Досліджено напружений стан прямокутних залізобетонних балок, підсилених нарощуванням розтягнутої стрижневої арматури при дії навантаження. На основі різних нормативних документів розроблено дві принципові методики оцінки надійності підсилених балок. Відповідно до розроблених методик, встановлено надійність досліджуваних конструкцій та отримано результати якісних і кількісних показників надійності, а саме індексів надійності та ймовірностей безвідмовної роботи. Також проаналізовано вплив на загальну оцінку надійності прийнятих в розрахунок стохастичних параметрів резерву несучої здатності нормальних перерізів підсилених балок.

Встановлення фактичних показників надійності балок, підсилених при дії навантаження, дозволить більш ефективно та економічно підходити до питання саме реконструкції елементів будівель і споруд. Зокрема, це стосується підсилення згинаних залізобетонних елементів, що знаходяться в експлуатації. Крім того, отримані результати дослідження надійності дозволяють в подальшому, за достатньої точності розрахунку, оперувати тими змінними параметрами, які мають максимальний вплив на дисперсію граничного згинального моменту досліджуваних балок. Розроблені принципові методики оцінки надійності також дають можливість проектувати підсилені залізобетонні згинані елементи із заданим рівнем надійності (економічність рішень) – ймовірністю безвідмовної роботи, що, в тому числі, може бути предметом майбутніх досліджень. Насамкінець, використовуючи отримані результати, виникає можливість ефективніше підходити до питання вибору методу підсилення.

Таким чином, пропонується методика оцінки надійності адаптовануа до чинних норм проектування України, яка містить в собі відносно нескладний математичний апарат розрахунку. Більше того, на відміну від результатів попередніх досліджень, отримані значення показників надійності є наочними, оскільки мають розподіл близький до пропорційності в залежності від рівня навантаження та діаметра арматури нарощування. Так, для індексів надійності β_i діапазон значень склав від 3,35 до 3,45, а для ймовірностей безвідмовної роботи $P(\beta)_i$ – від 0.999596 до 0,999720 (в сторону зростання рівня надійності при більшому діаметрі арматури нарощування та рівні навантаження в момент підсилення). При цьому розбіжність між ідентичними значеннями показників, знайденими відповідно до інженерної та деформаційної моделі розрахунку, склала лише до 8 %. Даний факт дозволяє використовувати розроблену методику в практиці проектування. Тому, враховуючи майже повну відсутність досліджень в області оцінки надійності залізобетонних згинаних елементів, підсилених при дії навантаження, одержані нами результати можна вважати актуальними

Ключові слова: залізобетонна балка, підсилення, стохастичні параметри, оцінка надійності, ймовірність безвідмовної роботи, рівень навантаження

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DEVELOPMENT OF THE PROCEDURE FOR THE ESTIMATION OF RELIABILITY OF REINFORCED CONCRETE BEAMS, STRENGTHENED BY BUILDING UP THE STRETCHED REINFORCING BARS UNDER LOAD

R. Khmil
PhD, Associate Professor*
E-mail: roman.y.khmil@lpnu.ua

R. Tytarenko
Postgraduate student*
E-mail: romantaren27@gmail.com

Y. Blikharskyy
PhD, Senior lecturer**
E-mail: yaroslav.z.blikharskyy@lpnu.ua

P. Vegera
PhD, Assistant*
E-mail: pavlo.i.vehera@lpnu.ua

*Department of building constructions and bridges***
Department of highways and bridges*
***Lviv Polytechnic National University
S. Bandery str., 12, Lviv, Ukraine, 79013

1. Introduction

The most widespread composite material for construction of extended bending elements of buildings and structures in the world is reinforced concrete. The material of most of building structures that bend during operation is the precast or monolithic reinforced concrete. Such structures include crossbars of frames, trusses, arches, beams, etc. Such structures have been in operation for at least 30...40 years by now. Therefore, they often become physically and morally outdated. Taking into consideration the above, a rational solution to the problem of the economic value of new construction is an effective use of the restored or reinforced elements of existing buildings and structures.

The effectiveness of application of different methods of strengthening of reinforced concrete bending elements is one of the urgent tasks of research at present. Works [1–4] investigate the real stressed state of reinforced concrete beams strengthened by different methods under the action of load. There is also a search for options to improve efficiency of strengthening methods. At the same time, there is hardly any research on the actual stressed state of the bent reinforced concrete elements strengthened by building up the stretched reinforcing bars under the action of load, which is almost always present during strengthening. The mentioned method of strengthening is extremely effective both in terms of time and complexity (economy), and its impact on the stressed state of a structure.

In turn, probabilistic methods for calculation of construction structures become now more widespread. In contrast to the semi-probabilistic method [5], they provide a possibility to assign a guaranteed level of reliability to a structure at the design stage and to establish a quantitative reliability estimate in the form of a probability indicator of failure-free operation (efficiency of solutions). Moreover, the use of a system of reliability coefficients according to paper [5], which takes into consideration the random nature of uncertain conditions of operation of structures, often leads to lowering the actual level of reliability for responsible structures. This level may be overestimated for buildings of CC1 class, according to paper [6]. However, at present, researchers almost do not apply probabilistic methods in the calculation practice, because of the complexity of a mathematical apparatus of these methods, as well as the absence of a single objective methodology for assessment of reliability. This is, to a large extent, due to a significant level of subjectivity in approach to project variables of parameters of the reserve of bearing capacity of structures. In addition, there is a lack of sufficient normative base for calculation, since a significant number of variables have large statistical variances.

Proceeding from the above, it is recommended to conduct calculation of reinforced concrete structures (including the strengthened ones) as systems, which contain stochastic parameters, in a probabilistic approach. Therefore, today the development of the problem of assessment of reliability of such systems is a very urgent task. It requires further development and improvement.

2. Literature review and problem statement

Paper [7] presented initial ideas of a statistical approach to assessment of reliability of building structures. They acquired further development and improvement in paper [8]. Despite the significance of the ideas outlined, there is no development of complete mathematical models for reliability calculation in these works. They show the statistical nature of strength characteristics of materials and load parameters only. In addition, the works proved a need to calculate structures in a probabilistic definition. Scientific paper [9] developed a principal position of the concept of safety of buildings and structures and derived a formula for a reserve coefficient or "a safety characteristic" of a structure, which depends on probability of its failure. Work [10] initiated development of the theory of reliability for building structures in the countries of Europe and North America. It proposed a formula for the determination of a so-called "security index", which made it possible to assess reliability of structures under laws of distribution of random variables, which are different from a normal one. The disadvantage of the above papers [9, 10] is the impossibility to take into consideration many stochastic parameters of the reserve of bearing capacity of structures, including the geometry of a cross section, in the theory of calculation of reliability. Moreover, the use of distribution laws, different from a normal one, complicates the mathematical apparatus for calculation significantly, which makes it almost impossible to apply them in the design practice. We should also add that the range of statistical variability of stochastic parameters of building structures lies within the limits of 5...25 %, which, in turn, makes it possible to linearize functions of random variables to the normal distribution law. Monographs [11, 12] optimized and described the accumulated experience of previous studies, as well as principles of implementation of the theory of reliability in building design. All the studies described in them relate only to the non-strengthened elements.

Papers [13, 14] described new approaches to assessment of reliability of steel structures. Authors proposed a description of vulnerability of a constructive element, as well as survivability of building structures. However, along with progressive solutions developed in the works, the problem of assessment of reliability of reinforced structures remained open. Work [15] considered only the general principles of assessment of reliability of buildings and structures, as well as possibilities of implementation of the theory of reliability in the design practice. Authors of paper [16] proposed a methodology for assessment of reliability of the non-strengthened trusses of reinforced concrete under conditions of incomplete information on their operation (unusual in the practice of construction and reconstruction).

