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Development of an Excel® Rocket Simulator for Application in Middle School, High-School, and University STEM Education

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DEVELOPMENT OF AN EXCEL® ROCKET SIMUATOR FOR APPLICATION IN MIDDLE SCHOOL, HIGH SCHOOL, AND UNIVERSITY STEM EDUCATION

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ABSTRACT

DEVELOPMENT OF AN EXCEL® ROCKET SIMULATOR FOR APPLICATION IN MIDDLE SCHOOL, HIGH SCHOOL, AND UNIVERSITY STEM EDUCATION

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Water rocket activities are one of the most popular STEM activities used in primary, secondary, and higher education yet are void of engineering, though engineering is heavily implied in the STEM acronym. This study investigated the amount of engineering present in water rocket activities, and options for emphasizing engineering more in water rocket activities using an open-platform flight simulator for use by educators to enable students to predict flight parameters of a water rocket they designed, and test those predictions against experimental data. The simulator was constructed in Excel® with many functions, but the function validated in this study was the prediction of maximum height. The simulator was able to predict maximum height of a water rocket at specific input parameters within 5.773% with 95% confidence using a calibration factor to account for unknown sources of error. Further validation of the simulator at other input parameters is needed to ensure the calibration factor enables the accurate prediction of maximum height with varied input parameters, as is common occurrence in STEM water rocket activities.

Keywords: water rockets, water rocket experimental data, simulator validation.

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CHAPTER I

INTRODUCTION

The science, technology, engineering, and math disciplines have been combined into integrated educational programs for the past two decades. *STEM* denotes the combined curricula (Marshall, 2015). It has been the aim of integrated STEM programs to ensure students gain top-level proficiency in STEM subjects through student and teacher interaction from preschool to university levels better preparing students for the STEM workforce (U.S. Department of Education, 2015). The U.S. government and the world of education sees students pursuing STEM careers as a necessity for the continued progression of the United States, especially in the areas of economy and technological advancement.

This progression is dependent on the integration of STEM because most technical positions require all four disciplines. Engineers require expertise in physics (science), instrumentation (technology), mechanics (engineering), and differential equations (math). Medical scientists require expertise in biology, testing technology, materials engineering, and linear algebra. Mathematicians require the ability to apply newfound mathematical theories and techniques to solve problems in science and engineering, and use sophisticated technology to do so. Production managers require expertise in using scientific experimentation, manufacturing equipment, engineering decisions, and mathematical control systems to improve production efficiencies. Technical professionals need to understand STEM on all levels.

To integrate STEM in the classroom, well-structured guidelines and standards have been created by the International Technology and Engineering Educators Association to facilitate technological literacy and readiness for STEM careers (Woodruff, 2013). These guidelines and standards assist the formulation of tools and structured activities for use in the classroom specific to education level. Among the most popular and broadly used are water rocket activities, the topic of this study.

This section addresses the context of the study. This includes the context of the problem, statement of the problem, null hypotheses, limitations, delimitations, assumptions, significance of the study, and research objectives pertaining to the topic of STEM water rocket activities.

The Context of the Problem

Water rocket activities used in K-12 and university educational programs often consist of 3 phases: defining concepts, rocket design and construction, and rocket flight-testing. The students learn rocket flight principles, such as Newton's laws and fluid mechanics. Then they design their own water rocket using plastic bottles and other low cost home items. With their design, they either predict the flight characteristics (depending on the decided testing arrangement) or determine the method of gathering the flight characteristics of their rocket during testing.

Water rocket activities are popular in STEM educational programs due to their ability to cover the full spectrum of STEM disciplines. Discussing the physics involved in rocketry, such as Newton's laws of motion, covers the science portion. Investigating the different tools used to make rockets aerodynamically stable, such as fins, ballasts, and nose cones, introduces students to technology. Activities often include computer programs for predicting rocket flight characteristics as well. Teaching design principles for constructing rockets and predicting rocket flight characteristics using the disciplines of fluid mechanics and dynamics easily covers engineering. Finally, determining the maximum height a rocket achieved during a flight incorporates trigonometry into the activity.

CHAPTER I

Although STEM water rocket activities have the capability to cover the full spectrum of STEM disciplines, they tend to ignore the engineering discipline. Designing the water rocket, if it can be called that without numbers being involved, may pass as engineering in elementary STEM programs, but cannot suffice for middle, high school or university programs.

Some have attempted to input an engineering and additional technology component of learning into water rocket activities using water rocket flight simulators (National Aeronautics and Space Administration, 2015c; De Podesta, 2007). However, the simulators still fail to offer appropriate engineering components to STEM water rocket activities. Since there is a lack of on-screen explanation of engineering principles, no indication of what computations produce outputs, and no definition of user-specified inputs, the user can easily make errors in input specification. The most significant reason for simulator failure, however, is the lack of validation of all water rocket simulators by experimental testing. Though the simulators predict water rocket flight characteristics based on user-specified inputs, the accuracy or precision of the simulators has scarcely been tested using standard experimental testing methods. Thus, the simulators are prone to error making them illegitimate sources for engineering predictions. All three of these factors severely limit the students' learning of engineering principles involved in water rocket activities.

Though water rocket simulators currently fail to offer engineering components to STEM water rocket activities, they are worthy candidates for use in middle school, high school, and university STEM education programs, the focus of this study.

CHAPTER I

Statement of the Problem

This study covers the creation and validation of a water rocket simulator. The author specifically cares about its ability to predict maximum height of water bottle rockets with specified input parameters for use in STEM water rocket activities.

Hypotheses

The *null* hypothesis for the study is as follows: The developed Excel® water rocket simulator will not predict maximum theoretical heights of water bottle rockets with specified input parameters within ten percent of experimental maximum heights with 95% confidence. In statistical form,

$$10\% \geq \bar{X} + 1.65s$$

Limitations and Delimitations

The limitations of the study all pertained to the limitations of the testing equipment used and extent of environmental control. Below are the limitations in detail.

- Air pump pressure gauge accuracy and precision: The air pump used for the launch testing portion of the study was a Specialized® pump equipped with a pressure gauge of unknown accuracy. The precision of the pressure gauge equipped with the air pump was one psig.
- Altimeter accuracy and precision: The altimeter used for the launch testing portion of the study was a Jolly Logic® Altimeter having a resolution of one foot and an accuracy of plus or minus two inches according to John Beans, the president of Jolly Logic (personal communication, October 6, 2016). The precision and accuracy limit the study because the

simulator validated by the study had a resolution of one thousandth of a meter, much finer than that of the altimeter. This limited the experimental height values collected during the launch testing portion to ranges.

- Launch setup: The launch setup used for the launch testing portion (first method) of the study was a nonstandard launcher that did not have a secure plug to prevent loss of water from the rocket's water storage during launch setup, and did not have an angle setting function. Because the launch setup did not prevent the loss of water, the actual volume of water in the rocket's water storage had a range. This was not the case for the second method. However, because both launch setups did not make the rocket rigid, the angle of the rocket at launch had a range of 5 degrees from the axis of the launcher. These uncontrollable inconsistencies added inconsistency to the results of the launch data, placing ranges on water volume and launch angle inputs for the first method, and launch angle inputs for the second method.
- Camera frame precision: The camera used for experimentally determining the drag coefficient of the water bottle rocket used in the launch testing portion of the study was a Canon® digital camera capable of recording video at 24 frames per second. The testing operator used the camera to time the duration of the water bottle rocket's fall from a specified height. Because the camera only had a resolution of 24 frames per second, the recorded durations of the rocket's fall from the specified height had a range of 83.3 milliseconds. A different camera was used, capable of 120 frames per second, for the second method of launch testing for acquisition of flight duration times. The range associated with this camera was 16.7 ms.

- Graduated cylinder precision: The graduated cylinder used during the launch-testing portion of the study to fill the water storage chamber of the water bottle rocket had a resolution of 10 milliliters. This resolution limited the total resolution of the launch testing data by applying a range to the water volume input of the water bottle rocket, expanding the total range of the launch testing data.
- Outside environment: The launch testing portion of the study took place outdoors in an uncontrolled climate and environment. Because of this, although the testing operator collected environmental data, the operator did not consider the effects of wind if wind speeds were higher than 5 mph. If wind speeds were higher than 5 mph, the operator postponed testing until wind speeds decreased.

The methods of the study considered the limitations listed above to ensure external validity of the study, with the sacrifice of internal validity, considering the limitations apply to the majority of STEM water rocket activities.

Below are listed the delimitations of the study that describe the scope and refine the purpose of the study.

- Water bottle rockets: The designed simulator predicted the performance of water bottle rockets due to the nature of its intended use. The simulator was intended for use in middle, high school, and university STEM educational activities. Most STEM activities are performed with low resource accessibility therefore disqualifying the use of hard-to-acquire resources such as actual fuel propulsion rocket equipment. Water bottle rockets are very cheap to construct and the materials are easily accessible anywhere in the United States.

- Height prediction validation: Although the simulator performs many functions and predicts many flight and design parameters, the study only validated the height prediction function. The purpose was to maintain a reasonable project schedule and focus on maximum attained height as the most valuable result in nearly all STEM water rocket activities.
- One simulator: The study considered only one simulator for construction and validation. All STEM water rocket activities require only one simulator for flight parameter prediction. The simulator utilized the open platform of Microsoft Excel®.
- Middle, high school, and university education: The study considered the three educational levels because of the lack of depth in the engineering discipline of STEM water rocket activities currently present. The simulator was designed to expand the depth of coverage of the engineering discipline in STEM water rocket activities. The level of engineering content in elementary-level water rocket activities is more than sufficient and graduate level education is outside of the scope of integrated STEM programs in the United States.
- Simulator functionality: The simulator contained ballast and body design aids, and predict height, velocity, acceleration, impulse, thrust force, drag force, and weight of the rocket at any time during the course of its flight. The simulator was also capable of predicting stress states of the rocket under its maximum pressure condition.
- The study took place in Ellensburg and Seattle Washington during the 2016 and 2017 year.

Assumptions

The assumptions considered over the course of the study were as follows:

- All testing equipment operated within their respective design parameters.
- The testing environments and climates effect on all recorded testing data of the study was negligible.
- The results of all testing were not influenced by any fluke phenomena.
- Successive trials of testing did not modify the aerodynamic parameters of the water bottle rocket, including heavy impacts between the rocket and the floor of the testing environment.
- The ranges that determined the design parameters of all testing equipment that did not have manufacturer’s design parameters, as judged by the author, were reasonable and approximate.
- Successive trials of testing did not affect the design parameters of all testing equipment during launch testing.

Significance of the Study

The outcomes of the study have vast implications for the future quality of STEM water rocket activities. With successful validation of the simulator, the STEM education community has access to an Excel® program adding significant engineering merit and technology merit to existing water rocket activities for use in all education levels, but especially middle, high school, and university levels. Being able to have students design a water rocket with optimum pressure and water volume parameters to achieve maximum height puts them in the engineer’s seat exposing them to career-level engineering design. Having the simulator in the Excel® platform allows the simulator to be free of charge and 100% reproducible for revalidation and improvement of the simulator. The outcomes of the study also open doors for expansion of the simulator to cover other types of rockets such as fuel propellant rockets and future commercial use of the simulator. Existing water rocket simulators currently do not offer these opportunities.

Research Objectives

The primary objective of the research study was to validate a simulator for the STEM teaching community to enhance the engineering and technology portions of water rocket activities used in the classroom. Alternative objectives were to develop a simulator program that was explanatory of engineering concepts and definitions present in the program to ensure enhancement of student engineering knowledge and clarity of required input variables for the program to deliver appropriate output variables desired by the user. Another alternative objective was to develop within the simulator a suitable means of determining drag coefficient data for designed rockets.

CHAPTER II

LITERATURE REVIEW

The literature review discusses a synopsis of water bottle rocket activities used in STEM education, the evidence of engineering within water bottle rocket analysis, how engineering can be emphasized in the activities using simulator programs, and the testing methodologies used for validating simulation models. This allows the reader to comprehend the nature, necessity, and related work of the study. The review is limited to education levels from middle school to undergraduate university.

Synopsis of Water Bottle Rocket Activities

All water bottle rocket activities used in STEM education have a common format as discussed in Chapter I. They often consist of three phases: discussion of engineering principles, rocket design and build, and launch testing.

The discussion of engineering principles is often brief and limited. Principles of aerodynamics and Newtonian physics are discussed with students and enforced by worksheets containing review questions and problems (Todd, Riskowski, Butler, & Skinner, 2007). Educators connect these principles to design decisions for rocket construction in order to explain to the students that rocket shape and construction heavily influence aerodynamic and drag characteristics.

The design and construction of the water bottle rocket simply consists of the students constructing the rockets. The students are provided with water bottles, tape, glue guns, cardboard, and/or foam to construct their rockets and are allowed creative freedom (Institute of

Electrical and Electronics Engineers, 2016; National Aeronautics and Space Administration, 2016; De Podesta, 2007; Todd, et al., 2007). Figure 1 shows a typical water rocket.

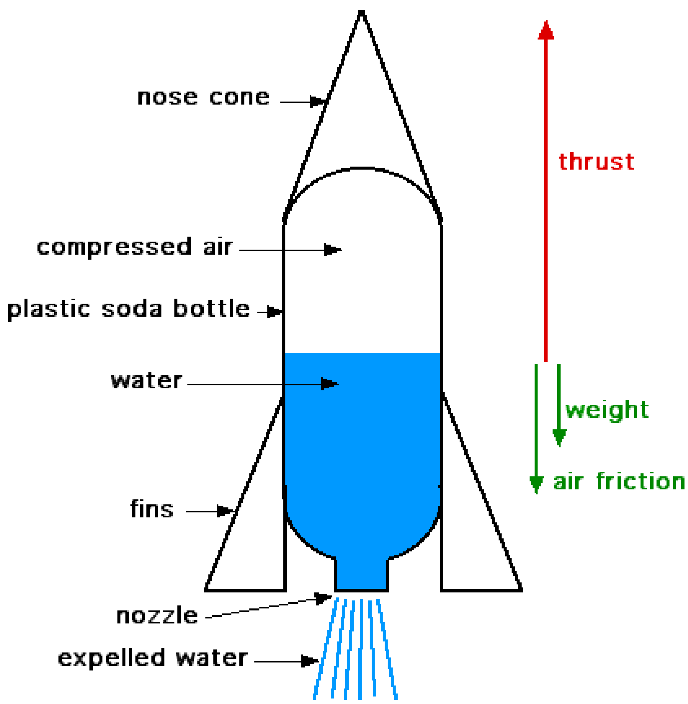

Figure 1. Typical water bottle rocket. (Halliday & Foley, 2016).

Students usually carve the fins from cardboard and tape or glue them onto the water bottle that holds the water and compressed air. A nose cone is made from another water bottle or a piece of paper and taped or glued onto the top of the former water bottle. The water bottle orifice serves as the rocket nozzle for the water bottle rocket (Institute of Electrical and Electronics Engineers, 2016; National Aeronautics and Space Administration, 2016; De Podesta, 2007; Todd, et al., 2007).

The launch testing phase of water bottle rocket activities consists of launching the rockets using a water bottle rocket launcher, as seen in figure 2, and collecting flight characteristic data using trigonometry, flight duration data, or an altimeter (Institute of Electrical and Electronics

Engineers, 2016; National Aeronautics and Space Administration, 2016; De Podesta, 2007; Todd, et al., 2007).

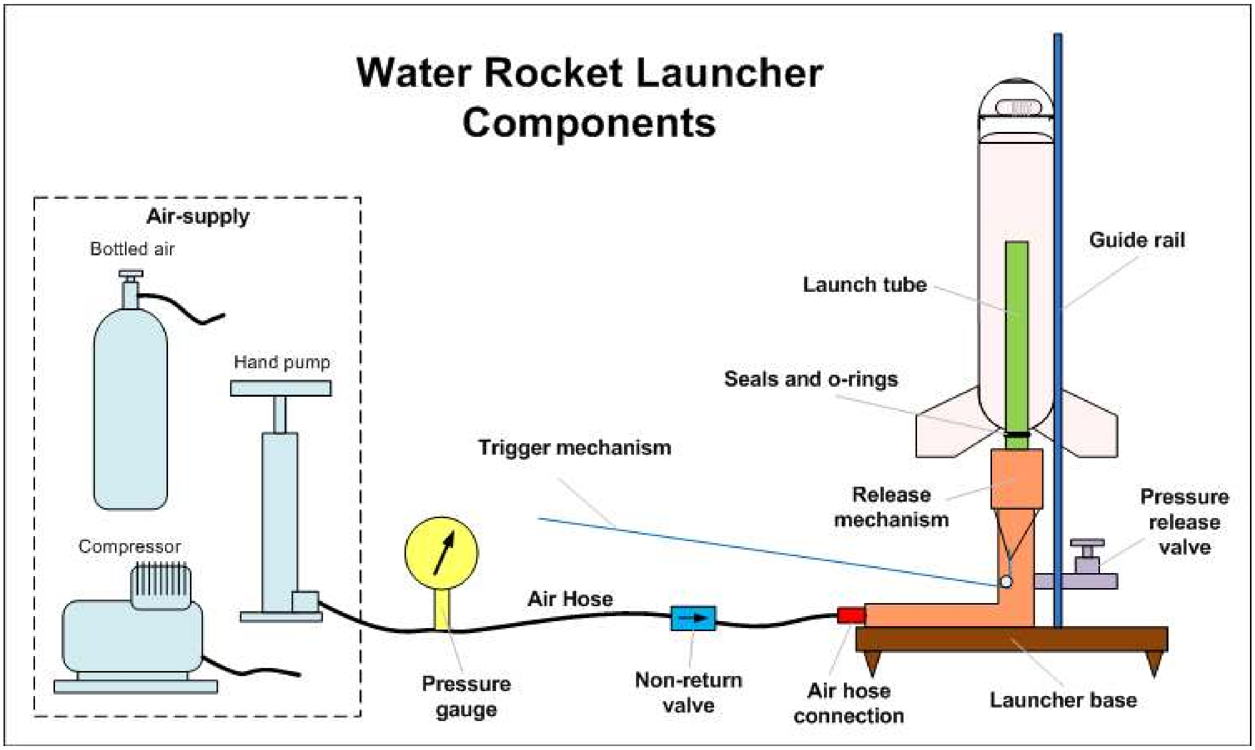

Figure 2. Typical water bottle launch setup. (Air Command Rockets, 2016).

Students record the distance between the rocket's launch point and landing point along with the launch angle for trigonometric derivation of the rocket's maximum height. Alternatively, the students may time the duration of the rocket's flight stopwatches from launch trigger to landing and correlate the duration with maximum height. An altimeter can also be stored in a payload area of the water bottle rocket to record maximum height of the rocket's flight. All of these different actions are common in STEM water rocket activities, but none of these actions demonstrates to the student how the rocket flight parameters relate to input parameters quantitatively. None of these activities involves engineering. The next section explains the engineering principles within water bottle rocket analysis so that the reader can articulate an understanding of the potential emphasis of engineering within water bottle rocket activities.

Evidence of Engineering in Water Rockets

Rocket flight is possible by three fundamental laws of physics: Newton's second law of motion, Newton's third law of motion, and the law of the conservation of energy. Newton's second law states that an unbalanced force acting on a mass will produce an acceleration in the direction of the force and proportional to the force. Newton's third law states that for every action, there is an equal and opposite reaction. The law of the conservation of energy simply states that all energy is conserved; energy cannot be created or destroyed.

What Newton's second law entails for rocket flight is that when there is an unbalanced sum of forces acting on a rocket, an acceleration a is produced. The forces that often act on a rocket mass m are the weight W, drag force F_D, and thrust force T_h. The thrust force has to be greater than the combined value of the weight and drag forces in order for rocket flight to take place. The mathematical relationship is as follows,

$$\Sigma F_y = T_h - F_D - W = ma. \tag{1}$$

With methods for obtaining thrust, drag forces, weight, and mass, equation 1 may be solved for acceleration at any time interval t. When acceleration is known, velocity and position of a rocket may be determined.

Drag Forces

The drag forces are characterized by the frictional forces that are caused by the sliding motion between the body of the rocket and the surrounding fluid and the resistance of the surrounding fluid to be displaced by the mass of the rocket (Cengel, Cimbala, & Turner, 2012; National Aeronautics and Space Administration, 2015d). Drag force is dependent on the density of the surrounding fluid ρ, the velocity of the mass u, the cross sectional area of the mass A, and the drag coefficient C_D which is determined experimentally. The drag coefficient represents the effect of fluid flow, mass shape, roughness of the mass surface, and mass orientation to flow of the surrounding fluid. Knowing the surrounding fluid for the rocket is air, equation 2 characterizes drag mathematically,

$$F_D = \frac{1}{2} C_D \rho_{air} u^2 A. \tag{2}$$

Thrust Force

The propulsion system generates the thrust force of the rocket where a rocket engine performs work on a fluid, the *working fluid*, accelerating the fluid through the propulsion system and finally exhausting the fluid in one direction. By Newton's third law of motion, the acceleration of the fluid in one direction causes an acceleration of the rocket in the opposite direction (National Aeronautics and Space Administration, 2015b). With a fuel propelled rocket system, the rocket engine performs the work and the working fluid is a fuel. With the water bottle rocket system, manually added pressure in the rocket system performs the work, and the fluid is water and air. This pressure is stored by the rocket system as *flow energy*, energy produced by a pressure acting on a fluid, until released to perform work on the working fluid. Equation 2 characterizes the thrust force mathematically by,

$$T_h = v_{D/e} \frac{dm_e}{dt}. \tag{3}$$

Here, $v_{D/e}$ is the exit exhaust velocity of the fluid as the fluid leaves the rocket system and dm_e/dt is the mass flow rate of the fluid as the fluid leaves the rocket system. These two variables multiplied together display the equivalent force applied to the rocket to accelerate the rocket. Because mass flow rate is equal to the fluid density ρ_{fluid} multiplied by the fluid velocity $v_{D/e}$ and nozzle exit area A_n, the equation for thrust can be simplified as shown below.

$$\frac{dm_e}{dt} = \rho_{fluid} A_n v_{D/e}. \tag{4}$$

$$T_h = \rho_{fluid} A {v_{D/e}}^2. \tag{5}$$

In order to derive the value for exit exhaust velocity of the working fluid, the work performed on the working fluid must be derived. The law of the conservation of energy is used to do this. The flow energy added to the rocket system by the operator and the air pump discussed above must be equal to the kinetic energy that is applied to the working fluid to exit the rocket propulsion system. The conservation of energy is used to derive the Bernoulli equation.

$$\frac{P_1}{\rho_{fluid}} + \frac{v_1^2}{2} + gz_1 = \frac{P_2}{\rho_{fluid}} + \frac{v_2^2}{2} + gz_2. \tag{6}$$

The Bernoulli equation takes into account flow energy, kinetic energy, and potential energy of a steady incompressible flow region of a fluid (Cengel, et al., 2012). The flow energy (the first portion of both sides of the Bernoulli equation) is dependent on pressure P and density ρ of the fluid. The kinetic energy (the middle portion of both sides of the Bernoulli equation) is dependent on the velocity of the fluid v and the potential energy is dependent on the elevation of the fluid z and gravity g. In the case of the rocket, v is equal to $v_{D/e}$, and ρ_{fluid} is equal to the density of water ρ_w. In terms of the rocket propulsion system, the potential energy of the system is negligible for the starting and ending conditions of the working fluid and the kinetic energy is negligible for the starting condition of the working fluid because it is infinitesimal compared to the ending condition of the fluid. This simplifies the Bernoulli equation for the rocket propulsion system as shown below.

$$\frac{(P-P_{atm})}{\rho_w} = \frac{v_{D/e}^2}{2}, \quad (7)$$

where P is absolute pressure.

Because pressure can be related to the velocity of the working fluid, the thrust can be determined. This allows for the prediction of rocket flight parameters as long as the pressure is known and constant, which is most often the case with engine driven rockets, but not so with water bottle rockets. Due to the limitations of the Bernoulli equation in equation 7, it is only valid until all of the water has left the water storage chamber of the rocket. A different form of the Bernoulli equation enables the derivation of the exhaust fluid velocity for the second phase of thrust as the excess air in the rocket storage chamber begins to exit. The air inside will stop exiting the chamber when its pressure has reached atmospheric pressure, and at this point, thrust will cease to be produced. These latter events are explained later in the chapter.

With water bottle rockets, an operator uses an air pump to apply an initial pressure to the rocket system by pumping air into the water storage chamber and blocking the nozzle exit with a rubber stop. Because water is an incompressible fluid, the air, as the mass increases, increases in density and energy causing a positive pressure to act on the surrounding walls of the rocket storage chamber and on the water since there is no room for the air to expand as more air is added to the chamber. Because the density of air is significantly less than that of water, the air sits on top of the water in the water storage chamber. This allows expanding air to push out water when the operator opens the nozzle exit. When this occurs, the volume of the air increases in the water storage chamber as the volume of the water decreases as it exits the water storage chamber. This process happens very quickly, in less than a second, and therefore transfers a negligible amount of heat, allowing for this process to be considered an adiabatic, and therefore isentropic, expansion within a closed system. In such processes, pressure of a fluid changes with respect to the fluid volume *V* and the ratio of the fluid's specific heat capacities k. This relationship mathematically shown as,

$$P = P_0(V_0/V)^k. \tag{8}$$

In the case of the water bottle rocket, the fluid causing the positive pressure is the air and therefore, the fluid of concern in the adiabatic expansion process is the air.

Because equation 8 contains two variables that are not constants, one of the variables must be determined before being able to solve it. Pressure and volume are unknown, but the derivative of volume, the volume flow rate *dV/dt*, can be obtained by the equation below.

$$\frac{dV}{dt} = A_n v_{D/e}. \tag{9}$$

The nozzle area is a constant and the exit exhaust velocity of the working fluid can be solved for by solving equation 7 for$v_{D/e}$, substituting equation 8 into equation 7, and then substituting equation 7 into equation 9. Equation 9 then becomes,

$$\frac{dV}{dt} = A_n\sqrt{\frac{2[P_0(V_0/V)^k - P_{atm}]}{\rho_w}}. \tag{10}$$

This equation must be solved by using an approximate numerical integration method. Using the Euler numerical integration method, equation 10 becomes,

$$V(t + \Delta t) \approx V(t) + \Delta t A_n\sqrt{\frac{2[P_0(V_0/V)^k - P_{atm}]}{\rho_w}}. \tag{11}$$

Equations 10 and 11 were derived with reference to works authored by professors from Ohio State University and the University of Queensland (Halliday & Foley, 2016; Nielson, 2005). *V(t+Δt)* is the current value for air volume to be evaluated, *V(t)* is the previous value, and *Δt* is a time step increment. The equation can be solved using time stepping for any duration of time until the water volume within the system becomes equal to 0 as long as an initial volume is known. The initial volume of air is the total volume of water placed in the water storage chamber subtracted from the total volume of the water storage container.

