 0.75 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} + \begin{bmatrix} w_k^1 \\ w_k^2 \\ w_k^3 \end{bmatrix} \quad (19a)$$

$$z_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} + v_k \quad (19b)$$

with initial conditions $\hat{x}_0 = 0$, $P_0 = 0.01\, I_3$, where x^1, x^2, and x^3 are the position, velocity and acceleration, respectively, of a flying object. In Equation 19, the system and measurement noise sequences $\{w_k\}$ and $\{v_k\}$ are pseudorandom sequences (i.e., uncorrelated zero-mean Gaussian white noise sequences) with covariance matrices:

$$Q = 0.02\, I_3 \ , \ R = \begin{bmatrix} 0.2 & 0 \\ 0 & 15 \end{bmatrix} \quad (20).$$

MATLAB code was developed to simulate the FL-AKF, the TKF and the TAKF. The position and velocity were considered as the characteristics being measured. The simulation and results obtained are presented in the following sections.

3.1 Adaptation of *R* and comparisons

Following the guidelines given in section 2.1, five fuzzy sets have been defined for *DoM*: *NM* = Negative Medium, *NS* = Negative Small, *ZE* = Zero, *PS* = Positive Small, and *PM* = Positive Medium. In a similar way five fuzzy sets have been defined for *ΔR*: *IL* = Increase Large, *I* = Increase, *M* = Maintain, *D* = Decrease, and *DL* = Decrease Large. Finally, five rules comprise the SISO FIS rule base,

1. If *DoM* = *NM*, then Δ*R* = *IL*
2. If *DoM* = *NS*, then Δ*R* = *I*
3. If *DoM* = *ZE*, then Δ*R* = *M*
4. If *DoM* = *PS*, then Δ*R* = *D*
5. If *DoM* = *PM*, then Δ*R* = *DL*.

The membership functions for *DoM* and Δ*R* are presented in Fig. 2.

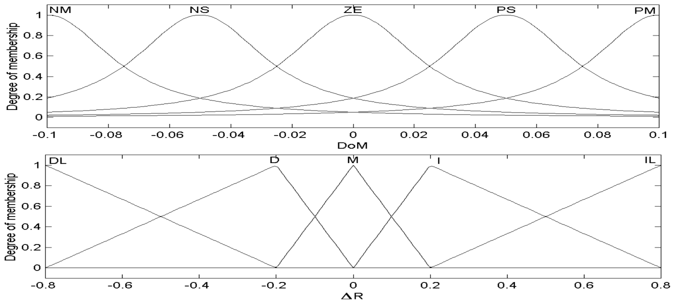

Fig. 2. Membership functions for *DoM* and Δ*R*.

The model described by Equation 19 was simulated for 500s with a sample time of 0.5s. *Q* was fixed at its actual value, $0.02I_3$. The actual value of *R* has been assumed unknown, but its initial value was selected as:

$$R_0 = \begin{bmatrix} 2.5 & 0 \\ 0 & 5.0 \end{bmatrix} \quad (21).$$

The values in the main diagonal of *R* are continuously adjusted as described in section 2.1.

For comparison purposes the following performance measures were adopted:

$$J_1 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (z_i - zv_i)^2} \quad (22)$$

$$J_2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (z_i - ze_i)^2} \quad (23),$$

where z_i is the actual value of the characteristic being measured (without noise); zv_i is the measurement (with noise added); and ze_i is the estimation of the characteristic being measured.

To make comparisons, the traditional adaptation procedure proposed by Mohamed and Schwarz [10] was used to adapt *R* and *Q*. In this method *R* or *Q* are estimated using the following equations,

$$\hat{R}_k = \hat{C}_{rk} - H_k^T P_k^- H_k^T \quad (24)$$

$$\hat{Q}_k = K_k \hat{C}_{rk} K_k^T \quad (25),$$

where $\hat{C}_{rk}$ is obtained with Equation 14. H_k^T, P_k^-, and K_k are those obtained in the Kalman filter algorithm.

Table 1 shows the performance measures obtained for each of the following cases:

1. TKF-ANS, traditional Kalman filter considering the actual values of the noise statistics (matrices *R* and *Q*).
2. TKF, traditional Kalman filter with incorrect fixed value of *R* and actual fixed value of *Q*.
3. TAKF, traditionally adapted Kalman filter with initial incorrect value of *R* and actual fixed value of *Q*. Here *R* is adaptively tuned using a traditional technique.

4. FL-AKF, fuzzy logic-based adaptive Kalman filter with initial incorrect value of *R* and actual fixed value of *Q*. Here *R* is adaptively tuned using a FIS.

