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Development of lumped-parameter mathematical models for a vehicle localized impact †

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Abstract

In this paper, we propose a method of modeling for vehicle crash systems based on viscous and elastic properties of the materials. This paper covers an influence of different arrangement of spring and damper on the models' response. Differences in simulating vehicle-to-rigid barrier collision and vehicle-to-pole collision are explained. Comparison of the models obtained from wideband (unfiltered) acceleration and filtered acceleration is done. At the end we propose a model which is suitable for localized collisions simulation.

Keywords: Data analysis; Filtering; Modeling; Vehicle crash

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1. Introduction

This paper deals with establishing an appropriate mathematical model representing vehicle soft impacts such as localized pole collisions. In simulation of the vehicle collision, elements which exhibit viscous and elastic properties are used. Models utilized by us consist of energy absorbing elements (EA) and masses connected to their both ends. We focus on finding a model with such an arrangement of springs, dampers and masses, which simulated, will give a response similar to the car's behavior during the real crash.

Due to the fact that real crash tests are complex and complicated events, their modeling is justified and advisable. Every car which is going to appear on the roads has to conform to the worldwide safety standards. However, crash tests consume a lot of effort, time and money. The appropriate equipment and qualified staff is needed as well. Therefore our goal is to make possible simulation of a vehicle crash on a personal computer.

When it comes to modeling the vehicle crash we can distinguish two main approaches. The first one utilizes CAE (Computer Aided Engineering) software including FEA (Finite Element Analysis) while the second one bases on the analytical method presented in this paper. Much research has been done so far in both of those areas. Refs. [1-3] provide a brief overview of different types of vehicle collisions.

Approach presented here - mathematical modeling of a crash event with the equations of motion which can be solved explicitly with closed form solutions - is different that the methods which have been shown in Refs. [4-7]. In order to simulate the collision of a car the software based on FEM (Finite Element Method) was utilized. After the creation of 3D CAD and FE models, the crash simulations were performed. Results obtained showed good correlation between the test and model responses. When it comes to determining crush stiffness coefficients, in Ref. [8] it is presented a method which employs CRASH3 computer program. Vehicle structure was modeled as a homogenous body and then the comparative analysis of the crash response of vehicles tested in both: full-overlap and partial-overlap collisions, was done.

A lumped parameter modeling (LPM) is another way of approximation of the vehicle crash. It is an analytical method of formulating a model which can be further used for simulation of a real event. It allows us to establish dynamic equations of the system - differential equations - which give the complete description of the model's behavior, see Refs. [9] and [10].

To be able to create a mathematical model of a vehicle collision, it is often enough (and more efficient) not to analyze the complicated crash pulse recorded during the full-scale experiment but just to study an approximation of the measured acceleration signal. Those approximated functions were compared to experimental pulses in Ref. [11]. Subsequently they were tested to obtain different models' responses which were compared to the original pulse. Results confirmed that the crash pulse approximation is a reasonable method to simplify the collision analysis. Recently, the Haar wavelet-based performance analysis of the safety barrier for use in a full-scale

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test was proposed in Ref. [12].

Refs. [13-17] talk over commonly used ways of describing a collision - e.g. investigation of tire marks or the crash energy approach. Vehicle crash investigation is an area of up-to-date technologies application. Refs. [18-22] discuss usefulness of such developments as neural networks or fuzzy logic in the field of modeling of crash events. It is extremely important to assess what factors have an influence on the crash severity for an occupant. As in the case of a vehicle crash simulation, also here we can distinguish two main ways of examining the occupant behavior during an impact. Ref. [23] focuses on finding the relationship between the car's damage and occupant injuries. On the other hand, Ref. [24] employs FEM software to closely study the crash severity of particular body parts.

In Refs. [25-28], basic mathematical models are proposed to represent a collision. The main part of this research is devoted to methods of establishing parameters of the vehicle crash model and to real crash data investigation, e.g. creation of a Kelvin model (spring and damper connected in parallel with mass) for a real experiment, its analysis and validation. After model's parameters extraction a quick assessment of an occupant crash severity is done. Finally, the dynamic response of such a system was similar to the car's real behavior in the time interval which corresponds to the collision's duration. Parameters of this assembly (spring stiffness and damping coefficient) were obtained analytically with closed-form solutions according to Ref. [29].

