1-1-1972

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Recommended Citation

King, Alton B.; Andersen, Jay C.; Clyde, Calvin G.; and Hoggan, Daniel H., "Development of Regional Supply Functions and a Least-Cost Model for Allocating Water Resources in Utah: A Parametric Linear Programming Approach" (1972). *Reports.* Paper 269.
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DEVELOPMENT OF REGIONAL SUPPLY FUNCTIONS AND A LEAST-COST MODEL FOR ALLOCATING WATER RESOURCES IN UTAH: A PARAMETRIC LINEAR PROGRAMMING APPROACH

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June 1972

PRWG100-2

ABSTRACT

The development and allocation of the water resources within a state require water planners to prepare plans far in advance of the actual time new facilities are required. It is not easy to identify and evaluate all the possible alternatives for providing water which incorporate broad objectives such as economic efficiency, social welfare regional development, recreation benefits, and conservation of environment.

Water resources development entails the modification of a natural hydrologic system to better meet man's needs. The interrelationships among elements of the hydrologic system are relatively simple in comparison to the social, legal, economic, and institutional interdependencies involved. The relationships are so complex as to require that planning of water resource development be accomplished on a systems basis. It has become apparent that water resource planning must consider mass transfer of water encompassing areas which have potential for economic growth competing with other areas already highly developed economically. The wisest political decisions and the greatest benefit to the public will result if a method is used to explore the probable consequences of alternative water resources development and management policies and plans. The objective of this study is to extend the capability of systems analysis and operations research to the problem of interregional planning of water resources allocation for the State of Utah.

The hydrologic characteristics and cost of water in each of the ten hydrologic study units of the state were determined. Hydrologic data from hydrologic inventories and estimates from the Utah Division of Water Resources were used to determine availability, reservoir storage-draft relationships, evaporation loss from reservoirs, agricultural use return flow, and municipal and industrial use return flow. Cost data were developed for storage facilities, diversion and canal works, artificial recharge facilities, treatment of waste water, and treatment of municipal supply.

Supply functions for water in each of the ten hydrologic study units of the state were determined. Two sets of functions were developed—one for agricultural use and one for municipal and industrial use. Parametric linear programming was employed to develop a functional map of the shadow price (marginal cost) of water for each of the two uses. The shadow price of imported water (value) to each of the study units was also determined to show the possible economic consequence of inter-basin transfers. In general, imported water was of little or no value if water presently being evaporated from Great Salt Lake is available for diversion upstream.

A statewide model was developed to determine a least-cost allocation of water resources to meet projected requirements. This linear programming allocation model was developed subject to constraints such as hydrologic characteristics, limits on inter-basin transfers, limits on artificial groundwater recharge, and existing water requirements. Parametric programming was utilized to determine the impact of changing availability which reflects policies regarding inflow requirements of the Great Salt Lake and interstate agreements, increased agricultural use and municipal and industrial use which reflects population increases projected for the future, and changing groundwater availability which reflects legal constraints. The primary factor affecting inter-basin transfer of Colorado River water is the degree to which evaporation from Great Salt Lake is reduced.

ACKNOWLEDGMENTS

The Utah Water Research Laboratory, under direction of Dr. Jay M. Bagley, on 1 July 1968, received a matching fund grant from the Office of Water Resources Research of the United States Department of the Interior to perform research studies directed toward the application of operations research techniques for allocation of Colorado River waters in Utah. A statewide allocation model was developed under this grant.

On 4 March 1971 the Corps of Engineers of the United States Army issued a grant to the Utah Water Research Laboratory to conduct studies related to interregional planning of water resource allocation using a systems analysis approach. Regional supply functions were developed under this grant. This paper contains results of the study performed for OWRR and part of the results of the Corps of Engineers study.

The cooperation and assistance of the Division of Water Resources, Utah Department of Natural Resources, is also gratefully acknowledged.

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INTRODUCTION

Nature of the Study

The development and allocation of water within a state calls for a long sequence of crucial decisions. Water planners are faced with the problem of identifying optimal development plans far in advance of the actual time the new facilities would be needed. This problem is confounded in Utah due to the state's location in the mountain west, a region of arid to semi-arid conditions. Human judgment alone is not sufficient to determine long-term plans which incorporate a broad overview of the state with objectives such as economic efficiency, social welfare, regional development, recreation benefits, and conservation of environment.

Fundamentally, water resources development entails the modification of a natural hydrologic system to meet man's needs. When modifications are made to certain parts of the system, the equilibrium of the system is changed and other components or elements are affected. Consequently, one of the main questions raised in connection with any development scheme is: What will be the effect on existing uses? The interrelationship among elements of the hydrologic system, though varied and complex, are relatively simple in comparison with the social, legal, economic, and institutional interdependencies involved. These relationships are so close and so strong as to require that planning of water development be accomplished on a systems basis. Although methodology has not been devised which can consider all of the variables and parameters involved, describe their interaction in time and space, and arrive at a simultaneous solution to the whole matrix, advances in the physical sciences and technology have made available a number of new and improved decision-making techniques for application to water resources planning. Operations research and systems analysis associated with advances in computer technology are particularly useful.

The nature of this study is to extend the capability of systems analysis to the problem of interregional planning of water resources allocation. The study is restricted to the State of Utah. However, the methodology is general and is applicable to other regions of the nation as well.

Timeliness to Utah Water Planning

Water resource developments in the State of Utah historically have followed a piecemeal approach and little consideration has been given to the entire economy. The Utah State Water Plan has taken significant steps in overcoming the problems of independent developments within the various regions. The research in this report should supplement studies under the State Water Plan by evaluating the impact of water resources developments on the entire state economy.

The total supply of water within the state is limited by amounts of precipitation received and interstate agreements. The general purpose of this research is to make a more adequate evaluation of the geographical allocation of water within and, in particular, between the various regions of the state and to determine the economic feasibility of interregional transfers. The future development of facilities to transport water from areas of excess supply to those of excess demand will be considered.

It has become apparent that water resource decisions must partly be based on an evaluation of mass transfers of water within regions which have potential for economic growth competing with other areas already highly developed economically. Clearly, the wisest political decisions and the greatest benefit to the public will result if a method is used to explore the probable consequences of alternative water resources development and management policies and plans. It is to the development and application of such an analytical method that this research is directed.

Objectives of the Study

The specific objectives of the study are outlined below:

1. *Determine the hydrologic characteristics and cost of water from various sources in each of the hydrologic study units of the state defined by the Utah Division of Water Resources.* Hydrologic data from water budget studies and estimates from the Division of Water Resources are used to determine availability, storage-draft relationships, evaporation loss from reservoirs, agricultural return flow, and municipal and industrial use return flow.

Cost data are developed in the study for storage facilities, diversion and canal works, artificial recharge facilities, treatment of waste water, and treatment of municipal supply.

2. *Determine supply functions for water in each of the hydrologic study units of the state.* Two sets of functions are developed—one set for agricultural use and one set for municipal and industrial use. Parametric linear programming is employed to facilitate the calculations.

3. *Determine a least-cost spatial allocation of Utah's water resources to meet requirements within the hydrologic study units of the state.* A statewide linear programming allocation model is developed which will minimize cost subject to constraints such as hydrologic characteristics, limits on inter-basin transfers, limits on artificial groundwater recharge, and water requirements. Parametric programming is utilized to determine the impact of changing availability of water, increasing municipal and industrial use, increasing agricultural use, and changing policy on such things as groundwater development laws.

REVIEW OF LITERATURE

Systems Analysis and Mathematical Programming Techniques

In recent years systems analysis has become increasingly useful as a tool in water resources planning. design and development, operating procedures, and management.

According to Drobney systems analysis is:

> ... A strategy for problem solving which relies heavily on mathematical modeling to assess the technical and economic optimality of alternative systems designs, policies, operating procedures, etc., for performing various functions and meeting various needs with limited resources. It is important to keep in mind that systems analysis *per se* does not provide these assessments which also must incorporate professional, legal, political, and social consideration. Rather systems analysis may be employed as a decision aid in assessing the technical and economic consequence of alternative courses of action. (Drobney, 1968, p. 534)

A mathematical model is defined as a set of equations which describe some physical, biological, or chemical process. James and Lee (1971) classify the models in three categories; (1) performance versus optimization models, (2) deterministic versus stochastic models, and (3) analytical versus simulation models. Drobney (1968) further distinguishes between the usefulness of the various models and states the type of problems which might be solved by each model. The optimization model using analytical definitions of the function to be optimized and based on deterministic technology has been used most often for water resource planning in the past (James and Lee, 1971, and Maass et al., 1962). Simulation models with stochastic hydrology are becoming increasingly popular.

A mathematical programming problem occurs when an analyst seeks to maximize or minimize an analytical function (called an objective function) of one or more variables subject to certain relationships involving the variables (called constraints) (Intriligator, 1971). Under certain limited conditions, a solution to this problem can be found using classical differential calculus, including Lagrangian multipliers and the calculus of variations. The complex engineering and economic aspects of current water resource problems with their multiplicity of variables are far beyond the computational adequacy of the classical methods and have motivated a keen interest in programming models (Drobney, 1968). Several programming models have been developed and computational algorithms exist for some of their solutions. There are linear programming (Hadley, 1962), non-linear programming (Hadley, 1964) including quadratic programming and geometric programming (Duffin, Peterson, and Zener, 1967), and dynamic programming (Hadley, 1964).

Linear programming is one of the most widely used of all systems analysis techniques. A statement of this problem might be:

> Given a set of m linear inequalities or equations in r variables ($r \lesseqgtr m$), we wish to find non-negative values of these variables which will satisfy the constraints and maximize or minimize some linear function of the variables. (Hadley, 1962)

Many applications have been made of the linear programming model to solve problems in water resources. Some of these are:

(1) Least-cost plan for waste treatment (Loucks, Revelle, and Lynn, 1967; Johnson, 1967; Rogers and Gemmel, 1966; Sobel, 1965; Thomann, 1965).
(2) Optimum operation of large dams considering benefits from hydropower and irrigation (Thomas and Nevelle, 1966).
(3) Sewage treatment plant design (Lynn, Logan, and Charnes, 1962).
(4) Conjunctive use of surface water and groundwater (Milligan, 1969).
(5) Water allocation between regions of a state (Gold, Milligan, and Clyde, 1969; Clyde, King, and Andersen, 1971).

Non-linear programming is similar to linear programming except the objective function and constraints are not required to be linear functions of the decision variables (Hadley, 1964). One form of this non-linearity for which numerical techniques have been developed to solve is known as quadratic programming in which the objective function has quadratic terms subject to linear constraints. Quadratic programming was used by Lynn (1966) to determine a least-cost pumping schedule for wells. A more general, and consequently harder to solve, form of non-linearity occurs when the objective function is non-linear to a higher degree than quadratic. This form is known as geometric programming (Duffin, Peterson, and Zener, 1967). Geometric programming is just in its infancy in water resources use but has been used successfully in other applications (Beightler, Crisp, and Meier, 1968, and Wilde and Beightler, 1967).

A tool that has been used quite successfully to solve sequential decision problems is dynamic programming. According to Drobney:

> A sequential decision problem is a problem in which a sequence of decisions (termed a policy) must be made and in which each decision affects future decisions ... unlike linear programming, there exists no standard mathematical model format according to which a problem may be structured for solution by dynamic programming. Rather dynamic programming is an approach oriented technique, and the particular equations to be used must be developed to fit the problems at hand. (Drobney, 1968, p. 543)

Examples of its use are:

(1) Design and operation of multi-reservoir systems (Amir, 1967; Buras, 1965; Meier and Beightler, 1967; and Schweig and Cole, 1968).
(2) Optimization of individual multi-purpose reservoirs (Hall, 1964; and Hall, Butcher, and Esogbue, 1968).
(3) Minimization of overall cost of waste treatment among discharges (Liebman and Lynn, 1966).
(4) Optimal use of groundwater over time (Burt, 1964).
(5) Optimization of conjunctive use of groundwater and suface water (Aron, 1969).

A combination of dynamic programming with linear programming has been used to study the problem of optimal future operation of a water resource system with random streamflows (Shailendra and Shepard, 1967).

Economic Analysis and Resource Allocation

Economics has been described both as an art and a science. According to Samuelson:

> Economics is the study of how men and society end up choosing, with or without the use of money, to employ scarce productive resources which could have alternative uses, to produce various commodities and distribute them for consumption, now or in the future, among various people and groups in society. (Samuelson, 1970, p. 4)

To be meaningful, economics must be able to describe, to analyze, to explain, and to correlate the behavior of production, unemployment, prices, and similar phenomena. Descriptions must be more than a series of disconnected narratives, they must fit in a systematic pattern, i.e., constitute true analysis. The phenomena associated with water resource development are a subset of the general set of phenomena associated with economics which may not include unemployment. Social factors such as legal constraints, environmental constraints, and costs of all kinds to society should be included in a general water resources analysis.

Water resources systems may be created in almost infinite variety through different combinations of system units, levels of output, and allocations of reservoir capacity, etc., to various uses. Maass et al. (1962) indicates the methodology of system design involves four related steps: (1) Identifying the objectives of the design, (2) Translating these objectives into design criteria, (3) Using the criteria to devise plans for the development of systems that fulfill the criteria in the highest degree, and (4) Evaluating the consequences of the plans that have been developed.

James and Lee (1971) state that the overall objective of water resource development is to meet human needs. It therefore fits into the category of welfare economics which seeks to develop better procedures (without bias toward either the public or private sector of the economy) for allocating the total resource base (labor, capital, land, etc.) among potential uses and users to meet individual and group needs. The ideal resource allocation would be achieved if the policy were to maximize some unanimously accepted index of total human welfare. Social goals related to water resource development are quite varied and there is a diversity of opinion on their relative desirability. A review of the following list of goals should indicate that an ideal resource allocation acceptable to all persons involved will never be available to planners of water resource developments.

1. Maximum national income
2. Ideal income distribution
3. Institutional stability
4. Public health
5. Regional development
6. Environmental enhancement

James and Lee (1971) designate first-order efficiency as social and to achieve such efficiency would require meeting all the social goals of water resources development. Since it is not possible to describe all these social goals in mathematical terms they suggest the next best that can be hoped for is second order or economic efficiency. The mathematical model describing economic efficiency is constrained by the social goals and the implication of these social goals can be determined by the manner in which they compromise economic efficiency. The optimum project then becomes one which is most effective in increasing national income or net benefits subject to constraints. In terms of mathematical programming then, the objective function is net benefit and the economic constraint is related to the technical feasibility of the project and is known as a production function. Thus if an input vector of resources is designated as X with an associated output vector as Y, then the problem can be stated as; maximize an objective function u(X,Y) subject to the constraint f(X,Y).

It would be beneficial at this point to provide insight into the relationships that would exist between inputs and outputs for the above statement of optimality. Use can be made of the calculus to find a maximum by differentiating the objective function with respect to each of the vector components, setting each differential to zero and solving the resulting equations. Since this would result in the problem being over-determined, an artificial unknown called the Lagrange multiplier is introduced. The details of this technique are given by several authors (e.g. Dorfman, in Maass et al., 1962, Chap. 3). The resulting relationship between inputs and outputs and their respective prices is:

$$\frac{\partial u/\partial x_i}{\partial u/\partial y_j} = -\frac{\partial y_j}{\partial x_i} = \frac{p_i}{p_j} = \text{MPP} \quad . \quad (1)$$

$$i = 1,\ 2,\ \ldots\ h,\ \ldots$$

$$\frac{\partial u/\partial x_i}{\partial u/\partial x_h} = -\frac{\partial x_h}{\partial x_i} = \frac{p_i}{p_h} = \text{MRS} \quad . \quad (2)$$

$$j = 1,\ 2.\ \ldots\ k,\ \ldots$$

$$\frac{\partial u/\partial y_j}{\partial u/\partial y_k} = -\frac{\partial y_k}{\partial y_j} = \frac{p_j}{p_k} = \text{MRT} \quad . \ . \ . \ . \ . \ . \ . \quad (3)$$

Analysis of these equations indicates:

$\partial u/\partial x_i$ = Marginal cost of input i = price of input i = p_i,

$\partial u/\partial y_j$ = Marginal benefit of output j = price of output j = p_j,

$-\partial y_j/\partial x_i$ = Marginal physical product (MPP), or the additional output which can be produced per unit of input,

$-\partial x_h/\partial x_i$ = Marginal rate of substition (MRS), or the marginal rate at which the h-th input can be substituted for the i-th input while holding production constant,

$-\partial y_k/\partial y_j$ = Marginal rate of transformation (MRT), or the marginal rate at which production can be shifted from the j-th output to the k-th output.

According to James and Lee the following set of rules applies:

> Rule 1. The optimum allocation of goods. Each consumer maximizes his satisfaction by ordering his consumption so that the marginal rate of distribution between any two goods is equal to the ratio of their prices....
>
> Rule 2. The optimum degree of specialization. Each firm maximizes its profit by making its marginal rate of transformation between any two outputs produced equal to the ratio of their prices.... [Equation 3]
>
> Rule 3. The optimum relationship between input and output. Each firm maximizes its profit by equating the marginal physical product of input in producing output with the ratio of their prices.... [Equation 1]
>
> Rule 4. The optimum allocation of inputs. Each firm maximizes its profit [minimizes its cost] by making its marginal rate of substitution between any two inputs used in production equal to the ratio of their prices.... [Equation 2] (James and Lee, 1971, p. 103)

Samuelson (1970) shows that the ideal market under conditions of pure competition would automatically achieve these optimum conditions. He further states that the allocation of these resources to different tasks in different ways is a problem in the theory of production. Dorfman, Samuelson, and Solow (1958) also state that this is a problem in linear economics since the restrictions on the problem are linear in that the total amount of any resource devoted to all tasks must not exceed the total amount available, thus each restriction is a simple sum.

New methods of analysis have been developed which depend on the linear characteristics of economics. The most noteworthy of these are linear programming, input-output analysis, and game theory. Linear programming is the core of linear economics.

Each of the three equations derived above has an equivalence in linear programming. Davidson, Smith, and Wiley (1962) describe how linear programming solves the problem of the choice of optimal production technique or process (Equation 2) and the problem of the choice of optimal product mix (Equation 3).

Consider the case of production possibilities for a product with only two variable inputs (X_1 and X_2) and three possible production techniques or processes (L, M, and N). As shown in Figure 1, lines of constant production rates (isoquants) are in straight line segments with slope changes occurring at the lines representing the process. The assumption is implicit that not only can the product be produced with any given process but that a *combination* of processes can be used, e.g., production at point K of 200 units reflects a combination of 100 units from process L and 100 units from process M. The production processes are assumed to have constant returns to scale, i.e., each input requirement is proportional to the output level and the processes can be operated in combination without altering the structure of input requirements. If the unit price of input X_1 is p_1 and of input X_2 is p_2 then

$$C = p_1 X_1 + p_2 X_2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

represents production cost as a function of X_1 and X_2 and is represented on Figure 1 as constant cost (iso-cost) lines for $p_1 = 6$ and $p_2 = 10$. Note that the slope of the iso-cost line is the inverse ratio of the input prices, p_1/p_2. Suppose it is desired to produce 200 units at lowest cost. The shaded area is the feasible production area. The problem in linear programming format becomes:

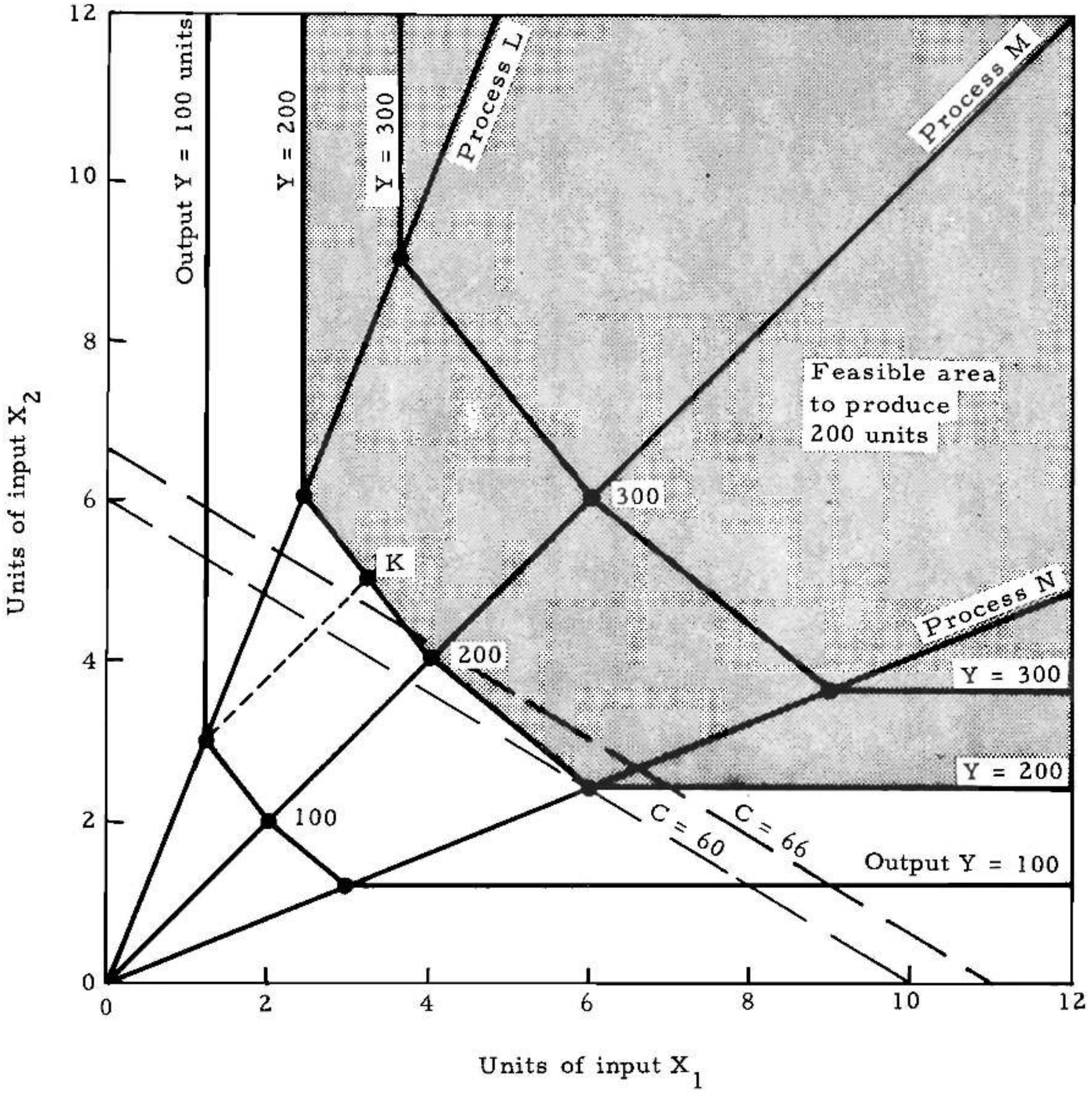

Figure 1. Graphical presentation of linear programming analysis to determine choice of inputs.

Objective function: minimize $C = 6X_1 + 10X_2$
Constraints: lines representing 200 unit isoquant

The solution can be found graphically to be:

1. Cost = 60
2. $X_1 = 6.0$ units
3. $X_2 = 2.4$ units
4. Process is N

In marginal analysis the isoquant would be a smooth curve and the contact with the iso-cost line would be a point of tangency. Thus the marginal rate of substitution (tangent line to the isoquant) would equal the price ratio of the two inputs (slope of the iso-cost line) and Equation 2 would be satisfied. In linear programming the isoquant is a series of straight line segments resulting in its slope (MRS) being discontinuous and undefined at the intersection of two segments. Thus the price ratio at a condition of optimality can vary between the values given by the MRS on each side of the optimal point.

Now consider the case of production possibilities for a firm having two possible products (Y_1 and Y_2) and two inputs (X_1 and X_2). Assume the inputs could not exceed 150 units each and the two production functions are:

$15Y_1 + 10Y_2 \le 150$ for input X_1

$10Y_1 + 15Y_2 \le 150$ for input X_2

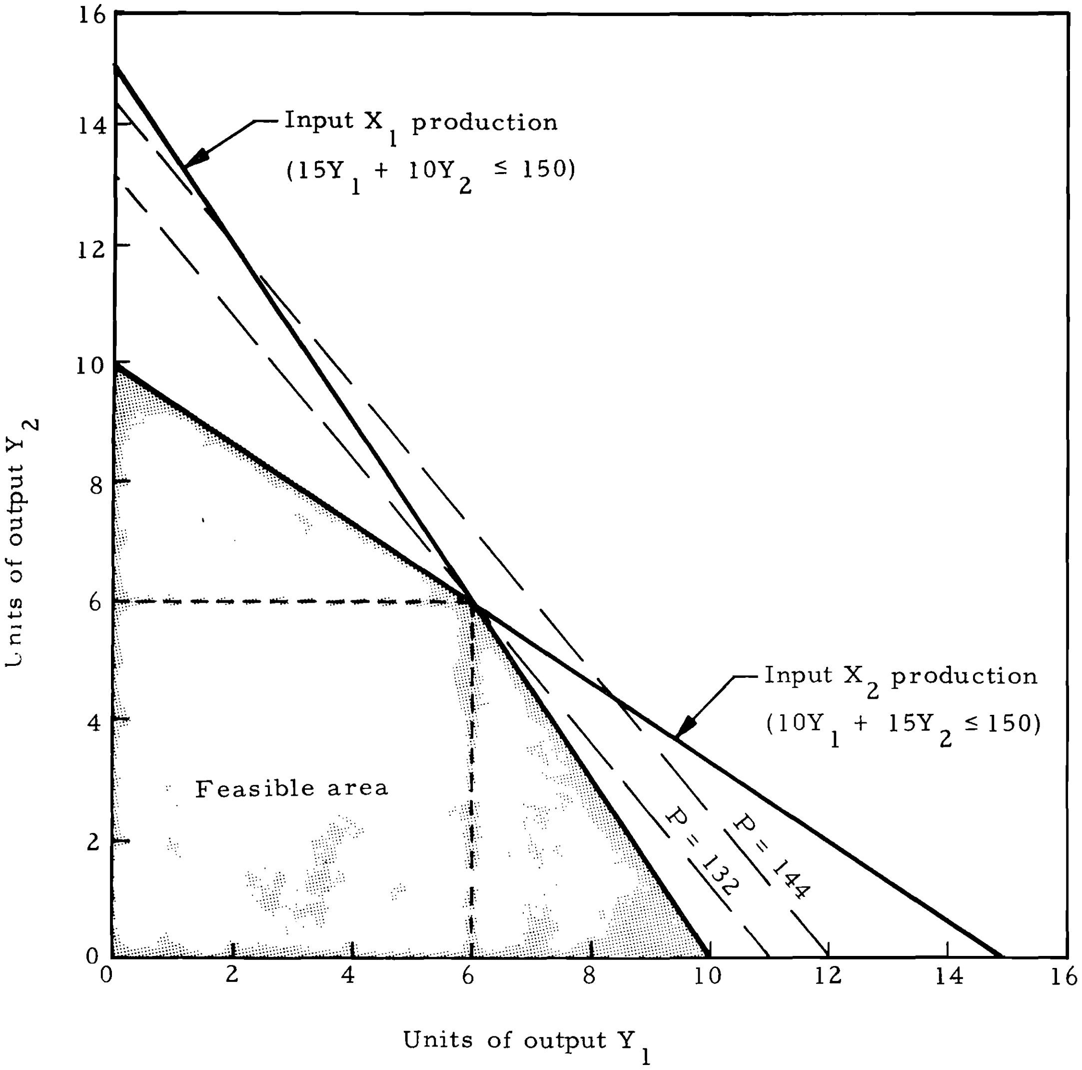

Figure 2. Graphical presentation of linear programming analysis to determine choice of outputs.

The shaded area in Figure 2 is the feasible production area common to both inputs If the unit profit from output Y_1 is p_1 and from Y_2 is p_2 then

$$P = p_1 Y_1 + p_2 Y_2 \quad \ldots\ldots\ldots\ldots \quad (5)$$

represents the profit as a function of Y_1 and Y_2 and is represented on Figure 2 as a constant profit (iso-profit) line for $p_1 = 12$ and $p_2 = 10$. The two lines shown represent a profit of 144 and a profit of 132.

Suppose it is desired to find the combination of outputs which bring the grestest profit. The problem in linear programming format becomes:

Objective function: maximize $P = 12Y_1 + 10Y_2$

Constraints:

$15Y_1 + 10Y_2 \le 150$

$10Y_1 + 15Y_2 \le 150$

The solution is found to be:

1. Profit = 132
2. Y_1 = 6 units
3. Y_2 = 6 units
4. X_1 = 150 units
5. X_2 = 150 units

An argument can be presented just as before to show that the price ratio is not equal to the marginal rate of transformation (tangent to production line) but can vary between the values given by the MRT on each side of the optimal point. This is the equivalent in linear programming to Equation 3.

It can also be shown how linear programming solves the problem of the optimal relationship between output and input (Equation 1). The production function for output Y and input X_1 can be determined from Figure 1 by relating the output at any given value of input X_2 to the input X_1 (found by taking horizontal cuts across the graph). After non-dimensionalizing with respect to X_2 (divide each term by X_2), the production function is shown on Figure 3. This curve holds for any value of X_1 due to the basic assumption of constant return to scale. If the unit price of input X_1 is p_1, of input X_2 is p_2, and of output Y is p, then

$$N = pY - p_1X_1 - p_2X_2 \quad \dots\dots\dots\dots \quad (6)$$

represents the net profit. After non-dimensionalizing with respect to X_2 this function becomes:

$$N(X_2) = \left(\frac{N}{X_2} + p_2\right) = p\left(\frac{Y}{X_2}\right) - p_1\left(\frac{X_1}{X_2}\right). \quad (7)$$

and is shown on Figure 3 as constant net-profit-per-unit-of-input-X_2 (iso-net) line. If $p = 3$ and $p_1 = 6$, then the two lines shown represent $N(X_2)$ of 220 and 235. Suppose it is desired to find the combination of output Y and input X_1 which would bring the greatest net profit. The problem in linear programming format becomes

Objective function:

$$\text{maximize} \quad N(X_2) = 3\left(\frac{Y}{X_2}\right) - 6\left(\frac{X_1}{X_2}\right)$$

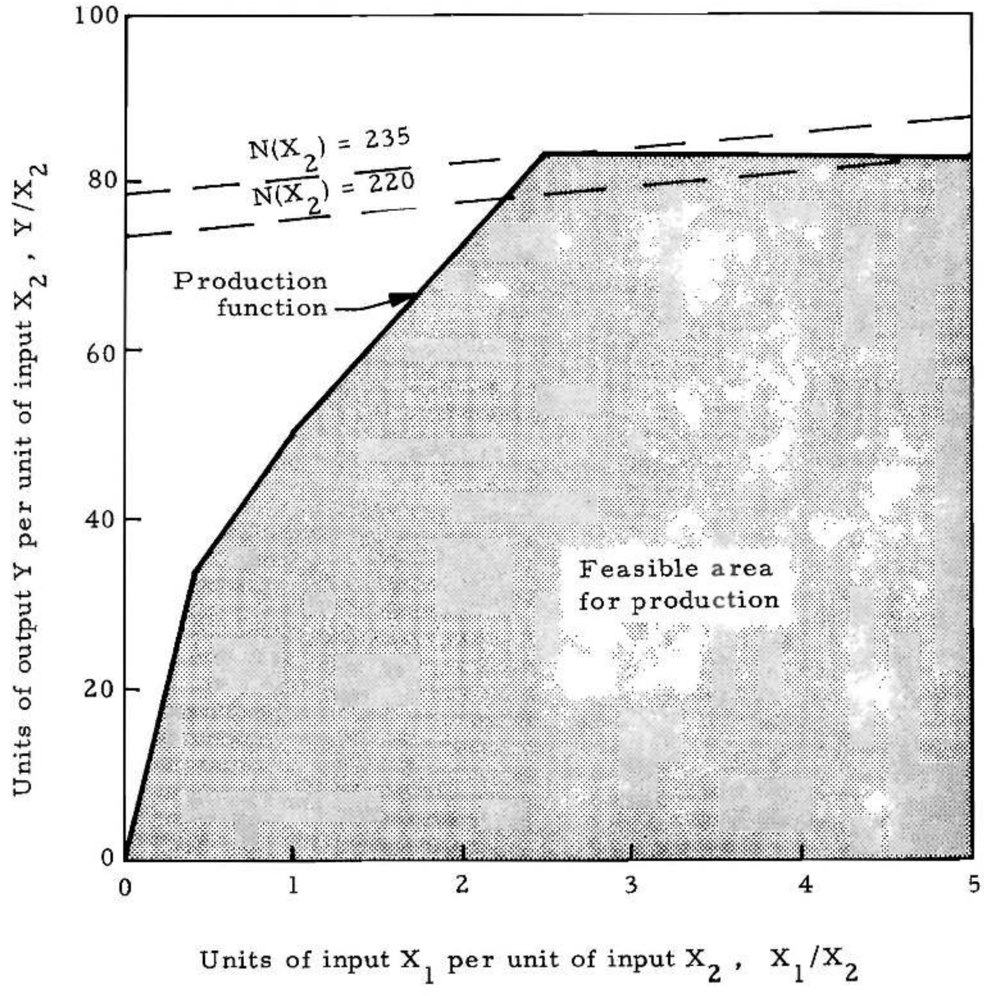

Figure 3. Graphical presentation of linear programming analysis to determine relationship between output and input.

Constraints: lines representing the production function

The solution is found to be

1. $N(X_2) = 235$
2. $Y/X_2 = 83.3$
3. $X_1/X_2 = 2.5$

Now the slope of the production function is the marginal physical product (MPP) of output Y with input X_1. Also, the slope of the iso-net lines is the inverse ratio of the prices, p_1/p. Using similar arguments as before it can be shown that the price ratio is not equal to the MPP but can vary between the values given by the MPP on each side of the optimal point. This is the equivalent in linear programming to Equation 1.

This problem could have been solved including the second input X_2 by working with a three-dimensional problem rather than a two-dimensional problem. The production function would be a surface rather than a line—likewise the iso-net function would be a plane. The optimum would occur at the point of tangency of the iso-net plane with the production function surface.

One of the additional benefits of the linear programming technique is called sensitivity analysis. Such an analysis performed for prices in the first equivalent linear problem discussed above would indicate the range over which each of the prices p_1 and p_2 could vary (holding the other price fixed) such that the optimal combination of inputs would remain unchanged. These ranges are determined from the range of the price ratio (slope of iso-cost line) for which the optimal point would remain unchanged. Similar arguments could be made for the second and third problems.

Systems Analysis Approach in Other States

Susquehanna River Basin - New York and Pennsylvania

Howes (1966) used linear programming to develop an interregional model which specifies economically feasible water resource investments. The model enabled simultaneous estimates of the benefits resulting from a project and market prices and generated a spatial economic equilibrium solution. Optimal solutions were generated for ranges of production costs and resource rents and values of agricultural commodities. The dual of the linear programming problem was developed to determine marginal values of water in agriculture. Demand functions for water were then generated. These data were utilized to determine the impacts of water development upon resource owners.

Santa Clara Valley - California

Aron (1969) developed a conceptual model of a regional water conservation and distribution system for conjunctive use of surface water and groundwater, and a set of procedures for establishing water allocation and import policies of maximum economic efficiency. Dynamic programming was chosen as the primary optimizing technique because of its flexibility of application. In particular, the sequence of operations necessary to arrive at an optimal operating policy made dynamic programming the best choice of mathematical tools. Limitations on the number of state variables were noted with the suggestion that simulation may be the only practical tool for developing an efficient water allocation policy in a complex, multisource, multipurpose system.

Statewide - California

Lofting and McGauhey (1968) used input-output linear programming analysis in a continuing study on the economic evaluation of water development on a statewide basis. Earlier Lofting and McGauhey (1963) had presented an input-output table as a first step in establishing a procedure for developing guidelines for a statewide water resources policy. In their later work these authors up-dated the model from 1947 economic data to 1958 data. Linear programming was used as an optimizing technique to identify the time path of shadow prices of water for 24 productive water dependent sectors of the California economy. A time series gross state product was developed for 1940 to 1966 in 1958 constant dollars and growth projections were made to the year 1990. Ranges of final demands for the model were set and solutions obtained so as to maximize value added given different levels of fresh water availability.

San Joaquin Valley - California

Moore (1962) estimated a demand schedule for irrigation water in a highly commercialized farm area by constructing linear programming models to represent five farms of different sizes with maximum farm income as the objective function. Cost of irrigation water was varied with the result that new combinations of crops became optimum, making it possible to trace quantity used versus price. In addition, the temporal distribution of water was studied by shifting the run-off pattern to successively later times and determining the net increase in farm income, thus estimating value of storage.

Pecos River Basin - New Mexico

Gisser (1970) applied the method of parametric linear programming to forecast the demand for imported irrigation water in the future. The objective was to maximize net return to land and management. Acreage and salinity constraints were incorporated with water application varying in unit increments from 0 to 4 ac-ft/ac.

River basin - Iowa

Baldwin (1970) used linear programming to model a river basin and determine an optimal water use pattern and value of water. Iowa's water permit system was a major constraint. Benefits were estimated for several major water users and combined with costs to give a net benefit objective function.

Trans-Texas Division, Texas
Water System - Texas

Orlob (1970) discussed the approach taken by planners for the Texas Water System. The Trans-Texas Division of the Texas Water System would be comprised of 18 reservoirs, more than 500 miles of canals, and pumping facilities to raise the water from near sea level to over 3000 feet elevation. The planning problem is:

Given:
1. The location of all reservoirs
2. The routes of connecting canals
3. The schedules of in-basin demand for each reservoir or major junction in the system
4. The hydrology of supply for each major storage element
5. The cost of imported water, and
6. The costs of construction and O & M for all elements

Find:

> The least costly alternative system and schedule for its construction to meet specified demands to the year 2020 within the prescribed legal, financial, contractual, and political constraints.

The approach was to seek "near optimum" solutions rather than an exact optimum to overcome limits on time and computer capability. The procedure was carried out in four phases:

1. Preliminary sizes of elements and operating rules for reservoirs were determined by a formal optimization procedure.
2. Initial screening was performed by simulation of the given hydrology, element sizes, and operating rules for each of a large number of alternative stage development schedules selected by random sampling of the cost "response surface." The most attractive schedules were improved by a method of successive perturbations.
3. Element sizes were refined by a second simulation procedure which constrained flows in some expensive canals.
4. Final screening was performed by a formal optimization of the most attractive systems and development schedules.

Sacramento Basin - California

Hall et al. (1967) discussed the development of analytical techniques for optimization of water resource systems. The study area discussed includes four major streams, ten reservoirs, and associated pumping plants, aqueducts, and power generation facilities. The objective maximized is financial gain based on deliveries of firm energy, firm water, off-peak energy, and off-season water. The procedure decomposes the complete system by a "master wholesaler"–"individual producer" relationship. Dynamic programming is used to optimize returns of individual reservoir operators based on a schedule of prices provided by the master. The corresponding outputs over the study period are reported to the master. Using these outputs as "available resources" linear programming is used to maximize the actual returns that could be obtained from water and power contracts. A new set of prices is generated which reflects the value of a modified output schedule for the operators. The cycle of calculations is repeated until the improvement is negligible.

Entire state - Texas

McKee (1966) developed a linear programming model for determining least cost of agricultural production for the entire state of Texas. Account was made of soil classification, acreage required per unit of production, and cost of production per unit in each soil class. Constraints were the acreage in each soil class and the demand for each crop. Cost data included the cost of supplying water for each soil class and each crop. Cost of drainage was also included. On-farm production costs were estimated. Requirements for crop production were projected to the year 1975 and the production allocation was determined by the linear programming algorithm. Marginal costs were derived for each of the crops.

Previous Studies for Utah

Gold, Milligan, and Clyde (1969) formulated a least-cost water allocation model to study alternate means of allocating Utah's water resources and the economic effects of imposing certain political and social decisions. This study was completed by Clyde, King, and Andersen (1971) including technological limitations of inter-basin transfer, artificial recharge, reservoir storage, etc. The study showed projection to year 2020 using demands for water as defined by the Utah Division of Water Resources. Cost data, return flow coefficients, inter-basin transfer information, etc. from this study formed the basis for developing the supply functions and least-cost model in the present study.

Table 1. Percentage of total income from var

Basic Physical Production	Perce Total I Utah	
Agriculture	3.0	
Mining	4.8	
Manufacturing	19.7	
Utilities and transportation	8.3	.4
Contract construction	8.8	.4
Subtotal production	44.6	.6
Wholesale and retail trade	19.7	1
Finance and insurance	4.3	2
Service	10.2	
Government	21.1	
Other Miscellaneous	0.1	
Subtotal service	55.4	
TOTAL	100.0	J

Source: Nelson and Harline, 1964.

[a]Total personal income (millions of doll the nation $461,610. This does not include unemployment insurance, welfare, etc.

(Nelson and Harline, 1964). For p economic growth and shifts in empl during this period see Cluff (1964).

The population of Utah was estim Bureau of Census to be 997,000 in 196 Census, 1966) and has continued to gr high rate. In the future, average populat Great Basin region, encompassing weste of Nevada, will probably be at 2.5 according to one estimate (U.S. Water F 1968). The eastern areas of the state are growth at a somewhat lower rate.

The greatest economic developme tion of population in the state occurs Lake City-Ogden-Logan area, a relativel eastern edge of the Great Basin. Incre of population and economic growth Wasatch Front area in the future indi shift of development toward urban, industrial activities.

Water Uses and Projected Re

As shown in Table 2 approximat the total precipitation over the stat grazing lands and watersheds, wastel parks and monuments. In addition, a consumed on arable grazing land, dry-farmed land, irrigated land, and municipalities. The remaining 13.8 percent is consumed by evaporation from water surface areas and outflow to interstate streams (McGuiness, 1963). The 7.7 percent mentioned above is contributing directly to the livelihood and well-being of man and is considered an available controllable resource. The 13.8 percent is not considered as completely available. There are compact agreements involving the outflow of the interstate streams which must be included in any analysis of the state's resources. The evaporation losses from water surface areas come predominantly from the Great Salt Lake. Policies and legal commitments concerning inflow to the lake must also be included in any analysis of the state's resources. The water totaling 21.5 percent appears in three forms: 1) Precipitation directly on the water and land areas, 2) surface runoff in rivers and streams originating in the watershed areas, and 3) groundwater in alluvial reservoirs and other aquifers which originated from percolation of precipitation and water bodies on the above ground surface and from groundwater interflow from the watershed areas.

Use of water as an available resource falls into three primary categories: 1) Agricultural, 2) municipal and industrial, and 3) recreation and maintenance of natural vegetation and wildlife. Water appearing in rivers and streams is diverted through canals and other irrigation works to irrigate croplands during the dry months of the year. In those areas where local surface water is not available in sufficient supply, pumps are installed to utilize the groundwater. Excess water not used by the crops either runs off as surface water back to the streams or percolates into the groundwater reservoir for use again. Likewise surface and groundwater resources are diverted through municipal and industrial systems. The sewage and other excess water can be treated before being returned to the sources. Water for recreation and maintenance of natural vegetation and wildlife primarily appears as part of

Table 2. Land use and water consumed in Utah.

Type of Land	Percent Total Area	Percent Water Consumed
Grazing land and watersheds	81.7	72.1
Arable but uncropped land used for grazing	2.6	1.9
Dry-farmed land	1.1	1.0
Irrigated land	2.1	4.6
Cities and towns, industrial sites	.5	.2
Wasteland, national parks, and monuments	9.0	6.4
Water area	3.0	9.5
	100.0	95.7
Outflow to interstate streams		4.3
		100.0

Source: McGuiness, 1963.

the water storage and conveyance systems. Some water used by phreatophytes could be made available for other use by management of wetlands.

Beginning with the settlement of the Mormon pioneers in the middle 1800's, irrigation has been one of the major uses of water in Utah. In fact, the practice of irrigation by pioneers in the Great Basin is held to be the first on an extensive scale by Anglo-Saxons in the United States.

Because of water scarcity and the development of needs other than irrigation, the annual amount diverted for irrigation has not increased greatly in recent years. This has occurred even though a considerable acreage of arable land remains undeveloped. The withdrawal uses estimated by the U.S. Geological Survey between 1950 and 1965 reflect only a 14 percent overall increase for this 15-year period (U.S. Geological Survey, 1951, 1968). Total arable land in the state has been estimated at approximately 5 million acres of which only about 1½ million are irrigated (Utah State University Agricultural Experiment Station, 1968). The breakdown of arable and irrigated lands by hydrologic study unit is presented in Table 3. Arable land is defined as land capable of productive cultivation.

In the foreseeable future, irrigation will undoubtedly maintain its position as the largest water user in the state despite a trend for rural areas in general not to keep pace economically with urban areas. While additional water alone will not reverse present trends, more water for supplemental irrigation and new irrigation in established agricultural communities will assist in establishing a more viable economy in rural areas. Water will be needed to eliminate present irrigation shortages and to bring new lands into cultivation as demands for agricultural products increase in the future.

Some other major water uses will probably increase faster than irrigation. In the Provo-Salt Lake City-Ogden-Logan area of relatively high population growth, demands for industrial and municipal water supplies will increase rapidly. Other areas of the state showing little urban growth in the past may experience such growth in the future as government policies designed to alleviate pressing problems of the cities may encourage development of sparsely populated regions and as technological advances allow development of oil shales, etc. Water supplies will be needed to enable and facilitate this growth. Population and municipal-industrial water use by hydrologic region in 1965 are shown in Table 4.

Table 4. Population and municipal and industrial demand.

Hydrologic Study Unit	Population	Municipal and Industrial Water Use (ac-ft/yr)
1	23,000	3,000
2	70,000	15,000
3	215,000	28,000
4	567,000	94,000
5	33,000	9,000
6	16,000	4,000
7	20,000	4,000
8	26,000	5,000
9	16,000	5,000
10	12,000	1,000
Total	997,000	168,000

Source: Utah Division of Water Resources, 1970.

With greater emphasis being placed on environmental and recreational goals by society, demands for water related to these goals will increase throughout the state. Managed waterfowl areas, for example, will require supplemental water supplies and additional supplies for expansion.

Table 3. Land use and water use in the hydrologic study units.

Hydrologic Study Unit	Arable Land (acres)	Irrigated Land (acres)	Water Consumed (ac-ft/yr)
1	1,483,200	52,000	59,000
2	445,400	246,000	354,000
3	194,100	166,700	236,000
4	448,400	207,200	310,000
5	1,022,200	293,000	436,000[a]
6	838,300	71,800	137,000
7	340,700	195,000	293,000
8	206,200	98,100	114,000
9	531,300	16,000	30,000
10	89,000	17,500	34,000
Total	5,598,800	1,363,300	2,003,000

Source: Wilson, Hutchings, and Shafer, 1968.

[a] Includes 105,000 ac-ft direct groundwater use.

Major Water and Related Land Resources Problems

Utah, generally considered an area of chronic water shortage, has access to only partial supplies for nearly two-thirds of its irrigated land. Yet, it has over 2 million acres of swamp land, marshes, mud flats, and valley bottoms suffering from an excess of water. In addition, water evaporation from reservoirs and lakes, as well as transpiration by phreatophytes amounts to far more than is withdrawn for public supplies. This may or may not be a misallocation when one considers the total environment. Herein lies the challenge for water planning and management in Utah (Utah Water and Power Board-Utah State University, 1963).

Even though there are more than 3 million acres of land in Utah that could be added to agricultural production if water were available, and industrial and urban areas in the state need water to sustain growth, a major share of Utah's portion of Colorado River water continues to flow out of the state and about 1½ million ac-ft/yr of water is evaporated from the Great Salt Lake. The determination of whether or not potential use of this water by Utah will be socially efficient is beyond the scope of this study. The assumption is simply made that this is water which is within the manageable capacity of man. By constraining (limiting) the economic efficiency model to use various amounts of this water, it is possible to determine the degree of compromise of economic efficiency that would result.

Maximum development of Utah's vast groundwater reservoirs will require changes or at least more realistic interpretations of present state statutes in harmony with natural hydrologic laws. In the past, well owners have commonly held the view that their rights involve a guarantee by the state to maintain given water pressures or water table levels in wells. Such restrictions, though not physically possible, would limit the use of groundwater to a fraction of the amount available in storage. Recent court decisions indicate that some improvement in this condition is imminent.

Despite the large sums of money invested in municipal and agricultural waterworks in Utah, many additional improvements are needed. Where positive net benefits can be shown, worn out and obsolete control and conveyance works should be replaced, new water projects should be constructed to meet growing demands, and some legal and institutional changes should be implemented if they improve social welfare. Problems of water quality are intimately interwoven with other development problems, and will require careful consideration. In general, in spite of aridity, Utah's major immediate concern in water development is not in deficiency of total supply, but in the maldistribution of water resources seasonally and geographically. The challenge first is to determine where water is available and then to store, transport, treat, and distribute the available water in an optimal manner.

HYDROLOGIC CHARACTERISTICS

Available Resources

There are four basic sources of water that may be more fully developed to provide for future requirements in Utah (Haycock, 1968):

1. Water resources along the Wasatch Front including Bear River. This means utilization of water currently evaporated from the Great Salt Lake.
2. The Virgin River and minor streams draining into the lower Colorado River.
3. Groundwater basins within the state.
4. Upper Colorado River water allocated to Utah.

Streams within the state have been measured or gaged extensively, and surface-water availability is well defined

Although there already has been considerable groundwater development in Utah, extensive groundwater supplies remain available. Water available for development in each hydrologic study unit (hereinafter referred to as HSU) is presented in Table 5.

One of the state's greatest sources of undeveloped water is in the Upper Colorado River Basin separated from the most significant population growth areas by the Wasatch Mountains. Because of this separation of present growth areas from potential supply, much of Utah's share of the Colorado River water currently flows out of the state unused. Even with the transfer of a sizable amount of Upper Colorado River Basin water to the Great Basin by the Central Utah Project, a large scale project of the U.S. Bureau of Reclamation, approximately a third of Utah's share of this water will still be unused (Haycock, 1968). Other projects must be developed to fully utilize this supply.

Several other means by which available supplies can probably be increased include: control of phreatophytes and evaporation, saline water conversion, waste water reclamation and reuse, and better watershed management. Weather modification and importation schemes also may eventually provide additional supplies.

Table 5. Available water resources in Utah.

	Water Availability		
Hydrologic Study Unit	Groundwater (ac-ft/yr)	Local Surface Water (ac-ft/yr)	Local Surface Water Plus Groundwater (ac-ft/yr)
1	187,000	613,000	800,000[a]
2	138,000	917,000	1,055,000[b]
3	65,000	660,000	725,000[c]
4	394,000	560,000	954,000[a,d]
5	356,000	417,000	773,000[e]
6	130,000	80,000	210,000[a]
7	40,000	1,319,000	1,359,000[f]
8	----	650,000*	650,000[a]
9	----	430,000*	430,000[a]
10	10,000	250,000*	260,000[a]
Total	1,320,000	5,896,000	7,216,000

*Much of this water considered as available for transfer.

Source:

[a] Utah Division of Water Resources, 1970.

[b] Utah State University - Utah Division of Water Resources, 1972.

[c] Utah State University - Utah Division of Water Resources, 1970b.

[d] Utah State University - Utah Division of Water Resources, 1969.

[e] United States Department of Agriculture - Utah Department of Natural Resources, 1969.

[f] Utah State University - Utah Division of Water Resources, 1970a.

Return Flow

Not all of the water diverted to agriculture is consumptively used by the crops. That part which is not consumptively used runs off the cropland as surface flow or seeps into the ground, and is known as return flow. Some of the water which seeps into the ground becomes part of the water called "inter-flow" in the water budget studies and essentially is available as surface water since streams, lakes, and reservoirs intercept it. The remainder becomes part of the groundwater supply by the process of deep percolation. Return flow coefficients, K_{RF}, have been determined from water budget studies and when multiplied by the diversion give the return flow as shown below.

$$\text{Return Flow} = K_{RF} \times \text{Agricultural diversion}$$

Coefficients were determined separately for return flow to surface water and for return flow to groundwater for each of the ten HSU. These coefficients are tabulated in Table 6.