To derive an equation for volume of the excess air when all the water has exited the water storage chamber, the Bernoulli equation for compressible, isentropic processes must be used as seen in equation 12.

$$\frac{P_1}{P_{atm}} = \left[1 + (k - 1/2)\left(\frac{v_{D/e}^2}{c}\right)\right]^{\frac{k}{k-1}}. \tag{12}$$

Here, *c* is the speed of sound for the air denoted by the relation between the ratios of the specific heat capacities, the gas constant *R*, and the temperature *T* at any point in time. The mathematical relation is shown below.

$$c = \sqrt{kRT}. \tag{13}$$

By solving equation 12 for $v_{D/e}$, substituting equation eight into equation 12, and then substituting equation 12 into equation nine, the equation for volume flow rate is derived.

$$\frac{dV}{dt} = A_n c\sqrt{\left[\left(\frac{P_0(V_0/V)^k}{P_{atm}}\right)^{\frac{k-1}{k}} - 1\right]\left[\frac{2}{k-1}\right]}. \tag{14}$$

The Euler approximate numerical integration yields,

$$V(t + \Delta t) \approx V(t) + \Delta t A_n c\sqrt{\left[\left(\frac{P_0(V_0/V)^k}{P_{atm}}\right)^{\frac{k-1}{k}} - 1\right]\left[\frac{2}{k-1}\right]}. \tag{15}$$

Because temperature of the air is also a function of the second phase of thrust, temperature must also be derived. The mathematical relationship between fluid temperature and volume is shown below.

$$T = T_0(V/V_0)^{-0.4}. \tag{16}$$

Mass

The instantaneous mass of the rocket *m(t)* varies as the volume of the water in the water storage chamber varies, and the mass of the air M_a within the chamber is calculated using the ideal gas law. The mass of the empty rocket M_s is known and is constant. The equation for mass is below.

$$m(t) = M_s + M_a + \rho_w(V_{tot} - V),\ for\ V < V_{tot}. \tag{17}$$

$$M_a = \frac{P_0 V_0}{R T_0}. \tag{18}$$

For the second phase of thrust, because water is no longer present in the water storage chamber, the excess air is able to exit the storage chamber at the instantaneous mass flow rate, and equation 17 becomes,

$$m(t + \Delta t) = m(t) - \left(\frac{dV}{dt}\right)\rho_{air}\Delta t,\ for\ V_{tot} < V < V_{atm}. \tag{19}$$

Acceleration

By consolidating the derived equations for mass, thrust, drag force, and weight, acceleration equations can be derived for the first phase of thrust where the volume of the air inside the water storage chamber is less than the total volume of the chamber. They can also be derived for the second phase of thrust where the volume of the air is greater than or equal to the volume of the chamber, but less than its volume at atmospheric pressure, and the third projectile phase where no thrust is being produced. The equations for the three different phases of water bottle rocket flight are displayed below. These equations also were developed with the help of Ohio State University and University of Queensland professors (Halliday & Foley, 2016; Nielson, 2005).

$$a_1 = \frac{2A_n[P_0(V_0/V)^k - P_{atm}] - \frac{1}{2}\rho_{air}A|u|u}{M_s + \frac{P_0 V_0}{RT_0} + \rho_w(V_{tot} - V)} - g, \qquad for\ V < V_{tot}. \tag{20}$$

$$a_2 = \frac{\rho_{air}A_n c^2\left(\left[\frac{P_0(V_0/V)^k}{P_{atm}}\right]^{\frac{k-1}{k}} - 1\right)\left(\frac{2}{k-1}\right) - \frac{1}{2}\rho_{air}A|u|u}{M_s + \frac{P_0 V_0}{RT_0}} - g, \qquad for\ V_{tot} < V < V_{atm}. \tag{21}$$

$$a_3 = \frac{-\frac{1}{2}\rho_{air}A|u|u}{M_s} - g, \qquad for\ V = V_{atm}. \tag{22}$$

Velocity and Height

Because acceleration is a derivative of velocity, and likewise velocity is a derivative of height, approximate numerical integration methods may be used to obtain rocket velocity *u* and height *h* using time stepping of minor increments. The equations for using these methods are displayed below.

$$u(t + \Delta t) \approx u(t) + \Delta t[a_1],\ for\ V < V_{tot}. \tag{23}$$

$$u(t + \Delta t) \approx u(t) + \Delta t[a_2],\ for\ V_{tot} < V < V_{atm}. \tag{24}$$

$$u(t + \Delta t) \approx u(t) + \Delta t[a_3],\ for\ V = V_{atm}. \tag{25}$$

$$h(t + \Delta t) \approx h(t) + \frac{\Delta t}{6}\left[u(t) + 4\left(\frac{u(t) + u(t + \Delta t)}{2}\right) + u(t + \Delta t)\right]. \tag{26}$$

Velocity calculations used Euler approximations and height calculations used a Simpson approximation. Launch angle is accounted for by taking the sine value of the angle and multiplying it by the thrust and drag force.

Impulse

Impulse by definition is the added momentum difference to an object with an initial momentum. The accumulated impulse of the water rocket is calculated by taking the difference in momentum from an instantaneous and previous value for momentum and then adding that value to a previous impulse. Initially, the impulse is zero. In the simulator, accumulated impulse is calculated by taking the difference between the products of an instantaneous and previous mass and velocity. The mathematical relationship is displayed below.

$$Ft = Ft(t) + [m(t + \Delta t)u(t + \Delta t) - m(t)u(t)], \tag{27}$$

where *Ft* is the accumulated impulse.

Engineering Emphasized in Water Rocket Activities

With the derivations above, the flight characteristics, such as maximum height, of a water bottle rocket can be fully predicted, allowing water bottle rocket engineering to be performed. This correlates with the design of NASA rockets to reach the height required for orbital insertion into the Earth's gravitational orbit. These equations must, however, be entered into a program platform capable of performing hundreds to thousands of calculations quickly in order for the flight characteristics of a water bottle rocket to be predicted along its flight path in a reasonable amount of time. The above numerical approximations are only reasonably accurate when they are time stepped by small increments such as one thousandth of a second. Rocket flights for common water bottle rockets are up to six seconds long, meaning that the simulator must perform 6000 independent calculations to predict the flight parameters of the rocket along its full path to find maximum values for velocity, height, and acceleration. It would take a very long time if these calculations were done by hand. The next section discusses the construction of a simulator program to perform the above calculations to characterize a water bottle rocket's flight path.

Water Bottle Rocket Simulators in STEM

Few have attempted to construct water bottle rocket simulator programs, and their use in water rocket STEM activities has been rare. Common water rocket activities almost suggest to not use the simulators to predict rocket flight characteristics as is expressed by the National Physical Laboratory in their *Guide to Building and Understanding the Physics of Water Rockets*,

> This software has ***not*** been developed under NPL quality procedures and is not warranted for any use whatsoever. Got that? I can't be clearer. The software comes with no guarantee that it will do anything at all. That said, we believe that it is pretty *Good for Nothing*™. (De Podesta, 2007, p. 36)

In other cases, water rocket simulator use, or any prediction of flight characteristics, is considered an optional task, if mentioned at all (Institute of Electrical and Electronics Engineers, 2016; National Aeronautics and Space Administration, 2016; Todd, et al., 2007).

The lack of use of the existing water rocket simulators is due in part to their lack of availability and lack of usefulness in portraying engineering principles. The National

Aeronautics and Space Administration, *NASA*, offers and supports a couple impressive water bottle rocket simulators, one of them even being validated with test data, but these programs are unavailable for easy download onto PC platforms (Seeds Software, 2016; National Aeronautics and Space Administration, 2015a). It is also important to note that some of these programs are only prototypes. Universities that have developed other simulator programs simply do not offer the program on the university websites (Halliday & Foley, 2016; Nielson, 2005). Other programs are readily available for download but are invalidated and have design flaws. These design flaws include ambiguous input variable explanations allowing for inappropriate entry of initial data, such as drag coefficients and nozzle efficiencies, by their users (De Podesta, 2007).

All of these programs, however, have appropriate design for their intended functions. All input variables needed for calculation have at least some relatedness to the input variables available for entry data on the program interfaces. In addition, help tips are given for each input variable that requires user entry, explaining the input variable's meaning (De Podesta, 2007).

The programs offer graphed data of rocket flight characteristics and maximum values for velocity, height, and acceleration (National Aeronautics and Space Administration, 2015a; Nielson, 2005; De Podesta, 2007; Seeds Software, 2016). Some of the simulators also offer results graphs of impulse, thrust, and drag over time, but no explanation is given on how those results were calculated (Seeds Software, 2016). Furthermore, the Seeds Software company provides experimental data for validation of the software, but no indication is given of the derivation methods or if calibration factors were used to align the predicted outputs to the experimental outputs.

The simulators described above operate in either C++ or Java coding languages, making it very difficult to see firsthand how the software works. Without the ability to see how the software works apart from provided derivations such as the ones in the previous section, instructors using the software for STEM activities cannot assess the predictions of the software and cannot show students the engineering behind the software. This also prevents the instructor, or the user, from troubleshooting the software if student launch data does not correlate with the predictions provided by the software. The instructor is also incapable of making changes to the software to fit the learning objectives he/she sets for the students.

Water Rocket Analyses

Although few and futile attempts have been made to construct water bottle rocket simulators for use in STEM activities, the flight of water rockets have been analyzed and compared to experimental data countless times in the physics and engineering literature (Gommes, 2010; Kagan, Buchholtz, & Klein, 1995; Romanelli, Bove, & Madina, 2013; Romrell, Harger, & Ross, 2016; Strutz, 2005). A particular case is that of Cedric J. Gommes' study where every known rocket flight phenomena is accounted for including generation of water vapor, transient flow effects, and the real shape of the water bottle rocket and still a large discrepancy between the experimental and the predicted height values appears as is seen in figure 3 (Gommes, 2010).

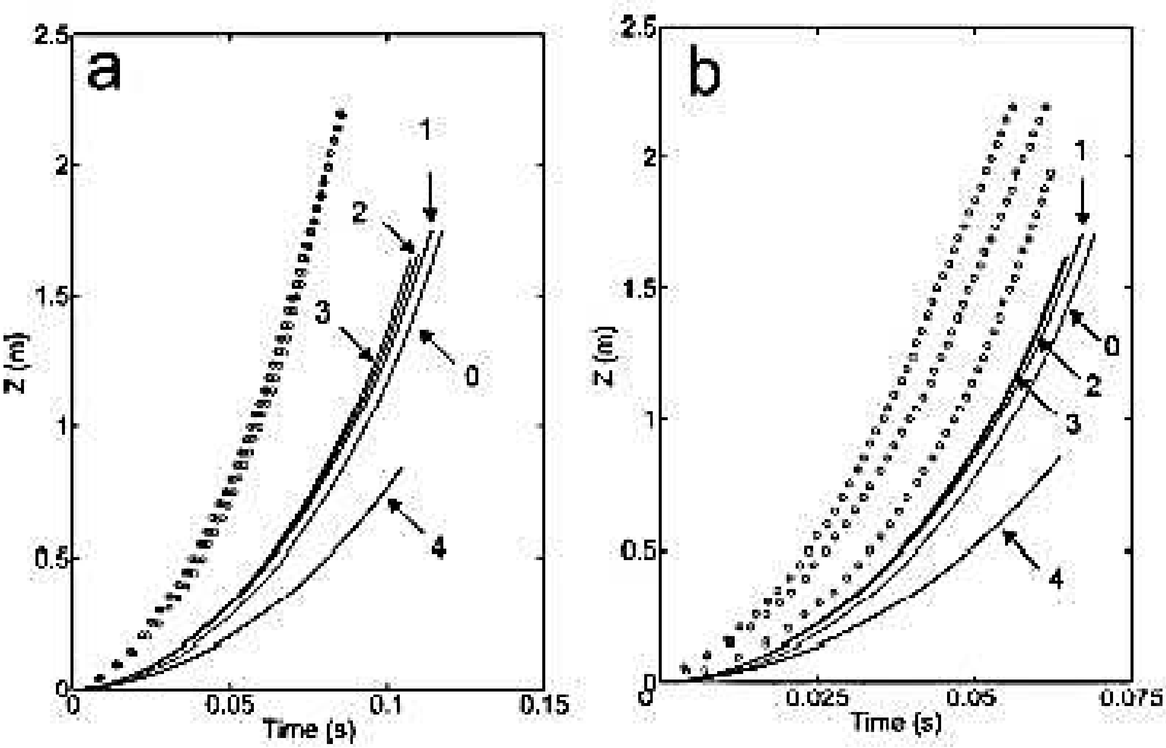

Figure 3. Literature Experimental data compared to theoretical predicted values at (a) 50 psi and (b) 100 psi for rocket height over time. Dotted lines are experimental test data, solid lines are theoretical predicted values (Gommes, 2010).

Simulation Validation Methods

For simulators that are based solely on analysis and assumptions and not statistical derivation, percent error analysis is the most common method for validating the theoretical predictions of simulators with experimental test data. There is no other appropriate method for validating theoretical prediction models. Hypothesis testing with t-scores, assessing the model accuracy as a range, or using statistical confidence intervals is only appropriate for simulators that simulate models by producing statistical values and that choose results based on the statistically produced values (Banks, Carson, Nelson, & Nicol, 2010; Sargent, 2010, 2011).

Though the author did not analyze the simulator statistically in this way, the experimental data was.

Previously performed comparisons of predicted and experimental height values have not compared maximum height, but the first few of flight points at respective times in the first phase of the water rocket flight. A high-speed camera recorded the initial launch with a measuring device near the water rocket to determine the height in relation to instantaneous time. The study did not consider this approach due to the expense of high-speed cameras and the unimportance of the initial set of height values in comparison to the maximum achieved height. In real world engineering scenarios, the maximum height is of primary concern to the engineer.

Due to the disadvantages of existing water bottle rocket simulation programs, it became necessary to construct a new water bottle rocket simulation software that would directly deal with these disadvantages and validate it with test data for use in future STEM water rocket activities. Chapter III describes the construction of the *Excel® Water Rocket Simulator* as well as the methods for testing and validating the Simulator.

CHAPTER III

CHAPTER III

METHOD

The following chapter discusses the construction of the Excel® water rocket simulator as well as the experimental method for validating the simulator. To validate the simulator, a test water bottle rocket was constructed resembling the common products of STEM water bottle rocket activities. The drag coefficient of the test rocket was experimentally determined, and the test rocket was launched a number of times with specified input parameters to gather experimental test data to be compared with the theoretical predictions provided by the simulator using percent error analysis. This chapter splits into four sections to discuss the factors of the simulator construction and validation. Those sections are the construction of the simulator, construction of the test rocket, drag coefficient test procedures, and launch test procedures.

Construction of the Excel® Water Rocket Simulator

The simulator has a number of features including flight parameter prediction and graphical display, drag coefficient prediction, and ballast and stress analysis design aids. An Excel® file held the different functions on different sheets with lists of procedures to guide the user in effective implementation of the simulator functions. The flight prediction function was the primary function of the simulator and the focus of this study and, therefore, described in detail below. The end of this section describes the other functions of the simulator in brief. Table 1 outlines the simulator functions.

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Table 1

Excel® Water Rocket Simulator Outline

Function	Component/Column	Results	Equations
Flight parameters	Table of Constants	-	User input
	Time	-	28 (shown below)
	Air volume	Thrust duration	11, 15
	Air temperature	-	16
	Thrust	-	29, 30 (shown below)
	Drag force	-	2
	Mass	-	17, 18, 19
	Velocity	Max velocity	31 (shown below)
	Height	Max height	26
	Acceleration	-	20, 21, 22
	Impulse	Max impulse	27
	Weight force	-	-
Graphed results	Velocity by Time	-	-
	Height by Time	-	-
	Acceleration by Time	-	-
	Weight by Time	-	-
	Drag Force by Time	-	-
	Thrust Force by Time	-	-
	Impulse by Time	-	-
Drag coefficient		-	-
Ballast and Stress Design Aids		-	-
Actual height	Time		
	Velocity		
	Height		
	Acceleration		
	Drag force		

Note. All equations referenced in the table are available in chapter II unless otherwise noted.

Equations 11, 15, and 20 through 26 in the chapter II construct the simulator. Because the Excel® platform had the capability to write “if” functions within the platform, equations 11 and 15 constructed an “if” function to calculate the volume of air during the first and second phase of thrust of the water rocket by time stepping increments. With volume known, the simulator could derive velocity, and then height, through the same methods. A table of constants consisting of input variables facilitated the values for each and constructing a table containing calculated columns for time, air volume, air temperature, thrust, drag force, and mass. With these table columns, velocity and height columns were constructed.

Table of Constants

The table of constants consisted of all the input variables that the simulator used to calculate rocket parameters. The user directly input some of these constants and some the program calculated based on user input. Figure 4 shows the table of constants. The blue cells denote inputs that require the attention from the user, the red cells denote inputs that may require attention from the user, and the colorless cells are inputs calculated based on user inputs.

Initial Parameters			***Calculated Parameters***		
Inputs	*Value*	*Units*	*Inputs*	*Value*	*Units*
Volume of Water	400	mL	*Mtot*	0.618961441	kg
Total Volume	1030	mL	*Fill Ratio*	0.388349515	
Mass of Empty Rocket	216	g	*V0 of air*	0.00063	m3
Launch Angle	90	Degrees	*Area Nozzle*	3.66096E-04	m2
Initial Pressure	40	psi	*Area Bottle*	0.00693683	m2
D of Nozzle	0.02159	m	*V0 of water*	0.0004	m3
D of Bottle	0.09398	m	*Mass of empty rocket*	0.216	kg
Temperature	5.56	C	*Vtot=*	0.001030	m3
density w	1000	kg/m3	*P0*	377115	Pa
k	1.4		*Temp0*	278.56	K
g	9.807	m/s2	*Launch Angle (Radians)*	1.570796327	
Atmospheric Pressure	100000	Pa	*End of Burst Volume*	0.001625951	m3
density a	1.2	kg/m3			
Drag C	0.345				
Time Interval	0.001	s			
Time Max	6	s			
R of air	288	kJ/kg*K			
Nozzle Coefficient	1				

Figure 4. Table of Constants from the Excel® Water Rocket Simulator.

Time

Although position in time had no place in the calculations for air volume, velocity, and height, the time column was needed for tracking the flight of the rocket's flight over time. The time column was constructed by using an equation to increment the time for each row in the column referencing an initial value, 0, and a constant specified by the user, Δt. The equation is,

$$t(t+\Delta t) = t + \Delta t. \tag{28}$$

The constant, Δt, specified by the user is the *Time Interval* value as can be seen in row 15 of figure 3. The first row of the column holds an initial value for volume and the equation in the second row applying to all proceeding rows until the column ends. By applying the equation to every proceeding row, the reference cell for "previous time value" continued to update accordingly. This pattern applied for all of the columns of the simulator table.

Air Volume

The air volume column was calculated by two "if" statements, one inside of another, to specify equations for use during the three different phases. Those phases were when the air volume was less than the total volume of the rocket's water storage chamber, in between the total volume and the volume of the air when it reached atmospheric pressure, and when the air volume reached atmospheric pressure and thereafter. After the air reached atmospheric pressure, the air volume within the water storage chamber remained at a constant value. Prior to that, air volume related to equations 11 and 15. As can be seen from the table of constants in figure 3, all of the variables required for the two equations are in the table of constants or derived from variables in the table of constants except for instantaneous air temperature. To accommodate this value, a reference loop was established between the air volume column and the air temperature column, meaning that the two instantaneous values referenced each other simultaneously to derive their values. Therefore, the completion of the air volume column depended on the completion of the air temperature column.

Air Temperature

The column for air temperature only needed equation 16. The equation referenced the table of constants for the initial temperature and volume and referenced the instantaneous volume from the air volume column for the respective time for the temperature calculation.

Thrust

For the thrust column of the simulator table, two “if” statements were used as with the air volume column. The thrust portions of equations 20 and 21 constructed the “if” statements, and a constant value, 0, characterized the air volume condition when it reached atmospheric pressure. Equations 29 and 30 show the mathematical relationships for review.

$$T_h = 2A_n C_{noz}[P_0(V_0/V)^k - P_{atm}],\ for\ V < V_{tot}\ . \tag{29}$$

$$T_h = \rho_{air} A_n C_{noz} c^2 \left(\left[\frac{P_0(V_0/V)^k}{P_{atm}} \right]^{\frac{k-1}{k}} - 1 \right) \left(\frac{2}{k-1} \right),\ for\ V_{tot} < V < V_{atm}\ . \tag{30}$$

The equations included the nozzle coefficient, C_{noz}, so that it could be factored into the derivation if a nozzle coefficient was available for the water bottle rocket. The thrust equations referenced all except the instantaneous volume from the table of constants.

Drag Force

Because drag force is dependent on rocket velocity, which in turn is dependent on drag force, the drag equation needed a reference loop between its value and rocket velocity. To do this, the rocket velocity column had to also be constructed so that the velocity column referenced the drag force column and vice versa. Once the two columns were constructed, correct values populated the columns. However, this did not occur every time. Excel® would experience a reference error once either column experienced any change. Therefore, the drag force referenced the velocity of the rocket a millisecond prior to avoid the error occurrence.

Mass

The mass column was constructed by two “if” statements where equations 17-19 were used. The column calculation initially only required reference to the instantaneous volume and the table of constants for the first phase of thrust. For the second phase, the column calculation required reference to the air volume column and a previous value for mass. The previous value for mass when the second phase of thrust first began was the mass of the empty rocket and the mass of the initial excess air. Using the previous value, the mass flow rate out of the storage chamber of the air and the time interval allowed the calculation of the amount of mass leaving for each interval of time.

Velocity and Height and Other Columns

The thrust, drag force, and mass columns could then facilitate the construction of a velocity column. Equation 31 simplifies the mathematical relationships of equations 20-25 in terms of velocity.

$$u(t+\Delta t) \approx u(t) + \Delta t\left(\frac{T_h - F_D}{m(t)} - g\right). \tag{31}$$

As can be seen in figure 3 above, *g* is a constant available in the table of constants.

With velocity known, equation 26 allowed for the calculation of height. The only variables needed for the height column to be calculated were the instantaneous velocity, previous velocity, and the time interval Δt.

Columns for acceleration, impulse, and weight were also constructed for educational viewing and comprehension of rocket flight and how these parameters are affected by input variables. These columns referenced the columns discussed above.

The program has a results table shown in figure 5 that returns the maximum values for velocity, height, and impulse, and returns the thrust duration of the first thrust phase. These values are most important for comparing experimental data with the theoretical values.

Results		
Velocity max	17.739548	m/s
Height max	16.485301	m
Impulse max	4.333873942	N*s
Thrust Duration	0.057	s

Figure 5. Results Table of Excel® Rocket Simulator.

The simulator also included a graph sheet within the Excel® program containing various graphs that characterized the water rocket's flight. The graphs included were velocity, height, acceleration, weight, drag force, thrust, and impulse over time for helping the user visualize the rocket flight simulation. Figures B12 and B13 show examples of these graphs.

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Other Program Functions

Other functions of the simulator aid the design process of the water rocket and show potential users the many different aspects that are part of rocket design. The additional functions include a drag coefficient finder, a ballast design aid, and a stress analysis aid.

The drag coefficient finder simulates a mass falling from a specified height. The forces acting on the mass are gravity and drag. The finder is located on a separate sheet and contains a table of constants similar to that shown in figure 3. Knowing a specified height and the time of fall of a mass from that height facilitates the calculation of a drag coefficient for the rocket. After putting in all known input variables into the table of constants, drag coefficient values can be input into the drag coefficient cell until the returned height in the results table matches that of the specified height. The same mathematical relationships used in the simulator construct the drag coefficient finder. The drag coefficient finder was used to determine the drag coefficient of the test water rocket used in this study. Figure 6 shows the table of constants and results for the drag coefficient finder.

Initial Parameters		
Value	*Inputs*	*Units*
251.3	*Mass of Empty Rocket*	g
9.807	*g*	m/s2
0.09398	*D of Bottle*	m
1.2	*density a*	kg/m3
0.055	*Drag C*	
0.001	*Time Interval*	s
1	Duration of Fall	s
Calculated		
0.006936825	*Area Bottle*	m2
0.2513	*Mass of empty rocket*	kg
Results		
4.88645791	Elevation	m

Figure 6. Drag Coefficient Finder Table of Constants and Results.

The ballast and stress design aids allow for the input of known variables about the rocket by the user in order to calculate aerodynamic and structural stability of the rocket. Figure 7 shows these variables.

Initial Parameters		
Inputs	*Value*	*Units*
Ballast		
Length of Rocket	0.4191	m
Length of Nose Cone	0.09525	m
Mass of Rocket without Ballast	80.7	g
Mass of Ballast	140	g
Stress		
Wall Thickness	0.2	m
Material Yield Strength in Tension	1600000	Pa
Material Shear Strength	500000	Pa

Figure 7. Ballast and Stress Design Aid Table of Constants. Material properties acquired from MatWeb™ for low strength polyethylene terephthalate.

Once the user enters these variables, the cell functions will either highlight the cell results green for acceptable or red for unacceptable. The cells also contain quantitative results such as stress states. The ballast results calculate the location of the center of mass and center of pressure of the rocket during flight. If the center of mass is greater than the center of pressure by a factor of 1.25, then a cell will be highlighted green and read, “GO” indicating that the ballast design is sufficient. The stress states of the rocket are calculated using pressure vessel analysis due to the similarities between the water storage chamber of the rocket and pressure vessels. Figure 8 shows the results table of the ballast and stress design aid.