In this paper, we present a process of improving the accuracy of the vehicle crash model. We start with simulation of the vehicle to pole impact by using the Kelvin model (spring and damper in parallel connected to mass). Afterwards, by filtering the crash pulse data, more accurate response of the system is obtained. Model establishment is done one more time. This allows us to compare what the crash models are for both: raw and filtered data, and to decide which of them is more suitable to represent vehicle to pole collision. In total, four different models created for the filtered data are elaborated here and it is being assessed which of them gives the most exact description of the car's behavior in the pole collision. The main contribution of this paper is the evaluation of the proposed modeling methodology results with the full-scale experimental data. When compared to the previous work which concerns the similar area of research [25], the current study presents more detailed insight into vehicle localized impact modeling. By the comparative analysis of different viscoelastic models responses, it is decided which of them is the most suitable to simulate vehicle-to-pole collision. To assess model's fidelity, its structural properties and dynamic responses are examined.

2. Experimental setup

In the experiment conducted by UiA [24] the test vehicle, a standard Ford Fiesta 1.1 L 1987 model was subjected to a central impact with a vertical, rigid cylinder at the initial impact velocity v_0 = 35 km/h. Mass of the vehicle (together with the measuring equipment and dummy) was 873 kg. Experiment's scheme is shown in Fig. 1.

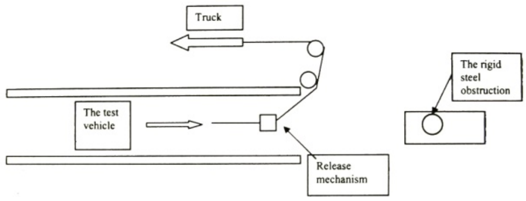

Fig. 1. Scheme of the test collision [30].

Fig. 2. The car is undergoing deformation.

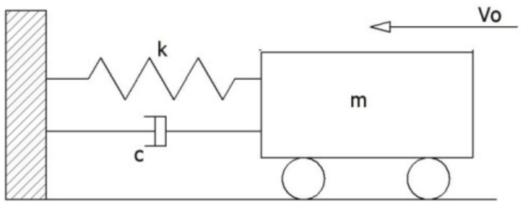

Fig. 3. Kelvin model.

Vehicle accelerations in three directions (longitudinal, lateral and vertical) together with the yaw rate at the center of gravity were measured. Using normal-speed and high-speed video cameras, the behavior of the obstruction and the test vehicle during the collision was recorded. Fig. 2 shows one of the crash stages.

3. Raw data analysis - Kelvin model

According to Ref. [25], the Kelvin model shown in Fig. 3 has been proposed to represent the vehicle to pole collision. Symbols used: k - spring stiffness, c - damping coefficient, m - mass, V_0 - initial impact velocity.

Known parameters of the model are:

m = *873 kg* - mass
V_0 = *10.8 m/s* - initial impact velocity.

Parameters which we obtain from the crash pulse analysis (acceleration of the car in the x-direction - longitudinal) shown

Table 1. Comparison between car's and Kelvin model's responses - raw data.

Parameter	Crash pulse analysis	Kelvin model
Dynamic crush C [cm]	57	50
Time of dynamic crush t_m [ms]	80	80

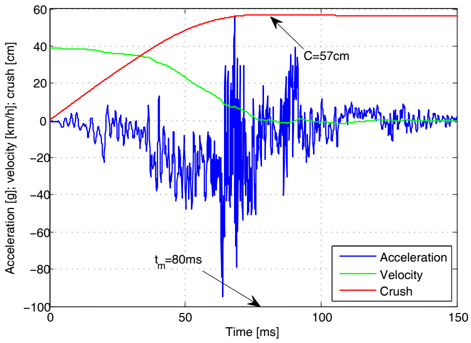

Fig. 4. Raw data analysis.

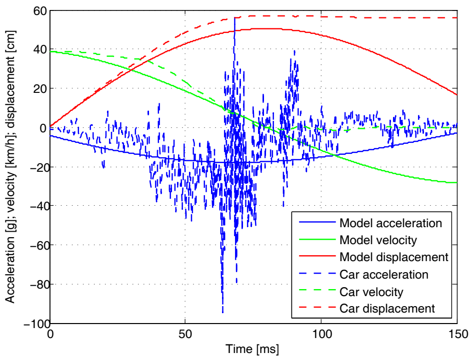

Fig. 5. Kelvin model's response - raw data.

in Fig. 4 are listed in Table 1. At the time when the relative approach velocity is zero, the maximum dynamic crush occurs. The relative velocity in the rebound phase then increases negatively up to the final separation (or rebound) velocity, at which time a vehicle rebounds from an obstacle. The contact duration of the two masses includes both contact times in deformation and restitution phases. When the relative acceleration becomes zero and relative separation velocity reaches its maximum recoverable value, separation of the two masses occurs.