To date, there is a number of scientific studies carried out on issues of assessment of reliability of the non-strengthened structures [17, 18] or analysis of deflected mode of damaged [19–21] reinforced concrete structures of various types. Despite the novelty of the obtained results, authors of works [17, 18] took into consideration strength variable parameters of the reserve of bearing capacity of structures only when calculating the reliability of columns and beams. In turn, papers [19–21] studied the actual deflected mode of damaged pillars, beams, and pipes – without assessment of reliability of elements under consideration. It is possible to use the obtained results in the studies for determination of the actual bearing capacity of structures. We should mention also the developed methods for assessment of reliability of reinforced concrete beams strengthened by external reinforcement [22–24]. Thus, although the above methods model a structure of strengthening of bending elements, but they do not take into consideration a load, which is almost always present during strengthening.

Paper [25] proposed and tested a method for assessment of reliability of reinforced concrete beams and plates strengthened by a composite tape under an action of load. The advantage of the paper is that it examined beams and plates with insufficient bending strength – that is, it considered a normal cross section of structures (a typical one for most cases of strengthening of bending elements). On the other hand, a complex mathematical apparatus for assessment of reliability based on statistical data from previous studies and adaptation to design standards [26] does not makes possible to apply this technique in design practice fully. In addition, the paper investigated reliability at only one load level at the moment of strengthening, which does not give possibility to construct a dependence of β_i indices on this parameter. Authors of work [27] obtained reliability indices for beams strengthened by a carbon-plastic tape under an action of load. Here, the advantage is the investigation of reliability at different load levels at the moment of strengthening, as well as different cross sections of a tape. The disadvantage is that considered beams are not sufficiently strong for cross section (sloping sections often do not require strengthening during reconstruction). There are some significant disadvantages in work [28]; despite it processed experimental data of more than 250 beams reinforced by composite reinforcement, as well as data on a use of different design norms in calculation of reliability. First of all, it considered only one load level at the moment of strengthening along with the study of beams with insufficient sectional strength. Second, the work registered a significant spread of safety coefficients into account from 43 to 57 % – for adjacent values in accordance with two design norms. It is possible to apply the β_i indices obtained in [29] in design practice in the context of reliability assessment of reinforced concrete extended bending elements (first of all, slab parts of bridges) strengthened with a carbon-plastic tape only. In addition, the work does not take into consideration the actual level of load fully at strengthening of real bridge elements. However, we should note that, in contrast to previous works, it proposed an obvious classification of β_i indices and corresponding probabilities of failure $p_{F,i}$, which gives more practical value to this study.

Thus, there are reasons to believe that the study of the problem of quantitative assessment of reliability of reinforced concrete bending elements reinforced by an action of load is insufficient. As it follows from the analysis of literature data, development of a level of influence of certain stochastic parameters on the investigated structures is not sufficient for now. First of all, because of complexity of parameters control under specific conditions (secondary parameters neglected), as well as their large statistical variances. In addition, there is a lack of a single objective criterion in approaches to assessment of reliability (at present, there is no sufficient regulatory framework on this issue). In turn, the increasing number of papers on the reconstruction of elements of buildings and structures, development of a general theory of reliability and continuation of work on adaptation of norms [30] in Ukraine indicate the need for the further development of this problem. Therefore, the above circumstances necessitate further research in this direction and indicate its relevance.

3. The aim and objectives of the study

The objective of the study is to develop a basic methodology for assessment of reliability of rectangular reinforced concrete beams strengthened by building up the stretched reinforcing bars under an action of load, which will reflect real conditions of operation of a structure. The principal feature of the methodology is the proposal to consider a level of load of beams at the moment of strengthening as a stochastic parameter. This will enable to simulate an actual impact of all stochastic parameters on the reserve of bearing capacity as accurately as possible.

We formulated the following tasks to achieve the objective:

– adaptation of the existing method for assessment of reliability of new design structures for strengthened reinforced concrete structures [13]. Determining the dependences for calculation of qualitative (so-called "safety characteristics" or "reliability indices") and quantitative (probability of failure-free operation) reliability indicators for reinforced beams;

– application of the following models of calculated cross section of a strengthened beam at creation of assessment methodology: a power model in accordance with norms [32] and a deformation model in accordance with the current norms of design [5];

– testing of suitability of the developed methodologies under an action of different levels of load at the moment of strengthening and different diameters of reinforcement extension;

– formulation of recommendations on a choice of the most rational method for assessment of reliability in dependence on the accepted model of a calculated cross section of beams strengthened under the action of load.

4. Materials and methods to study the reliability of beams strengthened under the action of load

4. 1. Development of stochastic parameters of the reserve of bearing capacity in a strengthened beam and a load level at the moment of strengthening

Based on the facts above, we describe variable parameters of the reserve of bearing capacity of a normal cross section of a strengthened beam, as well as a load level at the moment of strengthening. These parameters probably have the maximum impact on qualitative (reliability index β, level of safety) and, accordingly, quantitative assessment of reliability of the design – probability of failure-free operation $P(\beta)$.

Thus, we propose to take strength of materials, geometry of a strengthened cross section and a level of current load on a beam at the moment of strengthening as random, statistically independent parameters that are subject to the normal distribution law.

4. 2. Tools for theoretical study into reliability of strengthened beams

For now, the transition from design of reinforced concrete structures according to norms [32] based on the power calculation model to design according to norms [5] based on the deformation model of calculation of construction goes in Ukraine. Proceeding from this, we offer the basic methods of reliability assessment developed based on both of these models below. We use well-known theses of the theory of probabilities [33] and recommendations for application of these theses to building structures [34] to develop the methods.

4. 3. Development of the basic methodology for assessment of reliability based on the calculation model of the cross section in accordance with the norms [32]

We record a random value of the boundary bending moment $\tilde{M}_{ult}$ perceived by a beam strengthened by building up the stretched reinforcing bars under an action of load, taking into consideration the presence of reinforcement in the compressed zone of the cross section of the strengthened beam (for preservation of the condition $\xi \leq \xi_R$), as follows:

$$\tilde{M}_{ult} = f\left(\tilde{\sigma}_c, \tilde{\sigma}_s, \tilde{\sigma}_{s,add}, \tilde{\sigma}_{sc}, \tilde{\gamma}_{s,dis}^{add}, \tilde{b}, \tilde{d}_{red}\right) = = \tilde{\sigma}_c \tilde{b} \tilde{x}\left(\tilde{d}_{red} - 0{,}5\tilde{x}\right) + \tilde{\sigma}_{sc} A_s^{'}\left(\tilde{d}_{red} - a^{'}\right), \quad (1)$$

where $\tilde{\sigma}_c$ is the random value of concrete strength for compression for the first group of boundary states; $\tilde{\sigma}_s$, $\tilde{\sigma}_{s,add}$ are the random values of strength of the main reinforcing bars and additional reinforcing bars for tension, respectively; $\tilde{\sigma}_{sc}$ is the random value of strength of wire reinforcement for compression; $\tilde{b}$, $\tilde{d}_{red}$ are the random values of a width and a reduced useful height of the section of a strengthened beam, respectively (Fig. 1); $a^{'}$ is the distance from the center of gravity of compressed wire armature to the upper edge of the beam section (Fig. 1); $\tilde{x}$ is the random value of a height of the compressed zone of the section of a reinforced beam, which we find from formula:

$$\tilde{x} = \frac{\tilde{\sigma}_s A_s + \tilde{\sigma}_{s,add} A_{s,add} \tilde{\gamma}^{add}_{s,dis} - \tilde{\sigma}_{sc} A'_s}{\tilde{\sigma}_c \tilde{b}}, \quad (2)$$

where A_s, $A_{s,add}$ are the areas of the section of main reinforcing bars and an additional stretched reinforcing bars, respectively; $\tilde{\gamma}^{add}_{s,dis}$ is the random value of a coefficient of a use of the cross section of additional reinforcing bars, which depends on a load level on a beam before strengthening.