Calculated		
Pressure	40	psi
Outer Radius	0.04699	m
Center of Pressure	0.20955	m
Center of Mass	0.264	m
Ballast Sufficiency	**GO**	
Pressure	275792	Pa
Thin or Thick Walled?	THICK WALL	
Inner Radius	-0.15301	m
Thin Wall Hoop Stress	0	Pa
Thin Wall Axial Stress	0	Pa
Thin Wall Radial Stress	-275792	Pa
Thin Wall Shear Stress	0	Pa
Thick Wall Hoop Stress	-333230	Pa
Thick Wall Axial Stress	-304511	Pa
Thick Wall Shear Stress	-166615.1755	Pa

Figure 8. Ballast and Stress Design Aid Results Table.

Construction of the Test Rocket

The test rocket constructed for validating the simulator matched as close to typical STEM water rockets as possible to add external validity to the study. Chapter I discussed typical STEM rockets. Typical STEM water rockets consist of a water storage chamber, bay area, nose cone section, and fins. The water storage chamber, bay area, and nose cone section typically consist of cut and taped together water bottles. The fins are typically cardboard. Sand or dirt also fills the nose cone section to ensure that the center of gravity is higher on the rocket than the center of pressure. This design feature is necessary to ensure a stable flight of the rocket (National Aeronautics and Space Administration, 2014). Without this design feature, the water rocket spins out of control. Fins serve a similar purpose by preventing the buildup of turbulent airflow near the tail of the rocket, which can cause drastic change of trajectory (National Aeronautics and

Space Administration, 2014). Figure 1 showed a typical STEM water rocket, and chapter I gave a more thorough description. The materials the test rocket consisted of are as follows:

- One liter Dasani® water bottles
- Cardboard (unknown brand)
- Scotch® heavy duty tape
- Gorilla® glue
- Dirt (unknown brand)
- Paper towels (unknown brand)

The first step to constructing the water bottle test rocket was to cut a third of the length of one of the water bottles from the spout. The butt of the larger section of the bottle made up the altimeter bay for the rocket, and the spout section of the bottle made up the nose cone for the rocket.

The spout section was then force fit over the butt section of the major section of the cut water bottle. The cross sectional area of the major section expanded at the butt section making the force fit possible. The force fit ensured a sealed connection with the addition of glue and tape.

The latter end of the major section of the water bottle then slid over another water bottle that was uncut. The uncut water bottle served as the water storage chamber of the rocket. This connection fit was not a force fit but a close fit. This was acceptable because none of the materials held within the altimeter bay of the rocket were fine enough to fit through the crevices of the connection.

Gorilla® glue was then applied to both connections completely around their circumference while the rocket stood upright. All excess glue was wiped away with paper towels and the glue was left to dry for 2 hours. The glue ensured the connections were sealed. The Scotch® tape was then placed over the connections evenly to add rigidity to the connections. Rigidity of design ensured the connections did not fail during launch testing.

Then three identical fins carved from cardboard were taped with equal spacing near the spout of the water storage chamber exit nozzle, the spout of the uncut water bottle. Four layers of tape were applied to each fin to ensure rigidity of the joined connections.

Finally, a specified mass of dirt was added to the nose cone through its spout. The spout was sealed off using the water bottle cap that was originally a part of the water bottle. A slit was made into the altimeter bay of the rocket to allow the entrance and exit of an altimeter. Small millimeter sized holes were also punched around the circumference of the altimeter bay to allow proper ventilation for the altimeter. Two square foot pieces of paper towel were crumpled and placed toward the nose in the altimeter bay section of the rocket to allow protective cushion for the altimeter during the launch testing to ensure that the altimeter would not fail on impact of the water rocket during testing. A duplicate was also made. The first test rocket was named rocket *Alpha* (181.9 g), the second rocket made was rocket *Beta* (216g). Figure 9 shows the water bottle test.

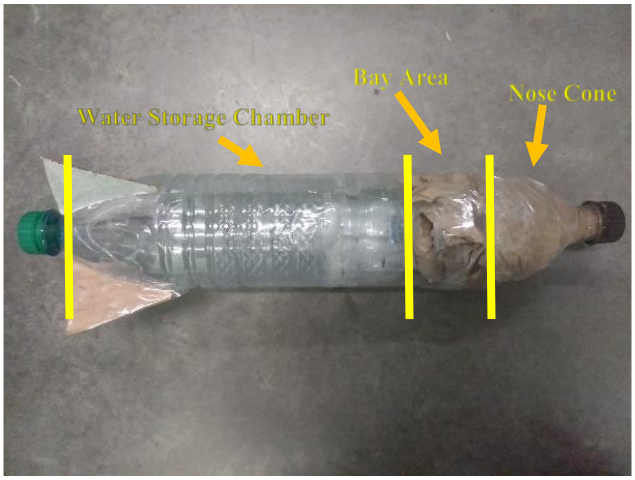

Figure 9. Water bottle test rocket.

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Drag Coefficient Test Procedures

After the water bottle test rocket was constructed, the drag coefficient of the rocket was determined through uncommon means of testing. Common methods for determining the drag coefficient of an object is by wind tunnel testing. However, the education environment demands a more accessible method. The drag coefficient consisted of dropping the rocket from a controlled height and recording the fall of the rocket for its duration using a digital camera. The testing took place on the Central Washington University campus at the Moore Apartments using the flights of stairs to acquire elevation for dropping the rocket. Three trials occurred at a controlled time of day. The materials used for the testing are as follows:

- Empty test rocket
- Digital camera (24 frames/second)
- Padding (coats/blankets)
- Venue (Moore Apartments)
- Tape Measure
- Computer with network

As noted above, the drag coefficient finder of the simulator and the time durations gathered from the digital camera with video editing software allowed the determination of the drag coefficient of the test rocket.

Two people conducted the testing: one person dropped the rocket from the control height and the other filmed the fall with the digital camera. The testing took place at 76ºF dry bulb temperature in a six mile per hour wind. First, blankets and coats were laid at the estimated impact point of the test rocket. Then, height of the drop was measured and recorded using a tape measure. The camera for recording the duration of the fall of the test rocket was prepared for each trial of the testing. Once the camera was ready, the camera operator signaled the rocket-dropping operator to drop the test rocket from the control height onto the blankets and coats at ground level. The guardrail of the flights of stairs were used as the starting point for the nose of the rocket for each drop. The rocket was dropped with its nose facing towards the ground level.

Once the footage of the rocket drops were gathered, they were observed in a video editing software, Adobe Premiere®, to determine the duration of the falls. By looking at each frame individually, the frames where the fall first began and where the fall ended were determined. This allowed for the duration of the fall to be determined in terms of video frames. Because the camera recorded footage at 24 frames per second, each frame was 41.67 ms in duration. This value was then multiplied by the duration of the fall in terms of video frames to obtain the duration of the fall in seconds, which could be input into the drag coefficient finder. The "Construction of the Water Rocket Simulator" section under "*Other Program Functions*" discusses the use of the drag coefficient finder of the simulator.

After experimentation to determine the drag coefficient of the test rocket, the range of the drag coefficient was assessed. Because the camera used to measure the duration of the fall recorded 24 frames per second and each frame was 41.67 ms, the range was plus or minus 41.67 ms of the time duration recorded. After using this range to calculate a range for the drag coefficient of the test rocket, the range was considered to be too large (-3 to 3.1) to be practical for use in the launch testing of the test rocket. Because of this, the calculated drag coefficient was not used. Instead, a literature value of 0.345 ±.004 was used (Barrio-Perotti, Blanco-Marigorta, Arguelles Diaz, & Fernandez-Oro, 2009). Inputting the literature value for drag coefficient into the drag coefficient finder infers a true duration of fall of 1.004, which is only four milliseconds off from the original value obtained from drop testing.

CHAPTER III

Launch Test Procedures

Before analysis of the launch-testing results occurred, the accuracy and precision of the launch testing equipment was assessed in order to acquire ranges of values for each input that was facilitated by each launch testing component. Table 2 summarizes the methods in acquiring the ranges associated with each piece of equipment.

The launch testing took place in a minimal interference environment on a morning where the wind speed was one to three miles per hour, and the temperature was 40-60ºF.

Table 2

Summarization of Methods in Acquiring Equipment Ranges

Value Type	Value Name	Associated Equipment	Acquisition Method
Input	Pressure	Air Pump Pressure Gauge	Resolution of Pressure Gauge
Input	Water Volume	Launch Setup	Qualitative Estimate[a]
Input	Launch Angle	Launch Setup	Qualitative Estimate
Input	Water Volume	Graduated Cylinder	Resolution of Graduations
Input	Drag Coefficient	Test Rocket	Literature Value
Output	Maximum Height	Altimeter	Supplier Email Correspondence
Output	Flight Duration	120 fps Camera	Resolution of Camera

Note: [a] The estimates were determined by the launch testing operator based off visual observations made during the launch testing. The observations that assumed the greatest variance of input values were used to estimate a range for the input parameter. Visual observations records for each launch are available in table A1 of appendix A of this study.

The first method for launch testing had a few issues that required a second method to yield data that was conclusive. The first method for testing allowed too much variation in the testing data where outlier data was unidentifiable. The data acquisition method was also insufficient in that the altimeter became damaged after multiple flights. This slowed data collection and provided less assurance in the accuracy of the altimeter after successive launches. This also prevented the collection of abundant data points due to the inability to purchase multiple altimeters when the altimeter sustained damaged. The next sections describe both the first and second methods for launch testing.

First Method

Ten launches were to take place with three different sets of launch parameters using rocket *Beta*, varying the water volume content of the water rocket and the initial pressure. Only nine launches occurred due to the altimeter sustaining damage on the tenth launch.

To launch, first, the altimeter had to be powered on, set to launch mode, and placed inside the altimeter bay of the water rocket. To ensure security of the altimeter in the altimeter bay, a rubber strap was tied to the altimeter and wrapped around the bottle. The strap was assumed to not affect the drag coefficient of the rocket significantly. Figures 10 and 11 show the strap mechanism.

Figure 10. Altimeter Strap Mechanism.

Figure 11. Altimeter Strap Mechanism Close Up.

Once the altimeter was strapped and set, the specified volume of water was added to the water storage chamber using a graduated cylinder and funnel. A rubber stop attached to the air pump plugged the storage chamber of the water rocket. The assembly was then set into a locking mechanism to hold the water rocket nose up and to retain the water rocket while the pressure was increased in the water storage chamber. Once the specified pressure was reached within the water storage chamber, a string was pulled to release the locking mechanism from the water rocket, allowing the water to be expelled from the water storage chamber, launching the rocket into the air.

After the water rocket landed, the maximum height was retrieved from the altimeter in feet and recorded. The height value was later converted to metric meters. This process was repeated for all trials. The materials used for the launch testing are as follows:

- Test rocket
- Launch setup

- Tap water
- Graduated cylinder
- Jolly Logic® Altimeter
- Specialized® air pump
- Funnel

Once the actual values for maximum height were collected, they were compared to the predicted values of the simulator by percent error analysis. The values were analyzed in light of their ranges determined by the accuracy and resolution of the launch testing instruments. Chapter IV discusses the results of the launch testing as well as the precision of the predicted and actual maximum height values.

Second Method

As was discussed briefly before, the differences of the second method from the first used an improved launch setup, a different data acquisition method, rocket *Alpha* as the test rocket, and consisted of more trials and less variation of input parameters.

Using a more precise launch setup prohibited any noticeable loss of water from the test rocket and reduced variation in the launch angle. These improvements also allowed the water chamber pressure to be set with zero fluctuation.

The different data acquisition method consisted of video recording the launches of the test rocket with a digital camera at 120 frames per second, using advanced film editing software (Adobe Premiere® 2015) to acquire the flight durations from the footage, and using the flight durations and the principles of the simulator to calculate the height. The height was attained using calculation methods similar to those used for the drag-coefficient-finder function of the simulator. The height calculations used gravity and drag force, which are two well-proven principles of motion, to acquire results. This version of the simulator was called the "Height

Finder" and is an included Excel® sheet in the Excel® simulator file. Because of this, the method was determined to be proficiently accurate. The range associated with the method was dependent on the amount of frames the digital camera was able to record per second. Because the camera had a maximum recording speed of 120 frames per second (8.33 ms long frames) and the height difference between one more and one less frame was .137 meters, .137 meters was determined to be the range of the output height. To acquire this range, the minimum and maximum flight durations, based on the flight duration range, were entered into the Height Finder.

Because a different data acquisition method was utilized for the second method of launch testing that allowed no restraint on number of launch trials, 42 trials were conducted with the same input parameters (40 psi and 400 mL) for every trial. This minimized variation in the data and made outlier groups easily recognizable.

Twelve trials of the launch testing took place in Ellensburg, WA using rocket Beta (216 g) and thirty trials took place in Seattle, WA using Rocket Alpha (181.9 g). The twelve trials conducted in Ellensburg helped identify any consistent error between the program and the experimental data. A calibration factor was then calculated using the experimental data for use in predicting maximum height and comparing those values to the experimental data collected in Seattle, WA. By changing the environment of testing and the rocket, the calibration factor was assessed for accuracy in maximum height prediction for different sets of data.

CHAPTER IV

CHAPTER IV

RESULTS

This chapter presents all aspects of data applied to the water rocket analysis. This includes the types of data gathered, the measurement methods, and any important aspects of data in the validation itself. Presented are the results from the first and second methods of launch testing, including the statistical analysis of the second method.

First Method

This section first displays the launch testing data along with the determined ranges of values associated with the accuracy and precision of the testing equipment used in launch testing. Then, the launch testing height data are provided using nominal input values and ranges of input values. Table 3 displays the range data.

Table 3

Range Data for Experimental Equipment

Value Name	Associated Equipment	Tolerance Range
Pressure	Pump Pressure Gauge	± 1 psi
Water Volume	Launch Setup	−40 mL
Launch Angle	Launch Setup	± 5º
Water Volume	Graduated Cylinder	± 5 mL
Drag Coefficient	Test Rocket	± .004
Output Height	Altimeter	±.1524 m

Note: m = meters; mL = milliliters; psi = pounds per square inch.

To calculate the minimum and maximum percent error of each trial, the two ends of each range for each parameter were used to calculate the minimum and maximum predicted and actual height. The ratios of each predicted and actual height were calculated (i.e. minimum predicted / minimum actual, minimum predicted / maximum actual, etc.) for each trial and the minimum and maximum ratios were used to calculate the minimum and maximum percent errors. Tables 4 and 5 show the launch testing height data, using nominal input values and ranges of input values.

Table 4

Nominal Launch Testing Results

Trial	Water Volume (mL)	Pressure (psi)	Recorded Height (m)	Predicted Height (m)	Percent Error (%)
1	450	55	21.64	19.78	8.60
2	450	50	18.59	17.31	6.89
3	450	48	16.46	16.61	.91
4	450	48	15.55	16.61	6.82
5	450	48	16.46	16.61	.91
6	400	48	14.63	16.61	13.53
7	400	48	17.68	16.61	6.05
8	400	48	16.15	16.61	2.85
9	400	48	18.59	16.61	10.65
				Average	6.36
				Standard Deviation	4.29

Table 5

Launch Testing Results Considering Ranges

Trial	Water Volume (mL)	Pressure (psi)	Recorded Height (m)	Predicted Height (m)	Minimum Percent Error (%)	Maximum Percent Error (%)
1	410-451	54-56	21.49-21.79	18.17-19.35	9.95	16.62
2	410-451	49-51	18.44-18.74	16.00-16.78	8.99	14.63
3	410-451	47-49	16.31-16.61	15.13-15.89	2.56	8.92
4	410-451	47-49	15.40-15.70	15.13-15.89	1.19	3.65
5	410-451	47-49	16.31-16.61	15.13-15.89	2.56	8.92
6	360-401	47-49	14.48-14.78	14.8-15.91	0.12[a]	9.89
7	360-401	47-49	17.53-17.83	14.8-15.91	9.23	17.01
8	360-401	47-49	16.00-16.30	14.8-15.91	0.55	9.22
9	360-401	47-49	18.44-18.74	14.8-15.91	13.71[a]	21.03
				Average	5.43	12.21
				Standard Deviation	5.03	5.43

Outlier groups and there causes were unidentifiable due to the large amount of variation associated with the launch testing factors (nearly 5% standard deviation for both the minimum and maximum percent errors, summing to nearly 18% allowable deviation). The testing operator made significant visual observations for all trials of testing as can be seen from the observations recorded in Table A2 in appendix A. These included loss of water from the water storage chamber of the rocket, as well as undergone damage to the rocket proceeding launches. Figure 12 presents the insufficiency of the data graphically.

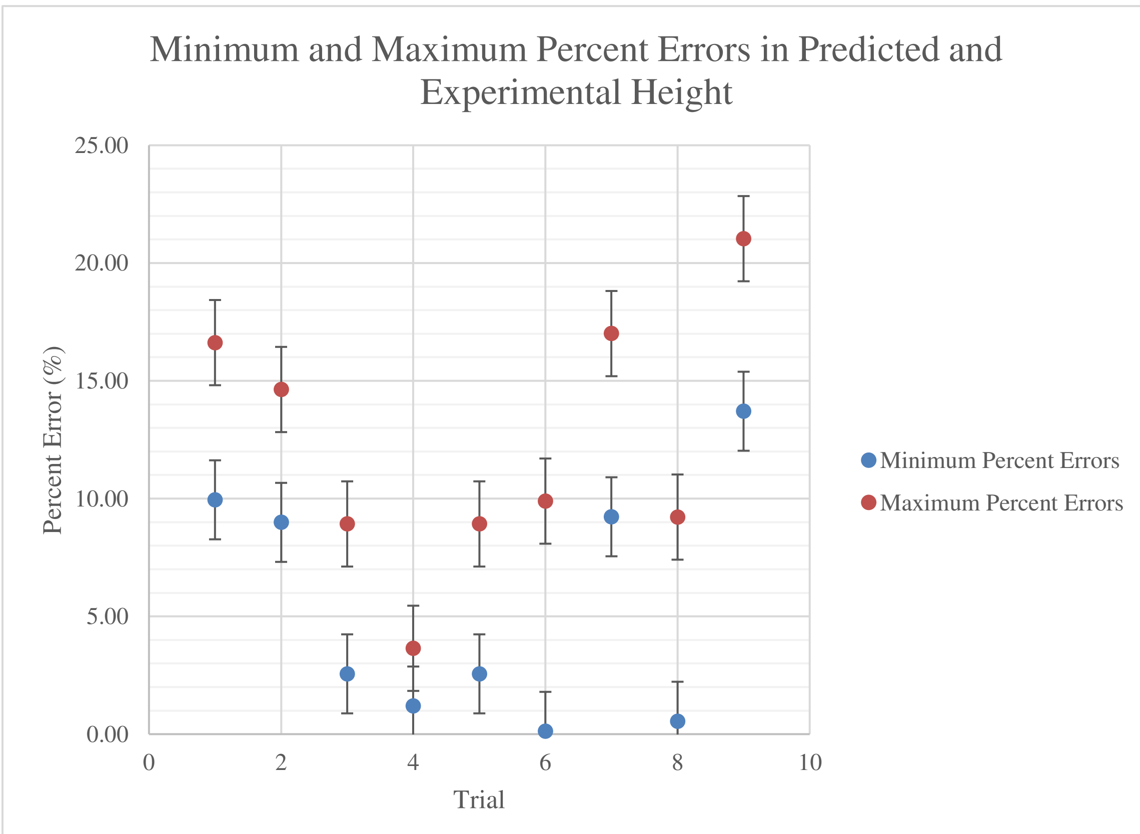

Figure 12. Minimum and maximum percent error in predicted and experimental maximum height for each trial of the first method of launch testing. 46ºF dry bulb temperature; 1-3 mph wind speed. Error bars represent the calculated standard error of the data set. The percent error varies more than 20%, which is too much deviation for validating the water rocket simulator.

CHAPTER IV

Second Method

Data collected from the second method of launch testing gives insight to the accuracy and precision of the testing method, the normality of the first grouping of data, the normality of the second grouping of data, and the accuracy and precision of the flight simulator as compared to the experimental data from the second grouping. As stated before, the first grouping was collected in Ellensburg, Washington for determining a calibration factor for the simulator to gain optimum accuracy, and the second grouping was collected in Seattle, Washington for comparing to the results of the flight simulator.

Batches were omitted from the second grouping of data due to drastic change in standard deviation of the batch and evidence of non-normal trends. Later shown is the reasoning behind the omissions. The data left from the omissions were then checked for normality and displayed in terms of percent error compared to the predicted values of the flight simulator. Finally displayed was the summary of the data in terms of the original hypothesis of the study and in terms of the mean and a calculated confidence interval. Table 6 displays the precision of the equipment used in the second method of launch testing. All precisions of the equipment are due to the amount of graduations available per unit measurement.

Table 6

Range Data for Experimental Equipment in Method 2

Value Name	Associated Equipment	Tolerance Range
Pressure	Pump Pressure Gauge	± 1 psi
Launch Angle	Launch Setup	± 5º
Water Volume	Graduated Cylinder	± 5 mL
Drag Coefficient	Test Rocket	± .004
Output Height	120 fps Camera	±.137 m

Note: m = meters; mL = milliliters; psi = pounds per square inch.

CHAPTER IV

First Grouping of Data: Ellensburg WA

As noted in chapter III, the raw data of the launch testing was in terms of flight durations and then the data were used to calculate the maximum height using similar, but more rigid, principles of the flight simulator. Because of this, it was necessary to ensure that the calculation did not transform the raw data in any way by the calculation method. This was done by comparing the trends of the calculated experimental heights to the trends of the experimental flight durations. Figures 13 and 14 displays the trends of both sets of data for the first grouping of data. The range of the predicted height was calculated using the extremes of the ranges for each parameter. These were then entered into the simulator to acquire the upper and lower limit of the predicted height.

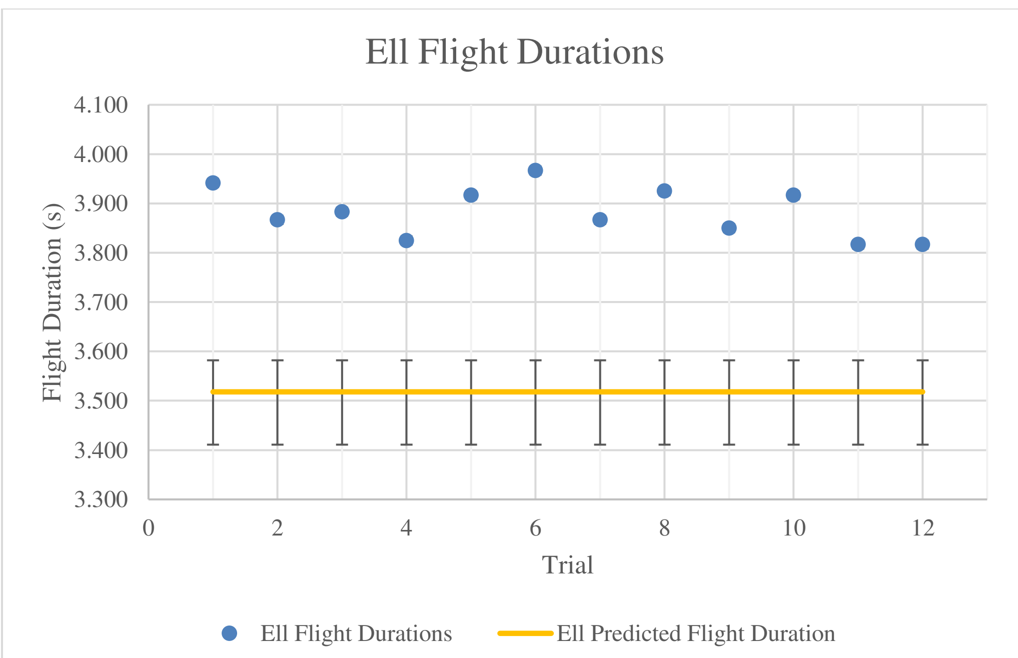

Figure 13. Ellensburg predicted and actual flight durations. 23°F dry bulb temperature; 0 mph wind speed. The error bars on each series represent the ranges of the data points as affected by the precision of the launch testing equipment. The error on the predicted flight duration had a higher range because there were more pieces of equipment affecting the value.

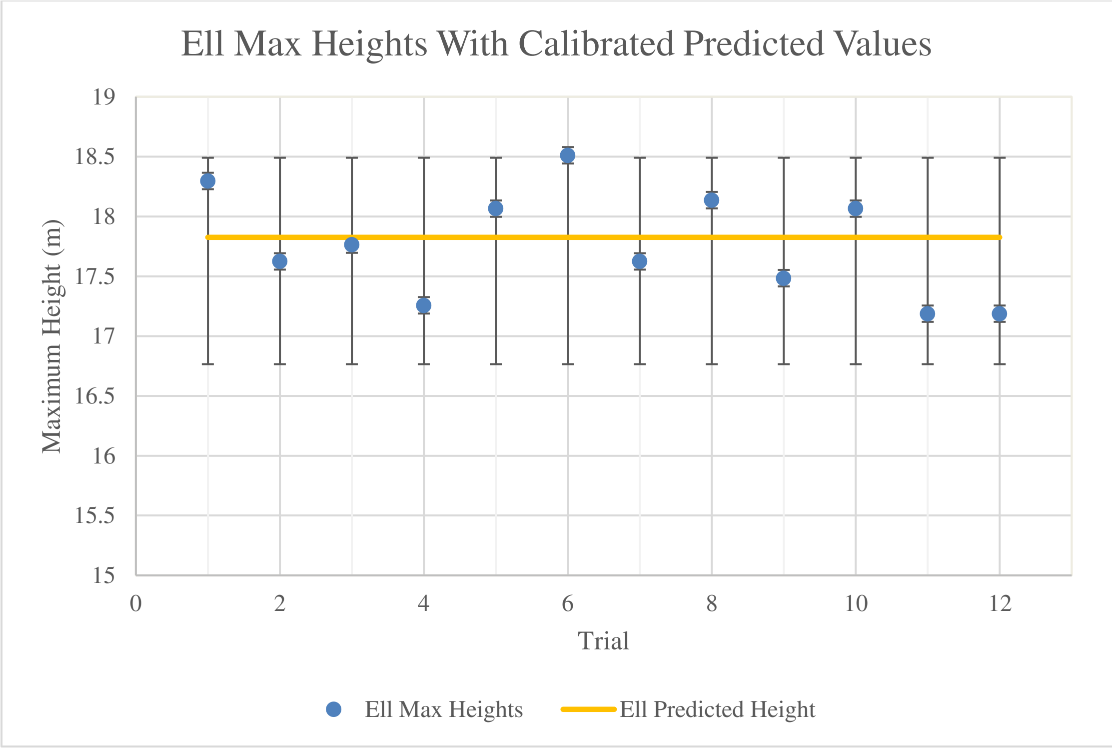

Figure 14. Ellensburg predicted and actual maximum heights. 23°F dry bulb temperature; 0 mph wind speed. Error bars indicate the ranges of the data as affected by the precision of the testing equipment. The predicted flight values have an additive calibration factor applied to them of 2.95. The calibration factor was attained by taking the average of the actual calculated heights and then subtracting the original predicted height value.

