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DEVELOPMENT OF LOW REYNOLDS NUMBER TWO EQUATION TURBULENCE MODELS FOR FOR PREDICTING EXTERNAL HEAT TRANSFER ON TURBINE BLADES*

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INTRODUCTION

A research effort has been underway to study the use of two equation low Reynolds number tubulence models in predicting gas side heat transfer on turbine blades. The major objectives of this ongoing work are basicly threefold.

(1) Study the predictive capabilities of two equation low Reynolds number turbulence models under the conditions characteristic of modern gas turbine blades.

(2) Explore potential improvements to the models themselves as well as to the specification of initial conditions

(3) Provide a comparison of the predictions of these models with the experimental data from a broad range of recently available turbine cascade experiments.

This work is particularly concerned with the problems associated with predicting the boundary layer transition from laminar to turbulent flow, as this may be the most serious deficiency of current modeling techniques.

The work has proceeded in a number of phases or steps, and several of these of been completed. The purpose of this report is to briefly discribe the results and conclusions of the first two phases of this work.

PHASE ONE

Evaluation of the transition prediction characteristics of current low Reynolds number two equation models for flat plate zero pressure gradient boundary layers under the influence of free stream turbulence.

Research has shown that the dominant factor influencing the location and length of transition is the free stream turbulence intensity [1]. It has also been shown that the effects of free stream turbulence on skin friction and heat transfer can be correlated reasonably well with two parameters, the free stream turbulence intensity (Tu_e), and the free stream turbulence length scale (L_e) [2,3]. These same two parameters are directly related to the two quantities whose behavior is modeled in most two equation turbulence models, ie. the turbulent kinetic energy (k), and the turbulence dissipation rate (ε). With the recent development of low Reynolds number versions of these models, it has seemed reasonable to hope that with refinement, these models would be capable of predicting the influence of free-stream turbulence on boundary layers in both the laminar and the turbulent regimes, and also on transition.

The application of some of these models by a number of independant workers has verified that at least qualitatively, these models do predict the major effects of free stream turbulence on transition [5,6]. However, a detailed quantitative study has not been available to the knowledge of the authors. The purpose of this phase of the research was to more carefully evaluate the transition prediction capabilities of two relatively popular low Reynolds number versions of the stardard k-ε two equation turbulence model. The models chosen were those of Lam and Bremhorst [7], and Jones and Launder [8].

The computations were performed using the Patankar Spalding method [9] for boundary layer flows. A variable grid using 88 nodes was used for all calculations and was found to be sufficiently fine to produce essentially grid independant results.

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Sensitivity to Initial Conditions

Figure 1 shows the results of varying the initial profiles of k and ε on the location of transition for a zero pressure gradient flat plate flow where Tu_e=3% (note that the subscript e will always refer to free stream conditions). The model used was the Lam-Bremhorst model. Run "A" follows the initial profile recommendations of Rodi and Scheuerer [4], where

$$k=k_e*(U/U_e)^n \quad , n=2 \tag{1}$$

$$\varepsilon=a_1*k*\partial U/\partial Y \quad , \varepsilon > (k_e)^{1.5}/L_e \quad , a_1(Tu=3.\%) = .375 \tag{2}$$

Run "B" was begun with essentially zero kinetic energy within the boundary layer. This was accomplished by setting n=8 and a_1=2.0. Both A and B were begun at $Re_x=2.27 \times 10^4$, corresponding to a momentum thickness Reynolds number (Re_θ) of 100. Runs C and D correspond to A and B except for the initial starting location, which was moved upstream to $Re_x=10^3$. This figure illustrates the following general characteristics of both low Reynolds number models.

(1) At a given starting location, a k=0 initial profile results in the onset of transition begining at the farthest downstream location.

(2) The sensitivity to initial profiles decreases with decreasing initial Re_x. Below some critical value, the location of transition becomes essentially independant of initial Re_x.

An important result to understand is that it is possible to specify profiles at $Re_x=2.27 \times 10^4$ such that the curves A and B are reproduced. However, it is not possible to specify any set of profiles at $Re_x=10^3$ and yield transition as per C or D.

Sensitivity to Initial Starting Location

To further explore the sensitivity of the prediction to the initial starting location, a set of calculations were made with identical initial profiles, but at different initial Renolds numbers ($Re_{x,i}$). The k=0 initial profile explained above was used for all cases, as this always yielded the higher limit on $Re_{x,trans}$. Figure 2 shows the results of these calculations. For the Lam Bremhorst model, the location of transition is strongly dependant on $Re_{x,i}$ for $Re_{x,i} > 10^3$, but basically independant for $Re_{x,i} < 10^3$. The Jones Launder model shows a somewhat lower critical value, with the location of transition not significantly changing until $Re_{x,i}<10^2$.

Sensitivity to different free stream turbulent intensities

Figure 3 shows the results of calculations at free stream turbulence intensities ranging from 1.0 to 6 %. The calculations were all started at $Re_x=10^3$ (where for the Lam Bremhorst model the initial profiles of k and ε were unimportant). A Tu_e=1% calculation for the Lam Bremhorst model is not shown because it was found that transition was not predicted by this model for Tu=1%. This agrees with the experience of Rodi and Scheuerer [5].

As can be seen, the qualitative characteristics of the variation of C_f during transition are predicted reasonably well. Also, the onset of transition moves progressively upstream with increasing Tu_e as it should. However, significant differences between the predictions of the two models occur at higher

Tu_e. In Figure 4 the momentum thickness Reynolds number at the start ($Re_{\theta S}$) and the end ($Re_{\theta E}$) of transition are plotted and compared with the correlation of Abu-Ghannam and Shaw [10]. Two major quantitative problems are apparent in this figure. First, the onset of transition is generally predicted too early for both models.This is especially true at higher Tu_e. Second, the predicted length of transition is much too short.