Likewise not all the water diverted for municipal and industrial use is consumptively used. Waste water from residential sewage and industrial plants after treatment is channeled into surface streams, and is also known as return flow This water is available for use again. Return flow coefficients have been determined from water budget studies for each of the ten HSU. As is the case for agriculture, the return flow is determined from the product of the coefficient and the diversion as shown below.

$$\text{Return Flow} = K_{RF} \times \text{Municipal and Industrial Diversion}$$

Coefficients were determined for each of the HSU and are also tabulated in Table 6.

Storage Requirements

Storage requirements, including amounts needed to adjust seasonal fluctuations in streamflow as well as to provide long-term carryover needed to meet extended series of dry years, were estimated for each of the 10 HSU.

Estimates of long-term carryover storage requirements are based upon the results of frequency mass-curve analyses completed for 76 streams located throughout the state and published in the "Hydrologic Atlas of Utah" (Utah State University-Utah Department of Natural Resources, 1968). A frequency mass-curve is obtained by plotting, for any selected probability of occurrence, the expected values of accumulated volumes of runoff during each of many sequences of consecutive months (through several years) against the carryover period in months. Separate frequency mass-curves are obtained for each probability of occurrence selected.

Since the volume of required storage can be considered a function of probability, carryover period, and demand level, frequency mass-curve analysis provides information necessary for plotting draft demanded vs. storage curves. A computer program developed to carry out the large amount of computation involved (Jeppson, 1967) was used to analyze monthly runoff data and provide the information necessary to compute draft vs. storage for the 76 streams considered in the Hydrologic Atlas. Draft was in percent of mean annual flow for values of 50, 65, 80, 95, and 110 percent. Storage was given in inches over the watershed. Probability values of .75, .90, and .95 were used.

The long-term storage required corresponds to the maximum values of storage as a function of the carryover period. These values were determined for each of the streams at each of the five draft values and three probability levels. The total long-term storage for each HSU was then determined by weighting each stream's watershed area to the total watershed area.

The seasonal storage was determined for each HSU by calculating the difference between the supply curve on a monthly basis and the draft requirement for each of the five draft values. Where hydrologic inventories were

Table 6. Return flow coefficients.

	Agricultural Use		Municipal and Industrial Use
HSU	To Surface	To Ground	To Surface Only
1	.4742	.0500	.7000
2	.6077	.0500	.6600
3	.5833	.0500	.4366
4	.5609	.0500	.6889
5	.6250	.0500	.4588
6	.4947	.0500	.6923
7	.6288	.0000	.6500
8	.6250	.0000	.3000
9	.8000	.0000	.2500
10	.5000	.0000	.3000

Source: Same as Table 5.

available (areas 2, 3, 4, 5, and 7), the draft curves were based on these data. Where water budgets were not available, the draft curves were based on calculations using Munson's Index (Munson, 1966). The supply curve was based on monthly stream flow data from the Hydrologic Atlas weighted for the watershed area as before.

The seasonal storage was added to the long-term storage to determine the total storage required for HSU 2 through 10. Insufficient stream flow data were available for HSU 1 to perform this type analysis. Figures 5 through 14 show the draft vs. storage required at probability levels of .75, .80, .85, .90, and .95 where the intermediate values were obtained by cross plots. The curves for HSU 1 shown on Figure 5 were obtained from Figures 15 and 16 which are a summary of HSU 2 through 10 in non-dimensional form. An average value through the shaded area for HSU 2, 3, and 5 was used to determine storage requirements for HSU 1 at a probability of 0.75 while an average value through the shaded area for HSU 2 through 6 was used to determine storage requirements at a probability of 0.95. Intermediate values were obtained by linear interpolation.

The use of these storage-draft curves can be illustrated by the following example using Figure 8.

Assume it is desired to know how much storage would be required in the Jordan River study unit (HSU 4) to meet a total draft in the area equal to 80 percent of the mean annual flow or 450,000 ac-ft/yr. From Figure 8 the required storage is seen to be 460,000 ac-ft at the 95 percent probability level. The interpretation of the probability level is that approximately 95 percent of the time one would expect to be able to provide the draft or 450,000 ac-ft/yr by building 460,000 ac-ft of storage. Both long-term holdover storage and annual storage requirements would be provided.

Groundwater Recharge Potential

The groundwater recharge potential or opportunity was assessed in each HSU in order to define the recharge constraint. The problem was to designate the areas where artificial recharge to the groundwater basin is practicable, provided the water table is low enough to permit recharge, and to estimate for each area the amount of water that could be put underground in basins and/or through wells.

In HSU 2, 3, and 4 the reservoirs are essentially alluvial fans intercalated with and overlapped by lake-bottom sediments of Pleistocene Lake Bonneville. The aquifers in these fans are sheets or trains of stream gravel

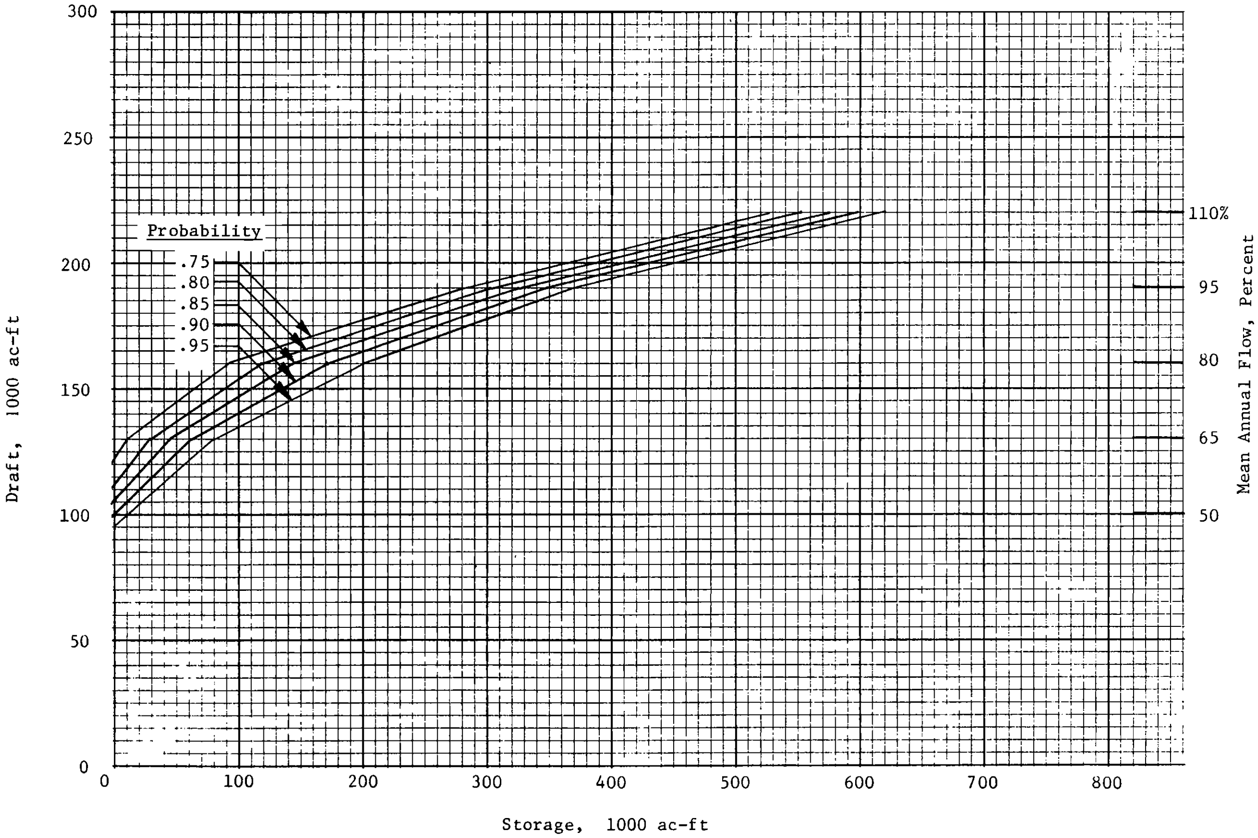

Figure 5. Reservoir storage requirement for the Great Salt Lake Desert hydrologic study unit.

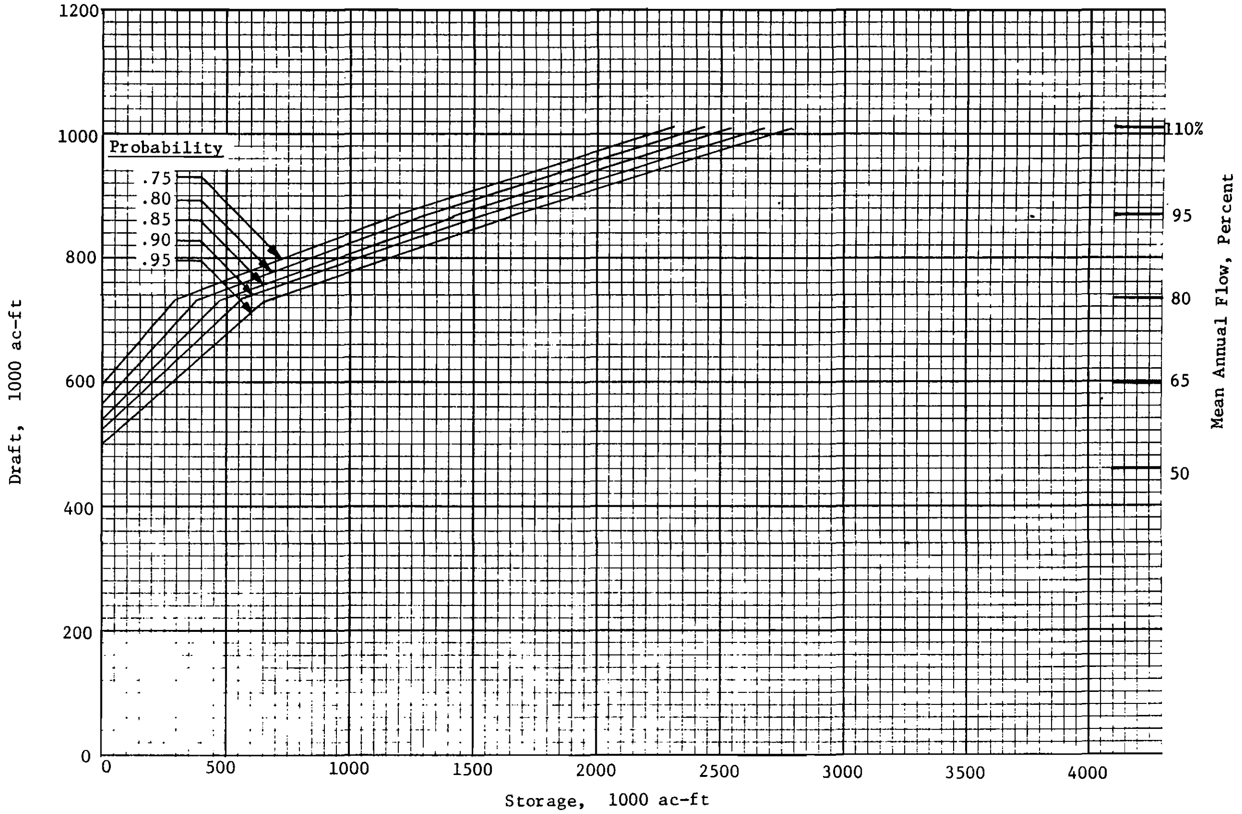

Figure 6. Reservoir storage requirement for the Bear River hydrologic study unit.

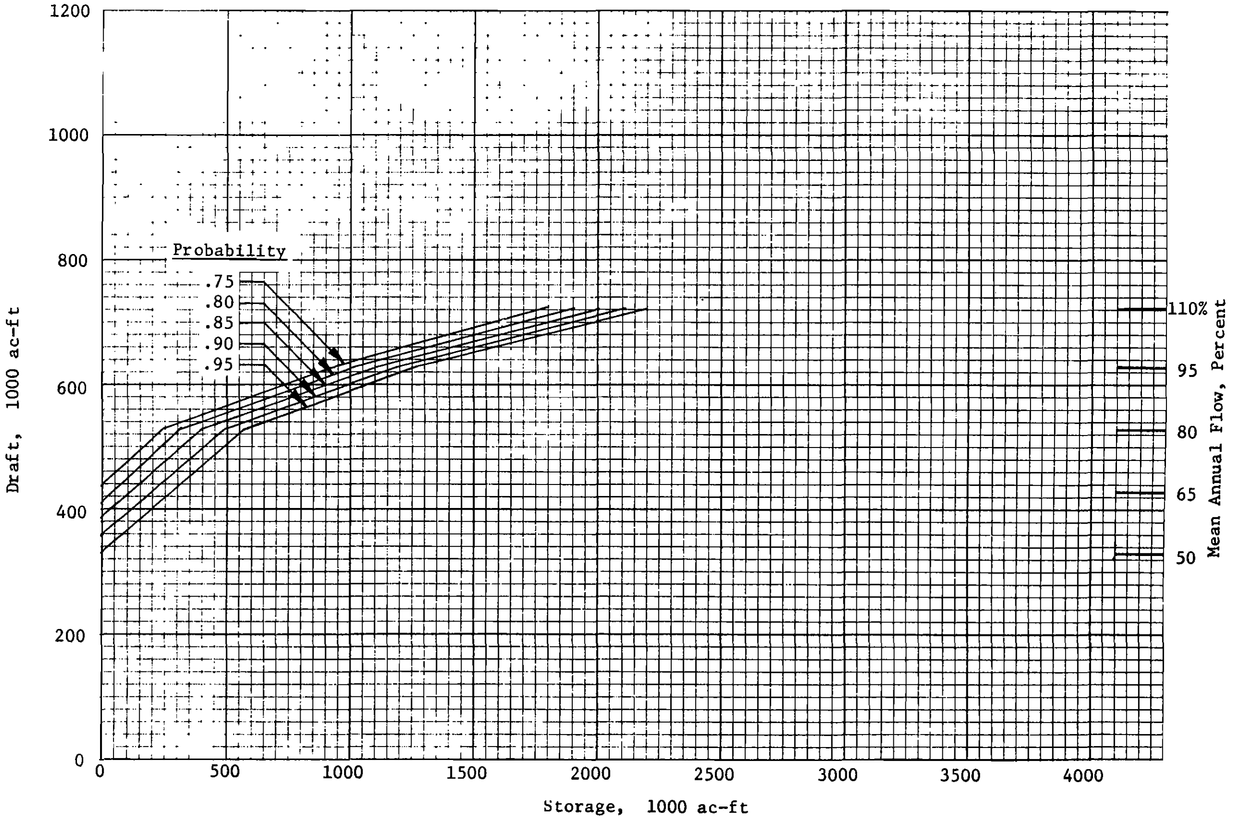

Figure 7. Reservoir storage requirement for the Weber River hydrologic study unit.

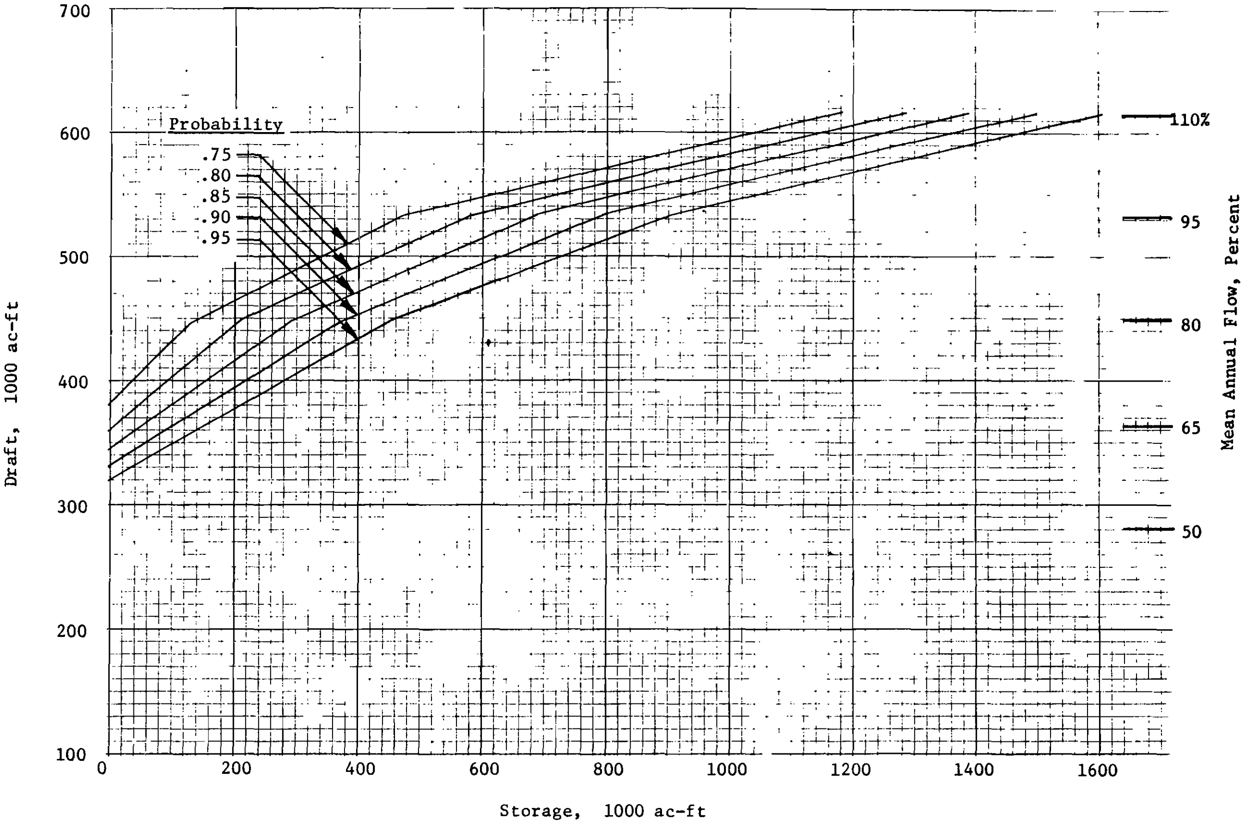

Figure 8. Reservoir storage requirement for the Jordan River hydrologic study unit.

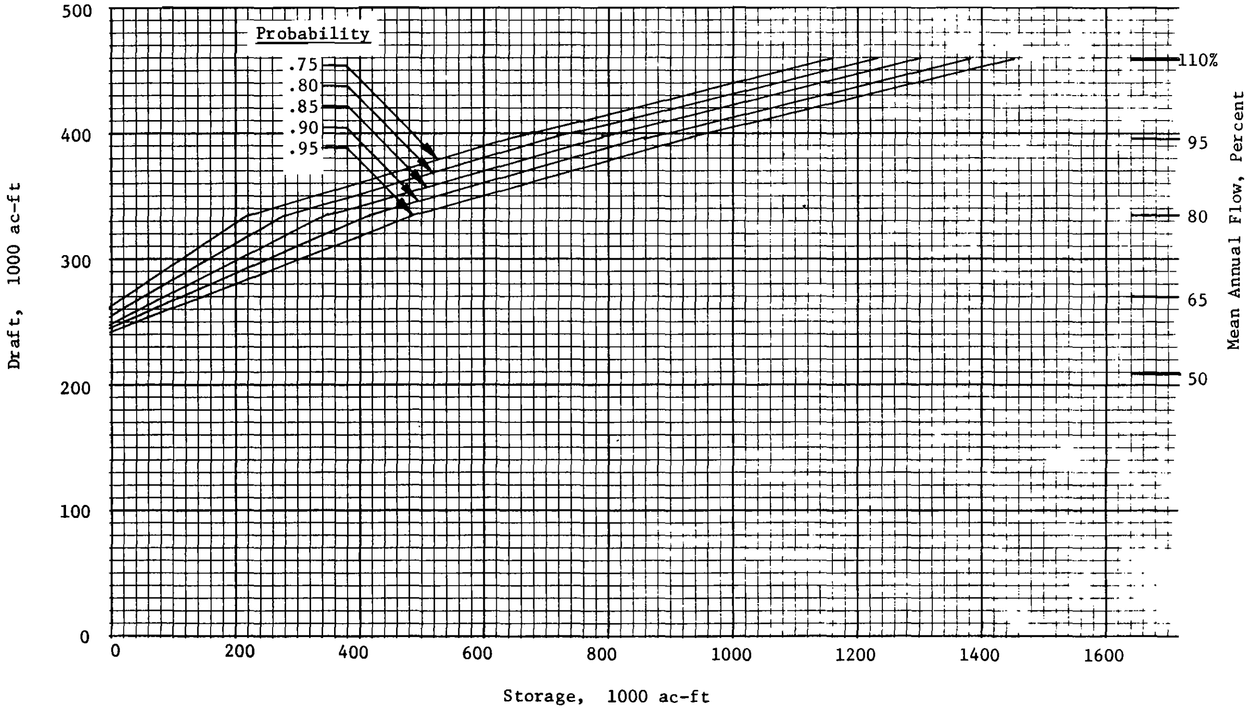

Figure 9. Reservoir storage requirement for the Sevier River hydrologic study unit.

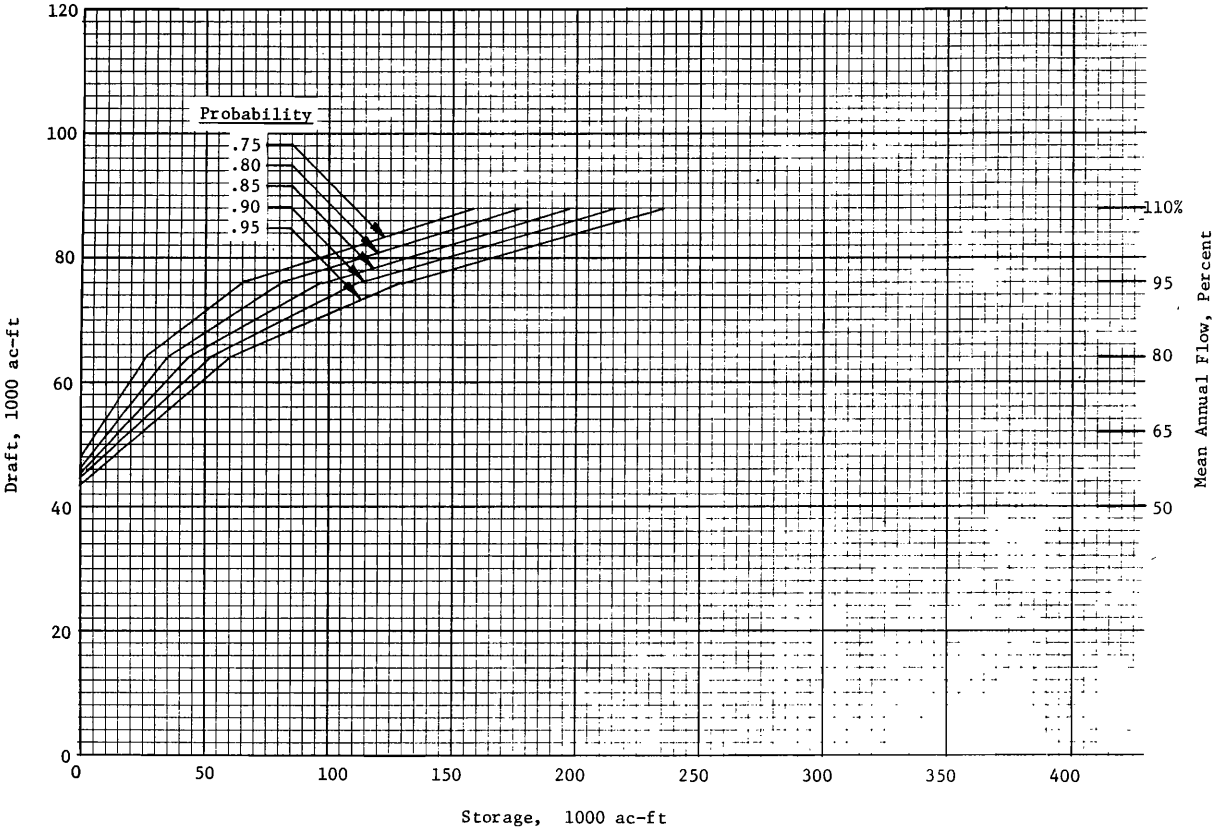

Figure 10. Reservoir storage requirement for the Cedar-Beaver hydrologic study unit.

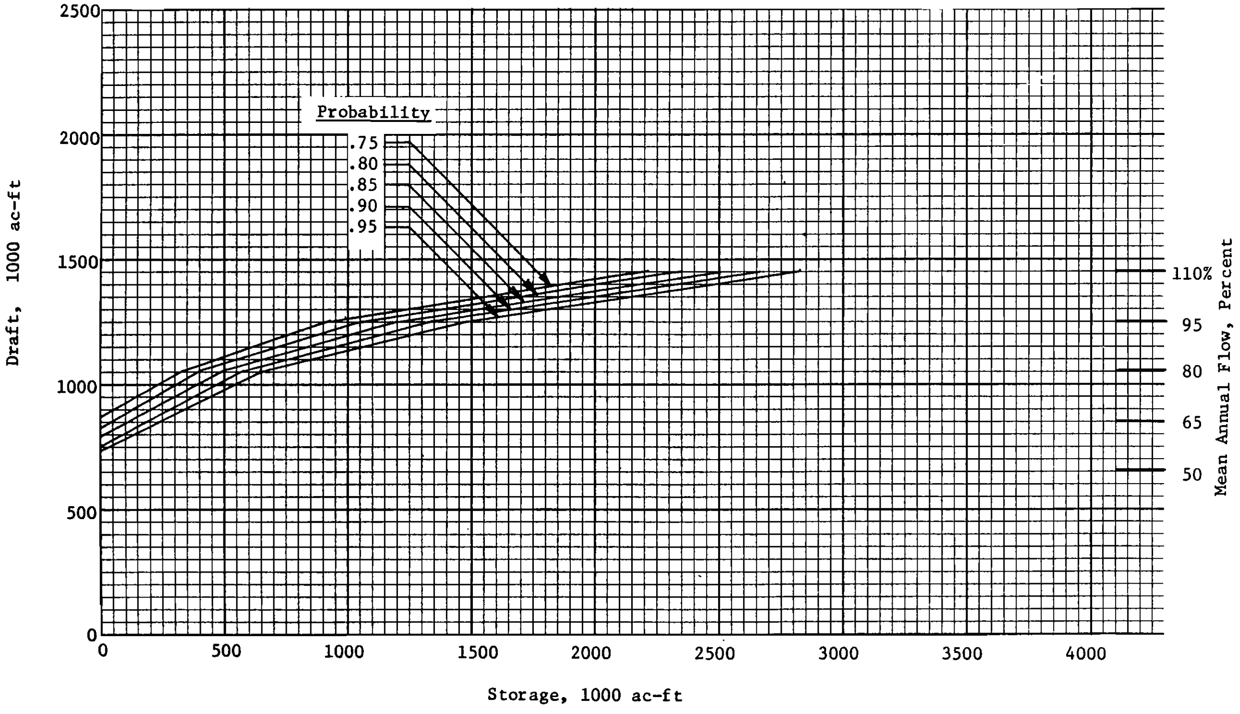

Figure 11. Reservoir storage requirement for the Uintah Basin hydrologic study unit.

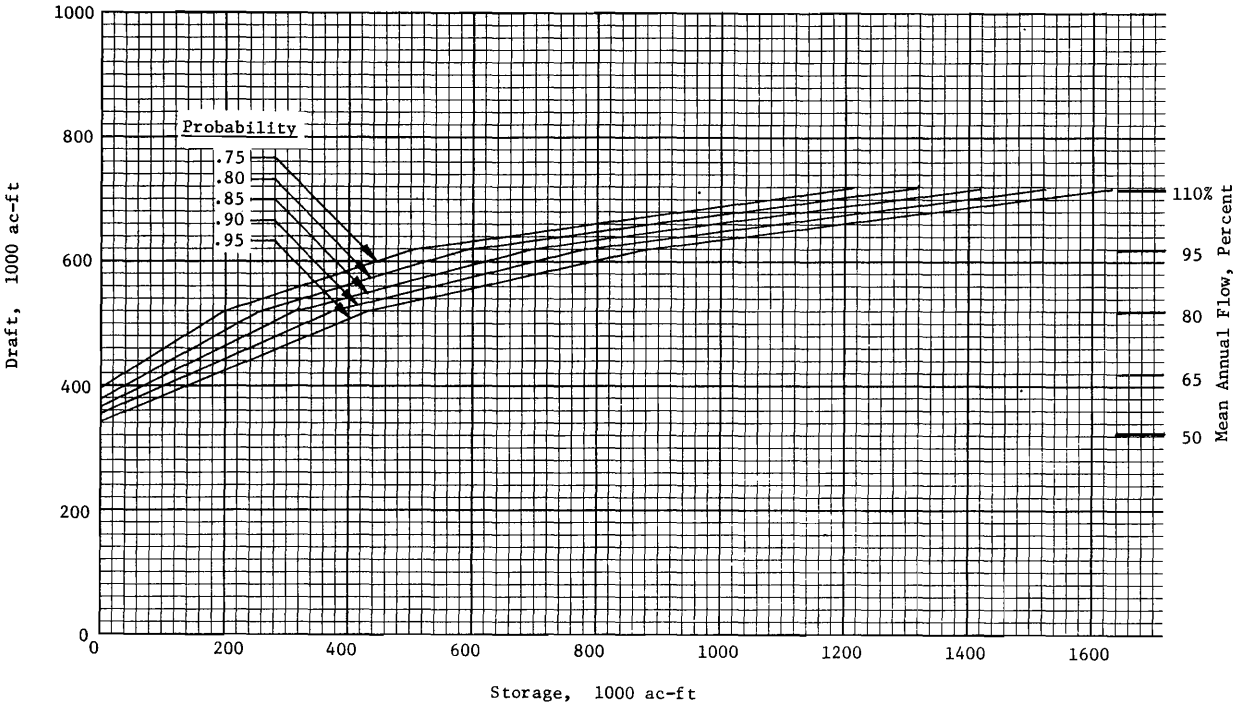

Figure 12. Reservoir storage requirement for the West Colorado hydrologic study unit.

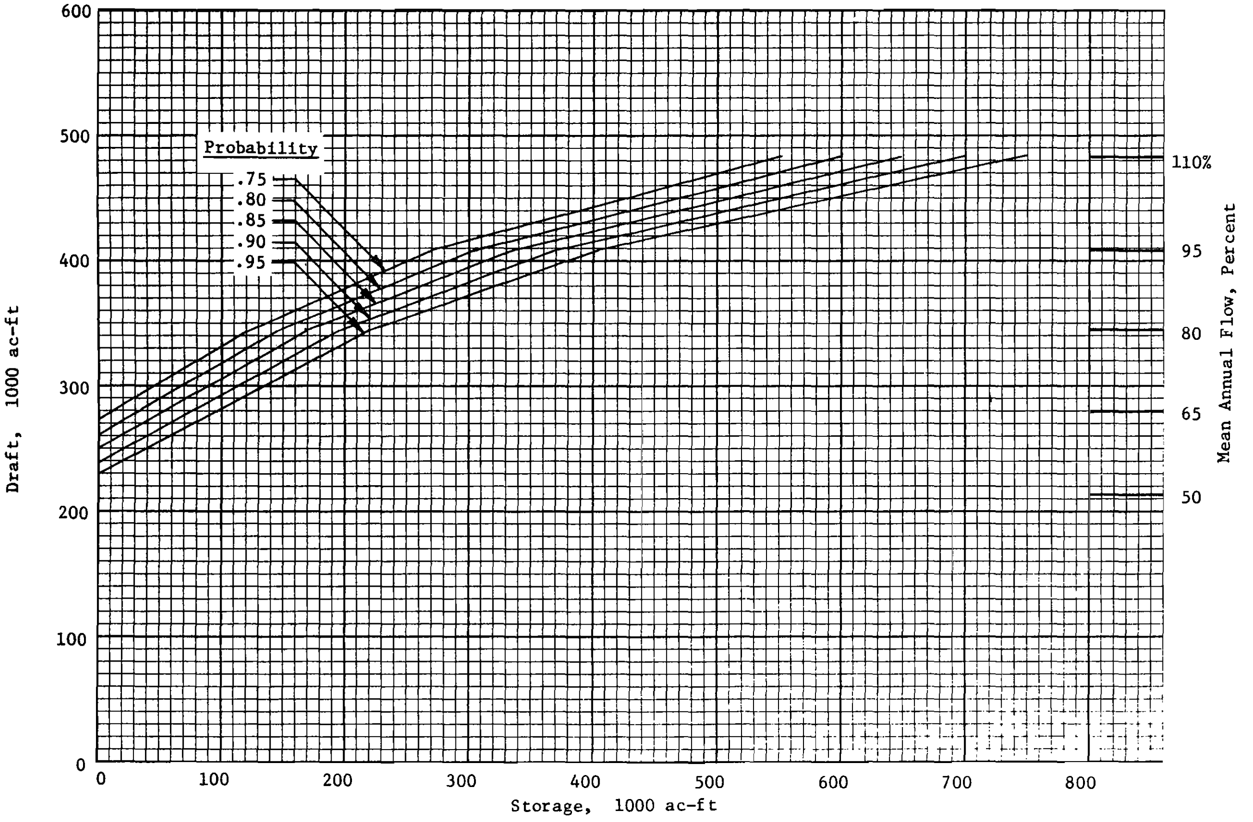

Figure 13. Reservoir storage requirement for the South and East Colorado hydrologic study unit.

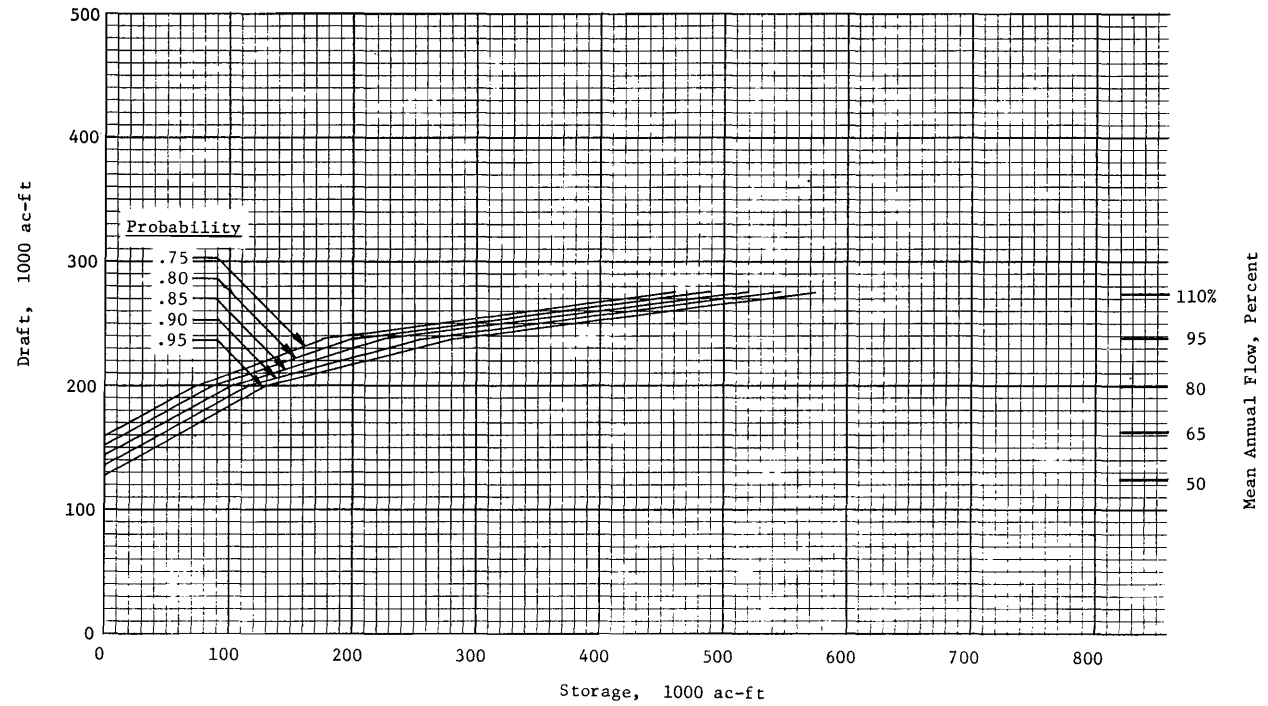

Figure 14. Reservoir storage requirement for the Lower Colorado hydrologic study unit.

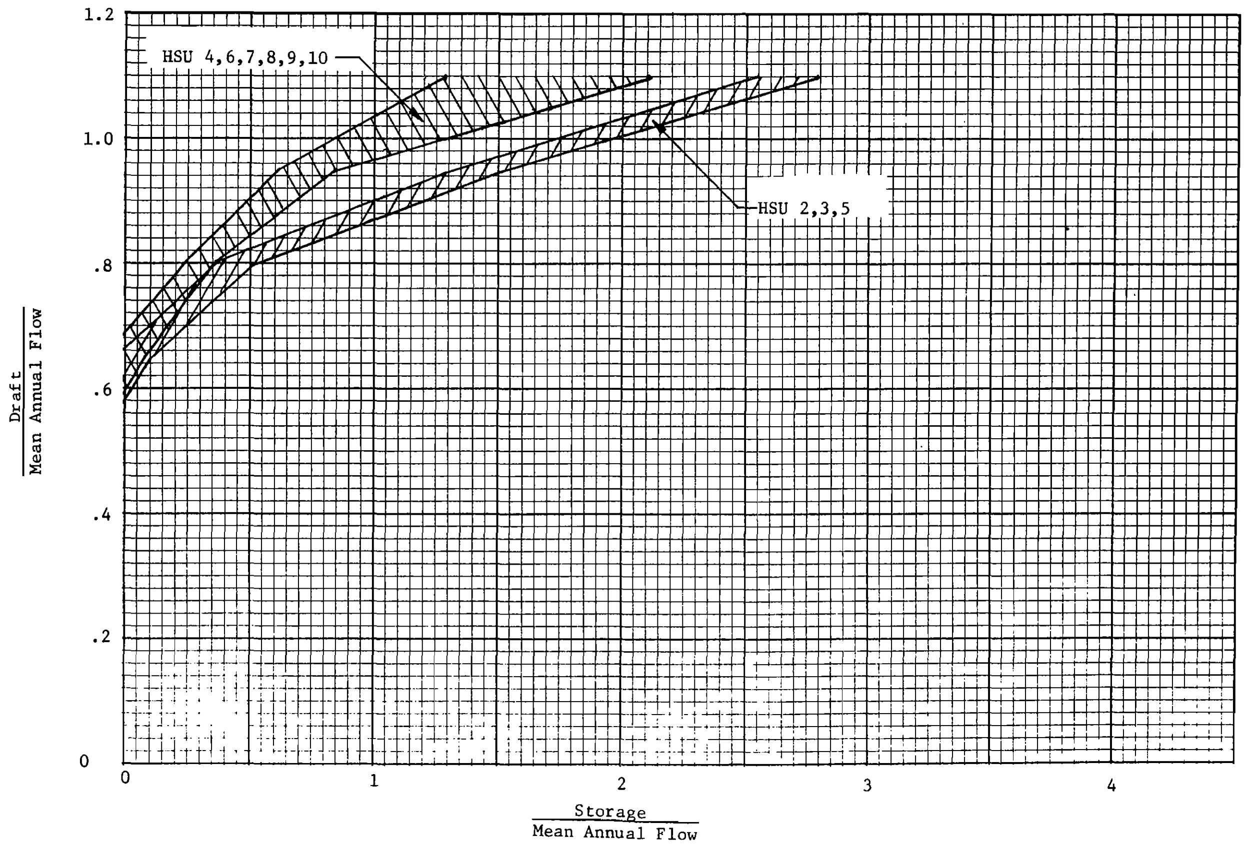

Figure 15. Reservoir storage requirement at a probability of 0.75.

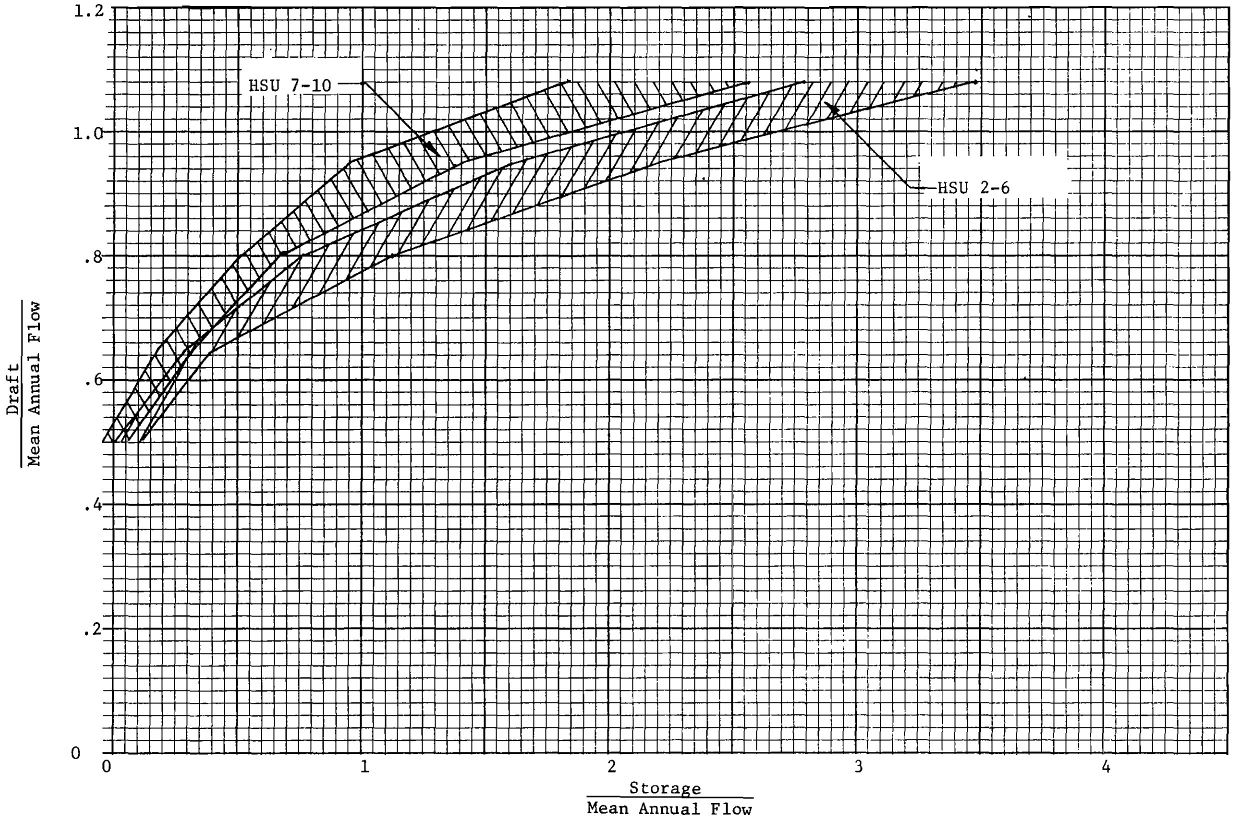

Figure 16. Reservoir storage requirement at a probability of 0.95.

that spread outward from the canyon mouths and thin out and decrease in particle size toward the valley bottom. Recharge to these reservoirs is largely at the apex of the alluvial fans where the stream gravel is coarse, and where lake-bottom sediments, deposited over the fan during high stages of the lake, have been stripped away by the stream after the lake lowered. These recharge areas are surrounded, valleyward, by the most productive parts of the artesian basins, where pressures, yields, and water quality are best. The areas near the apexes of the fans, where recharge basins are not perched on lake-bottom sediments, are small, and their position can be judged only partly by the present surface layer of coarse stream alluvium. In any case, these are limited areas very near the mouth of canyons from which the fan material came.

Based on results of the few artificial recharge experiments for ponds that have been conducted in Utah and experience elsewhere, a possible recharge rate of 2 feet per day for 300 days of the year was selected.

The most favorable location for recharge wells would also have to be high on the alluvial fan where the aquifers are relatively thick and coarse-grained. Based on experience in Utah and elsewhere, a value of 2500 gallons per minute per well was selected as a reasonable estimate, with the wells spaced one to a quarter section.

In HSU 5, reasonably thick and coarse grained aquifers appear to be quite wide-spread providing a relatively large area over which recharge by wells might be conducted.

In HSU 6 the alluvial fan is probably the reservoir unit, but here recharge is far less than production, so that artificial recharge could be achieved immediately, provided the water for recharge were available.

In eastern Utah, HSU 7, 8, and 9, where the only large aquifers are in bedrock, artificial recharge is not practicable.

Based on the above criteria, limits on the amount of water that can be artificially recharged each year in each HSU were determined and are given in Table 7. In practically all cases the fans are at present full or nearly full of water, and a program of artificial recharge would depend upon lowering of the water table in the fans so that additional recharge could be accommodated.

Present Status of Water Resource Development

A summary of the status of water resource development in the State of Utah is shown in Table 8.

Table 7. **Limits on annual artificial recharge to groundwater basins.**

Hydrologic Study Unit	Maximum Mean Annual Artificial Groundwater Recharge (ac-ft/yr)
1	0
2	60,000
3	366,000
4 (low cost)	434,000
4 (high cost)	100,000
5 (low cost)	52,000
5 (high cost)	52,000
6	65,000
7	0
8	0
9	0
10	0

Explanation and reference information are given in the following paragraphs.

a. Basin Yield–These data are the same as shown previously in Table 5.

b. Net Evaporation Loss–Large Lakes–These data show the loss of water as a result of evaporation from Bear Lake in HSU 2 and from Utah Lake in HSU 4. Account was taken of the precipitation on the lake surface to calculate the net loss. Since about one-half of the surface area of Bear Lake is in Idaho, only one-half the net evaporation loss was charged to Utah. Water budget studies were used to determine the loss which was divided between surface and groundwater.

c. Net Evaporation Loss–Other Major Reservoirs–These data were determined as discussed in b above except that in HSU 5 the loss was distributed 75 percent to surface water and 25 percent to groundwater and in HSU 7 and 8 where no groundwater is available.

d. Storage Capacity–The storage capacity data were taken from several sources:
   1. An early report on the state water plan (Utah State University–Utah Water and Power Board, 1963),
   2. Investigations by the Utah Division of Water Resources, and
   3. Investigations by the Pacific Southwest Inter-Agency Committee, U.S. Water Resources Council (Water Resources Work Group, 1971).

e. Direct Use of Groundwater by Croplands–It is recognized that this occurs in all HSU, however these data were only calculated in the water budget for the Sevier Basin (United States Department of Agriculture–Utah Department of Natural Resources, 1969). It was included there as a reduction in the available groundwater to make the data compatible in all HSU.

f. Excess Precipitation on Irrigated Croplands, October-April–These data were determined from the hydrologic inventories for HSU 2, 3, 4, 5, and 7. The values represent the amount of precipitation which is in excess of the amount consumptively used by the crops. This represents an addition to the water supply since it would appear as runoff in the streams or an addition to groundwater.

g. Transbasin Diversions–These data were obtained from the same sources as Table 5:
   1. Hydrologic inventories for HSU 2, 3, 4, 5, and 7, and
   2. Utah Division of Water Resources data.

h. Gross Supply–These data are the summation of: Basin Yield; Net Evaporation Loss Large Lakes; Net Evaporation Loss Other Major Reservoirs; Direct Use of Groundwater by Croplands; Excess Precipitation on Irrigated Croplands, October-April; and Net Imported Water from Transbasin Diversions.

i. In-Basin Water Availability–These data are the summation of: Basin Yield; Net Evaporation Loss Large Lakes; Direct Use of Groundwater by Croplands; and Excess Precipitation on Irrigated Croplands, October-April.

j. Diversions–The total diversions to agriculture and to municipal and industrial for HSU 2, 3, 4, 5, and 7 were taken from the hydrologic inventories referenced on Table 5. Total diversions to the other five HSU were based primarily on data from Utah Division of Water Resources except where modified to account for studies conducted by the Utah Water Research Laboratory on the return flow coefficient for agriculture and to approximate the return flow coefficient indicated for the year 2020. This latter modification was made since the linear programming model must hold the coefficient constant over time. Groundwater pumpage was determined by using the average figure from 1964-1968 given in the yearly reports on groundwater conditions in Utah (Utah Division of Water Resources-United States Department of the Interior, Geological Survey, 1965-1969). Surface water diversions were obtained by subtraction.

k. Return Flows–The return flows for HSU 2, 3, 4, 5, and 7 were obtained from the hydrologic inventories. Agriculture return flows for HSU 1, 6, 8, and 10 were based on Utah Division of Water Resources data while for HSU 9 were based on Utah Water Research Laboratory studies. Municipal and industrial return flows for HSU 1 and 6 were based on Utah Division of Water Resources data whereas for HSU 8, 9, and 10 were based on approximations to the expected return flow coefficients

Table 8. Status of water resource development in Utah. (Units in thousands of ac-ft/yr except storage.)

Hydrologic Study Unit	Basin Yield			Net Evaporation Loss Large Lakes			Net Evaporation Loss Other Major Reservoirs			Storage Capacity	Direct Use by Cropland	Excess Precipitation on Irrigated Croplands, Oct-Apr			Transbasin Diversions		
	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total	Surface Water	Ground water	Total	(ac-ft)	Ground-water	Surface Water	Ground water	Total	Imported Water	Exported Water	Net Imported Water
1	613	187	800				1	0	1	17	-				0	10	-10
2	917	138	1,055	42	41	83[a]	2	1	3	311		66	7	73	19	0	19
3	660	65	725				13	13	26	578		129	30	159	0	90	90
4	560	394	954	131	132	263[b]	13	13	26	416		85	10	95	182	0	182
5	417	356	773				45	15	60	481	105	37	4	41	11	4	7
6	80	130	210				3	1	4	56					3	0	3
7	1,319	40	1,359				12	0		428[c]		33	0	33	0	101	101
8	650	0	650				9		9	199					0	11	11
9	430	0	430			-		-		1					4	0	4
10	250	10	260			-	1	0	1	14					0	3	3
Total	5,896	1,320	7,216	173	173	346	98	43	142	2.501	105	350	51	401	219	219	0

Hydrologic Study Unit	Gross Supply			In-Basin Water Availability			Diversions							Return Flow				
							To Agriculture			To Municipal & Industrial				From Agriculture			From M&I	Total
	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total Ag	Surface Water	Ground-water	Total M&I	Total Diversion	To Surface	To Ground	Total Ag	Only to Surface	Return Flow
1	602	187	789	613	187	800	105	19	124	7	3	10	134	59	6	65	7	72
2	959	102	1,061	941	104	1,045	1,015	19	1,034	36	8	44	1,078	628	52	680	29	709
3	686	82	768	789	95	884	610	33	643	29	21	50	693	375	32	407	22	429
4	683	259	942	514	272	786	714	83	797	171	132	303	1,100	447	40	487	208	695
5	416	240	656	453	255	708	890	128	1,018	7	10	17	1,035	636	51	687	8	695
6	80	129	209	80	130	210	136	64	300	10	3	13	313	148	15	163	9	172
7	1,238	40	1,278	1,352	40	1,392	789	0	789	10	0	10	799	496	0	496	6	502
8	630	0	630	650	0	650	303	0	303	7	0	7	310	189	0	189	2	191
9	434	0	434	430	0	430	150	0	150	7	0	7	157	120	0	120	2	122
10	246	10	256	250	10	260	68	0	68	2	0	2	70	34	0	34	1	35
Total	5,974	1,049	7,023	6,072	1,093	7,165	4,780	446	5,226	286	177	463	5,689	3.134	196	3.329	294	3.623

Hydrologic Study Unit	Depletions Other Than Reservoir Evaporation												Outflow From Hydrologic Study Unit		
	For Agriculture			For Municipal & Industrial			For Wetlands			Total Depletions					
	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total	Surface Water	Ground-water	Total
1	46	13	59	0	3	3	549	165	714	595	181	776	7	6	13
2	387	-33	354	7	8	15	118	122	240	512	97	609	447	5	452
3	235	1	236	8	20	28	107	36	143	350	57	407	336	25	361
4	267	43	310	-38	132	94	274	76	350	503	251	754	180	8	188
5	254	77	331	-1	10	9	149	184	333	402	271	673	14	-31[d]	-17
6	-12	149	137	1	3	4	91	35	126	80	187	267	0	-58[d]	-58
7	293	0	293	4	0	4	315	0	315	611	0	611	627	40	667
8	114	0	114	5	0	5	36	0	36	155	0	155	475	0	475
9	30	0	30	5	0	5	8	0	8	43	0	43	391	0	391
10	34	0	34	1	0	1	9	10	19	44	10	54	202	0	202
Total	1,647	250	1,897	-8	176	168	1,657	627	2,284	3,295	1,053	4,348	2,679	-5	2,674[e]

[a] One-half of total Bear Lake net evaporation.