As can be seen from figures 13 and 14, a calibration factor of 2.95 was applied to the predicted maximum height. The simulator added this factor to the original predicted maximum height and effected none of the other outputs of the flight simulator. The trends of the maximum actual heights and the flight durations for the first grouping of data are identical. This shows that calculating the maximum height from the recorded flight duration or applying the calibration factor did not transform the data.

It was also essential to show the normality of the data to ensure applying a calibration factor to the flight simulator was appropriate at least for the range of input values that the study considered. Figure 15 shows the normality of the first grouping of data along with the square of the Pearson product-moment correlation coefficient.

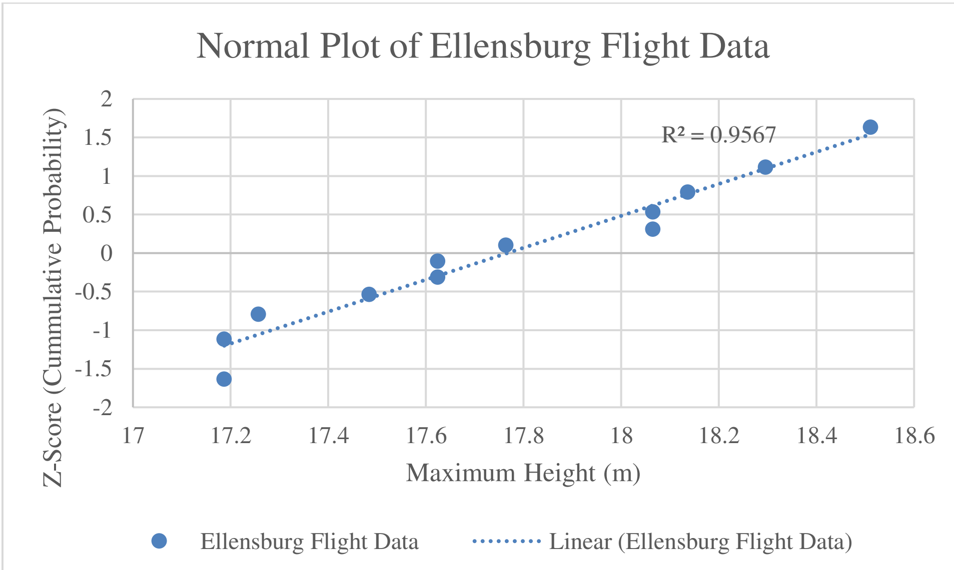

Figure 15. Normal plot of the Ellensburg maximum height data. The square of the Pearson coefficient is displayed on the figure showing a linear regression less than five percent.

Second Grouping of Data: Seattle WA

The second grouping of data also showed no transformation after calculation of maximum heights from flight durations or after the calibration factor determined from the first grouping of data was applied to the calculated flight data (figures B6-B8 of appendix B). However, certain batches of the second grouping of data showed slight variation in mean and drastic changes in standard deviation and trend. These batches were omitted from the second grouping on these grounds. The second grouping of data divided into three distinct batches: Trials 1-12, 13-25, and 26-30. Table 7 compares these batches.

Table 7

Comparisons of Seattle Batches of Data for Maximum Height

Trial/Statistical Value	Batch 1: Trials 1-12	Batch 2: Trials 13-25	Batch 3: Trials 26-30
Mean (m)	20.388	19.884	18.656
Standard Deviation (m)	1.113	.405	1.168
Trend	Slight Upward Linear	Normal	Shard Downward Linear

Batches 1 and 3 were omitted from the analysis of the data. Batch 1 was omitted on the grounds of having a standard deviation greater than 1 and a non-normal trend. Batch 3 was omitted on the grounds of having a standard deviation greater than 1, a mean more than one standard deviation away from the mean of all three batches, and a non-normal trend. Figure 16 supports these grounds further.

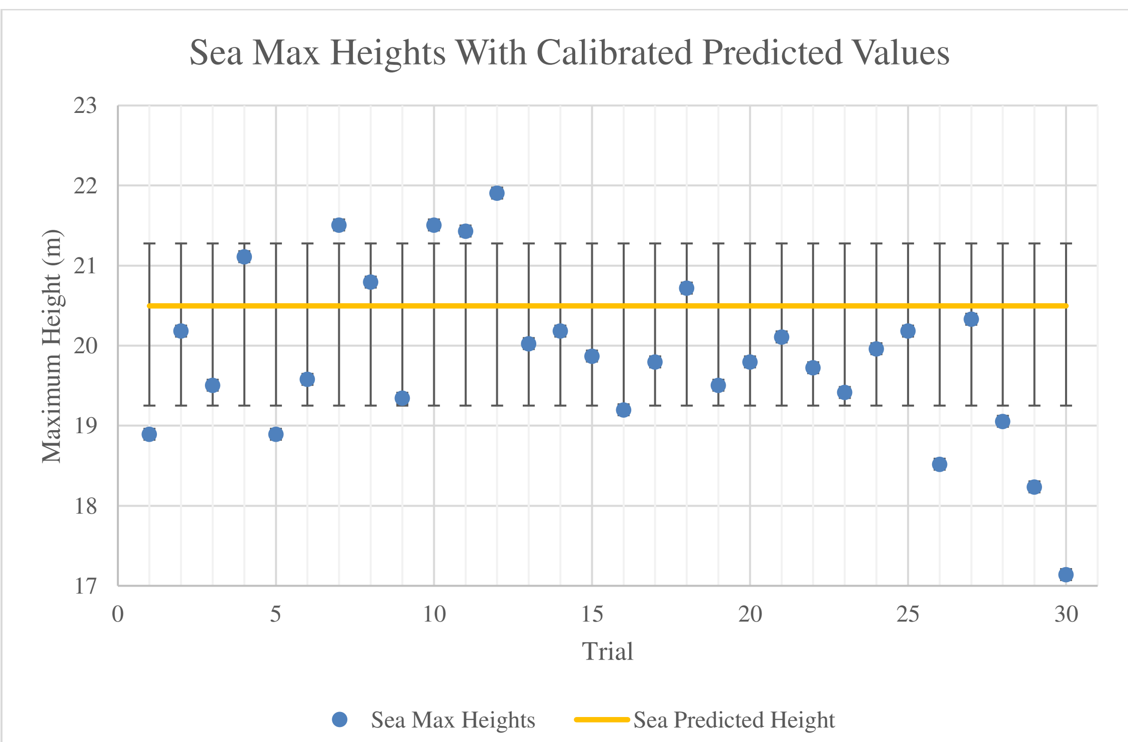

Figure 16. Seattle predicted and actual maximum heights. 36ºF dry bulb temperature; 3-5 mph wind speed. Error bars indicate the ranges of the data as affected by the precision of the testing equipment.

Because batches 1 and 3 are at the beginning and end of the second grouping of data, the change in the nature of the data was attributed to an initiation of a learning curve in the launch testing operator for batch one and fatigue of the operator for batch 3. The testing for collecting the second grouping of data spanned more than three hours in duration. The testing environment for the second grouping of data was also new to the operator, which the operator assumed caused the learning curve. The first grouping of data did not experience a learning curve effect in the testing operator because the operator was already very familiar with the testing environment from performing tests prior to the study as well as in the first method of launch testing that was a part of the study. The first grouping of data did not experience the effects of fatigue because the testing operator performed only 12 trials over the course of one hour.

Due to the omissions of batches 1 and 3, batch 2 was the only batch considered for analysis consisting of 13 trials. Batch 2 had a low amount of deviation and a mean maximum height that was slightly lower than the predicted maximum height.

The variation seen in the batch 2 data can be attributed to variation in input parameters including water volume, air pressure, launch angle, variations in weather, condition of the test rocket, and impulse imparted to the launch system from pulling the ignition tab of the launch setup. However, even with many sources of variations, as was said earlier, the variation in the data set is small. Figure 17 shows the normality of batch 2 from the second grouping of data.

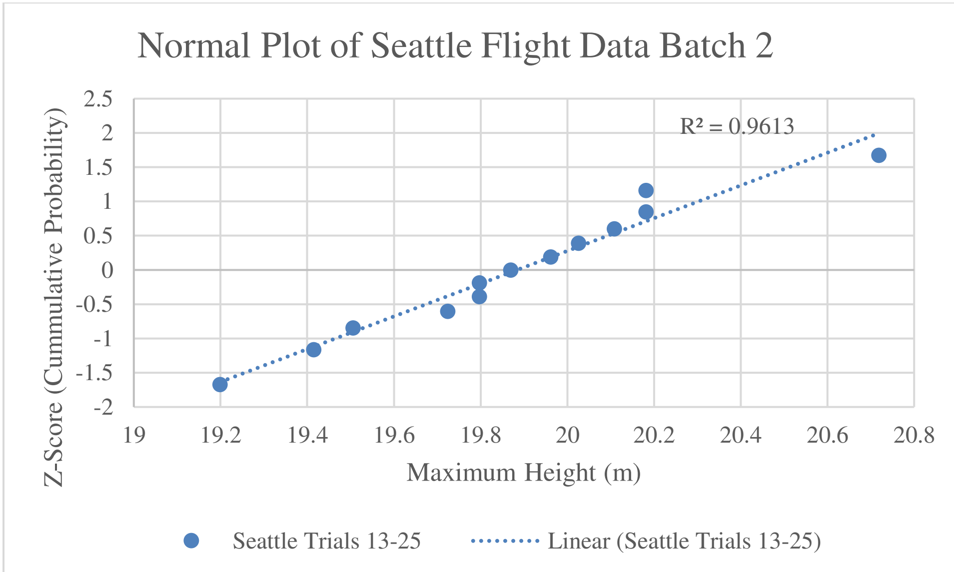

Figure 17. Normal plot of the Seattle maximum height data. The square of the Pearson coefficient is displayed on the figure showing a linear regression less than five percent.

Considering the ranges of the second grouping of data determined by the equipment used in the second method of launch testing, figure 15 showed that only 10 out of 30 trials had the potential for more than 0% error compared to the predicted maximum heights of the flight simulator. In addition, all of those trials were part of batches 1 and 3 that were omitted from the analysis of the second grouping of data. Looking at just the nominal maximum height data from the second grouping (not considering the associated ranges), the data can be compared to the predicted maximum height in terms of percent error. Figure 18 shows this relation, and table 8 shows the statistical data.

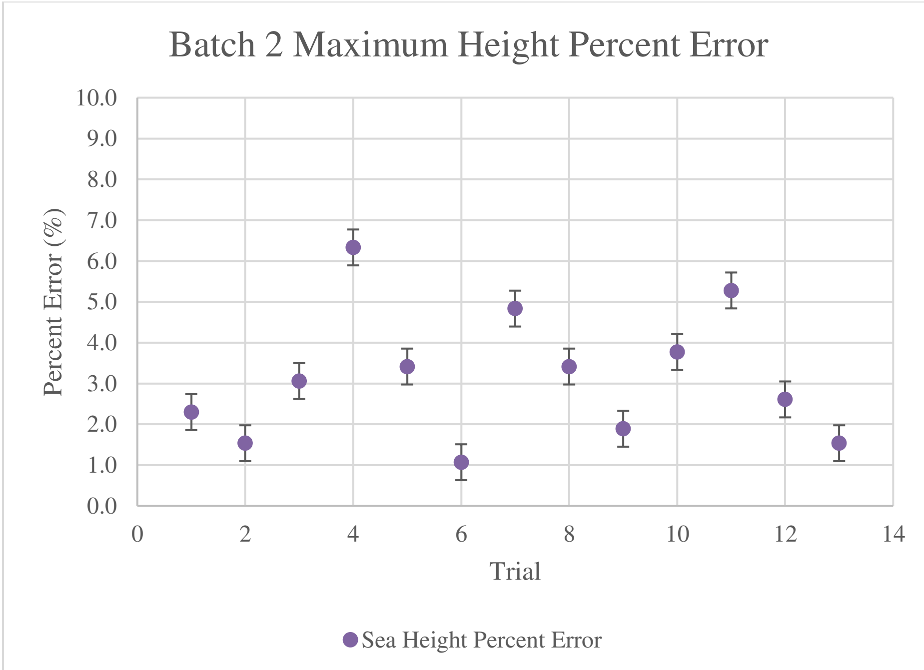

Figure 18: Seattle percent error between predicted and actual maximum height. The error bars indicate the calculated standard error of the batch 2 data.

Table 8

Statistical Data for the Batch 2 Maximum Height Percent Error

Statistical Value	Batch 2 (%)
Mean (m)	3.158
Standard Deviation (m)	1.585
Standard Error	.440
Range	1.5-6.3

From the statistical data calculated from observing the batch 2 data in terms of percent error, equation 32 describes the data in terms of the original hypothesis of the study. Because the hypothesis described a one-sided test with 95% confidence, the associated Z-score was 1.65. The equation describing the batch 2 data in terms of the original hypothesis is as follows.

$$10\% > 5.773\% = 3.158 + 1.65(1.585). \tag{32}$$

The confidence interval also describes the data in terms of the original hypothesis. The confidence interval is as follows:

$$Percent\ Error(\%) = 3.158 \pm 0.959. \quad (33)$$

Percent Error Data without Applied Calibration Factor

Figure 18 displays the percent error data without using a calibration factor with the flight simulator along with the statistical values of the data in table 9. Equations 34 and 35 also describe the data in terms of the original hypothesis of the study and a confidence interval for the data.

It can be seen from figure 19 and table 9 that although the standard deviation of the data increased slightly by 8.1%, the mean increased dramatically by 73.1%. This data shows that precision is not the problem, but accuracy is, therefore revealing a flaw in the prediction capabilities of the simulator without a calibration factor. Another interesting observation about the non-calibrated data is that the experimental values are higher than the predicted values, meaning that the error is not from inefficiencies or losses in the water rocket system. This shows that the simulator is not accounting for a resource of energy in the water rocket system, and that further development of the flight simulator can only determine the unaccounted energy. Chapter V discusses these issues further.

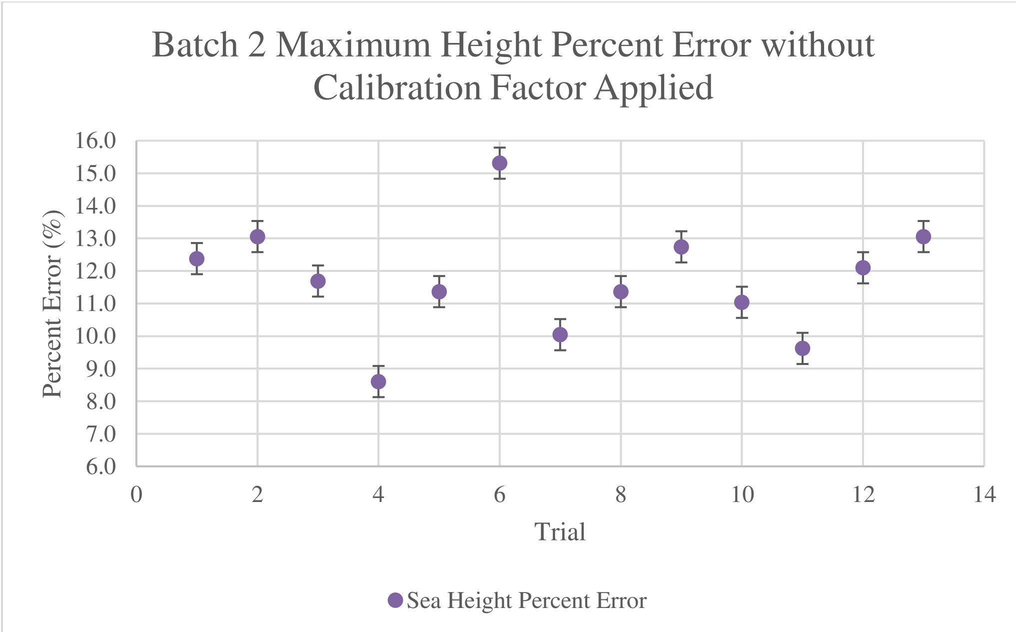

Figure 19: Seattle percent error not adjusted with a calibration factor. The error bars indicate the calculated standard error of just the batch 2 data.

Table 9

Batch 2 Maximum Height Percent Error without Calibration Factor

Statistical Value	Batch 2 (%)
Mean (m)	11.7
Standard Deviation (m)	1.725
Standard Error	.5
Range	8.6-15.3

$$10\% < 14.567\% = 11.721 + 1.65(1.725). \quad (34)$$

$$Percent\ Error(\%) = 11.721 \pm 1.09. \quad (35)$$

CHAPTER V

CHAPTER V

DISCUSSION

In this chapter, the implications of the results from the second method of launch testing are discussed, including the legitimacy of the results and areas of concern, the generalizability of the results to the use of the water rocket flight simulator in STEM water rocket activities, and further limitations and constraints of the findings. Conclusions and recommendations are also stated. The first grouping of data from the second method of launch testing is discussed first.

First Grouping of Data

The first grouping of data was collected and then analyzed to determine an appropriate calibration factor for the flight simulator. However, the initial discrepancy in the predicted maximum height value to the actual maximum height values brings significant concerns. By applying a calibration factor, although the maximum height may become closer in accuracy to the actual values for this particular study, that accuracy only applies to the experimental input conditions of the study. These values are not generalizable to the larger population of water rocket flights without further testing at different experimental input conditions using the same calibration factor.

The whereabouts of the discrepancy requiring the use of the calibration factor was also unknown. The discrepancy was predicted to be due to the lack of consideration of the generated water vapor at the end of the water rocket's thrust phase, external environmental influence (such as pressure driven flow of air upward) on the final output data, and generated lift from the flow of air past the rocket geometry during flight. However, analysis of previous studies, as discussed in *Water Rocket Analyses* in chapter II of this study, revealed that account for water vapor and transient flow of the air over the actual shape of the rocket only increased the accuracy of analysis slightly, still leaving a large difference in predicted and actual data. Induced drag by the change of direction of the rocket at apogee may have also caused the discrepancy. Because the raw data was collected in terms of flight duration, any factors affecting the flight duration, also affected the calculated maximum height. This discrepancy can only be resolved by conducting further testing and development of the flight simulator to account for water vapor at the end of the thrust phase and for environmental inputs, as well as the unknown effects that are causing most of the discrepancy between predicted and actual outputs. It is also necessary to explore other methods of construction of the flight simulator, using derivations that are more accurate. Doing this will eliminate the need for a calibration factor and allow higher accuracy to be attained in the simulator that is explainable using accepted theory, i.e. not an arbitrary calibration factor.

Second Grouping of Data

Although there was a large discrepancy in the experimental and predicted results of the flight simulator, a calibration factor allowed a high level of accuracy, as chapter IV discussed. However, the second grouping of data had to be manipulated to attain this accuracy by omitting batches of the data that showed signs of being affected by significant factors not a part of the original launch testing design.

Batches 1 and 3 were omitted due to attribution to the testing operator following a learning curve initially and the operator experiencing fatigue near the end of testing. These claims are not doubted, but the presence of a significant factor affecting batches 1 and 3 is, making them not normal and therefore disqualified from consideration in the study. However, the presence of the significant factor causing the need for the omissions was only present in the second grouping of data, where a different test rocket was tested in a completely new environment to the testing operator. Based on the assumption of the causes to the omissions in the study, this indicates that the omission of experimental data will not be necessary if the testing operator is familiar with the testing environment.

From the percent error data presented in chapter IV, it was determined with more than 95% confidence that the Excel® water rocket simulator will predict maximum height values within 5.773%. This value is almost half of the hypothesized value of 10%. However, the value has associated limitations as discussed previously. This level of accuracy is only generalizable with the initial input conditions as used in the study (40±1 psi initial pressure and 400±5 mL initial water volume, etc.). Further testing will need to validate the level of accuracy at different initial input conditions of the water rocket. Further testing will also be needed using different volume water storage chambers (2 L, 20 oz., etc.).

Limitations and Constraints of Findings

The limitations and constraints of the findings are summarized below concerning use of the Excel® Flight Simulator in STEM water rocket activities.

- The use of a calibration factor appropriate for the initial input parameters is needed to attain the accuracy shown in this study. For initial parameters significantly different from those of the study, the calibration factor presented in this study will need to be validated or changed.
- To ensure normality of all experimental data collected, the testing operator must be familiar and experienced in the launch testing environment. Simply said, the testing

operator should perform a minimum of 12 trials in the test environment before collecting data on proceeding trials.

- Actions must be taken to prevent fatigue of the testing operator either by taking periodic breaks in launch testing or using more than one testing operator to perform tests. Though doing so will add different sources of variation to the testing, accepted blocking techniques can block these (Anderson & Whitcomb, 2015).
- To ensure the accuracy of the simulator is within 5.773% of actual maximum heights, the same initial input parameters must be used as in this study. Using different input parameters will require revalidation of the calibration factor used in the flight simulator.
- The cause of the large discrepancy between predicted and actual maximum height data, that is before a calibration factor is applied, is unknown and therefore cannot be explained to participating students in STEM water rocket activities. Further testing and development of the simulator is needed to determine that cause. The predicted cause of the discrepancy is lift effects from the flow of air past the rocket geometry, pressure driven airflows upward to colder altitudes, and the induced drag caused by the water rocket changing direction at apogee.

Conclusions

The study determined that the Excel® Water Rocket Flight Simulator is able to predict maximum height within 5.773% of the actual maximum height of water bottle rockets with 95% confidence. This level of accuracy is only assured with the use of the initial input parameters of 40 psi and 400 mL water volume level in 1 L plastic water bottle rockets. It was not determined if the findings of the study are generalizable to different geometry water bottle rockets or different sets of initial input parameters. However, using the flight simulator and the same initial conditions used in the study will ensure that engineering can be a large part of STEM water rocket activities at the upper division level of STEM education. By having students predict the maximum height of a water rocket with predetermined input parameters, and then test those predictions against experimental results, the student is able to participate in the practice of engineering and enhance their knowledge of physics and fluid mechanics principles. The student is able to predict a quantitative value, compare that value to an experimental quantitative value, and determine the accuracy of the prediction. The student will also be able to make suggestions on how to make changes to the original rocket design to ensure that a desired outcome is achieved, i.e. maximum height. They will learn the importance of numerical methods in complex analyses and see how a number of theoretical principles collide in an engineering problem. Therefore, in conclusion, the use of the Excel® Water Rocket Flight Simulator will enable engineering to enter many STEM water rocket activities largely, hence accomplishing the original goal of the study.

Recommendations

The study suggests the need for further investigation. In addition, due to scope, the study did not discuss utilizing many of the other functions of the flight simulator in STEM water rocket activities. In context of these conditions, the author’s recommendations are summarized below.

- To make the simulator generalizable to other input parameters, further study of the variables affecting maximum flight is needed. Further development of the simulator is also needed, investigating more rigid forms of output parameter derivation of water rocket flights.

- To ensure the method of launch testing used in the study was legitimate and adequately controlled, validate the study using a different method of launch testing. The author suggests an improved and more rigid form of the first method discussed in chapters III and IV of the study.
- When performing STEM water rocket activities using the simulator, follow the limitations and constraints stated above.
- Modify the preferred water rocket activity into a design challenge, requiring structural analysis on the water rocket to determine a maximum pressure. This will allow utilization of the ballast and stress design check sheet of the simulator and the connection of strength of materials topics into the activity. Students will then be able to see how many engineering topics and principles are involved in a single engineering problem.

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APPENDIXES

Appendix A — Tables

Table A1

Drag Coefficient Testing Raw Data

Time of Fall (s)	Elevation (m)	Calculated C_D
1.00	4.88	.057
1.00	4.88	.057
1.00	4.88	.057
1.00	4.88	.057

Note: The time of fall data had a range of 83 milliseconds, too large to produce conclusive data.

Table A2

First Method Launch Testing Raw Data

Trial	Pressure (psi)	Water Volume (mL)	Max Height (m)	Comments
1	50	450	21.64	Loss of little water; angled
2	50	450	18.59	Loss of moderate water; angled
3	48	450	16.46	Loss of lots of water
4	48	450	15.55	Loss of moderate water
5	48	450	16.46	Loss of lots of water
6	48	400	14.63	Loss of moderate water; nose cone dented
7	48	400	17.68	Loss of little water
8	48	400	16.15	Loss of moderate water
9	48	400	18.59	Loss of little water
10	50	450	-	Altimeter died

Note: Date & time 10-12pm 06 October 2016. Weather conditions: 40 ºF; 1-3 mph wind speed. Rocket Beta was used, having a mass of 216g.

Table A3

Second Method Launch Testing Raw Data: Ellensburg WA Collection

Flight	End Frame	Start Frame	Flight Duration	Calculated Height
1E	8704	8231	3.942	18.296
2E	8403	7939	3.867	17.624
3E	7057	6591	3.883	17.764
4E	2428	1969	3.825	17.257
5E	5996	5526	3.917	18.065
6E	3652	3176	3.967	18.511
7E	1678	1214	3.867	17.624
8E	1909	1438	3.925	18.136
9E	1674	1212	3.850	17.484
10E	1985	1515	3.917	18.065
11E	1875	1417	3.817	17.187
12E	3290	2832	3.817	17.187

Note: Rocket Beta was used having a dented nose cone. The dented nose cone didn't appear to affect aerodynamic stability of the rocket. The testing operator did not make observations because all flights were filmed. Footage of testing is available upon request by contacting the author. Date & time: 1-3pm 13 December 2016. Weather conditions: 23 ºF; 0 mph wind speed; snow on the testing site. Initial pressure and water volume were set at 40 psi and 400 mL.