Summary

Tests have been made of the transition prediction characteristics of two low Reynolds number two equation turbulence models. The major items of interest learned include the following;

(1) Both models tested showed, as expected, the ability to correctly model the basic qualitative aspects of transition, ie. the continuous transition from laminar to turbulent flow, the onset of which moves upstream with increaseing Tu_e.

(2) The onset of transition is moderately sensitive to the initial profiles specified for k and ε. This sensitivity decreases with decreasing $Re_{x,i}$.

(3) For any given $Re_{x,i}$, there is a limit to how far downstream the onset of transition can be predicted. This limit is reached by specifying the initial profile of k=0.

(4) The onset of transition is very sensitive to the location at which the calculations are started. This sensitivity decreases with decreasing $Re_{x,i}$.

(5) For calculations started at low $Re_{x,i}$ (where the sensitivity to the initial profiles for k and ε becomes small), the onset of transition occurs at unrealistically early locations for both models tested.

(6) Both models predict transition lengths significantly shorter than experiment.

(7) The Lam Bremhorst model does not predict transition for free stream turbulence intensities of about 1.1% and lower.

(8) Because of the above deficiencies, the transition predictions of both models compare rather poorly with the correlation of Abu-Ghannam and Shaw.

PHASE TWO

Modifications to improve the transition prediction characteristics of the Lam Bremhorst low Reynolds number turbulence model.

A fairly ideal transition model for boundary layer flows would be one which, given any physically realistic velocity, pressure, free stream turbulence and length scale distribution with x , and the profiles for U, k, and ε at some given x_i, would consistantly predict the correct location and length of transition. The agreement with experiment should be at least as good as the correlations currently available relating the effects of these parameters on transition. Furthermore, the results should be invariant with the initial starting location (x_i), as long as the profiles for k and ε were specified correctly.

That the two models tested do not adequately approach this ideal is quite obvious from the results presented earlier. However, there is another difficulty with striving to achieve this ideal. That problem centers around the lack of experimantal data concerning the nature of the "correct" profiles for k and ε at any point prior to transition. Although our models require this as input, insufficient knowledge is currently known about the values of these turbulent quantities within the quasi-laminar region just prior to the onset of transition. Thus previous researchers have had to rely on add-hoc methods with little more than the known boundary conditions and intuition to guide them [5, 6].

In searching for ways to improve on the current models, we must be content (for the present) with "reasonable" profiles in this region, and try to minimize the sensitivity of the predictions to small variations in them.

The Lam Bremhorst model was chosen as the model to begin work with. This was done for basicly three reasons. First, the favorable results of the study by Patel et al [11] . Second, the previous use of this model by Rodi and Scheuerer in working on this same problem. And third, the simpler form of the source terms present in the k and ε equations, a result of the form of the dissipation rate variable used in this model.

Stability Considerations

The physical process by which an initially laminar boundary layer undergoes transition to a fully turbulent state is a very complex problem, but is unseparably tied to stability considerations. Fundamental to the process is the response of the flow to the introduction of small disturbances, from whatever source. Under some conditions, a disturbance will decay, it's small energy being absorbed into the mean flow. Under other conditions, a disturbance will be amplified, and energy will be extracted from the mean flow to feed this growth. It is only under these "unstable" conditions that the onset of transition can occur.

Linear stability theory gives some insight into the conditions underwhich a boundary layer becomes unstable. Solutions to the well known Orr-Sommerfield equation for a Blasius velocity profile yield a critical Reynolds number based on momentum thickness, $Re_{\theta,c}$, below which infinitesimal disturbances will not be amplyfied (Commonly quoted as 163 due to an approximate solution, more accurate solutions have shown it to be equal to 201).

Experiments have shown that under the infuence of high free stream turbulence, transition can begin to occur at Re_θ even less than this stability limit [10]. This is due the nonlinear behavior which the high Tu_e introduces. However, there does appear to be a lower limit, as the data seems to bottom out at about $Re_\theta=160$. Consequently, Abu-Ghannan and Shaw have proposed $Re_\theta=163$ as a lower limit below which transition will not occur even at high turbulence intensities.

Stability considerations are not a part of either of the low Reynolds number turbulence models that we have looked at. The k and ε equations are simple advection diffusion equations with a particular set of nonlinear source terms. From this context, it is not particulaly surprising that the deficiencies previously discribed exist.

Method of Rodi and Scheuerer

The most helpful previous work in this area that this author is aware of is the recent work presented by Rodi and Scvheuerer [5,6]. They apparently recognized many of the problems previously discussed and recomended a particular procedure to deal with it. They chose to begin all calculations at a momentum thickness Reynolds number of $Re_\theta=100$. This in essence is their answer to the stability problem discussed earlier. They then proposed particular forms for the k and ε profiles (see eq. 1,2) which seemed reasonable, and which gave them a simple constant (a_1) with which to tune their results (a_1 was correlated with Tu_e).

Figure 5 shows the results of following this procedure for a range of Tu_e of 1.5 to 6 %. These conditions are identical to those used for the runs presented in Figure 3, and the results show improvement as compared to the previous Lam-Bremhorst model calculations. However, the results are still relatively unsatisfactory when compared to the correlation of Abu-Ghannan and Shaw. Although some improvement has been made with respect to the onset of transition, the short length of transition is still a problem. Furthermore, since the model itself has not been changed, we are still left with the undesirable situation where, if we needed to start our calculations just a little farther upstream, we would be unable to find any profiles for k and ε which would yield similar results.

The mechanism by which the model simulates transition

Before attempting to consider ways to improve the transition prediction characteristics of the model, it is important to consider carefully how the process occurs in the model as it stands.

Figure 6 shows the typical development of the turbulent kinetic energy profiles as the model proceeds from a laminar to a turbulent state. Turbulent kinetic energy from the free stream initially begins to diffuse into the boundary layer. As this continues, the production term, $\mu_t(\partial u/\partial y)^2$, starts to become significant. This in turn increases the local value of k, and thus μ_t. Thus process feeds on itself, causing the rapid increase in k shown until the paramaters achieve a relatively stable state due to the low Reynolds number functions and the wall boundary conditions.

