DEVELOPMENT OF FRAGILITY CURVES USING HIGH DIMENSIONAL MODEL REPRESENTATION

U. Vipin Unnithan[1], A. Meher Prasad[2] and B. N. Rao[3]

[1] *Student, Dept. of Civil Engineering , Indian Institute of Technology Madras, Chennai, India*
[2] *Professor, Dept. of Civil Engineering, Indian Institute of Technology Madras, Chennai ,India*
[3] *Associate Professor, Dept. of Civil Engineering, Indian Institute of Technology Madras, Chennai ,India*
Email: unnithan_vipin@yahoo.com , prasadam@iitm.ac.in

ABSTRACT :

Fragility curves represent the conditional probability that a structure's response may exceed the performance limit for a given ground motion intensity. The use of fragility curves for the assessment of seismic losses is in increasing demand, both for pre-earthquake disaster planning as well as post-earthquake recovery and retrofitting programmes. Conventional methods for computing building fragilities are either based on statistical extrapolation of detailed analyses on specific buildings or based on Direct Monte Carlo simulation with these models. The Monte Carlo technique usually requires a relatively large number of simulations in order to obtain a sufficiently reliable estimate of the fragilities making it computationally expensive and time consuming. High Dimensional Model Representation (HDMR) response surface methodology on the other hand simplifies the process of fragility computation. It is used to replace the algorithmic performance-function with an explicit functional relationship, fitting a functional approximation, thereby reducing the number of expensive numerical analyses. After the functional approximation has been made, Monte Carlo simulation is used to obtain the fragility curve of the system.

KEYWORDS: Metamodel, Monte Carlo Simulation, High Dimensional Model Representation, Fragility Curves

1. INTRODUCTION

Fragility analysis was originally employed to evaluate the seismic safety of nuclear power plants and was later accepted as a reliable method for the evaluation of the seismic performance of civil infrastructures like bridges and buildings [3]. The probability of structural damage due to earthquakes are estimated as a function of ground motion indices or various design parameters, such as peak ground acceleration (PGA), elastic pseudo-spectral acceleration *(S_a)*, and elastic spectral displacement *(S_d)*. Fragility curves, which is an important tool in evaluating seismic damage in structures, represents the conditional probability that the response of a structure may exceed the performance limit for a given ground motion intensity. On the basis of these performance limits, the damage states of the building can be generally classified into four categories: slight damage (SD), moderate damage (MD), extensive damage (ED), and complete damage (CD).

Fragility curves can be derived using empirical or analytical methods, based on the source of the data and the type of analysis. The former is based on the interpretation of test data and engineering judgement while the latter uses the response of a structure to a set of ground motions. The response can be either from a linear or nonlinear time history analysis, elastic spectral analysis or nonlinear static analysis. Analytical fragility curves are used when the actual earthquake damage data is limited and when there is a lack of sufficient statistical information. Analytical fragility curves are developed from seismic response data obtained from the computational model of the structure analyzed using simulated ground motions. It generally involves the identification and incorporation of aleatory uncertainties (due to the randomness in construction material properties) and uncertainties from earthquakes incorporated into the structural model, followed by a statistical approach to obtain a probabilistic description of the structural response. In this work, we develop the fragility curve for a three bay six storey reinforced concrete plane frame designed according to Indian

Standard Code (IS-456) using the HDMR technique.

2. DEVELOPMENT OF FRAGILITY CURVES

There has been numerous works in literature for evaluating the fragility curves of structures, like Shinozuka et al. [6], Yamazaki et al.[5], and Kiremidjian et al. [6], however there is no unified approach available. Conventional means of generating fragility curve involves the use of Monte Carlo Simulation and other sampling techniques [12]. Monte Carlo (MC) simulation is one of the most direct and accurate means of statistical analysis to obtain probabilistic description of response [8]. It is a crude simulation technique, in which values for random input variables are generated based on its statistical parameters and probability distributions, are used to simulate different scenarios of the problem. In Monte Carlo simulations, the random selection process is repeated such that each time a value is randomly selected, it forms one possible scenario and the outcome of that scenario is to be evaluated. For probabilistic seismic response analysis, structural uncertainty parameters and seismic inputs are randomly selected based on their probability distributions, to form different structure-earthquake combinations. Seismic analysis performed on each combination (linear or nonlinear time history analysis, elastic-spectral analysis or nonlinear static analysis.) yields the seismic response of interest. When this entire process is carried out repeatedly for hundreds or thousands of times, probabilistic descriptions of seismic response can be formulated and probabilities of response exceeding certain performance limits can be obtained. Although MC Simulation is very accurate, it requires a relatively large number of simulations in order to obtain a sufficiently reliable estimate of the probability of damage [15]. Depending on the function being evaluated, the number of simulations should be of the order of 10,000 to 20,000 for approximately 95% confidence limit [11].