Table A4

Second Method Launch Testing Raw Data: Seattle WA Collection

Flight	End Frame	Start Frame	Flight Duration	Calculated Height
1S	995	512	4.025	18.893
2S	980	480	4.167	20.182
3S	1068	577	4.092	19.506
4S	1118	606	4.267	21.112
5S	1161	678	4.025	18.893
6S	1309	817	4.100	19.579
7S	1623	1106	4.308	21.508
8S	1487	979	4.233	20.794
9S	1326	837	4.075	19.343
10S	886	369	4.308	21.508
11S	1222	706	4.300	21.432
12S	1109	587	4.350	21.906
13S	1293	797	4.133	20.026
14S	962	462	4.167	20.182
15S	1038	542	4.133	19.87
16S	1316	829	4.058	19.199
17S	1540	1045	4.125	19.797
18S	1048	541	4.225	20.719
19S	1109	618	4.092	19.506
20S	1131	636	4.125	19.797
21S	1046	547	4.158	20.109
22S	949	455	4.117	19.724
23S	1133	643	4.083	19.415
24S	1080	583	4.142	19.962
25S	1104	604	4.167	20.182
26S	1142	664	3.983	18.519
27S	946	444	4.183	20.33
28S	1076	591	4.042	19.055
29S	1051	577	3.950	18.236
30S	1458	999	3.825	17.14

Note: The rocket used was Rocket Alpha and had a mass of 181.9 g. The testing operator filmed all flights and therefore did not need to make observations. Footage of testing is available upon request by contacting the author. Date & time: 12-4pm 15 December 2016. Weather conditions: 36 ºF; 3-5 mph wind speed; snow on the testing site. Initial pressure and water volume were set at 40 psi and 400 mL. The rocket sustained no observed damage during testing. The testing floor was composed of soggy and soft soil.

Table A5

Percent Error of Predicted and Flight Data: Ellensburg WA Collection

% Error Flight Duration	% Error Maximum Height
10.7	2.6
9.0	1.1
9.4	0.3
8.0	3.2
10.2	1.3
11.3	3.7
9.0	1.1
10.4	1.7
8.6	1.9
10.2	1.3
7.8	3.6
7.8	3.6

Note: Predicted flight duration: 3.5188 s; predicted maximum height: 17.825 m. Equation for calculating the percent error was the ratio of predicted and actual values subtracted from one. The predicted height without the calibration factor is 14.875 m.

Table A6

Percent Error of Predicted and Flight Data: Seattle WA Collection

% Error Flight Duration	% Error Maximum Height
5.1	7.8
8.3	1.5
6.7	4.8
10.5	2.9
5.1	7.8
6.9	4.5
11.4	4.7
9.8	1.4
6.3	5.6
11.4	4.7
11.2	4.4
12.2	6.4
7.6	2.3
8.3	1.5
7.6	3.1
5.9	6.3
7.4	3.4
9.6	1.1
6.7	4.8
7.4	3.4
8.2	1.9
7.2	3.8
6.5	5.3
7.8	2.6
8.3	1.5
4.1	9.7
8.7	0.8
5.5	7.0
3.3	11.0
0.2	16.4

Note: Predicted flight duration: 3.819 s; predicted maximum height: 20.497 m. Equation for calculating the percent error was the ratio of predicted and actual values subtracted from one. The predicted height without the calibration factor is 17.547 m.

Table A7

Percent Error of Flight Data without Calibration: Ellensburg WA

% Error Flight Duration	% Error Maximum Height
10.7	18.7
9.0	15.6
9.4	16.3
8.0	13.8
10.2	17.7
11.3	19.6
9.0	15.6
10.4	18.0
8.6	14.9
10.2	17.7
7.8	13.5
7.8	13.5

Note: The percent error for maximum height has a much larger range than for the flight durations. The error is also almost one and a half times as high on average. The two percent error groups do correlate however.

Table A8

Percent Error of Flight Data without Calibration: Seattle WA

% Error Flight Duration	% Error Maximum Height
5.1	7.1
8.3	13.1
6.7	10.0
10.5	16.9
5.1	7.1
6.9	10.4
11.4	18.4
9.8	15.6
6.3	9.3
11.4	18.4
11.2	18.1
12.2	19.9
7.6	12.4
8.3	13.1
7.6	11.7
5.9	8.6
7.4	11.4
9.6	15.3
6.7	10.0
7.4	11.4
8.2	12.7
7.2	11.0
6.5	9.6
7.8	12.1
8.3	13.1
4.1	5.2
8.7	13.7
5.5	7.9
3.3	3.8
0.2	2.3

Note: The percent error for maximum height has a much larger range than for the flight durations. The error is also almost one and a half times as high on average. The two percent error groups do correlate however.

Appendix B — Additional Figures

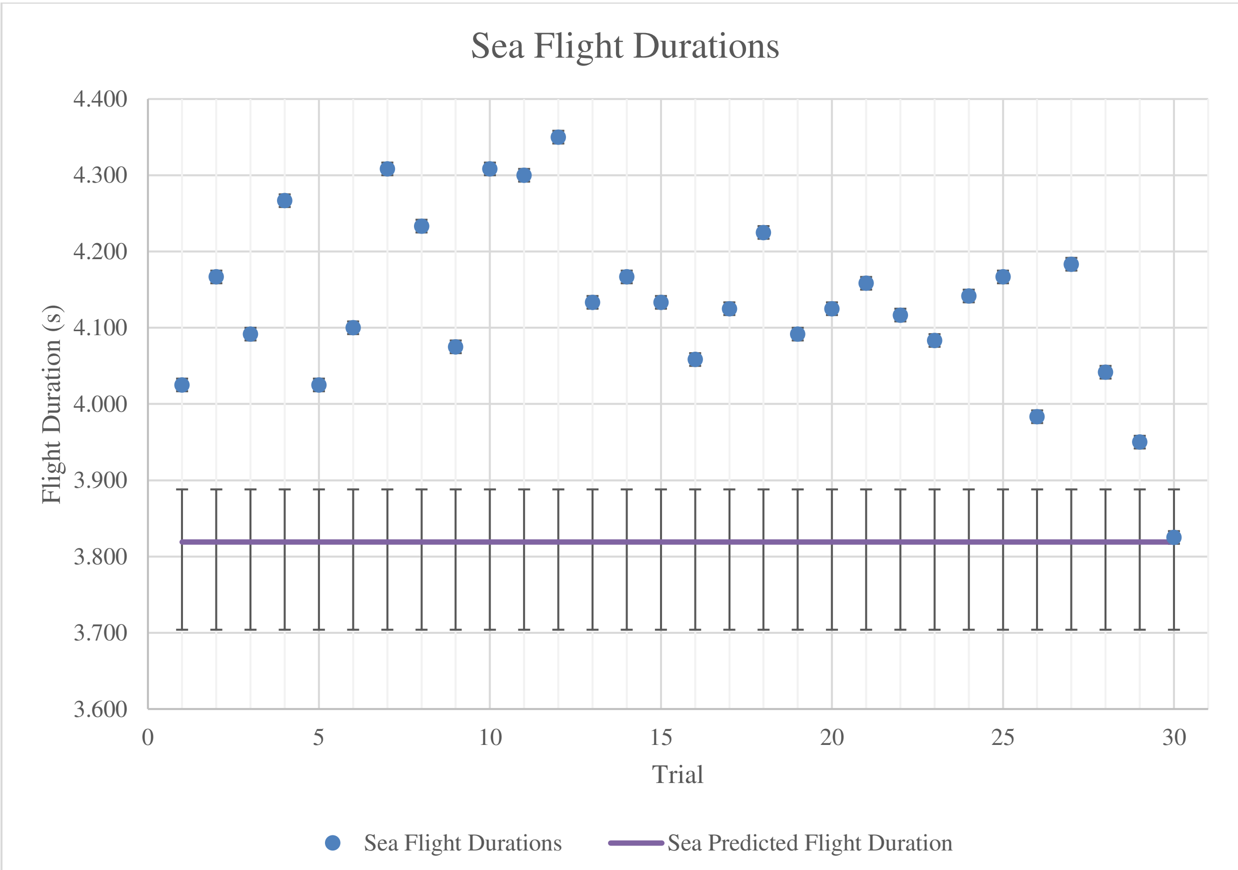


Figure B1: Seattle flight durations compared to the predicted. This is from the second method of launch testing. The trend matches that of the maximum heights attained during the launch testing.

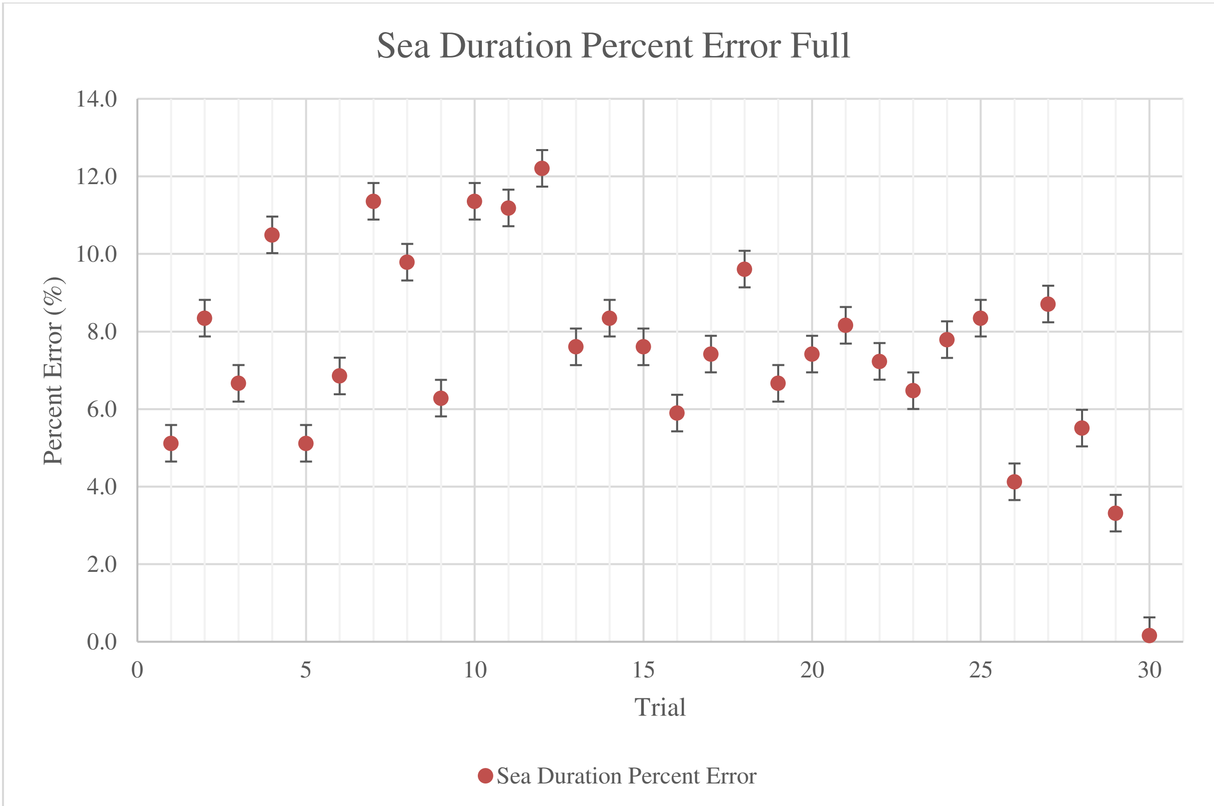


Figure B2: Percent error of Seattle flight durations all batches. The trend is consistent with the other sets of data from the second grouping.

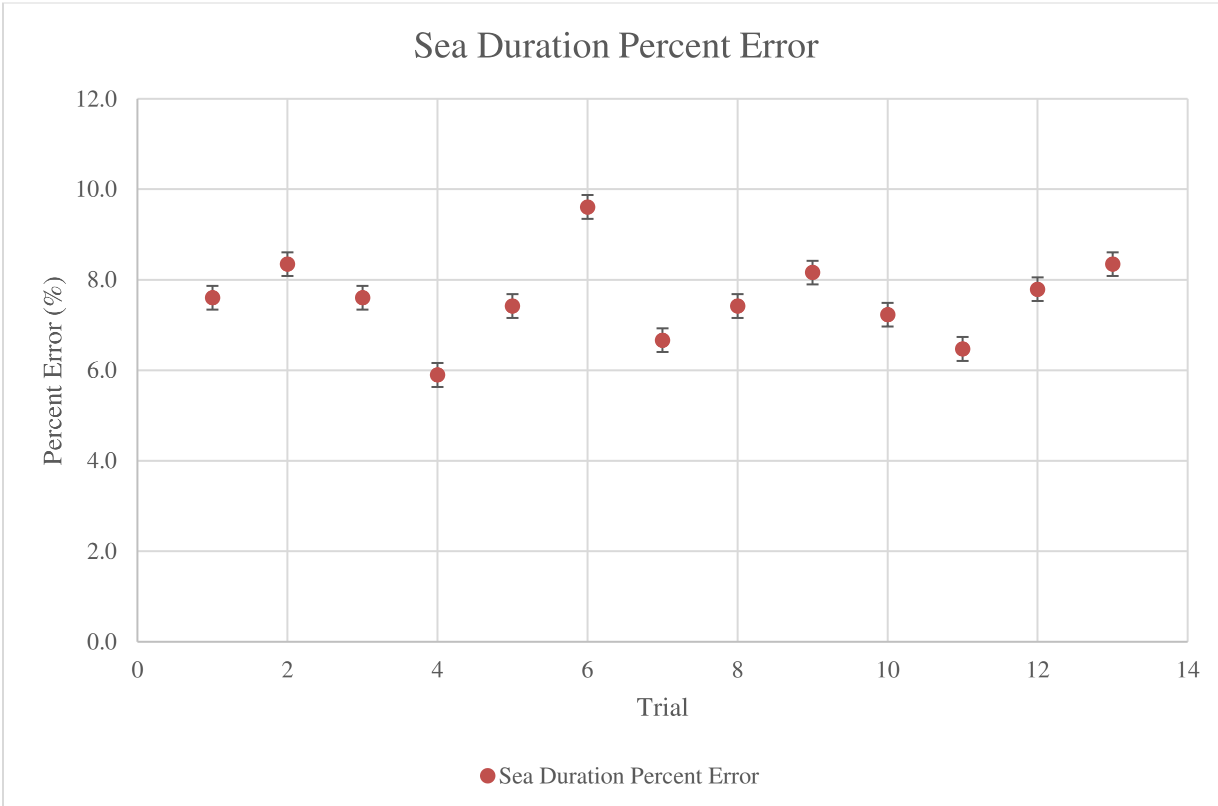


Figure B3: Seattle percent error of the flight durations batch two. These are from the second method of launch testing.

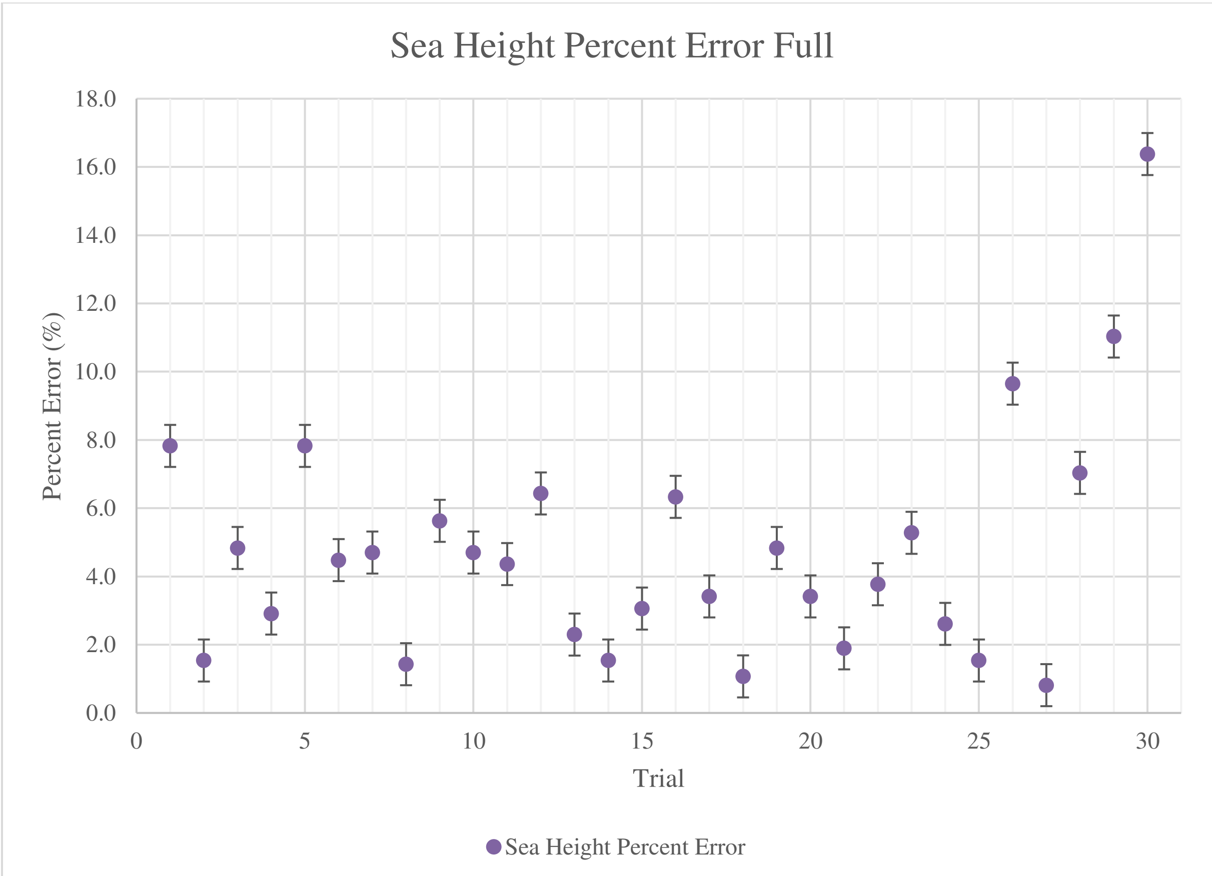


Figure B4: Seattle percent error of maximum heights attained all batches. These are from the second method of launch testing. The calibration factor polarized the trend from other data set trends from the Seattle data.

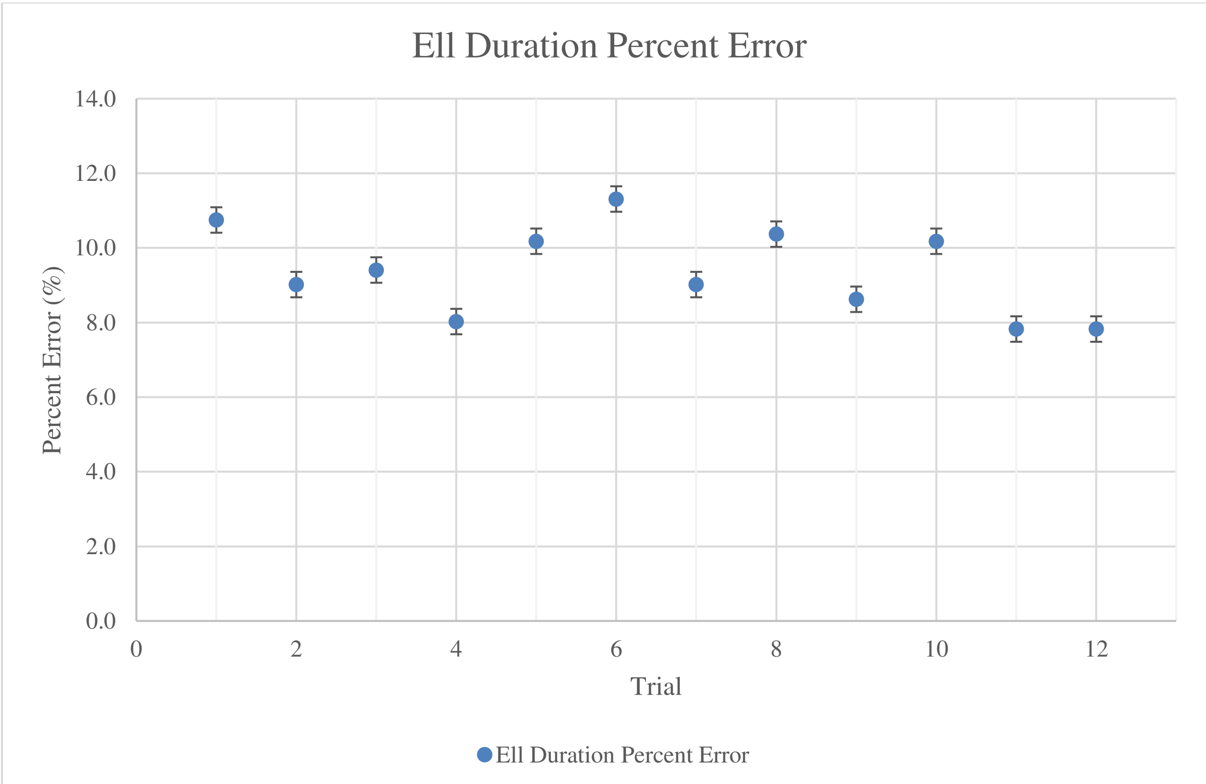


Figure B5: Ellensburg percent error of the flight durations. These are from the second method of launch testing.

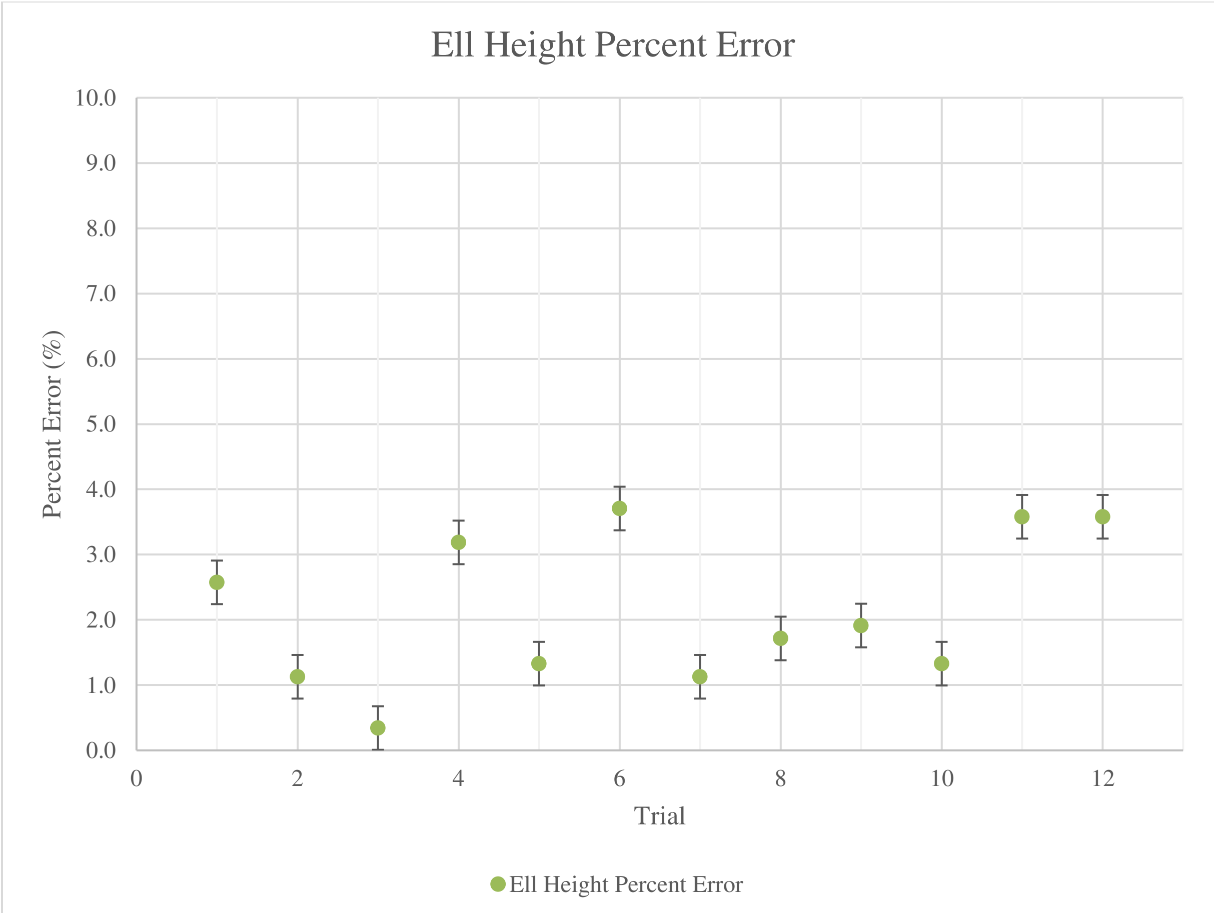


Figure B6: Ellensburg percent error of maximum height. This is from the second method of launch testing. This was with a calibration factor applied to the data set.

