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DEVELOPMENT OF THE THEORY OF SYNCHROTRON RADIATION AND ITS REABSORPTION

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Development of the Theory of Synchrotron Radiation and Its Reabsorption[1]

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I Introduction 1

2 Synchrotron Radiation in the Case of Noncircular (Spiral) Particle Movement 2

2.1 Elementary Analysis 2

2.2 Synchrotron Radiation of an Individual Particle Moving at Arbitrary Angle ϕ to a Magnetic Field 7

2 3 Radiation of System of Particles 14

3 Reabsorption of Synchrotron Radiation by Ultrarelativistic Particles 19

3 1 General Notes 19

3.2 Transfer Equation for Tensor $I_{\alpha\beta}$ 23

3 3 Usage of the Einstein Coefficient Method for Polarized Radiation 31

3.4 Reabsorption of Synchrotron Radiation in a Vacuum 36

3 5 The Reabsorption of Synchrotron Radiation in the Presence of a "Cold" Plasma 44

4. Some Problems Related to the Theory of Synchrotron Radiation 55

4 1 The Radiation of Sources Moving at Relativistic Velocities 55

4 2 Synchrotron Radiation of Protons 62

4 3 The Change in Magnetic Field Related to Deceleration (Energy Losses) of Particles Moving in a Field 65

4 4 Synchrotron and Other Mechanisms of Cosmic Radiation 76

References

[1] The bibliographic references for this review were gathered in May of 1968 This review was written for the *Annual Review of Astronomy and Astrophysics*, Vol 7; 1969

Introduction

Comparatively recently (1965) we published a rather detailed review [1] on the theory of synchrotron radiation[1] It seemed that this review concerned rather well established concepts and formulas Certain methods and problems in the area of further development of the theory of synchrotron radiation were, of course, quite clear even then However, one could consider that these problems were of no significance in principle Therefore, we did not believe that we would soon be returning to the same theme However, this occurred for a number of reasons First of all, it was discovered that there were errors in the theory of synchrotron radiation in the case of noncircular (spiral) movement of particles True, as applicable to the problems and conditions discussed in [1] (synchrotron galactic radiation and the radiation of discrete sources, the expansion and movement of which occurs at nonrelativistic speeds) all of the formulas used were actually correct However, the principal aspect of the matter is also rather important Also, conditions might be realized under which more general formulas would have to be used Secondly, the theory of reabsorption of synchrotron radiation underwent important development both with and without a "cold" plasma in the radiating area Third, it was discovered that it was possible to encounter radiation sources in space moving at relativistic speeds shells, jets and "clouds" of plasma ejected during explosions, an example might be explosions in galactic nuclei leading to the formation of

[1] Review [1] will be cited in the following as I, and formulas from this review will be represented, for example, as (I, 2.10). We note that the term synchrotron radiation arose by chance and seems to us an unfortunate selection Therefore, we have used broadly, particularly in I, the more significant term "magnetobremsstrahlung " However, it seems hardly possible at this late date to change the accepted terminology, so that we have decided to go along with the usage of the term "synchrotron radiation" for magnetobremsstrahlung of ultrarelativistic particles

radiogalaxies) Thus, the necessity has arisen of analyzing synchrotron radiation and its reabsorption for a rapidly moving cloud of relativistic particles Of course, this problem is closely related to that noted above

All of these problems, as well as certain related problems, will be discussed in this article, since their significance for astrophysics may be quite great (we will make broad usage in the following of materials contained in [2])

2. Synchrotron Radiation in the Case of Noncircular (Spiral) Particle Movement

2 1 Elementary Analysis

An ultrarelativistic electron[1] moving in a vacuum (this is the only case which will be of basic interest to us) radiates practically only in the direction of its instantaneous velocity or, more precisely, into a cone with an apex angle

$$\psi \sim \frac{mc^2}{E}, \quad E \gg mc^2. \tag{2.1}$$

In this section, in our qualitative analysis of the problem, we will consider the radiation to be acicular whenever possible, i e , we will consider angle ψ to be small. As it moves through a constant, homogeneous magnetic field with intensity $\vec{H}$, an electron generally moves along a spiral line with a velocity $v_{\parallel} = v \cos \theta$ in the direction of the field and velocity $v_{\perp} = v \sin \theta$ transverse to the field (of course, the total velocity $v = \sqrt{v_{\parallel}^2 + v_{\perp}^2}$). The rotational frequency ω_H depends only on v and is equal to

[1] For definition, we will speak of electrons However, of course, all of the literal expressions relate to particles with charge e and rest mass m

$$\omega_H = \frac{eH}{mc}\sqrt{1-\frac{v^2}{c^2}} = \frac{eH}{mc}\,\frac{mc^2}{E} \qquad (2\ 2)$$

If the movement is circular (i e , $v_{\parallel} = 0$, $v_{\perp} = v$), on the basis of the above, the ultrarelativistic electron radiates only in the plane of its orbit. An observer in this plane (recording device) will "see" bursts of radiation at those moments when the electron is moving exactly in the direction of the observer (of course, we must consider the delay, equal to the propagation time of the radiation and equal in a vacuum to r/c, where r is the distance from the electron to the observer) Obviously, the bursts will be repeated each period or, in other words, follow at time intervals $\tau = 2\pi/\omega_N = (2\pi mc/eH)(E/mc^2)$ As was shown in detail in 1, the characteristic duration of each burst under condition (2 2) is on the order of $\Delta t \sim \frac{mc}{eH}(\frac{mc^2}{E})^2$ and the observer will record a field as shown schematically on Figure 1[1]. Clearly, expansion of this field into a Fourier series will lead to a spectrum consisting of the overtones of frequency ω_N. All corresponding expressions for field intensity and the other quantities presented in 1 and a number of articles are true, and there is no reason to discuss them The improper formulas, as was noted above, are those for noncircular movement, when the longitudinal velocity component $v_{\parallel} = v \cos\theta \neq 0$, i e , angle $\theta \neq \pi/2$ The source of the error is particularly clear from the initial expression, for example in [3] and in 1, for the field intensity of the radiation, which was written in the form

---

[1] This figure corresponds with Figure 4 from 1 if movement occurs in a circle and $H_{\perp} = H \sin\theta = H$. However, as we will see in the following, where $\theta \neq \pi/2$, Figure 4 from 1 is incorrect, since the pulses follow with time separation τ^*, not $\tau = 2\pi/\omega_H$.

$$\vec{\mathcal{E}} = Re\left(\sum_{n=1}^{\infty} \vec{\mathcal{E}}_n e^{-in\omega_H t}\right) \tag{2.3}$$

The problem is that where $v_{\parallel} \neq 0$, the radiation pulses do not follow each other at time intervals $\tau = 2\pi/\omega_H$, but rather at intervals τ^*, which differ from τ as a result of the Doppler effect

Figure 1 Figure 2[1]

Time τ^* can be easily found, using Figure 2 For the observer selected, the radiation bursts arrive when the electron is located at points A, B, C, . (for simplicity here and below we will consider that the radiation is strictly acicular) In other words, at these points the electron "looks" at the observer The time intervals between moments when the electron passes through points A and B, naturally, is equal to the period $\tau = 2\pi/\omega_H$ The distance between points A and B is $v_{\parallel}\tau = v\tau \cos\theta$ (θ is the angle between $\vec{v}$ and $\vec{H}$), but an impulse emitted at point A will travel path $c\tau$ in this period of time. We can see from Figure 2 that an impulse emitted at point B will arrive at the position of the observer with a delay with respect to the first impulse of time

$$\tau^* = \tau\left(1 - \frac{v_{\parallel}\cos\theta}{c}\right) = \tau\left(1 - \frac{v}{c}\cos^2\theta\right) \simeq \tau \sin^2\theta = \frac{2\pi}{\omega_H}\sin^2\theta. \tag{2 4}$$

in which upon transition to the next to last expression, it is considered that

[1] Locations for figures are marked in the original text, but the figures themselves are not presented -- Tr.

the entire calculation is performed for the limit case $v \to c$. We note that the usage of a picture in which radiation approaches the observer in the form of individual pulses is suitable only for $\theta \gg \psi \sim mc^2/E$. Actually, however, expression $\tau^* = \tau(1 - \frac{v_{\parallel} \cos\theta}{c})$ is general in nature and is unrelated to the assumption of "acicularity" of the radiation or to the possibility of dividing the radiation into discrete pulses (in this case $v_{\parallel} \cos\theta$ is replaced by $v_{\parallel} \cos\theta'$, see section 2 2)

Thus, the field of an ultrarelativistic electron in the wave zone consists of the overtones of the frequency

$$\omega_H^* = \frac{2\pi}{\tau^*} = \frac{\omega_H}{\sin^2\theta} \qquad (2.5)$$

In itself, this fact is not very essential, if we consider that in the cases which interest us the overones are not resolved and we must concern ourselves with a continuous spectrum. The estimate of pulse widths $\Delta t \sim \frac{mc}{eH_{\perp}}(\frac{mc^2}{E})^2$ presented in 1, and therefore the characteristic frequency $\omega_m \sim 1/\Delta t$ are quite correct (here and in the following $H_{\perp} = H \sin\theta$). However, a change in the interval between pulses influences not only the spectrum, but also all characteristics of the radiation field, in particular its intensity recorded at the observation point. Actually, suppose the electron in each revolution (over time $\tau = 2\pi/\omega_H$) loses energy $\Delta E = P\tau$ to radiation. Then, on the basis of the above, it is obvious that this energy will arrive at the "observers" located on a certain fixed sphere at distance τ from the electron in time τ^* and, consequently, the mean observed radiation power (total energy flux) will be equal to

$$P^* = \frac{\Delta E}{\tau^*} = \frac{P\tau}{\tau^*} = \frac{P}{\sin^2\theta} \qquad (2\ 6)$$

At first glance it might seem that this is a contradiction with the law of conservation of energy. The electron loses energy P per unit time (the value of P is determined by the well known formula, see formula (1, 2 10) or formula (2 29) below) All of this energy goes over to radiation and, it would seem, should equal the total radiation flux through the sphere in question This approach is frequently used -- the radiation losses of the particle are calculated and set equal to the total radiation flux In the stationary case and for a radiator whose center of gravity is nonmoving, of course, this approach can be used In general, however, as is well known, the work performed by a radiator per unit time (power of losses P) is equal to the total flux through a certain surface plus the change in field energy $\frac{\partial}{\partial t}\int\left(\frac{E^2 + H^2}{8\pi}\right)dV$ in the volume enclosed by this space In the case of interest to us, the area of the space occupied by the radiation located between the moving electron and the surface, which is fixed in space, and over which the observation is performed, decreases continually The energy enclosed in this space also decreases, so that the power of the radiation received P* is greater than the power of the losses P (Incidentally, in number of work upon transition to spectral quantities the power of losses P has been used) This approach, of course, cannot lead to correct expressions for radiation intensity recorded on a certain nonmoving surface if movement of the radiator is taken into consideration However, if the radiatin particles are located in a fixed volume (for example, the envelope of a supernova star) or, more precisely, if the distribution function of the radiating particles does not change with time, the intensity of the radiation of the set of particles corresponds to the spectral power of the losses This conclusion is clear from the law of conservation of energy and, of course, is confirmed by direct calculation (see section 2 3).