Probabilistic seismic response analysis using MC simulations are computationally very expensive and time consuming, as thousands of nonlinear time-history analysis needs to be simulated. Alternate methods to obtain fragility curves have been proposed by Schotanus et al. (2002) and Towashiraporn et al. (2004), wherein the metamodels formulated using response surface methods are used to approximate seismic response analysis and MC simulation is applied over this metamodel to obtain the probability distribution of the response. This lead to significant reduction in the computational time required for the generation of fragility curves. [13 & 15].

2.1 High Dimensional Model Representation (HDMR)

High Dimensional Model Representation (HDMR) is a tool developed to express input-output relations of complex, computationally burdensome models in terms of hierarchical correlated function expansions [10]. Application of HDMR methodology to complex nonlinear model provides an efficient means to obtain an accurate reduced model of the original system. The uncertainty analysis of the outputs of the computationally burdensome model can then be well approximated by MC simulation of the corresponding reduced model outputs, which is thus performed at a much lower computational cost without compromising accuracy. It is a general set of quantitative model assessment and analysis tool for capturing the high dimensional relationships between sets of input and output model variables, where the order of input variables may go up to order of $10^2 \sim 10^3$ or more [12].

As an example, let an N-dimensional vector $x = \left\{x_1, x_2, x_3........x_n\right\}$ be the input variable and $f(x)$ be the output variable. HDMR expresses the output variable as a hierarchical correlated function expanded in terms of the input variables as given in Eqn 2.1

$$f(x) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1\le i\le j\le N} f_{ij}(x_i, x_j) + \sum_{1\le i\le j\le k\le N} f_{ij}(x_i, x_j, x_k) + ... + f_{1,2....N}(x_1, x_2, ..., x_N) \quad (2.1)$$

where, f_0 denotes the mean response or response of the system evaluated at a reference point. The function

$f_i(x_i)$ is a first-order term expressing the effect of variable x_i acting alone, although generally nonlinear, on the output $f(x)$. The function $f_{i_1 i_2}(x_{i_1}, x_{i_2})$ is a second-order term which describes the cooperative effects of the variables x_{i_1} and x_{i_2} on $f(x)$. The higher order terms gives the cooperative effects of increasing numbers of input variables acting together to influence the output $f(x)$. The last term $f_{12....N}(x_1, x_2, ..., x_N)$ contains any residual dependence of all the input variables locked together in a cooperative way to influence the output $f(x)$. Once all the relevant component functions in eqn (2.1) are determined and suitably represented, the component functions constitute HDMR, thereby replacing the original computationally expensive method of calculating $f(x)$ by the computationally efficient model.

Depending on the method adopted to determine the component functions in eqn. (2.1) HDMR expansions are classified into: ANOVA-HDMR and cut-HDMR [10]. ANOVA-HDMR is useful for measuring the contributions of the variance of individual component functions to the overall variance of the output. On the other hand, cut- HDMR expansion is an exact representation of the output *f(x)* in the hyper plane passing through a reference point in the variable space [12]. In the cut-HDMR method, first a reference point $c = \{c_1, c_2, ... c_N\}$ is defined in the variable space. In the convergence limit, cut-HDMR is invariant to the choice of reference point *c.* In practice, *c* is chosen within the neighborhood of interest in the input space. The expansion functions are determined by evaluating the input-output responses of the system relative to the defined reference point *c* along the associated lines, surfaces, sub volumes, etc. (i.e. cuts) in the input variable space. This process reduces to the following relationship for the component functions in the Eqn. 2.1

$$f_0 = f(c) \tag{2.2}$$

$$f_i(x_i) = f(x_i, c^i) - f_0 \tag{2.3}$$

$$f(x_i, c^i) = f(c_1, c_2 ...... c_{i-1}, x_i, c_{i+1}, ... c_N) \tag{2.4}$$

In Eqn. 2.4 all the input variables are at their reference point values except x_i, thus rendering a one dimensional form. The f_0 term is the output response of the system evaluated at the reference point c. Each first-order term is evaluated along its input variable space through the reference point. In this study first-order cut-HDMR is used for generating the response surface.

3. ASSESSMENT PROCEDURE

3.1 Modelling of Uncertainties

Uncertainties in modeling occur due to the materials, structural response, and also in the determination of the magnitude and nature of the seismic loads. The treatment of uncertainties considered in the present work is discussed below.