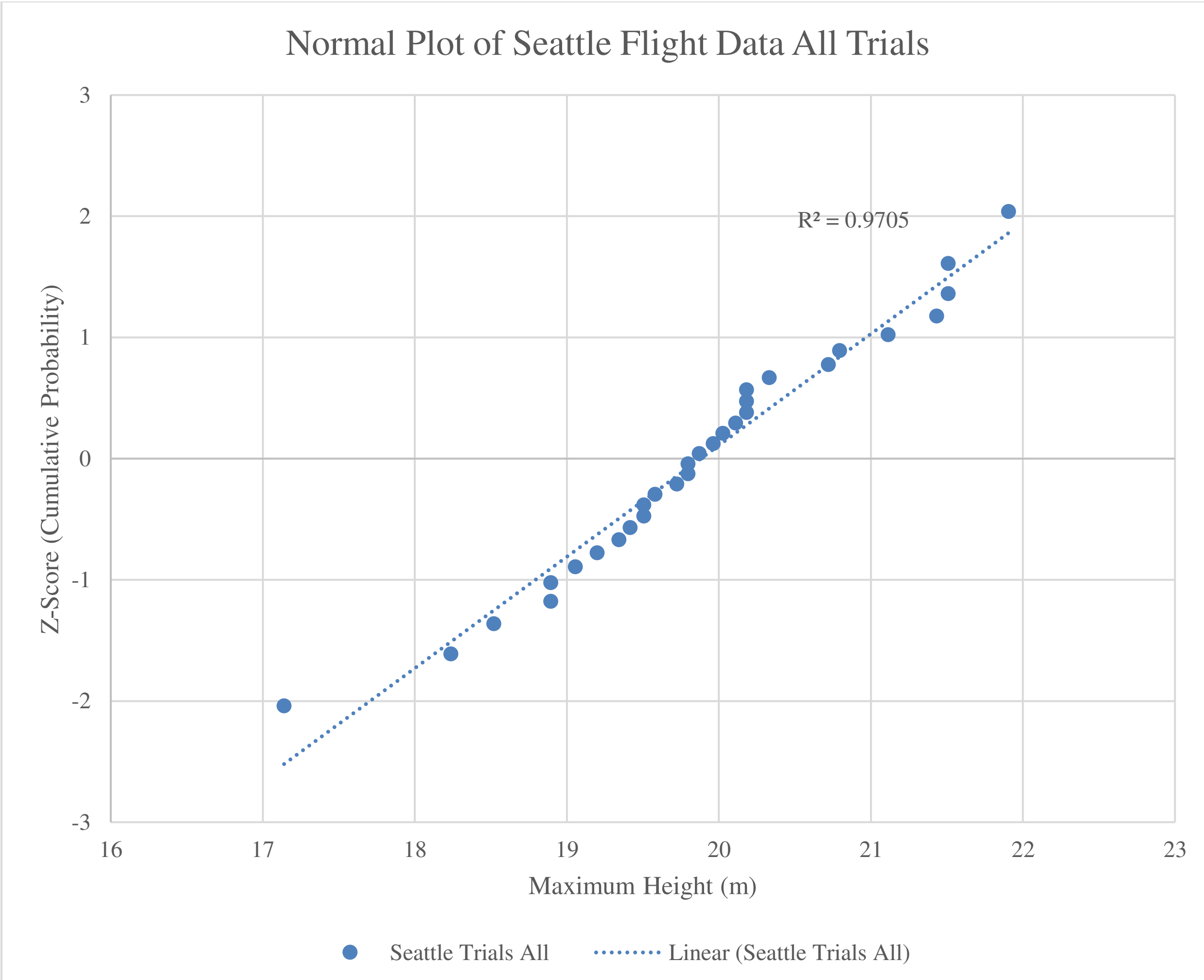


Figure B7: Normal plot of all batches from the second grouping of data. These are from the second method of launch testing. The middle portion of data seems to be normal with the low and high ends showing signs of significant factors being present.

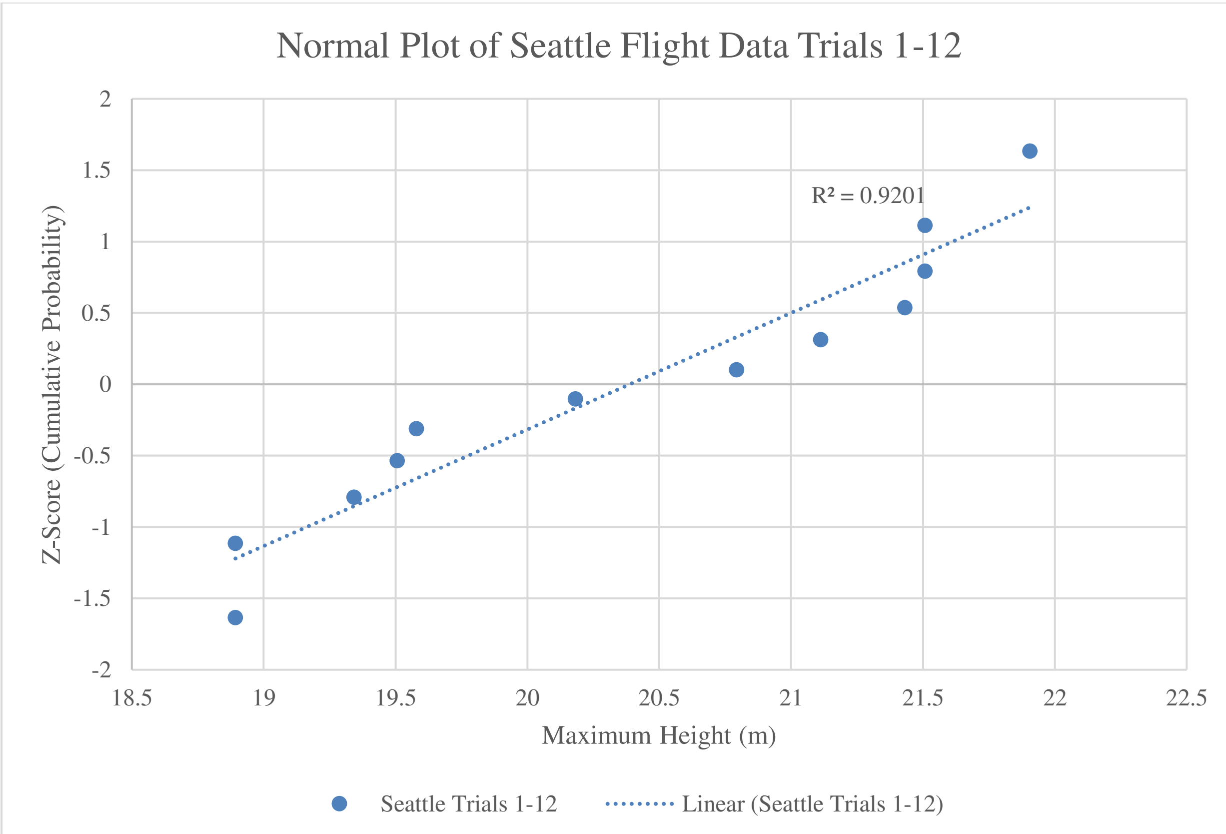


Figure B8: Normal plot of the first batch of the second grouping of data. These are from the second method of launch testing. This data shows little evidence of normality due to the low amount of data points forming a linear curve.

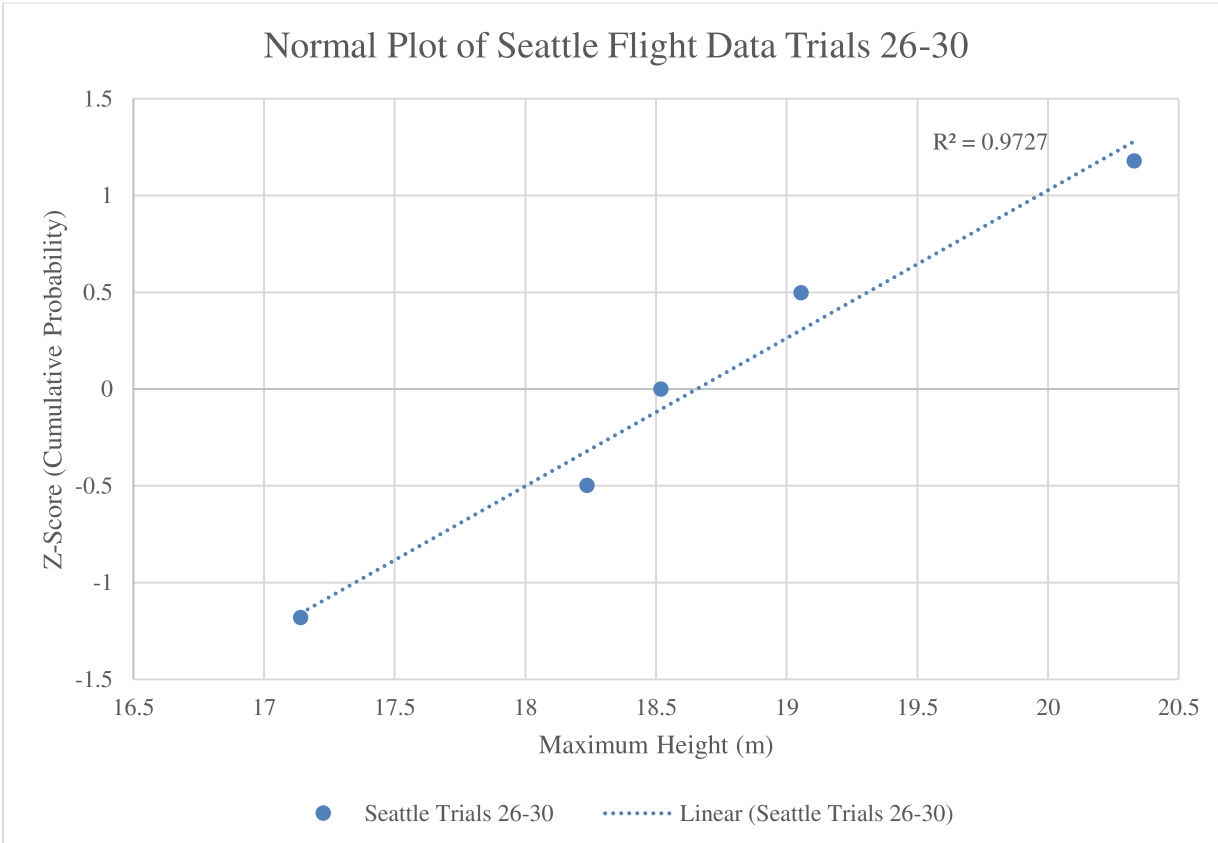


Figure B9: Normal plot of the third batch of the second grouping of data. These are from the second method of launch testing. This data shows little evidence of normality due to the low amount of data points forming a linear curve even though the R^2 value is over 95%. This value was calculated based only on a small portion of data points, resulting in a misleading value.

Initial Parameters		
Inputs	*Value*	*Units*
Volume of Water	400	mL
Total Volume	1030	mL
Mass of Empty Rocket	216	g
Launch Angle	90	Degrees
Initial Pressure	40	psi
D of Nozzle	0.02159	m
D of Bottle	0.09398	m
Temperature	5.56	C
density w	1000	kg/m3
k	1.4	
g	9.807	m/s2
Atmospheric Pressure	100000	Pa
density a	1.2	kg/m3
Drag C	0.345	
Time Interval	0.001	s
Time Max	6	s
R of air	288	kJ/kg*K
Nozzle Coefficient	1	
Calibration Factor	2.95	
Results		
Velocity max	17.494717	m/s
Height max	17.825157	m
Impulse max	3.848145214	N*s
Thrust Duration	0.062	s

Figure B10: Simulation results for the Ellensburg data. The calibration factor is applied only to the maximum height attained by the flight.

Initial Parameters		
Inputs	*Value*	*Units*
Volume of Water	400	mL
Total Volume	1030	mL
Mass of Empty Rocket	181.9	g
Launch Angle	90	Degrees
Initial Pressure	40	psi
D of Nozzle	0.02159	m
D of Bottle	0.09398	m
Temperature	5.56	C
density w	1000	kg/m3
k	1.4	
g	9.807	m/s2
Atmospheric Pressure	100000	Pa
density a	1.2	kg/m3
Drag C	0.345	
Time Interval	0.001	s
Time Max	6	s
R of air	288	kJ/kg*K
Nozzle Coefficient	1	
Calibration Factor	2.95	
Results		
Velocity max	19.416222	m/s
Height max	20.497371	m
Impulse max	3.699572956	N*s
Thrust Duration	0.062	s

Figure B11: Simulation results for the Seattle data. The calibration factor is applied only to the maximum height attained by the flight. The only difference between the input parameters of the first and second grouping of data is the rocket mass because a different rocket was used for the different groupings.

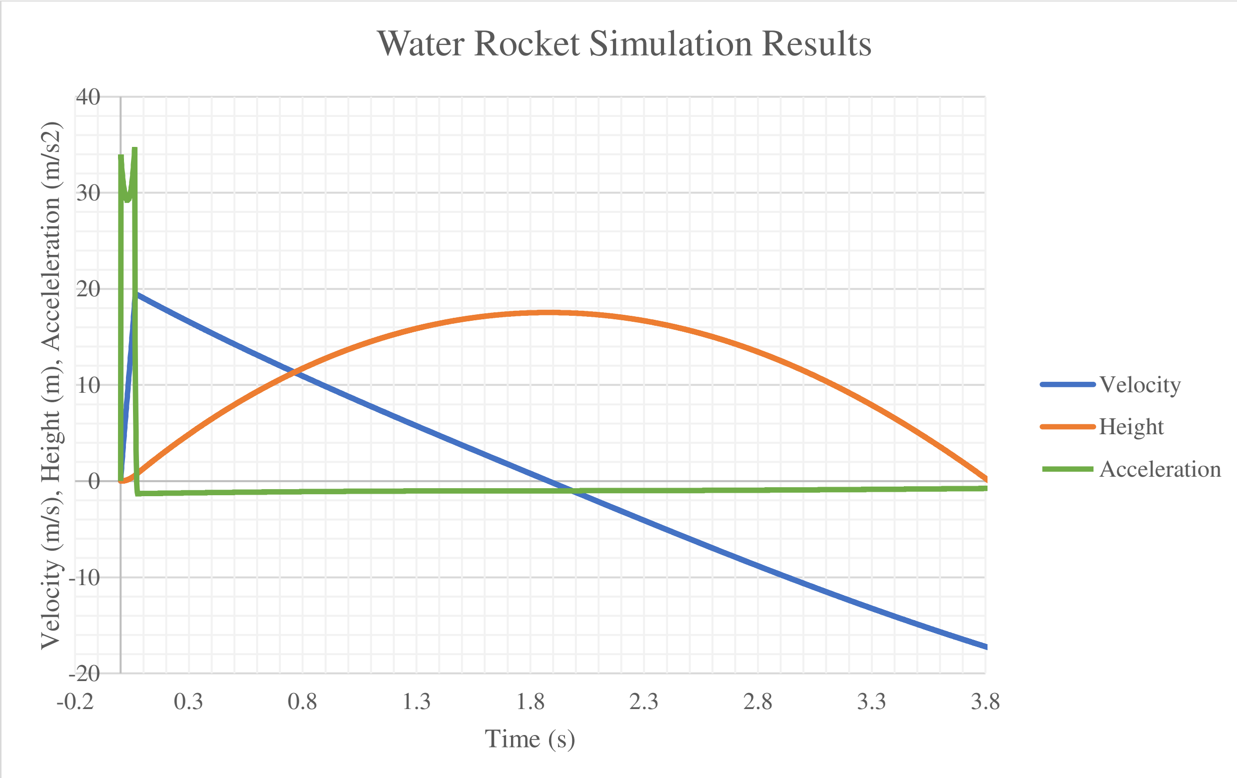


Figure B12: Simulation Results for Ellensburg predicted flight. Acceleration is initially very high, and decreases while water and pressure within the pressure chamber of the rocket decrease. When the second phase of thrust begins, acceleration increases briefly until all of the excess air is expelled. Then acceleration becomes only dependent on drag force and gravity. Acceleration then approaches zero as the rocket approaches apogee upwards and reaches terminal velocity downwards.

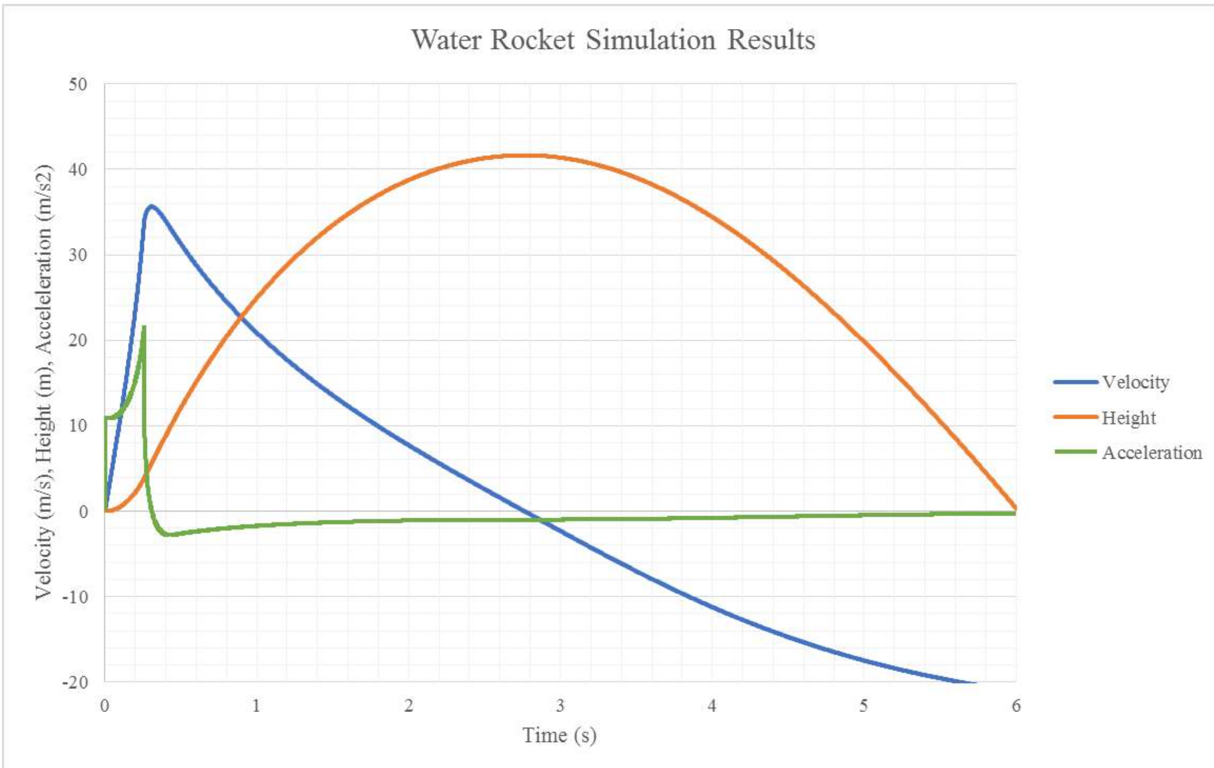


Figure B13: Simulation results with restricted nozzle, optimum mass and 78 psi. This graph helps to show how acceleration, velocity, and height change over time. Acceleration has a different behavior in this flight because the thrust forces are always greater than the weight and drag forces on the rocket. The acceleration exponentially increases until all of the water and air are expended.

Appendix C — Excel® Water Rocket Simulator Instructions

1. Determine the drag coefficient of the constructed rocket by either using a literature value or experimentally determining the value. Skip steps a-d if literature value is used.
    a. If experimentally determining the value, use a high speed camera that films to at least 1000 frames per second.
    b. Drop the rocket from a known height and film the drop with a high speed camera. Determine the fall duration from a film editing software.
    c. Plug in the input variables (blue cells) that were determined from the experimental testing into the first sheet of the simulator named "Drag Coefficient Finder". Also, ensure the values in the pink cells are appropriate.

Initial Parameters		
Value	*Inputs*	*Units*
251.3	*Mass of Empty Rocket*	g
1.008	*Duration of Fall*	s
4.886325	*Known Drop Height*	m
0.09398	*D of Bottle*	m
0.8	*Drag C*	
9.807	*g*	m/s2
1.2	*density a*	kg/m3
0.001	*Time Interval*	s

d. Change the drag coefficient until the known fall height matches the height in the results section of the simulator.

Initial Parameters		
Value	*Inputs*	*Units*
251.3	*Mass of Empty Rocket*	g
1.008	*Duration of Fall*	s
4.886325	*Known Drop Height*	m
0.09398	*D of Bottle*	m
0.8	*Drag C*	
9.807	*g*	m/s2
1.2	*density a*	kg/m3
0.001	*Time Interval*	s
Calculated		
0.006936825	*Area Bottle*	m2
0.2513	*Mass of empty rocket*	kg
Results		
4.876546599	*Height*	m

2. Enter all input variables (blue cells) into the second sheet of the simulator named "Ballast & Stress Design Check".

   a. Change the "Mass of Ballast" until the cell next to "Ballast Sufficiency" says, "GO" and is filled in green.

   b. To optimize for the weight of the ballast, decrease the mass of ballast value just before the ballast sufficiency cell turns red and reads "NO GO".

   c. Place this mass of sand/dirt into the nose cone of your rocket to act as the stabilizing ballast.

3. Switch over to the third sheet of the excel file named "Simulation Program NumMethods" and input the initial variables into the blue cells. Also verify the values in the pink cells are appropriate.

Initial Parameters		
Inputs	*Value*	*Units*
Volume of Water	400	mL
Total Volume	1030	mL
Mass of Empty Rocket	181.9	g
Launch Angle	90	Degrees
Initial Pressure	40	psi
D of Nozzle	0.02159	m
D of Bottle	0.09398	m
Temperature	5.56	C
density w	1000	kg/m3
k	1.4	
g	9.807	m/s2
Atmospheric Pressure	100000	Pa
density a	1.2	kg/m3
Drag C	0.345	
Time Interval	0.001	s
Time Max	6	s
R of air	288	kJ/kg*K
Nozzle Coefficient	1	
Calibration Factor	2.95	

a. Go back to sheet two and make sure that the stress values for the initial pressure are still all highlighted green. If any of the cells are red, then lower the initial pressure. If a different material then PET plastic is being used, determine maximum strength values for the material being used.

b. Note the values in the results section.

Results		
Velocity max	19.416222	m/s
Height max	20.497371	m
Impulse max	3.699572956	N*s
Thrust Duration	0.062	s

c. Note graphical relationships between variables of the rocket flight on the fourth sheet named "Graphed Results".

d. Perform experimental rocket flights recording each flight with a high speed camera with at least 120 fps capability.

e. Use the flight recordings to acquire flight durations using a film editing software that allows seeking frame by frame.

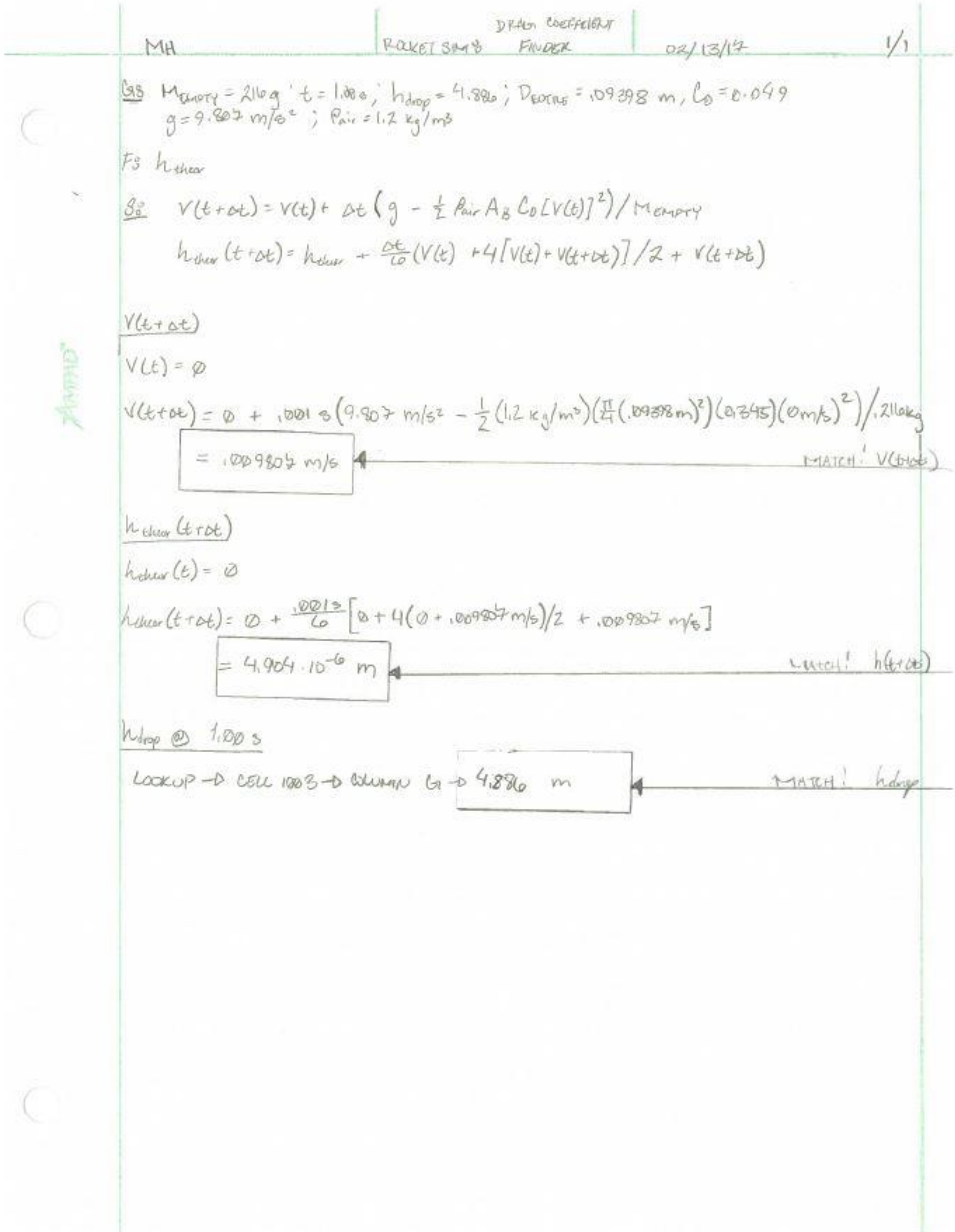
f. Use the fifth sheet of the excel file named "Height Finder" to determine the maximum height attained for each experimental launch. Do this by entering the duration into the indicated blue cell along with the other input parameters.

Initial Parameters		
Value	*Inputs*	*Units*
181.9	*Mass of Empty Rocket*	g
0.09398	*D of Bottle*	m
0.345	*Drag C*	
3.8333	*Duration of Flight*	s
1.2	*density a*	kg/m3
0.001	*Time Interval*	s
9.807	*g*	m/s2

Results		
17.20862158	*Height*	m

g. Compare results with experimental values and determine if a different calibration factor needs to be used. If so, retest in a different test environment to ensure the new calibration factor was appropriate.