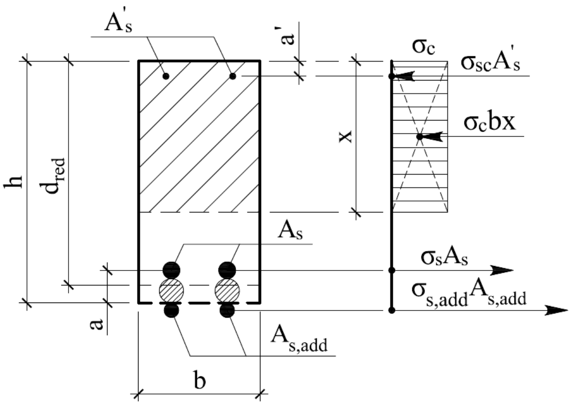

Fig. 1. Scheme of active forces and distribution of tensions in the normal cross section of the beam strengthened with additional reinforcing bars at $\xi \leq \xi_R$

We substitute expression (2) for $\tilde{x}$ in formula (1) for $\tilde{M}_{ult}$ with subsequent staged simplification and we obtain:

$$\tilde{M}_{ult} = \left(\tilde{\sigma}_s A_s + \tilde{\sigma}_{s,add} A_{s,add} \tilde{\gamma}^{add}_{s,dis}\right) \tilde{d}_{red} - \\ - \frac{0,5}{\tilde{\sigma}_c \tilde{b}} \left(\tilde{\sigma}^2_s A^2_s + \tilde{\sigma}^2_{s,add} A^2_{s,add} \tilde{\gamma}^{add\,2}_{s,dis}\right) - \\ - \frac{\tilde{\sigma}_s A_s \tilde{\sigma}_{s,add} A_{s,add} \tilde{\gamma}^{add}_{s,dis} - \tilde{\sigma}_{sc} A'_s \left(\tilde{\sigma}_s A_s + \tilde{\sigma}_{s,add} A_{s,add} \tilde{\gamma}^{add}_{s,dis}\right)}{\tilde{\sigma}_c \tilde{b}} - \tilde{\sigma}_{sc} A'_s a'. \quad (3)$$

We obtain the mathematical expectation of $\bar{M}_{ult}$ boundary bending moment by substitution of mathematical expectations of random arguments in the above simplified expression (3).

Next, we define coefficients for finding of a standard of boundary bending moment of a strengthened beam $\hat{M}_{ult}$ in the form of partial derivatives of function $\bar{M}_{ult} = f(x_1,...,x_n)$ by x_1,..., x_n variables.

Thus, for $\bar{\sigma}_c$, $\bar{\sigma}_s$, $\bar{\sigma}_{s,add}$, $\bar{\sigma}_{sc}$ mathematical expectations of strength parameters of materials, and $\bar{\gamma}^{add}_{s,dis}$ level of load, D_{σ_c}, D_{σ_s}, $D_{\sigma_{s,add}}$, $D_{\sigma_{sc}}$, $D_{\gamma^{add}_{s,dis}}$ coefficients acquire the following form:

$$D_{\sigma_c} = \frac{\partial \bar{M}_{ult}}{\partial \bar{\sigma}_c} = \frac{0,5}{\bar{\sigma}^2_c \bar{b}} \left[\left(\bar{\sigma}_s A_s\right)^2 + \left(\bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis}\right)^2 + \left(\bar{\sigma}_{sc} A'_s\right)^2\right] + \\ + \frac{\bar{\sigma}_s A_s \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis}\right)}{\bar{\sigma}^2_c \bar{b}};$$

$$D_{\sigma_s} = \frac{\partial \bar{M}_{ult}}{\partial \bar{\sigma}_s} = \\ = A_s \bar{d}_{red} - \frac{A_s}{\bar{\sigma}_c \bar{b}} \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s\right); \quad (4)$$

$$D_{\sigma_{s,add}} = \frac{\partial \bar{M}_{ult}}{\partial \bar{\sigma}_{s,add}} = A_{s,add} \bar{\gamma}^{add}_{s,dis} \bar{d}_{red} - \\ - \frac{A_{s,add} \bar{\gamma}^{add}_{s,dis}}{\bar{\sigma}_c \bar{b}} \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s\right);$$

$$D_{\sigma_{sc}} = \frac{\partial \bar{M}_{ult}}{\partial \bar{\sigma}_{sc}} = \\ = \frac{A'_s}{\bar{\sigma}_c \bar{b}} \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s\right) - A'_s a';$$

$$D_{\gamma^{add}_{s,dis}} = \frac{\partial \bar{M}_{ult}}{\partial \bar{\gamma}^{add}_{s,dis}} = \bar{\sigma}_{s,add} A_{s,add} \bar{d}_{red} - \\ - \frac{\bar{\sigma}_{s,add} A_{s,add}}{\bar{\sigma}_c \bar{b}} \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s\right).$$

Similarly, for $\bar{b}$, $\bar{d}_{red}$ mathematical expectations of parameters of the cross section geometry after strengthening, we obtain the following expressions for finding D_b, $D_{d_{red}}$ coefficients:

$$D_b = \frac{\partial \bar{M}_{ult}}{\partial \bar{b}} = \frac{0,5}{\bar{\sigma}_c \bar{b}^2} \left[\left(\bar{\sigma}_s A_s\right)^2 + \left(\bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis}\right)^2 + \left(\bar{\sigma}_{sc} A'_s\right)^2\right] + \\ + \frac{\bar{\sigma}_s A_s \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis} - \bar{\sigma}_{sc} A'_s \left(\bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis}\right)}{\bar{\sigma}_c \bar{b}^2}; \quad (5)$$

$$D_{d_{red}} = \frac{\partial \bar{M}_{ult}}{\partial \bar{d}_{red}} = \bar{\sigma}_s A_s + \bar{\sigma}_{s,add} A_{s,add} \bar{\gamma}^{add}_{s,dis}.$$

We should add that it is necessary to fulfill the condition of statistical independence of random parameters of the reserve of its bearing capacity for validity of the required values of indicators for assessment of reliability of a strengthened beam. In addition, we take strengthening class of the additional reinforcement as identical to the class of main reinforcement ($\bar{\sigma}_s = \bar{\sigma}_{s,add}$) in most cases under actual conditions. Therefore, proceeding from the above, we record the expression for finding the standard of the boundary bending moment $\hat{M}_{ult}$ as follows:

$$\hat{M}_{ult} = \sqrt{\begin{array}{l} D_{\sigma_c}{}^2 \hat{\sigma}_c{}^2 + D_{\sigma_s(tot)}{}^2 \hat{\sigma}_s{}^2 + D_{\sigma_{sc}}{}^2 \hat{\sigma}_{sc}{}^2 + \\ + D_{\gamma^{add}_{s,dis}}{}^2 \hat{\gamma}^{add\,2}_{s,dis} + D_b{}^2 \hat{b}^2 + D_{d_{red}}{}^2 \hat{d}_{red}{}^2, \end{array}} \quad (6)$$

where $D_{\sigma_s(tot)} = D_{\sigma_s} + D_{\sigma_{s,add}}$; $\hat{\sigma}_c$, $\hat{\sigma}_s$, $\hat{\sigma}_{sc}$, $\hat{\gamma}^{add}_{s,dis}$, $\hat{b}$, $\hat{d}_{red}$ are the standards of x_1,..., x_n variables (provided that $\hat{\sigma}_s = \hat{\sigma}_{s,add}$).