3.1.1 Uncertainty in Structural Parameters

Structural capacity is the maximum force, displacement, velocity, or acceleration that a member or a system can withstand without failure, or more specifically, without reaching a prescribed limit state. The capacity is therefore dependent on the material properties, member dimensions, and system configuration, the limit state under consideration, and methods and models used in describing the capacity.

3.1.2 *Uncertainty in Material Parameters*

An important source of uncertainty in the analysis of structures is due to the material variability. Uncertainty in the material properties can significantly affect the strength and ductility of structural members which inturn would affect the overall structural response. In the present study, the yield strength, ultimate strength, and modulus of elasticity of steel were modeled as deterministic factors and were set to their respective mean values. It is found from previous studies [1&2] that, while a fair amount of variability in both member strength and ductility existed solely owing to the uncertainty in basic material properties, the inclusion of the uncertainty in the estimation of the ultimate strain in confined concrete provided the most significant contribution to the response variability in member ductility. The random variables involved are the concrete yield strength *(f_c)*, the young's modulus of concrete *(E_c)*, the steel yield strength *(f_y)*, model uncertainty factor for ultimate compressive strain in concrete ductility *($X_{m,\varepsilon cu}$)* and their probabilistic modeling and the values used in this study are given in Table 1. [15]

Table 1. Statistics of Random Variables

	Variable	Mean	COV (%)	Distribution
Concrete	f_{ck}	19.54	21.0	Normal
	$X_{m,\varepsilon cu}$	1.56	49.7	Normal
	E_c	34100	20.6	Normal
Steel	f_y	469	10	Normal

In this work the peak compressive strength of concrete f_{ck} is taken as a normal variable. Of great importance is also the ductility of the confined concrete described through its ultimate strain ε_{cu}. The use of empirical models for the prediction of the stress-strain behaviour of confined concrete results in the uncertainty in the estimation of ε_{cu} and is accounted through the model uncertainty factor $X_{m,\varepsilon cu}$. The randomness associated with the variability in the confinement model using Fardis model has a mean of 1.5641 and a COV of 49.74% and the statistics of the uncertainty of the variable are given in Table 1. Panagiotakos and Fardis (2001) proposed modifications to Mander's model which incorporates the confinement effects for old non-ductile detailed components given by eqns (3.1) and (3.2). The model uncertainty factor for the estimation of ultimate concrete strain is arrived by comparing the experimental and analytical models from literature [1,2,8 & 9]

$$f'_{cc} = f'_{co}\left[1 + 3.7\left(\frac{0.5k_e\rho_s f_{yh}}{f'_{co}}\right)^{0.85}\right] \tag{3.1}$$

$$\varepsilon_{cu} = 0.004 + \frac{0.6\rho_s f_{yh}\varepsilon_{sm}}{f'_{cc}} \tag{3.2}$$

3.2.2 Uncertainty in ground motion

Random nature of ground motion is taken into account by using a number of earthquake records. For mid-rise buildings about ten to twenty ground motion records are usually enough to provide sufficient accuracy in the estimation of seismic demand [14].

3.3 Formulation of Metamodel

Towashiraporn et. al. (2004) has proposed a method to formulate the metamodel for fragility curve generation using response surface method, in which, the input variables are composed of two components: random variable and a control variable. Random variables are those that define uncertainties in structural properties, while the control variable is deterministic with its fixed values characterizing different response prediction models [15]. The control variable is the seismic intensity parameter that defines the earthquake intensity level. For formulating the metamodel the control variable is treated in a similar fashion as other random variables.

The first step in calculating the seismic fragility curves utilizing the metamodel concept is to define the input and output (response) variables. A response measure that best describes damage from seismic loadings should be selected. Damage limit states or performance limit states corresponding to the selected damage measure must also be identified. Parameters such as base shear, maximum roof displacement, peak inter-storey drift, damage indices, ductility ratio, and energy dissipation capacity can be used to identify the damage states depending on the types of structure being investigated. Input variables include aleatory uncertainties caused due to the randomness in construction material properties and uncertainties from earthquakes are defined together with their statistical parameters. Uncertainties from earthquakes are implicitly incorporated in the analysis by using a suite of ground acceleration records. Seismic intensity parameter is defined and the ground motion records in the suite are scaled to have the same level of intensity. Computational seismic analyses are performed on the models which represent different earthquake-structure scenarios. Scaled earthquake records are used as the loading inputs for these analyses and the chosen seismic response is extracted from each analysis. This is repeated for different combinations of input variables. The response resulting from the analysis is recorded for each ground motion, and the mean and standard deviation for each particular combination is calculated. Metamodels for the mean and standard deviation of the responses are formulated by applying the HDMR technique. Once the metamodel for mean and standard deviation are formed, they are combined to form the overall metamodel as given in Eqn 3.3.

$$y = y_{\mu} + N[0, y_{\sigma}] \tag{3.3}$$

The first term in Eqn 3.3 predicts an expected or a mean value of the maximum displacements due to a suite of ground motions, while the second term represents the earthquake-to-earthquake dispersion in response computation and consequently incorporates randomness in earthquake excitations [15].