[b] All of Utah Lake.

[c] Includes Strawberry Reservoir (283,000 ac-ft).

[d] Reflects groundwater mining.

[e] Reflects 1,014,000 ac-ft per year inflow to Great Salt Lake from Utah watersheds.

projected by Utah Division of Water Resources for the year 2020.

l. Depletions Other Than Reservoir Evaporation—Depletions for HSU 2, 3, 4, 5, and 7 were based on the hydrologic inventories while for HSU 1, 6, 8, 9, and 10 were based on Utah Division of Water Resources data. The division between surface and groundwater was determined using individual budgets for each knowing the groundwater outflow. It is recognized that much of the water in the upper areas of the river basins which is below ground may rise to the surface in the lower areas and be consumed by wetlands, etc. This fact is reflected by the large depletions of groundwater by wetlands.

m. Outflow from HSU—The groundwater outflow to Great Salt Lake from HSU 1, 2, 3, and 4 was estimated using the results of several studies conducted on this subject by Utah Water Research Laboratory and others. HSU 5 and 6 have groundwater mining which is shown by negative outflow. Groundwater outflow for HSU 7 was obtained from the water budget study. Surface water outflow was determined by balancing water availability, depletions, and groundwater outflow.

ALLOCATION PATTERN AND SUPPLY VARIABLES

Allocation Pattern

A definition of the potential pattern for water resource allocation within the state is necessary before a choice can be made of the variables to be used in the model. The potential allocation pattern is dependent upon: 1) the intended uses (demands), 2) the amount of excess available water above the current demands, 3) geographic limitations, and 4) presently structured water systems. The actual allocations are of course dependent also on cost and the value of water in one place compared to another.

Demands

For purposes herein water for agricultural demands is defined as the amount diverted onto the croplands. Return flow from agriculture is considered as available for diversion downstream. Water for municipal and industrial demand is the amount diverted to the water system. Return flow of waste water is considered as available for diversion after treatment. Water for wetlands is the amount consumptively used by evaporation from water surfaces and evapotranspiration from plants.

Availabilities

Water available to meet the demands is the net in-basin availability listed in the second group of data shown on Table 8. Depending upon cost factors, it may be economical to use local surface water to recharge a groundwater reservoir and consequently this alternative is allowed.

Transfer of excess water

The allocation pattern for Colorado River water used in the model is structured primarily according to the Central Utah Project of the Bureau of Reclamation. However, geographic considerations make it possible to transfer water from one HSU to another and such transfers are also included in the model. The allocation pattern as structured for this model is shown in Figure 17. The ten HSU are shown as the groups of five rectangles, located approximately according to the geographic pattern over the state. Major rivers are also shown. Within the rectangles the Arabic numeral corresponds to the numeral associated with each HSU as shown on page 11. The large horizontal rectangles represent available local surface water (LSW) and available groundwater (GW). The small rectangles between the large rectangles represent the demand for water by municipal and industrial (MI), wetlands (WL), and agriculture (AG). Other symbols are: AV–amount of water available for allocation, DR–required draft on stored water in surface reservoirs, EV–net evaporation loss from surface storage, ST–amount of surface water storage required to satisfy the required draft, and OF–outflow from the HSU. The three polygons near the Green River represent three inter-basin transfers; UI = Ute Indian Unit of the Central Utah Project, BU = Bonneville Unit of the Central Utah Project, and SA = Sevier Area. The lines connecting various geometric shapes represent an allocation of water. Some inter-basin transfer of local surface water is allowed in addition to the Colorado River water transfer. The present allocation (1965) as taken from Table 8 is shown on Figure 18.

Supply Variables

The mathematical model is set up so that the variables describe the allocation pattern of each HSU. A discussion of the variables is presented in the following paragraphs.

Colorado River water transfer

Provisions have been made in the model for the transfer of additional Colorado River water into the Great Basin. This water is supplied by two units of the Central Utah Project, the Bonneville unit, and the Ute Indian unit; and by an additional small amount from HSU 8 designated as the Sevier area. The water transferred by the Ute Indian unit can be used in HSU 3, 4, and 5 while that from the Bonneville unit and Sevier area is transferred to HSU 4 and 5. The transferred water is assumed to be released into the local surface water pool and is not specified to fill any particular demand–this decision being left to the model.

The variables representing the Colorado River transfers are:

$$\left.\begin{array}{l}\text{QBULSWY}\\\text{QBUMPT}\\\text{QUILSWY}\\\text{QUIMPT}\\\text{QSALSWY}\\\text{QSAMPT}\end{array}\right\}\quad Y = 3, 4, 5 \text{ (as applicable)}$$

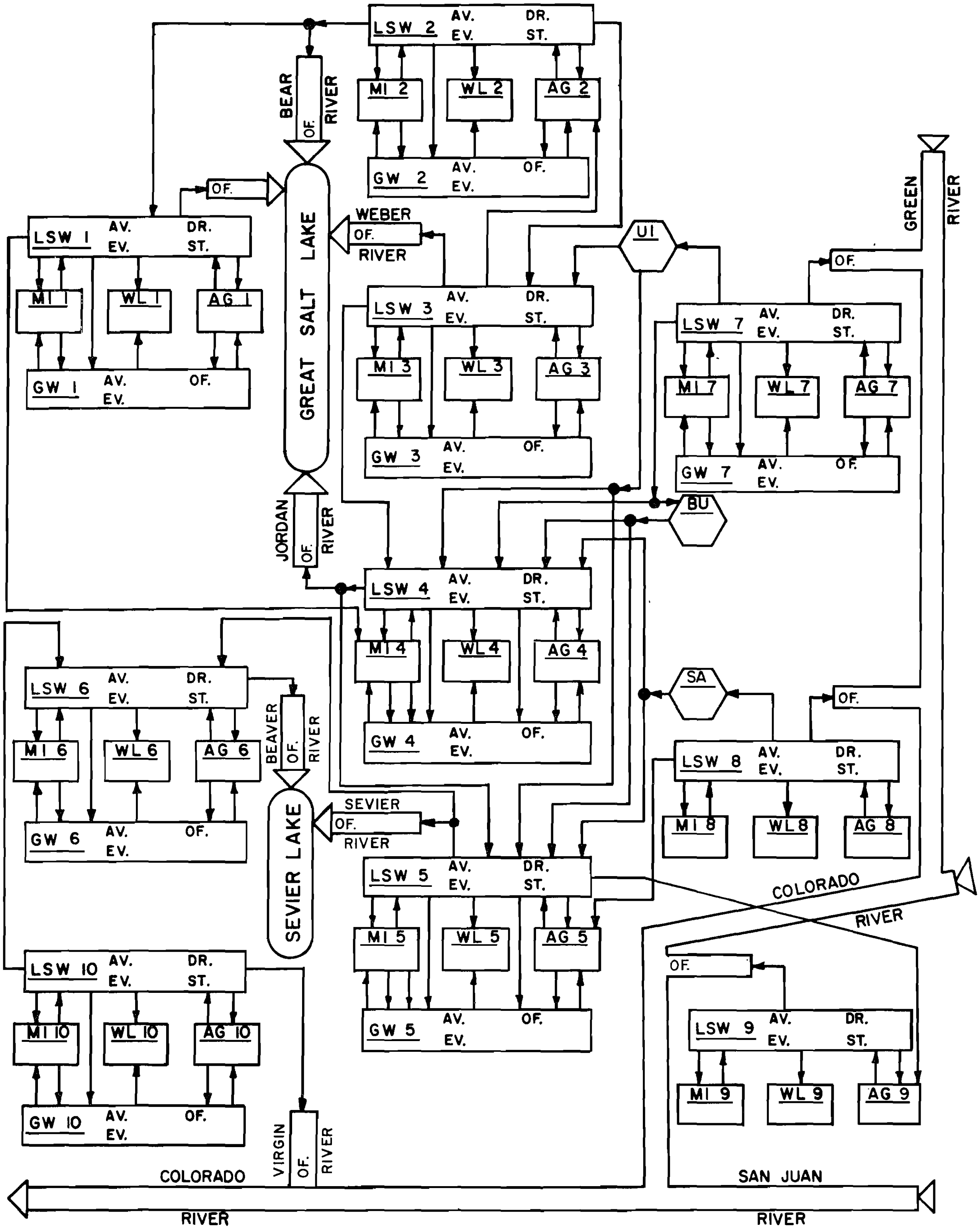

Figure 17. Flow diagram of the allocation model.

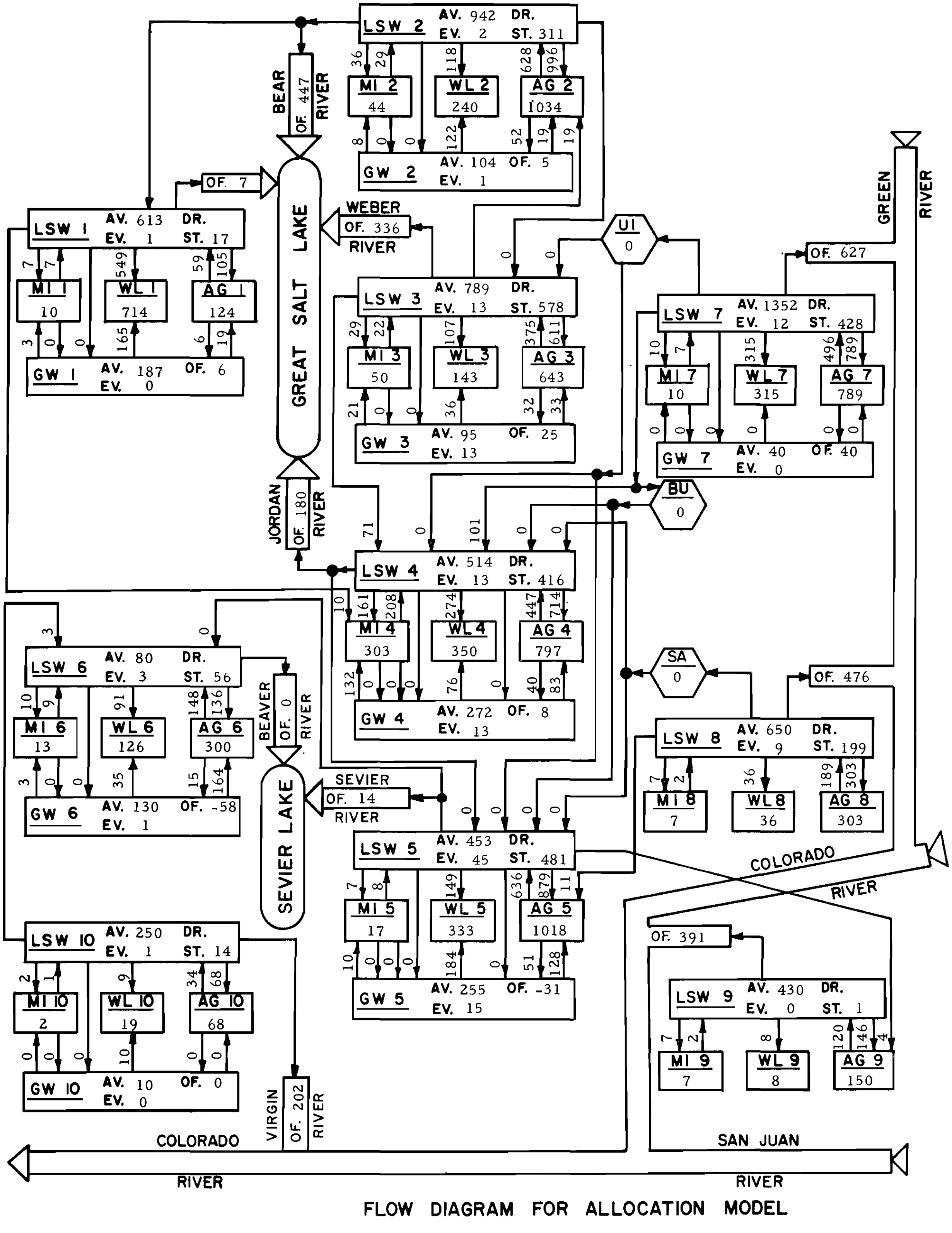

Figure 18. Present allocation of water in Utah (1965).

in which

Q indicates this is a quantity of water in ac-ft per year,
BU indicates water supplied by the Bonneville unit,
UI indicates water supplied by the Ute Indian unit,
SA indicates water supplied by the Sevier area,
LSW indicates the water is supplied to the local surface water pool,
Y indicates the HSU receiving the water, and
MPT indicates the total water accumulated by the particular transfer for all destinations.

As an example, QUILSW3 is the quantity of water transferred by the Ute Indian unit to the local surface water pool in HSU 3 (Weber River Basin).

Local surface water

The variables representing the quantity of local surface water allocated to meet the various demands within the same HSU are:

PLSWXAGX
QLSWXAGX
RLSWXAGX
PLSWXMIX
QLSWXMIX
RLSWXMIX
QLSWXWLX

X = 1, ..., 9, 0
(The symbol "0" is used in the computer to represent HSU 10 for convenience)

in which

P indicates this is an allocation presently in existence,
Q indicates new development for AG and MI but total allocation for WL,
R indicates present plus new development,
LSW indicates that local surface water is the source,
X indicates the HSU,
AG indicates the water is being allocated to satisfy agricultural demand,
MI indicates municipal and industrial demand, and
WL indicates wetlands demand.

In addition to the in-basin diversions discussed in the preceding paragraph, the model also allows for inter-basin transfer of local surface water. The variables representing these transfers are:

PLSWXSWY
QLSWXSWY
RLSWXSWY

X = 1, 2, ..., 9, 0
Y = 1, 3, 4, 5, 6 (as applicable)

in which

SW indicates the water is supplied to the local surface water pool, and

P,Q,R,LSW,X,Y are defined as before. Listed below are the transfers considered in the model.

X transferred to Y (HSU)	Variable
2 to 1	Q only
2 to 3	Q only
3 to 4	P, Q, and R
4 to 5	Q only
5 to 6	Q only
7 to 4	P only
10 to 6	P,Q, and R

Some inter-basin transfers presently in existence are allocated directly to satisfy a particular demand. The variables representing these transfers are:

PLS1MI4
PLSW3AG2
PLSW5AG9
PLSW8AG5

Groundwater

Sufficient quantities of groundwater are available to help meet demands in HSU 1 through 7 and 10. In HSU 8 and 9 the known groundwater aquifers are not large and are not considered in the model. In the other eight HSU, the groundwater can be allocated to meet the diversions required for agricultural, municipal and industrial, and wetland demands. The variables representing the quantity of groundwater allocated to meet the various demands within the same HSU are:

PGWXAGX
QGWXAGX
RGWXAGX
PGWXMIX
QGWXMIX
RGWXMIX
QFGWXWLX
QCGWXWLX

X = 1, 2, ..., 7, 0

in which

GW indicates this is groundwater,
FGW indicates this is groundwater freely available to wetlands,
CGW indicates this is groundwater which must be pumped to wetlands and the rest of the symbols are defined as before.

Stored local surface water

Because of the difference between the seasonal and long-term supply pattern and the demand pattern, storage is required in order to insure a sufficient supply of water. Storage of the local surface water is provided in the HSU for use both in-basin as well as for the inter-basin and Colorado River water transfers. The general forms of the variables for storage are:

QDREQX
QRECX
PLSWXSTX
QLSWXSTX
RLSWXSTX

X = 1, 2, ..., 9, 0

in which

ST indicates storage,

DREQ indicates draft requirement,

REC indicates excess water above the draft requirement which must be maintained to keep the reservoir at present levels for recreational purposes, and the rest of the symbols are defined as before.

The relationship between draft requirement and storage is highly non-linear. Since the functions are separable, use was made of a non-linear technique known as the "delta method" to represent these functions (Hadley, 1964). This method has been included with the linear programming capability of the Mathematical Programming System (MPS) 360 and is described as separable programming (IBM, 1971). This required the introduction of dummy variables as follows:

$$\mathrm{DXZ} \quad \left.\right\} \quad \begin{array}{l} X = 1, 2, \ldots, 9, 0 \\ Z = 1, 2, 3, 4 \end{array}$$

in which

D indicates the dummy variable relating draft and storage,

X indicates the HSU, and

Z is a counter to allow for more than one straight line segment in the fit to the non-linear curve.

Evaporation loss

Another group of variables in the model are those dealing with the net evaporation loss from major reservoirs. Bear and Utah Lakes are used as major storage reservoirs, however, the evaporation loss from these two bodies of water is deducted from the basin yield to obtain the water that is available for allocation to the various demands. The variables used to express evaporation loss from all other reservoirs are as follows:

$$\left.\begin{array}{l} \mathrm{QLSWXEVX} \\ \mathrm{QGWXEVX} \end{array}\right\} \quad X = 1, 2, \ldots, 9, 0$$

in which

EV indicates this is evaporation loss and the rest of the symbols are defined as before.

The relationship between evaporation loss and storage in HSU 2 and 4 is also non-linear and again dummy variables were introduced as follows:

$$\mathrm{EXZ} \quad \left.\right\} \quad \begin{array}{l} X = 2, 4 \\ Z = 1, 2, 3 \end{array}$$

in which

E indicates the dummy variable relating evaporation loss and storage and the rest of the symbols are defined as before.

Return flow

The return flow variables are another group of variables in the model. The return flows were considered as an available source of supply and were added to the right-hand side of the constraint equations of both the available local surface water supply and the available groundwater supply. The variables used to express the return flows from the agricultural diversions are as follows:

$$\left.\begin{array}{l} \mathrm{QARXLSWX} \\ \mathrm{QARXGWX} \end{array}\right\} \quad X = 1, \ldots, 9, 0$$

in which

AR indicates this is agricultural return flow and the other symbols are defined as before. The variables representing municipal and industrial return flow are:

$$\left.\begin{array}{l} \mathrm{QWWXLSWX} \\ \mathrm{QWWXRX} \end{array}\right\} \quad X = 1, 2, \ldots, 9, 0$$

in which

WW indicates this is waste water return flow from municipal and industrial,

R indicates the waste water is recharged into the groundwater aquifer, and the rest of the symbols are defined as before.

Groundwater recharge

Provision was made in the model to allow for recharge of the existing groundwater aquifers. The variables representing this type allocation are as follows:

$$\left.\begin{array}{l} \mathrm{QLSWXRX} \\ \mathrm{QLSWXRUX} \\ \mathrm{QWWXRX} \\ \mathrm{QWWXRUX} \end{array}\right\} \quad X = 1, 2, \ldots, 7, 0$$

in which

R indicates recharge in the groundwater aquifer,

RU indicates recharge in the upper region of the river basins with subsequent higher cost, and the rest of the symbols are defined as before.

Outflow

Another group of variables in the model are those showing the outflow from the various HSU. These are expressed as:

$$\left.\begin{array}{l} \mathrm{QLSWXOFX} \\ \mathrm{QGWXOFX} \end{array}\right\} \quad X = 1, 2, \ldots, 9, 0$$

in which

OF indicates this is outflow and the rest of the symbols are defined as before.

Miscellaneous variables

In addition to the variables discussed in the preceding paragraphs, there are also a few additional variables in the model which are included for convenience in writing the equations. These are:

AXLSWX	represents the local surface water from HSU X which is consumed in HSU X, X = 7 and 8
QAGXLSWX	represents the excess water that is allocated to the local surface water pool due to a reduction in agriculture demands over time in HSU 3, 4, and 8
QAGXGWX	represents the same allocation to groundwater in HSU 3, 4, and 8
QEVX	represents the evaporation loss in HSU X, X = 2 and 4

COST OF SUPPLYING WATER

The components of water cost are those costs associated with a particular function or process which when summed give the total cost associated with a particular allocation. These total costs are the cost coefficients which appear in the objective function of the linear programming problem. The following paragraphs discuss the components of cost and how these components are summed to determine the individual cost coefficients.

Components of Water Cost

Water transfer

Water transfers under consideration here are of three types: 1) New facilities to move Colorado River water to the Great Basin, 2) present facilities which move water from one basin to another, and 3) new facilities for other inter-basin transfers.

Colorado River water to surface water pool. The components are related primarily to elements of the Central Utah Project with a small amount of additional water identified as Sevier area. Since joint costs which occur when a project element contributes to the production of more than one output have not been allocated in the planning, the costs shown in columns 2, 3, and 4 of Table 9 are not precise estimates. They are based on generalized investigations of volume of water moved and distance covered. Note that these costs are not complete for moving and using water. Storage and collection costs at the point of origin and distribution and possible treatment costs at the point of use are added in the complete model. A single type of facility is assumed for moving water for whatever its final use might be. Differences in distribution costs or treatment are considered separately. The transferred water is assumed to be released into the surface water pool of the HSU indicated in column 1 and to become part of the available surface water.

Present diversions. Facilities have already been constructed to transfer some water from one basin to another. In some cases these transfers are distributed directly to agriculture. Column 5 indicates the HSU receiving the water from the HSU listed in column 1 and column 6 shows the cost. This cost is only that for operating and maintenance (O&M) since capital costs are considered as sunk costs and are not part of the optimization problem. Other facilities have been constructed to transfer water directly to municipal and industrial (M&I) use. Column 7 indicates the HSU receiving the water and column 8 the O&M cost. Additionally, facilities have already been constructed to transfer water from one HSU and release it in the surface water pool of another HSU. Column 9 indicates the HSU receiving the water and column 10 shows the associated O&M cost.

New diversions to surface water pool. New facilities which might be constructed to move water from one HSU to another are considered in the allocation problem. Column 11 indicates the HSU that feasibly could receive water from the HSU listed in column 1. Column 12 shows the total cost of building and operating the facilities for making the indicated transfers. Capital costs as well as O&M costs are included.

Storage

Present storage. Costs shown in column 13 represent the O&M costs only since capital costs associated with already constructed facilities are not part of the optimization problem.

New storage. Costs of new storage facilities shown in column 14 are based primarily on the estimates of size and quality of remaining reservoir sites. Storage at sites near collection points and sites nearer the point of use are included. The cost includes capital costs as well as O&M costs.

Agricultural distribution

These costs are for the diversion works and distribution facilities. Distribution costs for present diversions include only O&M whereas for new diversions the cost includes capital costs as well. Cost of storage facilities or on-farm ditches is not included. The on-farm costs are more logically determined as a function of acreage than ac-ft of water diverted. Table 10 shows the estimated on-farm cost in dollars per acre. Costs listed in Table 10 are not included in the supply model. If a model were constructed which included both supply and demand, then these costs would become part of the demand side since demand is related to cost per acre of land.

It is recognized that each water system will have a unique cost structure, but the data given in Table 9 represent averages for the size, terrain, and other factors that affect each HSU.

Table 9. Cost components for supplying water in Utah. (Annual cost in dollars per ac-ft.)

Hydrologic Study Unit	Transfer Costs[c]											Storage costs	
	Colorado River water to surface water pool[a]			Present diversions direct to agriculture		Present diversions direct to M&I		Present diversions to surface water pool		New diversions to surface water pool		Present storage	New storage[f]
Column No.	Bonneville Unit, CUP	Ute Indian Unit, CUP	Sevier area	To HSU	Cost (O&M)	To HSU	Cost (O&M)	To HSU	Cost (O&M)	To HSU	Cost	(O&M)	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Symbol[h] (X)[i]	CTBX	CTUX	CTSX	(Y)[j]	CTXAY	(Y)[j]	CTXMY	(Y)[j]	CTPXSY	(Y)[j]	CTNXSY	CPSX	CNSX
1						4	1.00					.10	11.00
2										1&3	4.00	.10	4.70
3		10.00		2	1.00			4	.40	4	4.00	.10	16.30
4	7.00	10.00	8.00							5	5.00	.10	13.00
5	10.00	13.00	4.00	9	1.00					6	4.00	.10	8.60
6												.10	14.00
7								4	.40			.10	10.80
8				5	1.00							10	7.20
9												.10	13.50
10								6	.40	6	4.00	10	14.30

Hydrologic Study Unit	Agricultural distribution costs				M&I distribution costs				M&I supply treatment costs[c]				Wastewater treatment costs[d]		Recharge groundwater basin costs[g]		
	Present diversions (O&M)		New diversions[b]		Present diversions (O&M)		New diversions[b]		Present diversions (O&M)		New diversions		Return to Local surface water	Return to Ground-water	Recharge	Collection system for local surface water	Transport from distant points
Column No.	Local surface water	Ground-water	Local surface water	Ground-water	Local surface water	Ground-water	Local surface water	Ground-water	Local surface water	Ground-water	Local surface water	Ground-water					
1	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Symbol[h] (X)[i]	CADPSX	CADPGX	CADNSX	CADNGX	CMDPSX	CMDNSX	CMDPGX	CMDNGX	CMTPSX	CMTPGX	CMTNSX	CMTNGX	CWTSX	CWTGX	CRCX	CCSX	CTRCX
1	.75	1.25	5.25	1.50	16.00	23.80	42.00	42.00	1.00	.20	5.00	.25	26.00	14.00	15.00	2.00	
2	.75	1.75	4.75	2.00	16.00	23.80	42.00	42.00	1.00	.20	5.00	.25	26.00	14.00	15.00	2.00	
3	.75	2.00	5.50	2.25	16.00	29.50	42.00	49.00	4.00	.50	17.00	.65	26.00	14.00	15.00	2.00	
4	.75	2.75	5.25	3.00	16.00	29.50	42.00	49.00	4.00	.50	17.00	.65	26.00	14.00	15.00	2.00	6.00
5	.75	1.75	4.75	2.00	16.00	23.80	42.00	42.00	2.00	.20	10.00	.25	26.00	14.00	15.00	2.00	6.00
6	.75	2.25	4.50	2.50	16.00	23.80	42.00	42.00	2.00	.20	10.00	.25	26.00	14.00	15.00	2.00	
7	.75	1.00	5.25	1.25	16.00	23.80	42.00	42.00	2.00	.20	10.00	.25	26.00	14.00	15.00	2.00	
8	.75	-	5.25	-	16.00	-	42.00	-	6.00		25.00		26.00		-		
9	.75	-	5.25	-	16.00	-	42.00	-	4.00	-	17.00		26.00				
10	.75	1.25	5.25	1.50	16.00	23.80	42.00	42.00	4.00	.20	17.00	.25	26.00	14.00	15.00	2.00	

[a] These values are only rough approximations. These costs are not strictly separable in the available data on this project. These costs do not include the storage at collection.

[b] These costs pertain to newly developed water supplies. They do not include storage costs.

[c] Treatment costs for surface water vary according to the amount of filtration and other measures required. Treatment of groundwater is only chlorination.

[d] Primary and secondary treatment is required for returning water to surface flows. Primary treatment only is required for returning to groundwater.

[e] Water transfer costs are based on average cost data for transporting water which depends on amount of water moved and the distance.

[f] Based on size and quality estimates of available reservoir sites.

[g] The recharging cost is for spreading ponds and pits for getting water into ground. The collection system is for bringing the water from various places to the point of recharge, except in areas 4 and 5 a portion of the water which could be recharged is at inconvenient and expensive places to recover. Hence the $6 charge applies to part of the water for extra transport and collection costs.

[h] This represents the symbol used in the summation of cost components for the cost coefficients of the variables in the objective function.

[i] The X represents the general form of the cost component as it appears in the cost coefficient summation equation and is for the HSU numbered below.

[j] The Y represents the general form of the cost component as it appears in the cost coefficient summation equation and is for the HSU receiving water.

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Weber Basin Water Conservancy District, 1969
Price Municipal Corporation, 1969
Department of Water Supply and Waterworks, Salt Lake City, 1969
Metropolitan Water District, Salt Lake City, 1969

Table 10. On-farm distribution cost. (Dollars per acre.)

Hydrologic Study Unit	Distribution Cost	
	Present Diversions (O&M)	New Diversions
1	1.00	3.00
2	1.00	4.00
3	1.00	5.00
4	1.00	6.00
5	1.00	4.00
6	1.00	5.00
7	1.00	3.00
8	1.00	3.00
9	1.00	3.00
10	1.00	3.00

Present diversions. Columns 15 and 16 show the costs of distributing water to agriculture using facilities already constructed. These costs are only O&M since capital costs are not included in the optimization model. Column 15 is for diversions from local surface water while column 16 is from groundwater. The costs for groundwater include the power cost of pumping. Cost differences for each HSU reflect the depth from which water must be pumped.

New diversions. Costs shown in columns 17 and 18 represent the total cost of constructing and maintaining new facilities. These costs include capital costs as well as O&M costs.

Municipal and industrial distribution

Present diversions. Columns 19 and 20 show the costs associated with distributing water for municipal and industrial use using facilities already constructed. O&M costs only are included. Diversions from local surface water are shown in column 19 whereas diversions from groundwater are shown in column 20. The costs for groundwater diversion include the cost of pumping and the cost required to boost to line pressure. For reasons possibly related to economies of scale and differential power rates for agriculture, the pumping for municipal and industrial supplies is always more expensive than the pumping for irrigation. The cost to boost to line pressure is essentially the same as for pumping to a higher elevation such as to storage tanks.

New diversions. Costs shown in columns 21 and 22 represent the total cost of constructing and maintaining new facilities. Capital costs are included with the O&M costs. Cost of pumping and boosting to line pressure is included in the groundwater costs.

Municipal and industrial supply treatment

Present diversions. Columns 23 and 24 show the costs of treating water using presently constructed facilities. Treatment costs for surface water shown in column 23 vary according to the amount of filtration and other measures needed to bring the water to acceptable standards. The values given represent averages. The only treatment for groundwater is chlorination, and only O&M costs are included.

New diversions. Costs shown in columns 25 and 26 reflect treatment costs associated with construction of new facilities. Capital costs as well as O&M costs are included.

Waste water treatment

Another element of treatment costs is the process of reclaiming waste water from municipal and industrial uses for recycling in the system. Recycling can be accomplished by 1) treating the waste water and returning it to the surface water pool where it is diluted, mixed, and eventually diverted into another M&I water supply system; 2) treating the waste water and returning it (by artificial recharge) to groundwater pool where it is diluted and, to an extent, purified and eventually pumped into another M&I water supply system; and 3) direct recycling by treating the waste water and returning it directly to the M&I water supply system. This third procedure is not considered in this study due to possible public aversion. Primary and secondary treatment is required for returning water to the surface water pool and is reflected in the costs shown in column 27. Primary treatment only is required for return to groundwater as reflected in the lower costs shown in column 28.

Recharging groundwater basin

The recharging cost shown in column 29 is for land acquisition, construction, and operation of spreading ponds and pits for getting water into the ground. The collection system, column 30, is for bringing the local surface water from various places to the point where recharge is to be made. In subareas 4 and 5, it has been determined that a part of the water which could be recharged is at inconvenient and expensive places to recover. Hence, the $6.00 charge in column 31 applies to part of the water for extra transport and collection costs. Note that in this case, too, recharge is only one of the components. Treatment costs as well as pumping and distribution costs would be incurred in order to use this water supply source.

Supply Variable Cost Coefficients

The cost components discussed in the previous paragraphs were combined to obtain the cost coefficients of the variables which make up the objective function. These cost coefficients together with their respective components are shown in the following paragraphs.

Inter-basin transfer

These variables have only one cost component. The general forms for Colorado River water transfer and other local surface water transfer are:

Variable	Component of Cost Coefficient
QBULSWX	CTBX
QUILSWX	CTUX
QSALSWX	CTSX
PLSWXAGY	CTXAY
PLSWXMIY	CTXMY
PLSWXSWY	CTPXSY
QLSWXSWY	CTNXSY

Diversion to agriculture

The general forms of the cost coefficients for diverting local surface water and groundwater to agriculture are:

Variable	Component of Cost Coefficient
PLSWXAGX	CADPSX
QLSWXAGX	CADNSX
PGWXAGX	CADPGX
QGWXAGX	CADNGX

Diversion to municipal and industrial

The general forms of the cost coefficient for diverting local surface water and groundwater to municipal and industrial use includes the cost of treatment. These forms are:

Variable	Component of Cost Coefficient
PLSWXMIX	CMDPSX + CMTPSX
QLSWXMIX	CMDNSX + CMTNSX
PGWXMIX	CMDPGX + CMTPGX
QGWXMIX	CMDNGX + CMTNGX

For example, the cost to divert water to meet M&I demands in HSU 4 from presently developed facilities (PLSW4MI4) is:

CMDPS4 = 16.00
CMTPS4 = 4.00
Coefficient = 20.00

Diversion of groundwater to wetlands

The cost coefficient has only a single component which is the same as cost of new diversions of groundwater to agriculture. The general form is:

Variable	Component of Cost Coefficient
QCGWXWLX	CADNGX

Groundwater recharge

The general forms for these cost coefficients are shown below. The municipal and industrial waste water must be treated before it can be used for recharge.

Variable	Component of Cost Coefficient
QLSWXRX	CRCX + CCSX
QLSWXRUX	CRCX + CCSX + CTRCX
QWWXRX	CRCX + CWTGX
QWWXRUX	CRCX + CWTGX + CTRCX

For example, the cost to recharge the groundwater in HSU 4 from local surface water from distant points (QLSW4RU4) is:

CRC4 = 15.00
CCS4 = 2.00
CTRC4 = 6.00
Coefficient = 23.00

Reclaim municipal and industrial waste water

These variables represent the reclamation of waste water when it is returned to local surface water. The general form of the cost coefficient is:

Variable	Component of Cost Coefficient
QWWXLSWX	CWTSX

Storage of local surface water

The general form of the cost coefficient is:

Variable	Component of Cost Coefficient
PLSWXSTX	CPSX
QLSWXSTX	CNSX

LEAST-COST ALLOCATION MODEL

Mathematical Model

The mathematical model used to study optimal allocation of water resources within the State of Utah falls generally in the category of linear-programming. According to Hadley (1962) the general linear-programming problem can be described as follows: Given a set of m linear inequalities or equations in r variables ($r \lesseqgtr m$), non-negative values of those variables are sought which will satisfy the constraints and maximize or minimize some linear function of the variables.

Objective Function

In the case at hand the linear function to be maximized or minimized (more commonly called the objective function) is an expression for the total cost in dollars of allocating (meeting demand for) the water resources of Utah. The variables are all the various alternatives of allocation which may combine to form the solution to the problem. These variables represent a quantity of water to be allocated to a given alternative use in acre-feet per year. Each variable has an associated cost coefficient which reflects the cost of allocating one acre-foot per year to the given alternative or activity. The objective function thus represents the total cost for allocation of Utah's water resources in dollars per year.

Variables

The variables described in the preceding chapter are grouped in eight general categories.

1. Variables showing the amount of water which would be transferred from the Upper Colorado River Basin to the Great Basin. This category is structured primarily on the basis of the Central Utah Project. This project proposes three transfer patterns: 1) via the Bonneville unit, 2) via the Ute Indian unit, and 3) that indicated as the Sevier area.
2. Variables showing the amount of local surface water used in the ten HSU.
3. Variables showing the amount of groundwater used in eight HSU. HSU 8 and 9 have insufficient groundwater to make its use economically feasible.
4. Variables dealing with the amount of local surface water which must be stored.
5. Variables dealing with the evaporation loss from the storage reservoirs.
6. Variables showing the amount of water that appears as return flow in each of the HSU.
7. Variables showing the amount of water used to recharge the groundwater basins.
8. Variables showing the amount of outflow from each of the ten HSU.

A generalized matrix which shows the various categories of the variables, the objective function, and the categories of constraint equations is shown in Figure 19. The complete model has 204 constraints and 348 variables.

Contraints

The model constraints consist of both equations and inequalities which are given names as required by the simplex algorithm utilized in the MPS 360 linear-programming computer solution (IBM, 1971). The structural coefficients and right-hand-side (RHS) coefficients are based upon the following assumptions for what is called the "basic" model:

1. Water availability for use in each of the HSU is the "In-Basin Availability" listed on Table 8. This includes: 1) excess precipitation on irrigated croplands for period October to April, 2) net evaporation loss from large lakes, and 3) direct use of groundwater by croplands in addition to the basin yield.
2. Water requirements for agricultural use in each of the HSU are listed as diversions to agriculture under "Total AG" on Table 8 for the present (1965) conditions.
3. Water requirements for municipal and industrial use in each of the HSU are listed as diversions to municipal and industrial under "Total M&I" on Table 8 for the present (1965) conditions.
4. Water requirements for wetlands use in each of the HSU are listed as depletions for wetlands under "Total" for the present (1965) conditions.
5. Probability of having sufficient surface water storage is 0.75.
6. Return flow coefficients are those given in Table 6.
7. Groundwater mining as present in HSU 5 and 6 is not allowed.
8. Artificial recharge is limited to that shown by the data in Table 7.

<table>
<tr><th rowspan="2">Model Variable
Constraints</th><th colspan="3">Colo. River Water Transfer</th><th rowspan="2">Local Surface Water</th><th rowspan="2">Groundwater</th><th rowspan="2">Stored LSW</th><th rowspan="2">Evaporation Loss</th><th rowspan="2">Return Flow</th><th rowspan="2">Groundwater Recharge</th><th rowspan="2">Outflow</th></tr>
<tr><th>Bonneville Unit</th><th>Ute I Unit</th><th>Sevier Area</th></tr>
<tr><td>Cost</td><td></td><td></td><td></td><td colspan="3">Objective Function</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Local Surface Water</td><td rowspan="2"></td><td colspan="7" rowspan="2">Water Availabilities within the Hydrologic Study Units</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>Groundwater</td></tr>
<tr><td>Municipal & Industrial</td><td rowspan="3"></td><td colspan="7" rowspan="3">Water Requirements within the Hydrologic Study Units</td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td>Agriculture</td></tr>
<tr><td>Wetlands</td></tr>
<tr><td>Draft Requirement</td><td rowspan="3"></td><td rowspan="3"></td><td colspan="5" rowspan="3">Reservoir Storage and Evaporation</td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td>Storage Requirement</td></tr>
<tr><td>Evaporation Loss</td></tr>
<tr><td>M & I Waste Water</td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td colspan="2" rowspan="2">Return Flow</td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>Agriculture</td></tr>
<tr><td>Free GW for Wetlands</td><td></td><td></td><td></td><td colspan="4">Free Groundwater for Wetlands</td><td></td><td></td><td></td></tr>
<tr><td>GW Recharge Limits</td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td colspan="3" rowspan="3">Other Limitations</td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td>Transfer Limits</td></tr>
<tr><td>Outflow Limits</td></tr>
</table>

Figure 19. Generalized matrix of the least-cost linear programming allocation model.

9. Minimum inflow to the Great Salt Lake from Utah drainage is 500,000 ac-ft/yr.
10. Minimum outflow to the Upper Colorado River from Utah drainage is 907,000 ac-ft/yr.
11. Growth projections of water requirement for agricultural use, municipal and industrial use, and wetlands use are based on Alternate 1 projections by the Utah Division of Water Resources (1970).

Water availability

Constraints dealing with the amount of water available for allocation within the various HSU are divided into two groups: (1) Those related to available local surface water shown in Figure 20 and (2) those related to available groundwater shown in Figure 21.

Water requirements

Constraints dealing with the amount of water demand to be met within the various HSU are divided into three groups: (1) Those related to diversion requirements for municipal and industrial shown in Figure 22, (2) those related to diversion requirements for agriculture shown in Figure 23, and (3) those related to depletion requirements for wetlands shown in Figure 24.

Reservoir storage and evaporation loss

These constraints deal with the amount of water which must be stored in order that the surface water which runs off in the spring can be available for use later in the year and for use in extended drought. Included in this category are constraints which allow for the evaporation loss from the reservoirs. These constraints are divided into three groups: (1) Those related to the storage draft requirements shown in Figure 25, (2) those related to the determination of the storage required shown in Figure 26, and (3) those related to determination of the net loss by reservoir evaporation shown in Figure 27.

Return flows

These constraints deal with the amount of water which appears as return flow. More water is diverted to agriculture than is used by the crops and the excess appears as agricultural return flow. More water is diverted for municipal and industrial use than is consumptively used and the remainder appears as waste water return flow.

These constraints are divided into two groups: (1) Those related to waste water return flow from municipal and industrial shown in Figure 28, and (2) those related to return flow from agriculture shown in Figure 29.

```
Constraint                                Constraint                                     Explanations and Comments
   Name

AVAILSW1     1.0 RLSW1AG1 + 1.0 QLSW1R1   + 1.0 RLSW1MI1 + 1.0 QLSW1WL1
           + 1.0 PLSW1MI4 + 1.0 QLSW1EV1  - 1.0 QWW1LSW1 - 1.0 QAR1LSW1
                          - 1.0 QLSW2SW1  + 1.0 QLSW1OF1                  =  613.0

AVAILSW2     1.0 RLSW2AG2 + 1.0 QLSW2R2   + 1.0 RLSW2MI2 + 1.0 QLSW2WL2
           + 1.0 QLSW2SW1 + 1.0 QLSW2SW3  + 1.0 QLSW2EV2 - 1.0 QWW2LSW2
                          - 1.0 QAR2LSW2  + 1.0 QLSW2OF2                  =  941.5

AVAILSW3     1.0 RLSW3AG3 + 1.0 QLSW3R3   + 1.0 RLSW3MI3 + 1.0 QLSW3WL3
           + 1.0 PLSW3AG2 + 1.0 RLSW3SW4  + 1.0 QLSW3EV3 - 1.0 QAG3LSW3
           - 1.0 QWW3LSW3 - 1.0 QAR3LSW3  - 1.0 QUILSW3  - 1.0 QLSW2SW3
                          + 1.0 QLSW3OF3                                  =  789.2

AVAILSW4     1.0 RLSW4AG4 + 1.0 QLSW4R4   + 1.0 QLSW4RU4 + 1.0 RLSW4MI4
           + 1.0 QLSW4WL4 + 1.25QLSW4SW5  + 1.0 QLSW4EV4 - 1.0 QAG4LSW4
           - 1.0 QWW4LSW4 - 1.0 QAR4LSW4  - 1.0 QBULSW4  - 1.0 QUILSW4       The equations calculate the
           - 1.0 QSALSW4  - 1.0 RLSW3SW4  - 1.0 PLSW7SW4 + 1.0 QLSW4OF4  =  513.6   maximum surface water outflow
                                                                                 in each of the HSU.  The RHS
AVAILSW5     1.0 RLSW5AG5 + 1.0 QLSW5R5   + 1.0 QLSW5RU5 + 1.0 RLSW5MI5          is the local surface water
           + 1.0 QLSW5WL5 + 1.0 QLSW5SW6  + 1.0 PLSW5AG9 + 1.0 QLSW5EV5          availability.
           - 1.0 QLSW4SW5 - 1.0 QBULSW5   - 1.0 QUILSW5  - 1.0 QSALSW5
           - 1.0 QWW5LSW5 - 1.0 QAR5LSW5  + 1.0 QLSW5OF5                  =  453.2

AVAILSW6     1.0 RLSW6AG6 + 1.0 QLSW6R6   + 1.0 RLSW6MI6 + 1.0 QLSW6WL6
           + 1.0 QLSW6EV6 - 1.0 QWW6LSW6  - 1.0 QAR6LSW6 - 1.0 RLSW0SW6
                          - 1.0 QLSW5SW6  + 1.0 QLSW6OF6                  =   80.0

AVAILSW7     1.0 QBUMPT   + 1.0 QUIMPT    + 1.0 Q7LSW7   + 1.0 PLSW7SW4
                          + 1.0 QLSW7EV7  + 1.0 QLSW7OF7                  = 1351.6

AVAILSW8     1.0 QSAMPT   + 1.0 Q8LSW8    + 1.0 PLSW8AG5 + 1.0 QLSW8EV8
                          - 1.0 QAG8LSW8  + 1.0 QLSW8OF8                  =  650.0

AVAILSW9     1.0 RLSW9AG9 + 1.0 RLSW9MI9  + 1.0 QLSW9WL9 + 1.0 QLSW9EV9
                          - 1.0 QWW9LSW9  - 1.0 QAR9LSW9 + 1.0 QLSW9OF9  =  430.0

AVAILSW0     1.0 RLSW0AG0 + 1.0 QLSW0R0   + 1.0 RLSW0MI0 + 1.0 QLSW0WL0
           + 1.0 QLSW0EV0 + 1.0 RLSW0SW6  - 1.0 QWW0LSW0 - 1.0 QAR0LSW0
                          + 1.0 QLSW0OF0                                  =  250.0

LSWU7        1.0 RLSW7AG7 + 1.0 QLSW7R7   + 1.0 RLSW7MI7 + 1.0 QLSW7WL7
                          - 1.0 QWW7LSW7  - 1.0 QAR7LSW7 - 1.0 Q7LSW7    =     0     These equations calculate the
                                                                                 surface water use in HSU 7 and
LSWU8        1.0 RLSW8AG8 + 1.0 RLSW8MI8  + 1.0 QLSW8WL8 - 1.0 QWW8LSW8          8 and are for convenience in
                          - 1.0 QAR8LSW8  - 1.0 Q8LSW8                    =     0     writing other constraints.
```

Figure 20. Constraints for availability of local surface water.

Constraints Name	Constraint		Explanation and Comments
AVAILGW1	1.0 RGW1AG1 + 1.0 RGW1MI1 + 1.0 QCGW1WL1 + 1.0 QFGW1WL1 + 1.0 QGW1EV1 - 1.0 QWW1R1 - 1.0 QAR1GW1 - 1.0 QLSW1R1 + 1.0 QGW1OF1	= 187.0	These equations calculate the maximum groundwater outflow in each of the HSU except 8 and 9 where groundwater is negligible. The RHS is the groundwater availability.
AVAILGW2	1.0 RGW2AG2 + 1.0 RGW2MI2 + 1.0 QCGW2WL2 + 1.0 QFGW2WL2 + 1.0 QGW2EV2 - 1.0 QWW2R2 - 1.0 QAR2GW2 - 1.0 QLSW2R2 + 1.0 QGW2OF2	= 103.5	
AVAILGW3	1.0 RGW3AG3 + 1.0 RGW3MI3 + 1.0 QCGW3WL3 + 1.0 QFGW3WL3 + 1.0 QGW3EV3 - 1.0 QWW3R3 - 1.0 QAR3GW3 - 1.0 QAG3GW3 - 1.0 QLSW3R3 + 1.0 QGW3OF3	= 94.9	
AVAILGW4	1.0 RGW4AG4 + 1.0 RGW4MI4 + 1.0 QCGW4WL4 + 1.0 QFGW4WL4 + 1.0 QGW4EV4 - 1.0 QWW4R4 - 1.0 QWW4RU4 - 1.0 QAR4GW4 - 1.0 QAG4GW4 - 1.0 QLSW4R4 - 1.0 QLSW4RU4 + 1.0 QGW4OF4	= 272.1	
AVAILGW5	1.0 RGW5AG5 + 1.0 RGW5MI5 + 1.0 QCGW5WL5 + 1.0 QFGW5WL5 + 1.0 QGW5EV5 - 1.0 QWW5R5 - 1.0 QWW5RU5 - 1.0 AR5GW5 - 1.0 QLSW5R5 - 1.0 QLSW5RU5 + 1.0 QGW5OF5	= 254.6	
AVAILGW6	1.0 RGW6AG6 + 1.0 RGW6MI6 + 1.0 QCGW6WL6 + 1.0 QFGW6WL6 + 1.0 QGW6EV6 - 1.0 QWW6R6 - 1.0 QAR6GW6 - 1.0 QLSW6R6 + 1.0 QGW6OF6	= 130.0	
AVAILGW7	1.0 RGW7AG7 + 1.0 RGW7MI7 + 1.0 QCGW7WL7 + 1.0 QFGW7WL7 + 1.0 QGW7EV7 - 1.0 QWW7R7 - 1.0 QLSW7R7 - 1.0 QAR7GW7 + 1.0 QGW7OF7	= 40.0	
AVAILGW0	1.0 RGW0AG0 + 1.0 RGW0MI0 + 1.0 QCGW0WL0 + 1.0 QFGW0WL0 + 1.0 QGW0EV0 - 1.0 QWW0R0 - 1.0 QAR0GW0 - 1.0 QLSW0R0 + 1.0 QGW0OF0	= 10.0	

Figure 21. Constraints for availability of groundwater.

Free groundwater for wetlands

Constraints dealing with the amount of groundwater that is used freely by wetlands are shown in Figure 30.

Limits

The constraints defining additional limits other than water availability and demands are divided into three groups: (1) Those limiting the amount of groundwater recharge shown in Figure 31, (2) those limiting the amount of the inter-basin transfers shown in Figure 32, and (3) those limiting the outflow from the various HSU shown in Figure 33. The limit on Colorado River outflow was established as follows:

	Present (1965) Depletions (ac-ft/yr)			Total Basin Yield
HSU	Man Caused	Total	Difference	(ac-ft/yr)
7	468,300	692,100	223,800	1,359,000
8	156,000	174,500	18,500	650,000
9	39,400	43,100	3,700	430,000
			246,000	2,439,000

Total Yield Upper Basin			2,439,000
Realistic Allocation to Utah	1,438,000		
Net Mainstem Evaporation	152,000		
Net Allocation to Meet Demands		1,286,000	
Additional Water Allocation Due to Definition that Only Man Caused Depletions are Chargeable Against the Allocation		246,000	
Total Allocation			1,532,000
Water that Must be Released as Colorado River Outflow			907,000

Variable Bounds

Bounds have been established on several groups of variables in the basic model. These groups are: 1) Inter-basin transfer, 2) additional surface water storage, and 3) surface and groundwater outflow from each of the HSU. In addition, an upper bound of unity was placed on each of the dummy variables as part of the separable programming algorithm.

Inter-basin transfer

Bounds on presently existing inter-basin transfers were established primarily from the water budget studies.

Constraint Name	Constraint		Explanation and Comments
MIREQ1	1.0 RLSW1MI1 + 1.0 RGW1MI1	≥ 10.0	These inequalities show the constraint on water to meet the diversion requirements for municipal and industrial use. The RHS is the 1965 M&I demand shown earlier.
MIREQ2	1.0 RLSW2MI2 + 1.0 RGW2MI2	≥ 44.0	
MIREQ3	1.0 RLSW3MI3 + 1.0 RGW3MI3	≥ 49.7	
MIREQ4	1.0 RLSW4MI4 + 1.0 RGW4MI4 + 1.0 PLSW1MI4	≥ 302.5	
MIREQ5	1.0 RLSW5MI5 + 1.0 RGW5MI5	≥ 17.0	
MIREQ6	1.0 RLSW6MI6 + 1.0 RGW6MI6	≥ 13.0	
MIREQ7	1.0 RLSW7MI7 + 1.0 RGW7MI7	≥ 10.0	
MIREQ8	1.0 RLSW8MI8	≥ 7.0	
MIREQ9	1.0 RLSW9MI9	≥ 6.8	
MIREQO	1.0 RLSWOMIO + 1.0 RGWOMIO	≥ 1.5	
TLSW1MI1	1.0 PLSW1MI1 + 1.0 QLSW1MI1 - 1.0 RLSW1MI1	= 0	These equations sum the diversion from present development to M&I from local surface water with the new development diversions to get the total diversion to M&I from local surface water.
TLSW2MI2	1.0 PLSW2MI2 + 1.0 QLSW2MI2 - 1.0 RLSW2MI2	= 0	
TLSW3MI3	1.0 PLSW3MI3 + 1.0 QLSW3MI3 - 1.0 RLSW3MI3	= 0	
TLSW4MI4	1.0 PLSW4MI4 + 1.0 QLSW4MI4 - 1.0 RLSW4MI4	= 0	
TLSW5MI5	1.0 PLSW5MI5 + 1.0 QLSW5MI5 - 1.0 RLSW5MI5	= 0	
TLSW6MI6	1.0 PLSW6MI6 + 1.0 QLSW6MI6 - 1.0 RLSW6MI6	= 0	
TLSW7MI7	1.0 PLSW7MI7 + 1.0 QLSW7MI7 - 1.0 RLSW7MI7	= 0	
TLSW8MI8	1.0 PLSW8MI8 + 1.0 QLSW8MI8 - 1.0 RLSW8MI8	= 0	
TLSW9MI9	1.0 PLSW9MI9 + 1.0 QLSW9MI9 - 1.0 RLSW9MI9	= 0	
TLSWOMIO	1.0 PLSWOMIO + 1.0 QLSWOMIO - 1.0 RLSWOMIO	= 0	
TGW1MI1	1.0 PGW1MI1 + 1.0 QGW1MI1 - 1.0 RGW1MI1	= 0	These equations sum the diversion from present developments to M&I from groundwater with the new development diversion to get the total diversion to M&I from groundwater.
TGW2MI2	1.0 PGW2MI2 + 1.0 QGW2MI2 - 1.0 RGW2MI2	= 0	
TGW3MI3	1.0 PGW3MI3 + 1.0 QGW3MI3 - 1.0 RGW3MI3	= 0	
TGW4MI4	1.0 PGW4MI4 + 1.0 QGW4MI4 - 1.0 RGW4MI4	= 0	
TGW5MI5	1.0 PGW5MI5 + 1.0 QGW5MI5 - 1.0 RGW5MI5	= 0	
TGW6MI6	1.0 PGW6MI6 + 1.0 QGW6MI6 - 1.0 RGW6MI6	= 0	
TGW7MI7	1.0 PGW7MI7 + 1.0 QGW7MI7 - 1.0 RGW7MI7	= 0	
TGWOMIO	1.0 PGWOMIO + 1.0 QGWOMIO - 1.0 RGWOMIO	= 0	

Figure 22. Constraints for water diversion requirements for municipal and industrial use.