The key term in this entire process appears to be the production term in the turbulent kinetic energy equation. This is the term which in the model, simulates the amplification of free stream disturbances and the resulting eventual transition to a turbulent state.

Proposed modification to the production term

A variety of different modifications to the model were explored and compared to try and find a method which would satisfactorily alleviate the problems previously identified. These will not be discussed individually. Only the method found to be the most satisfactory (at this point) will be discribed.

The method developed focuses on two ideas. First, that some means of incorporating stability considerations into the calculational procedure must be provided. Second, that the process by which the model simulates transition, once started, must proceed at a finite rate and in accord with experiment. It will be referred to as the "PTM3" modification (an acronymn for Production Term Modification 3).

The modification is based on the following hypothesis.

(1) Since the production term is the term in the model which simulates the amplification of purtibations, below some critical momentum thickness Reynolds number ($Re_{\theta,c}$), the production term in the k equation should be insignificant.

(2) The rate at which P_k can change is assumed to have some finite limit. The form of that limiting growth rate, $\partial P_k/\partial t$, is assumed to be a simple linear function of P_k , as per equation 3 below, and as shown in Figure 7.

$$\left[\partial P_k/\partial t\right]_{max} = A*P_k + B \qquad (3)$$

Guided by linear stability analysis and the results of experiments at a variety of free stream turbulent intensities, the value of $Re_{\theta,c}$ was set at 125, and was assumed constant. Although admittedly somewhat arbitary, it is based on the fact that no experiments known to this author have indicated the onset on transition occuring for Re_θ lower than about 130-140. Thus, in the model, for $Re_\theta<125$, the production term in the k equation (the ε equation remains unchanged) was set equal to zero.

The values of A and B are assumed to be functions of the free stream turbulence intensity, and were found by optimizing the results of numerous numerical experiments to the correlations of Abu-Ghannam and Shaw. Figure 8 illustrates the dependence of A and B on Tu_e found from this work.

It should be noted that the form of this modification is such that the fully turbulent predictions of the unmodified Lam-Bremhorst model are not affected, becoming completely transparent once transition has occured.

Results of the Proposed Modification

In Figure 9, the sensitivity of the model to initial starting location is shown. Once again, a series of calculations at Tu_e=3% were made at different initial locations, just as was done and presented in Figure 2. As can be seen, the sensitivity is greatly reduced, with all calculations started at $Re_{x,i}$ less than 10^4 being virtually identical. Variations due to initial starting profiles for k and ε were also negligible for runs initiated below this limit.

In Figure 4 the predicted momentum thickness Reynolds numbers at the start and end of transition for the PTM3 calculations, the correlation of Abu-Ghannam and Shaw, and the previous unmodified calculations presented earlier.

In Figure 10, the behavior of the predicted variation of C_f with Reynolds number is shown for free stream turbulence intensities of from 1.5 to 8 %. The improvement is excellent as compared to figures 3 and 5.

In Figure 11 the heat transfer results for calculations using the PTM3 form of the Lam-Bremhorst model are compared with an experiment of Wang,Simon and Buddhavarru. Also shown is a calculation by Park and Simon (paper submitted to 2nd ASME/JSME Thermal Engineering Conference, Mar. 1987) using standard mixing length type transition modeling as per Abu- Ghannam & Shaw [10] and Dhawan & Narasimha [12]. The agreement is excellent, and an improved simulation of the transition path as compared to mixing length type models is indicated.

In Figure 12, the heat transfer data from three experiments of Blair [3] are compared with the calculations. Excellent agreement is found except for the grid 2 case, where the calculations predict transition somewhat upstream of the experiments.

Although the difference between the calculation and the experiment for grid 2 is not greater than the scatter indicated in the original correlation by Abu-Ghannam and Shaw, it was nevertheless disappointing. A plausible explaination for this error relates to the difference between the "total" free stream turbulence intensity, and the three componants from which it is found, ie. u', v', w'. Blair's experiments report the variation with x of both the total value of Tu_e, as well as each of these components. For this experiment, the u' component is about 1/2 % lower than the average of them all. Since many of the results reported in the literature report only u', isotropic conditions must be assumed in order to obtain a value for k_e. Thus it may be that the model is slightly biased toward u'. This would naturally manifest itself most clearly in the medium Tu range for the following reasons. First, the assumption of isotropic turbulence generally improves significantly with decreasing turbulence intensities. And second, the sensitivity of the location of transition to the magnitude of Tu_e decreases very strongly as Tu becomes higher. Thus, at moderate levels of Tu_e, we would expect the most sensitivity to a potential bias of this sort. Although just conjecture at this point, it was confirmed that when the calculations were repeated by assumming a free stream turbulence based on u' only, the results were in much closer agreement. This is also shown in Figure 12.

Conclusions

It has been found that the proposed modification, as applied to the calculation of transition on zero pressure gradient boundary layers under the influence freestream turbulence, has the following improved characteristics.

(1) The model is insensitive to variations in starting location for $Re_{x,i} < 10^4$.

(2) The model is insensitive to any reasonable specification of the initial profiles for k and ε for starting locations below $Re_{x,i} < 10^4$.

(3) For free stream turbulence intensities of from 1.3 to 8 % the model predicts transition starting and ending at momentum thickness Reynolds numbers in accordance with the correlation of Abu-Ghannam and Shaw.

(4) The path by which transition occurs, as manifest in the variation in skin friction or surface heat transfer, is in closer agreement with experiment than standard mixing length type transition models.

It was also found that,

(5) The modifications become completely transparent after transition occurs, reverting to the standard Lam-Bremhorst model.