By following Ref. [29] (method of calculating damping factor ζ and natural frequency f is covered in Ref. [25]), spring stiffness k and damping coefficient c of the Kelvin model are determined to be:

$$k = 4\pi^2 f^2 m = 4\pi^2 \cdot (2.9375Hz)^2 \cdot 873kg$$
$$= 297392 N/m$$
$$c = 4\pi f \zeta m = 4\pi \cdot 2.9375Hz \cdot 0.1 \cdot 873kg$$
$$= 3223 N \cdot s/m.$$

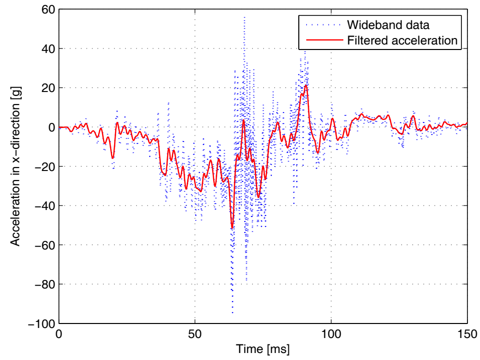

Fig. 6. Butterworth 3rd order filtering results.

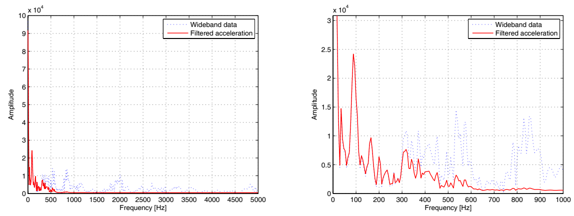

Fig. 7. Frequency analysis of crash pulses in linear scale (left - whole spectrum, right - cut-off frequency region).

Validation of the model has been done in Matlab Simulink software - the response of the Kelvin model with above estimated parameters is shown in Fig. 5.

Comparison of dynamic crush and time of dynamic crush from the crash pulse analysis and Kelvin model response is done in Table 1.

Remark 1. Since the raw data has been used above, the discrepancy between the real initial impact velocity (which is $V_0 = 9.86$ *m/s* = 35 *km/h*) and initial impact velocity obtained from the raw data analysis (which is $V_0 = 10.80$ *m/s* = 39 *km/h*) is visible. Therefore, to eliminate inaccuracies in modeling caused by this velocity difference we need to filter the acceleration measurements.

4. Acceleration measurements filtering

Digital filtering method has been used here [31]. Frequency response corridors for an appropriate channel class are specified in this standard. Since our goal is to analyze the crash pulse (i.e. integration for velocity and displacement) we select the channel class CFC 180. Filter utilized by us was Butterworth 3rd order lowpass digital filter with cut-off frequency $f_N = 300$ *Hz*. Comparison between the wideband data and data filtered with this method is shown in Fig. 6. In Fig. 7, the comparison in the frequency domain between the raw and filtered acceleration is presented.

Since the scale is linear, we clearly see that the filtering helped us to get rid of the high frequency components of the crash pulse. This makes its analysis more efficient and gives us results which better correspond to the reality than the ones obtained from wideband data (velocity and displacement).

Table 2. Comparison between car's and Kelvin model's responses - filtered data.

Parameter	Crash pulse analysis	Kelvin model
Dynamic crush C [cm]	52	43
Time of dynamic crush t_m [ms]	76	76

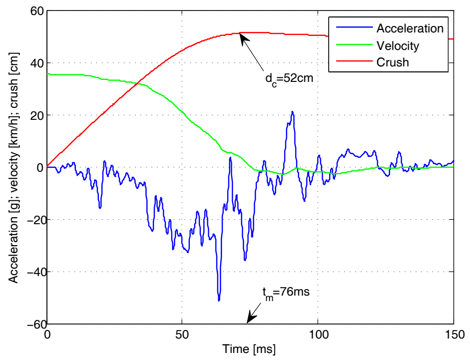

Fig. 8. Filtered data analysis.

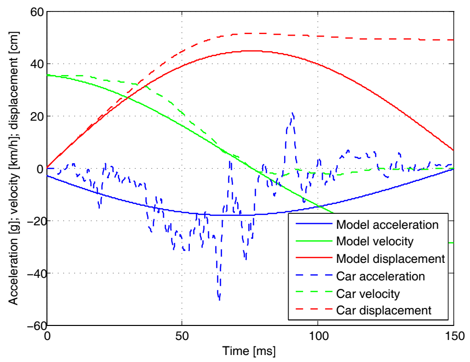

Fig. 9. Kelvin model's response - filtered data.