This is the essence of the matter We assume that the fact that all of this essentially quite elementary problem has remained so long unclear and has led to the usage (in particular, by us ourselves) of formulas not always of not completely correct, justifies this detailed explanation The corresponding general notes, naturally, relate to radiation of any nature, not only synchrotron radiation (as an example, [2] presents a discussion of the case of Cerenkov radiation arising when a particle passes through a flat plate of material).

2.2. Synchrotron Radiation of an Individual Particle Moving at Arbitrary Angle θ to a Magnetic Field

Figure 3

Let us select a system of coordinates in correspondence with Figure 3 such that the axis $\vec{e}_3$ is directed along the external magnetic field $\vec{H} = H\vec{e}_3$. The particle with charge e moves in field $\vec{H}$ along the trajectory

$$\begin{aligned} \vec{r}_0(t) &= \frac{c\beta_\perp}{\omega_H}\{-\vec{e}_1\cos\omega_H t + \vec{e}_2\sin\omega_H t\} + \vec{e}_3 c\beta_\parallel t \\ \vec{\beta}(t) &= \beta_\perp\{\vec{e}_1\sin\omega_H t + \vec{e}_2\cos\omega_H t\} + \vec{e}_3\beta_\parallel . \end{aligned} \tag{2.7}$$

Here $\vec{\beta}$ is the speed of the particle in units of the speed of light c, $\beta_\parallel$ and $\beta_\perp$ are the values of its projections in the direction of the field and in the direction transverse to the field, ω_H is defined by expression (2 2). For a negatively charged particle $\omega_H < 0$. Figure 3 shows that the trajectory of a negatively charged particle (for example an electron)

At large distances from the particle, in the wave zone, the Fourier components of the vector potential and the electric field intensities of the particle are, respectively (see [4], paragraph 66)

$$\vec{A}_\omega = e^{i\frac{\omega}{c}\vec{k}\vec{r}}\frac{e}{2\pi r}\int_{-\infty}^{+\infty}\vec{\beta}(t)e^{i\{\omega t-\frac{\omega}{c}\vec{k}\vec{r}_0(t)\}}dt,$$

$$\vec{\mathcal{E}}_\omega = i\frac{\omega}{c}[\vec{k}[\vec{A}_\omega\vec{k}]] = i\frac{\omega}{c}(\vec{A}_\omega - \vec{k}(\vec{A}_\omega\vec{k})), \qquad (2\ 8)$$

where $\vec{K}$ is a unit vector in the direction of the radiation (in the direction from the particle to the observer), $\vec{r} = r\vec{k}$, r is the distance between the observer and the position of the electron at a certain fixed moment in time, we consider that vector $\vec{k}$ lies in the plane $(\vec{e}_2, \vec{e}_3)$ and makes angle θ' with the direction of the magnetic field, i e , $\vec{k} = \{0, k_2, k_3\} = \{0, \sin\theta', \cos\theta'\}$.

We recall that the angle between $\vec{v}$ and $\vec{H}$ is represented by θ (see Figure 3)

As follows from (2 8) the expression for $\vec{\varepsilon}_\omega$ includes only the velocity component transverse to the direction of radiation

$$\vec{\beta}_t = \vec{\beta} - \vec{k}(\vec{\beta}\vec{k}) = \vec{e}_1\beta_\perp \sin\omega_H t + (\vec{e}_2 k_3 - \vec{e}_3 k_2)\times$$
$$\times(\beta_\perp k_3 \cos\omega_H t - \beta_\parallel k_2). \qquad (2.9)$$

It is convenient to introduce the three unit vectors $\vec{l}_1$, $\vec{l}_2$, $\vec{k}$ such that

$$\vec{l}_2 = \vec{e}_3 k_2 - \vec{e}_2 k_3, \quad \vec{l}_1 = [\vec{l}_2\vec{k}] = -\vec{e}_1 \qquad (2.10)$$

Vector $\vec{l}_2$ is directed along the projection of $\vec{H}$ on a plane perpendicular to the direction of observation (plane of the figure), i e , along the vector $\vec{e}_3 - \vec{k}(\vec{e}_3 \cdot \vec{k})$.

From expressions (2 8), (2 9) and (2 10) we derive

$$\vec{\mathcal{E}}_\omega = e^{i\frac{\omega}{c}r} \frac{ie\omega}{2\pi c r} \int_{-\infty}^{+\infty} \vec{\beta}_t(t) e^{i(\omega t - \frac{\omega}{c}\vec{k}\vec{r}_0(t))} dt \tag{2.11}$$

where

$$\vec{\beta}_t(t) = -\vec{l}_1 \beta_\perp \sin \omega_H t - \vec{l}_2(\beta_\perp k_3 \cos \omega_H t - \beta_\parallel k_2) \tag{2 12}$$

For the calculation of integral (2 11), we note that the exponent of the integrand is (see (2 7))

$$\omega t - \frac{\omega}{c}\vec{k}\vec{r}_0(t) = (1-\beta_\parallel k_3)\omega t - z \sin \omega_H t \;, \qquad z = \frac{\omega}{\omega_H}\beta_\perp k_2 \;. \tag{2.13}$$

Further, we use the representation

$$e^{-iz \sin \omega_H t} = \sum_{n=-\infty}^{\infty} \mathcal{I}_n(z) e^{-in\omega_H t} \;, \tag{2.14}$$

where $I_n(z)$ is a Bessell function of the first kind.

Integration with respect to t leads to the appearance of δ-functions.

$$\vec{\mathcal{E}}_\omega = e^{i\frac{\omega}{c}r}\,\frac{e\omega}{cr}\sum_{n=-\infty}^{\infty}\Big\{\vec{l}_1\beta_\perp J_n'(z) - i\vec{l}_2\Big(\beta_\perp k_3\frac{n}{z} - \beta_\parallel k_2\Big)J_n(z)\Big\}\,\delta\big\{(1-\beta_\parallel k_3)\omega - n\omega_H\big\}. \tag{2 15}$$

Consequently, the radiation has a discrete spectrum with frequencies

$$\omega = \omega_n \equiv n\,\frac{\omega_H}{1-\beta_\parallel k_3} \equiv \frac{\omega_H}{1-\beta\cos\theta\cos\theta'} . \tag{2 16}$$

In the ultraviolet case $\beta \to 1$ and the radiation, practically, is directed along the instantaneous particle velocity, i e , angle $\theta \approx \theta'$ and $\omega = \frac{n\omega_H}{1-\frac{v}{c}\cos^2\theta} \simeq \frac{n\omega_H}{\sin^2\theta} = n\omega_H^*$ which is in agreement with (2 5) The Fourier integral of the electric field of the radiation of the particle is thus reduced to the series

$$\vec{\mathcal{E}} = \int_{-\infty}^{\infty}\vec{\mathcal{E}}_\omega e^{-i\omega t}\,d\omega = \mathrm{Re}\sum_{n=1}^{\infty}\vec{\mathcal{E}}_n e^{i\omega_n\left(\frac{r}{c}-t\right)},$$

$$\vec{\mathcal{E}}_n = \frac{2e}{cr}\,\frac{n\omega_H\beta\sin\theta}{(1-\beta\cos\theta\cos\theta')^2}\left\{\vec{l}_1 J_n'(z_n) - i\vec{l}_2\,\frac{\cos\theta' - \beta\cos\theta}{\beta\sin\theta\sin\theta'}\,J_n(z_n)\right\}, \tag{2.17}$$

$$z_n = n\,\frac{\beta\sin\theta\sin\theta'}{1-\beta\cos\theta\cos\theta'}$$

This expression (2 17) completely defines the radiation field created at a certain sufficiently remote point in space by a particle moving at an arbitrary angle to the magnetic field In the following it will be convenient to use the "radiation tensor," which by definition is equal to[1]

[1] As far as we know, the name of the tensor $\tilde{\rho}_{\alpha\beta}$ has not been established, and we certainly shall not insist on the term "radiation tensor" which we have used.

$$\tilde{P}_{\alpha\beta}(n) = \frac{c}{8\pi} \mathcal{E}_{n,\alpha} \mathcal{E}^*_{n,\beta} \tag{2.18}$$

where $\alpha,\beta = 1,2$ and $\varepsilon_{n,\alpha}$ are the components of the electrical vector (12 17) Here, the mean energy flux density over the period (pointing vector) in the n-th harmonic is equal to

$$\tilde{P}_n = S_p \tilde{P}_{\alpha\beta}(n) \equiv \tilde{P}_{11} + \tilde{P}_{22} = \frac{c}{8\pi} |\vec{\mathcal{E}}_n|^2 \tag{2.19}$$