To evaluate the reliability of the reinforced beam we calculate the security characteristic (or reliability index), which takes the following form in this case:

$$\beta = \frac{\bar{M}_{ult} - M_{ult}}{\hat{M}_{ult}}, \quad (7)$$

where M_{ult} is the calculated bearing capacity of the normal cross section of a strengthened beam.

Thus, based on the above safety characteristics, we determine the quantitative assessment of reliability of the structure (in the form of an index of the probability of its failure) by the error function (better known as the Laplace function) $f(\beta)$:

$$Q(\beta) = 0,5 - f(\beta). \quad (8)$$

In turn, we determine the probability of failure-free operation of the strengthened beam (or its reliability) according to the following expression:

$$P(\beta)=0,5+f(\beta). \quad (9)$$

4. 4. Development of the basic method for assessment of reliability based on the calculation model of the cross section in accordance with acting norms [5]

We modify the dependence for ε_s, described in provisions [35] as for the non-strengthened beam (Fig. 2) and find a random value of the averaged true value of deformations of the entire stretched reinforcing bars in the strengthened beam $\tilde{\varepsilon}_{s,mid}$ from the following formula:

$$\tilde{\varepsilon}_{s,mid}=0,5\tilde{\gamma}_{s,inc}^{mid}\left[-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^2-4\left(-\frac{0,8\varepsilon_{cu,3}^2}{\tilde{\rho}\tilde{\alpha}}\right)}\right], \quad (10)$$

where $\tilde{\gamma}_{s,inc}^{mid}$ is the random value of the coefficient, which takes into consideration the percentage of inclusion of additional stressed reinforcement in operation in relation to the maximum use of strength of all stressed reinforcement (depends on a level of load on a beam before strengthening); $\varepsilon_{cu,3}$ is the fixed boundary value of deformations of outer fibers of a compressed zone of concrete; $\tilde{\alpha}$ and $\tilde{\rho}$ random parameters are the coefficients of reducing of an area of the entire stressed reinforcement to an area of concrete and reinforcement of the cross-section of a strengthened beam, respectively.

We find $\tilde{\alpha}$ and $\tilde{\rho}$, coefficients from the following formulas:

$$\tilde{\alpha}=\frac{\tilde{E}_{s,aver}}{\tilde{E}_{cm}\vartheta};\ \ \tilde{\rho}=\frac{A_{s,tot}}{\tilde{b}\tilde{d}_{red}}, \quad (11)$$

here, $\tilde{E}_{s,aver}$ is the random value of the averaged value of the elasticity modulus of the entire stretched reinforcing bars in a strengthened beam; $\tilde{E}_{cm}$ is the random value of the average value of the initial modulus of elasticity of concrete; $\vartheta=0.25$ is the coefficient, which takes into consideration a number of rods of the entire stressed reinforcement; $A_{s,tot}=A_s+A_{s,add}$ is the total area of the entire stretched reinforcing bars in a strengthened beam.

Using provisions developed in paper [35] and preconditions of the calculation method according to the current norms [5], we can record the expression for finding of a random value of the boundary bending moment $\tilde{M}_{ult}$ perceived by a strengthened beam (for preservation of the condition $x\leq x_R$) as follows:

$$\tilde{M}_{ult}=f\left(\tilde{E}_{cm},\tilde{E}_{s,aver},\tilde{f}_{yk},\tilde{E}_s^{'},\tilde{\gamma}_{s,inc}^{mid},\tilde{\gamma}_{s,dis}^{add},\tilde{b},\tilde{d}_{red}\right)=$$
$$=\left(A_s+A_{s,add}\tilde{\gamma}_{s,dis}^{add}\right)\tilde{f}_{yk}\tilde{d}_{red}\left(1-\frac{\lambda\tilde{x}}{2\tilde{d}_{red}}\right)+A_s^{'}\tilde{\sigma}_{sc}\left(0,5\lambda\tilde{x}-a^{'}\right), \quad (12)$$

where $\tilde{f}_{yk}$ is the random value of strength of the main and additional stretched reinforcing bars at the yield boundary; $\lambda=0.8$ is the coefficient of replacement of a curvilinear stress distribution in the compressed concrete zone of a strengthened beam, paper [35] gives its value; $\tilde{x}$ is the random value of the actual height of the compressed zone of the section of a strengthened beam, which we can find from formula:

$$\tilde{x}=\frac{\varepsilon_{cu,3}}{\varepsilon_{cu,3}+\tilde{\varepsilon}_{s,mid}}\tilde{d}_{red}. \quad (13)$$

We can record the dependence of the random value of strength of wire reinforcement on compression $\tilde{\sigma}_{sc}$ on its deformations as follows:

$$\tilde{\sigma}_{sc}=\tilde{\varepsilon}_{sc}\tilde{E}_s^{'}, \quad (14)$$

where $\tilde{E}_s^{'}$, $\tilde{\varepsilon}_{sc}$ are the random values of the elasticity modulus and the actual value of deformations of the compressed wire reinforcement, respectively.

In turn, the expression for finding of a random value of the actual value of deformations of the compressed wire reinforcement $\tilde{\varepsilon}_{sc}$ takes the following form:

$$\tilde{\varepsilon}_{sc}=\varepsilon_{cu,3}\left(1-\frac{a^{'}}{\tilde{x}}\right). \quad (15)$$

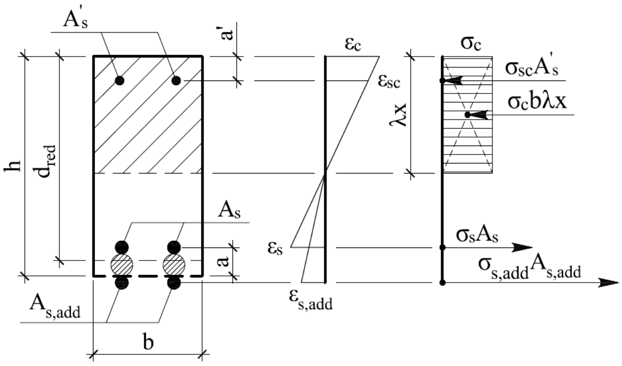

Fig. 2. The stressed-deformed state in the normal cross section of the beam strengthened with additional reinforcing bars under an action of load (in accordance with provisions [5, 35])

Finally, taking into consideration dependences (14), (15), we substitute the expression (13) for $\tilde{x}$ in formula (12) for $\tilde{M}_{ult}$, with further phased simplification. And we obtain:

$$\tilde{M}_{ult}=\left(A_s+A_{s,add}\tilde{\gamma}_{s,dis}^{add}\right)\tilde{f}_{yk}\tilde{d}_{red}-$$
$$-\frac{0,5\lambda\varepsilon_{cu,3}\tilde{d}_{red}\left[\left(A_s+A_{s,add}\tilde{\gamma}_{s,dis}^{add}\right)\tilde{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\tilde{E}_s^{'}\right]}{\varepsilon_{cu,3}+0,5\tilde{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^2+\frac{3,2\varepsilon_{cu,3}^2\tilde{b}\tilde{d}_{red}\tilde{E}_{cm}\vartheta}{A_{s,tot}\tilde{E}_{s,aver}}}\right)}-$$
$$-\varepsilon_{cu,3}A_s^{'}\tilde{E}_s^{'}a^{'}\left(0,5\lambda+1\right)+$$
$$+\frac{A_s^{'}\tilde{E}_s^{'}a^{'2}}{\tilde{d}_{red}}\left[\varepsilon_{cu,3}+0,5\tilde{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^2+\frac{3,2\varepsilon_{cu,3}^2\tilde{b}\tilde{d}_{red}\tilde{E}_{cm}\vartheta}{A_{s,tot}\tilde{E}_{s,aver}}}\right)\right]. \quad (16)$$

We obtain the mathematical expectation of the boundary bending moment $\bar{M}_{ult}$ by substitution the mathematical expectations of random arguments in the obtained simplified expression (16).