This has crucial influence on our further considerations because in order to develop a good model, we need to have at our disposal real parameters of the crash test (e.g. initial velocity).

5. Filtered data analysis

5.1 Kelvin model

Let us determine what the maximum dynamic crush and the time at which it occurs are for the filtered data.

Parameters which we obtain from the crash pulse analysis (acceleration of the car in the x-direction - longitudinal) shown in Fig. 8 are listed in Table 2.

Proceeding in the same manner as in Section 3, we obtain the following parameters of the Kelvin model:

$$k = 4\pi^2 f^2 m = 344150 N/m$$
$$c = 4\pi f \zeta m = 2427 N \cdot s/m.$$

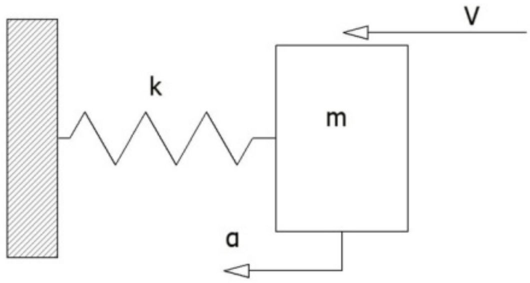

Fig. 10. Spring-mass model.

Kelvin model response for those parameters is shown in Fig. 9. Comparison between the model and reality for the filtered data is done in Table 2.

Filtering the data has improved our calculations - we have obtained the real value of the initial velocity $V_0 = 9.86$ *m/s* = *35 km/h*. However, we observe a larger discrepancy between the dynamic crush from the acceleration's integration and model's prediction than for the raw data.

This allows us to claim that since the method utilized in both of those cases remains the same and accuracy of our calculations has increased because of the data filtering, the Kelvin model is not suitable for modeling the impact examined by us. For that reason we investigate a simpler model which consists of spring and mass only.

5.2 Spring-mass model

The motion of this system is a non-decayed oscillatory one (sinusoidal) because there is no damping in it [29]. This arrangement is shown in Fig. 10. Symbols: k - spring stiffness, m - mass, a - absolute displacement of mass m.

Let us introduce the following notation:

V - initial barrier impact velocity [m/s]

f - structural natural frequency [Hz].

Response of this system is characterized by the following equations:

$$\ddot{\alpha}(t) = -V\omega_e \sin(\omega_e t) \tag{1}$$

$$\dot{\alpha}(t) = V\cos(\omega_e t) \tag{2}$$

$$\alpha(t) = \frac{V}{\omega_e}\sin(\omega_e t) \tag{3}$$

which represent deceleration, velocity and displacement, respectively. Furthermore we define:

$$C = \frac{V}{\omega_e} \tag{4}$$

$$t_m = \frac{\pi}{2\omega_e} \tag{5}$$

$$\omega_e = \sqrt{\frac{k}{m}} \tag{6}$$

Table 3. Comparison between car's and spring-mass model's responses - filtered data.

Parameter	Crash pulse analysis	Spring-mass model
Dynamic crush C [cm]	52	52
Time of dynamic crush t_m [ms]	76	83

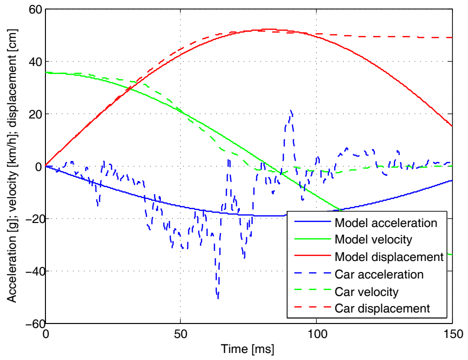

Fig. 11. Spring-mass model's response.

as maximum dynamic crush, time of maximum dynamic crush and system's circular natural frequency, respectively.

To investigate what the parameters C and t_m of such a model are, first we need to find the spring stiffness k. By substituting Eq. (6) to Eq. (4) and rearranging one gets:

$$k = \frac{V^2}{C^2} m. \tag{7}$$

From Fig. 8 it is obtained $C = 0.52\ m = 52\ cm$ and $V = 9.86\ m/s = 35\ km/h$ for filtered data. Therefore

$$k = \frac{(9.86 m/s)^2}{(0.52)^2} 873 kg = 313878 N/m$$

$$t_m = \frac{\pi}{2\omega_e} = \frac{\pi}{2\sqrt{\frac{k}{m}}} = \frac{\pi}{2\sqrt{\frac{313878 N/m}{873 kg}}}$$

$$= 0.083 s.$$

Spring-mass model's response for above spring stiffness k (initial velocity and mass of the car remain the same) is shown in Fig. 11. Let us compare what the dynamic crush and the time at which it occurs are for the car and model – see Table 3.