For ultrarelativistic particles

$$\xi = mc^2/E \ll 1 \tag{2 20}$$

and the main role is played by radiation in the higher harmonics $n \sim \frac{\omega_m}{\omega_H^*} \sim \frac{2\pi \nu_c}{\omega_H^*} \sim (\xi \sin\theta)^{-3} \gg 1$ (see 1 and (2.23)), concentrated within the small angle

$$\psi = \theta - \theta' \lesssim \xi = \frac{mc^2}{E} \tag{2.21}$$

The frequencies radiated (see (2.5) or (2.16)) with $\beta \simeq 1$ and $\theta \simeq \theta'$ are equal to

$$\nu = \frac{\omega}{2\pi} = \frac{n\,\omega_H}{2\pi \sin^2\theta}$$

In order to go over to the ultrarelativistic limit in (2.17), we can use the approximate expression for Bessell functions with high values of index and argument This leads to the following expression for the amplitude of the n-th harmonic of the electric radiation field of an ultrarelativistic electron[1]

$$\vec{\mathcal{E}}_n = \frac{2e\omega_H}{\sqrt{3}\pi c r}\,\frac{n}{\sin^5\theta}\Big\{\vec{l}_1(\xi^2+\psi^2)K_{2/3}(g_n)+i\vec{l}_2\psi(\xi^2+\psi^2)^{1/2}K_{1/3}(g_n)\Big\}, \qquad (2.22)$$

where

$$g_n = n\frac{(\xi^2+\psi^2)^{3/2}}{3\sin^3\theta} = \frac{\nu}{2\nu_c}\Big(1+\frac{\psi^2}{\xi^2}\Big)^{3/2}, \quad \nu = \frac{n\,\omega_H}{2\pi\sin^2\theta}, \qquad (2.23)$$

$$\nu_c = \frac{3\omega_H\sin\theta}{4\pi\xi^3} = \frac{3eH_\perp}{4\pi mc}\Big(\frac{E}{mc^2}\Big)^2$$

In the area of higher harmonics, the radiation spectrum is practically continuous and in place of the polarization tensor for radiation at the n-th harmonic (2.18), it is more expedient to use the "spectral density of the radiation tensor"

$$\tilde{P}_{\alpha\beta}(\nu) = \tilde{P}_{\alpha\beta}(n)\frac{dn}{d\nu} = \frac{2\pi\sin^2\theta}{\omega_H}\,\tilde{P}_{\alpha\beta}(n) \qquad (2.24)$$

[1] Here and in the following we will consider that $\omega_H > 0$ and, consequently, in expression (2.17) for the field of the electron frequency $\omega_n > 0$ Here, the change of the sign of the charge e corresponds to transition to complex-conjugate amplitude $\vec{\mathcal{E}}^*_n$ in (2.17) Therefore, for a positively charged particle (for example a positron), the amplitude is complex-conjugate with respect to (2 22), corresponding to opposite direction of rotation of the electrical vector

From this and from expressions (2 18), (2 22) we can find the spectral density of the radiation fluxes with two main directions of polarization

$$\tilde{P}^{(1)}_{\nu} \equiv \tilde{P}_{11}(\nu) = \frac{3e^2\omega_H}{4\pi^2 r^2 c\,\xi^2 \sin^2\theta}\left(\frac{\nu}{\nu_c}\right)^2\left(1+\frac{\psi^2}{\xi^2}\right)^2 K^2_{2/3}(g_\nu) \tag{2 25}$$

$$\tilde{P}^{(2)}_{\nu} = \tilde{P}_{22}(\nu) = \frac{3e^2\omega_H}{4\pi^2 r^2 c\,\xi^2 \sin^2\theta}\left(\frac{\nu}{\nu_c}\right)^2 \frac{\psi^2}{\xi^2}\left(1+\frac{\psi^2}{\xi^2}\right) K^2_{1/3}(g_\nu) \tag{2.26}$$

$$\tilde{P}_{12}(\nu) = \tilde{P}^{*}_{21}(\nu) = -i\,\frac{3e^2\omega_H}{4\pi^2 r^2 c\,\xi^2 \sin^2\theta}\left(\frac{\nu}{\nu_c}\right)^2\left(1+\frac{\psi^2}{\xi^2}\right)^{3/2}\frac{\psi}{\xi} K_{1/3}(g_\nu)\, K_{2/3}(g_\nu) \tag{2.27}$$

where $g_\nu = g_n$ (see (2 23))

We note here that formulas (2 22), (2 23) and (2 25)-(2 27) are easily generalized to cover the case when the radiating particle is located in a "cold" plasma, the index of refraction of which can with good approximation be considered equal to $\tilde{n} = 1 - \frac{\omega_0^2}{2\omega^2}$, where $\omega_0 = \sqrt{4\pi N_e e^2/m}$, N_e is the concentration of electrons in the plasma This approximation is correct if $\omega \gg \omega_0$ and $\omega \gg \omega_H^{(0)} = eH/mc$. Under these conditions in formulas (2.20)-(2 23) and (2.25)-(2.27) we should replace the quantity ξ where it appears explicitly by

$$\eta = \sqrt{\xi^2 + \left(\frac{\omega_0}{\omega}\right)^2} \simeq \sqrt{1-\tilde{n}^2\beta^2} \tag{2.28}$$

As is clear from the preceding, it is assumed in this case that $\eta \ll 1$

considered that $(1-\tilde{n}^2\beta^2)=[1-(1-\omega_0^2/\omega^2)v^2/c^2)]\simeq(mc^2/E)^2+\omega_0^2/\omega^2=\eta^2$ Expressions (2 25)-(2 27) and, correspondingly, the Stokes parameters for radiation of an individual electron differ from those used in 1 (see (1, 2 17) and (1, 2 18)) in the appearance of the factor $\sin^2\theta$ in the denominator (this conclusion was reached by us [2] and a number of other authors, for example, see [5; 6a] , it is in this respect that the expressions for the intensity and Stokes parameters presented in 1 and a number of other articles are incorrect, if we are considering the radiation of an individual particle or combination of particles moving in space

However, if, as occurs in most cases, we are interested in the radiation of particles from a volume fixed in space, we must use the expressions presented in 1. Let us now go over to analysis of this problem

2.3 Radiation of System of Particles

If we use (2 25)-(2 26) to calculate the total energy flux of radiation through a fixed surface, i e , calculate the integral of the flux density with respect to all particles and directions, we will find that it is $1/\sin^2\theta$ times greater than the known expression for energy losses of an ultrarelativistic particle

$$P=-\frac{dE}{dt}=\frac{2e^4H_\perp^2}{3m^2c^3}\left(\frac{E}{mc^2}\right)^2 \qquad (2.29)$$

As was indicated in section 2.1, this difference is caused by the nonstationary nature of the radiation field. Namely, the total energy flux through the fixed surface

$$P^{*} \equiv \oint_{S} \tilde{p}\, dS = P - \frac{\partial}{\partial t} \int_{V} \frac{E^2 + H^2}{8\pi}\, dV$$

is determined not only by work P performed by the particle, but also by the change in field energy within volume V, limited by surface S. The change in field energy obviously is related to the forward movement of the particle and becomes essential when the velocity of the forward movement of the particle is comparable to the speed of light.

Actually, this result is caused by the delay resulting from the finite speed of propagation of the electromagnetic field. Actually, let us analyze the radiation of an individual electron intersecting an element of volume $r^2 dr d\Omega$ at distance r from the observer (Figure 4). The electron is located in the volume element in question during time $dt' = dr/v_r$, where v_r is the projection in the direction of the observer of the mean velocity of the forward movement of the particle $v_{\parallel}$. Obviously, $v_r = v_{\parallel} \cos\theta' = v \cos\theta''$ $\cos\theta'$. If r(t) is the variable distance to the particle, moment of observation t is related to the moment of radiation t' by the relationship $t = t' + r(t')/c$ (having in mind radiation in a vacuum). Therefore the radiation emitted by an electron in time dt', corresponding to movement over distance dr will be received by the observer over time

$$dt = dt'\left(1 - \frac{v_r}{c}\right) = \left(1 - \frac{v_r}{c}\right)\frac{dr}{v_r} \tag{2.30}$$

It follows from this that the energy radiated over time dt' and passing through a unit surface at the observation point in time dt is equal to ($\tilde{p}_\nu = \tilde{p}_\nu^{(1)} +$ $+ \tilde{p}_\nu^{(2)}$. (see (2 25) and (2 26))

$$\tilde{p}_\nu dt = \tilde{p}_\nu \left(1 - \frac{v_r}{c}\right) dt' = p_\nu dt', \qquad (2.31)$$

where p_ν represents the quantity

$$p_\nu = \tilde{p}_\nu \left(1 - \frac{v_r}{c}\right) = \tilde{p}_\nu \left(1 - \frac{v}{c} \cos\theta \cos\theta'\right) \qquad (2\ 32)$$

As follows from (2 31), this quantity p_ν has the sense of the flux density of the energy radiated by the electron per unit time It is not difficult to see that the integral of p_ν with respect to all frequencies and directions leads to the proper expression for the energy losses (i e , in the ultrarelativistic case to expression (2 29))

Thus, relationship (2 32) establishes the connection between the observed flux $\tilde{p}_\nu$ of radiation and the "power" radiated by the electron p_ν. Obviously, a similar relationship can be written for all components of the radiation polarization tensor (see (2.18) and (2 24)):

$$p_{\alpha\beta}(\nu) = \tilde{p}_{\alpha\beta}(\nu) \left(1 - \frac{v}{c} \cos\theta \cos\theta'\right). \qquad (2.33)$$

In the ultrarelativistic case ($v \approx c$, $\theta \approx \theta'$) it follows that

$$P_{\alpha\beta}(\nu) = \tilde{P}_{\alpha\beta}(\nu)\sin^2\theta \qquad (2\ 34)$$

Figure 4

Let us now show that if we are concerned with the radiation of particles from a fixed volume, quantity $P_{\alpha\beta}(\nu)$ should be used. Actually, this is clear from (2 31), since this relationship (2 31) shows that the energy received by an observer from trajectory sector dr is determined by the value of $p_{\alpha\beta}$ and time $dt' = dr/v_r$, during which the electron passes through this sector Let us now analyze this problem in somewhat more detail, in order to produce an expression for the intensity and other Stokes parameters