Next, we define the coefficients for finding of the standard of the boundary bending moment of the strengthened beam $\hat{M}_{ult}$ in the form of partial derivatives of function $\bar{M}_{ult}=f\left(x_1,...,x_n\right)$ by $x_1,..., x_n$ variables.

Thus, for the mathematical expectations of parameters of strength and deformability of materials $\bar{E}_{cm}$, $\bar{E}_{s,aver}$, $\bar{f}_{yk}$, $\bar{E}_s^{'}$, as well as for the level of loading $\bar{\gamma}_{s,inc}^{mid}$, $\bar{\gamma}_{s,dis}^{add}$, the coefficients acquire the following form:

$$D_{E_{cm}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{E}_{cm}}=\pm\frac{\dfrac{0,4\lambda\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{3}\bar{b}\bar{d}_{red}^{2}\vartheta\left[\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\bar{E}_s^{'}\right]}{A_{s,tot}\bar{E}_{s,aver}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}}{\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]^{2}}\pm$$

$$\pm\frac{0,8\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{2}\bar{b}\vartheta A_s^{'}\bar{E}_s^{'}a^{'2}}{A_{s,tot}\bar{E}_{s,aver}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}};$$

$$D_{E_{s,aver}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{E}_{s,aver}}=\mp\frac{\dfrac{0,4\lambda\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{3}\bar{b}\bar{d}_{red}^{2}\bar{E}_{cm}\vartheta\left[\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\bar{E}_s^{'}\right]}{A_{s,tot}\bar{E}_{s,aver}^{2}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}}{\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]^{2}}\mp$$

$$\mp\frac{0,8\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{2}\bar{b}\bar{E}_{cm}\vartheta A_s^{'}\bar{E}_s^{'}a^{'2}}{A_{s,tot}\bar{E}_{s,aver}^{2}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}};$$

$$D_{f_{yk}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{f}_{yk}}=\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{d}_{red}-$$

$$-\frac{0,5\lambda\varepsilon_{cu,3}\bar{d}_{red}\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)}{\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)};\quad(17)$$

$$D_{E_s^{'}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{E}_s^{'}}=$$

$$=\frac{0,5\lambda\varepsilon_{cu,3}^{2}\bar{d}_{red}A_s^{'}}{\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)}-$$

$$-\varepsilon_{cu,3}A_s^{'}a^{'}\left(0,5\lambda+1\right)+$$

$$+\frac{A_s^{'}a^{'2}\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]}{\bar{d}_{red}};$$

$$D_{\gamma_{s,inc}^{mid}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{\gamma}_{s,inc}^{mid}}=\frac{-0,5\varepsilon_{cu,3}\pm0,5\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}{\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]^{2}}\times$$

$$\times\left[0,5\lambda\varepsilon_{cu,3}\bar{d}_{red}\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-0,5\lambda\varepsilon_{cu,3}^{2}\bar{d}_{red}A_s^{'}\bar{E}_s^{'}\right]+$$

$$+\frac{A_s^{'}\bar{E}_s^{'}a^{'2}}{\bar{d}_{red}}\left(-0,5\varepsilon_{cu,3}\pm0,5\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right);$$

$$D_{\gamma_{s,dis}^{add}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{\gamma}_{s,dis}^{add}}=A_{s,add}\bar{f}_{yk}\bar{d}_{red}-$$

$$-\frac{0,5\lambda\varepsilon_{cu,3}\bar{d}_{red}A_{s,add}\bar{f}_{yk}}{\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)}.$$

In turn, for mathematical expectations of parameters of the cross section geometry, after strengthening $\bar{b}$, $\bar{d}_{red}$, the expressions for finding of D_b, $D_{d_{red}}$ coefficients are:

$$D_b=\frac{\partial\bar{M}_{ult}}{\partial\bar{b}}=\pm\frac{\dfrac{0,4\lambda\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{3}\bar{d}_{red}^{2}\bar{E}_{cm}\vartheta\left[\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\bar{E}_s^{'}\right]}{A_{s,tot}\bar{E}_{s,aver}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}}{\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]^{2}}\pm$$

$$\pm\frac{0,8\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{2}\bar{E}_{cm}\vartheta A_s^{'}\bar{E}_s^{'}a^{'2}}{A_{s,tot}\bar{E}_{s,aver}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}};$$

$$D_{d_{red}}=\frac{\partial\bar{M}_{ult}}{\partial\bar{d}_{red}}=\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-$$

$$-\frac{0,5\lambda\varepsilon_{cu,3}\left[\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\bar{E}_s^{'}\right]}{\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)}\pm$$

$$\pm\frac{\dfrac{0,4\lambda\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{3}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta\left[\left(A_s+A_{s,add}\bar{\gamma}_{s,dis}^{add}\right)\bar{f}_{yk}-\varepsilon_{cu,3}A_s^{'}\bar{E}_s^{'}\right]}{A_{s,tot}\bar{E}_{s,aver}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}}{\left[\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\right]^{2}}-$$

$$-\frac{A_s^{'}\bar{E}_s^{'}a^{'2}}{\bar{d}_{red}^{2}}\left[\begin{array}{l}\varepsilon_{cu,3}+0,5\bar{\gamma}_{s,inc}^{mid}\times\\ \times\left(-\varepsilon_{cu,3}\pm\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}\right)\end{array}\right]\pm$$

$$\pm\frac{0,8\bar{\gamma}_{s,inc}^{mid}\varepsilon_{cu,3}^{2}\bar{b}\bar{E}_{cm}\vartheta A_s^{'}\bar{E}_s^{'}a^{'2}}{A_{s,tot}\bar{E}_{s,aver}\bar{d}_{red}\sqrt{\varepsilon_{cu,3}^{2}+\dfrac{3,2\varepsilon_{cu,3}^{2}\bar{b}\bar{d}_{red}\bar{E}_{cm}\vartheta}{A_{s,tot}\bar{E}_{s,aver}}}}.\quad(18)$$

Here, we define the standard of boundary bending moment $\hat{M}_{ult}$ as

$$\hat{M}_{ult}=\sqrt{\begin{array}{l}D_{E_{cm}}{}^{2}\hat{E}_{cm}{}^{2}+D_{E_{s,aver}}{}^{2}\hat{E}_{s,aver}{}^{2}+\\+D_{f_{yk}}{}^{2}\hat{f}_{yk}{}^{2}+D_{E_s^{'}}{}^{2}E_s^{'2}+\\+D_{\gamma_{s,inc}^{mid}}{}^{2}\hat{\gamma}_{s,inc}^{mid\,2}+D_{\gamma_{s,dis}^{add}}{}^{2}\hat{\gamma}_{s,dis}^{add\,2}+\\+D_b{}^{2}\hat{b}^{2}+D_{d_{red}}{}^{2}\hat{d}_{red}{}^{2},\end{array}}\quad(19)$$

where $\hat{E}_{cm}$, $\hat{E}_{s,aver}$, $\hat{f}_{yk}$, $\hat{E}_s^{'}$, $\hat{\gamma}_{s,inc}^{mid}$, $\hat{\gamma}_{s,dis}^{add}$, $\hat{b}$, $\hat{d}_{red}$ are the standards of $x_1,\ldots, x_n$ variables.