Results obtained in this step are good. The dynamic crush estimated by the spring-mass model is exactly the same as the reference dynamic crush of a real car. When it comes to the time when it occurs, the difference between the model and reality is less than 1%. This model gives us good approximation of the car's behavior during the crash. It is a particular case of a Kelvin model in which damping has been set to zero as well as of a Maxwell model in which damping goes to infinity.

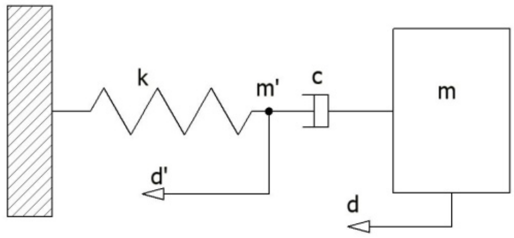

Fig. 12. Maxwell model - m' designates zero-mass.

6. Maxwell model - introduction

The arrangement in which spring and damper are connected in series to mass is called Maxwell model - Fig. 12. To derive its equation of motion it is proposed to place small mass m' between spring and damper. By doing this, the inertia effect which occurs for the spring and damper is neglected and the system becomes third order differential equation which can be solved explicitly [29]. According to Fig. 12 we define d and d' as absolute displacement of mass m and absolute displacement of mass m', respectively. We establish the following equations of motion (EOM):

$$m\ddot{d} = -c(\dot{d} - \dot{d}') \tag{8}$$

$$m'\ddot{d}' = c(\dot{d} - \dot{d}') - kd'. \tag{9}$$

By differentiating Eq. (8) and Eq. (9) w.r.t. time and setting $m' = 0$ we obtain:

$$m\dddot{d} = -c(\ddot{d} - \ddot{d}') \tag{10}$$

$$0 = c(\ddot{d} - \ddot{d}') - k\dot{d}'. \tag{11}$$

We sum up both sides of Eqs. (10) and (11) and rearrange:

$$\dot{d}' = \frac{-m}{k}\dddot{d}. \tag{12}$$

We substitute Eq. (12) into Eq. (8) and finally obtain the EOM found below:

$$\dddot{d} + \frac{k}{c}\ddot{d} + \frac{k}{m}\dot{d} = 0. \tag{13}$$

Therefore, characteristic equation of the Maxwell model is:

$$s[s^2 + \frac{k}{c}s + \frac{k}{m}] = 0. \tag{14}$$

In this system, the rebound of the mass depends on the sign of discriminant Δ of the quadratic equation in brackets. For positive Δ there is no rebound, i.e.:

$$\left(\frac{k}{c}\right)^2 > 4\frac{k}{m}.$$

In this case, roots of the characteristic equations (Eq. (14)) are, respectively:

$$s_0 = 0$$
$$s_1 = a + b$$
$$s_2 = a - b$$

where:

$$a = \frac{-k}{2c}$$
$$b = \sqrt{\left(\left(\frac{k}{2c}\right)^2 - \frac{k}{m}\right)}.$$

On the other hand, for negative Δ the rebound occurs when:

$$\left(\frac{k}{c}\right)^2 < 4\frac{k}{m}.$$

In this case, roots of the characteristic equation (Eq. (14)) are, respectively:

$$s_0 = 0$$
$$s_1 = a + ib$$
$$s_2 = a - ib$$

where:

$$a = \frac{-k}{2c}$$
$$b = \sqrt{\left(\frac{k}{m} - \left(\frac{k}{2c}\right)^2\right)}.$$

Since this case is of our greater interest than the previous one (due to the fact that in the experiment rebound occurred) we will describe in details its response. Displacement of a mass is given by the formula:

$$\alpha = d_0 e^{s_0 t} + e^{at}[d_1 \sin(bt) + d_2 \cos(bt)]. \quad (15)$$

Initial conditions ($t = 0$) are:

$$\alpha = 0$$
$$\dot{\alpha} = v$$
$$\ddot{\alpha} = 0.$$

where v is the initial impact velocity. Constants are:

$$d_2 = \frac{2av}{a^2 + b^2}$$
$$d_1 = \frac{v - ad_2}{b}$$
$$d_0 = -d_2.$$

However, in a Maxwell model, the mass may not rebound from the obstacle. It means that its displacement increases with time to an asymptotic value. The parameter, which determines whether the rebound will occur or not, is damping coefficient. When it is less than a limiting one (named transition damping coefficient c*), the mass will be constantly approaching an obstacle, whereas when it is higher, there will exist a dynamic crush at a finite time. Another boundary situation is for damping coefficient $c = \infty$. Then the Maxwell model degenerates into spring-mass system. To determine the value of transition damping coefficient we assume that c = 0, or equivalently

$$\frac{k}{c^*} = 2\sqrt{\frac{k}{m}}$$

and

$$c^* = \frac{\sqrt{km}}{2}. \quad (16)$$

Indeed, for $c < c^*$ we have $\Delta > 0$ - it means no dynamic crush at a finite time.