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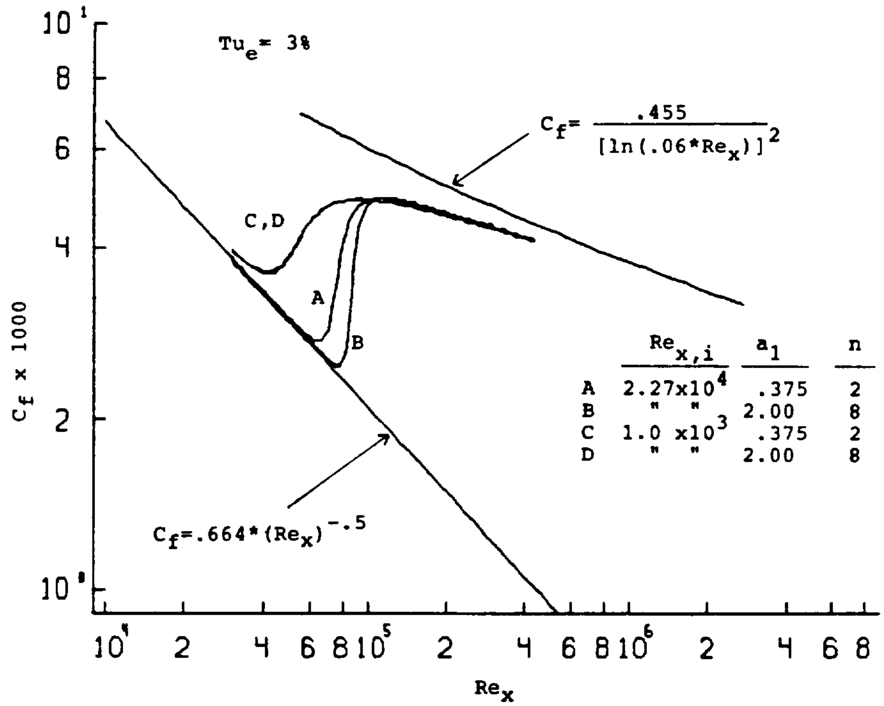
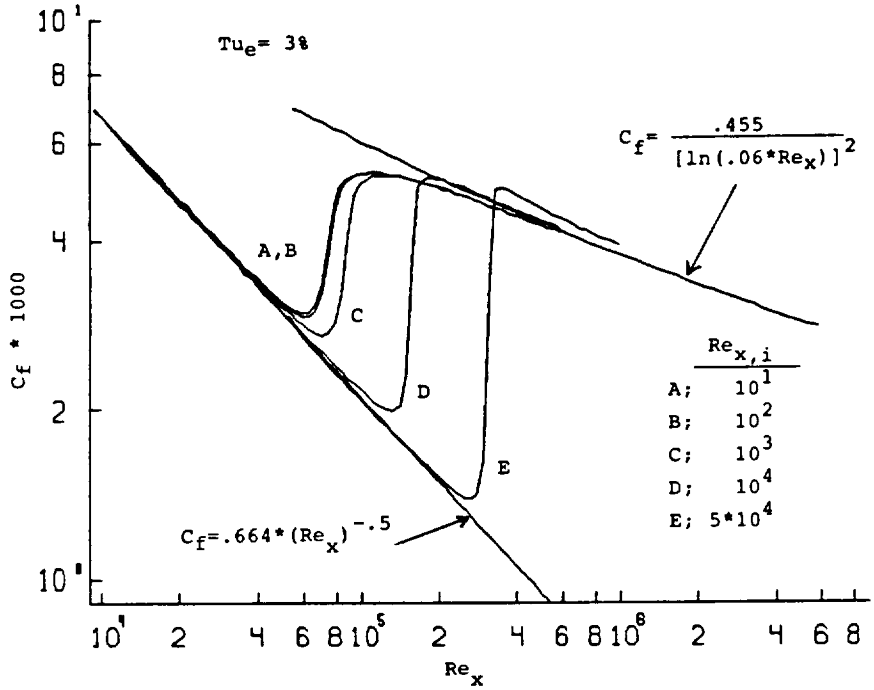
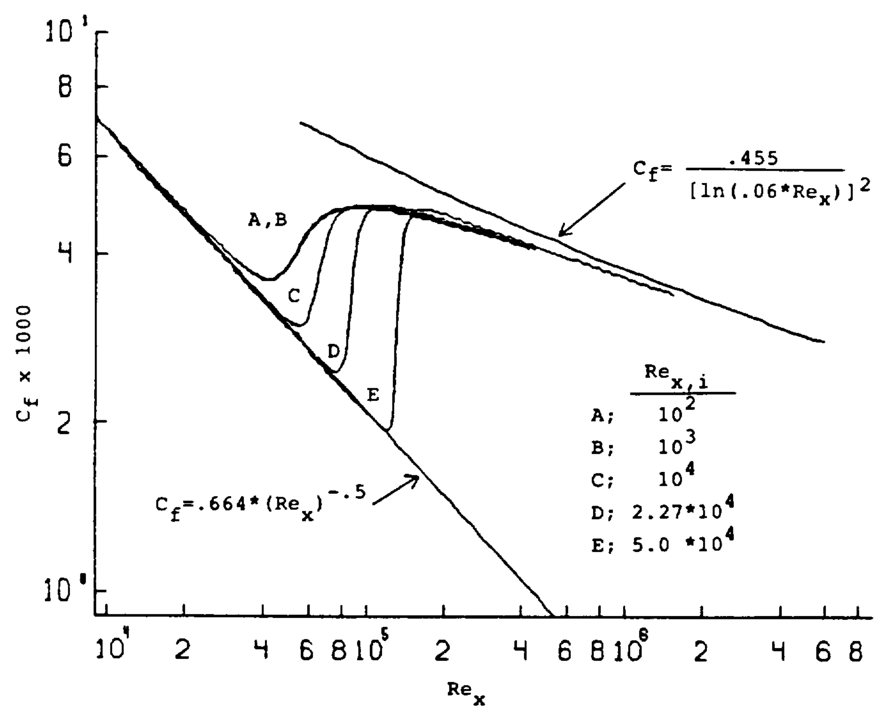