We find parameters (7) to (9) to evaluate the reliability of the reinforced beam.

4. 5. Materials and equipment used in the experimental-theoretical study

We performed calculation of bearing capacity of normal cross sections of strengthened beams based on data of experimental and theoretical studies. We carried out strengthening by building up the stretched reinforcing bars under the action of different load levels.

We manufactured a series of eight beams made of concrete of C45/55 class reinforced with two flat frames in factory conditions. Reinforcement of the cross section of a beam was double and symmetrical. We used reinforcing bars of 2Ø14 mm of A400C class, compressed wire reinforcement of 2Ø5 mm as working stressed reinforcement. We installed the additional reinforcing bars for strengthening, respectively, with 2Ø10, 2Ø12 and 2Ø14 mm of A400C class, by welding it to the existing beam reinforcement through Ø20 mm shorters. The design dimensions of a beam cross section were 100×200 mm (Fig. 1, 2).

Testing of beams by step loading went according to the scheme of “pure bending” (concentrated forces applied in one-thirds of span of a beam with the transfer of load through the traverse) at the age of 28 days and older. We performed strengthening of a beam by building up the stretched reinforcing bars at active load levels of 0.0; 0.3; 0.5; 0.75 from $M_{ult,0}$ – bearing capacity of the normal cross section of an non-strengthened beam. We studied operation and strained-deformed state of reinforced concrete beams in accordance with this load scheme.

5. Results of studying the reliability of beams strengthened under the action of load

We performed testing of the proposed methods for assessment of reliability based on the initial data for implementation of experimental studies [31]. To do this, we established calculated characteristics of strength and deformability of materials, load levels and geometry of cross sections of strengthened beams:

1. Concrete of C45/55 class: f_{cd}=30 MPa; E_{cm}=39.5 GPa; $\varepsilon_{cu3,cd}$=0.00219; ϑ=0.25 ξ_R=0.557 [35].

2. Stressed reinforcement with 2Ø14 mm of A400C class: A_s=308 mm^2=3.08 cm^2; $f_{yd}=f_{yk}/\gamma_s$=400/1.1=363.6 MPa; E_s=210 MPa.

3. Compressed wire reinforcement with 2Ø5 mm:

$$A_s^{'} = 39{,}2\,\text{mm}^2 = 0{,}39\,\text{cm}^2;$$

$f_{yd}=f_{yk}/\gamma_s$=395/1.1=359.1 MPa; E_s=170 MPa.

4. Additional stressed reinforcement, respectively, with 2Ø10, 2Ø12 and 2Ø14 mm of A400C class: $A_{s,add}$=157 mm^2= =1.57 cm^2 (2Ø10); $A_{s,add}$=226 mm^2=2.26 cm^2 (2Ø12); $A_{s,add}$= =308 mm^2=3.08 cm^2 (2Ø14); $f_{yd}=f_{yk}/\gamma_s$=400/1.1=363.6 MPa; E_s=210 MPa.

5. Design dimensions of the cross-section of the strengthened beam (Fig. 1, 2): $b\times h$=100×200 mm; d_{red}= =183.8 mm=18.38 cm (additional reinforcement – 2Ø10); d_{red}= =187.0 mm=18.7 cm (2Ø12); d_{red}=190.0 mm=19.0 cm (2Ø14); $a = 27$ mm; $a^{'} = 12{,}5$ mm.

We determine the given useful height of the cross section of a strengthened beam d_{red} in accordance with the following expression:

$$d_{red} = d + a_{red}, \tag{20}$$

where d is the useful height of the cross section of a beam before strengthening; a_{red} is the distance from the center of gravity of the existing stressed reinforcement (A_s) to the center of gravity of the entire stressed armature in a strengthened beam ($A_{s,tot}$):

$$a_{red} = \frac{\sigma_{s,add} A_{s,add} \left(d_{add} - d\right)}{\sigma_s A_s + \sigma_{s,add} A_{s,add}}, \tag{21}$$

here, d_{add} is the distance from the upper compressed side of concrete of a beam to the center of gravity of reinforcement strengthening.

We find the calculation value of the coefficient $\gamma_{s,inc}^{mid}$, which takes into consideration the presence of an increase of a certain level of load, from the following formula:

$$\gamma_{s,inc}^{mid} = \frac{\sigma_s A_s + \sigma_{s,add} A_{s,add}}{\sigma_s A_s + \sigma_{s,add} A_{s,add} \gamma_{s,dis}^{add}}, \tag{22}$$

where $\gamma_{s,dis}^{add}$ is the coefficient of using the cross section of the additional reinforcement (a coefficient of operation conditions), the calculation (minimum theoretical or experimental) value obtained based on research [31].

We should add that the value of $\gamma_{s,inc}^{mid}$ coefficient depends also on the percentage of inclusion of additional stressed reinforcement ($A_{s,add}$) in operation in relation to the maximum use of strength of the entire stressed reinforcement in a strengthened beam ($A_{s,tot}$).

Tables 1, 2 give calculation values of $\gamma_{s,dis}^{add}$, $\gamma_{s,inc}^{mid}$, coefficients as well as the bearing capacity of the normal cross sections of beams in dependence on a level of load at the moment of strengthening and an area of the cross section of the additional reinforcement.

Table 1

Calculated values of $\gamma_{s,dis}^{add}$ and $\gamma_{s,inc}^{mid}$ coefficients

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0{,}0M_{ult,0}$	$0{,}3M_{ult,0}$	$0{,}5M_{ult,0}$	$0{,}75M_{ult,0}$
1	Ø10 mm	1.000/1.000	0.900/1.035	0.830/1.060	0.750/1.090
2	Ø12 mm	1.000/1.000	0.870/1.060	0.790/1.100	0.680/1.155
3	Ø14 mm	1.000/1.000	0.850/1.080	0.750/1.145	0.625/1.230

Note: $\gamma_{s,dis}^{add}$, *– before the slope,* $\gamma_{s,inc}^{mid}$ *– after the slope*

Table 2

Bearing capacity of normal cross sections of strengthened beams M_{ult}, kNm

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0{,}0M_{ult,0}$	$0{,}3M_{ult,0}$	$0{,}5M_{ult,0}$	$0{,}75M_{ult,0}$
1	Ø10 mm	26.91/24.59	26.15/23.86	25.59/23.34	24.97/22.76
2	Ø12 mm	30.72/28.56	29.36/27.00	28.50/26.17	27.29/24.96
3	Ø14 mm	35.05/33.43	32.99/30.94	31.53/29.29	29.71/27.45

Note: before the slope there are values calculated in accordance with [32], after the slope there are values calculated in accordance with [35]

We determine statistical characteristics (mathematical expectations and standards) of strength and deformability of materials, levels of load and geometry of cross sections of strengthened beams based on the corresponding calculation characteristics.