We are able to assess what the minimal damping should be, which we add to the simple spring-mass model mentioned above, which will produce the dynamic crush not extended in an infinite period of time. According to Eq. (16), for the model and crash test being analyzed in Section 5.2, we calculate the transition damping coefficient:

$$c^* = \frac{\sqrt{313878 N/m \cdot 873 kg}}{2} = 8277 N \cdot s/m.$$

For every damping greater than this value, the Maxwell model formed from the spring-mass model from Section 5.2, will give us the response more and more similar to the spring-mass model characteristics presented in Fig. 11, as it is shown in Fig. 13.

It is noting that the final displacement (or asymptotic value - for transition damping coefficient) achieved by the mass in this model is characterized by the equation (V_0 - initial impact velocity, m - mass, c - damping coefficient):

$$crush = \frac{V_0 m}{c}. \quad (17)$$

This system is appropriate for simulating soft impacts or offset impacts because the time of dynamic crush is longer than for Kelvin model. We assume the same parameters for

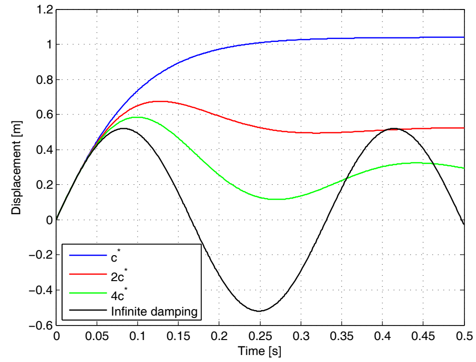

Fig. 13. Maxwell model responses for different values of damping.

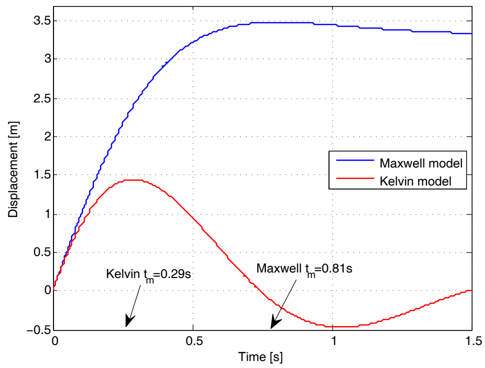

Fig. 14. Maxwell and Kelvin models' responses comparison.

both models, e.g.:

$k = 100\ N/m,\ c = 15\ N\text{-}s/m,\ m = 5\ kg,\ v_0 = 10\ m/s.$

In Fig. 14, it is seen that for the Maxwell model the dynamic crush occurs later than for the Kelvin model.

This is an analog situation to the real crash: in a vehicle-to-rigid barrier collision (Kelvin model) the whole impact energy is being consumed faster, therefore the crash is more dynamic than the vehicle-to-pole collision (Maxwell model) - under the assumption that we compare the same cars with the same initial impact velocities - as in the example above. It is noting that we do not investigate here the magnitude of the displacement of both models - as we can see for the same parameters it is higher for the Maxwell model. Above example just illustrates the dynamic responses of those two systems and in order to apply those two models to the real crash one needs to assess what spring stiffness and damping coefficient of both of them are separately.

7. Maxwell model analysis

When it comes to Maxwell model, we will discuss just two cases for which $\Delta < 0$, i.e. when the rebound occurs, because that is what happens during the experiment. We are going to start with the simplification of this situation, in which damping coefficient of this model has a limiting, transitional value. Then we proceed to the full Maxwell model's analysis.