Figure 1 Plot of C_f vrs. Re_x showing the sensitivity to the initial profiles of k and ε of the Lam-Bremhorst model

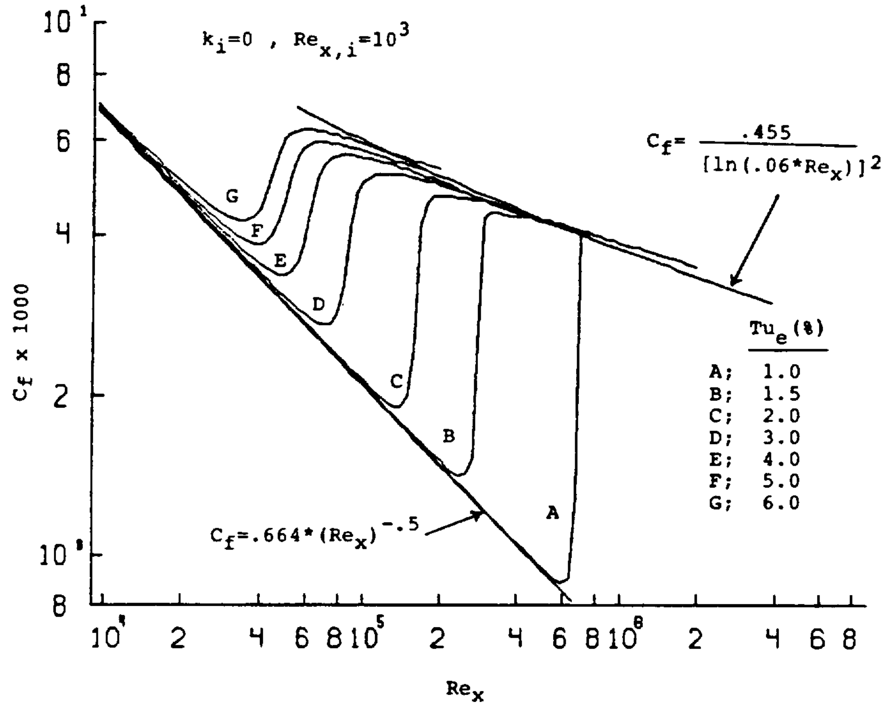


a) Jones Launder model

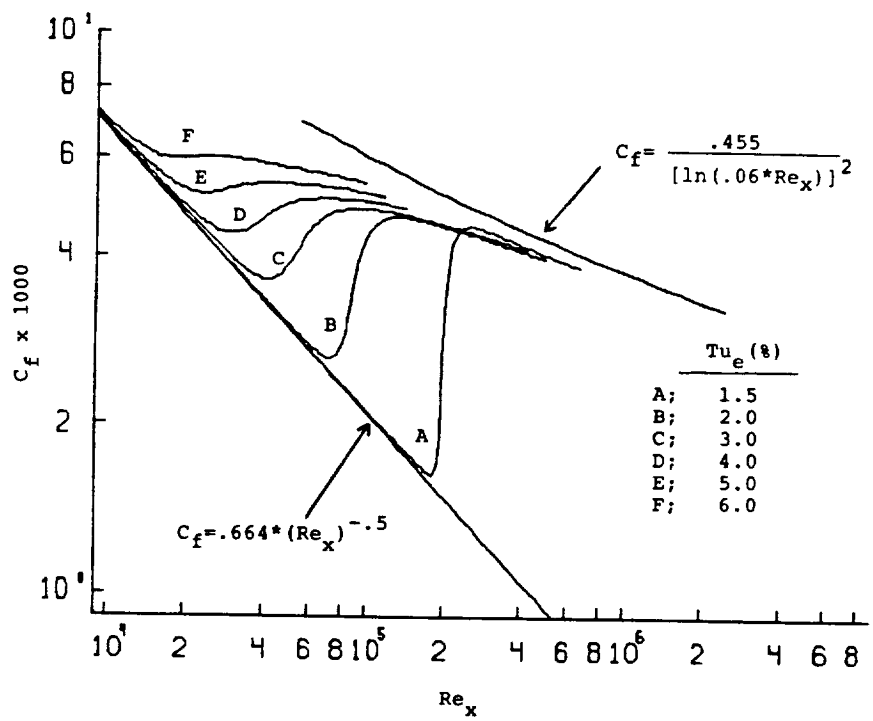


b) Lam Bremhorst model

Figure 2 Plot of C_f vrs. Re_x showing the sensitivity to the initial starting location



a) Jones Launder model



b) Lam Bremhorst model

Figure 3 Plot of C_f vrs. Re_x showing effect of free stream turbulence intensity on the prediction of transition