Suppose we are interested in the intensity of radiation of a set of particles, the distribution function of which is $N(E,\vec{\tau},\vec{r},t)$ By definition, quantity $N(E,\vec{\tau},\vec{r},t)dEd\Omega_\tau dv$ is equal to the number of particles with energies in the interval E,E + dE and velocity directions within the solid angle $d\Omega_\tau$ contained at moment t in the volume element $dV = r^2 dr d\Omega$

The volume element being analyzed (see Figure 4) receives $v_r N(E,\vec{\tau},\vec{r},t - \frac{r}{c})dEd\Omega_r \times r^2 d\Omega$ particles per unit time, here t is the moment of observation, $t' = t - \frac{r}{c}$ is the moment of radiation from the fixed point in space Each particle radiates from* the volume element in question an energy of (see (2 31)) $p_\nu dt' = p_\nu \frac{dr}{v_r}$ As a result, the total flux and intensity of radiation received are

$$F_\nu = \int P_\nu N(E,\vec{\tau},\vec{r},t-\frac{r}{c}) r^2 dr\, d\Omega\, dE\, d\Omega_r \,|,$$
$$I_\nu = \frac{dF}{d\Omega} = \int P_\nu N(E,\vec{\tau},\vec{r},t-\frac{r}{c}) r^2 dr\, dE\, d\Omega_r \tag{2.35}$$

Analogous expressions obviously occur for all components of the tensor

$$I_{\alpha\beta} = \int P_{\alpha\beta}(\nu) N(E,\vec{\tau},\vec{r},t-\frac{r}{c}) r^2 dr\, dE\, d\Omega_r =$$
$$= \frac{1}{2}\begin{pmatrix} I+Q & U\pm iV \\ U\mp iV & I-Q \end{pmatrix}, \quad \alpha,\beta = 1,2 \tag{2.36}$$

where I,Q,U,V are the Stokes parameters of the radiation received

For a stable distribution function, under conditions such that $N(E,\vec{\tau},\vec{r},t) \equiv N(E,\vec{\tau},\vec{r})$, expressions (2 35) and (2 36) correspond to those presented in 1

If we are analyzing an area (cloud) of moving particles, the observed intensity I_ν (or flux F_ν) is essentially determined by the dependence of the distribution function on time In particular, for an individual electron $N(E,\vec{\tau},\vec{r},t-\frac{r}{c}) \sim \delta(r+v_r(t-\frac{r}{c}))$ and as a result of integration with respect to r in (2 35) we produce

$$F_\nu = \frac{P_\nu}{1-(v_r/c)} = \widetilde{P}_\nu = \widetilde{P}_\nu^{(1)} + \widetilde{P}_\nu^{(2)} \tag{2 37}$$

as should be in correspondence with (2.25) and (2 26) Let us now assume that we are concerned with the stationary "cloud" of particles moving as a unit whole with velocity $\vec{V}$ and projection of velocity in the direction toward the observer V_r. This means that in (2 35) the function $N(E,\vec{\tau},\vec{r},t) = N_0(E,\vec{\tau},\vec{r}-\vec{V}t)$. The intensity of radiation from such a cloud is equal to

$$I_\nu = I_\nu^{(0)}\left(1-\frac{V_r}{c}\right)^{-1}, \quad I_\nu^{(0)} = \int P_\nu N_0(E,\vec{\tau},\vec{r})\, r^2\, dr\, dE\, d\Omega_\tau \quad . \qquad (2\ 38)$$

Here $I_\nu^{(0)}$ is the radiation intensity of the nonmoving cloud with distribution $N_0(E,\vec{\tau},\vec{r})$ the same as for a moving cloud at fixed moment t

Reabsorption of Synchrotron Radiation by Ultrerelativistic Particles[2]

3 1 General Notes

If there is a sufficiently large number of particles over the ray of vision, absorption and forced (induced) radiation by the radiating particles themselves begin to have an influence. This process is usually called reabsorption. Reabsorption can in principle change the intensity and polarization of radiation quite essentially. Furthermore, under certain conditions negative reabsorption is possible, i e , amplification of radiation. Of course, the nature of reabsorption is closely related to the nature of the radiator in question. Here we will be interested in reabsorption of synchrotron radiation,

[1] At this point for simplicity we are using the velocity averaged over the period of movement, i e , velocity $v_\parallel$. In this connection, $N(E,\vec{\tau},\vec{r}t)$ should be taken to mean the mean expression over the period, so that the dependence of N on $\vec{\tau}$ is reduced to the dependence on angle θ alone

[2] The authors are indebted to V V Zheleznyakov and V N Sazonov for their help in writing this section of the article and their permission to use their unpublished results

i e , the radiators will be considered to be charged relativistic particles moving in a magnetic field This radiation and its reabsorption may change essentially if the radiating area contains a "cold" plasma in addition to the relativistic electrons (see 1, section 4 3 and (2 28) above) For example, in the case of radiation in a vacuum, reabsorption in any system of relativistic electrons with isotropic velocity directions is positive (i e , under these conditions absorption occurs, see [7, 8] and below) When a "cold" plasma is present, reabsorption of synchrotron radiation may become negative [8, 9] This means that the corresponding system (for example layer or cloud) of relativistic electrons with isotropic distribution of the velocities will act like a maser

In investigating reabsorption earlier (see 1 and the bibliographic references thereto) expressions for intensity of radiation of an individual particle were used which were averaged with respect to all directions The conditions of acceptability and even the very nature of such an approach is not known in advance, and it is not suitable for determination of changes in polarization. Suffice it to say that the radiation has finite angular distribution, and its polarization properties depend essentially on angle $\psi = \theta - \theta'$ between the direction of the velocity and the direction of the radiation (see (2 25)-(2 27) Therefore, in an investigation of reabsorption (and particularly negative reabsorption) considering the polarization of the radiation, a stricter analysis of the angular and polarization properties of synchrotron radiation is necessary It should be added that a "cold" plasma in a magnetic field is anisotropic (magnetoactive) and in many cases, even in a weak field, can be considered isotropic with index of refraction $\tilde{n} = 1 - \omega_0^2/2\omega^2$ with sufficient accuracy The polarization characteristics are particularly sensitive in this respect, since the rotation of the polarization plane (Faraday effect) is integral, increasing with length of the path traveled by the wave (for example, see formula (1, 4.6))

The overall problem requiring investigation in individual particular cases is as follows Within a certain area ("source"), the distribution functions of relativistic electrons $N(\vec{p},\vec{r})$, concentration of "cold" plasma $Ne(\vec{r})$ and magnetic field intensity $\vec{H}(\vec{r})$ are fixed We must determine the radiation field both within this area (at the source) and in particular at some distance from it Usually we speak in this case of the radiation of the source itself, but the necessity may arise of determining the influence of this "source" on radiation passing through it from another source located farther from the reception point (for this reason, the term source is conditional in nature).

In the preceding we considered the source stationary and therefore time t has no part to play We cannot use this limitation for moving or expanding sources (see section 4 1) In practice, other limitations are possible in addition to the assumption of stability For example, under space conditions, due to the existence of a number of instabilities, the anisotropy of electron distribution by velocities rather rapidly disappears or, in any case, is sharply reduced (for example, see [10]). In this connection in most cases it can be considered that the distribution function for relativistic electrons depends only on their energy, i e., we can use concentration $N(\vec{E},\vec{r})$ Furthermore, the dependence of N, Ne and $\vec{H}$ on the coordinates is always extremely slow in comparison to the radiation wavelength under space conditions. Therefore, generally speaking, the approximation of geometric optics can be used and frequently we can simply consider all quantities constant over the ray of vision in an area of length L Another possibility is to consider that over length L concentrations N and Ne are constant, but field $\vec{H}$ is chaotic with intensity H

In order to describe the radiation in the general case we must use Stokes parameters I, Q, U and V, related to tensor $I_{\alpha\beta}$ by the following relationships (α = 1,2)

$$I_{\alpha\beta} = \frac{1}{2}\begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}; \quad \begin{matrix} I = I_{11}+I_{22}, V = i(I_{21}-I_{12}) \\ Q = I_{11}-I_{22}, U = I_{21}+I_{12} \end{matrix} \tag{3.1}$$

The indexes 1 and 2 here correspond to the x and y axes perpendicular to the ray of vision

Expression (3 1) was used above (2 36), but the concrete expression of $I_{\alpha\beta}$ through N in (2 36) relates to the case of radiation in a vacuum without consideration of reabosorption The relationship between the Stokes parameters and the intensity of radiation I, degree of polarization Π, ratio of axes of polarization ellipse p and angle x, determining the orientation of this ellipse is such that (for more detail see, for example, [11] and [18], paragraph 6, as well as 1)

$$I = I, \ \Pi = \frac{\sqrt{Q^2+U^2+V^2}}{I}, \ \sin 2\beta = \frac{V}{\sqrt{Q^2+U^2+V^2}}, \tag{3.2}$$
$$\beta = \operatorname{arctg} p, \ \operatorname{tg}(2\chi) = U/Q .$$

The Stokes parameters used (and any quantities expressed through them) relate to radiation in a certain frequency interval $\partial\nu \ll \nu$ and correspond to averaging of the squared field expressions over time $\Delta t \gg 1/\Delta\nu$. In an anisotropic medium or in particular in a magnetoactive medium, electric field $\vec{E}$ generally speaking, is not perpendicular to $\vec{k}$, whereas the induction vector $\vec{D}$ is always orthogonal to the wave vector $\vec{k}$ In this connection it is more convenient in an anisotropic medium to define the tensor $I_{\alpha\beta}$ [12] as $I_{\alpha\beta} = D_\alpha D^*_\beta$, $\alpha = 1,2$ The Stokes parameter in quantity (3 2) will now also relate to the vector $\vec{D}$, not to $\vec{E}$ It should be kept in mind that intensity $I = SpI_{\alpha\beta} \equiv D_{11}D^*_{11} + D_{22}D^*_{22}$ in the general case is not proportional to the energy flux. When radiation is received far from its source (in a vacuum or, more precisely, outside an anisotropic medium) this factor is generally unimportant.