From experimentation, it was noticed that the best results were obtained with a window size of 50 samples for cases 3 and 4.

Table 1

Performance measure	TKF-ANS	TKF	TAKF	FL-AKF
J_1 - *Position*	0.4490	0.4490	0.4490	0.4490
J_1 - *Velocity*	3.8512	3.8512	3.8512	3.8512
J_2 - *Position*	0.2256	0.3688	0.3020	0.2274
J_2 - *Velocity*	0.2914	0.4168	0.3665	0.2933

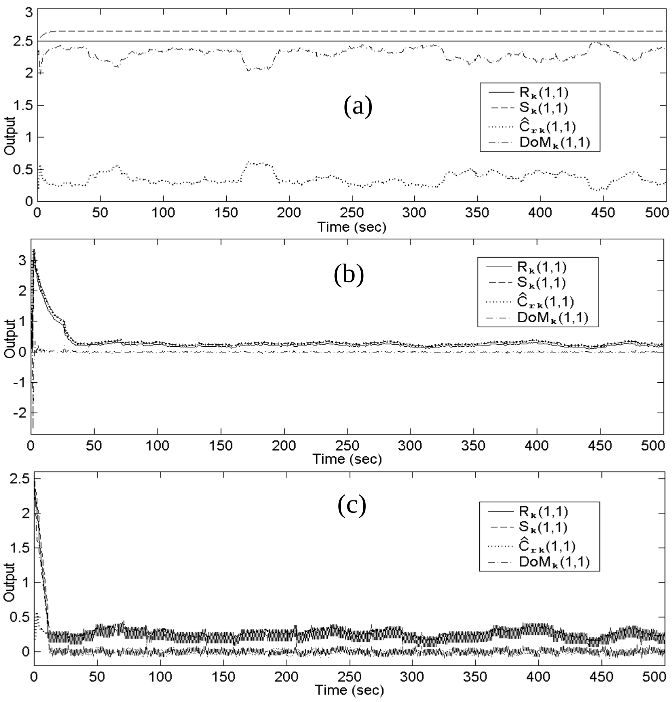

Fig 3. (a) Outputs for case 2; (b) Outputs for case 3, and (c) Outputs for case 4.

Analysing the data in table 1 it is deduced that the best estimation of the position and velocity of the flying object is that obtained with the FL-AKF as it has the best performance measures for both parameters. A more explicit performance measure is obtained calculating the percentage of degradation in the estimation with respect to the estimation obtained with the traditional Kalman filter (TKF) considering the actual noise statistics (case 1). These percentages are shown in Table 2. It is remarkable that only a slightly degradation in the estimations is observed in the FL-AKF (case 4). As expected, the worst degradation is observed in the TKF without adaptation (case 2). An intermediate level of degradation is observed in the TAKF (case 3).

Figure 3 shows the plots of the elements (1,1) of each matrix: *R*, S_k, $\hat{C}_{rk}$, and *DoM*. This data was obtained for cases 2, 3 and 4 described previously. In Fig. 3(a), corresponding to case 2, a discrepancy can be seen between the actual covariance of the residual $\hat{C}_{rk}$ and its theoretical value S_k. Because both *Q* and *R* are fixed *DoM* remains a large value. In Fig. 3(b), corresponding to case 3, a continuous adjustment of *R* can be observed. In this case *DoM* remains at a very small value while *R* almost reaches its true value. This explains the filter performance improvement. In Fig. 3(c), corresponding to case 4; similarly to the previous case, a continuous adjustment of *R* is noted; *DoM* oscillates around zero and *R* oscillates around its true value. Because we are dealing with uncertain information about the noise statistics, this oscillation around the true values explains the better estimation obtained in this last case.

Table 2

Percentage of degradation on the estimation of:	TKF-ANS	TKF	TAKF	FL-AKF
Position	0%	63.5%	33.9%	0.8%
Velocity	0%	43.0%	25.8%	0.65%

3.2 Adaptation of *Q* and comparisons

Following the guidelines given in section 2.2, five fuzzy sets have been defined for *DoM* and five for ΔQ with the same labels but different membership functions than those in the previous case (see Fig. 4). Thus, five fuzzy rules comprise the rule base,

1. If $DoM = NM$, then $\Delta Q = IL$
2. If $DoM = NS$, then $\Delta Q = I$
3. If $DoM = ZE$, then $\Delta Q = M$
4. If $DoM = PS$, then $\Delta Q= D$
5. If $DoM = PM$, then $\Delta Q = DL$.