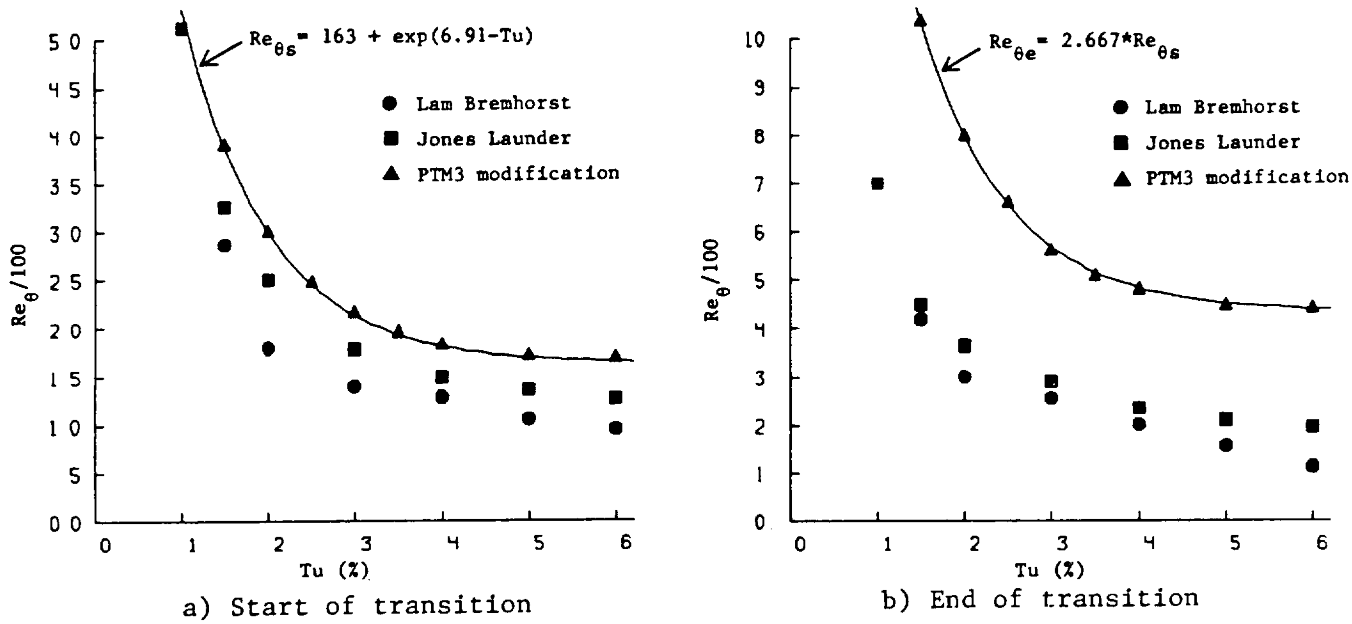


Figure 4 Momentum thickness Reynolds numbers at the start ($Re_{\theta s}$) and end ($Re_{\theta e}$) of transition.

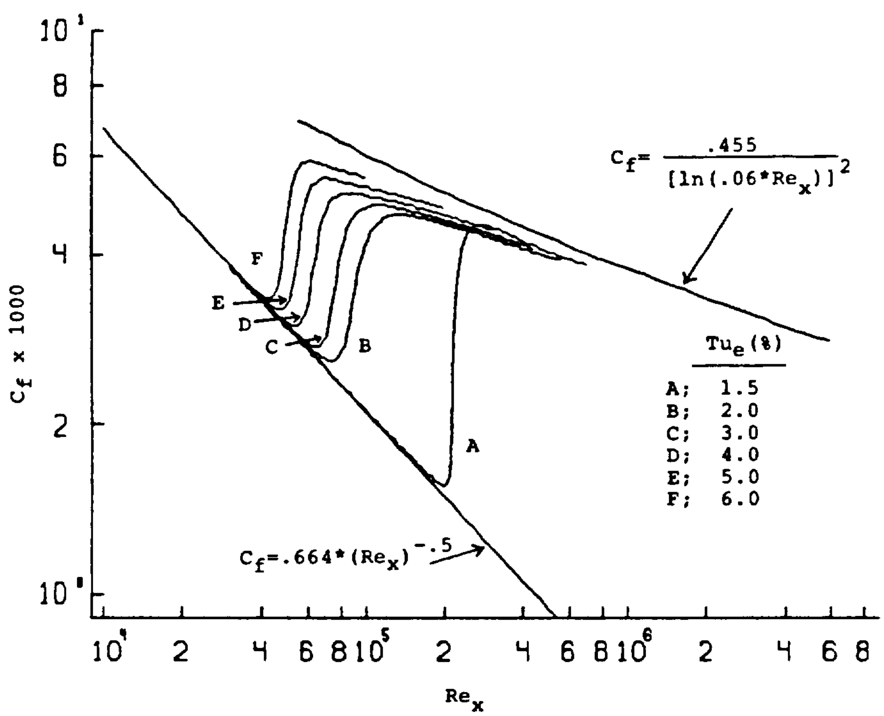


Figure 5 Plot of C_f vrs. Re_x showing effect of free stream turbulence intensity on the location of transition predicted by the Rodi-Scheuerer application of the Lam-Bremhorst model

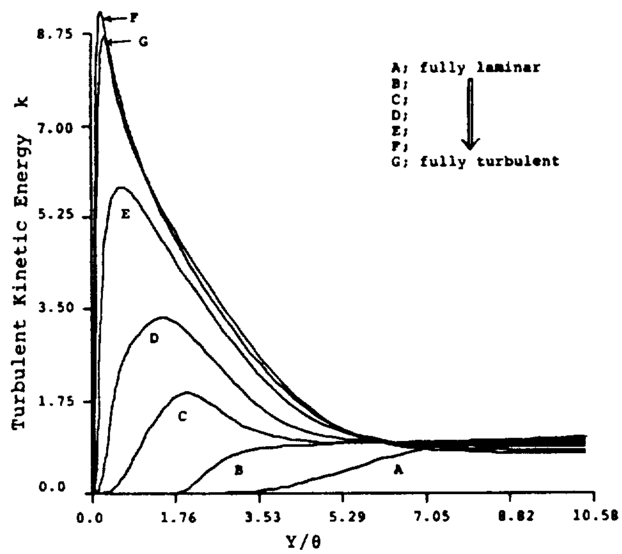


Figure 6 Typical development of the turbulent kinetic energy profiles through transition. Lam Bremhorst model