Constraint Name	Constraint		Explanations and Comments
AGREQ1	1.0 RLSW1AG1 + 1.0 RGW1AG1	≥ 124.0	These inequalities show the constraints on water to meet the diversion requirements for agricultural use. The RHS is the 1965 agriculture demand shown earlier.
AGREQ2	1.0 RLSW2AG2 + 1.0 RGW2AG2 + 1.0 PLSW3AG2	≥ 1034.0	
AGREQ3	1.0 RLSW3AG3 + 1.0 RGW3AG3	≥ 643.4	
AGREQ4	1.0 RLSW4AG4 + 1.0 RGW4AG4	≥ 796.7	
AGREQ5	1.0 RLSW5AG5 + 1.0 RGW5AG5 + 1.0 PLSW8AG5	≥ 1017.9	
AGREQ6	1.0 RLSW6AG6 + 1.0 RGW6AG6	≥ 300.0	
AGREQ7	1.0 RLSW7AG7 + 1.0 RGW7AG7	≥ 789.1	
AGREQ8	1.0 RLSW8AG8	≥ 303.0	
AGREQ9	1.0 RLSW9AG9 + 1.0 PLSW5AG9	≥ 150.0	
AGREQ0	1.0 RLSW0AG0 + 1.0 RGW0AG0	≥ 68.0	
TLSW1AG1	1.0 PLSW1AG1 + 1.0 QLSW1AG1 - 1.0 RLSW1AG1	= 0	These equations sum the diversion from present developments to agriculture from local surface water with the new development diversions to get the total diversions to agriculture from local surface water.
TLSW2AG2	1.0 PLSW2AG2 + 1.0 QLSW2AG2 - 1.0 RLSW2AG2	= 0	
TLSW3AG3	1.0 PLSW3AG3 + 1.0 QLSW3AG3 - 1.0 RLSW3AG3	= 0	
TLSW4AG4	1.0 PLSW4AG4 + 1.0 QLSW4AG4 - 1.0 RLSW4AG4	= 0	
TLSW5AG5	1.0 PLSW5AG5 + 1.0 QLSW5AG5 - 1.0 RLSW5AG5	= 0	
TLSW6AG6	1.0 PLSW6AG6 + 1.0 QLSW6AG6 - 1.0 RLSW6AG6	= 0	
TLSW7AG7	1.0 PLSW7AG7 + 1.0 QLSW7AG7 - 1.0 RLSW7AG7	= 0	
TLSW8AG8	1.0 PLSW8AG8 + 1.0 QLSW8AG8 - 1.0 RLSW8AG8	= 0	
TLSW9AG9	1.0 PLSW9AG9 + 1.0 QLSW9AG9 - 1.0 RLSW9AG9	= 0	
TLSW0AG0	1.0 PLSW0AG0 + 1.0 QLSW0AG0 - 1.0 RLSW0AG0	= 0	
TGW1AG1	1.0 PGW1AG1 + 1.0 QGW1AG1 - 1.0 RGW1AG1	= 0	These equations sum the diversion from present developments to agriculture from groundwater with the new development diversions to get the total diversions to agriculture from groundwater.
TGW2AG2	1.0 PGW2AG2 + 1.0 QGW2AG2 - 1.0 RGW2AG2	= 0	
TGW3AG3	1.0 PGW3AG3 + 1.0 QGW3AG3 - 1.0 RGW3AG3	= 0	
TGW4AG4	1.0 PGW4AG4 + 1.0 QGW4AG4 - 1.0 RGW4AG4	= 0	
TGW5AG5	1.0 PGW5AG5 + 1.0 QGW5AG5 - 1.0 RGW5AG5	= 0	
TGW6AG6	1.0 PGW6AG6 + 1.0 QGW6AG6 - 1.0 RGW6AG6	= 0	
TGW7AG7	1.0 PGW7AG7 + 1.0 QGW7AG7 - 1.0 RGW7AG7	= 0	
TGW0AG0	1.0 PGW0AG0 + 1.0 QGW0AG0 - 1.0 RGW0AG0	= 0	
AGEXC3	1.0 QAG3LSW3 + 1.0 QAG3GW3	= 0	These equations are for use in transferring excess water from agriculture where these depletions reduce with time.
AGEXC4	1.0 QAG4LSW4 + 1.0 QAG4GW4	= 0	
AGEXC8	1.0 QAG8LSW8	= 0	

Figure 23. Constraints for water diversion requirements for agricultural use.

Constraint Name	Constraint	Explanation and Comments
WLREQ1	1.0 QLSW1WL1 + 1.0 QCGW1WL1 + 1.0 QFGW1WL1 = 713.8	These equations show the constraint on water to meet the depletion requirement for wetland use. The RHS is the 1965 wetland demand shown earlier.
WLREQ2	1.0 QLSW2WL2 + 1.0 QCGW2WL2 + 1.0 QFGW2WL2 = 240.0	
WLREQ3	1.0 QLSW3WL3 + 1.0 QCGW3WL3 + 1.0 QFGW3WL3 = 143.1	
WLREQ4	1.0 QLSW4WL4 + 1.0 QCGW4WL4 + 1.0 QFGW4WL4 = 350.0	
WLREQ5	1.0 QLSW5WL5 + 1.0 QCGW5WL5 + 1.0 QFGW5WL5 = 332.6	
WLREQ6	1.0 QLSW6WL6 + 1.0 QCGW6WL6 + 1.0 QFGW6WL6 = 126.1	
WLREQ7	1.0 QLSW7WL7 + 1.0 QCGW7WL7 + 1.0 QFGW7WL7 = 315.0	
WLREQ8	1.0 QLSW8WL8 = 36.0	
WLREQ9	1.0 QLSW9WL9 = 8.0	
WLREQ0	1.0 QLSW0WL0 + 1.0 QCGW0WL0 + 1.0 QFGW0WL0 = 19.0	

Figure 24. Constraints for water depletion requirements for wetland use

Constraint Name	Constraint	Explanation and Comments
DREQ1	1.0 RLSW1AG1 + 1.0 RLSW1MI1 + 1.0 PLSW1MI4 - 1.0 QLSW2SW1 - 0.0 QWW1LSW1 - 0.1 QAR1LSW1 - 1.0 QDREQ1 = 0	These equations calculate the amount of draft required from water in storage reservoirs. Provision is made to include a portion of the M&I waste water return flow and agriculture return flow in the equation. This portion of the return flow is that which is available for re-use downstream. The coefficient for M&I return flow was estimated to be zero since the geographic location of the major cities and towns indicated negligible re-use of waste water downstream. The coefficient for agriculture return flow was estimated from an examination of the present relationship between draft and storage.
DREQ2	1.0 RLSW2AG2 + 1.0 RLSW2MI2 + 1.0 QLSW2SW1 + 1.0 QLSW2SW3 - 0.0 QWW2LSW2 - 0.62 QAR2LSW2 - 1.0 QDREQ2 = 0	
DREQ3	1.0 RLSW3AG3 + 1.0 RLSW3MI3 + 1.0 PLSW3AG2 + 1.0 RLSW3SW4 - 1.0 QLSW2SW3 - 1.0 QUILSW3 - 0.0 QWW3LSW3 - 0.56 QAR3LSW3 - 1.0 QDREQ3 = 0	
DREQ4	1.0 RLSW4AG4 + 1.0 RLSW4MI4 + 1.25 QLSW4SW4 - 1.0 RLSW3SW4 - 1.0 QBULSW4 - 1.0 QUILSW4 - 1.0 QSALSW4 - 1.0 PLSW7SW4 - 0.0 QWW4LSW4 - 0.49 QAR4LSW4 - 1.0 QDREQ4 = 0	
DREQ5	1.0 RLSW5AG5 + 1.0 RLSW5MI5 + 1.0 QLSW5SW6 + 1.0 PLSW5AG9 - 1.0 QLSW4SW5 - 1.0 QBULSW5 - 1.0 QUILSW5 - 1.0 QSALSW5 - 0.0 QWW5LSW5 - 0.75 QAR5LSW5 - 1.0 QDREQ5 = 0	
DREQ6	1.0 RLSW6AG6 + 1.0 RLSW6MI6 - 1.0 QLSW5SW6 - 1.0 RLSW0SW6 - 0.0 QWW6LSW6 - 0.4 QAR6LSW6 - 1.0 QDREQ6 = 0	
DREQ7	1.0 RLSW7AG7 + 1.0 RLSW7MI7 + 1.0 PLSW7SW4 + 1.0 QBUMPT + 1.0 QUIMPT - 0.0 QWW7LSW7 - 0.1 QAR7LSW7 - 1.0 QDREQ7 = 0	
DREQ8	1.0 RLSW8AG8 + 1.0 RLSW8MI8 + 1.0 PLSW8AG5 + 1.0 QSAMPT - 0.0 QWW8LSW8 - 0.1 QAR8LSW8 - 1.0 QDREQ8 = 0	
DREQ9	1.0 RLSW9AG9 + 1.0 RLSW9MI9 - 0.0 QWW9LSW9 - 0.1 QAR9LSW9 - 1.0 QDREQ9 = 0	
DREQ0	1.0 RLSW0AG0 + 1.0 RLSW0MI0 + 1.0 RLSW0SW6 - 0.0 QWW0LSW0 - 0.2 QAR0LSW0 - 1.0 QDREQ0 = 0	

Figure 25. Constraints for reservoir draft requirements.

Constraint Name	Constraint	Explanation and Comments
GRID1	123. D11 + 7. D12 + 30. D13 + 30. D14 - 1.0 QDREQ1 - 1.0 QREC1 = 0	These equations calculate the amount of storage required as function of the draft required. The draft-storage relationship is highly non-linear and these equations represent the approximation for the separable programming algorithm in the MPS 360.
LSW1ST1	0. D11 + 10. D12 + 80. D13 + 190. D14 - 1.0 RLSW1ST1 = 0	
GRID2	596. D21 + 138. D22 + 137. D23 + 138. D24 - 1.0 QDREQ2 - 1.0 QREC2 = 0	
LSW2ST2	0. D21 + 300. D22 + 880. D23 + 1140. D24 - 1.0 RLSW2ST2 = 0	
GRID3	435. D31 + 93. D32 + 99. D33 + 99. D34 - 1.0 QDREQ3 - 1.0 QREC3 = 0	
LSW3ST3	0. D31 + 240. D32 + 690. D33 + 870. D34 - 1.0 RLSW3ST3 = 0	
GRID4	382. D41 + 66. D42 + 84. D43 + 84. D44 - 1.0 QDREQ4 - 1.0 QREC4 = 0	
LSW4ST4	0. D41 + 130. D42 + 340. D43 + 710. D44 - 1.0 RLSW4ST4 = 0	
GRID5	262. D51 + 71. D52 + 63. D53 + 62. D54 - 1.0 QDREQ5 - 1.0 QREC5 = 0	
LSW5ST5	0. D51 + 220. D52 + 430. D53 + 510. D54 - 1.0 RLSW5ST5 = 0	
GRID6	48. D61 + 16. D62 + 12. D63 + 12. D64 - 1.0 QDREQ6 - 1.0 QREC6 = 0	
LSW6ST6	0. D61 + 26. D62 + 38. D63 + 94. D64 - 1.0 RLSW6ST6 = 0	
GRID7	870. D71 + 185. D72 + 198. D73 + 198. D74 - 1.0 QDREQ7 - 1.0 QREC7 = 0	
LSW7ST7	0. D71 + 320. D72 + 600. D73 + 1280. D74 - 1.0 RLSW7ST7 = 0	
GRID8	394. D81 + 126. D82 + 98. D83 + 97. D84 - 1.0 QDREQ8 - 1.0 QREC8 = 0	
LSW8ST8	0. D81 + 200. D82 + 300. D83 + 710. D84 - 1.0 RLSW8ST8 = 0	
GRID9	272. D91 + 72. D92 + 65. D93 + 64. D94 - 1.0 QDREQ9 - 1.0 QREC9 = 0	
LSW9ST9	0. D91 + 120. D92 + 150. D93 + 280. D94 - 1.0 RLSW9ST9 = 0	
GRID0	160. D01 + 40. D02 + 38. D03 + 37. D04 - 1.0 QDREQ0 - 1.0 QREC0 = 0	
LSW0ST0	0. D01 + 75. D02 + 100. D03 + 285. D04 - 1.0 RLSW0ST0 = 0	
TST1	1.0 PLSW1ST1 + 1.0 QLSW1ST1 - 1.0 RLSW1ST1 = 0	These equations sum the present developed storage and new development of storage to get the total storage.
TST2	1.0 PLSW2ST2 + 1.0 QLSW2ST2 - 1.0 RLSW2ST2 = 0	
TST3	1.0 PLSW3ST3 + 1.0 QLSW3ST3 - 1.0 RLSW3ST3 = 0	
TST4	1.0 PLSW4ST4 + 1.0 QLSW4ST4 - 1.0 RLSW4ST4 = 0	
TST5	1.0 PLSW5ST5 + 1.0 QLSW5ST5 - 1.0 RLSW5ST5 = 0	
TST6	1.0 PLSW6ST6 + 1.0 QLSW6ST6 - 1.0 RLSW6ST6 = 0	
TST7	1.0 PLSW7ST7 + 1.0 QLSW7ST7 - 1.0 RLSW7ST7 = 0	
TST8	1.0 PLSW8ST8 + 1.0 QLSW8ST8 - 1.0 RLSW8ST8 = 0	
TST9	1.0 PLSW9ST9 + 1.0 QLSW9ST9 - 1.0 RLSW9ST9 = 0	
TST0	1.0 PLSW0ST0 + 1.0 QLSW0ST0 - 1.0 RLSW0ST0 = 0	

a) Probability of 0.75.

Constraint Name	Constraint	Explanation and Comments
GRID1	96. D11 + 34. D12 + 30. D13 + 30. D14 - 1.0 QDREQ1 - 1.0 QREC1 = 0	
LSW1ST1	0 D11 + 80. D12 + 120. D13 + 170. D14 - 1.0 RLSW1ST1 = 0	
GRID2	500. D21 + 234. D22 + 137. D23 + 138. D24 - 1.0 QDREQ2 - 1.0 QREC2 = 0	
LSW2ST2	0 D21 + 660. D22 + 1000. D23 + 1140. D24 - 1.0 RLSW2ST2 = 0	
GRID3	330. D31 + 198. D32 + 99. D33 + 99. D34 - 1.0 QDREQ3 - 1.0 QREC3 = 0	
LSW3ST3	0 D31 + 570. D32 + 690. D33 + 940. D34 - 1.0 RLSW3ST3 = 0	
GRID4	320. D41 + 128. D42 + 84. D43 + 84. D44 - 1.0 QDREQ4 - 1.0 QREC4 = 0	
LSW4ST4	0 D41 + 450. D42 + 450. D43 + 710. D44 - 1.0 RLSW4ST4 = 0	
GRID5	242. D51 + 91. D52 + 63. D53 + 62. D54 - 1.0 QDREQ5 - 1.0 QREC5 = 0	
LSW5ST5	0 D51 + 480. D52 + 450. D53 + 520. D54 - 1.0 RLSW5ST5 = 0	
GRID6	44. D61 + 20. D62 + 12. D63 + 12. D64 - 1.0 QDREQ6 - 1.0 QREC6 = 0	
LSW6ST6	0 D61 + 60. D62 + 68. D63 + 107. D64 - 1.0 RLSW6ST6 = 0	
GRID7	730. D71 + 325. D72 + 198. D73 + 198. D74 - 1.0 QDREQ7 - 1.0 QREC7 = 0	
LSW7ST7	0 D71 + 650. D72 + 820. D73 + 1350. D74 - 1.0 RLSW7ST7 = 0	
GRID8	340. D81 + 180. D82 + 98. D83 + 97. D84 - 1.0 QDREQ8 - 1.0 QREC8 = 0	
LSW8ST8	0 D81 + 430. D82 + 450. D83 + 750. D84 - 1.0 RLSW8ST8 = 0	
GRID9	228. D91 + 116. D92 + 65. D93 + 64. D94 - 1.0 QDREQ9 - 1.0 QREC9 = 0	
LSW9ST9	0 D91 + 220. D92 + 185. D93 + 345. D94 - 1.0 RLSW9ST9 = 0	
GRID0	127. D01 + 73. D02 + 38. D03 + 37. D04 - 1.0 QDREQ0 - 1.0 QREC0 = 0	
LSW0ST0	0 D01 + 130. D02 + 150. D03 + 295. D04 - 1.0 RLSW0ST0 = 0	

b) Probability of 0.95.

Figure 26. Constraints for water storage requirements.

Constraint Name	Constraint	Explanations and Comments
EVLSW1	0.070 RLSW1ST1 - 1.0 QLSW1EV1 = 0	These equations calculate the amount of evaporation loss from the major reservoirs (except Bear and Utah lakes) as function of the reservoir storage. In HSU 2 and 4 the evaporation loss-storage relationship is highly non-linear and is calculated using the separable programming algorithm of MPS 360.
EVLSW2	0.50 QEV2 - 1.0 QLSW2EV2 = 0	
EVLSW3	0.02257 RLSW3ST3 - 1.0 QLSW3EV3 = 0	
EVLSW4	0.50 QEV4 - 1.0 QLSW4EV4 = 0	
EVLSW5	0.0934 RLSW5ST5 - 1.0 QLSW5EV5 = 0	
EVLSW6	0.0525 RLSW6ST6 - 1.0 QLSW6EV6 = 0	
EVLSW7	0.028 RLSW7ST7 - 1.0 QLSW7EV7 = 0	
EVLSW8	0.045 RLSW8ST8 - 1.0 QLSW8EV8 = 0	
EVLSW9	0.070 RLSW9ST9 - 1.0 QLSW9EV9 = 0	
EVLSW0	0.070 RLSW0ST0 - 1.0 QLSW0EV0 = 0	
EVGW1	0.0 RLSW1ST1 - 1.0 QGW1EV1 = 0	
EVGW2	0.5 QEV2 - 1.0 QGW2EV2 = 0	
EVGW3	0.02257 RLSW3ST3 - 1.0 QGW3EV3 = 0	
EVGW4	0.5 QEV4 - 1.0 QGW4EV4 = 0	
EVGW5	0.0311 RLSW5ST5 - 1.0 QGW5EV5 = 0	
EVGW6	0.0175 RLSW6ST6 - 1.0 QGW6EV6 = 0	
EVGW7	0.0 RLSW7ST7 - 1.0 QGW7EV7 = 0	
EVGW0	0.0 RLSW0ST0 - 1.0 QGW0EV0 = 0	
EV2ST2	208.0 E21 + 103.0 E22 + 1500.0 E23 - 1.0 RLSW2ST2 = 0	These equations calculate the amount of evaporation loss as function of storage in HSU 2 and 4.
EV2	0.0 E21 + 3.0 E22 + 105.0 E23 - 1.0 QEV2 = 0	
EV4ST4	220.0 E41 + 196.0 E42 + 1500.0 E43 - 1.0 RLSW4ST4 = 0	
EV4	0.0 E41 + 25.5 E42 + 105.0 E43 - 1.0 QEV4 = 0	

Figure 27. Constraints for net evaporation loss from reservoirs (other than Bear and Utah Lakes).

Constraint Name	Constraint	Explanations and Comments
WWRF1	.7000 RLSW1MI1 + .7000 RGW1MI1 - 1.0 QWW1LSW1 - 1.0 QWW1R1 = 0	These equations calculate the amount of waste water return flow from municipal and industrial uses. The return flow can go either to local surface water or ground water depending upon economics and need. The non-unity coefficients are called the return flow coefficients.
WWRF2	.6600 RLSW2MI2 + .6600 RGW2MI2 - 1.0 QWW2LSW2 - 1.0 QWW2R2 = 0	
WWRF3	.4366 RLSW3MI3 + .4366 RGW3MI3 - 1.0 QWW3LSW3 - 1.0 QWW3R3 = 0	
WWRF4	.6889 RLSW4MI4 + .6889 RGW4MI4 + .6889 PLSW1MI4 - 1.0 QWW4LSW4 - 1.0 QWW4R4 - 1.0 QWW4RU4 = 0	
WWRF5	.4588 RLSW5MI5 + .4588 RGW5MI5 - 1.0 QWW5LSW5 - 1.0 QWW5R5 - 1.0 QWW5RU5 = 0	
WWRF6	.6923 RLSW6MI6 + .6923 RGW6MI6 - 1.0 QWW6LSW6 - 1.0 QWW6R6 = 0	
WWRF7	.6500 RLSW7MI7 + .6500 RGW7MI7 - 1.0 QWW7LSW7 - 1.0 QWW7R7 = 0	
WWRF8	.3000 RLSW8MI8 - 1.0 QWW8LSW8 = 0	
WWRF9	.2500 RLSW9MI9 - 1.0 QWW9LSW9 = 0	
WWRF0	.3000 RLSW0MI0 + .3000 RGW0MI0 - 1.0 QWW0LSW0 - 1.0 QWW0R0 = 0	

Figure 28. Constraints for waste water return flow from municipal and industrial use.

Constraint Name	Constraint		Explanation and Comments
AGRFSW1	.4742 RLSW1AG1 + .4742 RGW1AG1 - 1.0 QAR1LSW1	= 0	These equations calculate the amount of agriculture return flow that goes to local surface water. The non-unity coefficient is called the return flow coefficient to surface water.
AGRFSW2	.6077 RLSW2AG2 + .6077 RGW2AG2 + .6077 PLSW3AG2 - 1.0 QAR2LSW2	= 0	
AGRFSW3	.5833 RLSW3AG3 + .5833 RGW3AG3 - 1.0 QAR3LSW3	= 0	
AGRFSW4	.5609 RLSW4AG4 + .5609 RGW4AG4 - 1.0 QAR4LSW4	= 0	
AGRFSW5	.6250 RLSW5AG5 + .6250 RGS5AG5 + .6250 PLSW8AG5 - 1.0 QAR5LSW5	= 0	
AGRFSW6	.4947 RLSW6AG6 + .4947 RGW6AG6 - 1.0 QAR6LSW6	= 0	
AGRFSW7	.6288 RLSW7AG7 + .6288 RGW7AG7 - 1.0 QAR7LSW7	= 0	
AGRFSW8	.6250 RLSW8AG8 - 1.0 QAR8LSW8	= 0	
AGRFSW9	.8000 RLSW9AG9 + .8000 PLSW5AG9 - 1.0 QAR9LSW9	= 0	
AGRFSWO	.5000 RLSWOAGO + .5000 RGWOAGO - 1.0 QAROLSWO	= 0	
AGRFGW1	.0500 RLSW1AG1 + .0500 RGW1AG1 - 1.0 QAR1GW1	= 0	These equations calculate the amount of agriculture return flow that goes to groundwater. The non-unity coefficient is called the return flow coefficient to groundwater.
AGRFGW2	.0500 RLSW2AG2 + .0500 RGW2AG2 + .0500 PLSW3AG2 - 1.0 QAR2GW2	= 0	
AGRFGW3	.0500 RLSW3AG3 + .0500 RGW3AG3 - 1.0 QAR3GW3	= 0	
AGRFGW4	.0500 RLSW4AG4 + .0500 RGW4AG4 - 1.0 QAR4GW4	= 0	
AGRFGW5	.0500 RLSW5AG5 + .0500 RGW5AG5 + .0500 PLSW8AG5 - 1.0 QAR5GW5	= 0	
AGRFGW6	.0500 RLSW6AG6 + .0500 RGW6AG6 - 1.0 QAR6GW6	= 0	
AGRFGW7	.0000 RLSW7AG7 + .0000 RGW7AG7 - 1.0 QAR7GW7	= 0	
AGRFGWO	.0000 RLSWOAGO + .0000 RGWOAGO - 1.0 QAROGWO	= 0	

Figure 29. Constraints for return flow from agricultural use.

Constraint Name	Constraint		Explanations and Comments
FGWAVWL1	1.0 QFGW1WL1 - 0.50 QAR1GW1	= 162.20	These equations calculate the amount of groundwater that is used from natural sources by wetlands. These sources are; 1) the groundwater that returns to the surface in the wetlands by natural conditions and 2) the groundwater which is available for wetland consumption which had as its source the agriculture return flow to the groundwater. The coefficient of 0.50 for the return flow and the RHS were estimated using present conditions based on water budgets and accounting for groundwater outflow.
FGWAVWL2	1.0 QFGW2WL2 - 0.50 QAR2GW2	= 95.85	
FGWAVWL3	1.0 QFGW3WL3 - 0.50 QAR3GW3	= 19.535	
FGWAVWL4	1.0 QFGW4WL4 - 0.50 QAR4GW4	= 56.065	
FGWAVWL5	1.0 QFGW5WL5 - 0.50 QAR5GW5	= 158.09	
FGWAVWL6	1.0 QFGW6WL6 - 0.50 QAR6GW6	= 27.32	
FGWAVWL7	1.0 QFGW7WL7 - 0.50 QAR7GW7	= 0.00	
FGWAVWLO	1.0 QFGWOWLO	= 10.00	

Figure 30. Constraints for free groundwater for wetlands.

Constraint Name	Constraint		Explanations and Comments
GWRC1	1.0 QLSW1R1 + 1.0 QWW1R1	≤ 0.0	These inequalities show the con-constraint on groundwater recharge. The RHS was estimated from geologic and hydrologic considerations discussed earlier in this report.
GWRC2	1.0 QLSW2R2 + 1.0 QWW2R2	≤ 60.0	
GWRC3	1.0 QLSW3R3 + 1.0 QWW3R3	≤ 366.0	
GWRC4	1.0 QLSW4R4 + 1.0 QWW4R4	≤ 434.0	
GWRCU4	1.0 QLSW4RU4 + 1.0 QWW4RU4	≤ 100.0	
GWRC5	1.0 QLSW5R5 + 1.0 QWW5R5	≤ 52.0	
GWRCU5	1.0 QLSW5RU5 + 1.0 QWW5RU5	≤ 52.0	
GWRC6	1.0 QLSW6R6 + 1.0 QWW6R6	≤ 65.0	
GWRC7	1.0 QLSW7R7 + 1.0 QWW7R7	≤ 0.0	
GWRCO	1.0 QLSWORO + 1.0 QWWORO	≤ 0.0	

Figure 31. Constraints for groundwater artificial recharge limits.

Constraint Name	Constraint		Explanation and Comments
BUMPT	1.0 QBULSW4 + 1.25 QBULSW5 - 1.0 QBUMPT	= 0	These equations calculate the total water imported to the Great Basin from each of the three sources in the CUP. The 1.25 coefficient accounts for transport losses.
UIMPT	1.0 QUILSW3 + 1.0 QUILSW4 + 1.25 QUILSW5 - 1.0 QUIMPT	= 0	
SAMPT	1.0 QSALSW4 + 1.0 QSALSW5 - 1.0 QSAMPT	= 0	
TLSW3SW4	1.0 PLSW3SW4 + 1.0 QLSW3SW4 - 1.0 RLSW3SW4	= 0	These equations show the constraint on inter-basin transfer in those basins presently having some transfer.
TLSWOSW6	1.0 PLSWOSW6 + 1.0 QLSWOSW6 - 1.0 RLSWOSW6	= 0	

Figure 32. Constraints for inter-basin transfer limits.

Constraint Name	Constraint		Explanation and Comments
INFLOGSL	1.0 QLSW1OF1 + 1.0 QLSW2OF2 + 1.0 QLSW3OF3 + 1.0 QLSW4OF4 + 1.0 QGW1OF1 + 1.0 QGW2OF2 + 1.0 QGW3OF3 + 1.0 QGW4OF4	≥ 201.0	This inequality shows the constraint on total inflow to the Great Salt Lake. The RHS will change depending upon the ground rules for the particular run being made. The number 201.0 is simply the sum of the individual minimum inflows.
CROUT	1.0 QLSW7OF7 + 1.0 QLSW8OF8 + 1.0 QLSW9OF9 + 1.0 QGW7OF7	≥ 907.0	This inequality shows the constraint on the Colorado River water which is allocated to Utah from the Upper Basin Compact. The RHS was calculated as shown in the text.

Figure 33. Constraints for inflow and outflow limits.

Average values to represent approximate 1965 conditions were used in the model. Bounds on new development were taken from Utah Division of Water Resources data and from consultation with Bureau of Reclamation personnel associated with the Central Utah Project. New development bounds are shown in Table 11.

Additional surface water storage

These bounds were established from data supplied by the Utah Division of Water Resources, the Pacific Southwest Inter-Agency Committee, and from studies conducted at the Utah Water Research Laboratory. The results from combining the various data are shown in Table 12.

Surface and groundwater outflow

These bounds were established from a consideration of minimum river flow to achieve a salt balance and on studies made at UWRL. The bounds are shown in Table 13.

Table 11. Variable bounds on new inter-basin transfers.

Variable	Bound (ac-ft/yr)	Type of Bound
QBULSW5	29,000	Upper
QBUMP5	136,600	Upper
QUILSW3	20,000	Upper
QUILSW5	57,000	Upper
QUIMPT	420,000	Upper
QSALSW4	15,000	Upper
QSAMPT	22,400	Upper
QLSW2SW1	90,000	Upper
QLSW2SW3	130,000	Upper
QLSW3SW4	146,000	Upper
QLSW4SW5	69,000	Upper
QLSW5SW6	60,000	Upper
QLSWOSW6	47,000	Upper

Source: Utah Division of Water Resources, 1970.

Table 12. Variable bounds on additional surface water storage.

Variable	Bound (ac-ft/yr)	Type of Bound
QLSW1ST1	25,000	Upper
QLSW2ST2	1,200,000	Upper
QLSW3ST3	125,000	Upper
QLSW4ST4	1,050,000	Upper
QLSW5ST5	125,000	Upper
QLSW6ST6	100,000	Upper
QLSW7ST7	1,500,000	Upper
QLSW8ST8	285,000	Upper
QLSW9ST9	140,000	Upper
QLSWOSTO	280,000	Upper

Source: Utah State University - Utah Water and Power Board, 1963; Water Resources Work Group, 1971.

Table 13. Variable bounds on surface and groundwater outflow.

Variable	Bound (ac-ft/yr)	Type of Bound
QLSW1OF1	7,000	Lower
QLSW2OF2	50,000	Lower
QLSW3OF3	50,000	Lower
QLSW4OF4	50,000	Lower
QLSW5OF5	13,700	Lower
QLSW6OF6	0.0	Lower
QLSW7OF7	100,000	Lower
QLSW8OF8	100,000	Lower
QLSW9OF9	100,000	Lower
QLSWOOFO	100,000	Lower
QGW1OF1	6,000	Lower
QGW2OF2	5,000	Lower
QGW3OF3	25,000	Lower
QGW4OF4	8,000	Lower
QGW5OF5	0.0	Lower
QGW6OF6	0.0	Lower
QGW7OF7	40,000	Lower
QGWOOFO	0.0	Lower

REGIONAL SUPPLY FUNCTIONS

Economic Background

In economic terms for a firm, supply may be defined as a schedule relating product prices and quantities which a firm is willing and able to produce during some time period. In a resource market, the definition of supply is a schedule relating resource prices and quantities which a resource owner is willing and able to supply in the market during some time period.

In a purely competitive market, marginal analysis shows that profit maximization for a firm in the short run occurs when the marginal revenue equals the marginal cost. Since in pure competition the marginal revenue equals the product price, the firm short run supply curve can be further defined as that portion of the marginal cost curve which lies above its average variable cost curve. The industry supply curve then is simply a horizontal summation (sum on quantity) of all the individual firm supply curves.

When dealing with the supply and demands for natural resources such as water one must recognize that competitive firms are not actually supplying the water. Federal agencies, such as the Bureau of Reclamation, who are the builders of a given water project usually are coordinating with state organizations, such as water districts, who distribute and sell the water. These agencies then are the resource owners and supply water to municipalities and farmers. In an economic sense they are more analogous to monopolies than to firms in a purely competitive market. Under these circumstances the supply curve or function is defined as the cost borne by the suppliers of the water when they make available various quantities of water for various purposes.

The "primal" problem of linear programming as used herein is a problem in resource allocation, i.e. the allocation of scarce resources (water) to meet certain requirements imposed by the model for AG diversions, M&I diversions, and wetland depletions at minimum cost. The corresponding "dual" problem is a problem in resource valuation, i.e. the change in the objective function (cost or shadow price) for each unit change in resource or requirement. Many authors have discussed the parallelism and inseparability of these two problems. See for example Dorfman, Samuelson, and Solow, 1958, Chapter 7. Linear programming can be used to generate the shadow prices of the resources and the requirements from the solution of the dual. The shadow prices of the requirements (AG and M&I) represent the additional cost which must be borne to supply each unit increase in the requirement. If other things are held constant and the requirement is varied from zero to the maximum possible, a schedule of shadow price is generated which represents the supply curve or function for that requirement. Use is made of a technique in linear programming which will vary or parameterize a right-hand-side of any constraint. The IBM MPS360 computer package has this capability. This technique was used to generate supply functions for agricultural water use when holding municipal and industrial use constant. As the RHS for the agricultural use requirement increases, the computer determines the optimum activities and shadow prices each time the optimum set of variables changes. The supply curve thus generated consists of flat segments of constant shadow price connected by vertical steps at each change. When the RHS for the municipal and industrial requirement is varied parametrically, a similar function is generated. The two data combined on a single plot with AG requirement as the abscissa and M&I requirement as the ordinate showed shadow prices constant over an area within the graph. The data thus presented is called a supply function map.

The cost to supply water for Ag use and for M&I use are the two major interests. Supply function maps for each can be generated by plotting the respective shadow prices as described.

Another shadow price of interest is that associated with new imported water. This shadow price represents the value of the new imported water which is its worth in reducing the total cost of supply. This shadow price is generated as part of the solution discussed above and likewise can be represented in a supply function map.

The maps thus generated for the ten HSU of the state indicate what would be the marginal costs of supplying water at any level of development represented by the combined values of the AG and M&I diversion requirements. If one were examining the shadow price of AG, one would view the map as representing what it would cost to supply an additional ac-ft of water for AG at a given level of M&I development. Shadow prices for M&I development are viewed likewise. Thus these maps may be thought of as development maps. Particularly is this concept realistic when diversions are above the present level of development. The supply function maps for new imported water become significant in showing when it is efficient to construct new importation facilities as development increases.

Model Definition

The development of a model for use in each of the ten HSU of the state is accomplished by disaggregating the statewide model discussed in the preceding chapter. The following ground rules are applicable:

1. Water availability for use in each of the HSU is the in-basin availability listed on Table 8. This includes: 1) excess precipitation on irrigated croplands for period October to April, 2) net evaporation loss from large lakes, and 3) direct use of groundwater by croplands.
2. Present (1965) inter-basin transfers and diversions are fixed at the given levels as shown on Table 8 with costs shown as O&M on Table 9. No new inter-basin transfers are allowed.
3. Present (1965) diversions for agricultural use and for municipal and industrial use are available at the cost shown as O&M on Table 9. New diversions are allowed at the costs shown on Table 9.
4. Present (1965) surface water storage is fixed at the levels given on Table 8 with costs shown as O&M on Table 9. This necessitates maintaining storage with its subsequent evaporation loss even though the storage draft may not require it. New storage is allowed up to the limits shown on Table 12 at the costs shown on Table 9.
5. Probability of having sufficient surface water storage is 0.75.
6. Wetlands depletions are fixed at the present (1965) levels given on Table 8.
7. Minimum outflow from each HSU is given by the data shown on Table 13.
8. No groundwater mining is allowed as is presently the case in HSU 5 and 6. Artificial recharge is limited to that given by the data shown on Table 7.
9. The determination of the shadow price or value of any new imported water is accomplished by introducing ten dummy variables QMPTX(X = 1,2, ..., 0). A fixed bound of zero is placed on these variables. The optimal solution thus reflects a condition of zero new imported water but the value of this water is calculated by the solution of the dual problem.

Results from the Models

Supply models were developed for each of the ten HSU of the state. Ground rules as defined in the preceding paragraph formed the basis for the models, and parametric linear programming was used to develop the supply function maps. Maps (for each of the HSU) are shown in Appendix C and are in six parts:

Part a. Agricultural development map. This graph shows how increasing the agricultural diversions is accomplished under the assumption of minimum cost. The arrow from each boundary indicates the direction of increasing diversion starting from zero at the boundary.

Part b. Shadow price of agricultural diversions. Areas within the development map are shown for constant shadow price in dollars per ac-ft diverted.

Part c. Municipal and industrial development map. This graph shows how increasing the municipal and industrial diversions is accomplished under the assumption of minimum cost. The arrow from each boundary indicates the direction of increasing diversion starting from zero at the boundary.

Part d. Shadow price of municipal and industrial diversions. Areas of the development map are shown for constant shadow price in dollars per ac-ft diverted.

Part e. Development map for surface storage and limiting conditions. This graph shows how increasing development introduces requirements for groundwater recharge and for new surface storage. The arrow from each boundary indicates the direction of increasing recharge or new storage starting from zero at the boundary. The outermost diagonal line from upper left to lower right is the limit of further development. This limit can be reached either due to requirements of minimum outflow or to the upper bound on new storage which can be constructed.

Part f. Shadow price of imported water. Areas within the development map are shown for constant shadow price in dollars per ac-ft imported.

General and specific comments about the results of the generation of the supply function maps for each of the HSU are made in the following paragraphs.

General comments

The assumption is made in this study that minimum cost is the criterion which determines the order that various facilities are utilized as AG or M&I diversions increase. For example, suppose it is cheaper to supply water to AG initially using facilities already developed for local surface water. The model will reflect the minimum cost assumption by showing the use of presently developed surface water facilities starting at zero diversion on the development map. As the AG diversions increase, the model will show use of more and more of the presently developed surface water facilities until either: 1) all of these facilities are required, or 2) some other source of water is cheaper, or 3) no further development can occur due to limiting conditions such as storage or outflow requirements.

Several general comments are made which apply to HSU, such as:

1. Present local surface water developed for municipal and industrial (M&I) purposes is

used completely before presently developed groundwater is used.

2. Present groundwater developed for M&I purposes is used completely before any new water is developed.
3. Where groundwater is available, new groundwater is developed for M&I use before new surface water is developed.
4. In HSU 7 and 10 no new groundwater is developed for either agricultural (AG) use or M&I use.
5. Present local surface water developed for AG purposes is used before presently developed groundwater is used.

Specific comments

Some comments are not generally applicable to all HSU. The figure number shown in parenthesis identifies the source of the comment.

HSU 1.

1. Most of the presently developed or old surface water for AG is used before old groundwater starts to be used (Figure C-1a).
2. New groundwater is developed for AG use before new surface water when M&I diversions are less than their present levels (Figure C-1a).
3. New surface water is developed for AG use when M&I diversions are above their present levels. No new groundwater is developed (Figure C-1a).
4. New surface water is developed for AG to replace old groundwater when M&I diversions are above about twice their present levels (Figure C-1a).
5. Very little further AG development can be made and this at the expense of reducing present M&I diversions. Cost of AG diversion increases from $1.00 to $5.16 per ac-ft/yr. No new storage is required (Figure C-1b).
6. A tremendous increase in M&I development (up to about 14-fold) can be made with a cost increase of M&I diversion from $41.85 to $65.20 per ac-ft/yr without new storage. New storage is required for maximum M&I development but only for about the upper 10 percent when AG diversions are less than about 75 percent present diversions (Figure C-1d).
7. Maximum development is limited by the minimum outflow requirements at high AG diversions while the maximum new storage limits development at high M&I diversions (Figure C-1e).
8. Imported water has a value of zero over most of the development map except at high AG diversion requirements and when new storage is required (Figure C-1f).

HSU 2.

1. Old groundwater for AG use is used before new surface water or new groundwater is developed when M&I diversions are below their present levels. From present M&I levels up to about three times present levels, new surface water is developed conjunctively with the use of old groundwater. Above about three times present M&I levels, new surface water is the only development for AG use (Figure C-2a).
2. For M&I diversions below present levels, new groundwater is developed for AG use before new surface water; while above present levels. new surface water is developed for AG use before new groundwater (Figure C-2a).
3. A moderate increase in AG development (up to about 30 percent) can be made for a cost increase of AG diversion from $1.75 to $13.21 per ac-ft/yr for no new storage. New storage is required for maximum AG development over about the last 15 percent of AG diversion (Figure C-2b).
4. A tremendous increase in M&I development (up to about 18-fold) can be made with a cost increase of M&I diversion from $41.16 to $64.16 per ac-ft/yr without new storage. New storage is required for maximum M&I development but only for about the last 15 percent of M&I diversion when AG diversions are low. This increases to about the last 40 percent when AG diversions are at their present levels (Figure C-2d).
5. M&I waste water is used to recharge the groundwater aquifer when M&I diversions exceed about their present levels. This applies for AG diversions up to about 120 percent their present levels. For AG diversions above this level, local surface water replaces waste water to recharge the groundwater aquifer but only for M&I diversion up to about twice their present levels (Figure C-2e).
6. Maximum development is limited by the upper limit on new storage (Figure C-2e).
7. The value of imported water is zero over a substantial portion of the development map. High values occur only when surface water is being recharged and/or when new storage is required (Figure C-2f).

HSU 3. 1. Old groundwater for AG is used before new surface water or new groundwater is developed when M&I diversions are below about three times their present level. From there on up to higher M&I diversion levels, new surface water is developed before the old groundwater for AG is used (Figure C-3a).

2. New groundwater is developed for AG use before new surface water when M&I diversions are below about twice their present level, while the reverse is true above this level (Figure C-3a).

3. A substantial increase in AG development (up to about 100 percent) can be made for a cost increase of AG diversion from $2.00 to $14.33 per ac-ft/yr with no new storage. New storage is required for maximum AG development but only for about the last 2 percent of AG diversion (Figure C-3b).

4. A tremendous increase in M&I development (up to about 18-fold) can be made with a cost increase of M&I diversion from $41.35 to $71.89 per ac-ft/yr with no new storage. New storage is required for maximum M&I development but only for about the last 2 percent of M&I diversion when AG diversions are low. This increases to about 4 percent at the present level of AG diversions (Figure C-3d).

5. M&I waste water is used to recharge the groundwater aquifer starting almost immediately after the present development is exceeded (Figure C-3e).

6. Local surface water is used to recharge the groundwater aquifer anytime the sum of the two diversions exceeds about 800,000 ac-ft/yr (Figure C-3e).

7. Maximum development is limited by upper limit on new storage (Figure C-3e).

8. The value of imported water is zero over a substantial portion of the development map. Moderate values occur only when surface water is being recharged. High values occur when new storage is required (Figure C-3f).

HSU 4. 1. Old groundwater for AG is used before new surface water or new groundwater is developed when M&I diversions are below about their present levels. From there on up to higher M&I diversion levels, new surface water is developed before old groundwater is used or new groundwater is developed for AG use (Figure C-4a).

2. New groundwater is developed for AG use before new surface water when M&I diversions are lower than their present levels, while the reverse is true above this level (Figure C-4a).

3. A moderate increase in AG development (up to about 75 percent) can be made for a cost increase of AG diversion from $2.75 to $15.52 per ac-ft/yr with no new storage (Figure C-4b).

4. A substantial increase in M&I development (up to about 370 percent) can be made with a cost increase of M&I diversion from $47.91 to $76.91 per ac-ft/yr with no new storage. New storage is required for maximum M&I development but only for about the last 5 percent of M&I diversion when AG diversions are lower than present levels (Figure C-4d).

5. M&I waste water is used to recharge the groundwater aquifer over about the upper 1/2 to 3/4 of the M&I development area. Low-cost recharge is used entirely before high-cost recharge is used (Figure C-4e).

6. Local surface water is used to recharge the groundwater aquifer at the extreme upper area of AG development when M&I development is lower than the present level (Figure C-4e).

7. Maximum development is limited by the minimum outflow requirements (Figure C-4e).

8. The value of imported water is zero over a substantial portion of the development map. High values occur only at the extreme ends of the development area when either AG or M&I diversions are high (Figure C-4f).

HSU 5. 1. Considerable mining of groundwater for AG uses in the past has developed facilities to the extent that no new surface water or groundwater development for AG is required. Maximum cost for AG is $9.89 per ac-ft/yr except for a very minor area when new storage is required for maximum M&I development (Figure C-5a and b).

2. A tremendous increase in M&I development (up to about 30-fold) can be made with a cost increase of M&I diversion from $35.47 to $63.93 per ac-ft/yr with no new storage (Figure C-5d).

3. M&I waste water is used to recharge the groundwater aquifer over about the upper 3/4 of the M&I development area. Low-cost recharge is used entirely, then high-cost recharge is used entirely. After the maximum recharge is achieved, the waste water from additional M&I development is returned to the local surface water (Figure C-5e).

4. Local surface water is used to recharge the groundwater aquifer at the extreme upper end of the AG development range (Figure C-5e).

5. Maximum development is limited by the minimum outflow requirements (Figure C-5e).

6. Imported water has a value of zero over almost all the development map except for high AG diversion requirements (Figure C-5f).

HSU 6 1 Old groundwater is used for AG before old surface water when M&I diversions are less than about 3½ times their present levels, while the reverse is true when M&I diversions are above this level (Figure C-6a).

2. Considerable mining of groundwater for AG in the past has developed facilities to the extent that no new surface water or groundwater development for AG is required. Maximum cost for AG is $4.28 per ac-ft/yr except for a very minor area when new storage is required for maximum M&I development (Figure C-6a and b).

3 A tremendous increase in M&I development (up to about 20-fold) can be made with a cost increase of M&I diversion from $40.38 to $70.12 per ac-ft/yr with no new storage (Figure C-6d).

4 M&I waste water is used to recharge the groundwater aquifer over about the upper half of the M&I development area (Figure C-6e).

5 Groundwater is pumped to supply wetlands in the roughly triangular area in the lower left corner of the development map bounded by a M&I diversion of about 45,000 ac-ft/yr and an AG diversion of about 75,000 ac-ft/yr (Figure C-6e).

6. Maximum development is limited by the minimum outflow requirements (Figure C-6e).

7 Imported water is valuable over about the lower 1/3 of the M&I development map. Above this area it has a value of zero except for a very small area near maximum M&I development where new storage is required (Figure C-6f).

HSU 7 1 Old surface water for AG is used entirely before new surface water is developed (Figure C-7a).

2 A moderate increase in AG development (up to about 30 percent) can be made with a cost increase of AG diversion from $.75 to $5.25 per ac-ft/yr for no new storage (Figure C-7b).

3 A tremendous increase in M&I development (up to about 100-fold) can be made with a cost increase of M&I diversion from $34.90 to $68.90 per ac-ft/yr. If the present AG development is maintained, the M&I development can be increased up to about 20-fold at the same cost increase for no new storage (Figure C-7d).

4. A substantial increase in development can be made without new storage however new storage is required for maximum development. Maximum development is limited by the upper bound on new storage (Figure C-7e).

5. The value of imported water is zero over much of the development map. Imported water only becomes of value when new storage is required (Figure C-7f).

HSU 8. 1. Old surface water for AG is used entirely before new surface water is developed (Figure C-8a).

2. A substantial increase in AG development (up to about 80 percent) can be made with a cost increase of AG diversion from $.75 to $5.25 per ac-ft/yr for no new storage (Figure C-8b).

3. A tremendous increase in M&I development (up to about 70-fold) can be made with a cost increase of M&I diversion from $29.80 to $74.80 per ac-ft/yr. If present AG development is maintained, the M&I development can be increased up to about 30-fold at the same cost increase for no new storage (Figure C-8d).

4. A substantial increase in development can be made without new storage, however new storage is required for maximum development. Maximum development is limited by the upper bound on new storage (Figure C-8e).

5. The value of imported water is zero over much of the development map. It only becomes of value when new storage is required (Figure C-8f).

HSU 9. 1. Old surface water for AG is used entirely before new surface water is developed for AG (Figure C-9a).

2. A substantial increase in AG development (up to about 2-fold) can be made with a cost increase of AG diversion from $.75 to $5.25 per ac-ft/yr with no new storage (Figure C-9b).

3. A tremendous increase in M&I development (up to about 40-fold) can be made with a cost increase of M&I diversion from $26.50 to $65.50 per ac-ft/yr. If the present AG development is maintained, then M&I development can be

increased up to about 20-fold at the same cost increase with no new storage (Figure C-9d).

4. A substantial increase in development can be made without new storage, however new storage is required for maximum development. Maximum development is limited by the upper bound on new storage (Figure C-9e).

5. The value of imported water is zero over much of the development map. It only becomes of value when new storage is required (Figure C-9f).

HSU 10. 1. Old surface water for AG is used entirely before new surface water is developed for AG (Figure C-10a).

2. A substantial increase in AG development (up to about 3-fold) can be made with a cost increase of AG diversion from $.75 to $5.25 per ac-ft/yr with no new storage (Figure C-10b).

3. A tremendous increase in M&I development (up to about 100-fold) can be made with a cost increase of M&I diversion from $27.80 to $66.80 per ac-ft/yr. If the present AG development is maintained, then M&I development can be increased about 70-fold at this same cost increase with no new storage (Figure C-10d).

4. A substantial increase in development can be made without new storage, however new storage is required for maximum development. Maximum development is limited by the minimum outflow requirements (Figure C-10e).

5. The value of imported water is zero over much of the development map. It only becomes a value when new storage is required (Figure C-10f).

RESULTS FROM THE STATEWIDE ALLOCATION MODEL

Results from the model can be classified in three general categories: 1) those which are available as part of the optimum solution to the linear-programming problem, 2) those available in a post-optimal analysis, and 3) those which can be obtained only through a manipulation of the structural coefficients, constraint right-hand-side values (RHS), and variable bounds. Included in the first category are the optimal solution and the determination of the shadow prices of the various resources. Included in the second category are the results of the sensitivity analysis of the cost coefficients and the parametric analysis of the right-hand-side In the third category are included the effect of changing irrigation efficiency, and effect of various policies such as groundwater restrictions, interbasin transfer limitations, changing growth projections with time, etc.

Computer print-outs of the control cards and data cards are shown in Tables A-1 and A-2 of Appendix A. The example includes the necessary control cards and data cards to systematically vary (or parameterize) the right-hand-side. The parameterized RHS values are the estimated values as time passes from the year 1965 to the year 2020. This 55 year time interval was divided into 5.5 year increments. The symbol θ (Theta) is the time parameter and takes values between 0 and 10. Thus the optimum allocation can be found for the year 1965 ($\theta = 0$) and at each 5.5 year time interval thereafter to the year 2020 ($\theta = 10$). A computer print-out of the optimum allocation for 1965 is also shown in Table A-3 of Appendix A.

Results from the Optimal Solution

Solution to the linear-programming problem consists of several parts including the optimum value of the objective function, the optimal activity levels or values of the real and slack variables, and the solution of the dual to the linear-programming problem.