3 2 Transfer Equation for Tensor $I_{\alpha\beta}$

In order to determine the tensor $I_{\alpha\beta}$ we must use the transfer equation which has been investigated and discussed in recent years in a number of works [12-16, 2] (a particularly detailed discussion is in [12]) In a homogeneous medium for the stationary case ($I_{\alpha\beta}$ independent of t), the transfer equation has the form

$$\frac{dI_{\alpha\beta}}{dz} = \mathcal{E}_{\alpha\beta} + (R_{\alpha\beta\gamma\delta} - K_{\alpha\beta\gamma\delta})\, I_{\gamma\delta} \qquad (3.3)$$

Here

$$\mathcal{E}_{\alpha\beta} = \int r^2 p_{\alpha\beta}\, N(E, \vec{\tau}, \vec{r})\, dE\, d\Omega_{\vec{r}} \qquad (3.4)$$

is the radiative capacity of a unit volume, i e , the power of spontaneous radiation from a unit volume per unit solid angle and unit frequency interval. For synchrotron radiation, which we will discuss for concreteness, expressions $p_{\alpha\beta}$ are determined by formulas (2 25)-(2 27) and (2 34), here, as concerns quantity ε_{12}, it may be necessary to supplement expression (2.27) with terms of higher order with respect to $\xi = mc^2/E$ (see [15]). In the presence of a plasma, we can also replace ξ by η in these formulas (see (2 28))

Furthermore, tensors $R_{\alpha\beta\gamma\delta}$ and $K_{\alpha\beta\gamma\delta}$ in (3.3) characterize the change in $I_{\alpha\beta}$ due to Faraday rotation and absorption of radiation respectively The tensors $R_{\alpha\beta\gamma\delta}$ and $K_{\alpha\beta\gamma\delta}$ are expressed through parameters characterizing the "normal" waves capable of propagation in the medium in question

In an anisotropic medium in which spatial dispersion is ignored, two "normal" waves can propagate, which in the case of monaxial crystals and magnetoactive plasma are called ordinary (0 or index 2) and extraordinary (e or index 1) waves Al quantities (of the field $\vec{E},\vec{D},\vec{H}$) in normal waves in a homogeneous medium depend on t and $\vec{r}$ according to an exponential rule and, for example,

$$\vec{\mathcal{D}}_{o,e} = A_{o,e}\, \vec{\gamma}_{o,e}\, e^{-\varkappa_{o,e} z} \cdot e^{-i(\omega t - \alpha_{o,e} - k_{o,e} z)} \tag{3.5}$$

Here, as in (3.3), the waves are considered to propagate along the z axis, $\kappa_{0,e}$ is the absorption coefficient with respect to amplitude (absorption coefficient with respect to power $\mu_{0,e}$ is equal to $2\kappa_{0,e}$, frequently κ is used to represent the absorption index $c\mu/2\omega$), $\omega = 2\pi\nu$ and $\vec{\kappa}_{0,e}$ is the wave vector ($\kappa_{0,e} = \frac{\omega}{c}\tilde{n}_{0,e}$, where $\tilde{n}_{0,e}$ is the index of refraction). The complex vectors $\vec{\gamma}_{0,e}$

characterize the polarization of normal waves ($A_{0,e}$ and $\alpha_{0,e}$ are the arbitrary amplitudes and phases of these waves) In a magnetoactive plasma when absorption is ignored (i e , in practice with rather weak absorption) we can assume

$$\gamma_{\alpha o}\gamma_{\alpha o}^{*} = \gamma_{\alpha e}\gamma_{\alpha e}^{*} = 1, \quad \gamma_{\alpha o}\gamma_{\alpha e}^{*} = \gamma_{\alpha o}^{*}\gamma_{\alpha e} = 0 \tag{3.6}$$

where summation is performed with respect to $\alpha = 1,2$, as is done throughout with the Greek indexes encountered twice (in other words, for example $\bar{\gamma}_{\alpha 0}\gamma_{\alpha 0} = S_p\gamma_{\alpha 0}\gamma_{\beta 0}$, for a more detailed presentation on normal waves in a magnetoactive plasma see, for example, [17, 18])

The induction component of the arbitrary radiation field in the frequency interval $\Delta\omega$ has the form

$$D_\alpha(z,t) = \int_{\Delta\omega} A_e\gamma_{\alpha e}\exp\{-\varkappa_e z - i(\omega t - \alpha_e - k_e z)\}\,d\omega + \int_{\Delta\omega} A_o\gamma_{\alpha o}\exp\{-\varkappa_o z - i(\omega t - \alpha_o - k_o z)\}\,d\omega$$

Forming the tensor $D_\alpha D_\beta^*$ from these components, and also calculating the derivative fraction $\frac{d}{dz}(D_\alpha D_\beta^*)$, after averaging with respect to time over the rather narrow frequency interval $\Delta\omega$, we can arrive at equation (3.3) [12], with which (not confusing index γ with the polarization vector $\overline{\gamma}$!)

$$R_{\alpha\beta\gamma\delta} = -i(\kappa_e - \kappa_0)(\gamma_{\alpha e}\gamma^*_{\beta 0}\gamma^*_{\gamma e}\gamma_{\delta 0} - \gamma_{\alpha 0}\gamma^*_{\beta e}\gamma^*_{\gamma 0}\gamma_{\delta e}) \tag{3.7}$$

$$\begin{aligned} K_{\alpha\beta\gamma\delta} = {} & (\varkappa_e + \varkappa_0)(\gamma_{\alpha e}\gamma^*_{\beta 0}\gamma^*_{\gamma e}\gamma_{\delta 0} + \gamma_{\alpha 0}\gamma^*_{\beta e}\gamma^*_{\gamma 0}\gamma_{\delta e}) + \\ & + 2\varkappa_e\gamma_{\alpha e}\gamma^*_{\beta e}\gamma^*_{\gamma e}\gamma_{\delta e} + 2\varkappa_0\gamma_{\alpha 0}\gamma^*_{\beta 0}\gamma^*_{\gamma 0}\gamma_{\delta 0} \end{aligned} \tag{3.8}$$

We note that expression (3 7) was produced in [13], but the expressions like (3 8) from [13] are inaccurate, since the radiation absorbed was not expanded into normal waves

If the absorption is sufficiently great, the normal waves cannot be considered orthogonal (see (3 6)) and formulas (3 7)-(3 8) are no longer accurate This occurs, in particular, under conditions when the relativistic particles ("hot" plasma and "cold" plasma) make a comparable contribution to the real and (or) imaginary anisotropic parts of the dielectric premeability tensor ε_{ij}. Transfer equation (3 3) without assumption (3 6) is analyzed in [15, 41], although only under conditions when the influence of the plasma can be considered fairly weak

If the medium includes radiation of only one type (ordinary or extraordinary), i e , tensor $I_{\alpha\beta}$ consists only of fields type e or type 0, then $R_{\alpha\beta\gamma\delta}I_{\gamma\delta} = 0$ This result can easily be produced formally, but is clear from the beginning, since according to the definition of normal waves in a homogeneous medium their polarization is unchanged. It is also obvious that for one normal wave $K_{\alpha\beta\gamma\delta}I_{\gamma\delta} = -2\kappa_{e,0}I_{\alpha\beta}$ and the transfer equation (3 3) takes on the following form where there are no radiation sources

$$\frac{dI_{\alpha\beta}^{(e,o)}}{dz} = -2\varkappa_{e,0} I_{\alpha\beta}^{(e,o)} \equiv -\mu_{e,0}(\vec{k}) I_{\alpha\beta}^{(e,o)} \tag{3.9}$$

Relationship (3 9) is obvious from the beginning, since it reflects the fact that in normal waves the field vectors (in particular vector $\vec{D}$), due to the influence of absorption, change according to the rule $e^{-\kappa_{e,0}z}$ (see (3 5)) The quantities $2\kappa_{e,0} = \mu_{e,0}(\vec{k})$ are the absorption coefficients with respect to power (intensity) along the wave vector $\vec{k}$ If the direction of the phase and group velocities (direction of vectors $\vec{k}$ and $\vec{v}_{gr} = d\omega/d\vec{k}$) correspond, then of course quantities $2\kappa_{e,0}$ correspond to the coefficients of absorption along the rays $\mu_{e,0}$ In the general case $\mu_{e,0} = 2\kappa_{e,0} \cos\vartheta_{e,0}$, where $\vartheta_{e,0}$ are the angles between $\vec{k}_{e,0}$ and $\vec{v}_{gr,e,0}$ Under conditions (3 9), only the intensity of radiation $I = I_{xx} + I_{yy}$ will change along $\vec{K}$ (i.e , along the z axis), since for the intensity $\frac{dI^{(e,0)}}{dz} = -2\kappa_{e,0}I^{(e,0)}$ As concerns the quantities Π, ρ (or β) and x, as was stated, they remain unchanged for normal waves Formerly, the same thing follows from (3 2) and (3.9), and is related with the fact that the quantities Π, ρ and x depend only on the ratio of the Stokes parameters. It is also obvious that constancy of Π, ρ and x occurs in the case when the medium contains only one type of radiation source In this case

$$\frac{dI^{(e,o)}}{dz} = \mathcal{E}_{e,0} - 2\varkappa_{e,o} I^{(e,o)} \equiv \mathcal{E}_{e,o} - \mu_{e,o}(\vec{k}) I^{(e,o)} \tag{3.10}$$

This equation can be generalized to the case of a heterogeneous medium if the approximation of geometrical optics is justified and, consequently, we can use the concept of rays (the possibility of a ray interpretation is limited also by the condition of weak absorption [17]) The corresponding transfer equations for intensity $I^{(e,0)}$ of waves of one type has the form (for the conclusion see [19])

$$\frac{1}{v_{gr}}\frac{\partial I}{\partial t}+\frac{\kappa^2}{|\cos\vartheta|}\frac{\partial}{\partial e}\left(\frac{I/\cos\vartheta/}{\kappa^2}\right)=\varepsilon-\mu I \qquad (3\ 11)$$