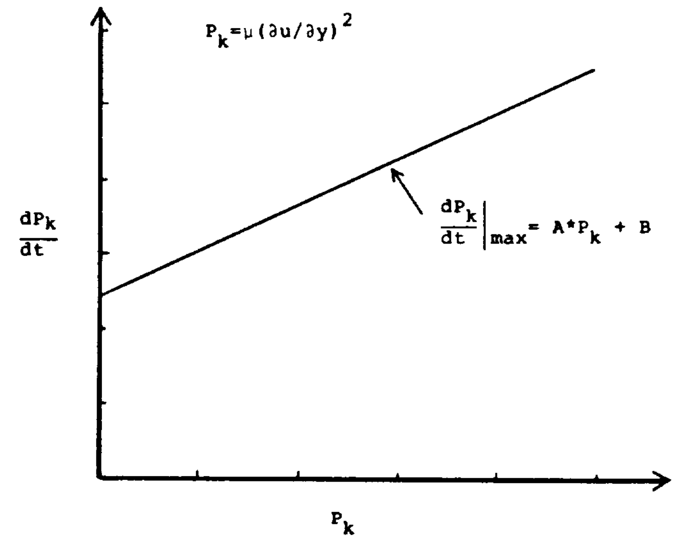


Figure 7 Limiting the growth rate of the production term in the k equation by the proposed model

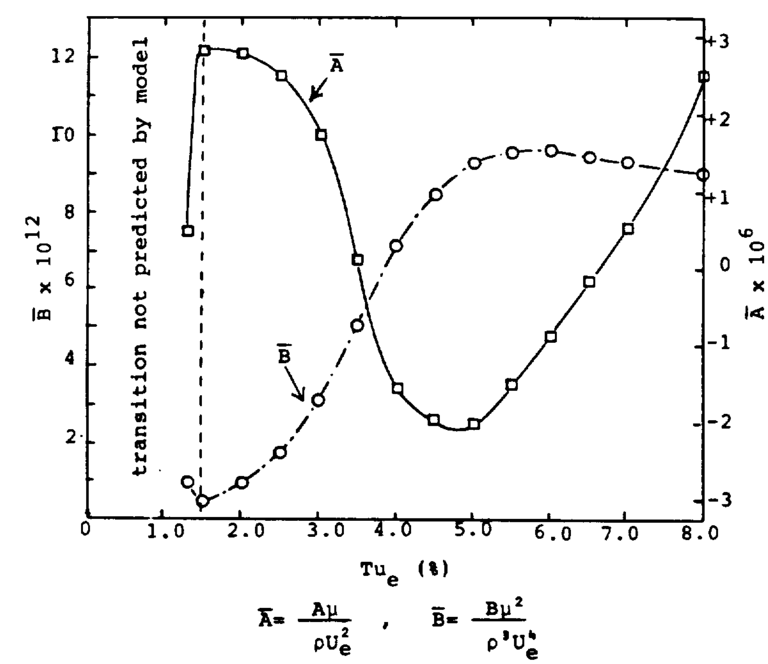


Figure 8 Variation of "A" and "B" with free stream turbulent intensity in the "PTM3" model

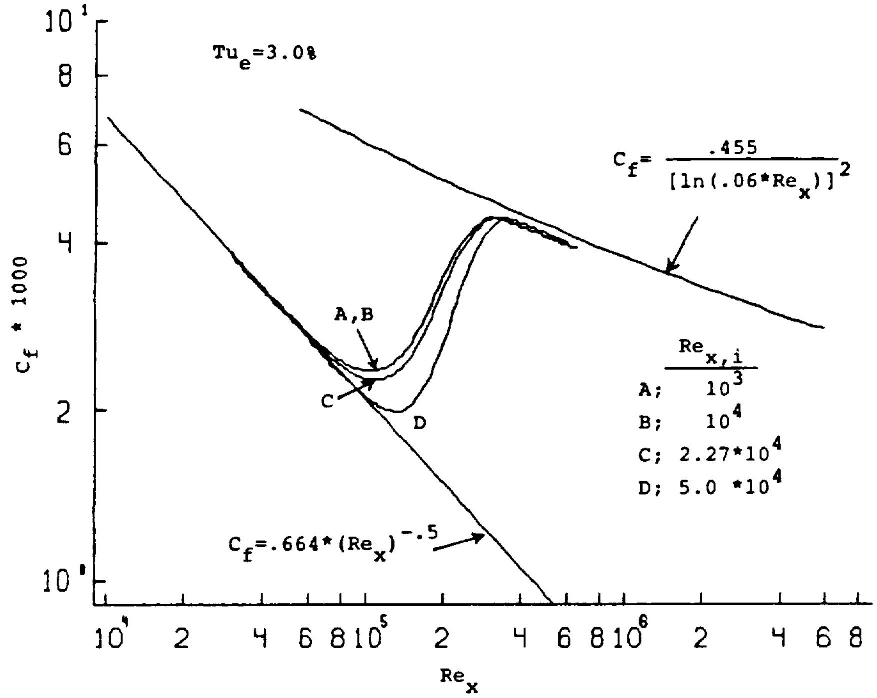


Figure 9 Plot of C_f vrs. Re_x showing the sensitivity to the initial starting location. "PTM3" modified form of the Lam Bremhorst model. Initial conditions set as per Rodi and Scheuerer