1. C45/55 concrete:

$$\bar{\sigma}_c = \frac{f_{cd}}{1-1,64V_{\sigma_c}} = \frac{30}{1-1,64\cdot 0,135} = 38,5\,\text{MPa} = 3,85\,\text{kN/cm}^2,$$

where the variation coefficient is

$$V_{\sigma_c} = 0,135\,[5] \rightarrow$$
$$\rightarrow \hat{\sigma}_c = 0,135\bar{\sigma}_c = 0,135\cdot 3,85 = 0,52\ \text{kN/cm}^2;$$

$$\bar{E}_{cm} = \frac{E_{cm}}{1-1,64V_{E_{cm}}} = \frac{39,5}{1-1,64\cdot 0,044} = 42,6GPa = 4,26\cdot 10^3\,\text{kN/cm}^2,$$

where the variation coefficient is

$$V_{E_{cm}} = 0,044\ \ [34] \rightarrow$$
$$\rightarrow \hat{E}_{cm} = 0,044\bar{E}_{cm} = 0,044\cdot 4,26\cdot 10^3 = 0,19\cdot 10^3\,\text{kN/cm}^2.$$

2. Main and additional reinforcing bars of A400C class:

$$\bar{\sigma}_s = \bar{\sigma}_{s,add} = \frac{f_{yk}}{1-1,64V_{\sigma_s}} = \frac{390}{1-1,64\cdot 0,0437} =$$
$$= 420,1\,\text{MPa} = 42,01\ \text{kN/cm}^2,$$

where the variation coefficient is

$$V_{\sigma_s} = 0,0437\ \ [34] \rightarrow$$
$$\rightarrow \hat{\sigma}_s = 0,0437\bar{\sigma}_s = 0,0437\cdot 42,01 = 1,84\,\text{kN/cm}^2;$$

$$\bar{E}_{s,aver} = 191,3\,\text{GPa} = 19,13\cdot 10^3\,\text{kN/cm}^2\ \ [34],$$

where the variation coefficient is

$$V_{E_{s,aver}} = 0,062\ \ [34] \rightarrow$$
$$\rightarrow \hat{E}_{s,aver} = 0,062\bar{E}_{s,aver} = 0,062\cdot 19,13\cdot 10^3 = 1,19\cdot 10^3\,\text{kN/cm}^2.$$

3. Wire reinforcement:

$$\bar{\sigma}_{sc} = \frac{f_{yk}}{1-1,64V_{\sigma_{sc}}} = \frac{395}{1-1,64\cdot 0,06} =$$
$$= 438,1\,\text{MPa} = 43,81\ \text{kN/cm}^2,$$

where the variation coefficient is

$$V_{\sigma_{sc}} = 0,06\ \ [34] \rightarrow$$
$$\rightarrow \hat{\sigma}_{sc} = 0,06\bar{\sigma}_{sc} = 0,06\cdot 43,81 = 2,63\,\text{kN/cm}^2;$$

$$\bar{E}_s^{'} = E_s\left(1-1,64V_{E_s^{'}}\right) = 170\cdot(1-1,64\cdot 0,062) =$$
$$= 152,7\,\text{GPa} = 15,27\cdot 10^3\,\text{kN/cm}^2,$$

where the coefficient of variation $V_{E_s^{'}} = 0,062$ – as for the reinforcement of A4000C class

$$[34] \rightarrow$$
$$\rightarrow \hat{E}_s^{'} = 0,062\bar{E}_s^{'} = 0,062\cdot 15,27\cdot 10^3 = 0,95\cdot 10^3\,\text{kN/cm}^2.$$

4. Parameters of levels of load ($\gamma_{s,dis}^{add}$, $\gamma_{s,inc}^{mid}$ coefficients): $\bar{\gamma}_{s,dis}^{add}$, $\bar{\gamma}_{s,inc}^{mid}$ mathematical expectations of $\gamma_{s,dis}^{add}$, $\gamma_{s,inc}^{mid}$, coefficients as well as $\hat{\gamma}_{s,dis}^{add}$, $\hat{\gamma}_{s,inc}^{mid}$, standards in dependence on the level of load at the moment of strengthening and the area of the cross section of the additional reinforcement are in Tables 3, 4, respectively.

Table 3

$\bar{\gamma}_{s,dis}^{add}$ and $\bar{\gamma}_{s,inc}^{mid}$ mathematical expectations

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0,0M_{ult,0}$	$0,3M_{ult,0}$	$0,5M_{ult,0}$	$0,75M_{ult,0}$
1	Ø10 mm	1.000/1.000	0.905/1.035	0.835/1.060	0.760/1.090
2	Ø12 mm	1.000/1.000	0.870/1.060	0.790/1.100	0.685/1.155
3	Ø14 mm	1.000/1.000	0.850/1.080	0.760/1.135	0.645/1.215

Note: before the slope – $\bar{\gamma}_{s,dis}^{add}$, *after the slope –* $\bar{\gamma}_{s,inc}^{mid}$

Table 4

$\hat{\gamma}_{s,dis}^{add}$ and $\hat{\gamma}_{s,inc}^{mid}$ standards (variances)

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0,0M_{ult,0}$	$0,3M_{ult,0}$	$0,5M_{ult,0}$	$0,75M_{ult,0}$
1	Ø10 mm	–/–	0.005/–	0.005/–	0.01/–
2	Ø12 mm	–/–	–/–	–/–	0.005/–
3	Ø14 mm	–/–	–/–	0.01/0.01	0.02/0.015

Note: before the slope – $\hat{\gamma}_{s,dis}^{add}$, *after the slope –* $\hat{\gamma}_{s,inc}^{mid}$

We determine mathematical expectations of $\bar{\gamma}_{s,dis}^{add}$ coefficients as an arithmetic average of $\bar{\gamma}_{s,dis}^{add} = \gamma_{s,dis(mid)}^{add}$ calculated values for the twin beams – based on experimental and theoretical studies [31].

5. Parameters of geometry of the cross section of a strengthened beam (Fig. 1, 2): $\bar{b} = 99,2\,\text{mm}$, where the coefficient of variation is

$$V_b = 0,008 \rightarrow \hat{b} = 0,008\bar{b} = 0,008\cdot 99,2 = 0,79\,\text{mm};$$

$\bar{d}_{red} = 185,7\,\text{mm}$ (additional reinforcement 2Ø10), where the coefficient of variation is

$$V_{d_{red}} = 0,0103 \rightarrow$$
$$\hat{d}_{red} = 0,0103\bar{d}_{red} = 0,0103\cdot 185,7 = 1,91\,\text{mm};$$

$\bar{d}_{red} = 188,9\,\text{mm}$ (2Ø12), where the variation coefficient is

$$V_{d_{red}} = 0,0102 \rightarrow$$
$$\hat{d}_{red} = 0,0102\bar{d}_{red} = 0,0102\cdot 188,9 = 1,93\,\text{mm};$$

$\bar{d}_{red} = 191,9\,\text{mm}$ (2Ø14), where the variation coefficient is

$$V_{d_{red}} = 0,01 \rightarrow \hat{d}_{red} = 0,01\bar{d}_{red} = 0,01\cdot 191,9 = 1,92\,\text{mm}.$$

We determine $\bar{b}$, $\bar{d}_{red}$ mathematical expectations based on the data on natural measurements of a width and a height of beams [31]. We executed measurements in ten cross sections along a length of each beam (the accuracy was 0.1 mm).

Consequently, in accordance with the principles developed above, we calculate qualitative and quantitative reliability indices of strengthened beams – reliability index

β_i and index of probability of fail-free operation $P(\beta)_i$. We present the obtained results of these indices in Tables 5, 6.