Here $I = I^{(e,0)}$, and all remaining expressions also relate to waves of types e or 0 at frequency ν (here v_{gr} is the group velocity, ϑ is the angle between $\vec{k}$ and $\vec{v}_{gr}$, $k = \frac{\omega}{c}\tilde{n}$ is the wave vector and $\mu = 2\kappa \cos\vartheta$ is the coefficient of absorption along the ray, the element of ray length is ∂e) Here, whereas in (3 10) the quantity $I^{(e,0)}$ in the magnetoactive plasma is generally not proportional to the energy flux (see above), in (3 11) we are concerned with the intensity in the true sense of the word, i.e , the energy flux per unit solid angle

Generalization of equation (3 11) to the case of simultaneous presence of radiation of two types, as far as we know, has never been done In a homogeneous and stable medium this generalization evolves to equation (3.3) This equation is doubtless correct for the functions $\varepsilon_{\alpha\beta}$, $R_{\alpha\beta\gamma\delta}$ and $K_{\alpha\beta\gamma\delta}$, which depend rather slowly on the coordinates However, as we can see from comparison of equations (3.11) and (3 3), this latter equation in a heterogeneous medium can be correct only if we ignore refraction (curving of rays) and the derivatives of $d\tilde{n}/dz$ in comparison with $dI_{\alpha\beta}/dz$ Also, of course, the usual approximation of geometric optics should be correct, i e , all quantities should change little over one wavelength in the medium $\lambda = 2\pi s/\tilde{n}\omega$ For example, the condition $\lambda\frac{d\varepsilon_{\alpha\beta}}{dz} \ll \varepsilon_{\alpha\beta}$ should be observed However, generally speaking, the more rigid condition $\lambda\left|\frac{d\tilde{n}_{e,o}}{dz}\right| \ll |\tilde{n}_e - \tilde{n}_o|$ is also fulfilled. This inequality, like the condition of correctness of the geometrical optics approximation, is typical for weakly anisotropic media in the calculation of polarization (see [17], paragraph 26 and [18], paragraph 24)

We attempted above to illuminate the problem of the transfer of radiation from a rather general point of view It is quite obvious that highly complex or at least cumbersome and difficult solutions may be produced for $I_{\alpha\beta}$ or the Stokes parameters. The situation is even more complicated if the "cold" plasma is rather dense and the magnetic fields rather strong Under these conditions, a consideration of the influence of the plasma can not be done by replacing the quantity $\xi = mc^2/E$ by $\eta = \sqrt{\xi^2 + \omega_0^2/\omega^2}$ (see (2 28)) In connection with this problem, see [20-24] The specific nature of the problem also appears if the distribution function of the relativistic electrons with respect to velocities is anisotropic [15, 16, 25] Further, even for isotropic distribution of electrons with respect to velocities, special analysis is required for the case when the function N(E) depends rapidly on energy In this case, function N(E) can be considered rather smooth and the expressions presented below for the coefficient of reabsorption can be used if N(E) changes little over the interval of energies ΔE corresponding to radiation of neighboring overtones of frequency $\omega_H^* = eH/mc \cdot mc^2/e \sin^2\theta$ The radiated frequency $\omega = n\omega_H^*$ and, consequently, $/\Delta\omega/ =$

$$|\Delta\omega| = \hbar\frac{eH}{mc}\cdot\frac{mc^2}{E\sin\theta}\quad\frac{\Delta E}{E}\sim\omega_H^* \quad \text{if} \quad \Delta E\sim\frac{E}{\hbar} =$$

$$=\frac{\omega_H^* E}{\omega} = \frac{eH}{mc\sin^2\theta}\,mc^2 = \frac{eH\lambda}{2\pi\sin^2\theta}, \quad \text{when} \quad \lambda=\frac{2\pi c}{\omega}$$

is the wavelength (radiation in a vacuum) This condition of smoothness of change of the function N(E), therefore, has the form

$$\frac{dN}{dE}\Delta E\sim\frac{dN}{dE}\frac{eH}{mc\omega\sin^2\theta}mc^2=\frac{dN}{dE}\frac{eH\lambda}{2\pi\sin^2\theta}\ll N \qquad (3.12)$$

This condition is necessary where $\theta \simeq \pi/2$ Where $\theta < \pi/2$, condition (3 12) is sufficient, but not necessary due to the dependence of ω_H^* on θ (for more detail, see [16])

Condition (3 12) can hardly be disrupted in most cases encountered in astrophysics (energy interval $\Delta E = \frac{eH}{2\pi}\lambda_0$ even in the meter wave band is less than or on the order of 10^5 Nev and may be rather great only in areas with strong fields $H \gg 10e$)

Discussion of the entire range of problems which we have touched upon would require at least a special review A number of problems related to this area have not yet been analyzed. We will limit ourselves in the following, therefore, to a discussion of the two narrowest problems concerning the reabsorption of synchrotron radiation in a vacuum and in a plasma with quasi-longitudinal distribution These cases, however, are in all probability the most important from the point of view of application to radio astronomy Before discussing these calculations, it would be expedient to make several notations concerning the usage of the method of Einstein coefficients for polarized radiation.

3 3 Usage of the Einstein Coefficient Method for Polarized Radiation

Both in an investigation of the transfer equation (3.3), and in other similar equations for the intensity of normal waves or Stokes parameters, it is necessary to calculate the coefficients $\varepsilon_{\alpha\beta}$, $R_{\alpha\beta\gamma\delta}$, $K_{\alpha\beta\gamma\delta}$ for (3.3), the coefficients and $\mu_{e,0} = 2\kappa_{e,0}$ in the case of equations (3 10), etc As concerns the radiating capacity $\varepsilon_{\alpha\beta}$, the basis used must be formula (3 4). The other quantities can in the general case be calculated by the kinetic equation method [15, 25, 26] Here, if we are speaking of the classical area (condition $h\nu \ll E$), we must use the classic relativistic kinetic equation The corresponding calculations are rather cumbersome Both for this reason and due to the natural tendency to produce results by the simplest and most obvious method, a significant role in analysis of reabsorption is played by the method of Einstein coefficients This method is generally well known, but its application to the case of a medium and particularly an anisotropic medium, and also when polarization of the radiation is taken into consideration is somewhat specific Therefore it is expedient here to make a few notations concerning the method of Einstein coefficients as applicable to radiation in a medium (see [27], [18] paragraph 27, [17] paragraph 12)

In a weakly absorbing (formally, in a transparent) medium, energy quanta in normal waves have energy $\hbar\omega$ and momentum $\hbar\vec{k}_j = \frac{\hbar\omega}{c}\tilde{n}_j(\omega,\vec{s})\vec{s}$, where $\vec{k}_j = k_j\vec{s}$, $|\vec{s}| = 1$ and the index j corresponds to the given wave (in a magnetoactive plasma, we are concerned with ordinary, extraordinary and plasma waves) In the classical area, the results of calculations are independent of the quantum constant $\hbar = h/2\pi$, but there is no reason not use quantum concepts if they are convenient. The energy flux and energy density in type j waves are equal to $I_j\,d\omega d\Omega$ and $\rho_j\,d\omega d\Omega$, where $d\Omega$ is an element of the solid angle and for convenience we shall temporarily use the spectral densities related to the interval $d\omega = 2\pi d\nu$. The following relationship also occurs ($\vec{v}_{gr,j} = d\omega/d\vec{k}_j$ is the group velocity for type j waves)

$$I_j = \rho_j v_{gr,j} = \rho_j \left| \frac{d\omega}{d\vec{k}_j} \right| = \rho_j \frac{c}{|\cos \vartheta_j| \left| \frac{\partial \omega \tilde{n}_j}{\partial \omega} \right|} \tag{3.13}$$

Let us introduce the Einstein coefficients A_m^n, B_m^n and B_n^m, such that $A_m^n d\omega d\Omega$ is the probability of spontaneous radiation per unit time upon transition between states $m \to n$ with radiation of a quantum of the given normal wave in the intervals $d\omega$ and $d\Omega$. Further, $B_m^n \rho d\omega d\Omega$ is the probability of the same induced transition and $B_m^n \rho d\omega d\Omega$ is the probability of absorption of a quantum upon transition $n \to m$. The coefficients A_m^n, B_m^n and B_n^m are connected by the relationships

$$B_m^n = B_n^m, \quad B_m^n = \frac{(2\pi c)^3}{\tilde{n}_j^2 \hbar \omega^3 \left| \frac{\partial (\omega \tilde{n}_j)}{\partial \omega} \right|} A_m^n \tag{3.14}$$

From this in a vacuum we produce the ordinary relationship

$$B_m^n = \frac{(2\pi c)^3}{\hbar \omega^3} A_m^n = \frac{2\pi c^3}{h \nu^3} A_m^n \tag{3.15}$$

Here n and m mean any two states in the momentum space for which the energy difference $E_m - E_n = \hbar\omega = h\nu$. If we were concerned with the transition between energy levels, we would be required to consider the statistical weights of these levels. Essentially, relationship (3.15) is concerned with waves with a single polarization. If we define the probability of an induced transition in a vacuum as $\tilde{B}^n_m I_\nu d\nu d\Omega$ (as is done in 1, section 4.2), then $\tilde{B}^n_m = \frac{c^2}{h\nu^3}\tilde{A}^n_m$, where $\tilde{A}^n_m d\nu d\Omega = 2\pi A^n_m d\nu d\Omega$ is the probability of spontaneous emission in the intervals $d\nu$ and $d\Omega$. Finally, if $\tilde{A}^n_m d\nu d\Omega$ is taken to mean the probability of emission of waves with both possible polarizations, we can utilize the relationship $\tilde{B}^n_m = \frac{c^2}{2h\nu^3}\tilde{A}^n_m$, which was used in 1. However, this sets up a source of insufficient completeness and definition of expressions. First of all, this method of transition to nonpolarized radiation is not well founded, although it might be expected that this produces the mean value of μ for both possible polarizations. Secondly, in a vacuum or an isotropic medium, polarization degeneration occurs (possibility of selection of normal waves with any polarization), as a result of which the polarization relationship can be produced only by additional analysis.