7.1 Maxwell model with transition damping coefficient

This is the particular case of a Maxwell model in which mass' displacement reaches an asymptotic value given by Eq. (17). For

$$c^* = \frac{\sqrt{km}}{2}$$

parameters of Eq. (15) degenerate into:

$$a = -\omega$$
$$b = 0$$
$$d_1 = -\frac{v}{b}$$
$$d_2 = -2\frac{v}{\omega}$$
$$d_0 = 2\frac{v}{\omega}$$

where

$$\omega = \sqrt{\frac{k}{m}}.$$

We take advantage of the following trigonometric relationships:

$$\lim_{b\to 0}\frac{\sin(bt)}{b} = t$$
$$\lim_{b\to 0}\cos(bt) = 1.$$

Finally we come up with the following equation of mass' displacement:

$$\alpha = \frac{v}{\omega}[2 - (\omega t + 2)e^{-\omega t}]. \tag{18}$$

To establish the parameters of the Maxwell model (spring stiffness k and damping coefficient c) we just substitute to Eq. (18) values of initial impact velocity v, maximum dynamic crush α and time of maximum dynamic crush t_m taken from the acceleration measurements analysis shown in Fig. 8 - we obtain $\omega = 37.52\ rad/s$. Knowing circular natural frequency ω and mass of the whole vehicle $m = 873\ kg$, we calculate spring stiffness k and transition damping coefficient c^*:

$$k = \omega^2 m = 37.24^2 \cdot 873 = 1228966 N/m$$
$$c^* = \frac{\sqrt{km}}{2} = \frac{\sqrt{1228966 \cdot 873}}{2} = 16377 N \cdot s/m$$

Response of the model with above computed parameters

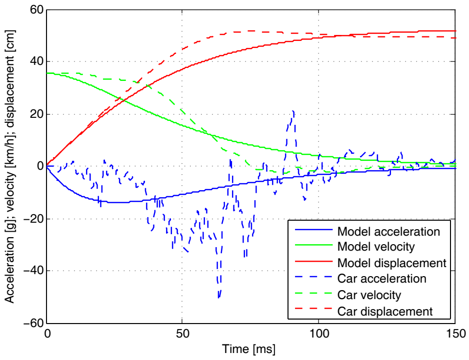

Fig. 15. Maxwell model's (transition damping coefficient) response.

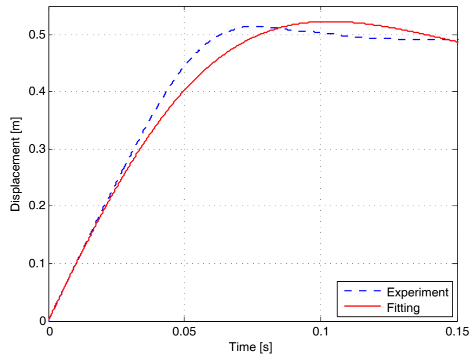

Fig. 16. Fitting the Maxwell model's response to the real experiment's displacement.

and initial impact velocity $v = 9.86$ *m/s* $= 35$ *km/h* is shown in Fig. 15.

As we can see the value of maximum dynamic crush is the same as the one obtained from the experiment's data analysis ($C = 0.52$ *m*) and time when it occurs is much longer - approximately $t_m = 0.2$ *s* (compared to experiment's $t_m = 0.076$ *s*).

7.2 Maxwell model - rebound

Response of the Maxwell model is described by Eq. (15). Having the car's displacement curve from the experiment we can establish parameters of the model (spring stiffness k and damping coefficient c) just by fitting the curve defined by Eq. (15) to that real graph. However parameters which we obtain by fitting Eq. (15) are: a, b, d_0, d_1 and d_2. Since d_1 and d_2 (we do not discuss d_0 separately because $d_0 = -d_2$) are functions of v as well, it is not guaranteed that the model's parameters which we obtained would be correct - in another words, v wouldn't be fixed if we fit Eq. (15) to the experiment's displacement. Therefore we express Eq. (15) only in terms of a and b (which are just functions of k, c and mass $m = 873$ *kg*) and set initial impact velocity to $v = 9.86$ *m/s*. The equation which we obtain has the following form:

$$\alpha = -\frac{2av}{a^2+b^2} + e^{at}\left[\frac{v - \frac{2a^2 v}{a^2+b^2}}{b}\sin(bt) + \frac{2av}{a^2+b^2}\cos(bt)\right]. \tag{19}$$

Fitting Eq. (19) to the experiment's results has been done in Matlab Curve Fitting Toolbox and is shown in Fig. 16. Fitting Eq. (19) not Eq. (15) resulted in loss of approximation's accuracy but on the other hand, we are sure that the initial impact velocity has the correct value. From the above operation we obtain parameters a and b of Eq. (19) which are equal to:

$$a = \frac{-k}{2c} = -14.79\frac{1}{s}$$

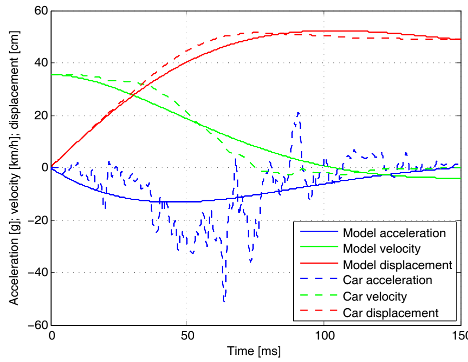

Fig. 17. Complete Maxwell model's response.