Optimum value of the objective function

The optimum value of the objective function is used primarily to compare the dollar value of one optimum solution with another. In this research, the value represents the minimum annual cost to meet the specified demands for water under a particular set of assumptions. For example, the computer print-out shown in Appendix A lists the optimum value (scaled in thousands) of the objective function as $24726.32817. This solution is based on the water demands for the year 1965 and the assumption is made in the model that groundwater mining is not permitted. Since facilities existing in 1965 are in the model at their O&M cost, the value of the objective function in this case represents the yearly cost of supplying water in the state and of developing new facilities to eliminate groundwater mining in HSU 5 and 6. Cost projections over time are made by examining the changes in the value of the objective function as the right-hand-side values of the demand constraints are changed as shown in a later paragraph.

Optimal allocation

The optimal allocation of water in the state is given by the activity levels or values of the variables in the optimal solution. As an aid in the analysis of the allocation pattern, these activity levels are transferred to flow diagrams as shown in Figure 17. For example, the data from the computer print-out in Appendix A was transferred to the flow diagram shown in Figure 34. As discussed in the previous paragraph, the allocations represent those values of the variables which bring about the minimum cost to meet the water demands for the year 1965 in the state and to develop new facilities to eliminate groundwater mining in HSU 5 and 6. The actual water allocations existing in 1965 are shown on the flow diagram in Figure 18. A comparison of these two flow diagrams shows that the water which is being mined can be replaced by importing additional water from HSU 4, 7, 8, and 10. This imported water together with M&I waste water is used to recharge the groundwater aquifers at an annual rate equal to the present mining rate so that presently existing pumping facilities can be continued. The additional imported water totals about 99,000 ac-ft/yr whereas only about 89,000 ac-ft/yr is presently being mined. An examination of the flow diagram shows that this extra water is released down the Sevier River. The reason for this apparent discrepancy or waste lies in the storage probability. One of the assumptions made in generating the data for the no groundwater mining case was that the probability of having sufficient surface water storage was 0.75. Since the runoff in the Sevier Basin is highly variable from year to year, some of the runoff in high flow years will be lost down the river to the Sevier Dry Lake. The difference between the average outflow of about 14,000 ac-ft/yr under 1965 conditions and the calculated outflow of 24,000 ac-ft/yr under conditions that would eliminate groundwater mining must then represent a difference in the probability of having sufficient storage.

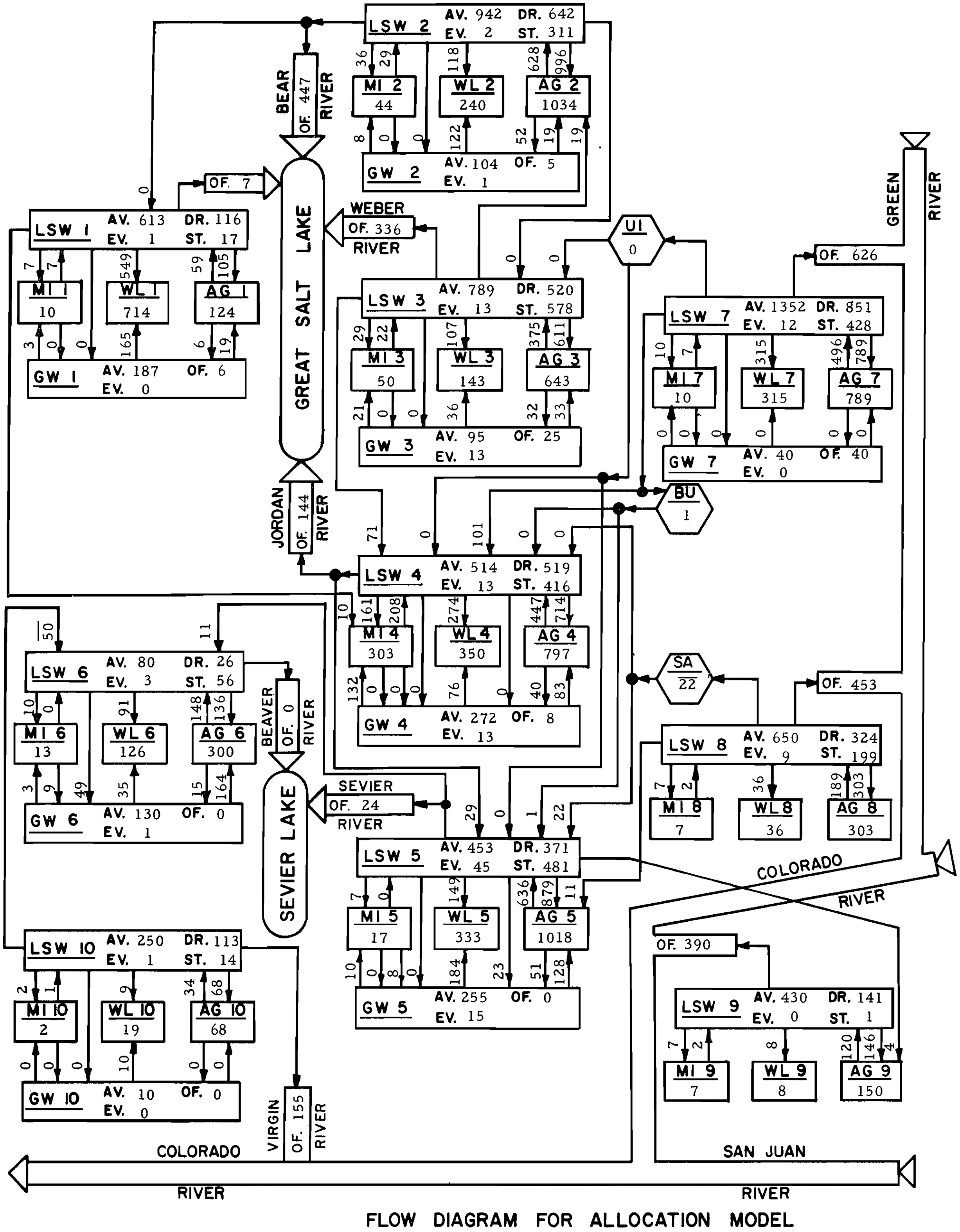


Figure 34. Flow diagram for the basic model (1965).

Resource shadow prices

Resource shadow prices are determined from the solution of the dual of the linear-programming problem. The economic interpretation of the dualism property of linear–programming lies in the concept that resource allocation and pricing are two aspects of the same problem. The dual problem is formulated as follows:

a) Re-structure primal so that all constraints become inequalities in the same sense.
b) Transpose rows and columns of the constraint matrix.
c) Transpose the right-hand-side of constraints with the objective function coefficients.
d) Change the sense of the inequality signs in the constraints.
e) Change the sense of the objective function (e.g. maximize instead of minimize).

The optimal solution to this dual problem gives the values of the dual variables which are referred to as shadow prices and indicates the rate at which costs increase or decrease for a corresponding increase or decrease in the amount of resource given by the right-hand-side value of the resource constraint. These values are listed under the heading ‘dual activity" of the rows section of the computer print-out as shown in Appendix A. For example, the shadow price or value of the resource "Available Surface Water in HSU 6, AVAILSW6" (shown on line 7). is $14.00 per ac-ft/yr. This says that the value of the objective function would change by $14.00 per year if the available surface water in HSU 6 were changed one ac-ft/yr; thus the value of this resource is defined.

Post-Optimal Analysis

Analysis of the linear-programming problem after an optimal solution has been achieved is referred to as post-optimal analysis and consists primarily of two possible phases of analysis; sensitivity analysis and parametric analysis.

Sensitivity analysis

Practical problems formulated in the linear-programming framework are seldom completely "solved" by the optimal solution. The coefficients of the model (objective function coefficients, structural coefficients of the constraint matrix, and constraint right-hand-side values) are seldom known with the desired degree of certainty. Also, the linear relationships assumed for a given problem formulation may not hold in the range indicated by the model solution. Therefore, it is usually desirable to carry out some sort of sensitivity analysis to determine the effect on the optimal solution of changing certain coefficients or constants to other possible values. If such an analysis indicates the optimal solution is very sensitive to small changes in the coefficients or constants, then special care should be taken in checking the values of these coefficients or constants. Thus one of the greatest helps that can come from a sensitivity analysis is the identification of those coefficients or constants which are critical to the solution, thereby reducing the number which must be reexamined. For example, an examination of the sensitivity analysis shown in sections 2 and 4 in Table A-4 of Appendix A reveals three variables for which a change in their related cost coefficients of less than 10 percent would change the allocation pattern. These variables are:

a) QLSW3SW4 (new imported water from HSU 3 to HSU 4)
b) QBULSW5 (water imported to HSU 5 via Bonneville Unit of the Central Utah Project)
c) QLSW4SW5 (new imported water from HSU 4 to HSU 5)

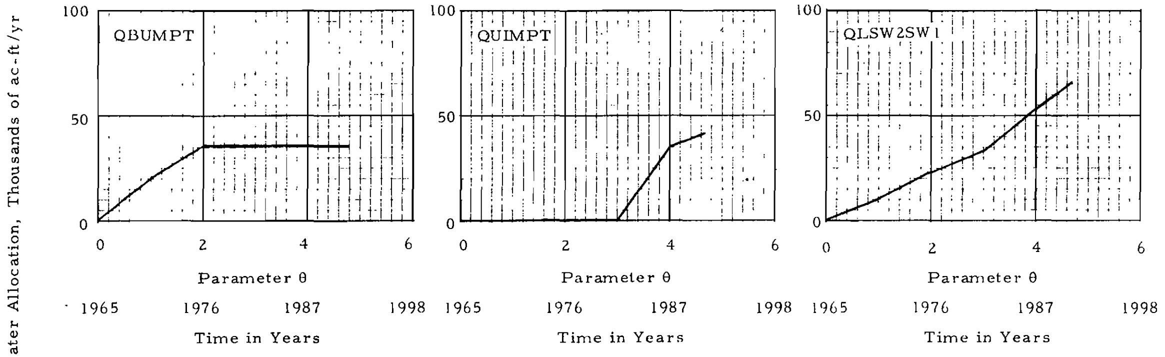
Further examination of the activity range over which the solution is valid for each of these three variables reveals very narrow ranges for each, thus leading to the conclusion that these three variables have critical cost coefficients which should be determined as accurately as possible.

Similar analyses can be made for the constraint right-hand-side values using data from sections 1 and 3 of the sensitivity analysis. Thus the constraint RHS values describing surface water availability, groundwater availability, AG diversion requirements, M&I diversion requirements, wetland requirements, reservoir draft requirements, evaporation loss, return flow, artificial recharge, inter-basin transfer limits, inflow or outflow limits, etc. can be investigated to see which RHS values impose critical limitations on the optional solution. The critical RHS values would deserve careful review and checking. Review of all these possible combinations is beyond the scope of this study.

Parametric analysis

Parametric analysis is a procedure for generating new optimal solutions from an original optimal solution while allowing one or more parameters (constants or coefficients) to vary systematically over a specified range of values. Either the objective function coefficients or the constraint right-hand-side values or both can be varied over a desired range either singularly or in any combination. Use is made of this procedure to vary the right-hand-side values of some of the constraint equations, in particular those showing the demand for water. Thus projections of demand over time can be inserted in the model and new optimal solutions generated quite easily.

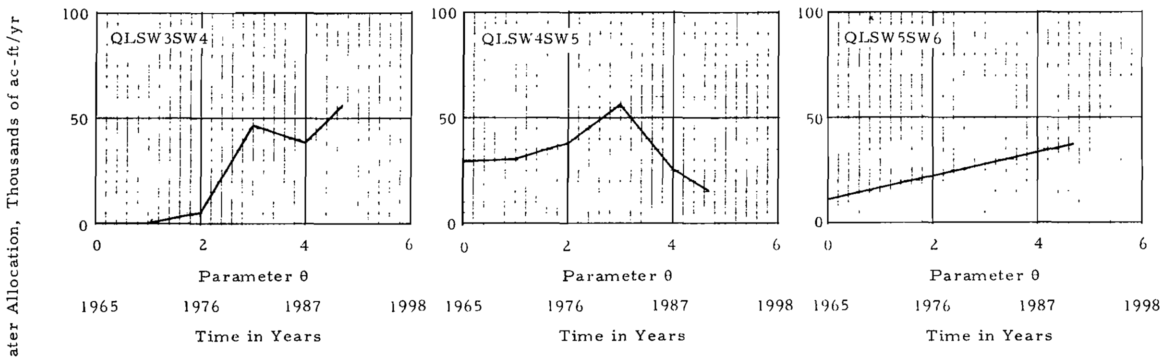
The Division of Water Resources Alternate 1 projections of increasing demand in the future were inserted into the model as changing values with time and the resulting optimal allocations are shown in Figures B-1a through B-1d of Appendix B. Some of the more significant allocation changes of this basic model are plotted versus time (or the parameter θ) in Figure 35. These data show for the assumptions of no groundwater mining and a minimum inflow to the Great Salt Lake of



a) Bonneville Unit Imports

b) Ute Indian Unit Imports

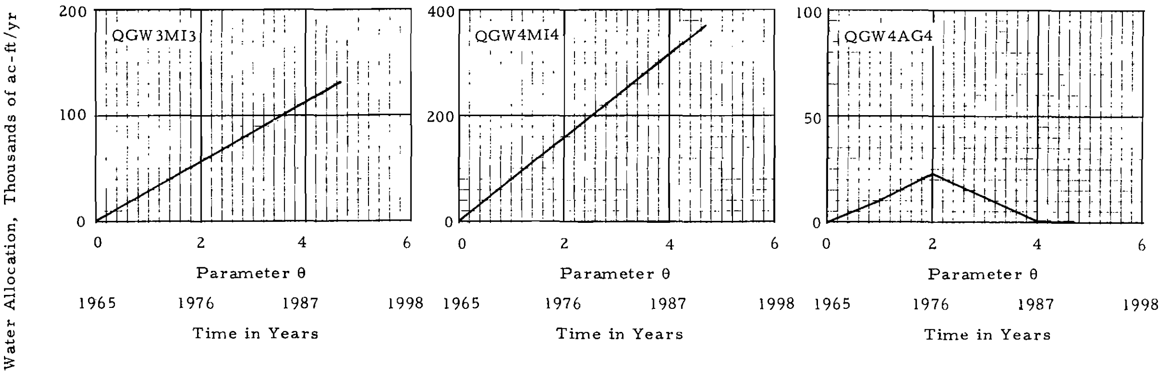
c) Surface Water 2 to Surface Water 1



d) Surface Water 3 to Surface Water 4

e) Surface Water 4 to Surface Water 5

f) Surface Water 5 to Surface Water 6



g) Groundwater 3 to M&I 3

h) Groundwater 4 to M&I 4

i) Groundwater 4 to AG 4

Figure 35. Allocations for the basic model as function of time.

500,000 ac-ft/yr that these activities generally increase as time passes except for QLSW4SW5 (HSU 4 import to HSU 5) and QGW4AG4 (groundwater 4 to AG4). Examination of the data from the computer print-out indicates the reason the computation stopped about the year 1991 (θ = 4.68) instead of continuing to the year 2020 was that the maximum surface water storage was reached in HSU 2. Other significant data from this example are shown in Figure 36. This plot shows the excess water above the minimum required for outflow of the Upper Colorado River drainage and inflow to the Great Salt Lake. As indicated, the excess inflow to the Great Salt Lake above the minimum assumed goes to zero about the year 1991 (θ = 4.68). Over 400,000 ac-ft/yr of water is still available at that time for use from the Upper Colorado River allocation. This indicates further development can take place provided the problem of surface water storage in HSU 2 can be resolved or the demands in HSU 2 are assumed to stop growing. Thus the first place to look for improving the model would be to determine more accurately just what can be done about storage in HSU 2. Since storage in HSU 2 is critical in the solution, the cost should be reexamined to be sure it is accurate. Possibly a non-linear cost relationship could be developed which would be more accurate than the linear approximation.

Other Results

The effects of such things as 1) changing groundwater policy, 2) giving up some present diversion, 3) changing the probability on storage, 4) changing policy on maintaining Great Salt Lake level, 5) changing the limits on inter-basin transfer, 6) changing growth projections, and 7) changing irrigation efficiency can be determined by manipulating the model structural coefficients, right-hand-side values, and variable bounds.

Effect of changing groundwater policy

There are two rather obvious groundwater policy changes that were investigated: 1) no groundwater recharge allowed and 2) no further development of the groundwater allowed. Both policies included the condition of not allowing groundwater mining as presently occurs in HSU 5 and 6. The effect of a policy of no groundwater recharge can be determined by simply setting to zero the right-hand-side values of the recharge constraints shown in Figure 31. The effect of a policy of no additional groundwater development (i.e. no increased pumpage) can be determined by setting zero bounds on the variables representing future groundwater diversions. These policies were combined and the results are plotted in Figures B-2a through B-2c of Appendix B. Some of the more significant data are summarized in Figure 37. A comparison with data from the basic model plotted in Figure 35 shows the Bonneville and Ute Indian units of the Central Utah Project to be required at greater levels earlier in time and to reach considerably higher levels. Imports of water from HSU 3 to HSU 4 and from HSU 4 to HSU 5 showed a reversal of the general trend. The model stopped about 1983 (θ = 3.32) due to the upper limit on new storage development in HSU 7.

Effect of eliminating some present diversions

It may be more efficient to give up some of the presently developed facilities and replace them with larger or different facilities in later years. The effect of this policy can be determined by changing the bounds on the variables representing present development from fixed bounds (which forces the model to keep all present developments) to upper bounds (which allows the model to choose how much of the present development should be kept for minimum cost). The results of this condition are plotted in Figures B-3a through B-3d of Appendix B. A comparison with the data from the basic model shows the only significant difference between the two models is that this new model does not recharge the groundwater in HSU 6 to as high a level but chooses to give up some of the present pumpage.

Effect of changing the probability on storage

It may be desired to determine the effect on the allocation pattern of changing the probability of having sufficient storage to supply the required draft. This effect can be determined by changing the draft-storage relationship coefficients as given in Figure 26. The basic model assumed a probability of 0.75 and used the coefficients from Figure 26a. Coefficients for other probability levels can be determined using the non-linear curves shown in Figures 5 through 14. These coefficients have been determined for a probability of 0.95 and are shown in Figure 26b. The results of assuming a probability of 0.95 are plotted in Figures B-4a through B-4c of Appendix B. A comparison with the data from the basic model shows greatly increased storage is required earlier in HSU 2 and 7. As a result, the model could only go to about the year 1983 (θ = 3.35) before reaching a limit on new storage in HSU 2.

Effect of changing policy of maintaining Great Salt Lake level

Requirements for mineral rights, recreation, and ecological demands may require maintaining the level of Great Salt Lake at some particular elevation. The average inflow to Great Salt Lake from Utah drainage over recent years, has been about 1,014,000 ac-ft/yr. The effect of having some particular inflow requirement can be determined by simply changing the right-hand-side value of the inflow constraint as given in Figure 33. The results of this policy are plotted in Figures B-5a through B-5d for an inflow $\geq$ 201,000 ac-ft/yr, in Figures B-6a through B-6d for an inflow $\geq$ 800,000 ac-ft/yr, and in Figures B-7a through B-7c for an inflow $\geq$ 1,014,000 ac-ft/yr. A

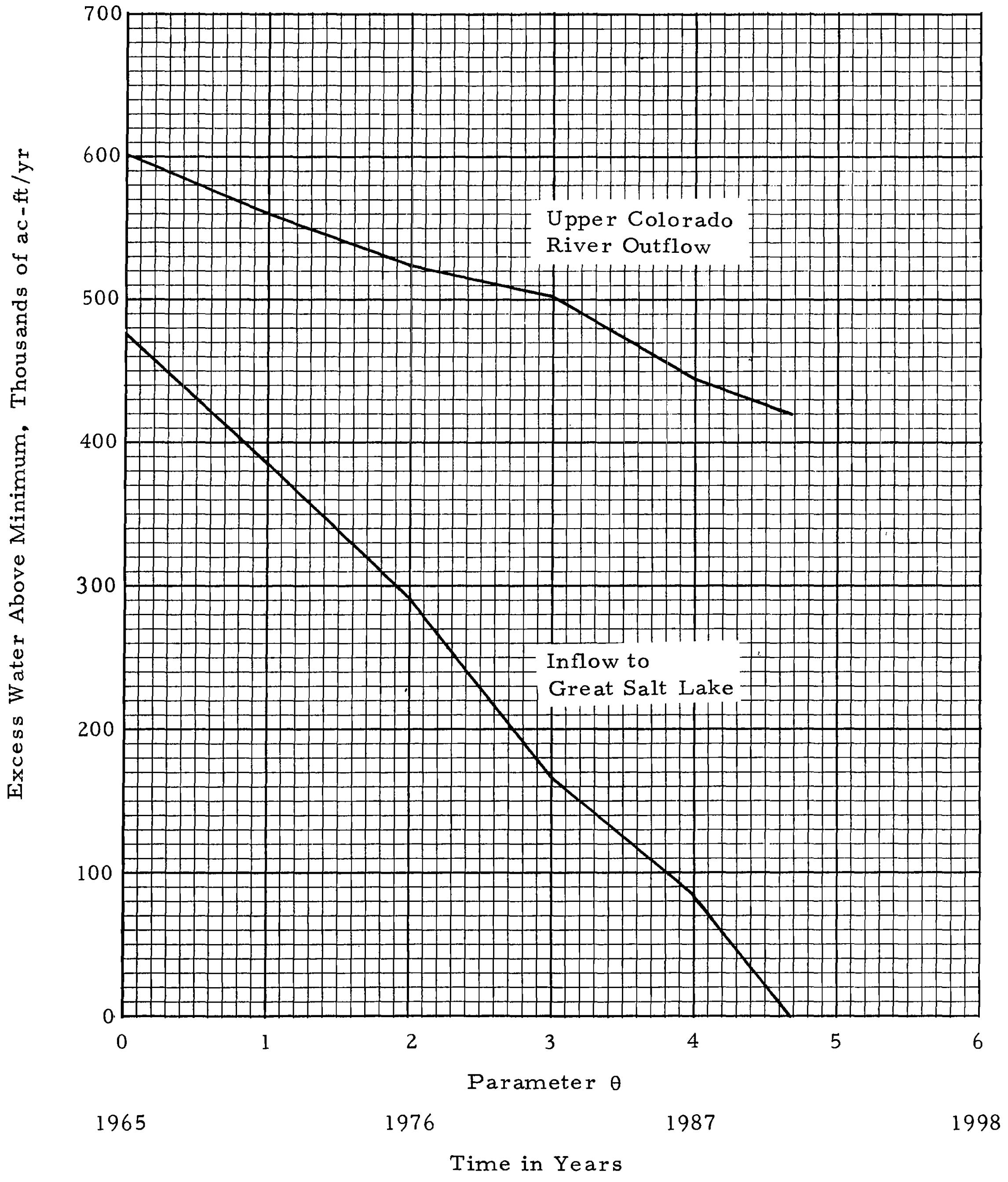
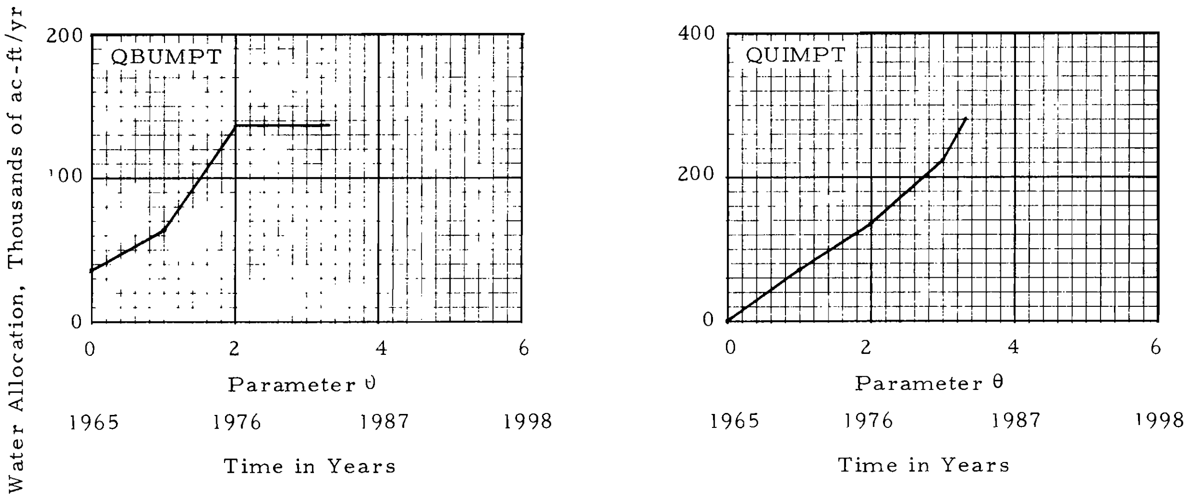


Figure 36. Excess water for the basic model as function of time.

comparison with data from the basic model (which assumes an inflow ≥ 500,000 ac-ft/yr) shows no change from the basic model in early years for the 201,000 ac-ft/yr model. This same condition is observed except shifted later for the 800,000 ac-ft/yr model. The 201,000 ac-ft/yr model stopped at the same time and for the same reason as the basic model. The 800,000 ac-ft/yr model required more import from HSU 7 and stopped at about the year 1990 (θ = 4.59) due to limitations on new storage in HSU 7. Results from the 1,014,000 ac-ft/yr case showed the requirement for greater import from HSU 7 started even earlier than the 800,000 ac-ft/yr inflow model. This computation stopped in about the year 1984 (θ = 3.46) due to limitations on new storage in HSU 7. A comparison of some of the more significant allocations is shown in Figure 38. Notice that the effect of varying inflow on QGW4MI4 (groundwater 4 to M&I 4) is only to stop development at different levels.

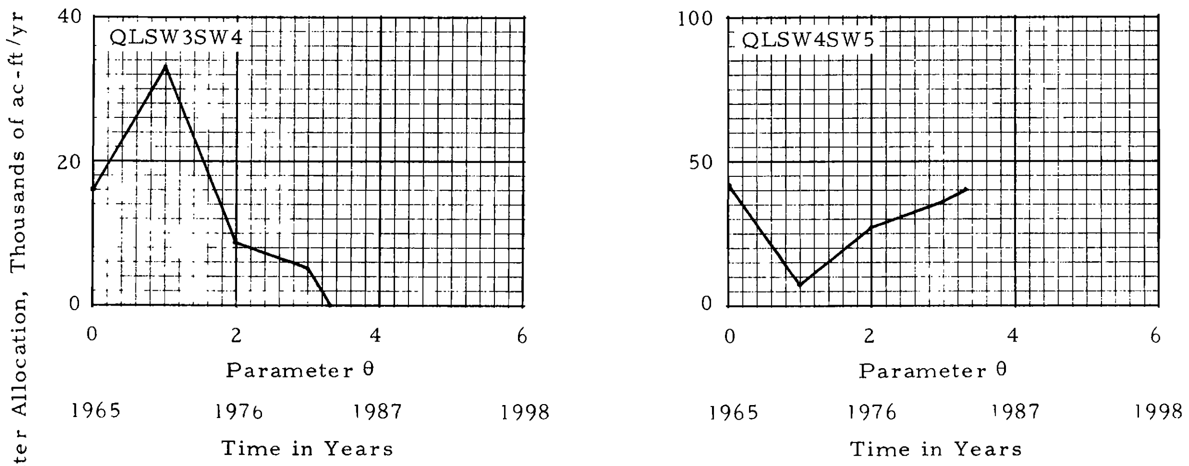
Effect of limitation on inter-basin transfer

There are many limitations on inter-basin transfer which could be examined. One of interest is to assume no further transfer will be allowed from the Upper Colorado River Basin to the Great Basin other than through the Bonneville unit of the Central Utah Project. The effect of this limitation on inter-basin transfer can be determined by setting zero bounds on the two variables representing the other transfers. The results of this condition are plotted in Figures B-8a through B-8d. Some of the more significant data are summarized in Figure 39. A comparison with the data for the basic model shows that Bonneville unit does not reach maximum size before the computation stopped at about 1989 (θ = 4.44) and that more water was exported from HSU 3 to HSU 4 and from HSU 4 to HSU 5 in the latter stages of development. The



a) Bonneville Unit Imports

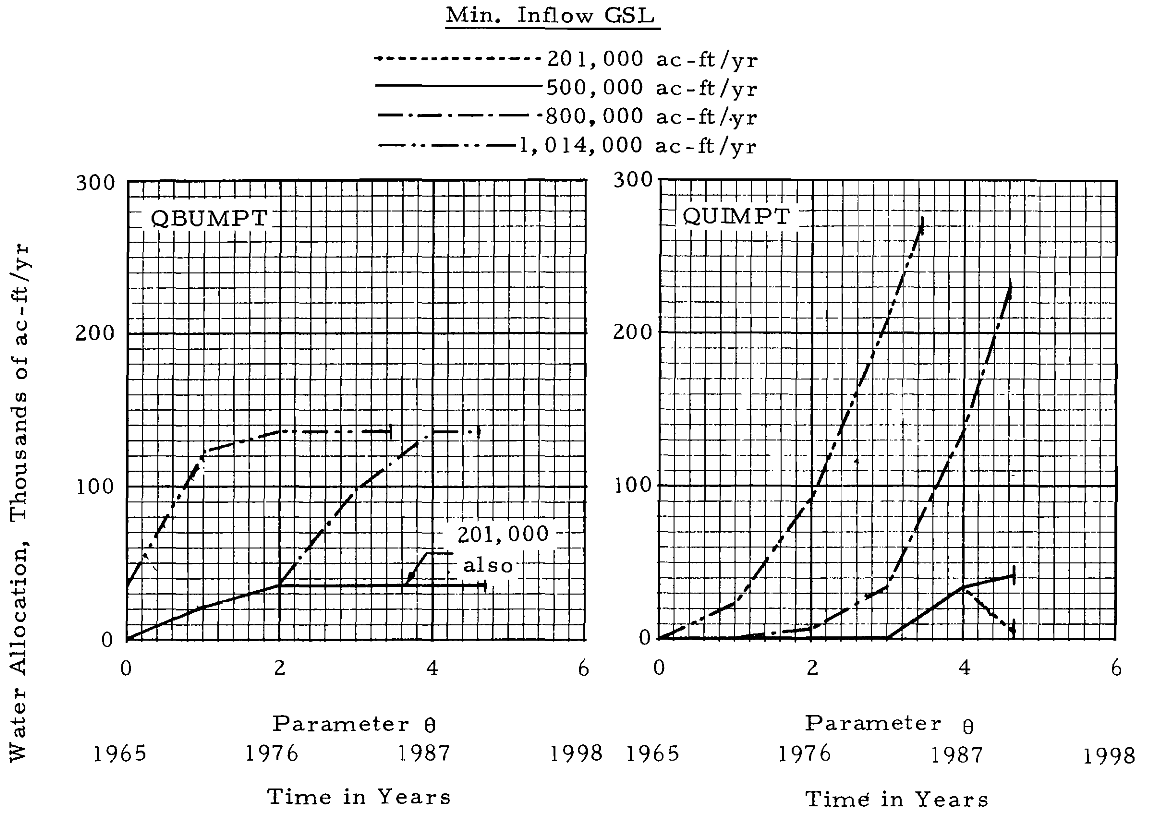
b) Ute Indian Unit Imports



c) Surface Water 3 to Surface Water 4

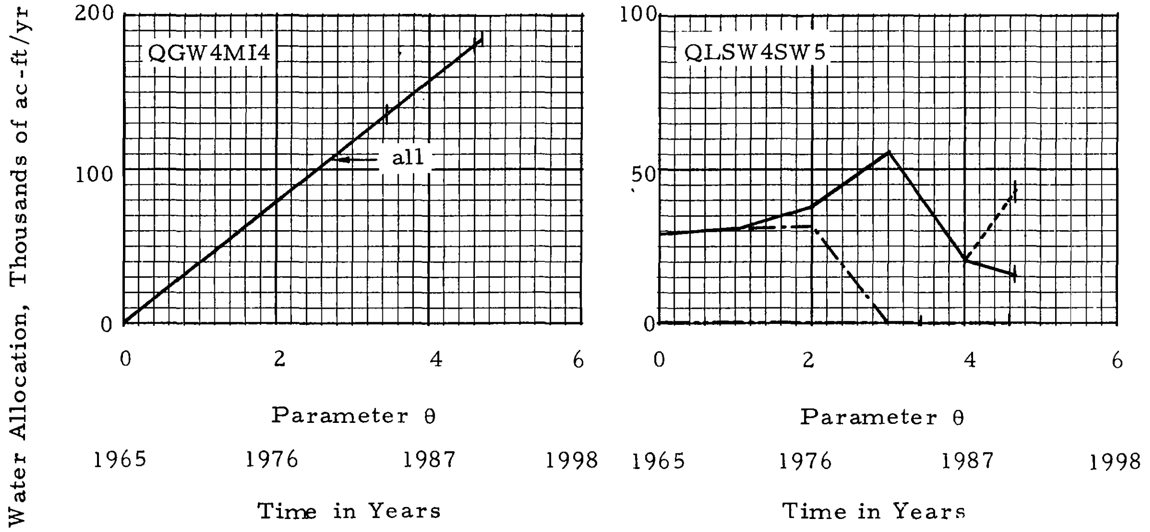
d) Surface Water 4 to Surface Water 5

Figure 37. Allocations for the no groundwater recharge or further development model as function of time.



a) Bonneville Unit Imports

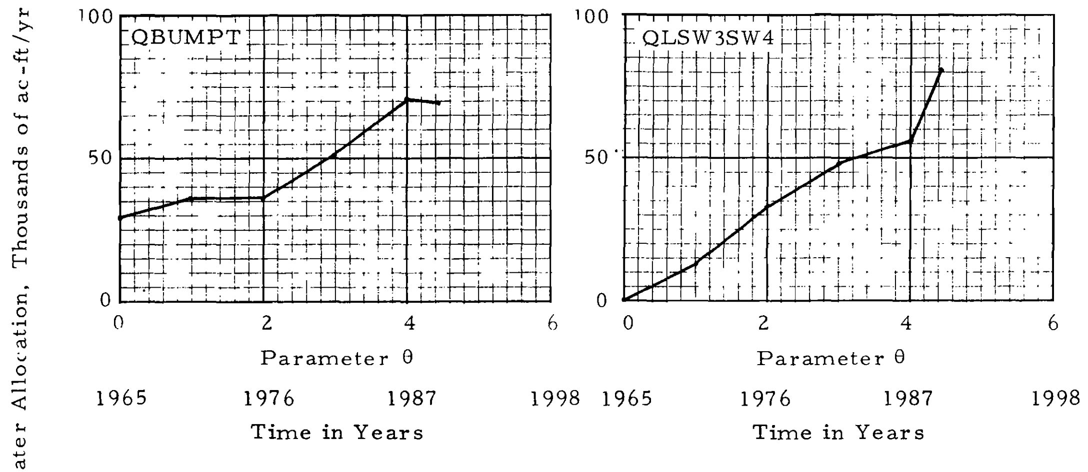
b) Ute Indian Unit Imports



c) Groundwater 4 to M&I 4

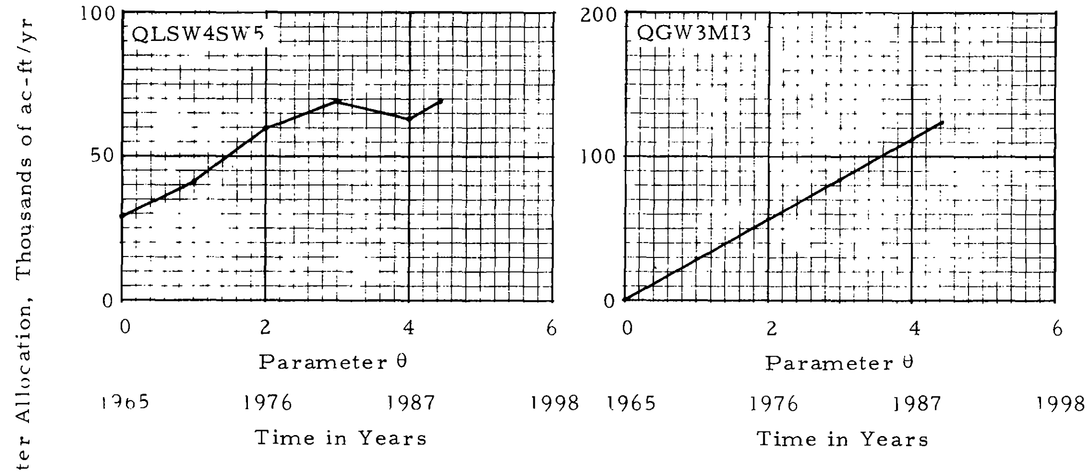
d) Surface Water 4 to Surface Water 5

Figure 38. Allocations as affected by time and inflow to the Great Salt Lake.



a) Bonneville Unit Imports

b) Surface Water 3 to Surface Water 4



c) Surface Water 4 to Surface Water 5

d) Groundwater 3 to M&I 3

Figure 39. Allocation for the Bonneville unit import only model as function of time.

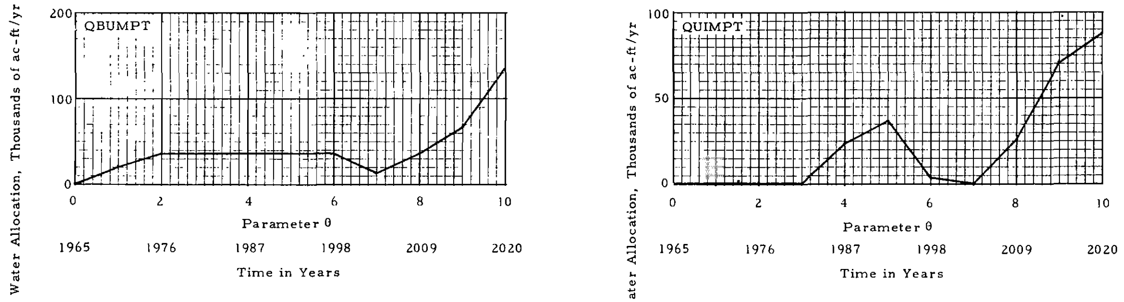
model computation stopped due to reaching an upper limit on imports to HSU 5.

Effect of changing growth projections

The projected growth defined by the Division of Water Resources as Alternate 1 (Utah Division of Water Resources, 1970) is higher for almost all uses than earlier projections made about June 1969. Likewise the Alternate 2, 3, and 4 projections in this same reference are significantly different from Alternate 1 projections and reflect different possibilities of growth and different means to meet the water demands of the growth. The effect of changing the growth projections to those of the earlier estimate of June 1969 can be determined by changing the increments used in parameterizing the right-hand-side of the water demand constraints shown in Figures 22, 23, and 24. The results of this change are plotted in Figures B-9a through B-9f. Some of the more significant allocations are summarized in Figure 40. A comparison of this data with the data for the basic model shows the lower growth projection allowed the computation to run to almost the year 2020 ($\theta = 9.98$). The Bonneville unit reached maximum development at the end, however the Ute Indian Unit did not. The excess water above the minimum assumed for inflow to the Great Salt Lake and the Upper Colorado River is shown in Figure 41. These data show that the inflow to the Great Salt Lake reached its minimum about the year 2009 and that by the year 2020 only about 40,000 ac-ft/yr is left from the Upper Colorado River compact allocation.

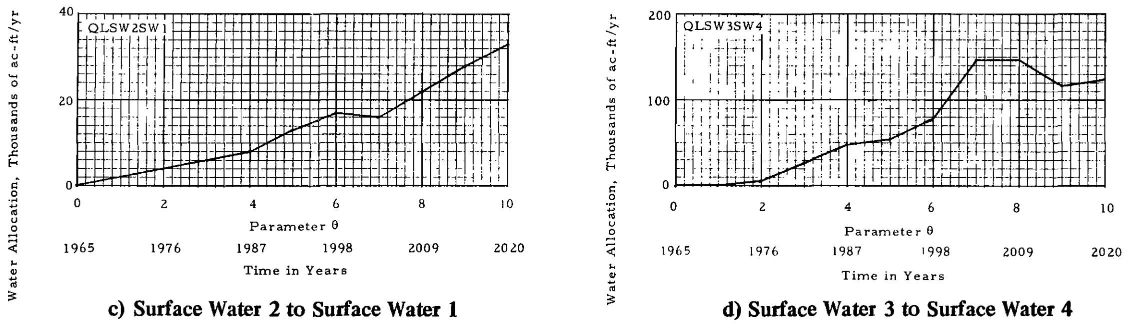
Effect of changing irrigation efficiency

This effect can be determined by changing the agricultural return flow coefficients in the constraints



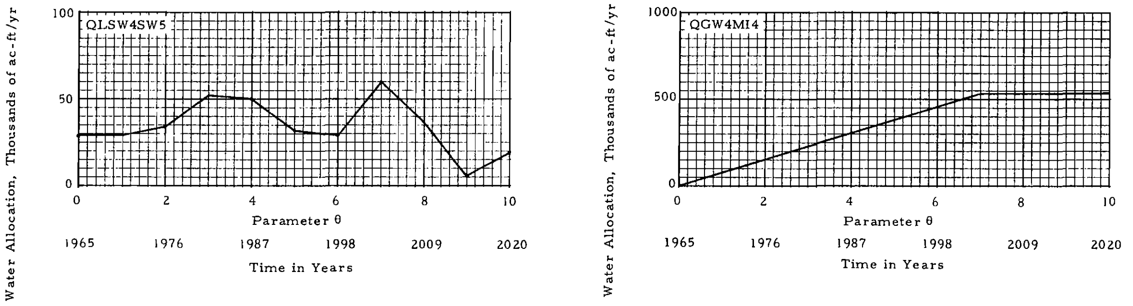
a) Bonneville Unit Imports

b) Ute Indian Unit Imports



c) Surface Water 2 to Surface Water 1

d) Surface Water 3 to Surface Water 4



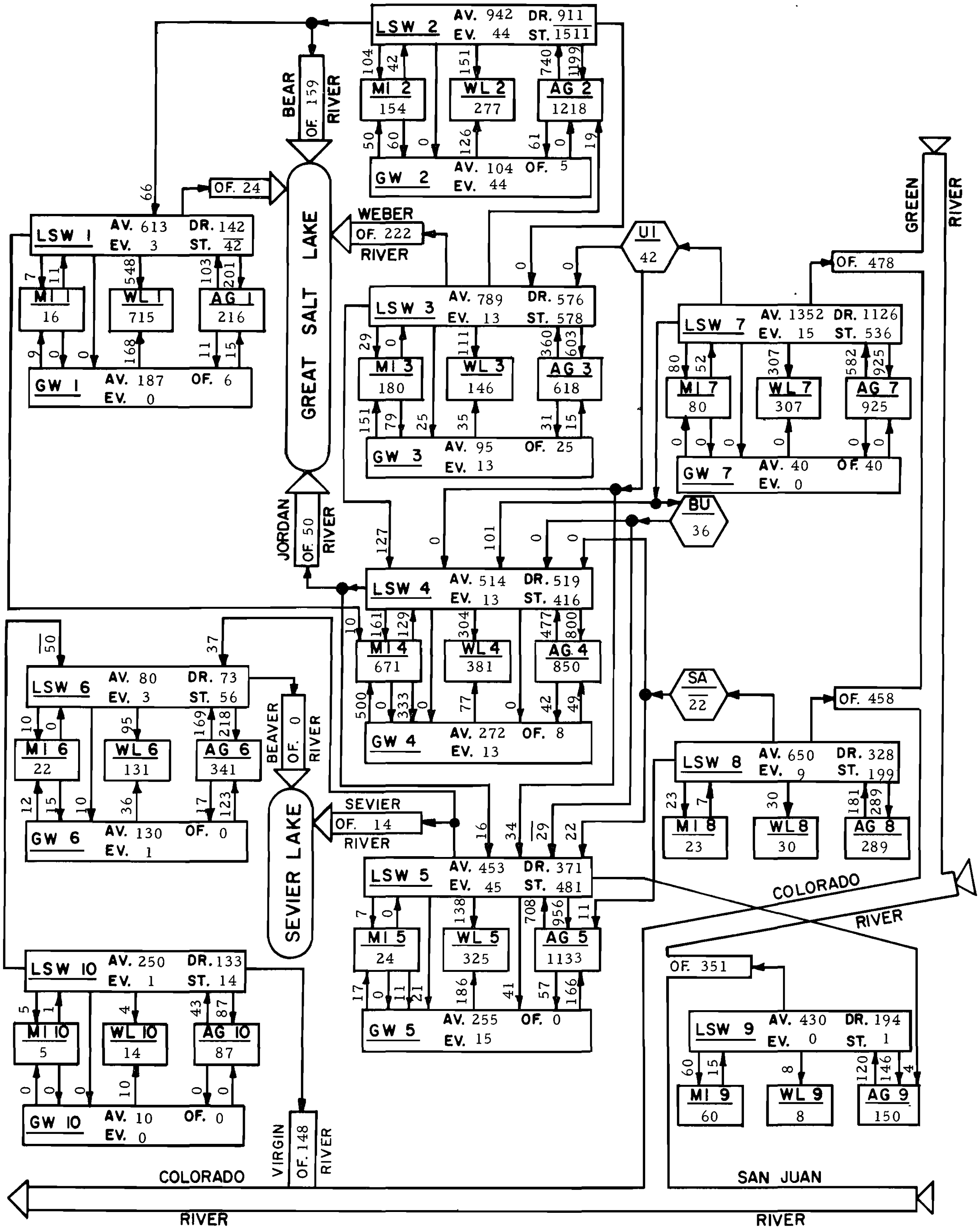
e) Surface Water 4 to Surface Water 5

f) Groundwater 4 to M&I 4

Figure 40. Allocation for the growth projections of June 1969 model as function of time.

shown in Figure 28 and the right-hand-side values of the constraints shown in Figure 22. Return flow coefficients to local surface water and to groundwater must be redetermined by considering the possible changes in irrigation efficiency due to such practices as land leveling, canal and ditch lining, pipeline installations, sprinkler irrigation, and trickle irrigation. Areas affected by each improved practice must also be known and then the new return flow coefficients could be estimated and applied to the model to test the effects of improved irrigation efficiency. The effort involved to determine these data is beyond the scope of this study.

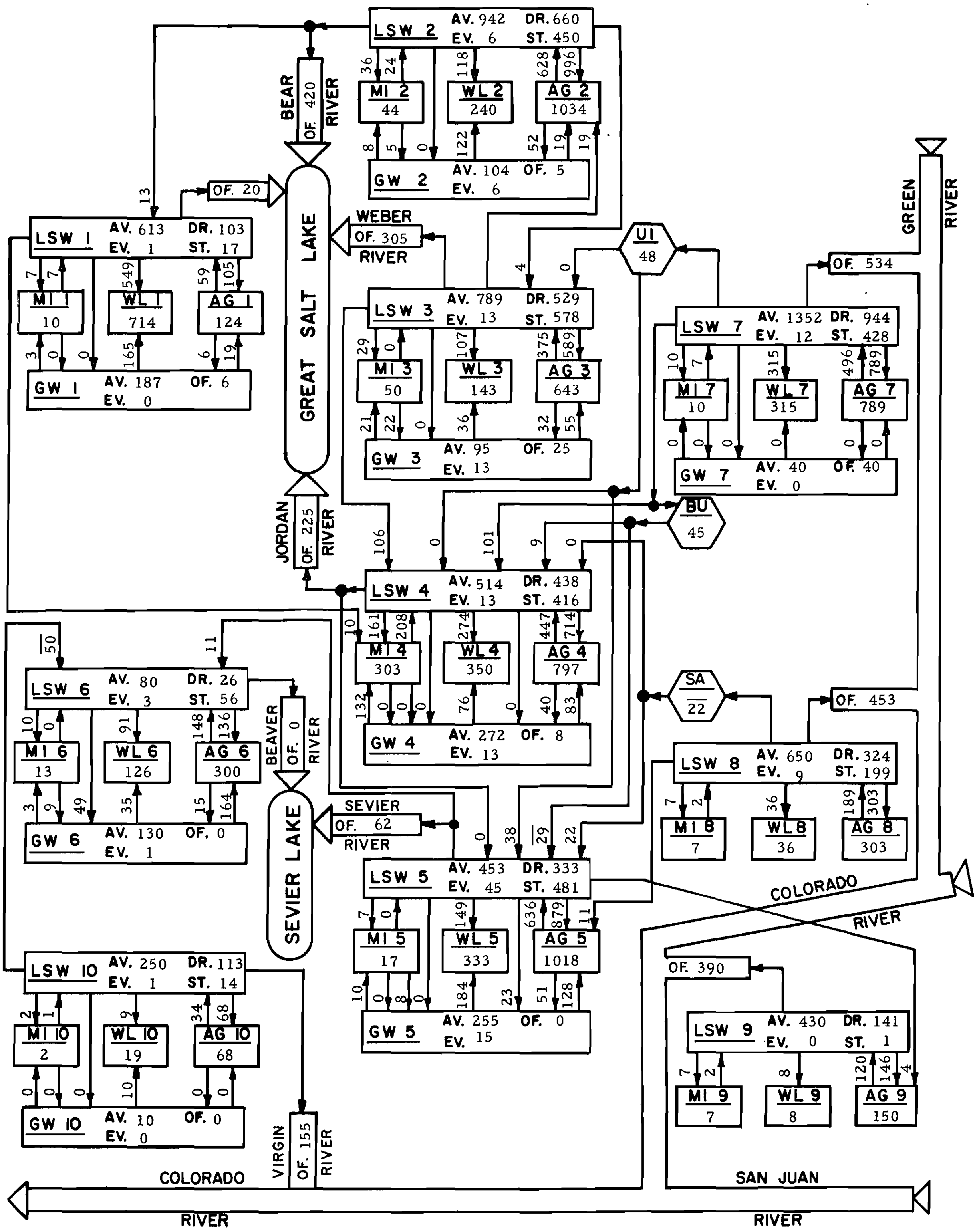
The data discussed in the preceding paragraphs should be considered as examples only and not final results. Many more investigations can be made for other policies or for any combination of policies. Such studies would be needed before a thorough picture of future development for the State of Utah can be determined.



FLOW DIAGRAM FOR ALLOCATION MODEL

(d) Theta = 4.67875 (Time = 1991).