Let us represent by N_n and N_m the concentration of electrons in states n and m with energies E_n and E_m, such that $E_m - E_n = \hbar\omega \equiv h\nu$. Then, on the strength of (3.14), the absorption coefficient μ_j along the ray for a wave of type j will be

$$\mu_j = -\frac{\Delta I_j}{I_j} = \frac{\hbar\omega \sum (N_n B_n^m \rho_j - N_m B_m^n \rho_j)}{\rho_j \left| \frac{d\omega}{d\vec{k}_j} \right|} = \frac{8\pi^3 c^2}{\omega^2 \tilde{n}_j^2} |\cos \vartheta_j| \sum_{m \geq n} A_m^n (N_n - N_m) \tag{3.16}$$

For simplicity we will immediately consider in the following that $|\tilde{n}_j - 1| \ll 1$ and $|\cos \vartheta_j| \approx 1$. Further, analyzing the ultrarelativistic case (acicular radiation, i e , radiation only in the direction of the particle velocities) and considering the distribution function isotropic, we can produce $N_n = N_m =$

$$N_n - N_m =$$

$$= N(\vec{p} - \hbar\vec{k}) - N(\vec{p}) = N\left(p - \frac{h\nu}{c}\right) - N(p) = -\frac{h\nu}{c} dN/dp.$$

Here it is considered that in the classical case being analyzed $h\nu \ll cp \approx E$. Finally, the radiating capacity in the interval $d\nu$ is equal to

$$\mathcal{E}_j = \sum \tilde{A}_m^n N_m h\nu = \sum 2\pi A_m^n N_m h\nu$$

and by comparison with (3 4) it is clear that $A_m^n = \tilde{A}_m^n/2\pi$ in (3 16) can be replaced by $\frac{r^2}{2\pi h\nu} p_j(\nu, E)$, where $p_j(\nu, E)$ is a function of the type of $p_{\alpha\beta(\nu)}$ in (2 34), but related to a type j wave The significance of this will be analyzed below We present now the final expression for μ_j, under the assumptions which we have made

$$\mu_j = -\frac{c}{\nu^2}\int \frac{dN(p)}{dp} q_j(\nu, E) p^2 dp =$$
$$= -\frac{c^2}{4\pi\nu^2}\int E^2 \frac{d}{dE}\left(\frac{N(E)}{E^2}\right) q_j(\nu, E) dE, \quad (3.17)$$

where

$$q_j(\nu, E) = \int r^2 p_j(\nu, E) d\Omega \quad (3.18)$$

and where the equalities $E = cp$ and $N(p)4\pi p^2 dp = N(E)dE$, it should also be explained that when summation in (3 16) is replaced by integration, the element of the phase volume is equal to $p^2 dp d\Omega$, where $d\Omega$ is the element of the solid angle in which spontaneous radiation occurs (by definition, angle Ψ between $\vec{p}$ and $\vec{k}$ is small). Formula (1, 4.17) for the reabsorption coefficient is produced from (3 18), if we assume $q_j = 1/2p(\nu,E)$, where $p(\nu,E)$ is the spectral density of the power of the total radiation of one electron (1, 2 21) As was already emphasized, there is no particular basis for this assumption, according to (3.17)-(3 18), the problem of calculation of μ_j consists of clarification of the sense of the quantities $p_j(\nu,E)$ or $q_j(\nu,E)$ In an anisotropic medium, this procedure is quite clear, since q_j is the spectral density of the power radiated by an electron in the form of normal type j waves However, in a vacuum or in an isotropic medium, where polarization degeneration occurs, we must clarify just what sort of waves are to be considered normal in calculating reabsorption coefficient μ_j. At first glance, it is true, it might seem that the result of calculations should be independent of the selection of polarization of normal waves, since this independence is the essence of polarization degeneration. Of course, with sequential performance of calculations by the kinetic equation method, this is how it is: definite selection of polarizations of normal waves in the case of a vacuum, and in principle the usage of normal waves itself in any medium, is not obligatory. However, in the method of Einstein coefficients, we are concerned only with probabilities (intensities), not with amplitudes of probability (fields). Therefore, coherence of various normal waves, which generally occurs in the case of degeneration, cannot be taken into consideration in the Einstein coefficients method. In other words, based on the very essence of this method its usage generally involves determination of the type of waves for which the absorption coefficient is being calculated.

3.4 Reabsorption of Synchrotron Radiation in a Vacuum

For a true vacuum, of course, it is impossible to state unambiguously the types of waves which are normal. However, in this case the problem of reabsorption does not occur. If we are speaking of reabsorption in a vacuum, we have in mind only the possibility of ignoring the influence of a "cold" plasma on radiation and reabsorption. A relativistic plasma at the source influences absorption of waves according to the very sense of the problem of reabsorption. This plasma should also have some influence on the index of refraction, in that the medium is anisotropic. This is obvious, since we are concerned with relativistic particles (a plasma) in a magnetic field and, consequently, there is a physically distinguished direction in the system -- the direction of the field. As we know (see 1), if the distribution function of ultrarelativistic particles is not sharply anisotropic, their radiation is linearly polarized, and the electrical vector in the waves is maximal in the direction perpendicular to projection $\vec{H}_\perp$ of vector $\vec{H}$ to the plane of the figure (in the following these waves will be called polarized perpendicular to the field for brevity, while waves with vector $\vec{E}$ parallel to $\vec{H}_\perp$ will be called waves polarized along the field). Under these conditions it is natural to expect that normal waves will also be polarized along the field and perpendicular to the field (we recall that we are limiting ourselves to angles $\theta \gg \frac{mc^2}{E}$, i.e., we are not analyzing the radiation of particles whose velocity directions make the small angle $\theta \lesssim \frac{mc^2}{E}$ with the direction of the field, in this case linear polarization also occurs only under this condition ($\theta \gg mc^2/E$). Calculations [41] confirm this assumption. Thus, when formulas (3.17)-(3.18) are used to calculate the coefficients of reabsorption of synchrotron radiation by ultrarelativistic particles in a vacuum, we must calculate the coefficients $\mu_\perp$ and $\mu_\parallel$ for polarizations across the field and along the field. Here as $p_\perp(\nu,E)$ and $p_\parallel(\nu,E)$ in (3.18), as is clear from the above, we must take expression (1, 2.20) multiplied by $2\pi \sin\theta$. Consequently,

$$\left.\begin{aligned} q_\perp(\nu,E) &= \frac{\sqrt{3}\,e^2\omega_H \sin\theta}{2c\eta}\,\frac{\nu}{\nu_c}\left[\int_{\nu/\nu_c}^{\infty} K_{5/3}(z)\,dz + K_{2/3}\left(\frac{\nu}{\nu_c}\right)\right] \\ q_\parallel(\nu,E) &= \frac{\sqrt{3}\,e^2\omega_H \sin\theta}{2c\eta}\,\frac{\nu}{\nu_c}\left[\int_{\nu/\nu_c}^{\infty} K_{5/3}(z)\,dz - K_{2/3}\left(\frac{\nu}{\nu_c}\right)\right] \end{aligned}\right\} \quad (3.19)$$

$$\nu_c = \frac{3\sin\theta}{4\pi}\cdot\frac{\omega_H}{\eta^3_{\xi}},\ \omega_H = \frac{eH}{mc}\cdot\frac{mc^2}{E};\ \eta = \sqrt{\left(\frac{mc^2}{E}\right)^2 + \frac{\omega_0^2}{\omega^2}};\ \omega_0^2 = \frac{4\pi e^2 N_e}{m}. \tag{3.20}$$

We have presented here for convenience expressions which are correct in the presence of an isotropic plasma with $\tilde{n} = 1 - \omega_0^2/2\omega^2$, $|1 - \tilde{n}| \ll 1$, although in the remainder of this section we will assume $\eta = \xi = mc^2/E$ In the ultrarelativistic case in question with isotropic (or weakly anisotropic) distribution of radiating particles by velocity directions, waves with elliptical polarization are not considered (with an accuracy to terms on the order of

$$\eta = \sqrt{(mc^2/E)^2 + \omega_0^2/\omega^2}.$$

Due to this fact in the analysis of natural radiation of the source we can limit ourselves to the Stokes parameters ___ and ___[1] or the intensities

$I_\perp = 1/2(I_q)$ and $I_\parallel = 1/2(I - Q)$

The spectral density of the total radiated power

$$q(\nu, E) = q_\perp + q_\parallel = P(\nu, E) = \frac{\sqrt{3}e^2\omega_H \sin\theta}{c\eta}\,\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(z)\,dz \tag{3.21}$$

In a vacuum

[1] Omitted in original text -- Tr.

$$P(\nu, E) = \frac{\sqrt{3}e^3 H_\perp}{mc^2} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(z)\,dz \qquad (3.22)$$

which, of course, corresponds with (1, 2.21)

Let us introduce the representation

$$\mu_\perp(\theta) = \mu(\theta) + \lambda(\theta), \quad \mu_\parallel = \mu(\theta) - \lambda(\theta) \qquad (3.23)$$

it is easy to see that $\mu(\theta) = \frac{\mu_\perp + \mu_\parallel}{2}$ corresponds precisely to expression (1, 4 17), used earlier as a coefficient of reabsorption. This result is natural, since $\mu(\theta)$ is the arithmetic mean of $\mu_\perp + \mu_\parallel$ For the power-law spectrum $N(E) = K_e E^{-\gamma}$, we have [2, 14]

$$\mu(\theta) = \frac{\gamma + 10/3}{\gamma + 2} \lambda(\theta) = g(\gamma) \frac{e^3}{2\pi m} \left(\frac{3e}{2\pi m^3 c^5}\right)^{\gamma/2} K_e H_\perp^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}} \qquad (3\ 24)$$

This formula corresponds with (1, 4.18), in which $g(\gamma)$ is determined by formula (1, 4.19) Here we present once more only the numerical values of $g(\gamma)$ (see Table 1)