Appendix D — Simulator Check Calculations



MH | ROCKET SIM 8 | DRAG COEFFICIENT FINDER | 02/13/17 | 1/1

GS $M_{EMPTY} = 216g$; $t = 1.00s$; $h_{drop} = 4.886$; $D_{EXTRE} = .09398$ m, $C_D = 0.049$
$g = 9.807\ m/s^2$; $\rho_{air} = 1.2\ kg/m^3$

FS h_{theor}

SS $V(t+\Delta t) = V(t) + \Delta t\left(g - \frac{1}{2}\rho_{air} A_B C_D [V(t)]^2\right)/M_{EMPTY}$

$h_{theor}(t+\Delta t) = h_{theor} + \frac{\Delta t}{6}(V(t) + 4[V(t) + V(t+\Delta t)]/2 + V(t+\Delta t))$

$V(t+\Delta t)$

$V(t) = \emptyset$

$V(t+\Delta t) = \emptyset + .001\,s\left(9.807\ m/s^2 - \frac{1}{2}(1.2\ kg/m^3)\left(\frac{\pi}{4}(.09398\ m)^2\right)(0.345)(0\ m/s)^2\right)/.216\ kg$

$= .009807\ m/s$ ← MATCH! $V(t+\Delta t)$

$h_{theor}(t+\Delta t)$

$h_{theor}(t) = \emptyset$

$h_{theor}(t+\Delta t) = \emptyset + \frac{.001\,s}{6}\left[\emptyset + 4(\emptyset + .009807\ m/s)/2 + .009807\ m/s\right]$

$= 4.904 \cdot 10^{-6}$ m ← MATCH! $h(t+\Delta t)$

h_{drop} @ 1.00 s

LOOKUP → CELL 1003 → COLUMN G → 4.886 m ← MATCH! h_{drop}

Figure D1. Drag-coefficient-finder check calculation.

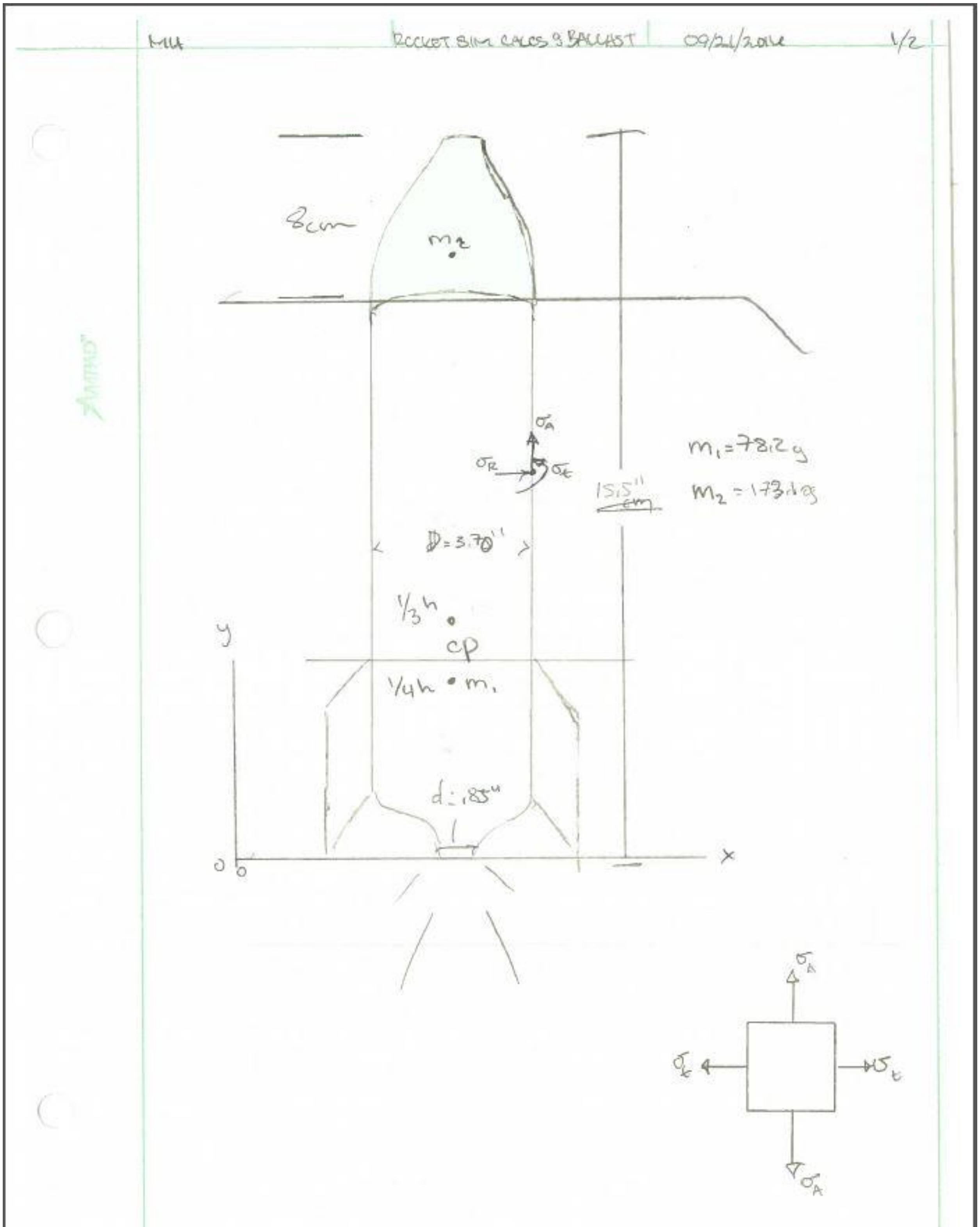


Figure D2. Water rocket geometry.

MH — ROCKET SIM CALCS & BALLAST — 09/21/2014 — 2/2

CENTER OF PRESSURE

$$C_p \approx 1/3(\text{HEIGHT OF ROCKET}) = \boxed{1/3\, y}$$

$1/3(22\text{cm}) = \underline{7.33\text{cm}}$

CENTER OF MASS

$$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} = \frac{(y_{m_1} \cdot m_1) + (y_{m_2} \cdot m_2)}{m_1 + m_2}$$

$$= \frac{(1/4\, y)\cdot m_1 + (y - y_{2}/2)\, m_2}{m_1 + m_2}$$

SAMPLE CALC

$$= \frac{(1/4(22\text{cm}))(20\text{g}) + \left(22\text{cm} - \frac{6.5\text{cm}}{2}\right)(100\text{g})}{20\text{g} + 100\text{g}}$$

$\bar{y} = \underline{16.54\text{ cm}}$

*IF $\bar{y} > C_p$, THEN $\boxed{\text{GOOD}}$

ROCKET STRESS STATES

THIN WALLED → $\sigma_t = \dfrac{P_i\left(\frac{r_i + r_o}{2}\right)}{t}$ $\quad \sigma_a = \dfrac{P_i\left(\frac{r_i + r_o}{2}\right)}{2t}$ $\quad \sigma_R = -P_i$

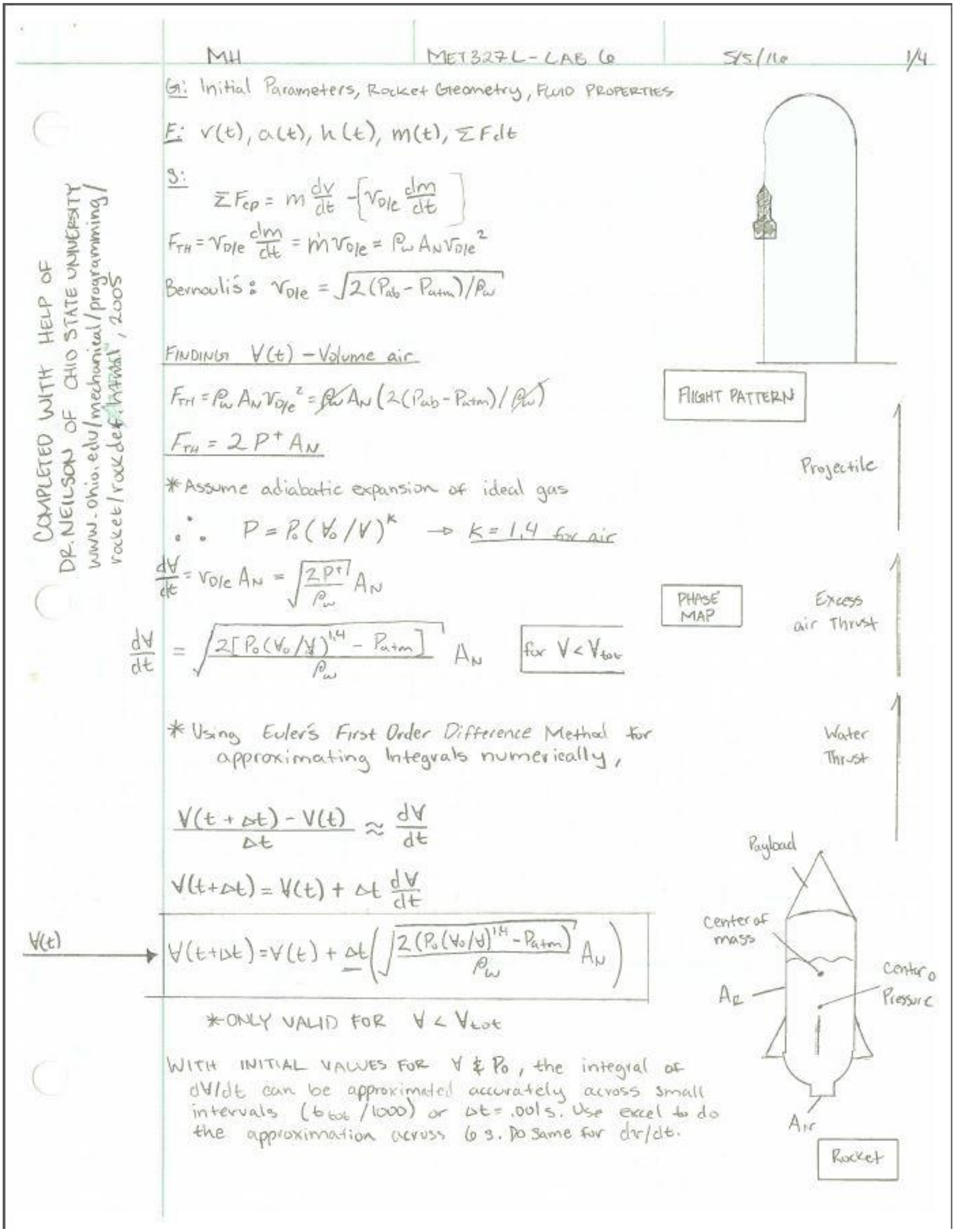
$\sigma_t = \dfrac{(80\text{psi})\left(\frac{2.7 + 2.75}{2}\right)}{.05''} = \underline{4360\text{ psi}}$ $\quad \sigma_a = \dfrac{(80\text{psi})\left(\frac{2.7+2.75}{2}\right)}{2(.05'')} = \underline{2180\text{ psi}}$

THICK WALLED → $\sigma_t = P_i\left(\dfrac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$ $\quad \sigma_a = P_i\left(\dfrac{r_i^2}{r_o^2 - r_i^2}\right)$ $\quad \sigma_r = -P_i$

$\sigma_t = (80\text{psi})\left(\dfrac{2.75^2 + 2.25^2}{2.75^2 - 2.25^2}\right) = \underline{404\text{ psi}}$ $\quad \sigma_a = (80\text{psi})\left(\dfrac{2.25^2}{2.75^2 - 2.25^2}\right) = \underline{162\text{ psi}}$

$\tau = 1/2\, \sigma_{max} = 1/2\,(404\text{psi}) = \underline{202\text{ psi}}$

Figure D3. Ballast and stress check calculations.



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COMPLETED WITH HELP OF DR. NEILSON OF OHIO STATE UNIVERSITY www.ohio.edu/mechanical/programming/rocket/rocket/.html, 2005

G: Initial Parameters, Rocket Geometry, FLUID PROPERTIES

F: $v(t), a(t), h(t), m(t), \Sigma F dt$

S:

$$\Sigma F_{cp} = m\frac{dv}{dt} - \left\{ v_{D/e}\frac{dm}{dt} \right\}$$

$$F_{TH} = v_{D/e}\frac{dm}{dt} = \dot{m} v_{D/e} = \rho_w A_N v_{D/e}^2$$

Bernoulli's: $v_{D/e} = \sqrt{2(P_{ab} - P_{atm})/\rho_w}$

FINDING $V(t)$ - Volume air

$$F_{TH} = \rho_w A_N v_{D/e}^2 = \rho_w A_N \left(2(P_{ab} - P_{atm})/\rho_w\right)$$

$$F_{TH} = 2P^+ A_N$$

*Assume adiabatic expansion of ideal gas

$$\therefore \quad P = P_0 (V_0 / V)^k \rightarrow k = 1.4 \text{ for air}$$

$$\frac{dV}{dt} = v_{D/e} A_N = \sqrt{\frac{2P^+}{\rho_w}} A_N$$

$$\frac{dV}{dt} = \sqrt{\frac{2[P_0(V_0/V)^{1.4} - P_{atm}]}{\rho_w}} A_N \quad \text{for } V < V_{tot}$$

* Using Euler's First Order Difference Method for approximating Integrals numerically,

$$\frac{V(t+\Delta t) - V(t)}{\Delta t} \approx \frac{dV}{dt}$$

$$V(t+\Delta t) = V(t) + \Delta t \frac{dV}{dt}$$

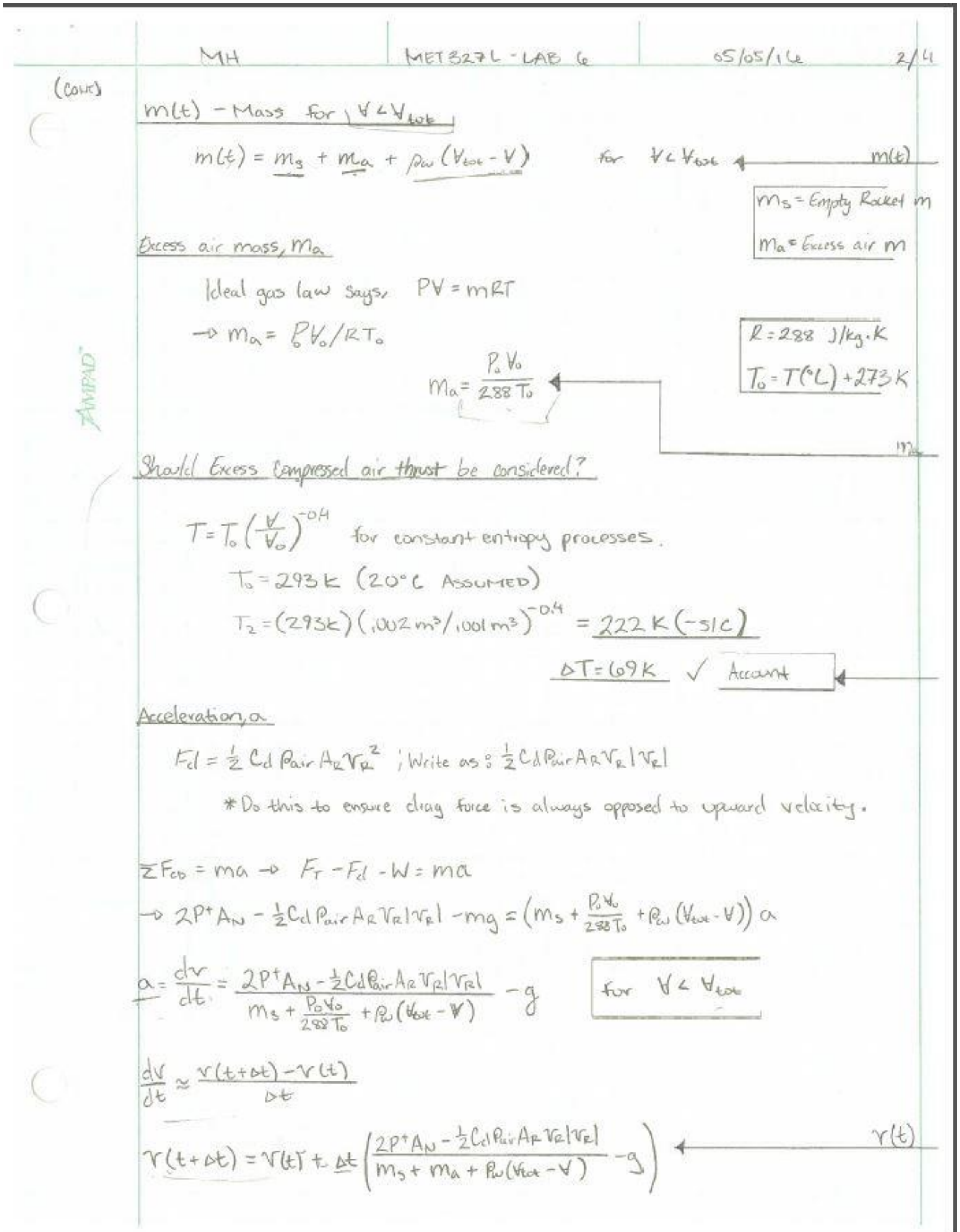
$V(t)$ →

$$V(t+\Delta t) = V(t) + \Delta t \left(\sqrt{\frac{2(P_0(V_0/V)^{1.4} - P_{atm})}{\rho_w}} A_N \right)$$

*ONLY VALID FOR $V < V_{tot}$

WITH INITIAL VALUES FOR V & P_0, the integral of dV/dt can be approximated accurately across small intervals ($t_{tot}/1000$) or $\Delta t = .001$ s. Use excel to do the approximation across 6 s. Do same for dv/dt.

Figure D4. Volume derivations for simulator.



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(cont)

m(t) - Mass for $\forall < \forall_{tot}$

$$m(t) = m_s + m_a + \rho_w(\forall_{tot} - \forall) \quad \text{for } \forall < \forall_{tot}$$

m(t)

m_s = Empty Rocket m

m_a = Excess air m

Excess air mass, m_a

Ideal gas law says, $P\forall = mRT$

$\rightarrow m_a = P_o\forall_o / RT_o$

$R = 288$ J/kg·K

$T_o = T(°C) + 273$ K

$$m_a = \frac{P_o \forall_o}{288\, T_o}$$

m_a

Should Excess compressed air thrust be considered?

$$T = T_o\left(\frac{\forall}{\forall_o}\right)^{-0.4} \text{ for constant entropy processes.}$$

$T_o = 293$ K (20°C ASSUMED)

$$T_2 = (293\text{K})(.002\,m^3/.001\,m^3)^{-0.4} = 222\text{ K } (-51\text{C})$$

$\Delta T = 69$ K ✓ Account

Acceleration, a

$$F_d = \tfrac{1}{2} C_d \rho_{air} A_R V_R^2 \text{ ; Write as: } \tfrac{1}{2} C_d \rho_{air} A_R V_R |V_R|$$

*Do this to ensure drag force is always opposed to upward velocity.

$$\Sigma F_{cv} = ma \rightarrow F_T - F_d - W = ma$$

$$\rightarrow 2P^+A_N - \tfrac{1}{2}C_d\rho_{air}A_R V_R|V_R| - mg = \left(m_s + \frac{P_o\forall_o}{288\,T_o} + \rho_w(\forall_{tot} - \forall)\right)a$$

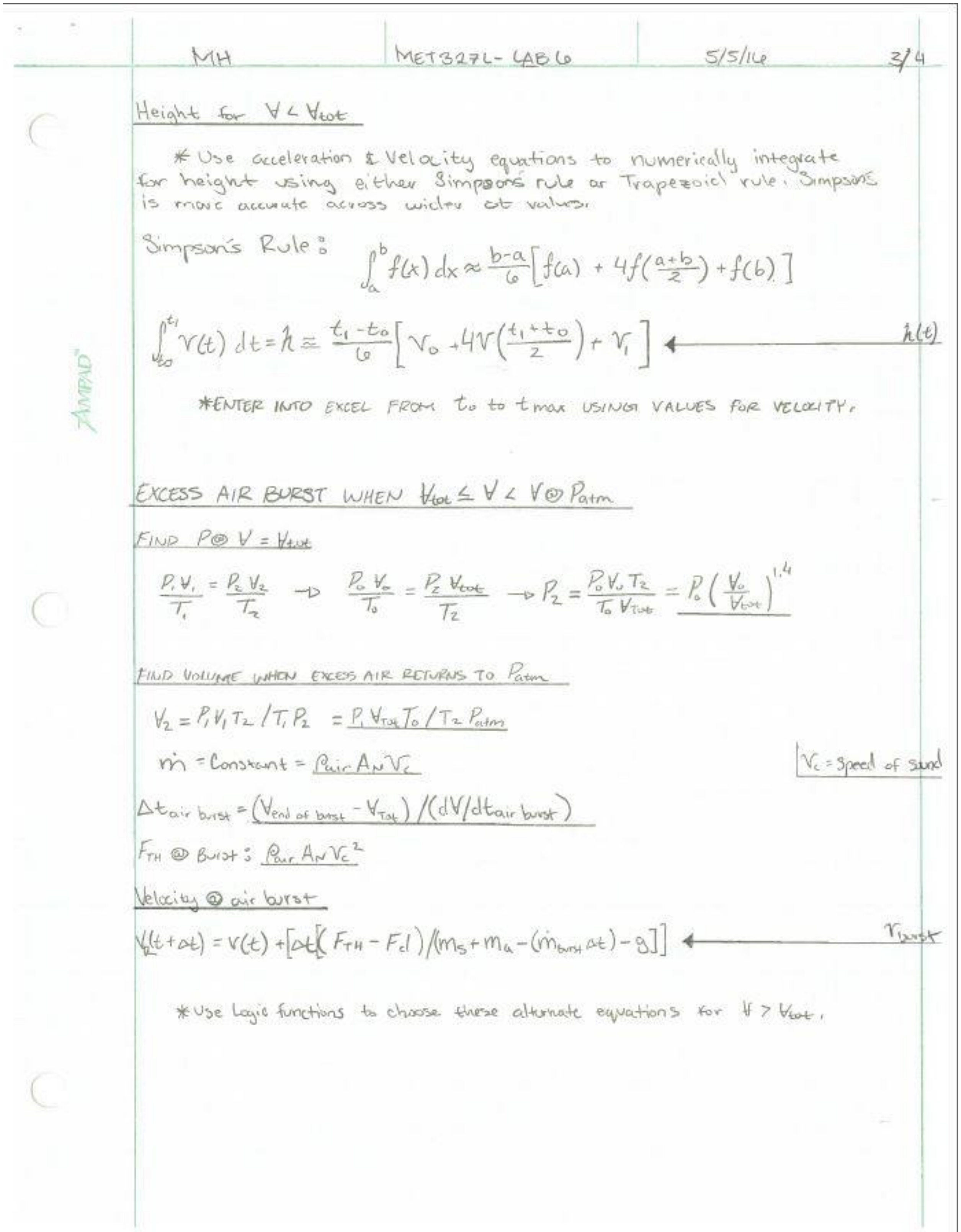
$$a = \frac{dv}{dt} = \frac{2P^+A_N - \tfrac{1}{2}C_d\rho_{air}A_R V_R|V_R|}{m_s + \frac{P_o\forall_o}{288\,T_o} + \rho_w(\forall_{tot} - \forall)} - g \quad \text{for } \forall < \forall_{tot}$$

$$\frac{dv}{dt} \approx \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

$$v(t+\Delta t) = v(t) + \Delta t\left(\frac{2P^+A_N - \tfrac{1}{2}C_d\rho_{air}A_R V_R|V_R|}{m_s + m_a + \rho_w(\forall_{tot} - \forall)} - g\right)$$

v(t)

Figure D5. Mass and acceleration derivations.



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Height for $\forall < \forall_{tot}$

* Use acceleration & velocity equations to numerically integrate for height using either Simpson's rule or Trapezoid rule. Simpsons is more accurate across wider Δt values.

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$\int_{t_0}^{t_1} v(t)\,dt = h \approx \frac{t_1 - t_0}{6}\left[V_0 + 4V\left(\frac{t_1+t_0}{2}\right) + V_1\right] \leftarrow h(t)$$

*ENTER INTO EXCEL FROM t_0 to t_{max} USING VALUES FOR VELOCITY.