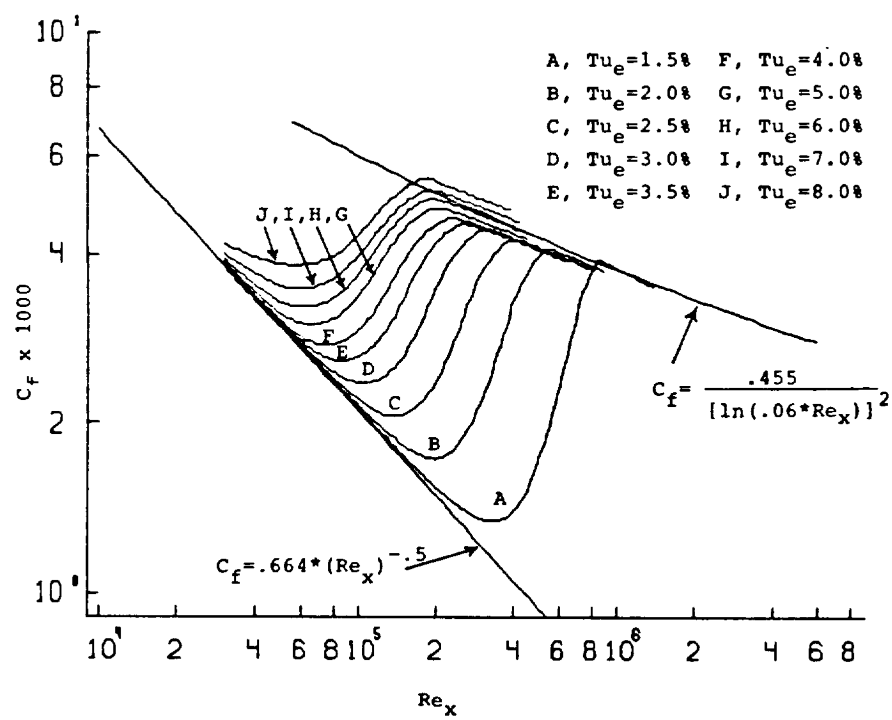


Figure 10 Plot of C_f vrs. Re_x showing the effect of free stream turbulence intensity on the location of transition as predicted by the "PTM3" modified form of the Lam Bremhorst model

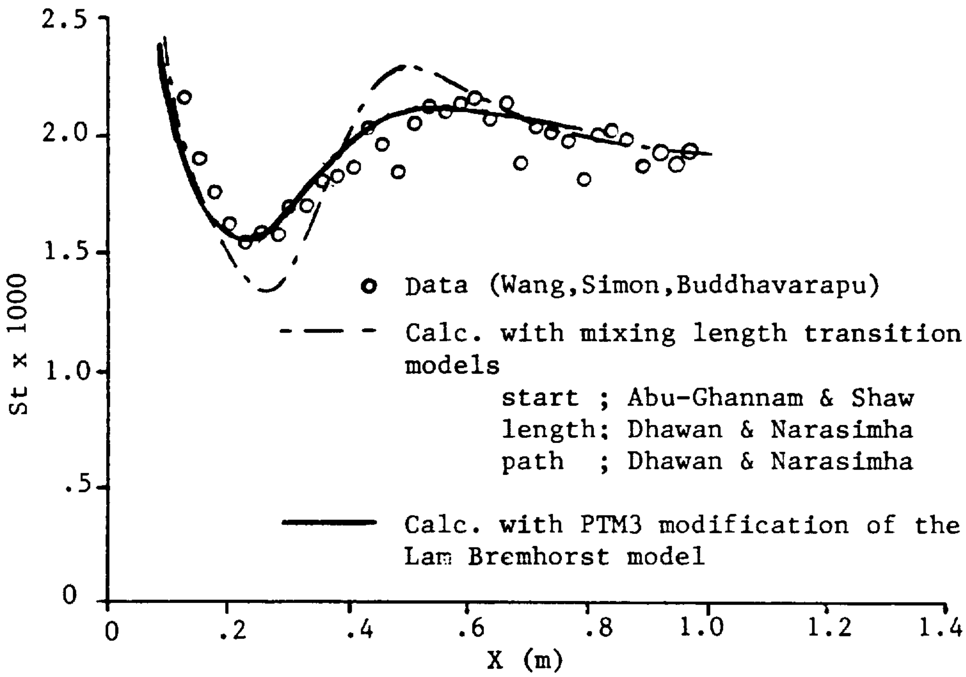


Figure 11 Comparison of the predicted heat transfer through transition with the data of Wang, Simon, and Buddhavarapu

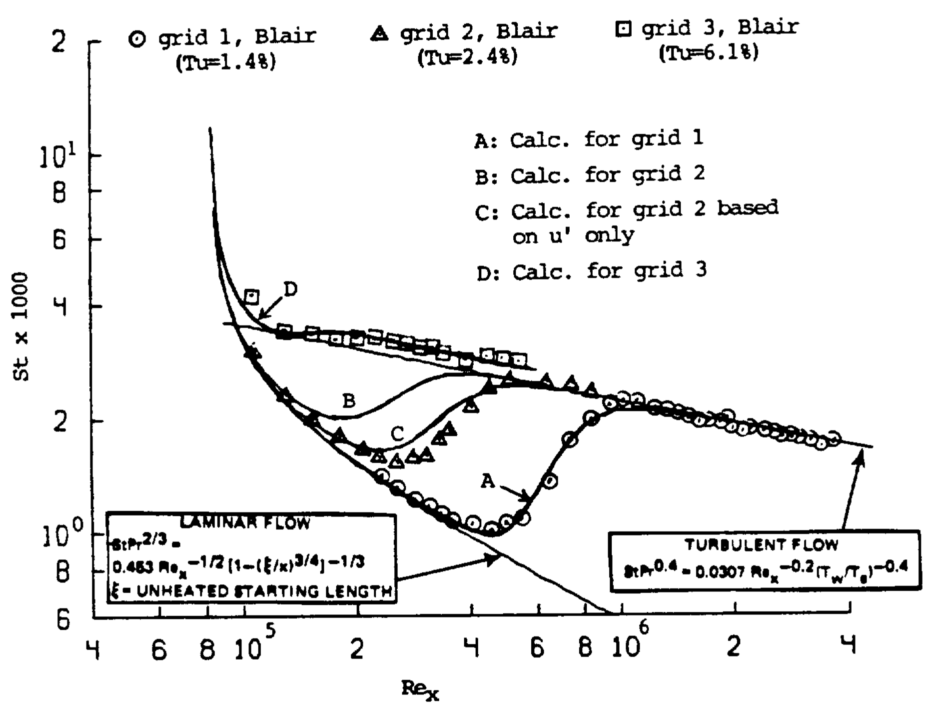


Figure 12 Comparison of the predicted heat transfer during transition with the data of Blair. "PTM3" modification of the Lam-Bremhorst model