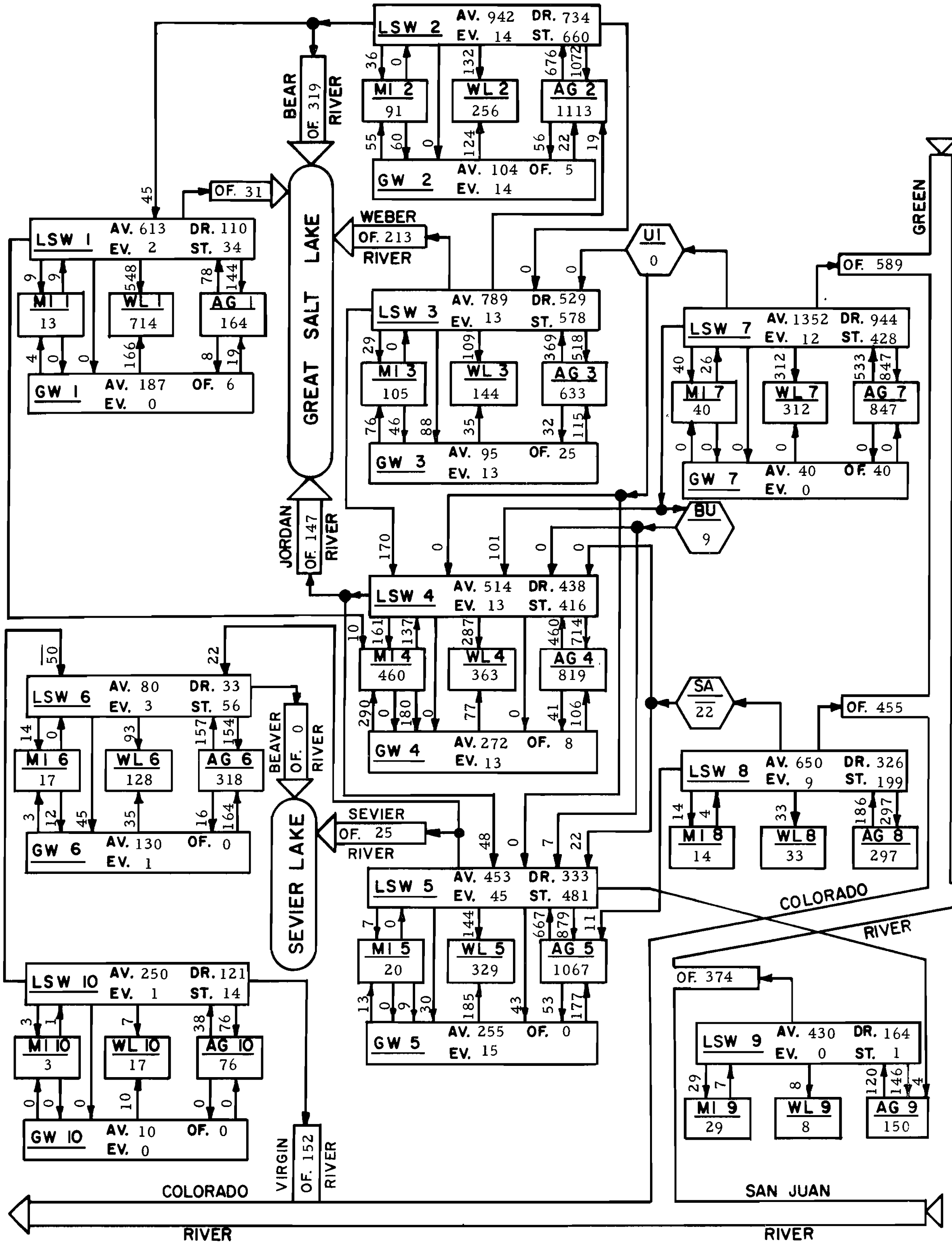
Figure B-3. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

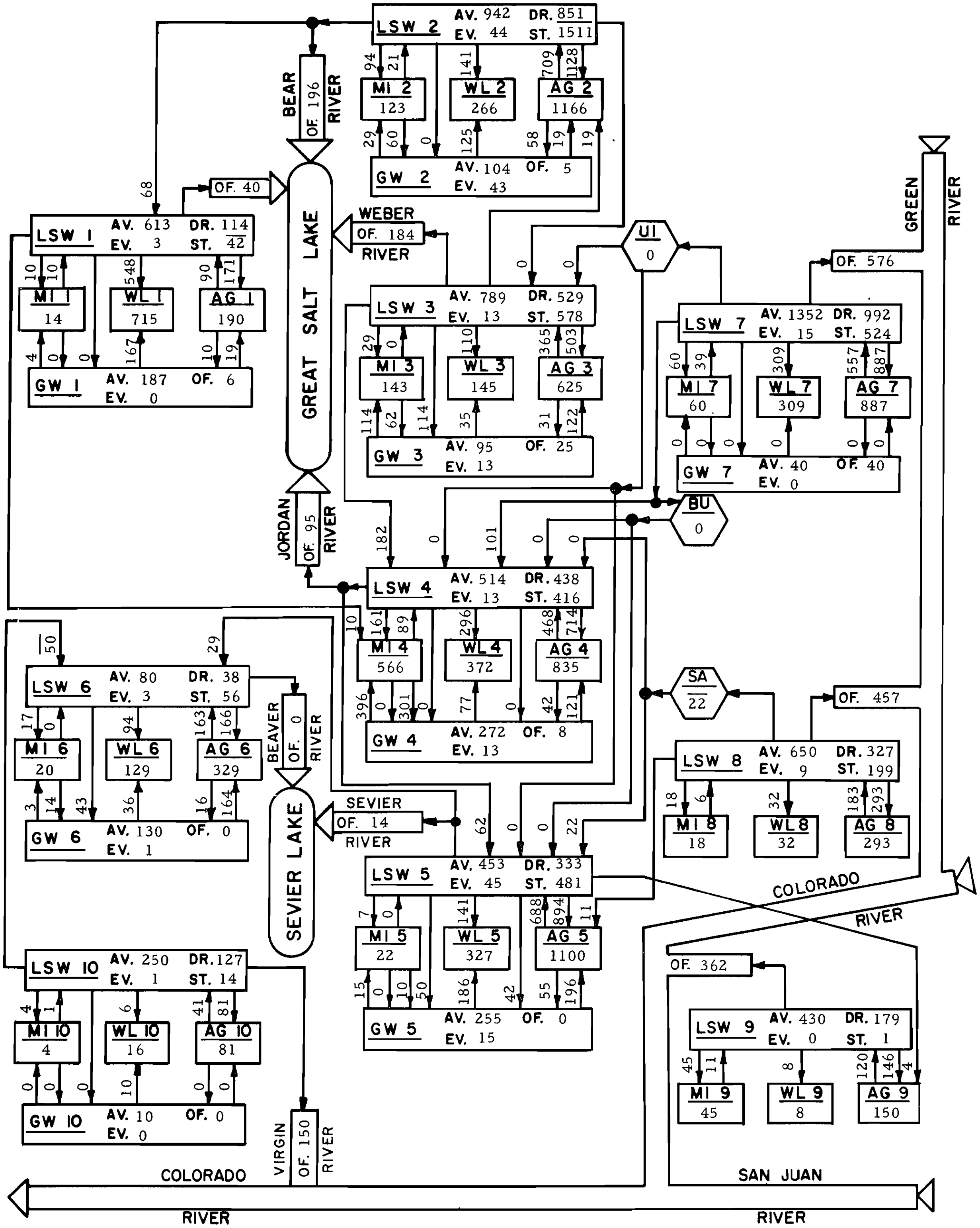
Figure B-4. Storage probability of 0.95 model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

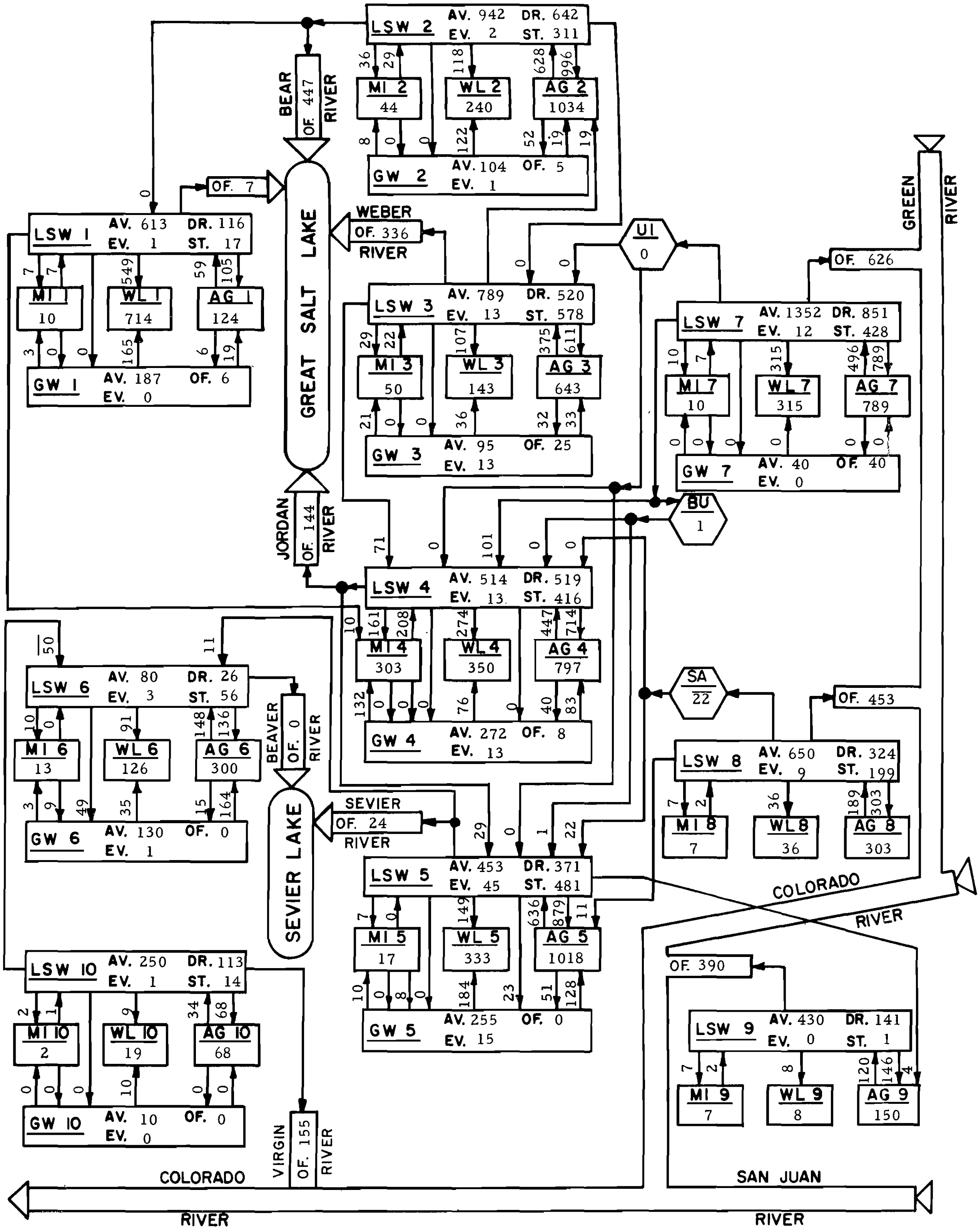
Figure B-4. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 3.35147 (Time = 1983).

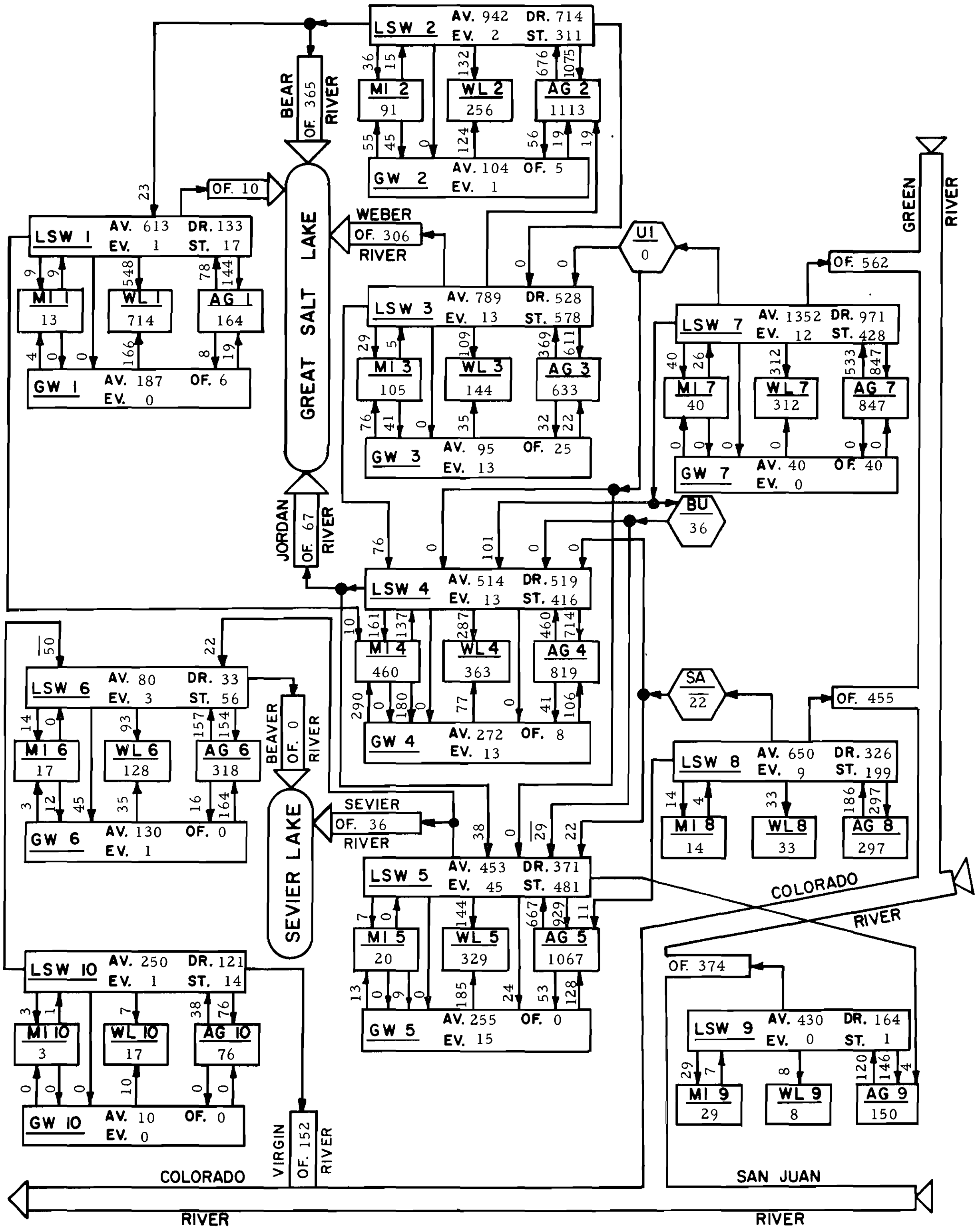
Figure B-4. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

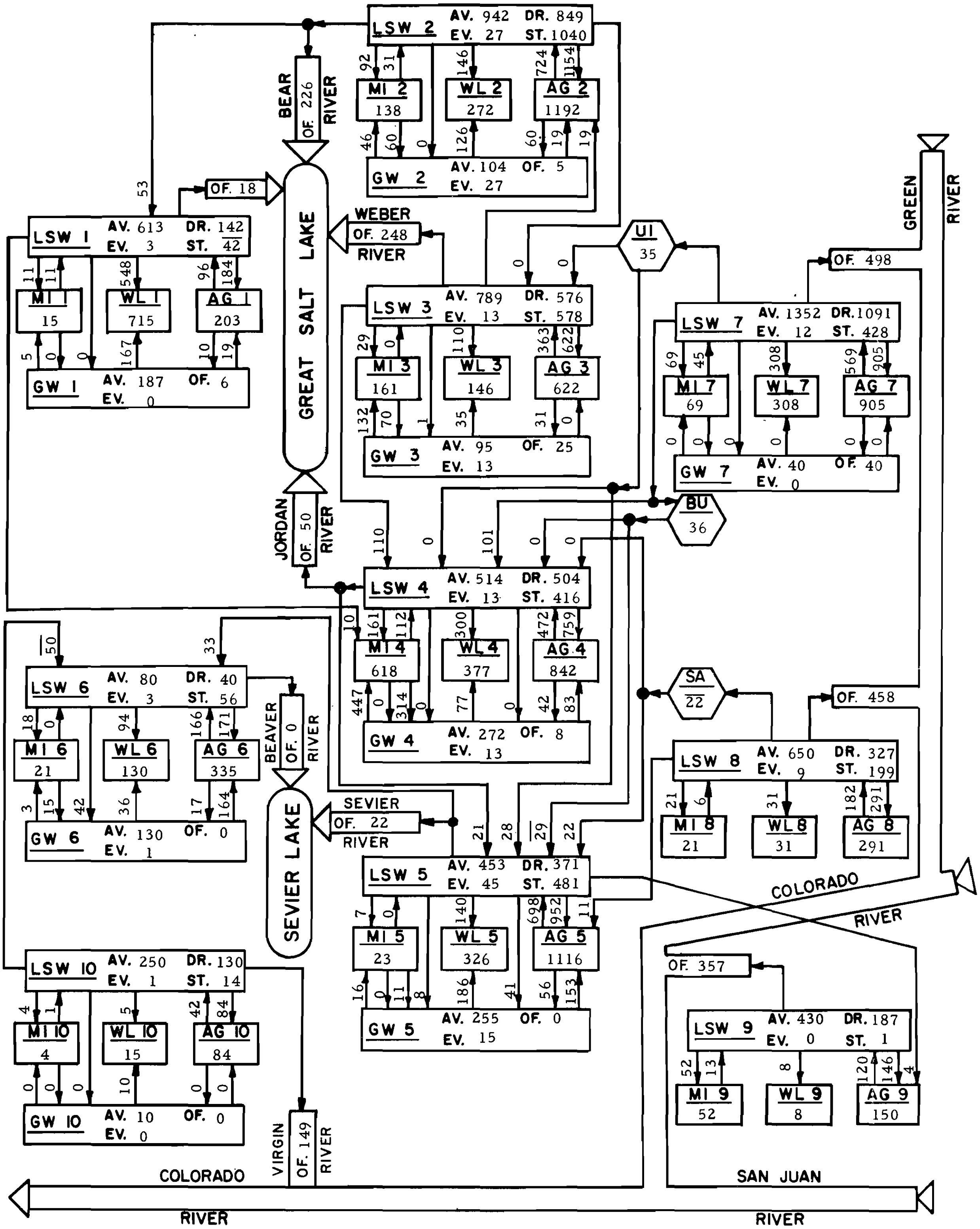
Figure B-5. Inflow to Great Salt Lake ≥ 201,000 ac-ft/yr model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

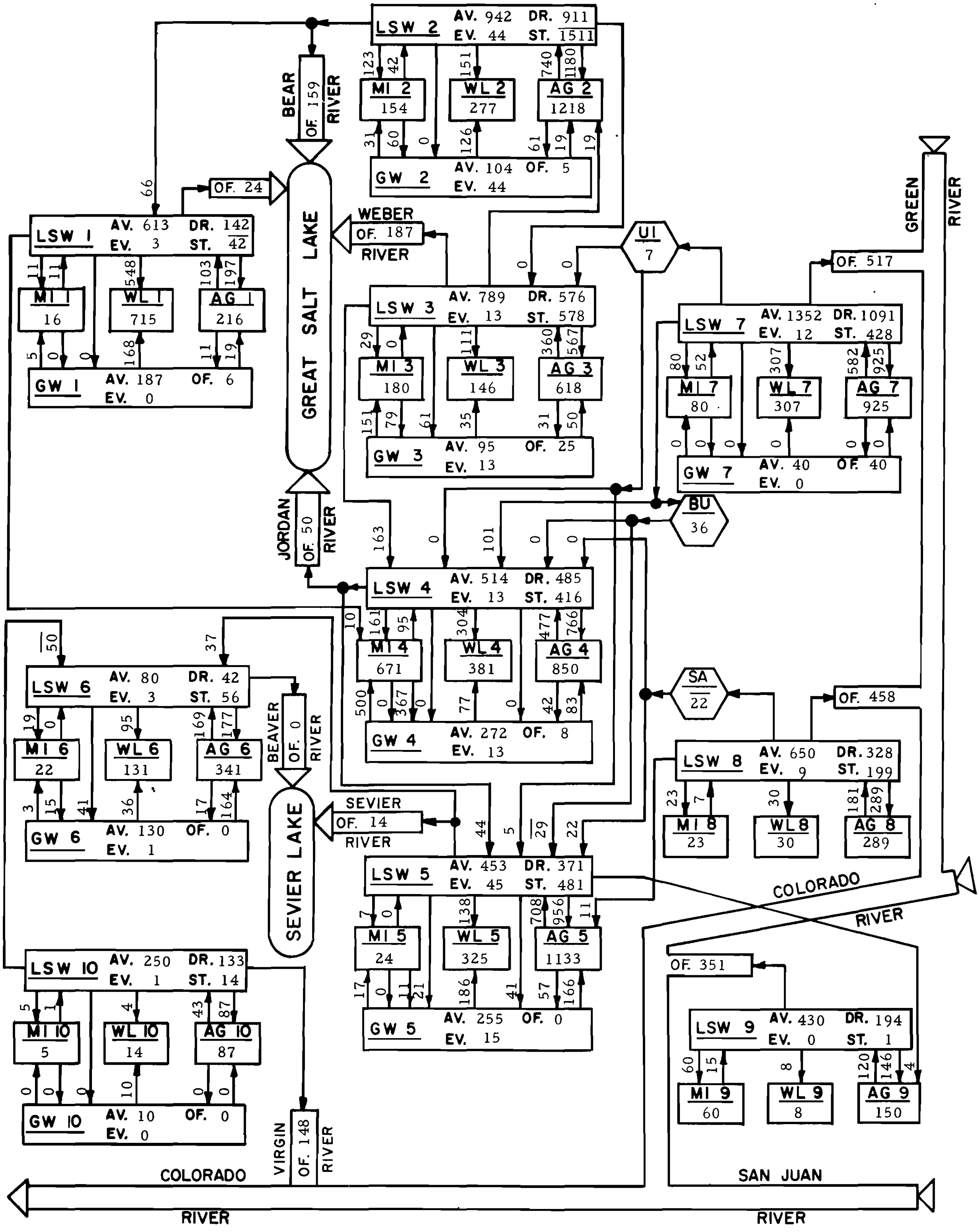
Figure B-5. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 4 (Time = 1987).

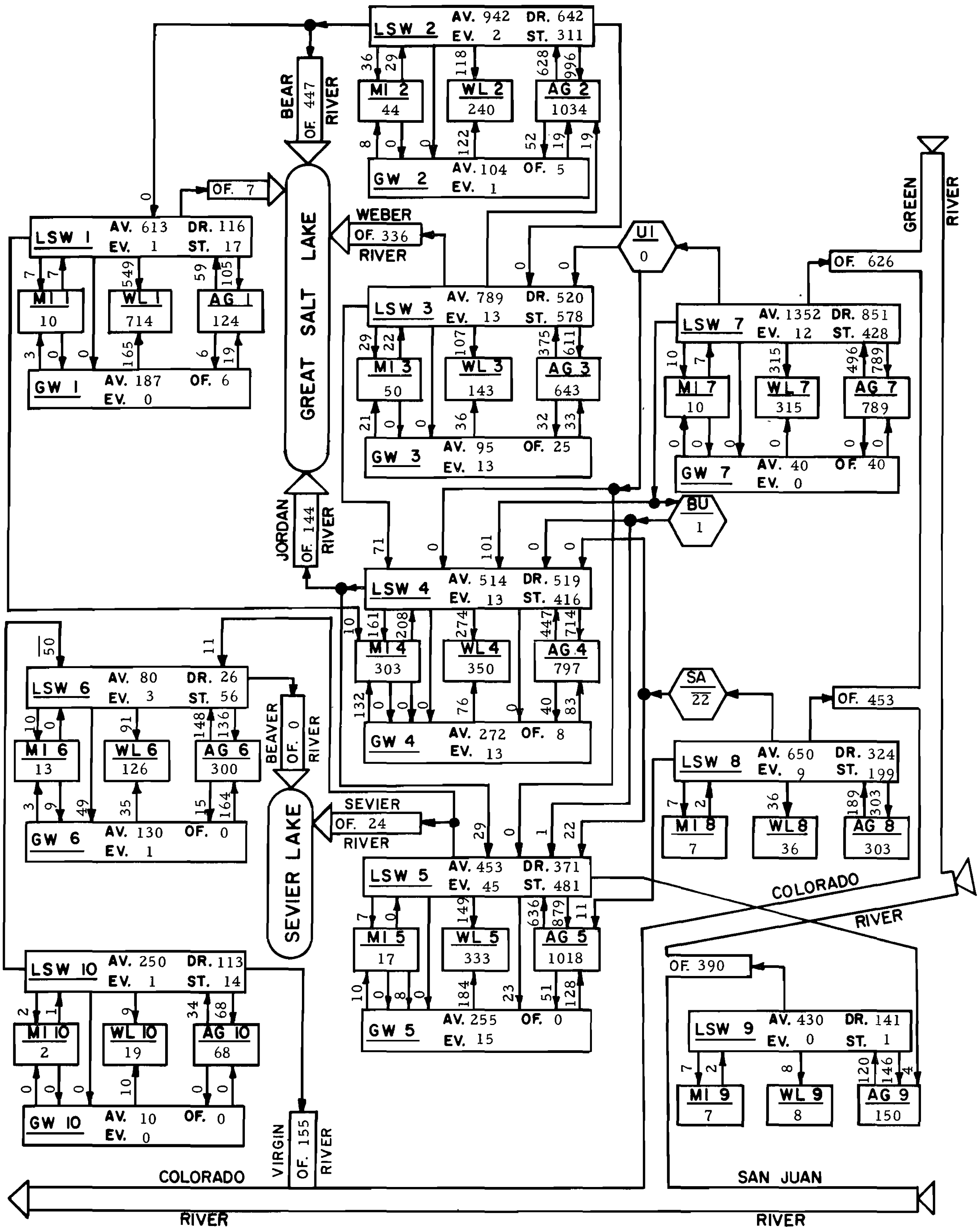
Figure B-5. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(d) Theta = 4.67875 (Time = 1991).

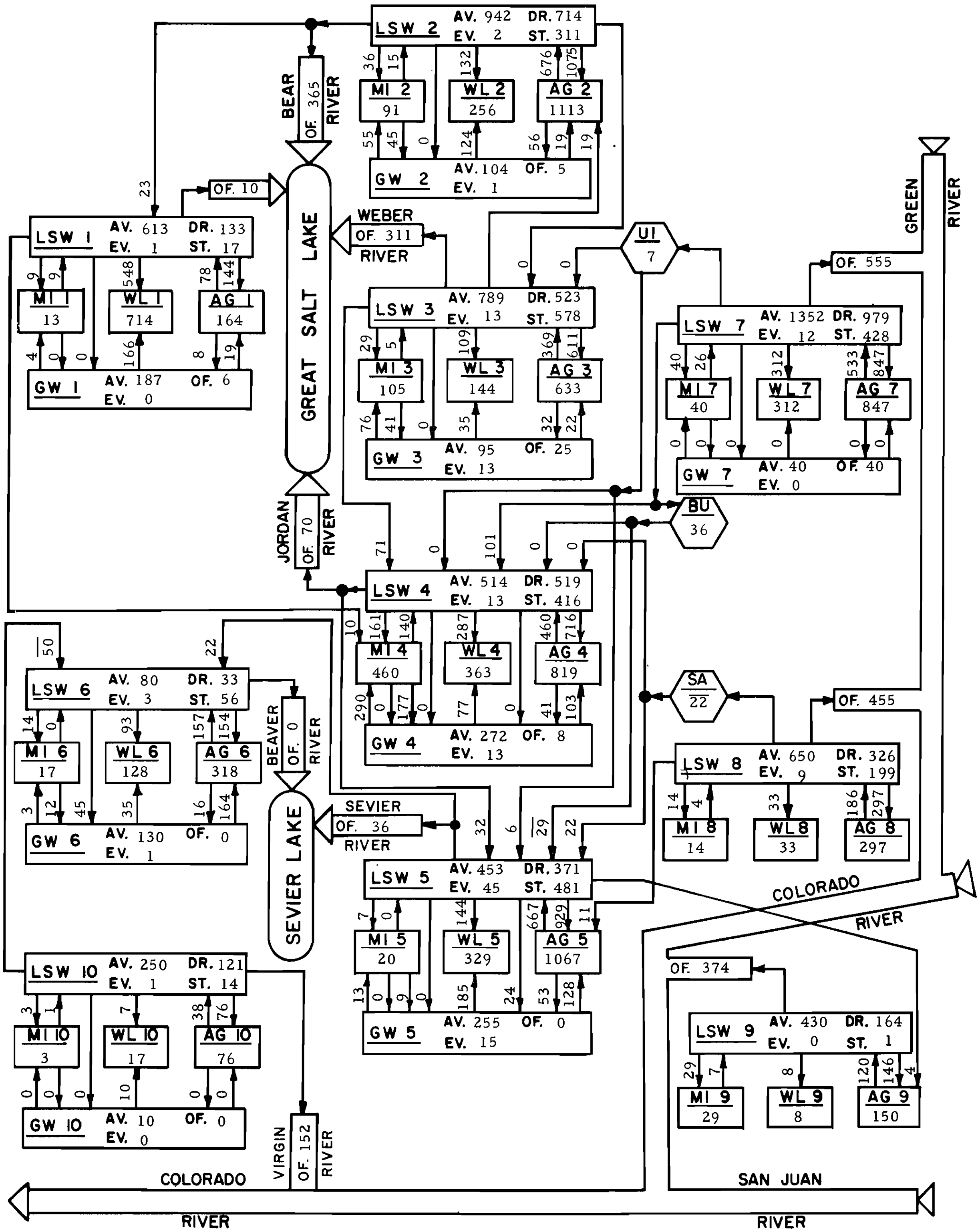
Figure B-5. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

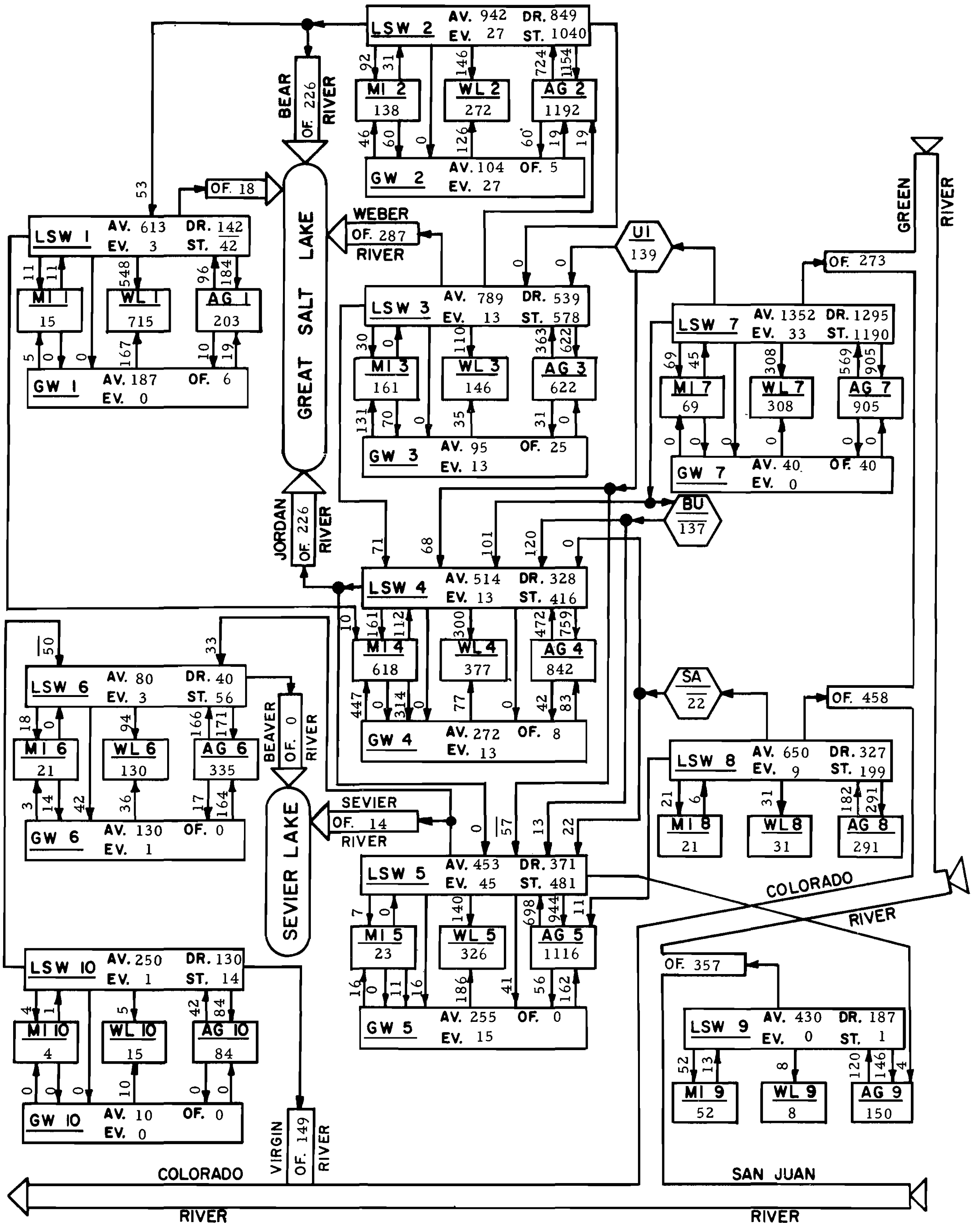
Figure B-6. Inflow to Great Salt Lake ≥ 800,000 ac-ft/yr model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

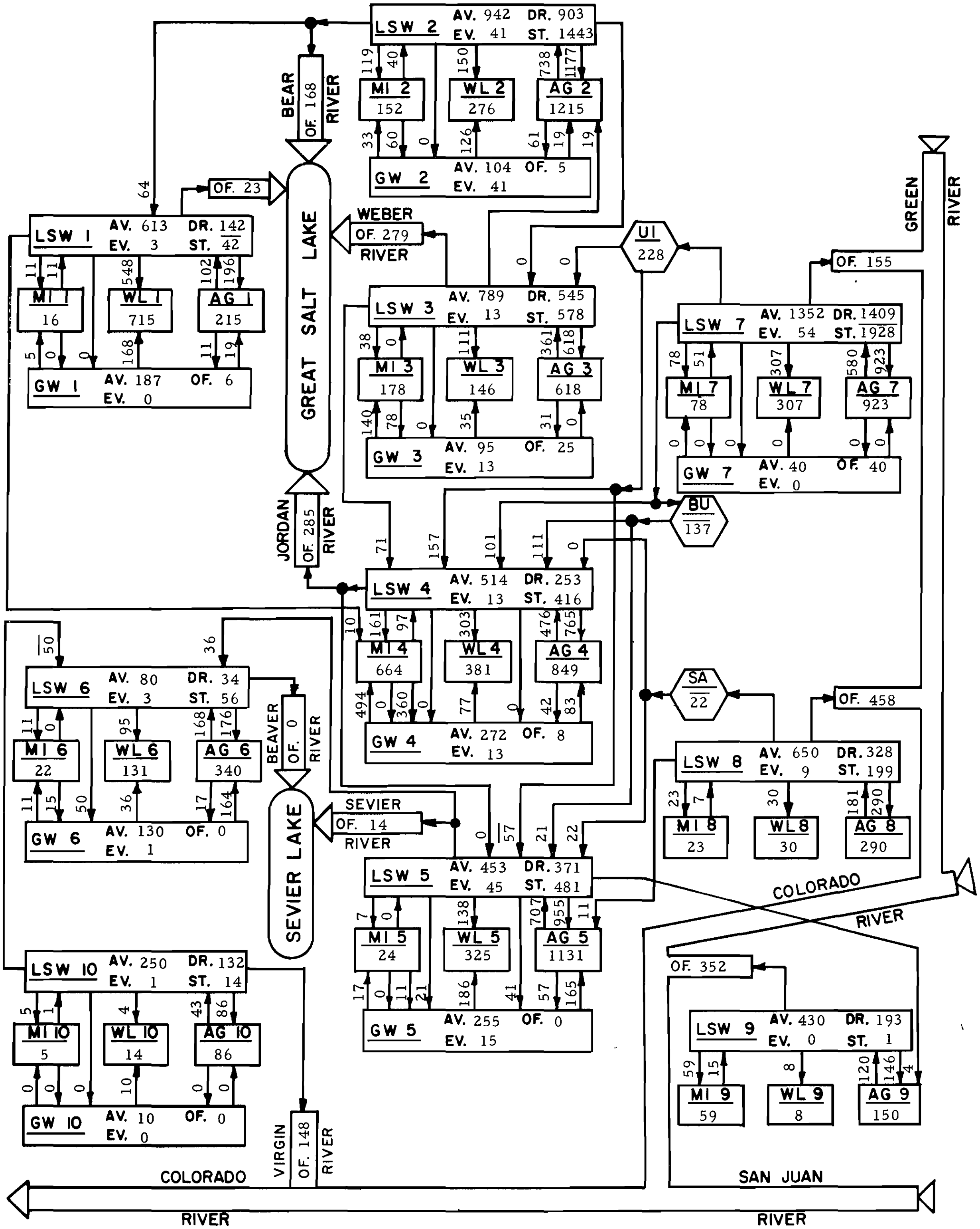
Figure B-6. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 4 (Time = 1987).

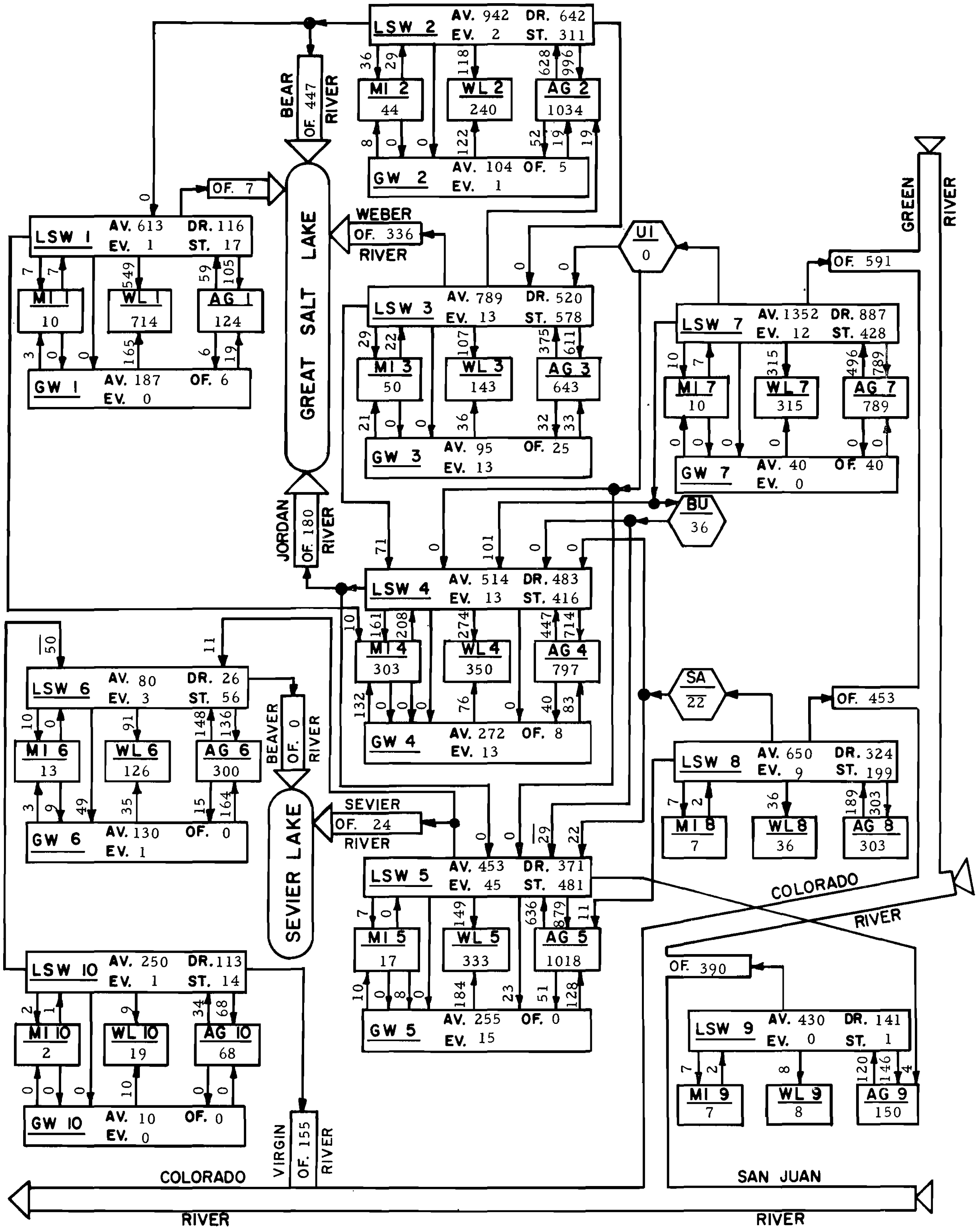
Figure B-6. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(d) Theta = 4.59149 (Time = 1990).

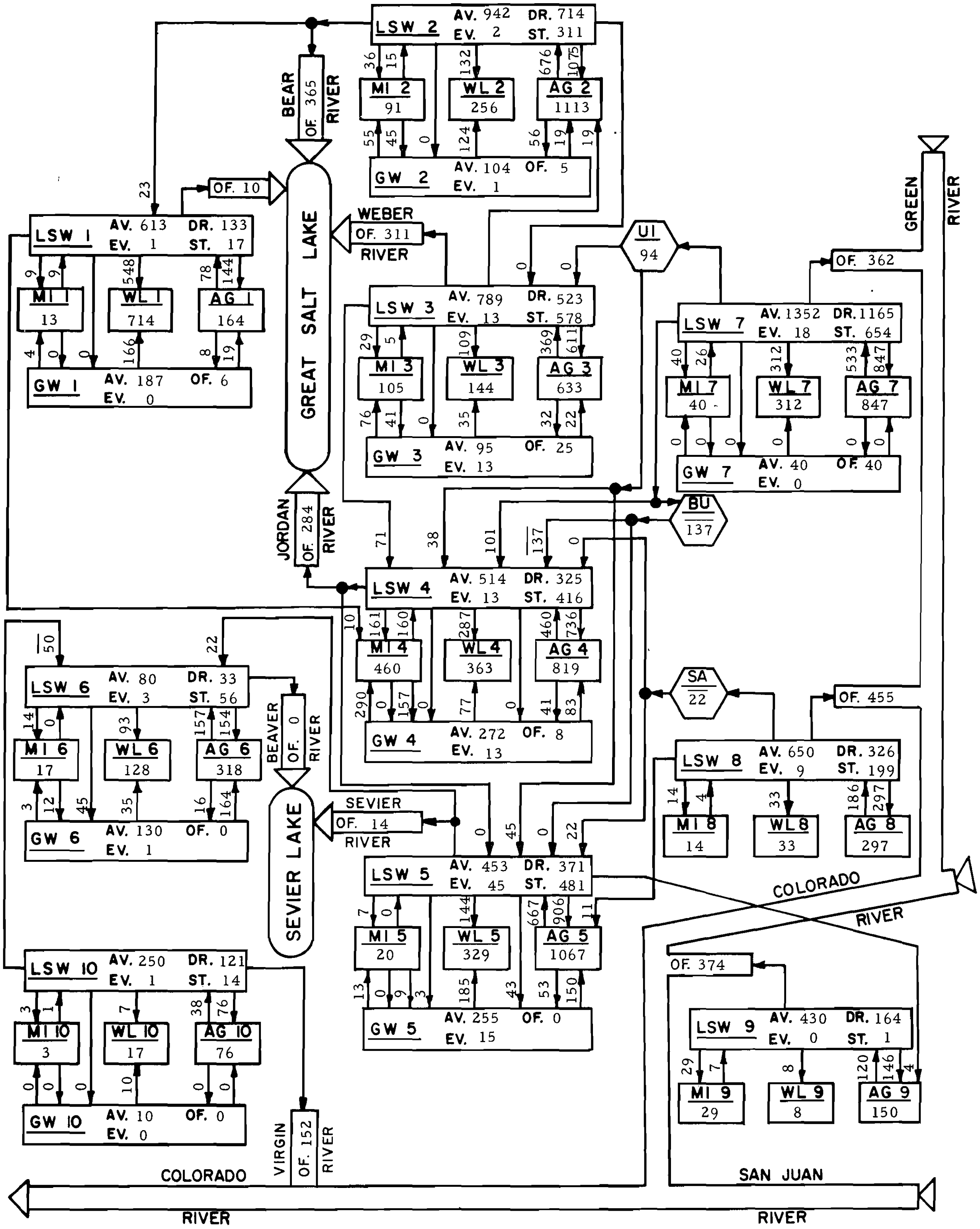
Figure B-6. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

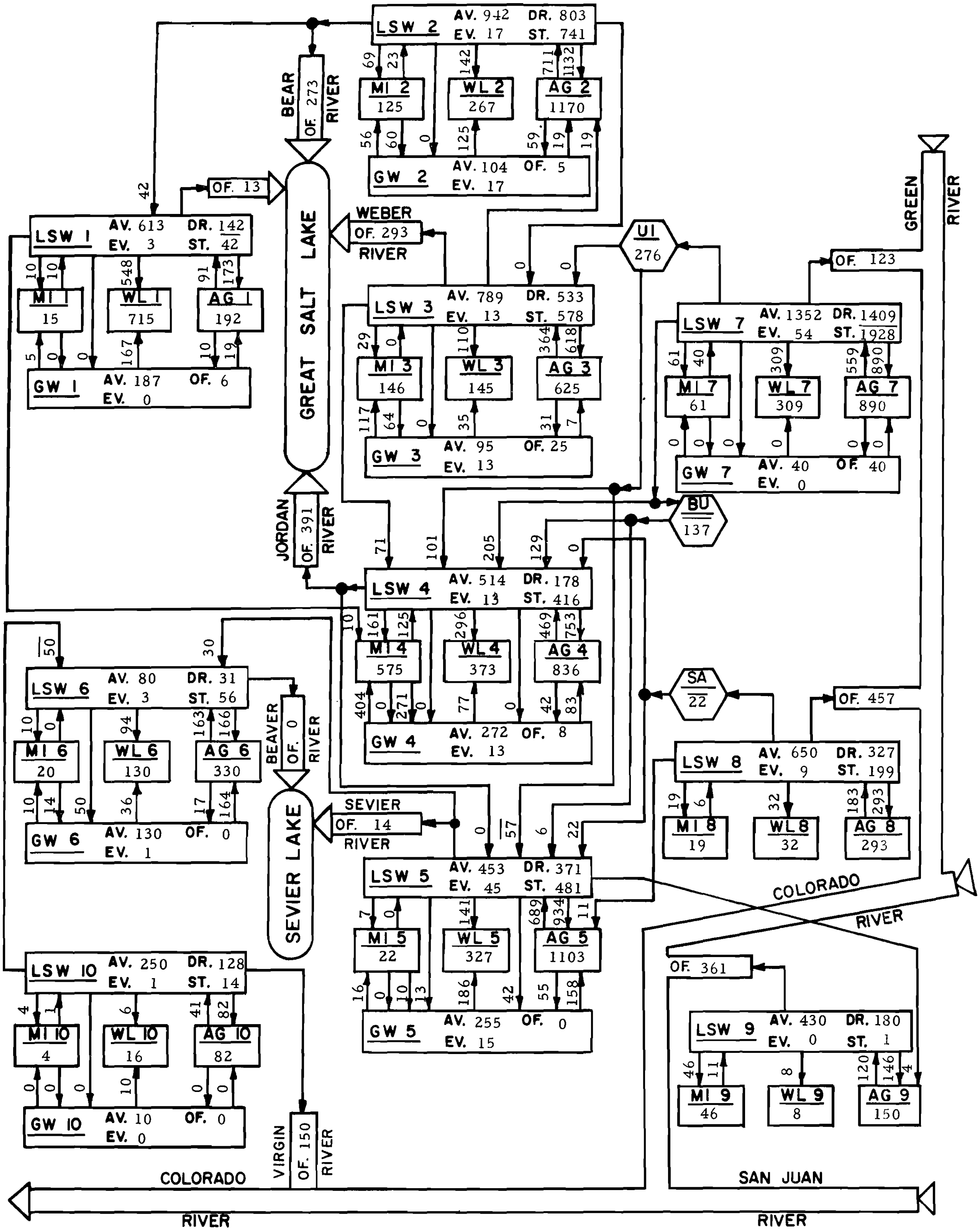
Figure B-7. Inflow to Great Salt Lake ≥ 1,014,000 ac-ft/yr model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

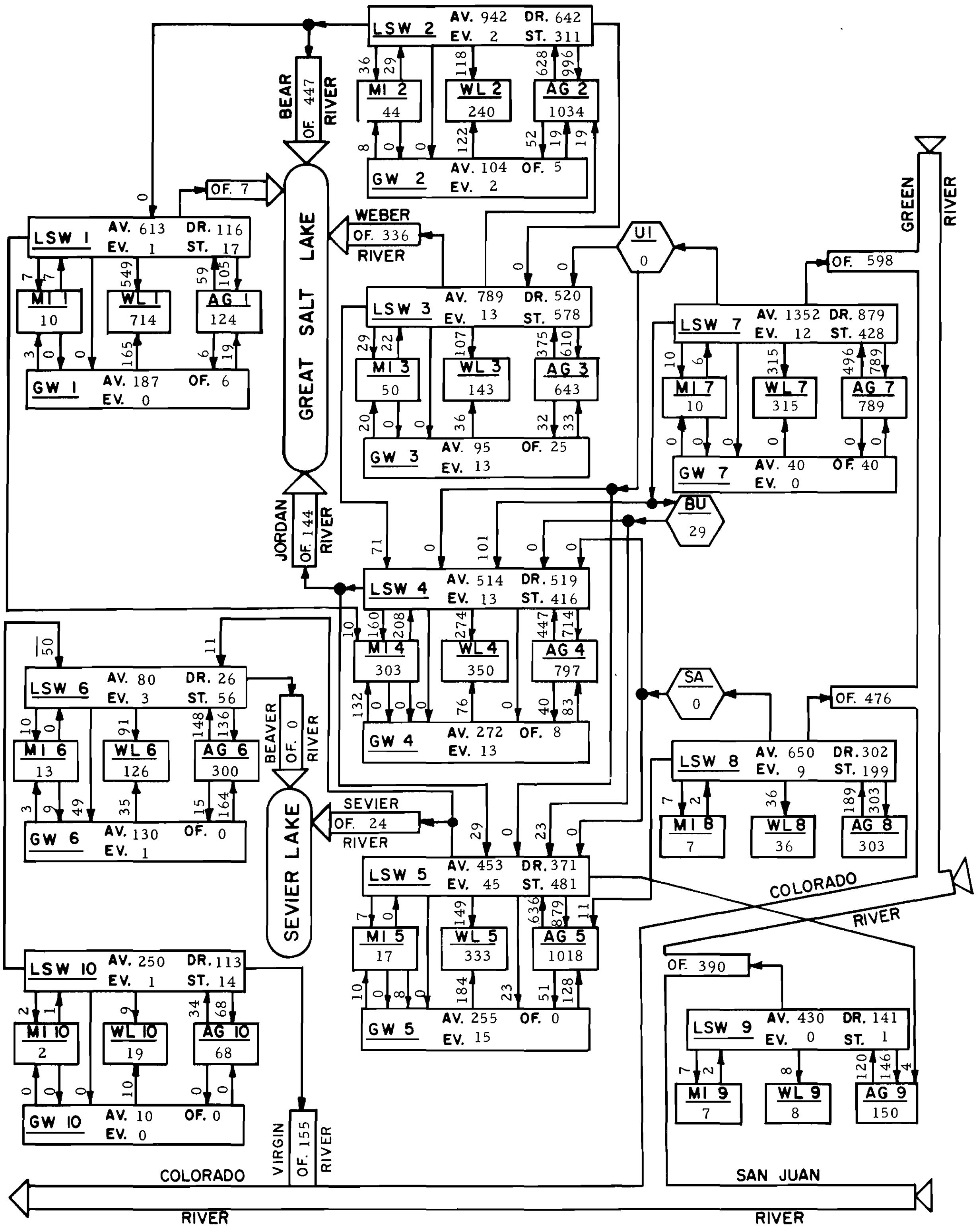
Figure B-7. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 3.45805 (Time = 1984).