3.4 Failure Criteria and Performance Limit States

The top storey displacement is often used by many researchers as a failure criterion because of the simplicity and convenience associated with its estimation. The limit states (immediate occupancy, life safety, and collapse prevention) associated with various performance levels of reinforced concrete frames as given in FEMA 356 is given in Table 3.

Table 2. Limits associated with various structural performance levels

Structural Performance Level	**Permissible Top storey Drift**
Immediate Occupancy	1%
Life Safety	2%
Collapse Prevention	4%

4. CASE STUDY – THREE BAY SIX STOREY RC FRAME

In this case study, we analyze a three-bay, six storeyed reinforced concrete frame designed according to IS 456-2000, to estimate the fragility curve. The details of the building elevation and reinforcement details of column are shown in fig 1. The height of each storey is 3.6m and the width of the bay is 5m and the supports are assumed to be fixed. The computational analysis for the time history determination is carried out using the commercially available SAP2000 software. The beams and columns are modeled as 3D frame elements and nonlinearity is introduced by defining moment hinges at the member ends with the M-θ relation obtained from the Fardis model [9]. Geometric nonlinearity effects in the form of P-Δ effects are also considered.

The random variables defined in Table 3 are used as the input variables, spectral acceleration as the seismic intensity parameter and top storey displacement as the output or response variable to form different earthquake-structure combinations, satisfying the sampling conditions of the HDMR technique as shown in Table 3. Uncertainties from the earthquakes are taken into account by the use of a suite of earthquake records of 20 ground acceleration records. Earthquake records are scaled such that they have the same level of intensity. The time history analysis is carried out on these combinations and the maximum response (top storey displacement) is calculated for each ground motion and the mean and standard deviation for each of these cases are calculated.

Table 3. Range of input variables used for sampling in HDMR

Variables	μ-2σ	μ-σ	μ	μ+σ	μ+2σ
f_{ck}	11.33	15.43	19.54	23.64	27.75
f_y	375.2	422.1	469	515.9	562.8
E_c	20050.8	27075.4	34100	41124.6	48149.2
X_m	0.04	0.8	1.56	2.32	3.08
S_a/g	0.1	0.325	0.55	0.775	1.0
Normalized	-2	-1	0	1	2

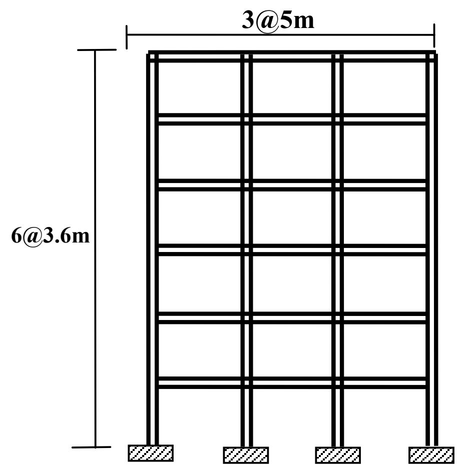

Figure 1. Elevation of Building

The response of the structure for 20 ground acceleration records are estimated at each of the 5 sampling points, for each random variable as shown in Table 3. The sampling is carried out such that only one random variable is estimated at a given time, while the other random variables are kept at their mean points. The metamodel for mean and standard deviation of the response is then formulated by applying HDMR technique and the overall metamodel is then formed as in Eqn. 4.1. Monte Carlo simulation is performed subsequently on the overall metamodel by randomly generating values for the input variables based on their probability distributions and the response is calculated from the overall metamodel. Fragility curves as shown in figure 2 are generated for different performance levels by comparing the response from the above simulations with the limit states associated with those performance levels.