$$b = \sqrt{\left(\frac{k}{m} - \left(\frac{k}{2c}\right)^2\right)} = 21.06\frac{1}{s^2}.$$

Now we have all we need to calculate damping coefficient c and spring stiffness k of the Maxwell model, respectively:

$$c = \frac{m(a^2+b^2)}{-2a} = 19546 N \cdot s / m$$
$$k = -2ac = 578171 N / m.$$

Response of the model is shown in Fig. 17. As we can see the value of maximum dynamic crush is exactly the same as the one obtained from the experiment's data analysis ($C = 52$ *cm*) and time when it occurs is longer - $tm = 104$ *ms* (compared to the experiment's $tm = 76$ *ms*). However, as it is going to be shown in the following Section, the overall response of the Maxwell model is the most similar to the car's behavior during collision with a pole.

8. Models comparison

To represent vehicle to pole collision we established in total four models here (spring-mass model, Kelvin model, Maxwell model with transition damping coefficient and complete Maxwell model). Let us compare their responses with the car's

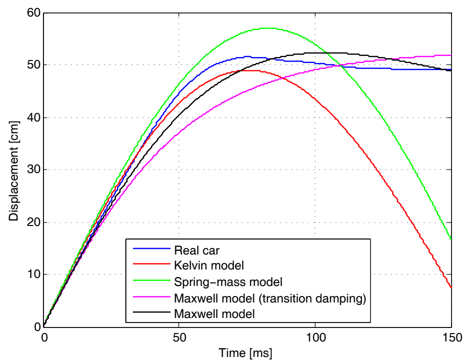

Fig. 18. Models' responses comparison.

behavior during the experiment analyzed by us - see Fig. 18.

Characteristics which the best represents the overall car's behavior during the crash period belongs to the Maxwell model. Although Kelvin and spring-mass models give good approximation in the beginning of the crash (up to the time of maximum dynamic crush), they completely fail when it comes to the crash representation after the rebound. Maxwell model with transition damping coefficient shows correctly just the maximum dynamic crush, not its time at all. Therefore the Maxwell model gives the best overall outcome - there is no difference in maximum displacement and about 27% of divergence for time of maximum dynamic crush comparing to the reality. And the entire shape of the Maxwell model's response resembles closely the real car's crush.

9. Conclusion and future works

In this paper, we studied a process of improving the accuracy of the vehicle crash model. First, we simulated the vehicle under a pole impact by using the Kelvin model. Afterwards, by filtering the crash pulse data, more accurate response of the system was obtained. Model establishment was done one more time. Finally, we compared the crash models and it was concluded which of them is more suitable to represent vehicle to pole collision.

The obtained results indicate that the Kelvin model is not appropriate for simulation of the collision which we deal with. Based on Section 6, for the data prepared in the proper way, we establish a proper model. Results obtained from studying Maxwell model provided us with satisfactory results. Comparative analysis of the model's and real car's responses turned out to be appropriate. Therefore if one wants to simulate a vehicle to pole collision it is advisable to use Maxwell model.

It is desirable to verify whether the other viscoelastic models which were not discussed in this paper are capable of vehicle crash simulation. In particular, so called hybrid models (systems composed of two springs, one damper, and a mass) may be promising for this application. Furthermore, a two-mass-spring-damper model can be used to represent interactions between fore- and aft-frame of a vehicle. On top of that, it is advisable to examine methods for nonlinear system parameters identification. Since all the models presented in the current study are lumped parameter ones which are valid only for the data which were used for their creation, they cannot be used to simulate e.g. a high-speed vehicle collision. However, the capabilities of mathematical models with nonlinear parameters (stiffness and damping) to simulate a variety of crash events are required to be assessed.

Nomenclature

k : Spring stiffness
c : Damping coefficient
m : Mass
V_0; V : Initial impact velocity
ζ : Damping factor
f : Structural natural frequency
C : Maximum dynamic crush
t_m : Time of maximum dynamic crush
f_N : Cut-off frequency
a; d : Absolute displacement of mass m
α : Model displacement
ω_e; ω : Circular natural frequency
m' : Zero-mass
d' : Absolute displacement of mass m'
c^* : Transition damping coefficient
Δ : Discriminant

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