EXCESS AIR BURST WHEN $\forall_{tot} \leq \forall < \forall$ @ P_{atm}

FIND P @ $\forall = \forall_{tot}$

$$\frac{P_1 \forall_1}{T_1} = \frac{P_2 \forall_2}{T_2} \rightarrow \frac{P_0 \forall_0}{T_0} = \frac{P_2 \forall_{tot}}{T_2} \rightarrow P_2 = \frac{P_0 \forall_0 T_2}{T_0 \forall_{Tot}} = P_0\left(\frac{\forall_0}{\forall_{tot}}\right)^{1.4}$$

FIND VOLUME WHEN EXCESS AIR RETURNS TO P_{atm}

$\forall_2 = P_1 \forall_1 T_2 / T_1 P_2 = P_1 \forall_{Tot} T_0 / T_2 P_{atm}$

$\dot{m}$ = Constant = $\rho_{air} A_N V_c$ | V_c = speed of sound

$\Delta t_{air\,burst} = (\forall_{end\,of\,burst} - \forall_{Tot}) / (d\forall / dt_{air\,burst})$

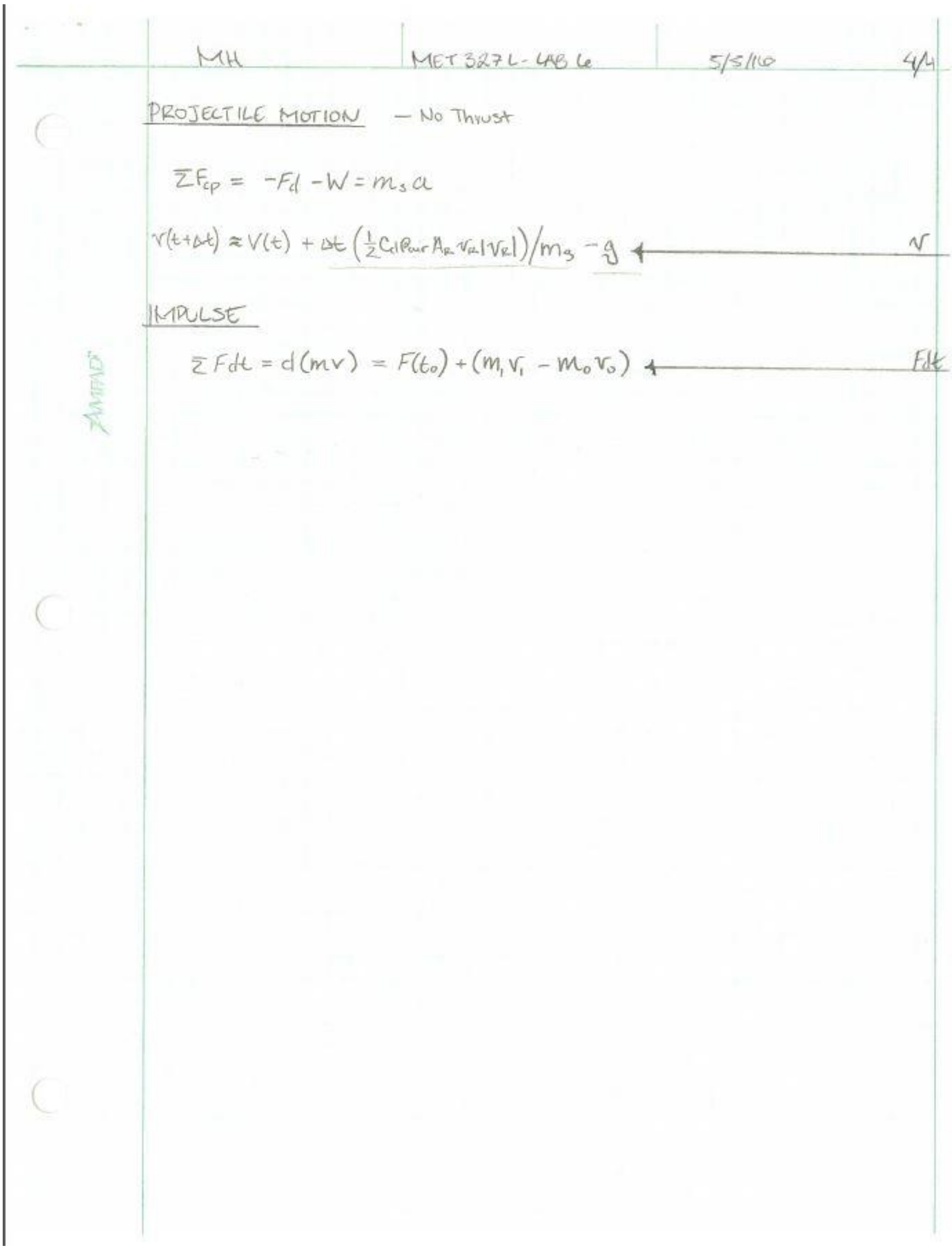
F_{TH} @ Burst: $\rho_{air} A_N V_c^2$

Velocity @ air burst

$$V_2(t+\Delta t) = v(t) + \left[\Delta t\left(\left(F_{TH} - F_d\right) / \left(m_s + m_a - (\dot{m}_{burst}\Delta t\right) - g\right)\right] \leftarrow v_{burst}$$

* Use Logic functions to choose these alternate equations for $\forall > \forall_{tot}$.

Figure D6. Height and phase 2 thrust derivations.



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PROJECTILE MOTION — No Thrust

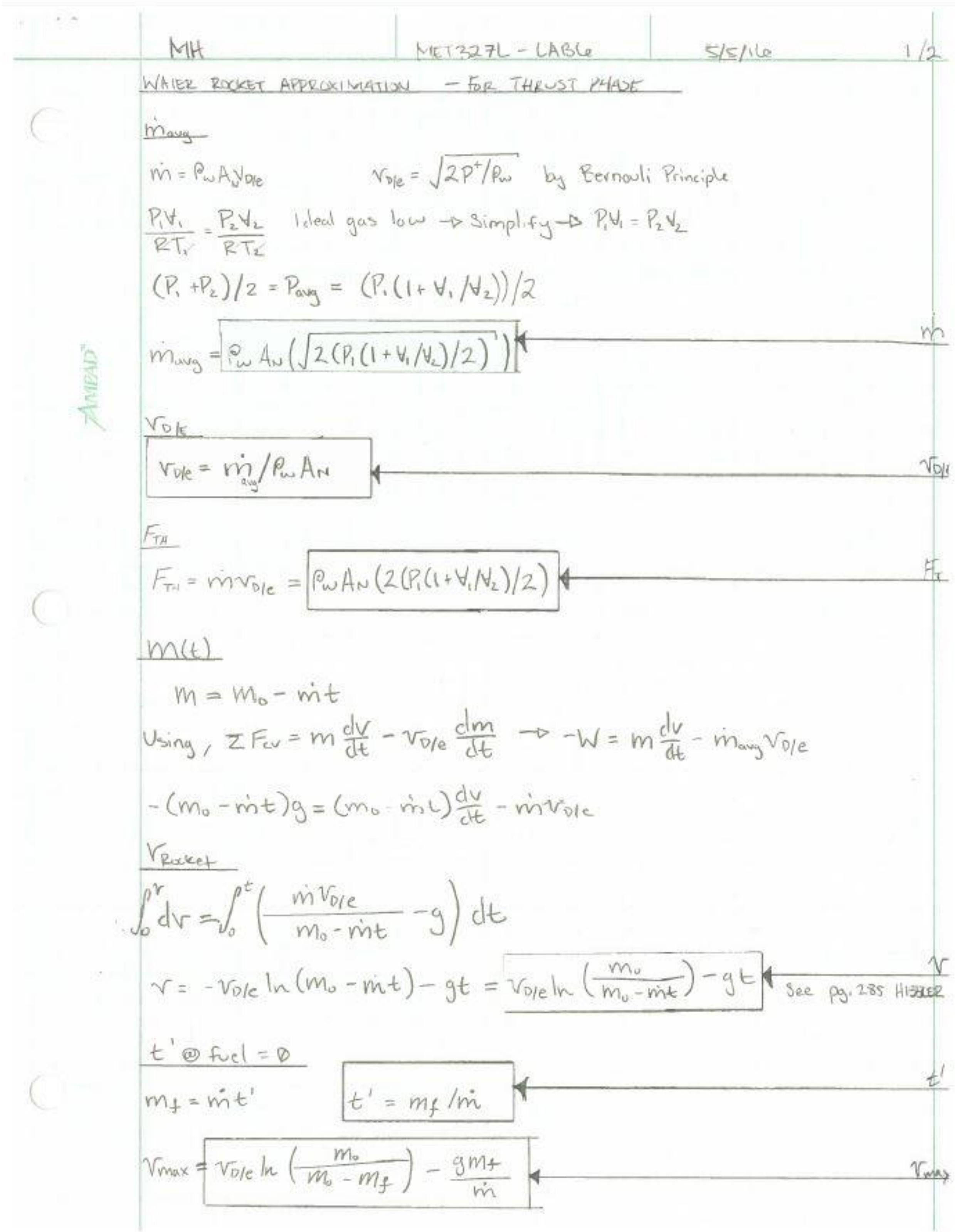
$$\Sigma F_{cp} = -F_d - W = m_s a$$

$$v(t+\Delta t) \approx V(t) + \Delta t\left(\tfrac{1}{2} C_d \rho_{air} A_R v_{rel}|v_{rel}|\right)/m_s - g \leftarrow v$$

IMPULSE

$$\Sigma F dt = d(mv) = F(t_0) + (m_1 v_1 - m_0 v_0) \leftarrow Fdt$$

Figure D7. Phase 3 projectile motion and impulse derivations.



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WATER ROCKET APPROXIMATION — FOR THRUST PHASE

$\dot{m}_{avg}$

$\dot{m} = \rho_w A_N v_{D/e}$ $v_{D/e} = \sqrt{2P^+/\rho_w}$ by Bernoulli Principle

$\frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$ Ideal gas law → Simplify → $P_1V_1 = P_2V_2$

$(P_1 + P_2)/2 = P_{avg} = (P_1(1 + V_1/V_2))/2$

$\dot{m}_{avg} = \boxed{\rho_w A_N\left(\sqrt{2(P_1(1+V_1/V_2)/2)}\right)}$ ← $\dot{m}$

$v_{D/e}$

$\boxed{v_{D/e} = \dot{m}_{avg}/\rho_w A_N}$ ← $v_{D/e}$

F_{TH}

$F_{TH} = \dot{m} v_{D/e} = \boxed{\rho_w A_N (2(P_1(1+V_1/V_2)/2)}$ ← F_T

$m(t)$

$m = m_0 - \dot{m}t$

Using, $\Sigma F_{cv} = m\frac{dv}{dt} - v_{D/e}\frac{dm}{dt}$ → $-W = m\frac{dv}{dt} - \dot{m}_{avg} v_{D/e}$

$-(m_0 - \dot{m}t)g = (m_0 - \dot{m}t)\frac{dv}{dt} - \dot{m}v_{D/e}$

V_{Rocket}

$$\int_0^v dv = \int_0^t \left(\frac{\dot{m}v_{D/e}}{m_0 - \dot{m}t} - g\right) dt$$

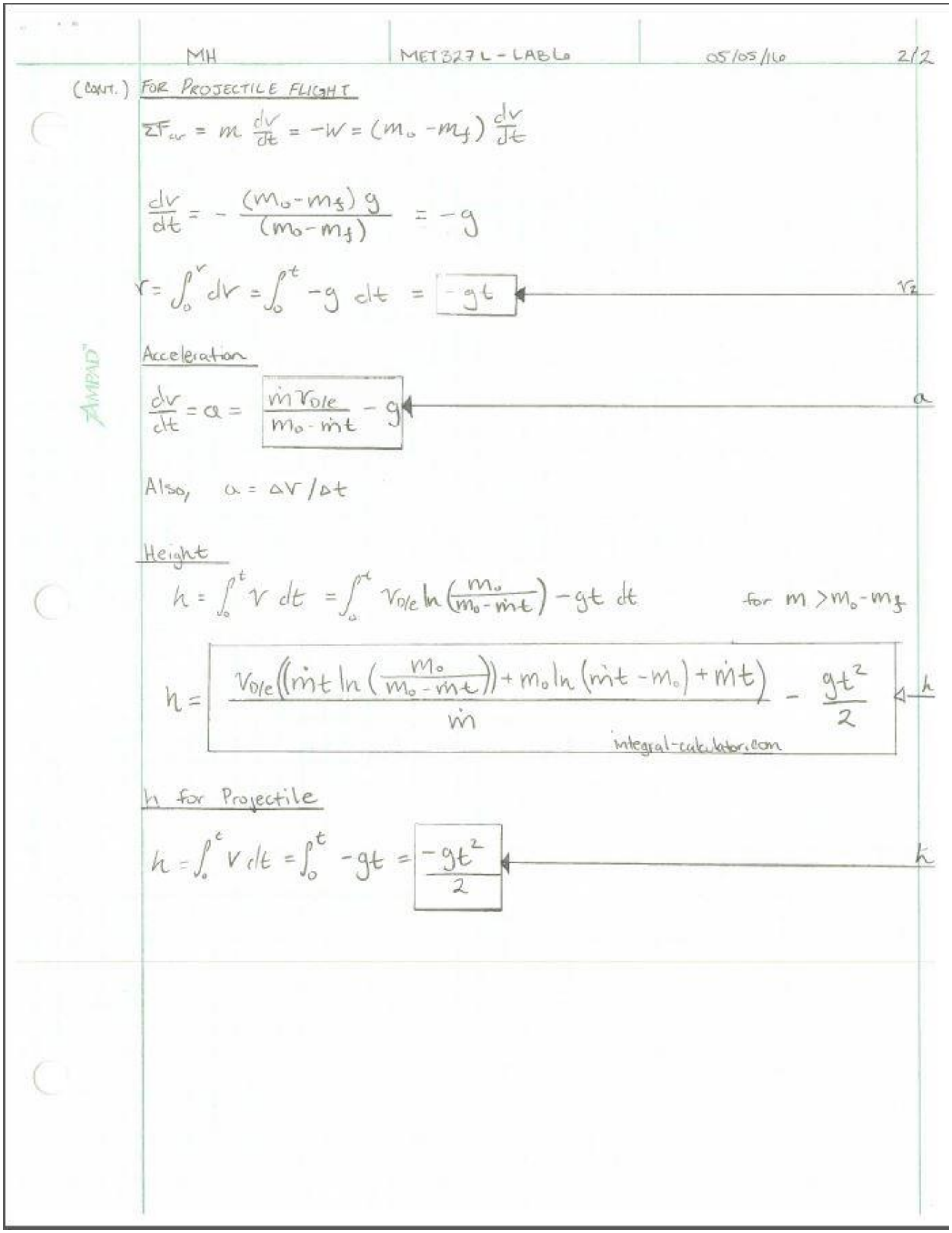
$v = -v_{D/e}\ln(m_0 - \dot{m}t) - gt = \boxed{v_{D/e}\ln\left(\frac{m_0}{m_0 - \dot{m}t}\right) - gt}$ ← v See pg. 285 HIBBELER

t' @ fuel = Ø

$m_f = \dot{m}t'$ $\boxed{t' = m_f/\dot{m}}$ ← t'

$v_{max} = \boxed{v_{D/e}\ln\left(\frac{m_0}{m_0 - m_f}\right) - \frac{gm_f}{\dot{m}}}$ ← v_{max}

Figure D8. Approximated rocket derivations 1.



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(CONT.) FOR PROJECTILE FLIGHT

$$\Sigma F_{cv} = m\frac{dv}{dt} = -W = (m_o - m_f)\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{(m_o - m_f)g}{(m_o - m_f)} = -g$$

$$v = \int_0^v dv = \int_0^t -g\,dt = -gt$$ ← v_2

Acceleration

$$\frac{dv}{dt} = a = \frac{\dot{m}v_{o/e}}{m_o - \dot{m}t} - g$$ ← a

Also, $a = \Delta v / \Delta t$

Height

$$h = \int_0^t v\,dt = \int_0^t v_{o/e}\ln\left(\frac{m_o}{m_o - \dot{m}t}\right) - gt\,dt \quad \text{for } m > m_o - m_f$$

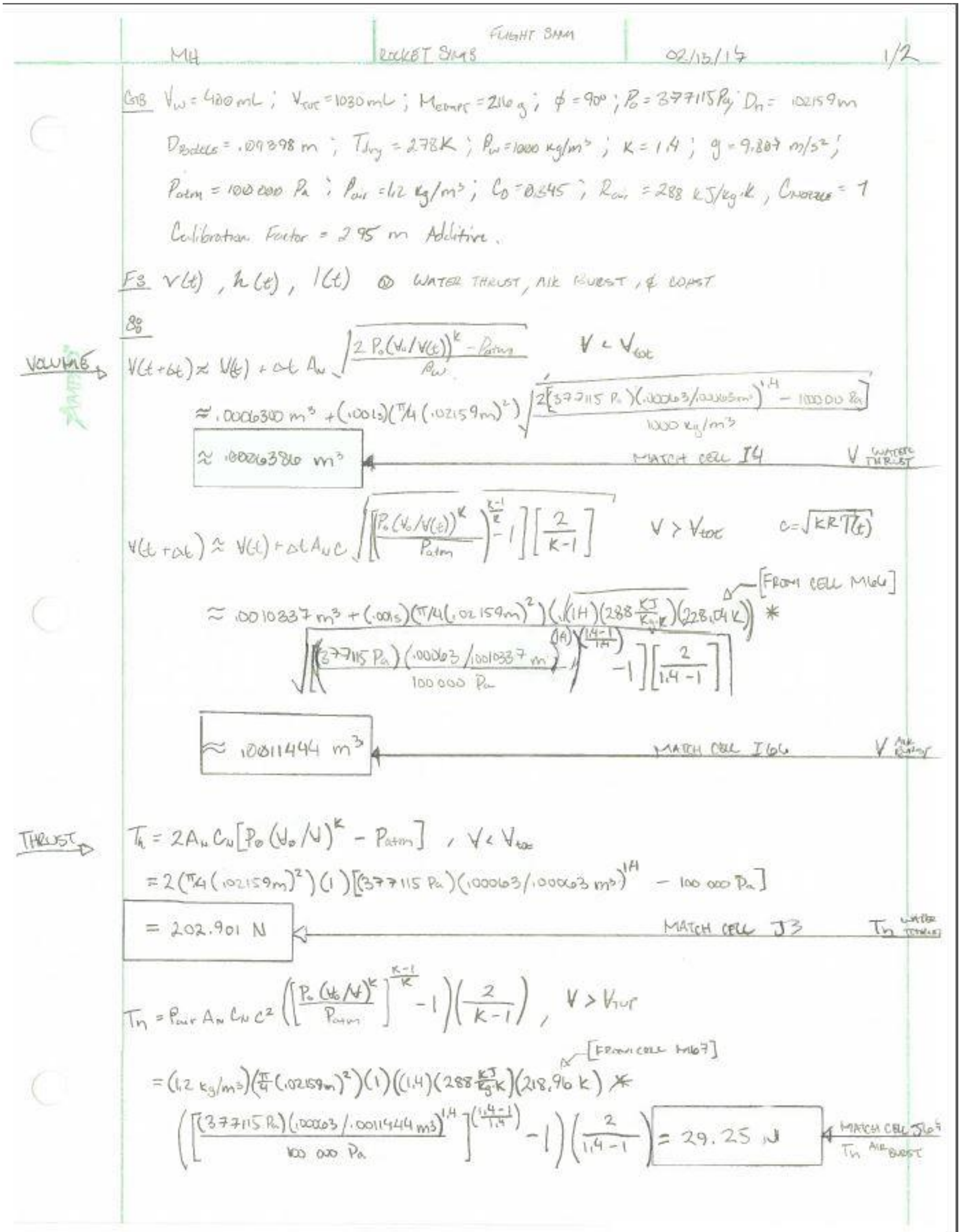
$$h = \frac{v_{o/e}\left(\left(\dot{m}t\ln\left(\frac{m_o}{m_o - \dot{m}t}\right)\right) + m_o\ln(\dot{m}t - m_o) + \dot{m}t\right)}{\dot{m}} - \frac{gt^2}{2}$$ ← h

integral-calculator.com

h for Projectile

$$h = \int_0^t v\,dt = \int_0^t -gt = \frac{-gt^2}{2}$$ ← h

Figure D9. Approximated rocket derivations 2.



MH | ROCKET SIMS (FLIGHT 3MM) | 02/15/17 | 1/2

GB $V_w = 400\,mL$; $V_{tot} = 1030\,mL$; $M_{empty} = 216\,g$; $\phi = 90°$; $P_0 = 377115\,Pa$; $D_n = .02159\,m$

$D_{bottle} = .09398\,m$; $T_{dry} = 278\,K$; $\rho_w = 1000\,kg/m^3$; $k = 1.4$; $g = 9.807\,m/s^2$;

$P_{atm} = 100\,000\,Pa$; $\rho_{air} = 1.2\,kg/m^3$; $C_D = 0.345$; $R_{air} = 288\,kJ/kg{\cdot}K$, $C_{nozzle} = 1$

Calibration Factor = 2.95 m Additive.

FS $v(t)$, $h(t)$, $l(t)$ @ WATER THRUST, AIR BURST, & COAST

SS

VOLUME → $V(t+\Delta t) \approx V(t) + \Delta t\, A_n \sqrt{\dfrac{2\left[P_0 (V_0/V(t))^k - P_{atm}\right]}{\rho_w}}$ $V < V_{tot}$

$\approx .0006300\,m^3 + (.0013)(\pi/4\,(.02159\,m)^2)\sqrt{\dfrac{2\left[(377115\,Pa)(.000063/.000063\,m^3)^{1.4} - 100\,000\,Pa\right]}{1000\,kg/m^3}}$

$\approx .0006386\,m^3$ ← MATCH CELL I4 V WATER THRUST

$V(t+\Delta t) \approx V(t) + \Delta t\, A_n c \sqrt{\left[\left(\dfrac{P_0 (V_0/V(t))^k}{P_{atm}}\right)^{\frac{k-1}{k}} - 1\right]\left[\dfrac{2}{k-1}\right]}$ $V > V_{tot}$ $c = \sqrt{kRT(t)}$

$\approx .0010337\,m^3 + (.0015)(\pi/4(.02159\,m)^2)\left(\sqrt{(1.4)(288\,\tfrac{kJ}{kg\,K})(228.04\,K)}\right)$ * [FROM CELL M66]

$\sqrt{\left[\left(\dfrac{(377115\,Pa)(.00063/.0010337\,m^3)^{(1.4)}}{100\,000\,Pa}\right)^{\left(\frac{1.4-1}{1.4}\right)} - 1\right]\left[\dfrac{2}{1.4-1}\right]}$

$\approx .0011444\,m^3$ ← MATCH CELL I66 V AIR BURST

THRUST → $T_h = 2A_n C_n\left[P_0 (V_0/V)^k - P_{atm}\right]$, $V < V_{tot}$

$= 2(\pi/4\,(.02159\,m)^2)(1)\left[(377115\,Pa)(.00063/.00063\,m^3)^{1.4} - 100\,000\,Pa\right]$

$= 202.901\,N$ ← MATCH CELL J3 T_h WATER THRUST

$T_h = \rho_{air} A_n C_n c^2 \left(\left[\dfrac{P_0 (V_0/V)^k}{P_{atm}}\right]^{\frac{k-1}{k}} - 1\right)\left(\dfrac{2}{k-1}\right)$, $V > V_{tot}$

$= (1.2\,kg/m^3)(\tfrac{\pi}{4}(.02159\,m)^2)(1)((1.4)(288\,\tfrac{kJ}{kg}K)(218.96\,K))$ * [FROM CELL M67]

$\left(\left[\dfrac{(377115\,Pa)(.00063/.0011444\,m^3)^{1.4}}{100\,000\,Pa}\right]^{\left(\frac{1.4-1}{1.4}\right)} - 1\right)\left(\dfrac{2}{1.4-1}\right) = 29.25\,N$ ← MATCH CELL J67 T_h AIR BURST

Figure D10. Simulator check calculations 1.

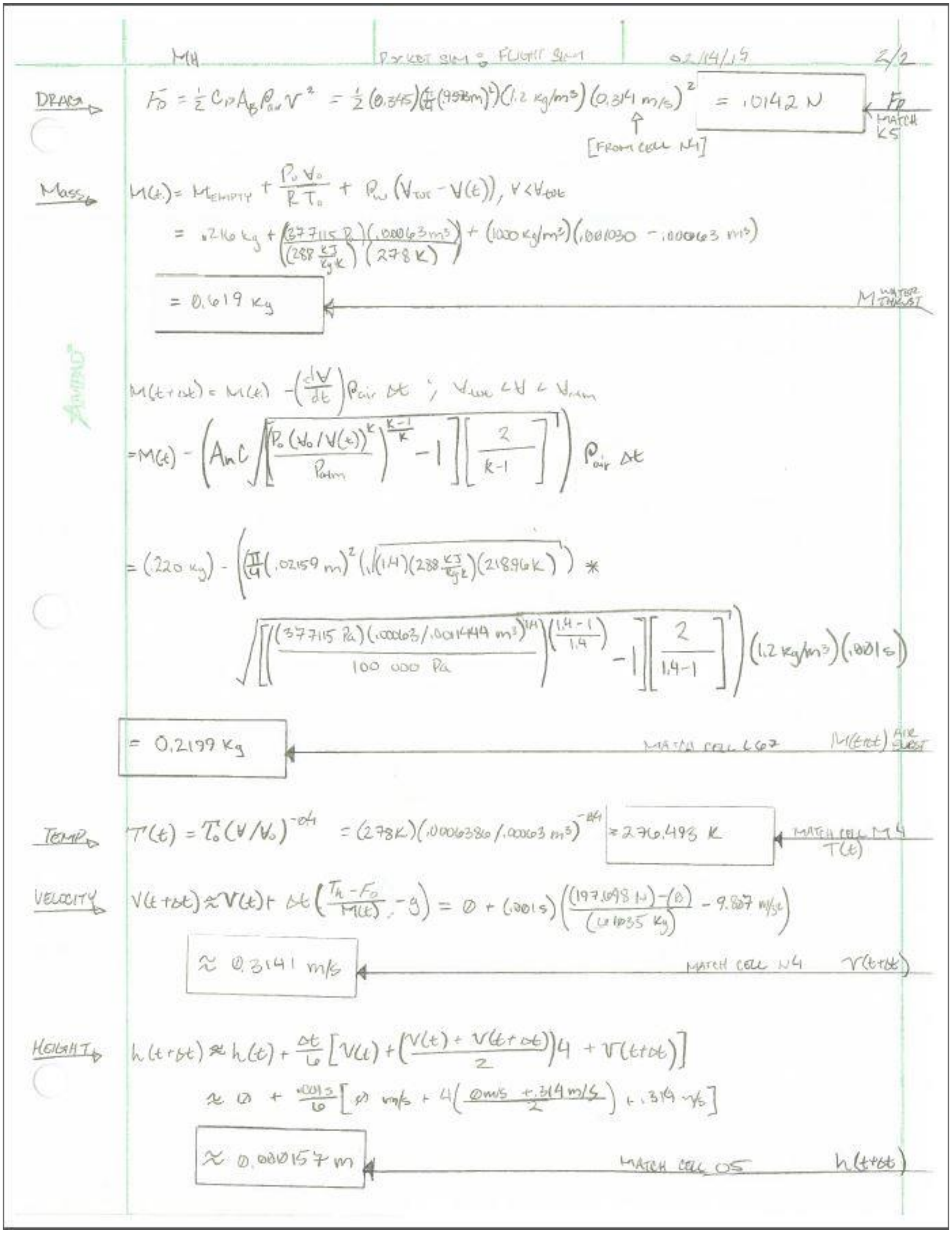


Figure D11. Simulator check calculations 2.