The model described by Equation 19 was simulated for 500s with a sample time of 0.5s. In this case *R* was fixed as its actual value. The value of *Q* has been assumed to be unknown, but its initial value was selected as,

$$Q_0 = 10Q \qquad (26).$$

In this case a direct correspondence between the dimension of the matrices *R*, *S*, *Q* and *DoM* does not exist; all matrices, except *Q* are of dimension 2 x 2 (only the position and velocity are being measured). To overcome this problem a heuristic consideration was incorporated to the tuning process. This consists of estimating the component (3, 3) of *DoM* as:

$$DoM_k(3,3) = [DoM_k(1,2)]^2 \qquad (27),$$

this value then enters the FIS and the procedure of adjustment continues as specified.

Table 3 shows the performance measures obtained for each of the following cases:

1. TKF-ANS, traditional Kalman filter considering the actual values of the noise statistics (matrices *R* and *Q*).
2. TKF, traditional Kalman filter with incorrect fixed value of *Q* and actual fixed value of *R*.

3. TAKF, traditionally adapted Kalman filter with initial incorrect value of Q and actual fixed value of R. Here Q is adaptively tuned using a traditional technique.
4. FL-AKF, fuzzy logic-based adaptive Kalman filter with initial incorrect value of Q and actual fixed value of R. Here Q is adaptively tuned using a FIS.

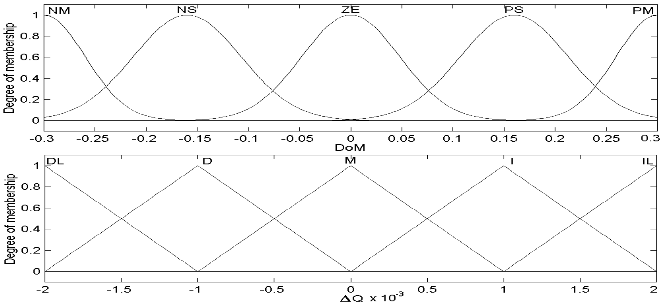

Fig. 4. Membership functions for *DoM* and ΔQ.

Table 3

Performance measure	TKF-ANS	TKF	TAKF	FL-AKF
J_1 - *Position*	0.4490	0.4490	0.4490	0.4490
J_1 - *Velocity*	3.8512	3.8512	3.8512	3.8512
J_2 - *Position*	0.2256	0.2990	0.2495	0.2433
J_2 - *Velocity*	0.2914	0.3410	0.3243	0.3141

Table 4

Percentage of degradation on the estimation of:	TKF-ANS	TKF	TAKF	FL-AKF
Position	0%	32.5%	10.6%	7.8%
Velocity	0%	17.0%	11.3%	7.8%

From the analysis of the data in table 3 it is seen that, in this case, the improvement of performance obtained with both TAKF and FL-AKF is almost the same, but they are significant with respect to the non-adaptive case. This is more explicit in the data of table 4. As can be seen the degradation in the estimation for both adaptive cases is between 7% and 11%. However the estimation in the performance in the TKF is degraded by up to 32.5%.

4. Conclusions

In this paper a fuzzy logic-based adaptive Kalman filter was presented (FL-AKF). The adaptation, carried out by a fuzzy inference system, is in the sense of adaptively tuning the measurement noise covariance matrix R or the process noise covariance matrix Q. An example showing the efficiency of this method was presented. It is remarkable that only five rules were needed to carry out the adaptation in each case. The analysis of the results from the example presented show that a greater improvement in Kalman filter performance is obtained with the FL-AKF when compared with its traditional counterpart (TAKF). Also, they show that the adaptation of Q is harder to obtain. Nevertheless the improvement on the Kalman filter performance, for this last case, is not as good as that for R, it is comparable to that obtained with its traditional counterpart.

The main advantage of the proposed FL-AKF is the weaker reliance on the *a priori* statistical information. In this adaptive formulation, the a priori statistical information is only of secondary importance because the new measurement and process noise covariance matrices are adaptively tuned according to the new information given each instant of time by the Kalman filter innovation sequence. This information is successfully manipulated by a FIS to carry out the adaptation procedure. Thus, the good results obtained in the illustrative example show that the general idea of exploring the use of fuzzy logic approaches to achieve adaptive Kalman filtering appears to be a promising avenue of investigation.

The FL-AKF approach shares the problem of fuzzy systems about the lack of tools that allow its mathematical analysis and demonstrations. One way of overcome this problem is to benchmark the fuzzy logic-based approach against some representative traditional scheme, like was done in this work; that do allow some confidence on the approach.

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