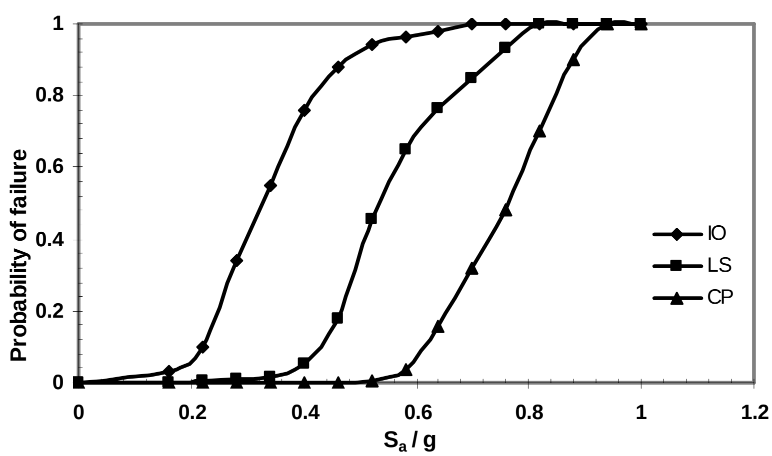

Fig.2 Fragility curves for different performance limits.

5. CONCLUSIONS

In this work, it is seen that High Dimensional Model Representation (HDMR) response surface methodology simplifies the process of fragility computation. It is used to replace the algorithmic g-function with an explicit functional relationship, fitting a second order polynomial, thereby reducing the number of expensive numerical analyses. The number of time history analyses required to generate the fragility curve is significantly reduced. Application of MC simulation directly on the structural model would require about 10000 time history analyses, and the use of MC simulation on the metamodel formulated using response surface method requires 860 time history analysis, on the other hand the proposed method require only 420 time history analysis to generate the curve.

REFERENCES

1. A J Kappos, M K Chryssanthopoulos and C Dymiotis (1999). Uncertainty analysis of strength and ductility of confined rein-forced concrete members. Engineering Structures, **21**:195-208.
2. Christiana D,. Kappos A. J. and Chryssanthopoulos, M. K (1999). Seismic reliability of RC frames with uncertain drift and member capacity. *Journal of Structural Engineering*, **19:**455 – 466.
3. Cimellaro, G. P., Reinhorn, A. M., Bruneau M. and Rutenberg, A. (2006). Multi-Dimensional Fragility of Structures: Formulation and Evaluation. Technical Report MCEER-06-0002.
4. Chowdhury, R., Rao, B. N., and Prasad, A. M., (2008). High Dimensional Model Representation Based Higher Order Limit State Function for Reliability Analysis. *Journal of Structural Engineering, SERC*. **36. 6**. 393–405.
5. Karim C. and Yamazaki, F. (2001) Effect of earthquake ground motions on fragility curves of highway bridge piers based on numerical simulation, *Earthquake Engineering & Structural Dynamics*, **30(12)**:1839-56
6. Singhal, A. and Kiremidjian. (1998). A Bayesian updating of fragilities with application to RC frames. *Journal of Structural Engineering, ASCE,* **124(8)**:922-929
7. Shinozuka, M., .Feng, M. Q., Kim H. K. & Kim S. H. (2000). Nonlinear static procedure for fragility curve development, *Journal of Engineering Mechanics, ASCE*, **126 (No.12)**: 1287 -1295.
8. Sheikh, S. A., & Uzumeri S. M. (1982). Analytical model for concrete confinement in tied columns, *Journal of structural Engineering, ASCE*, **108 (No. ST12)**: 2703 – 2722.
9. Telemachos, Panagiotakos, B. & Michael N. Fardis (2001). Deformations of reinforced concrete members yielding and ultimate, *ACI Structural Journal*, **98 (No.2)**:135 – 148.
10. Kleijnen, J. P. C. (1974), Statistical Techniques in Simulation: Part 1, Marcel Dekker, Inc., New York, NY.
11. Mann, N. R., Schafer, R. E., and Singpurwalla, N. D. (1974), Methods for Statistical Analysis of Reliability and Life Data, John Wiley & Sons, Inc., New York., NY.
12. Chowdhury R., Rao. B.N. and Prasad A.M. (2008), High Dimensional Model Representation for Structural Reliability Analysis. Ph.D. dissertation. Indian Institute of technology Madras, India.
13. Schotanus, M. I. and Pinto, P.E. (2002) Fragility Analysis of Reinforced Concrete Structures using a Response Surface Approach, Ph.D. dissertation. Rose School, Italy.
14. Shome N, Cornell CA. (1999) Probabilistic seismic demand analysis of nonlinear structures. Ph.D. dissertation. Stanford: Stanford University.
15. Towashiraporn, P., Goodno B. and Craig J (2004).. Building Seismic Fragilities Using Response Surface Metamodels. Ph.D. dissertation .Georgia Institute of Technology.
16. Ranganathan. (1990). Reliability analysis and design of structures, Tata McGraw Hill, NewDelhi.