Table 5

Reliability index β_i

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0,0M_{ult,0}$	$0,3M_{ult,0}$	$0,5M_{ult,0}$	$0,75M_{ult,0}$
1	Ø10 mm	3.64/3.35	3.65/3.38	3.68/3.39	3.70/3.41
2	Ø12 mm	3.66/3.36	3.67/3.38	3.66/3.37	3.68/3.40
3	Ø14 mm	3.63/3.34	3.64/3.35	3.75/3.44	3.73/3.45

Note: before the slope there are values calculated according to the algorithm developed in chapter 4.3, after the slope there are values calculated according to the algorithm developed in chapter 4.4

Table 6

Probability of failure-free operation $P(\beta)_i$

No.	Strengthening reinforcement	Level of load at the moment of strengthening			
		$0,0M_{ult,0}$	$0,3M_{ult,0}$	$0,5M_{ult,0}$	$0,75M_{ult,0}$
1	Ø10 mm	0.999864/0.999596	0.999869/0.999638	0.999883/0.999651	0.999892/0.999675
2	Ø12 mm	0.999874/0.999610	0.999879/0.999638	0.999874/0.999624	0.999883/0.999663
3	Ø14 mm	0.999858/0.999581	0.999864/0.999596	0.999912/0.999709	0.999904/0.999720

Note: before the slope there are values calculated according to the algorithm developed in chapter 4.3, after the slope there are values calculated according to the algorithm developed in chapter 4.4

Thus, we can state the following regularity based on the obtained results (Tables 5, 6): a level of reliability of most reinforced beams increases with an increase in a load level at the moment of strengthening, as well as a diameter of strengthening reinforcement. In addition, we can notice one more regularity after analysis of deviations between values of reliability indices β_i obtained based on two developed techniques. It is as follows: for all beams, the results obtained with a use of the method based on the norms [32] are higher above the results obtained with a use of the method based on the current norms [5] up to 8 %. That is, to assess reliability of reinforced beams, it is desirable to use a method based on the calculation model of reinforced concrete structure given in the norms [5]. The technique gives less importance to reliability index β_i, but it leads to the need to design a more reliable reinforcement structure and, accordingly, achievement of a higher value of the reliability index.

6. Discussion of results of studying the reliability of beams strengthened under the action of load

An important criterion (along with the quality of junction of bars) in the study on effectiveness of strengthening of reinforced concrete beams with additional reinforcement is its maximum integration into operation with reinforcement in the existing beam. This criterion is best provided at low load levels – $0.3M_{ult,0}$ and $0.5M_{ult,0}$ (Table 1). At the same time, the reliability level is proportional inversely – we obtain the maximum values of reliability indices β_i at high load levels – $0.75M_{ult,0}$ (Table 5). Obviously, we can explain this fact by the incomplete use of strength of stressed reinforcement extension at an increase of a level of current load.

In contrast to the results of studies published in papers [25, 27–29], the obtained values of qualitative and quantitative reliability estimates of reinforced beams (reliability indexes β_i and probability of failure-free operation $P(\beta)_i$, respectively) are obvious. This statement has the right to exist, since reliability estimates obtained above have distribution close to the proportionality in dependence on a load level and a diameter of the additional reinforcement (Tables 5, 6). In addition, we adapted the basic methodology developed in Section 4.4 to the current design rules [5]. It contains a relatively simple mathematical algorithm for calculation, which does not limit its use in individual cases of design of reinforced concrete beams strengthened by expansion of stressed reinforcement under an action of load. Also, the calculation model of reinforced concrete structure given in the norms [5] is more effective in assessment of reliability of a strengthened structure, since it leads to the need to design a more reliable strengthening structure.

The standard deviations $\hat{x}_i$ (see statistical characteristics in Section 5) of variable parameters of the reserve of bearing capacity of strengthened beams and a percentage ratio of products $(D_i \times \hat{x}_i)$ to the general standard of bending moment $\hat{M}_{ult}$ have a significant impact on the objective assessment of reliability. We observe the largest spread of values in the variances $\hat{\gamma}_{s,dis}^{add}$, $\hat{\gamma}_{s,inc}^{mid}$ (Table 4) for bars of the additional reinforcement of adjacent diameters. Because we tested only two beams-twins for only one level of load and one diameter of reinforcement strengthening in laboratory conditions. This, in turn, entailed the calculation of the mathematical expectations of coefficients of a level of load $\overline{\gamma}_{s,dis}^{add}$, $\overline{\gamma}_{s,inc}^{mid}$ (Table 3) based on the arithmetic average of only two $\gamma_{s,dis}^{add}$ values – the coefficients of operation conditions obtained based on a paper [31]. A significant variance of variance values $\hat{\gamma}_{s,dis}^{add}$, $\hat{\gamma}_{s,inc}^{mid}$ leads to an increase in values of the standards $\left(\hat{M}_{ult}\right)_i$ (6), (19) and, accordingly, to the obvious understatement of the reliability indices β_i (Table 5) for the beams strengthened under an action of high levels of load. This is especially true for $0.75M_{ult,0}$ load level with the installation of additional reinforcement of strengthening of Ø14 mm. Understatement of reliability indices β_i leads also to a decrease in the overall reliability level $P(\beta)_i$ (Table 6) for these beams. It is obvious that it is necessary to avoid significant variance of values in variances of variable parameters of the reserve of bearing capacity of strengthened reinforced concrete structures (in our case, coefficients of a load level) in future studies of reliability. Therefore, in the future, we recommend to test as many twin elements as possible (which, is, unfortunately, rather labor-intensive at a laboratory,) for maximally objective assessment of reliability of the investigated structures.

We should add that the recommendations of the current norms [6] propose to use certain values of failure time (here, reliability indices β_i) of a structure in calculation situations for the first group of boundary states. The range of values given there (from 3.89 for CC1 class of consequences to 4.76 for CC3 class of consequences) is slightly higher than the values obtained in our study for individual strengthened elements.

However, based on the analysis of the results of the study on reliability of structures of strengthened reinforced concrete bridges [29], we can note the following. The interval of values obtained there for β (from 3.3 to 4.2) has the average probability of failure (average reliability level) of the design.

Thus, we can consider the obtained reliability indicators β_i, which fall within this interval, as adequate and recommendatory ones. At the same time, the research is limited to a use of the normal law of distribution of random variables, the results of previous theoretical and experimental studies of the strained state of beams, as well as the small number of twin elements in the processing of statistical data. The stability of the obtained solutions is ensured under conditions of use as a strengthening element of the reinforcing bars, as well as the study of beams with insufficient bending strength.

Finally, the analysis of the results of theoretical studies based on approbation of the methods developed above makes possible to draw the appropriate conclusion. Thus, the least influence on a level of reliability of strengthened beams has such parameters of the reserve of bearing capacity of structures as strength and deformability of compressed reinforcement ($\overline{\sigma}_{sc}$ and $\overline{E}_s^{'}$ accordingly), as well as a width of a cross section $\overline{b}$. Therefore, we recommend to neglect these parameters (with sufficient accuracy of calculation) for further studies on reliability of strengthened reinforced concrete bending structures.

7. Conclusions

1. We developed the basic methods for assessment of reliability of reinforced concrete beams strengthened by expansion of stressed reinforcement under an action of load based on various normative documents [5, 32]. The principle of the methodology is also the ability to control calculations of taken into consideration stochastic parameters of a load level at the moment of strengthening of beams.

2. We tested the developed methods and obtained qualitative and quantitative reliability indices of structures using the data of previous experimental and theoretical studies of the stressed state of reinforced beams. The indices are: the reliability index β_i and the index of probability of failure-free operation $P(\beta)_i$, respectively.

3. We investigated an influence of stochastic parameters that exist under real conditions of strengthening on reliability of beams, namely: strength and deformability of materials, parameters of geometry of a cross section, and also, what is important, a level of load at the moment of strengthening. The obtained results of theoretical studies make us state that we can achieve the maximum level of reliability under conditions of a higher level of load at strengthening and a maximum diameter of reinforcement by extension – $0.75M_{ult,0}$ and Ø14 mm, respectively.

4. We recommend using a method based on the calculation model of reinforced concrete structure given in the current norms [5] to assess reliability of reinforced beams. The methodology gives less importance to the reliability index β_i (in average up to 8 %), but it leads to the need to design a more reliable strengthening structure and, accordingly, to achieve a higher value of the reliability index.

5. The basic methodology of reliability assessment developed above gives possibility to design structures with a given level of reliability – probability of failure-free operation $P(\beta)$, which may be the subject for future research.

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