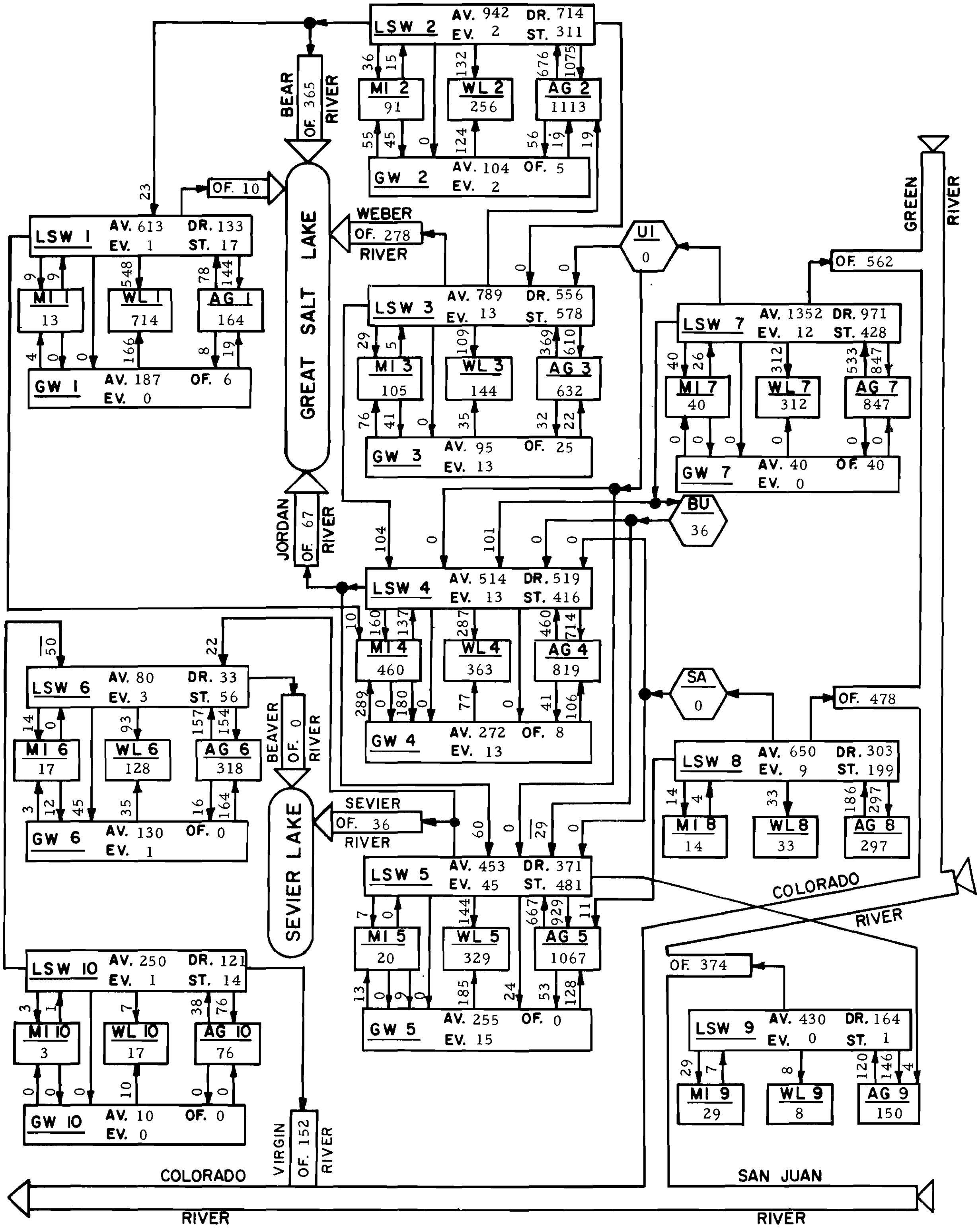
Figure B-7. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

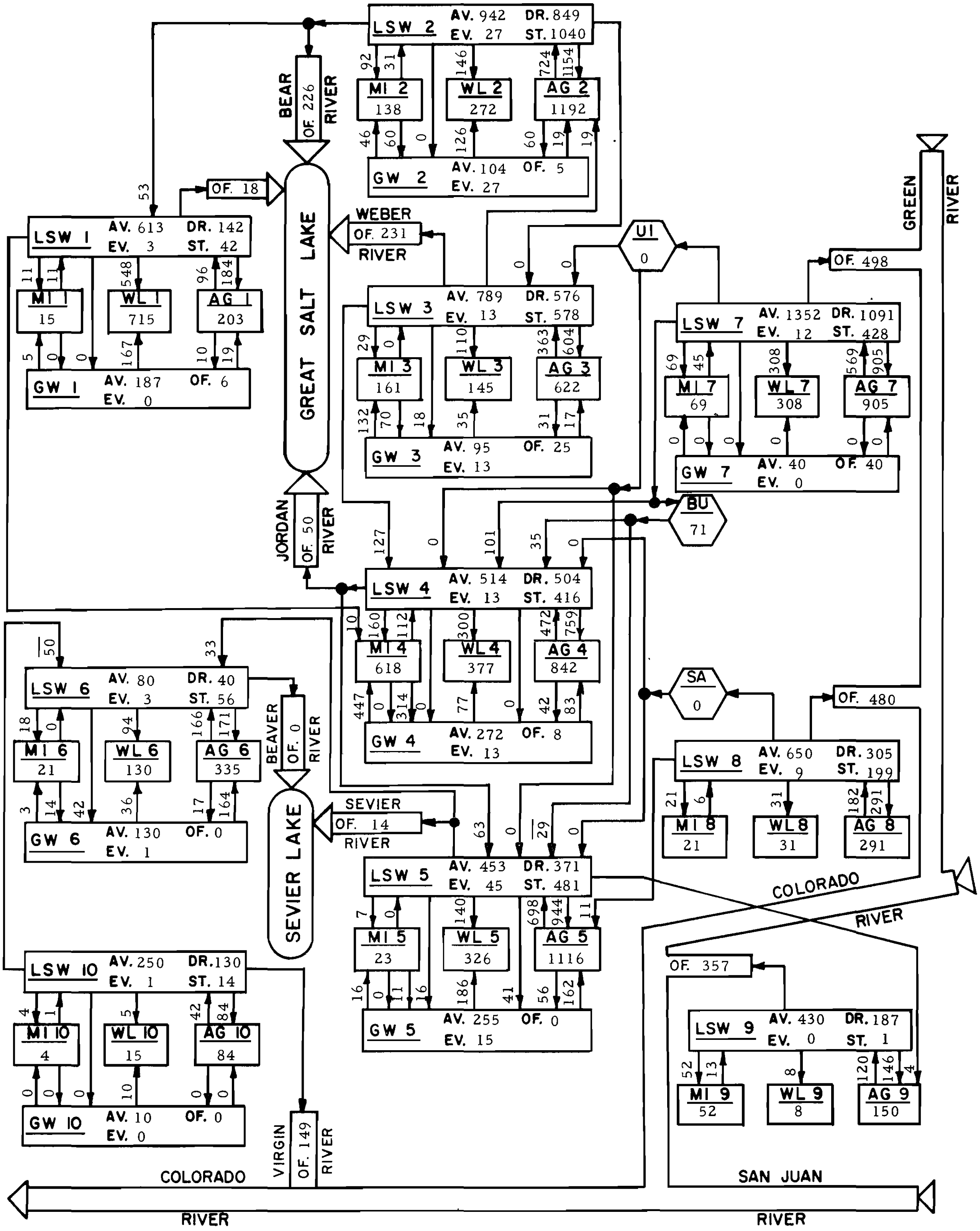
Figure B-8. Bonneville unit import only model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

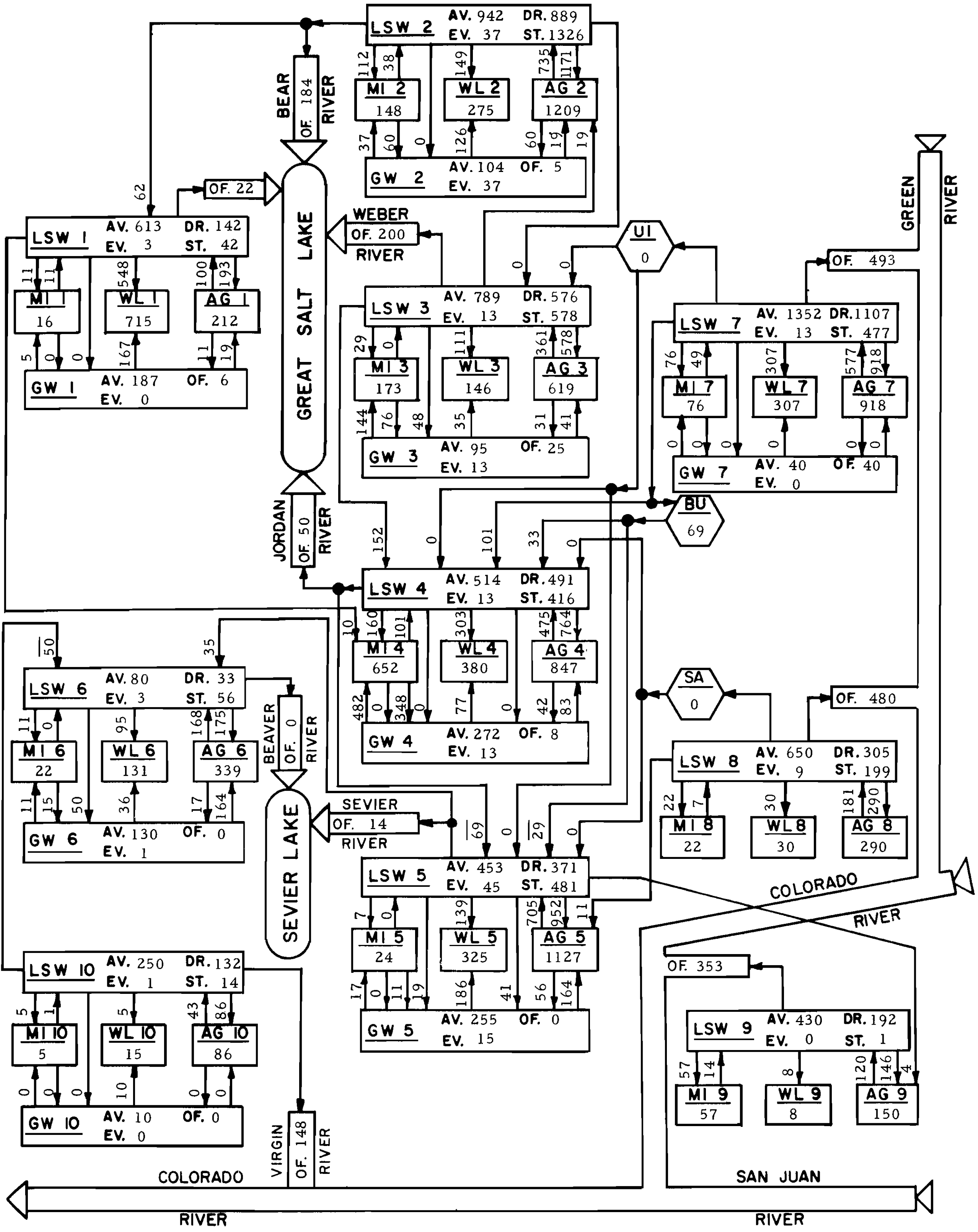
Figure B-8. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 4 (Time = 1987).

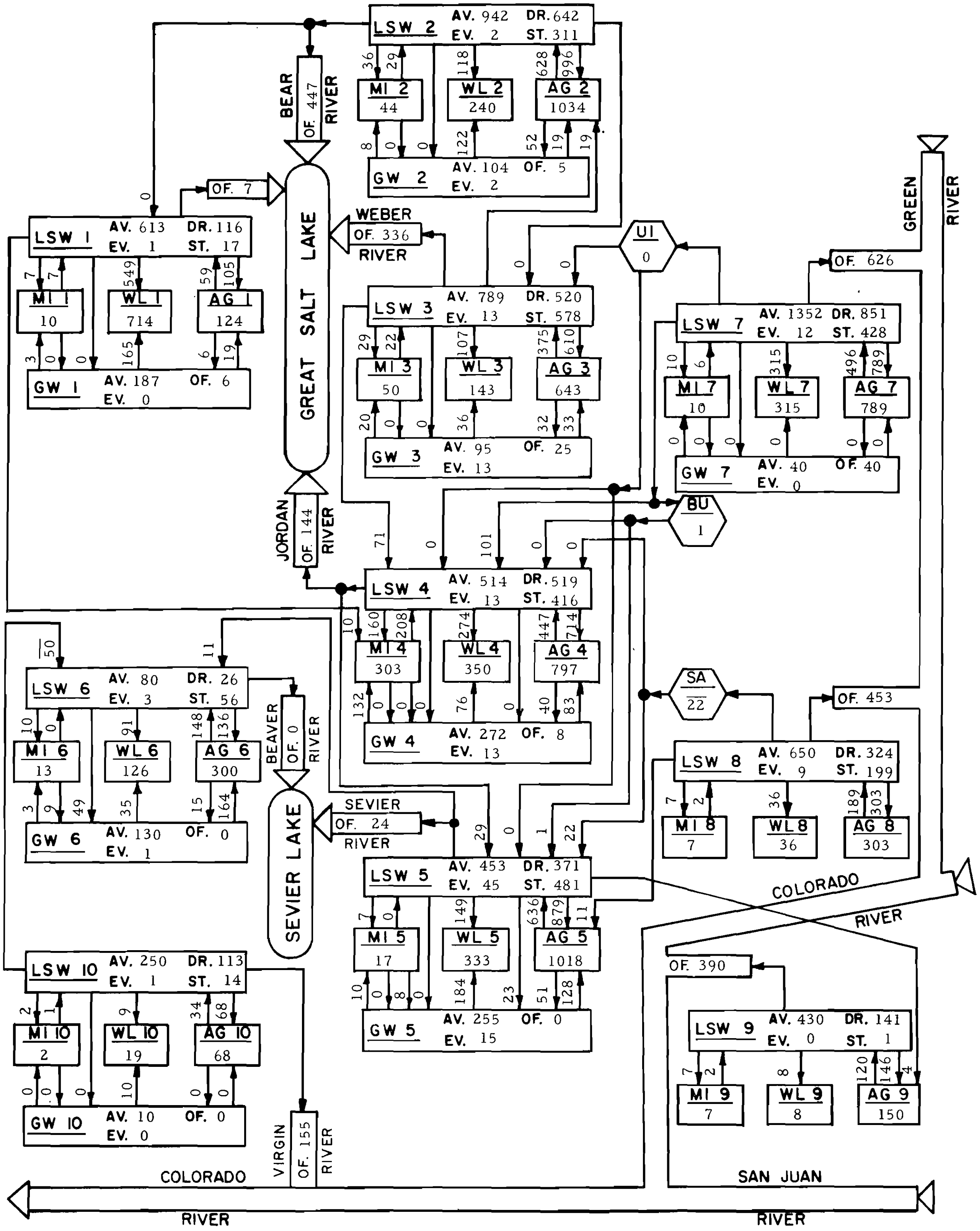
Figure B-8. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(d) Theta = 4.44009 (Time = 1989).

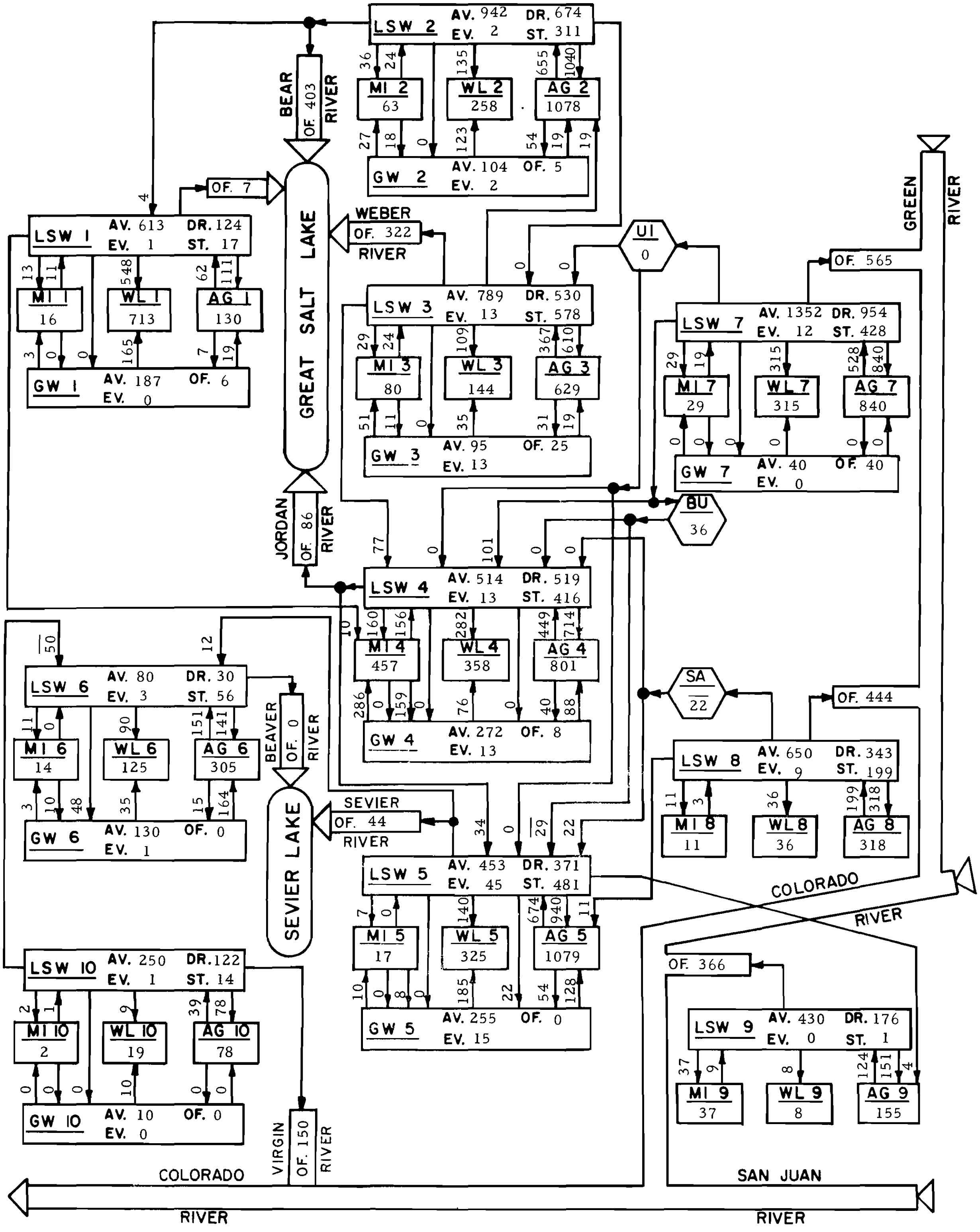
Figure B-8. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(a) Theta = 0 (Time = 1965).

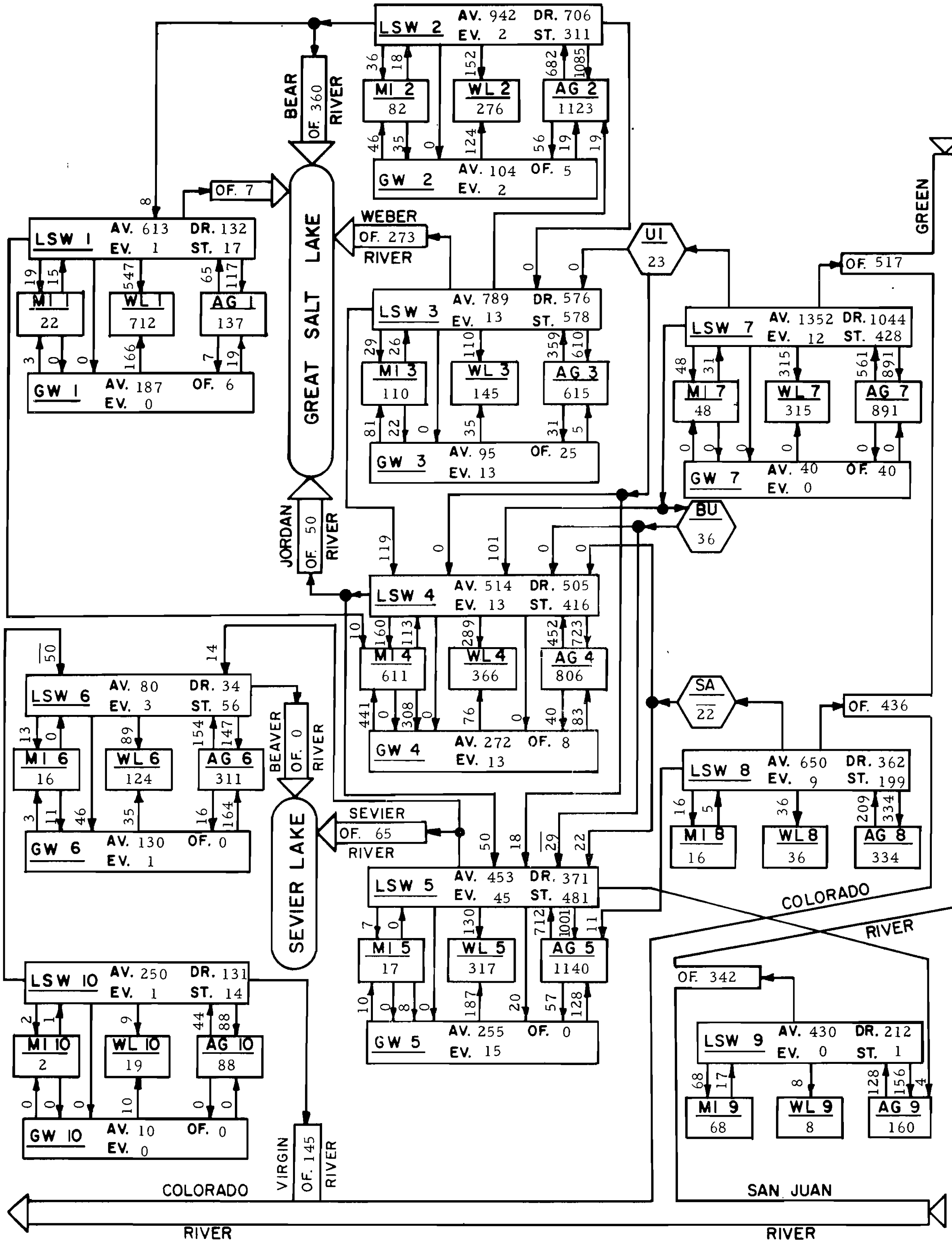
Figure B-9. Growth projections of June 1969 model.



FLOW DIAGRAM FOR ALLOCATION MODEL

(b) Theta = 2 (Time = 1976).

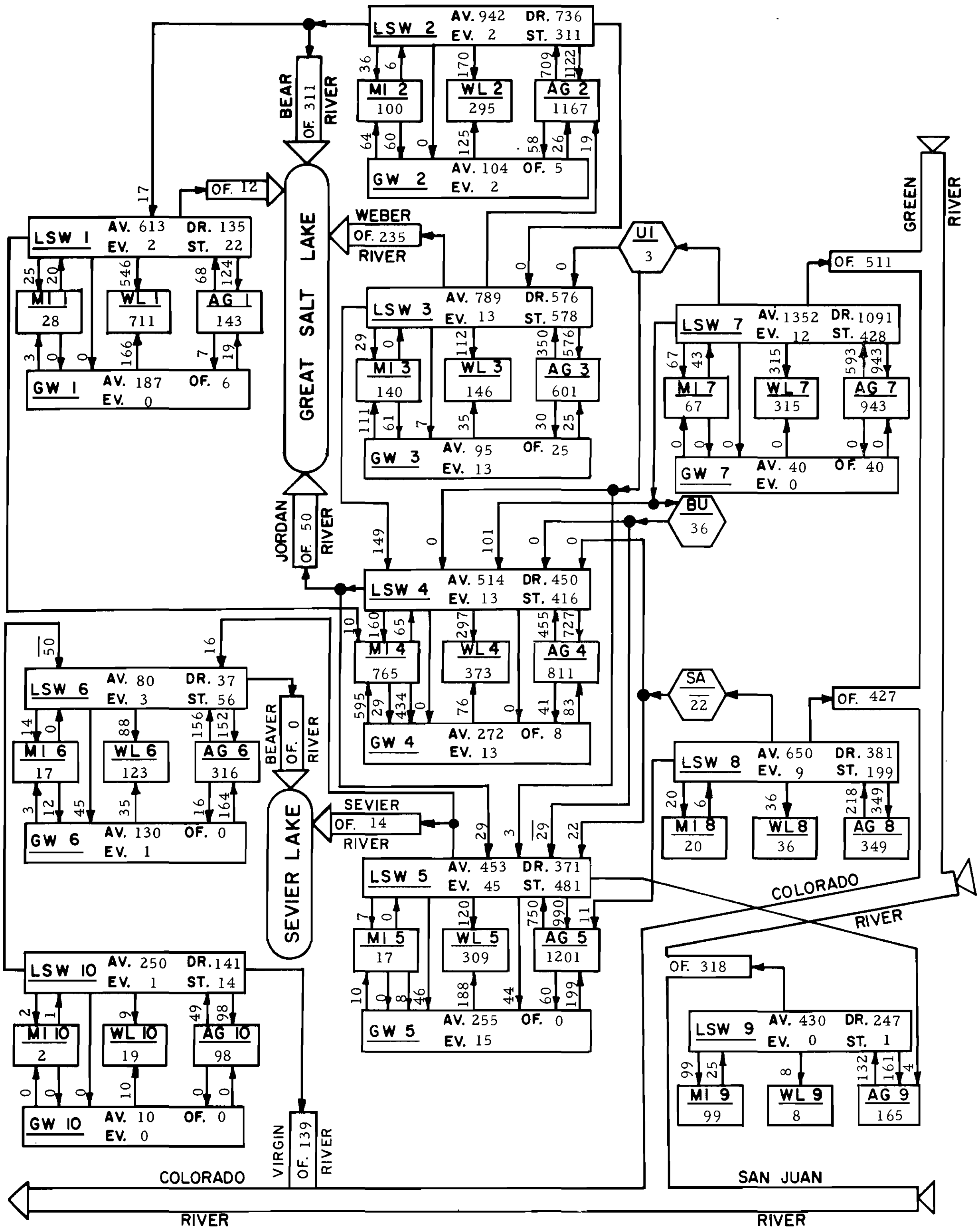
Figure B-9. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(c) Theta = 4 (Time = 1987).

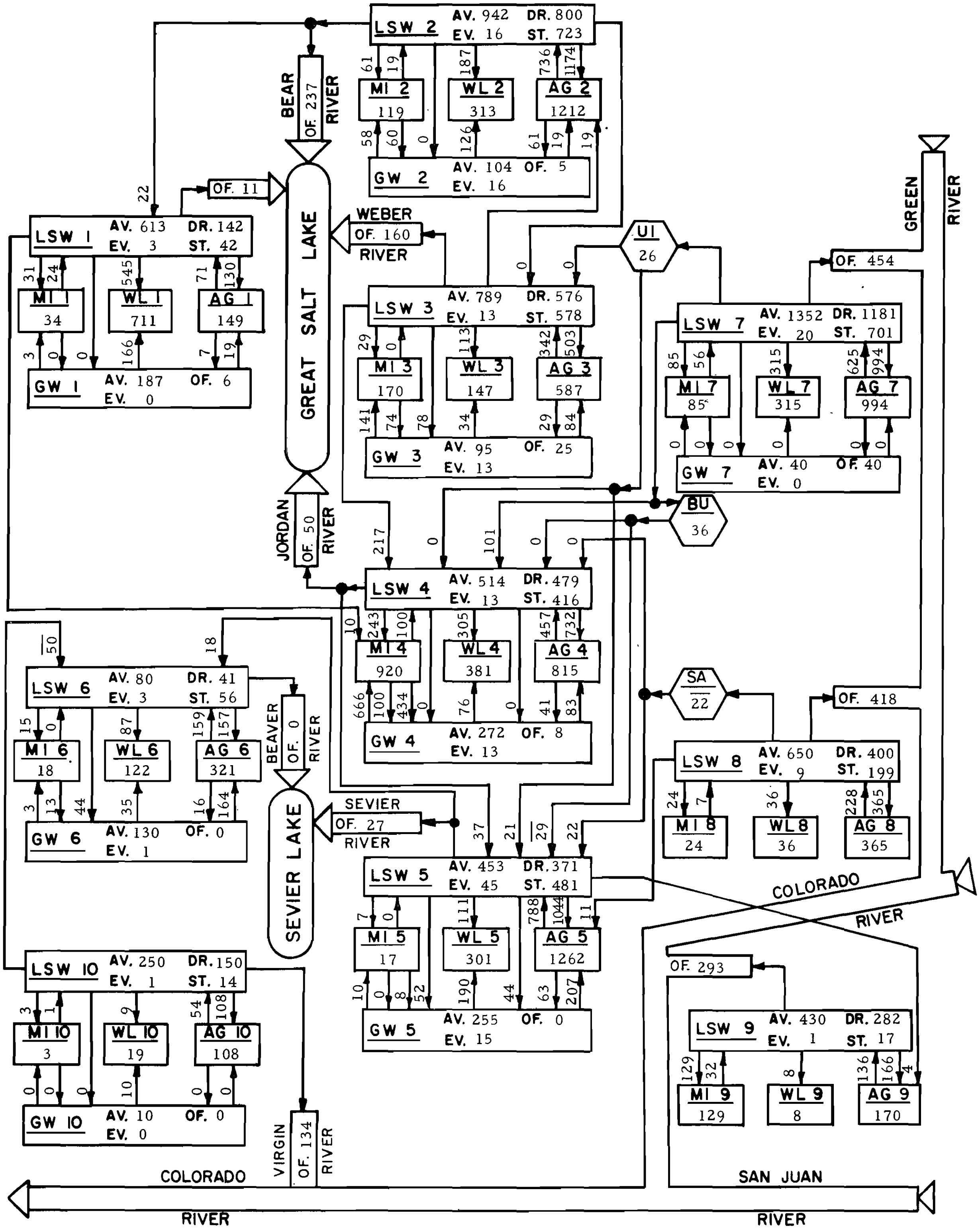
Figure B-9. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(d) Theta = 6 (Time = 1998).

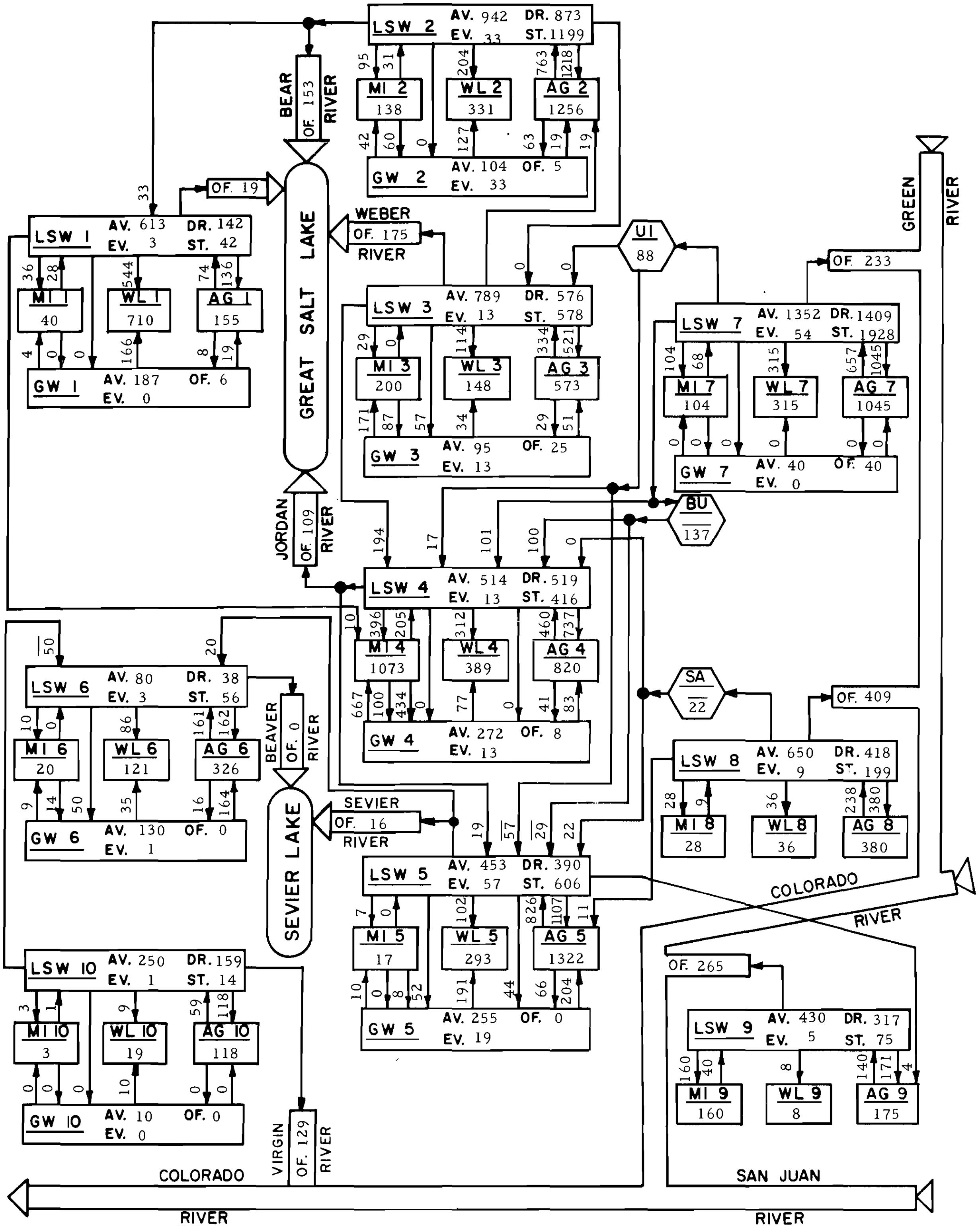
Figure B-9. Continued.



FLOW DIAGRAM FOR ALLOCATION MODEL

(e) Theta = 8 (Time = 2009).

Figure B-9. Continued.



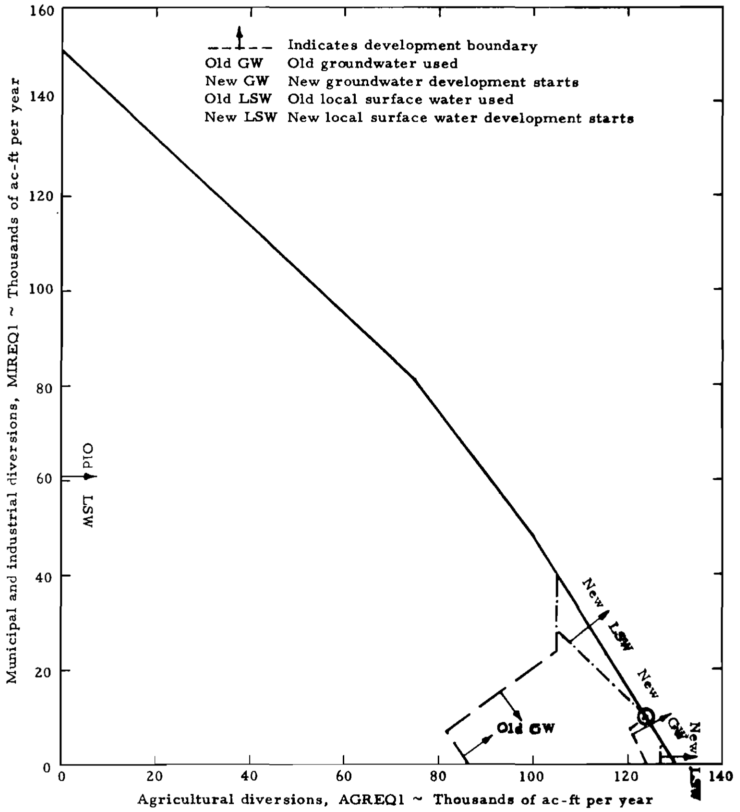
FLOW DIAGRAM FOR ALLOCATION MODEL

(f) Theta = 9.98374 (Time = 2020).

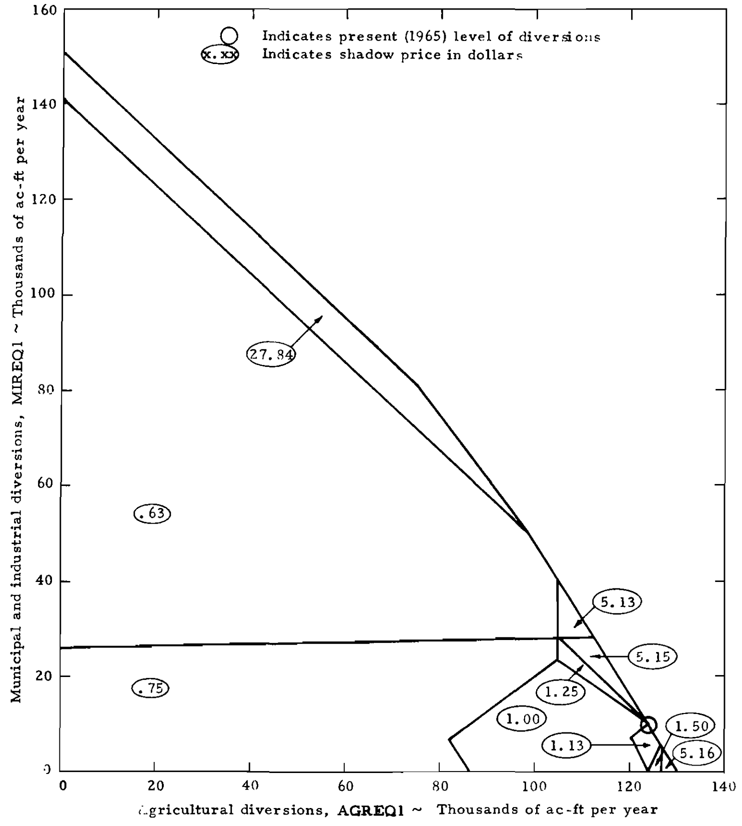
Figure B-9. Continued.

Appendix C

Supply Function Maps for the Hydrologic Study Units

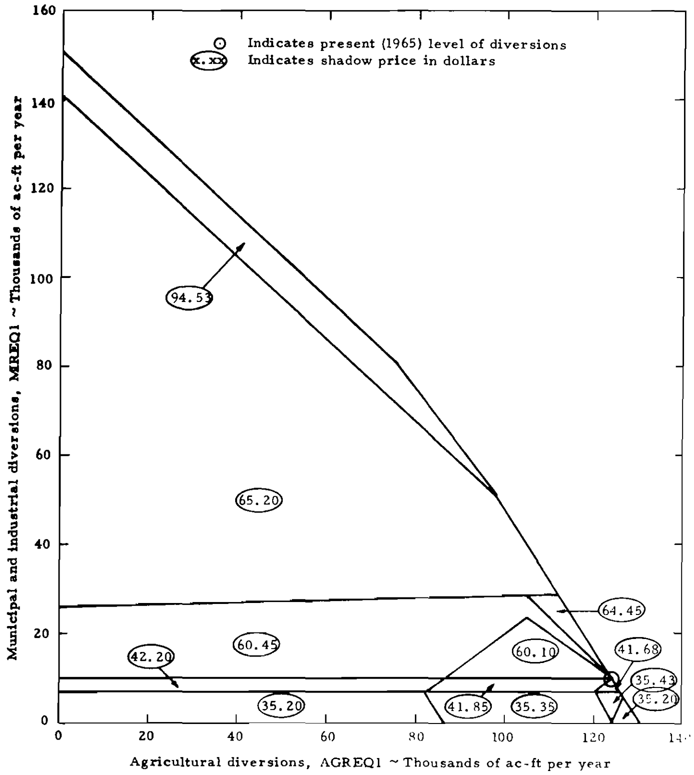


a) Agricultural development map.

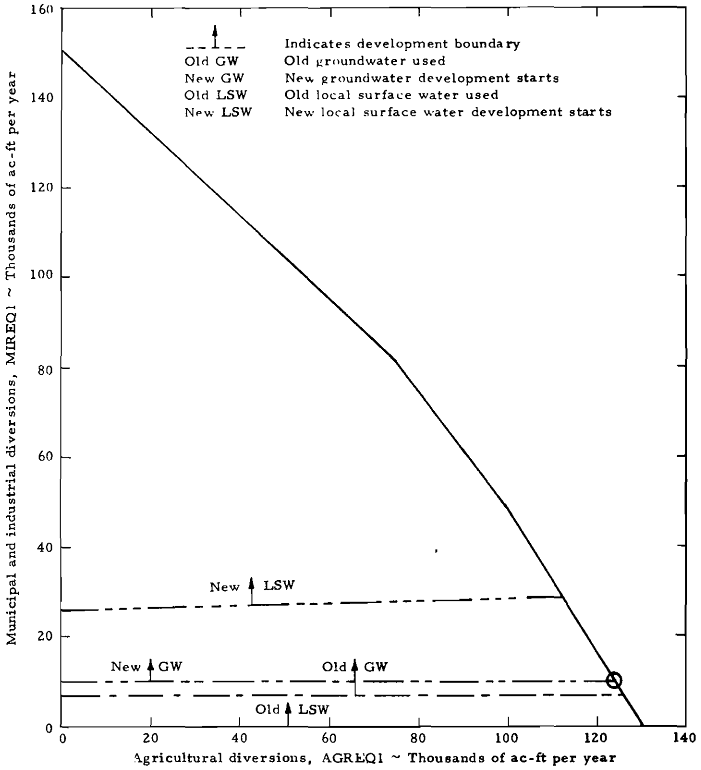


b) Shadow price of agricultural diversions.

Figure C-1. Supply function maps for hydrologic study unit 1.

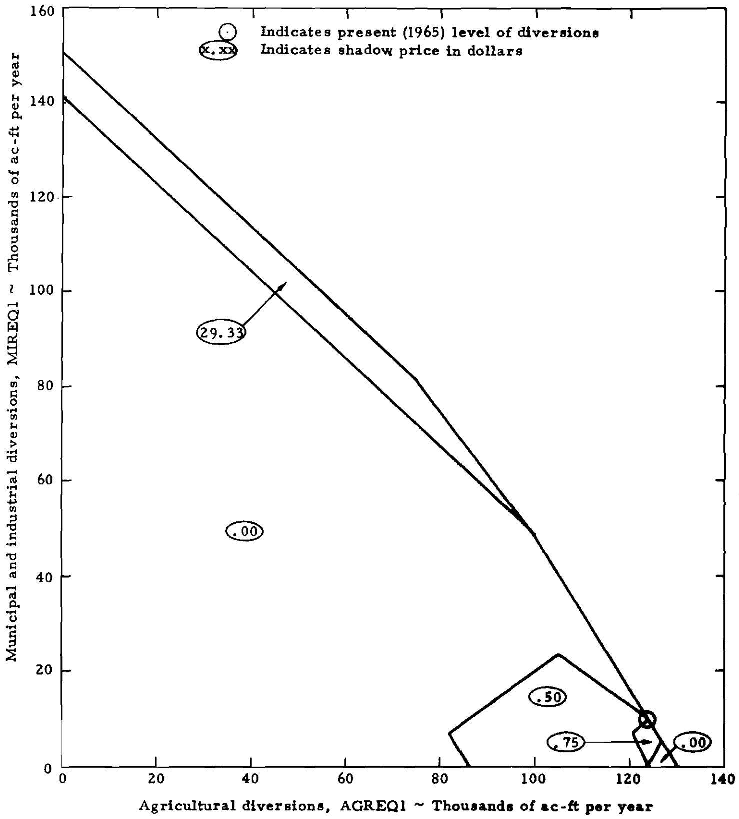


c) Municipal and industrial development map.

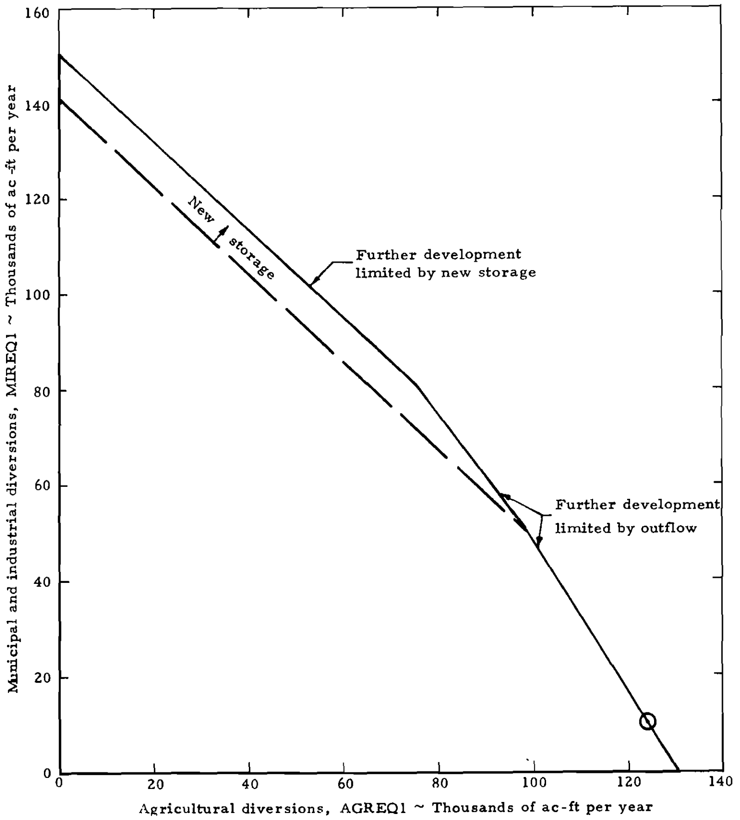


d) Shadow price of municipal and industrial diversions.

Figure C-1. Continued.

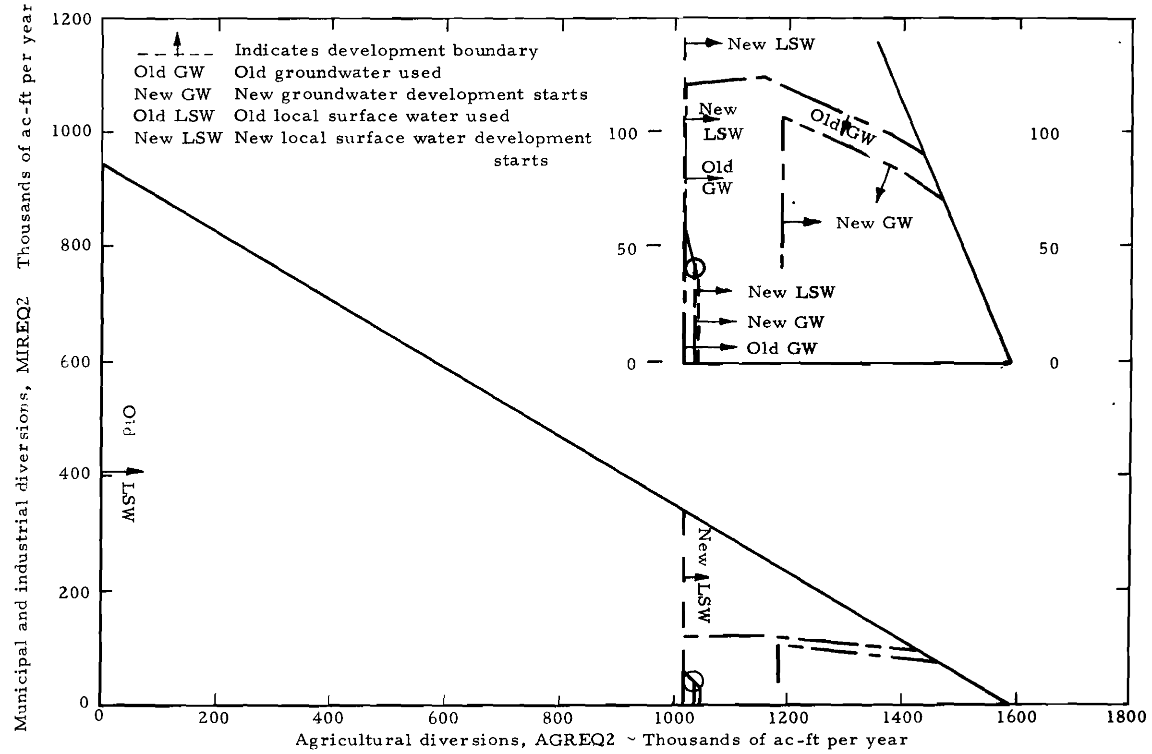


f) Shadow price of imported water.

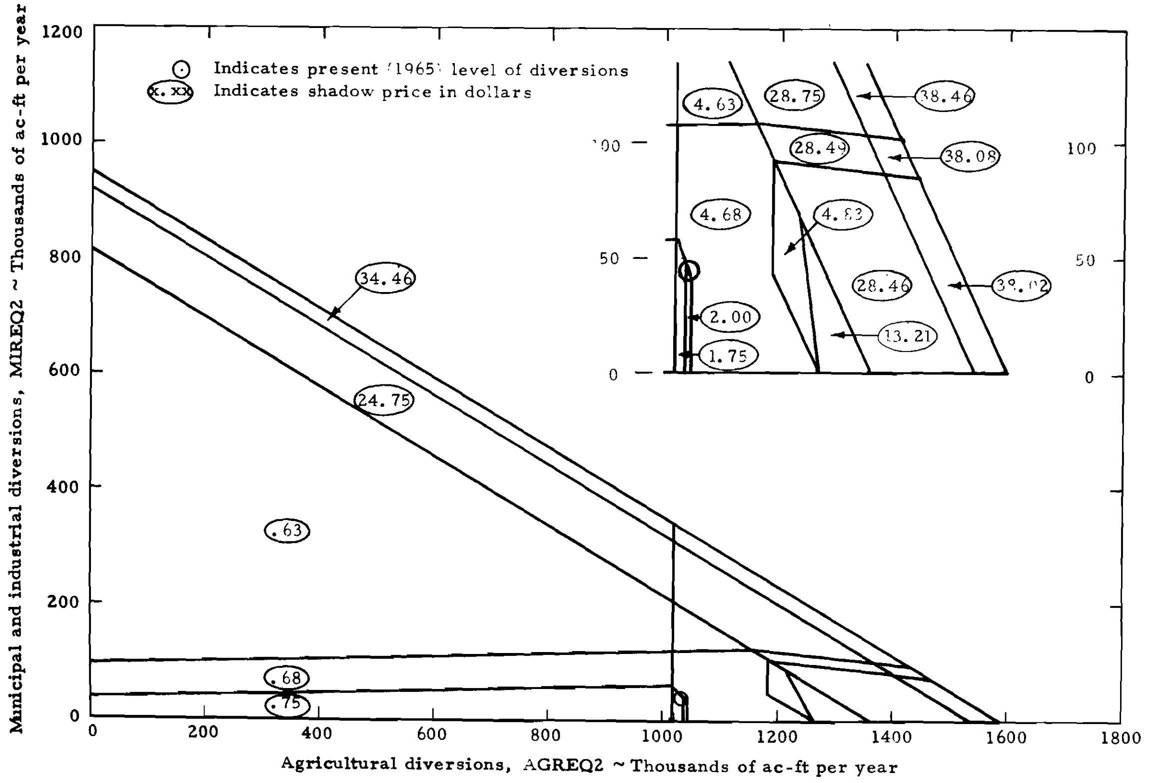


e) Development map for surface storage and limiting conditions.

Figure C-1. Continued.

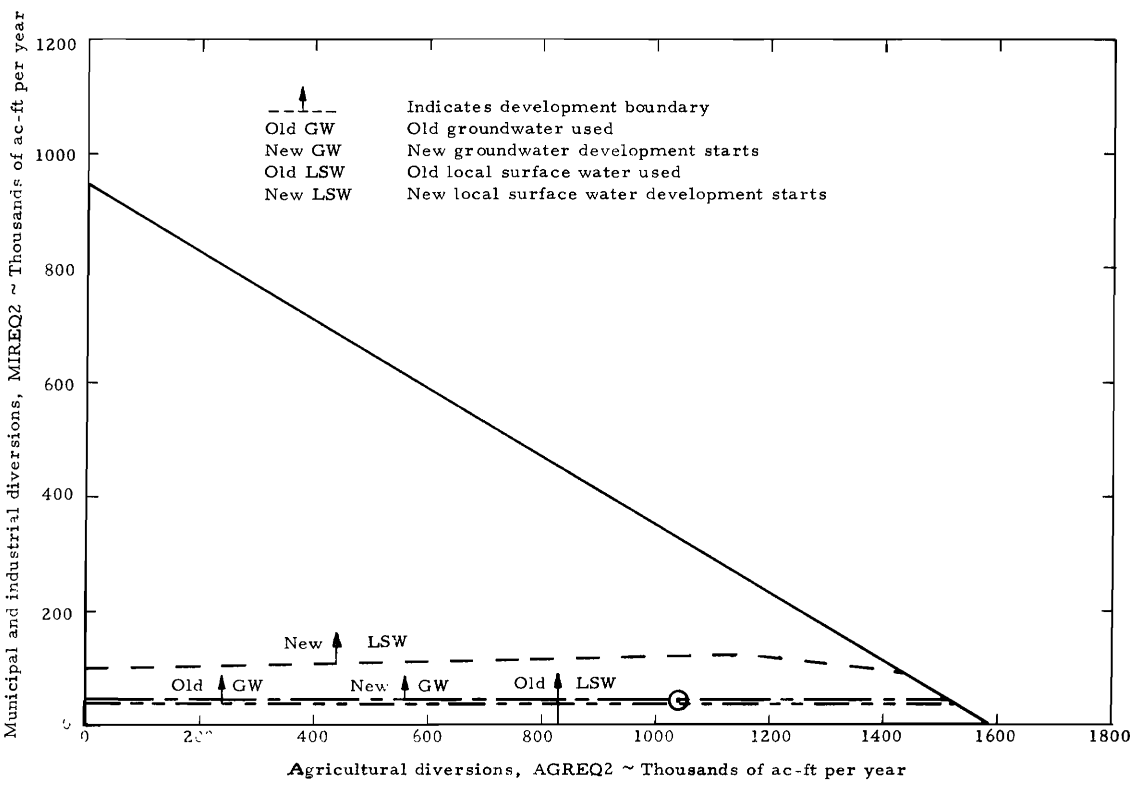


a) Agricultural development map.

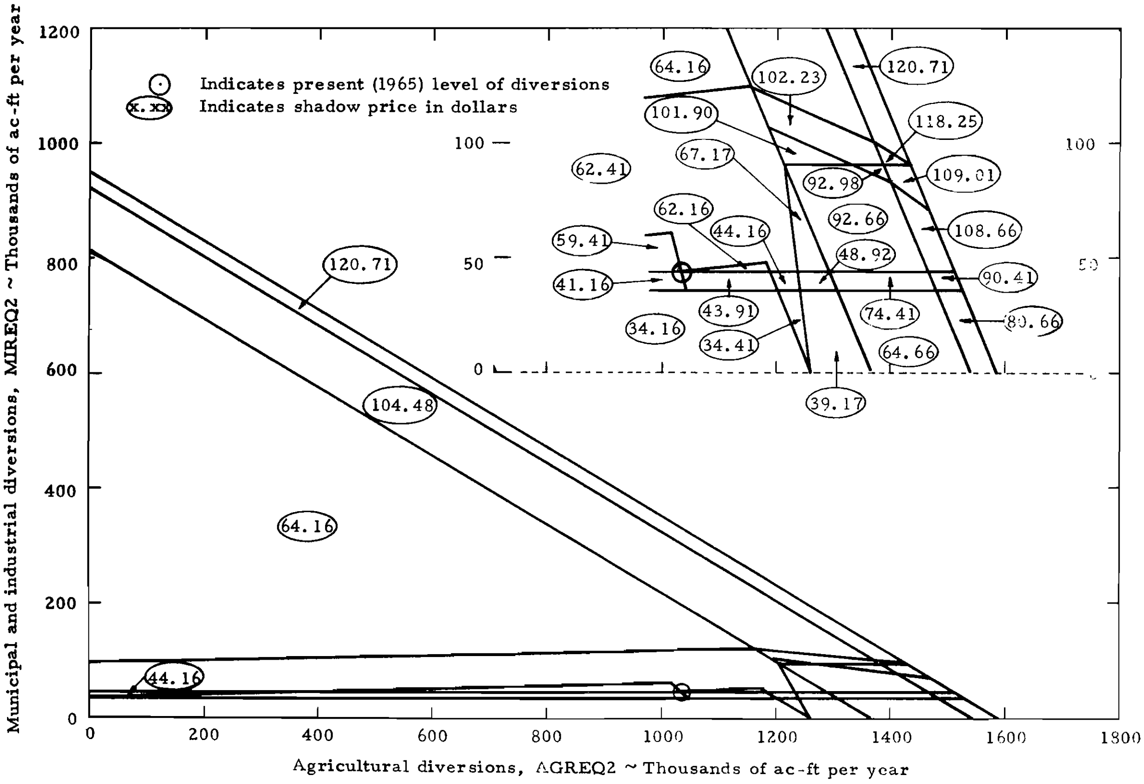


b) Shadow price of agricultural diversions.

Figure C-2. Supply function maps for hydrologic study unit 2.

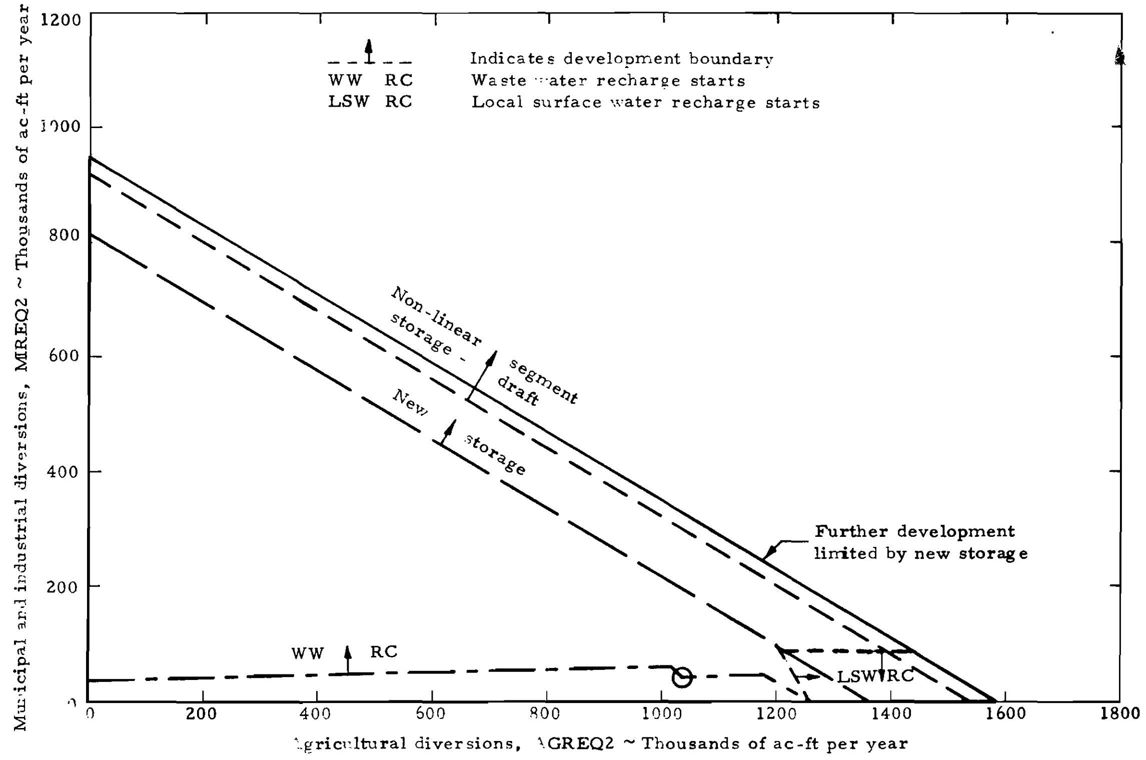


c) Municipal and industrial development map.

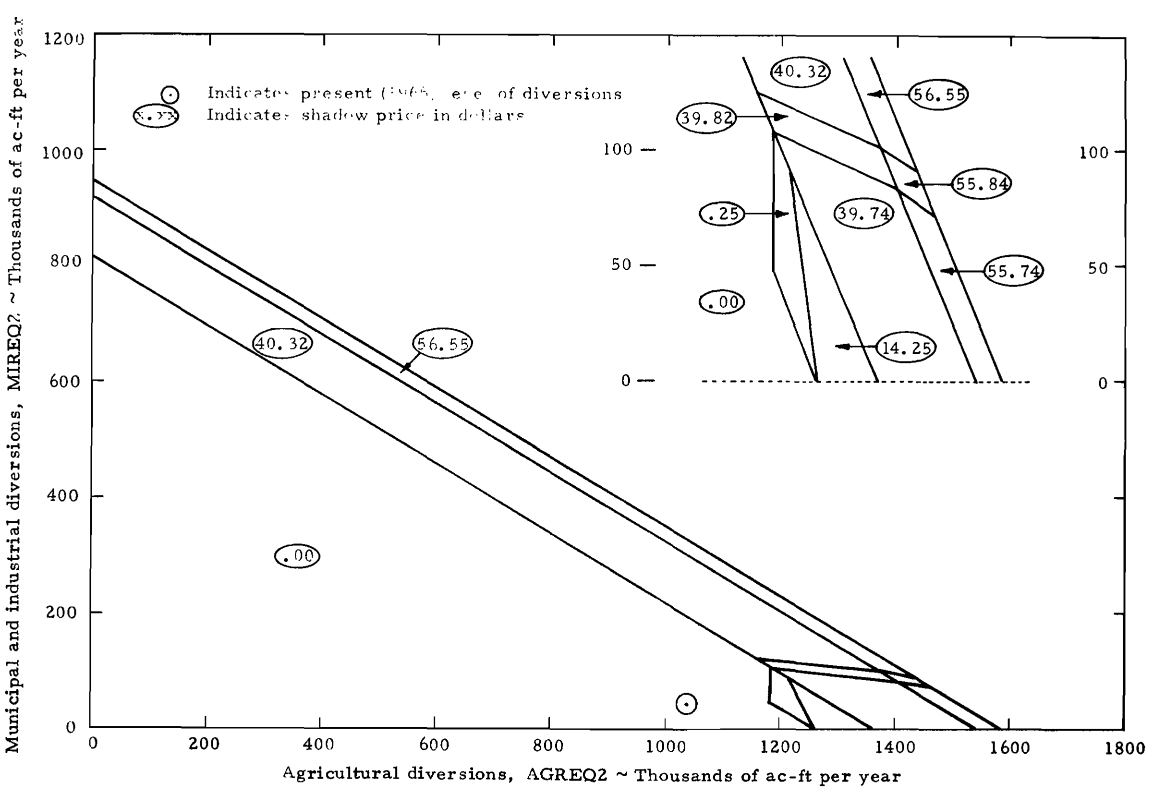


d) Shadow price of municipal and industrial diversions.

Figure C-2. Continued.

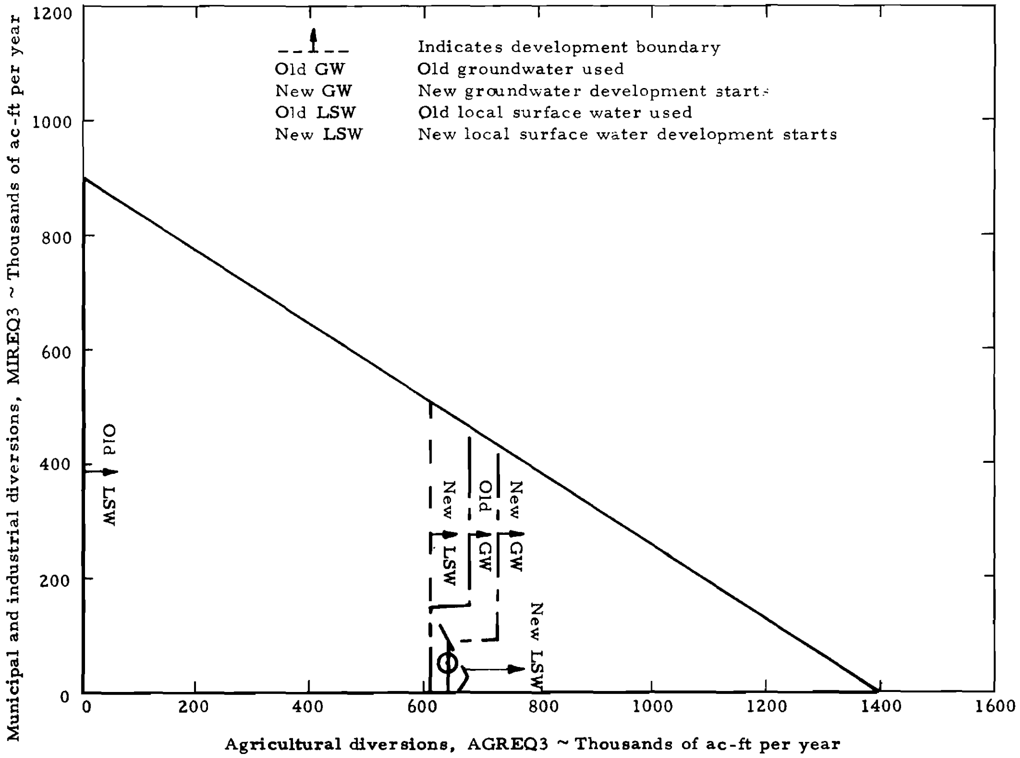


e) Development map for surface storage and limiting conditions.

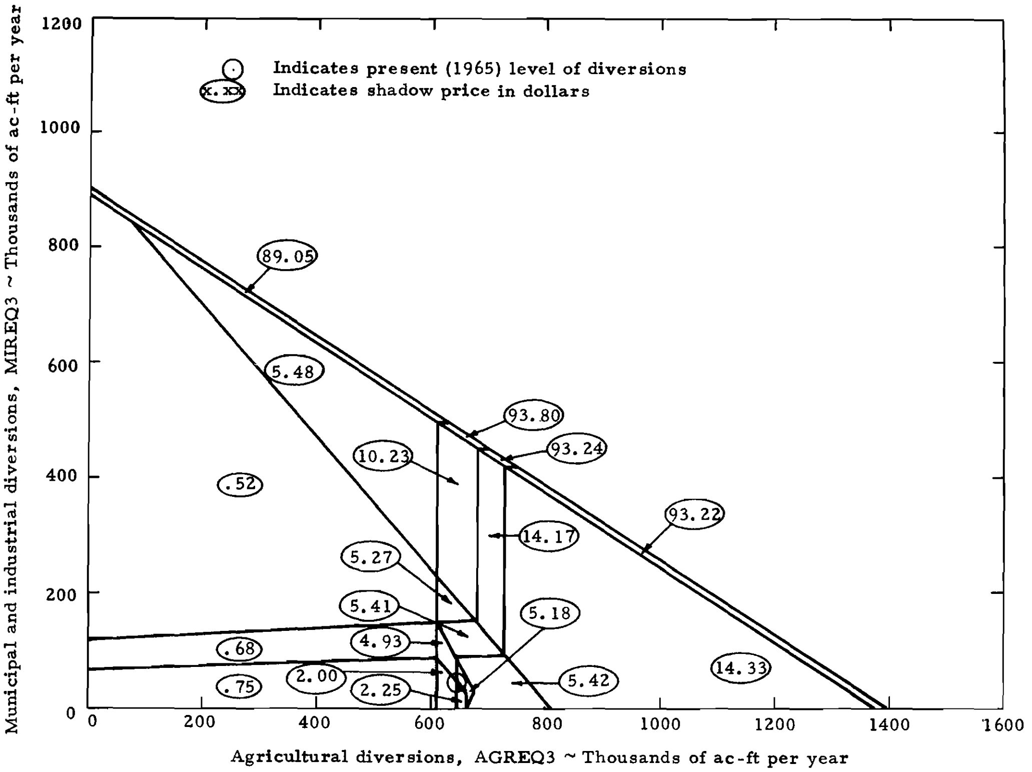


f) Shadow price of imported water.

Figure C-2. Continued.

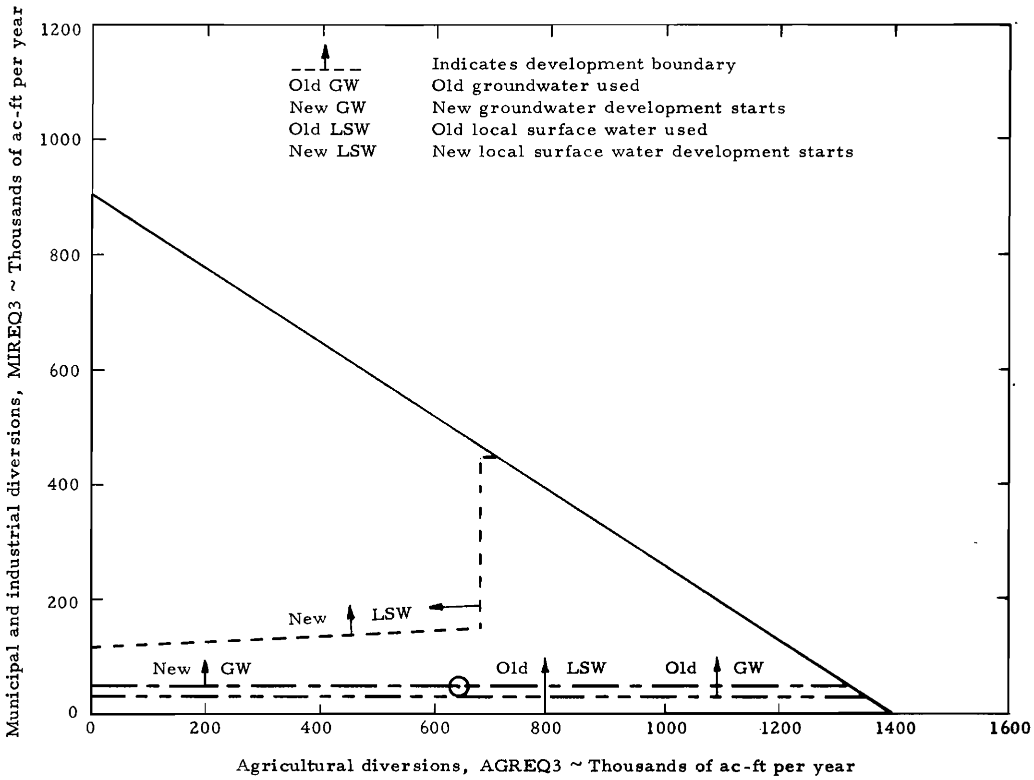


a) Agricultural development map.

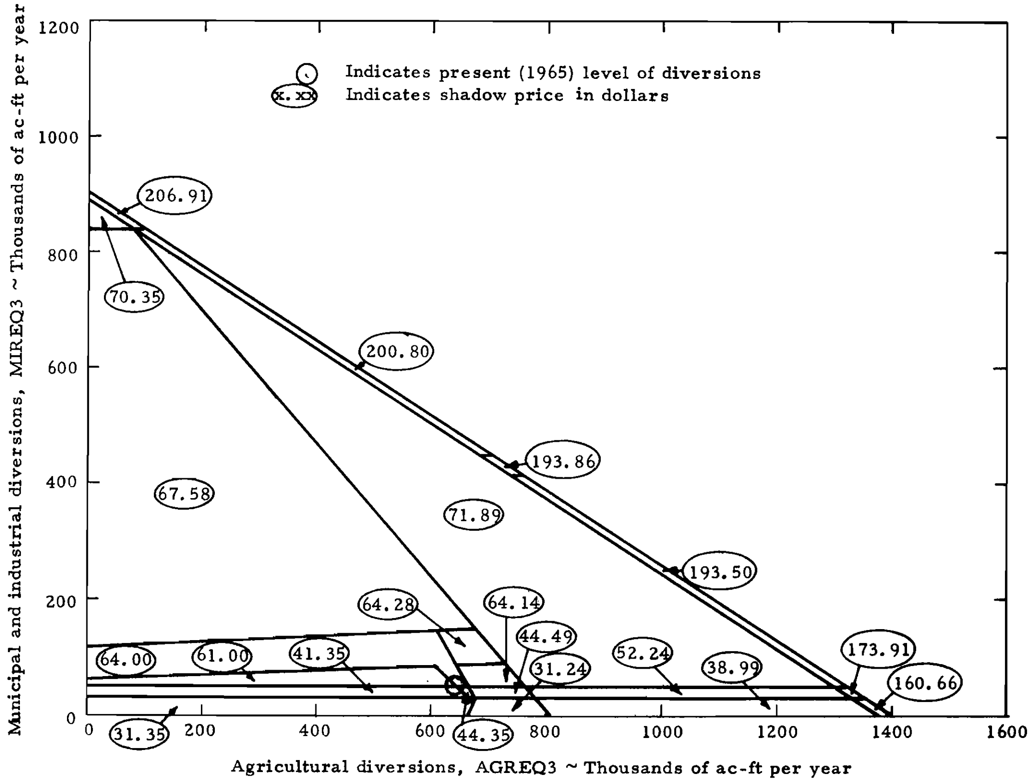


b) Shadow price of agricultural diversions.

Figure C-3. Supply function maps for hydrologic study unit 3.

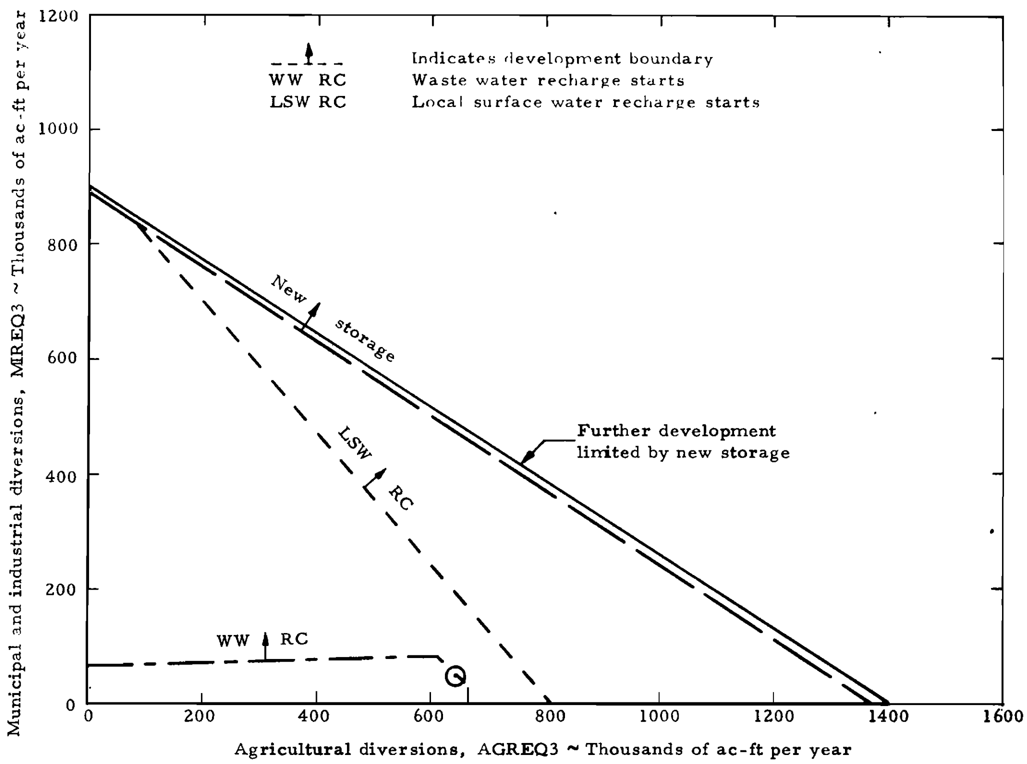


c) Municipal and industrial development map.

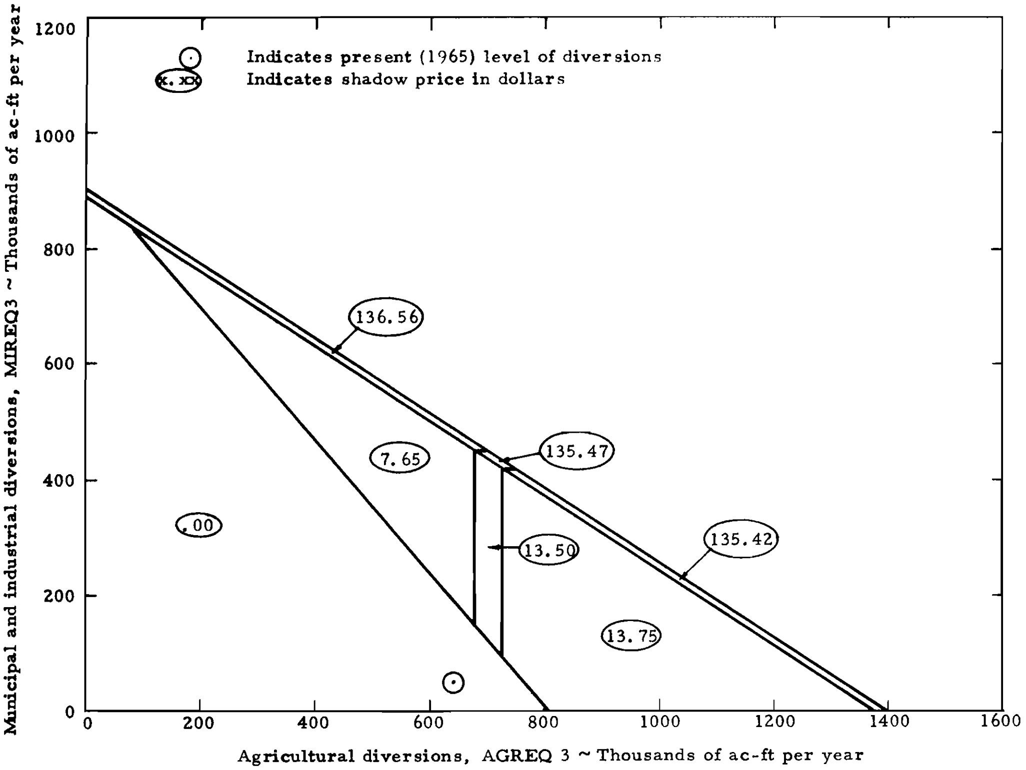


d) Shadow price of municipal and industrial diversions.

Figure C-3. Continued.

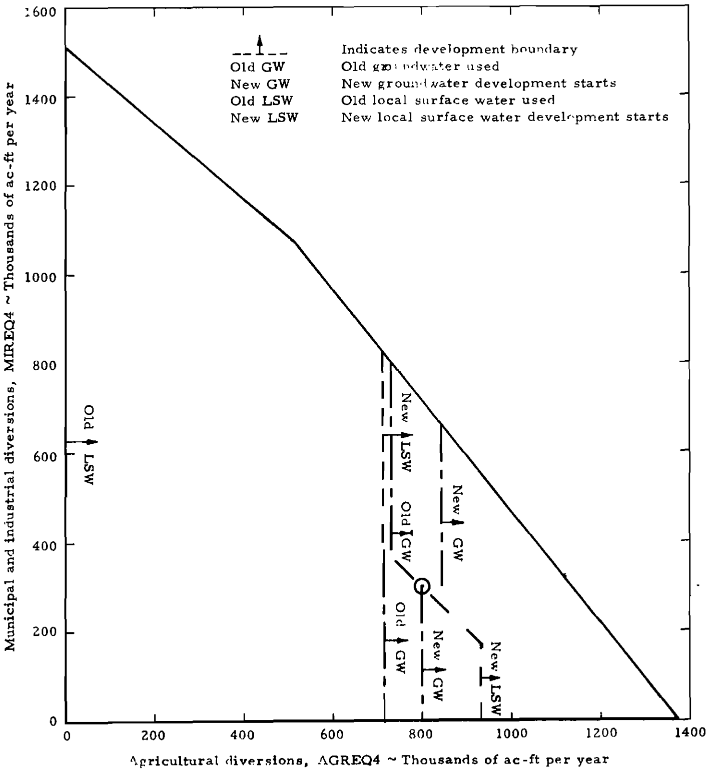


e) Development map for surface storage and limiting conditions.

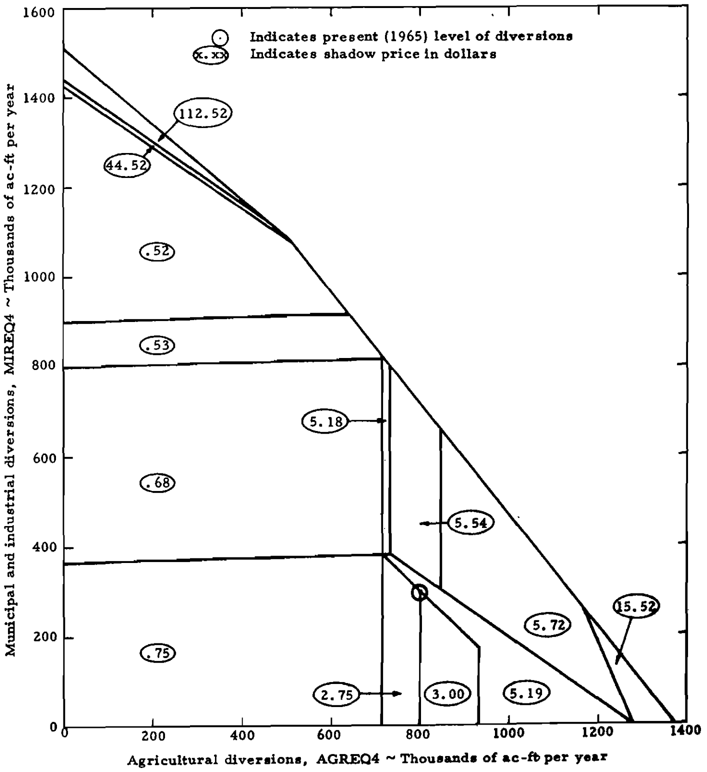


f) Shadow price of imported water.

Figure C-3. Continued.

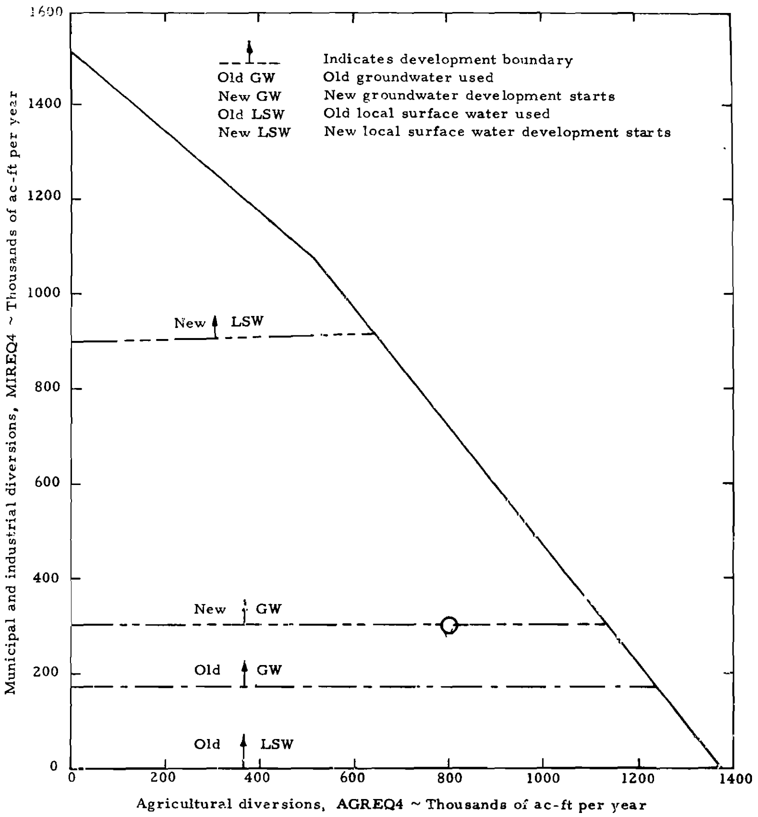


a) Agricultural development map.

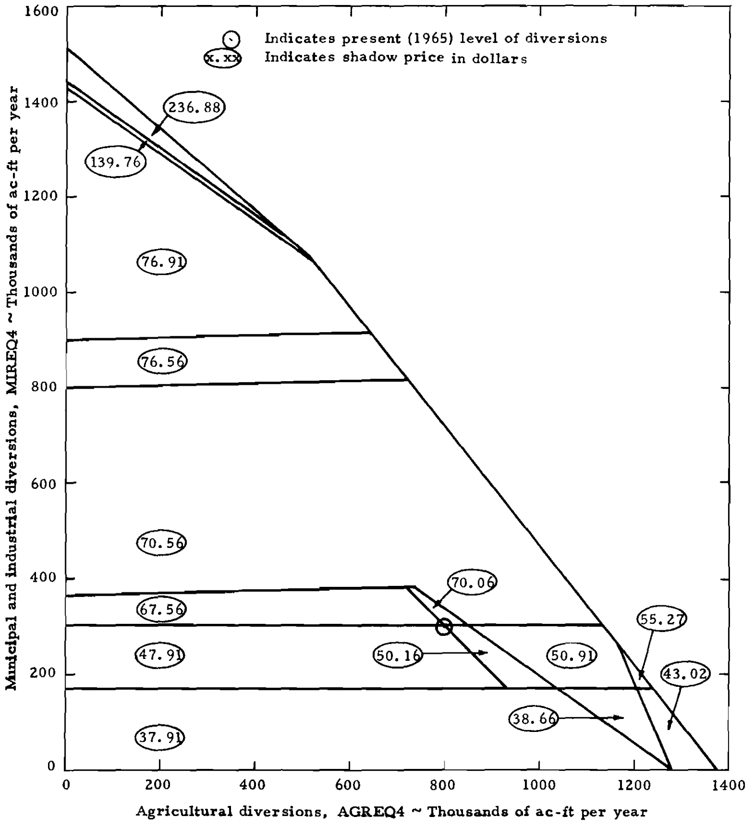


b) Shadow price of agricultural diversions.

Figure C-4. Supply function maps for hydrologic study unit 4.

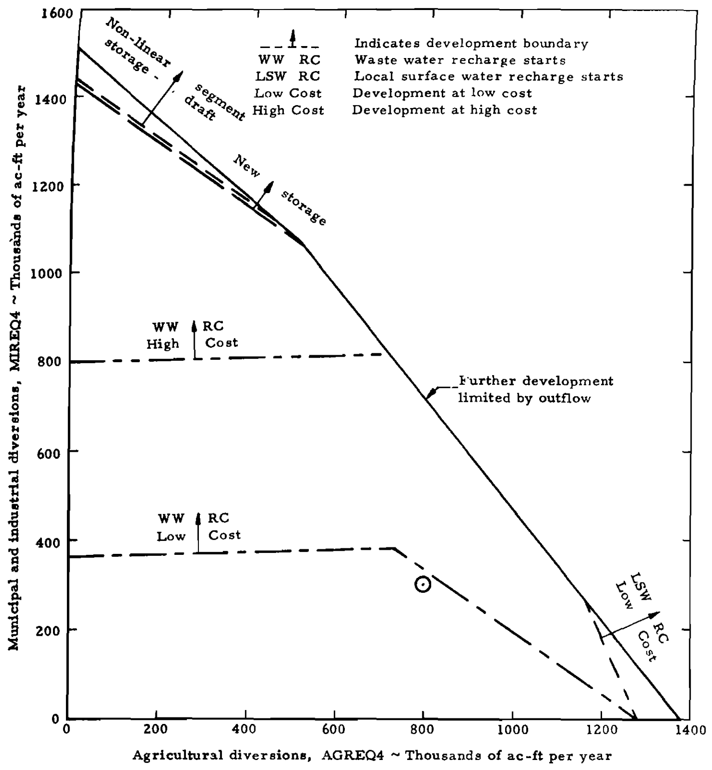


c) Municipal and industrial development map.

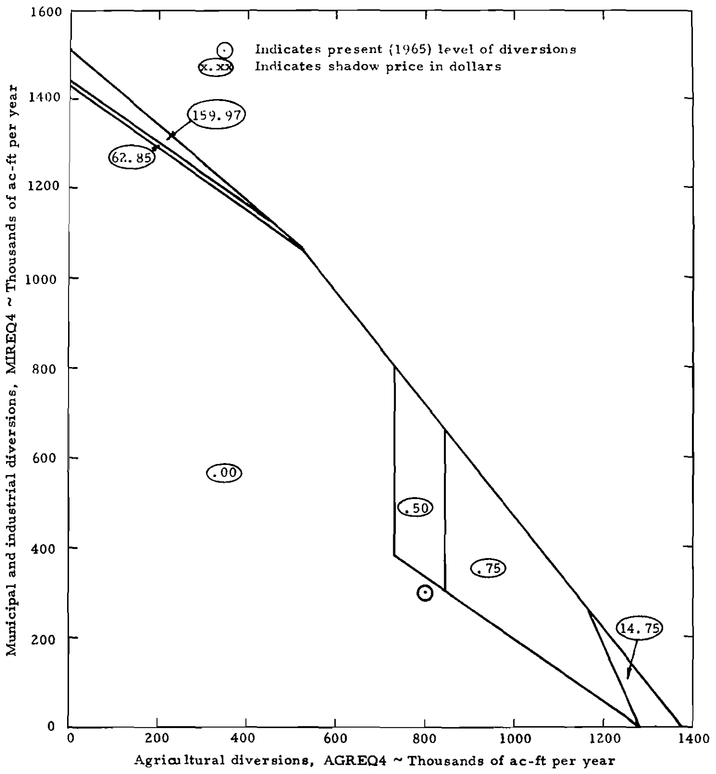


d) Shadow price of municipal and industrial diversions.

Figure C-4. Continued.

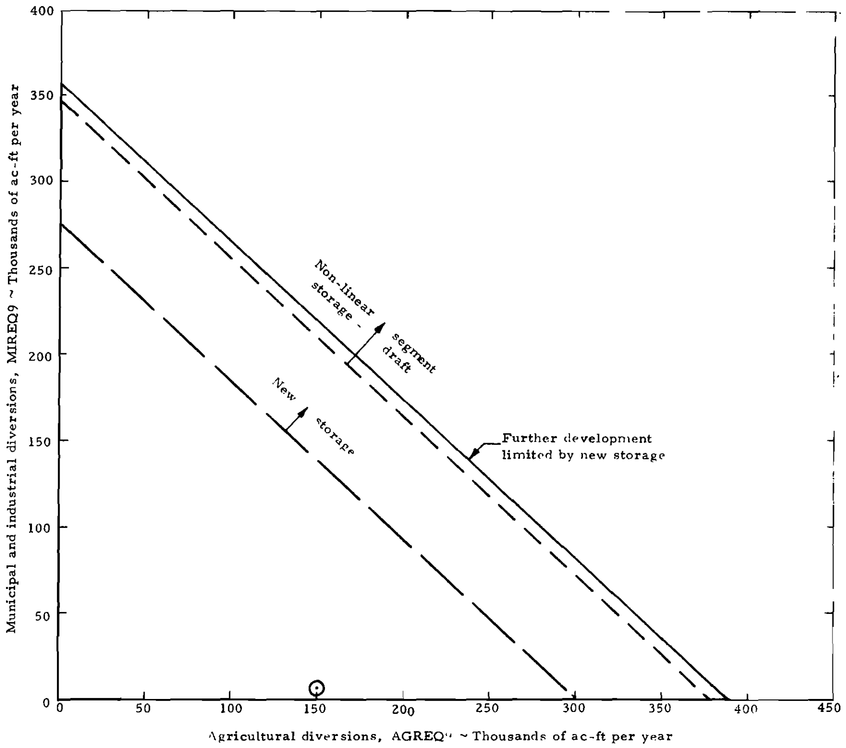


e) Development map for surface storage and limiting conditions.

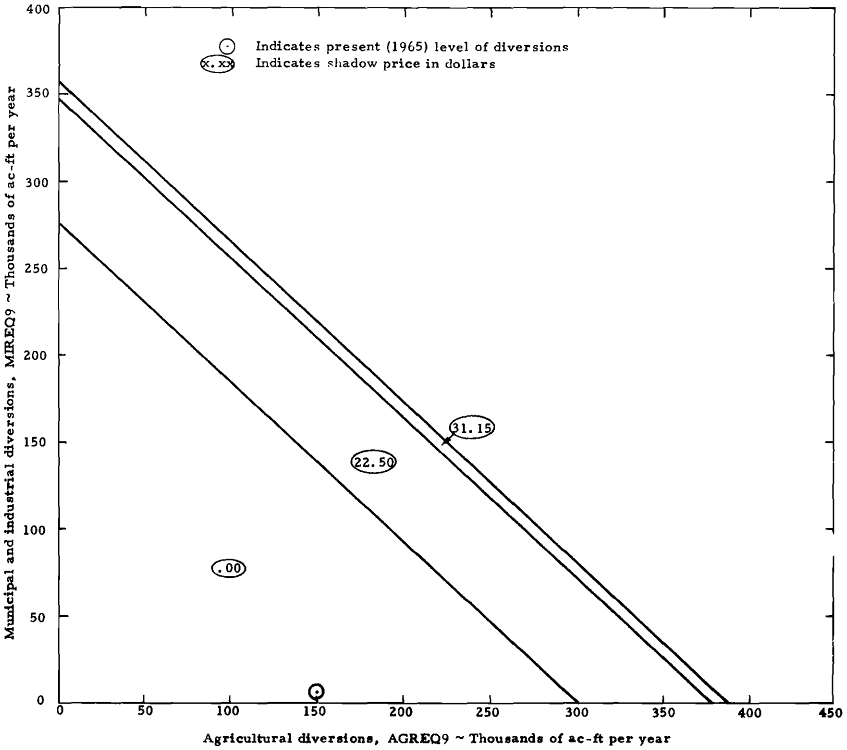


f) Shadow price of imported water.

Figure C-4. Continued.

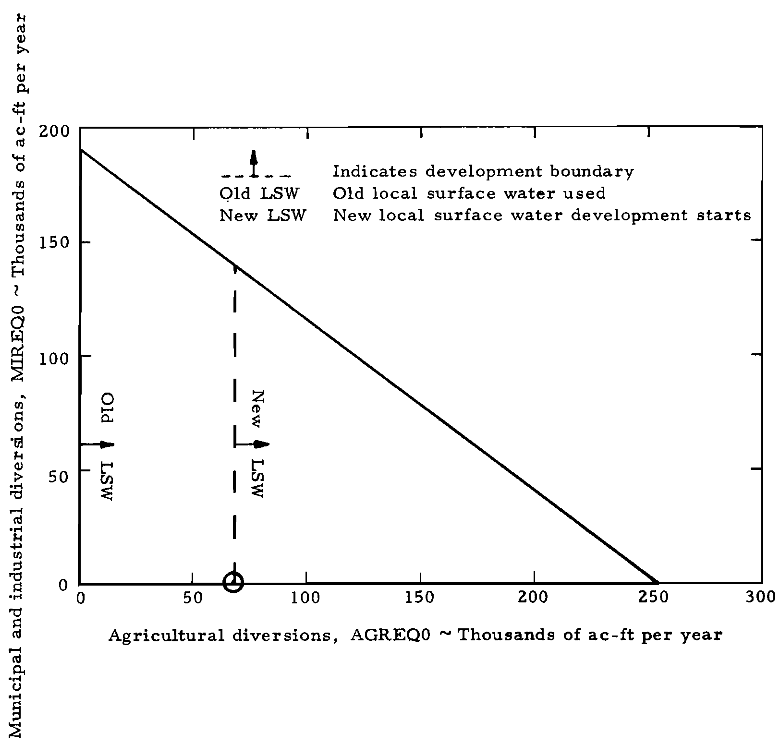


e) Development map for surface storage and limiting conditions.

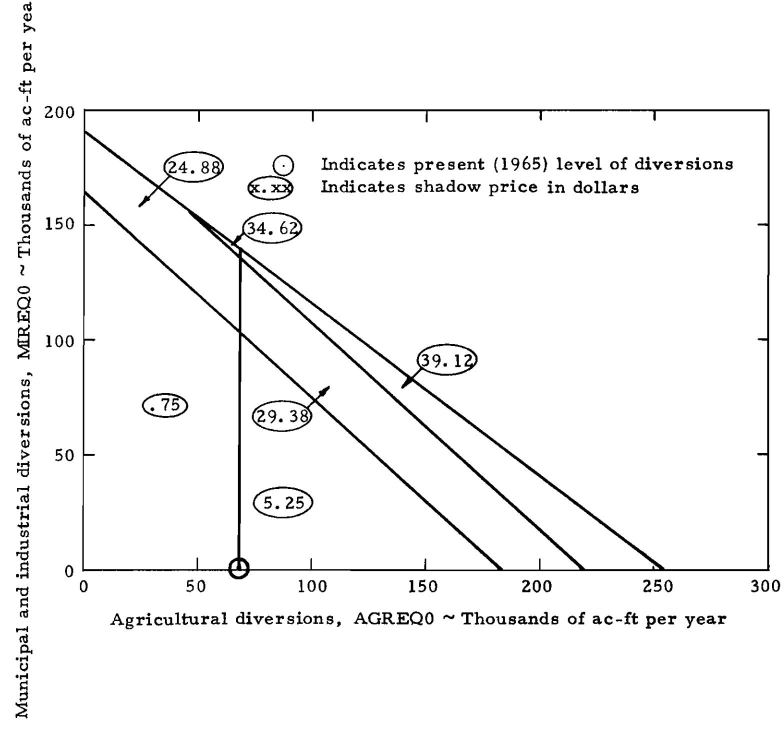


f) Shadow price of imported water.

Figure C-9. Continued.

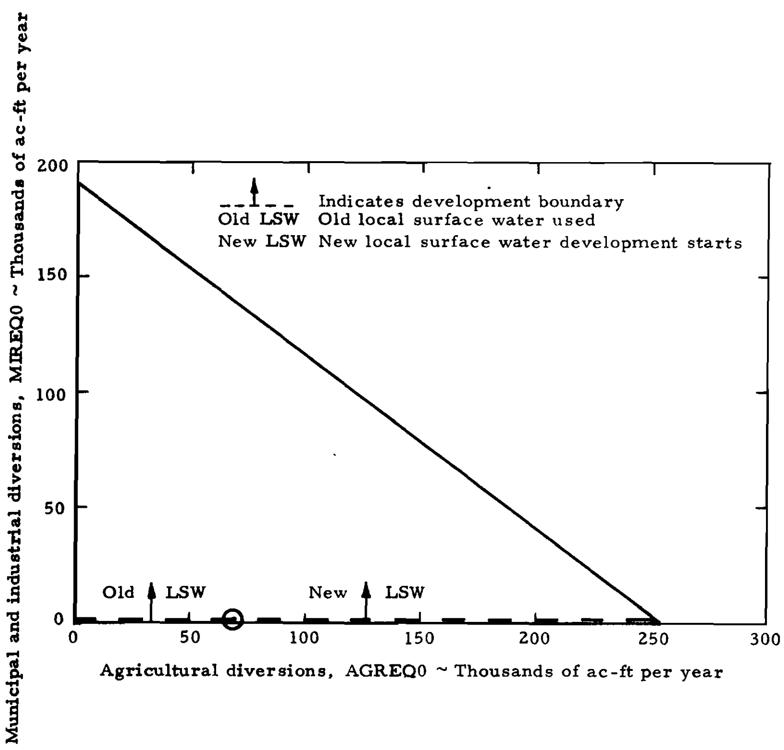


a) Agricultural development map.

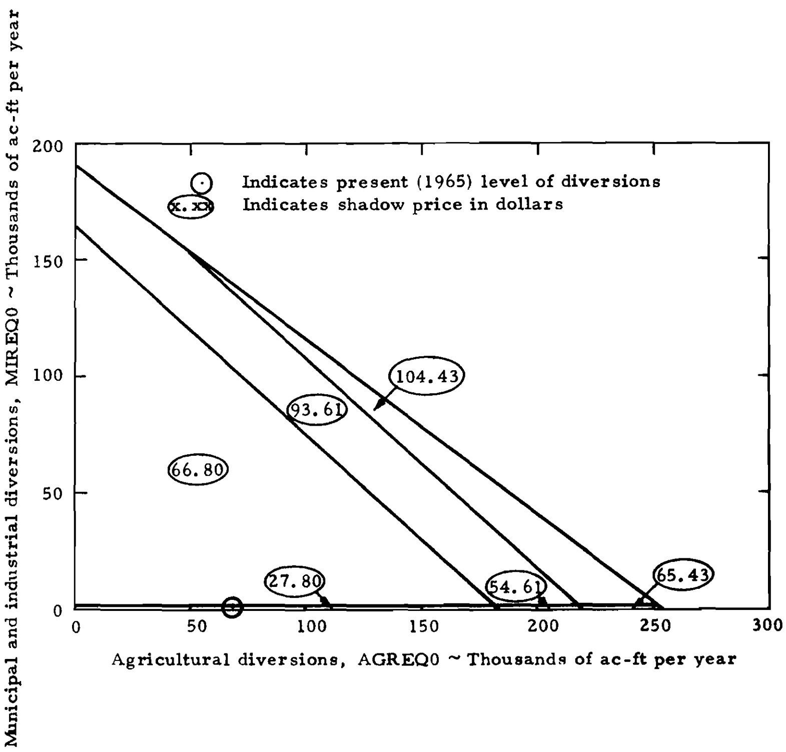


b) Shadow price of agricultural diversions.

Figure C-10. Supply function maps for hydrologic study unit 10.

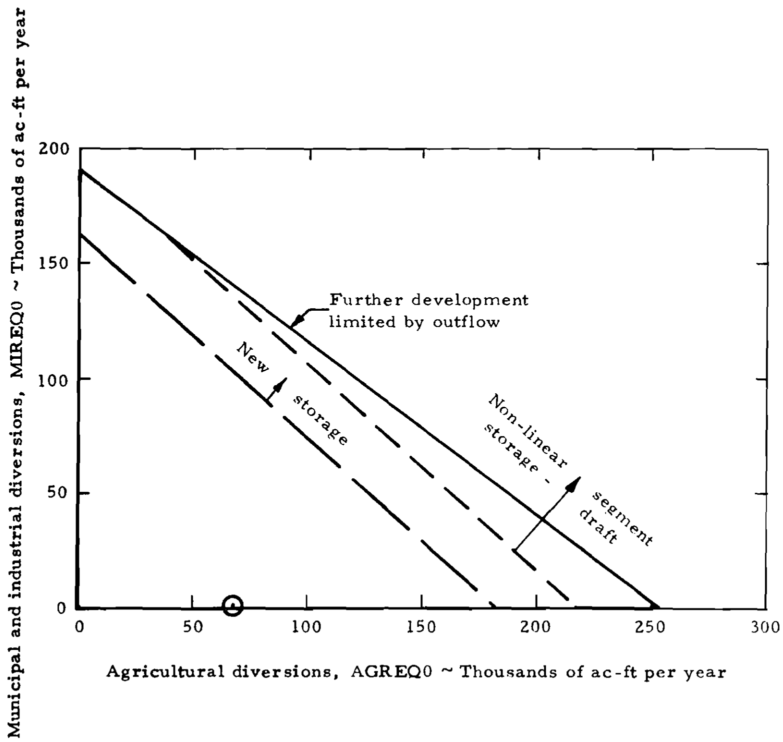


c) Municipal and industrial development map.

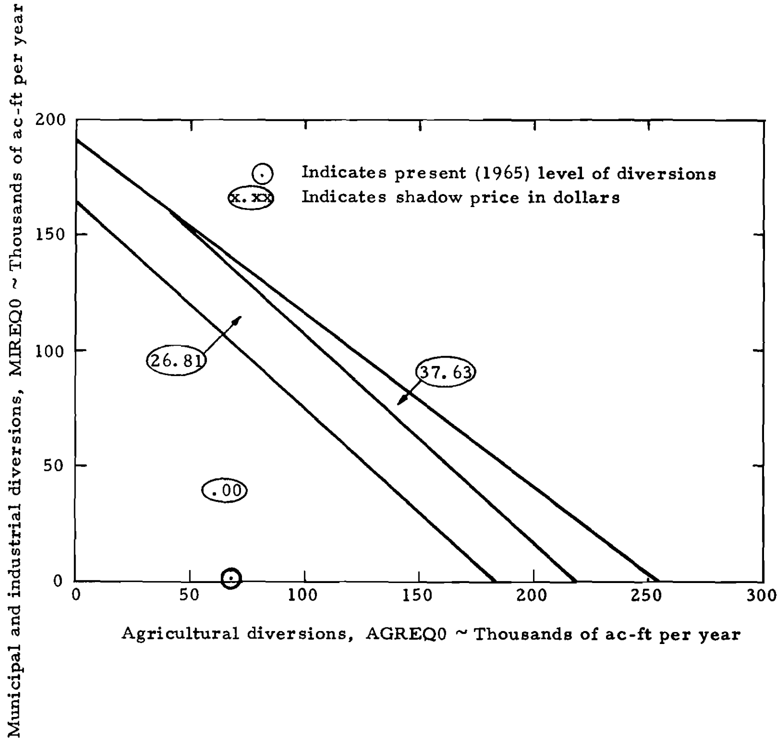


d) Shadow price of municipal and industrial diversions.

Figure C-10. Continued.



e) Development map for surface storage and limiting conditions.



f) Shadow price of imported water.

Figure C-10. Continued.