The polarization of synchrotron radiation in a vacuum without considering reabsorption for the case of a power-law spectrum of electrons (see 1, 3 28) is

$$\Pi_0 = \frac{I_\perp^{(0)} - I_\parallel^{(0)}}{I_\perp^{(0)} + I_\parallel^{(0)}} = \frac{\gamma+1}{\gamma+7/3}, \quad \frac{I_\perp^{(0)}}{I_\parallel^{(0)}} = \frac{1+\Pi_0}{1-\Pi_0} = \frac{3\gamma+5}{2} \tag{3.25}$$

Table 1

γ	I	2	3	4	5
$g(\gamma)$	0,96	0,70	0,65	0,69	0,83
$\overline{g(\gamma)}$	0,69	0,47	0,40	0,44	0,46

At the same time, according to (3 23) and (3 24)

$$\mu_\perp = \mu + \lambda = \frac{6\gamma+16}{3\gamma+10}\mu, \quad \mu_\parallel = \mu - \lambda = \frac{4}{3\gamma+10}\mu \tag{3.26}$$

$$\mu_\parallel / \mu_\perp = \frac{\mu-\lambda}{\mu+\lambda} = \frac{2}{3\gamma+8}$$

The transfer equation like (3 10) obviously has the following form in this case

$$\frac{dI_{\perp,\parallel}}{dz} = \mathcal{E}_{\perp,\parallel} - \mu_{\perp,\parallel} I_{\perp,\parallel} ; \tag{3.27}$$

where

$$\mathcal{E}_{\perp,\parallel}(\nu) = \int q_{\perp,\parallel}(\nu, E)\, N(E)\, dE \tag{3.28}$$

The radiating capacity (3 28) can be easily calculated for a power-law spectrum by using expressions (3 19) and (1, 3 25) Let us limit ourselves at this time to the note that where there is no reabsorption for the natural radiation of a homogeneous source with dimensions L

$$I^{(0)}_{\perp,\parallel} = \mathcal{E}_{\perp,\parallel} L, \quad \frac{I^{(0)}_{\perp}}{I^{(0)}_{\parallel}} = \frac{\mathcal{E}_{\perp}}{\mathcal{E}_{\parallel}} = \frac{3\gamma+5}{2}, \tag{3 29}$$

This clear from (3.25)

In consideration of reabsorption, integrating equation (3.27) under the condition that at the beginning of the layer (where z = 0) $I_{\perp,\parallel} = 0$, we produce

$$I_{\perp} = \frac{\mathcal{E}_{\perp}}{\mu_{\perp}}\left(1 - e^{-\mu_{\perp} z}\right), \quad I_{\parallel} = \frac{\mathcal{E}_{\parallel}}{\mu_{\parallel}}\left(1 - e^{-\mu_{\parallel} z}\right) \tag{3.30}$$

For a thin layer (source with dimension L, $\mu_{\perp,\parallel} L \ll 1$ and

$$\frac{I_{\perp}}{I_{\parallel}} = \frac{I^{(0)}_{\perp}}{I^{(0)}_{\parallel}} = \frac{\mathcal{E}_{\perp}}{\mathcal{E}_{\parallel}} = \frac{3\gamma+5}{2}, \quad \Pi = \frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}} = \Pi_0 = \frac{\gamma+1}{\gamma+7/3} \tag{3.31}$$

For a thick layer $\mu_{\perp,\parallel} L \gg 1$ and

$$\frac{I_\perp}{I_\parallel} = \frac{\mathcal{E}_\perp}{\mathcal{E}_\parallel} \frac{\mu_\parallel}{\mu_\perp} = \frac{3\gamma+5}{3\gamma+8} < 1, \quad \Pi = \left|\frac{I_\perp - I_\parallel}{I_\perp + I_\parallel}\right| = \frac{3}{6\gamma+13} \quad (3.32)$$

Of course, those expressions from (3 31) and (3 32) which do not contain the γ have general significance, in addition to their significance for a power-law spectrum We recall that when a power-law spectrum is used it is assumed in the calculations that $\gamma > 1/3$ (see 1)

We assume that on the ray of vision, the magnetic field is directionally chaotic on the average Let us assume further that as the waves propagate in this field the polarization of the waves does not change with a change in direction of the field (this occurs if the approximation of geometric optics is inapplicable for the description of the polarization of normal waves due to nonobservation of conditions such as $\lambda|\partial n_{e,0}/dz| \ll |n_e - n_0|$, mentioned above in section 3.2, for more detail, see [18], paragraph 24) Under these conditions when the waves are propagated in a chaotic field the anisotropy of absorption disappears and waves with any polarization will be absorbed identically with a certain absorption coefficient μ With the given angle θ, the mean absorption coefficient $\frac{\mu_\perp + \mu_\parallel}{2} = \mu(\theta)$ In order to produce $\overline{\mu}$, i e., the mean value of $\mu(\theta)$ with respect to angles θ between the field $\vec{H}$ and the ray of vision (velocity of radiating electrons), we must form the expression

$$\bar{\mu} = \frac{1}{2}\int_0^{\pi} \mu(\theta)\sin\theta\, d\theta = \overline{g(\gamma)}\, \frac{e^3}{2\pi m}\left(\frac{3e}{2\pi m^3 c^5}\right)^{\gamma/2} K_e H^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}}$$

$$\overline{g(\gamma)} = \frac{\sqrt{3\pi}}{8}\, \frac{\Gamma\left(\frac{\gamma+6}{4}\right)}{\Gamma\left(\frac{\gamma+8}{4}\right)} \cdot \Gamma\left(\frac{3\gamma+2}{12}\right) \cdot \Gamma\left(\frac{3\gamma+22}{12}\right) \qquad (3.33)'$$

In [2] it is shown that expression (3 33), not concluded strictly above, is actually the coefficient of reabsorption for a chaotic field The numerical values of the function $\overline{g(\gamma)}$ are shown in Table 1 For convenience, we also present the following expression (see 1, 4 20)

$$\bar{\mu} = \overline{g(\gamma)} \cdot 0.019\,(3.5\,10^{9})^{\gamma} K_e H^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}}\ \text{см}^{-1} \qquad (3\ 34)$$

Concerning reabsorption in a heterogeneous field, see [40] The formula for μ in the case of a "monointergetic" spectrum of electrons is presented below (see 3 47)

The question naturally arises of the area of applicability of these formulas as concerns the possibility of ignoring the influence of a "cold" plasma In order for this influence to be ignored, it is required first of all that the "cold" plasma have no influence on the radiation of the relativistic electrons. From this, we come to the conclusion (see (1, 4 26)) and the following section 3 5) that

$$\nu \gg \frac{4ecN_e}{3H_\perp} = \frac{4\omega_0^2}{3\pi\omega_H^{(0)}\sin\theta} \sim 20\frac{N_e}{H_\perp} = 20\frac{N_e}{H\sin\theta} \tag{3 35}$$

Secondly, it is required that the rotation of the plane of polarization of the "cold" plasma be slight, from which we come to the conclusion (see (1, 4 6))

$$\nu \gg 10^2\sqrt{N_e H L \cos\theta} \tag{3 36}$$

This condition, of course, is not required if the polarization of normal waves is determined by the relativistic particles (this occurs if the inequality the inverse of inequality (3 41) is observed Third, normal waves are linearly polarized only with observation of the same condition of the inverse inequality to (3 41) All of these three conditions together are sufficient for the influence of the plasma to be completely ignored. However, this is also possible in certain cases with less rigid requirements

3.5 The Reabsorption of Synchrotron Radiation in the Presence of a "Cold" Plasma

If there is also a "cold" plasma in the radiating area, we must first of all consider the influence of the cold" plasma on the process of radiation and secondly consider its influence on the propagation of waves. It was stated above that under the conditions

$$\omega \gg \omega_H^{(0)} = \frac{eH}{mc} = 1{,}76\cdot 10^7 H, \quad \omega \gg \omega_0 = \sqrt{\frac{4\pi e^2 N_e}{m}} = 5{,}64\cdot 10^4\sqrt{N_e} \tag{3.37}$$

in calculating the radiation, the plasma can be generally considered isotropic, where

$$\tilde{n}_e = \tilde{n}_o = \tilde{n} = 1 - \frac{\omega_0^2}{2\omega}, \quad |1 - \tilde{n}| \ll 1 \tag{3 38}$$

In this case the influence of the plasma on radiation is reflected, for example, in formulas (3 19)-(3 20)

As concerns the propagation of waves, in order to ignore the anisotropy, conditions (3 37) are of course insufficient However, an essential simplification can be achieved under these conditions, first of all as a result of the possibility in most cases of considering wave propagation quasi-longitudinal, in which case

$$\tilde{n}_e = 1 - \frac{\omega_0^2}{2\omega(\omega - \omega_L)}, \quad \tilde{n}_o = 1 - \frac{\omega_0^2}{2\omega(\omega + \omega_L)}, \tag{3 39}$$
$$\tilde{n}_e - \tilde{n}_o = \frac{\omega_0^2 \omega_L}{\omega^3}, \quad \omega_L = \omega_H^{(0)} \cos\theta$$

It is assumed here that $|n_{e,0} - 1| \ll 1$. The e and zero waves are both polarized circularly with opposite direction of rotation of the field vectors. In the extraordinary wave e, these vectors rotate in the same direction as the electron located in the magnetic field The conditions of applicability of the quasi-longitudinal approximation (3 39) under the conditions of interest to us are as follows (see [17], paragraph 2):

$$\frac{[\omega_H^{(0)}]^2 \sin^4\theta}{4\omega^2\cos^2\theta} \ll 1, \quad \frac{[\omega_H^{(0)}]^2}{2\omega^2}\sin^2\theta \ll 1 \tag{3 40}$$

It is easy to see that in radio astronomy formulas (3 39) are practically always applicable if the influence of relativistic particles on the index of refraction is slight in comparison to the influence of the "cold" plasma considered in (3 39)

As a result of the influence of relativistic particles [41] $|\tilde{n} - 1| \sim \frac{c}{2\omega}\mu(\theta) = \frac{\lambda}{4\pi}\mu(\theta)$, where $\mu(\theta)$ is the coefficient of reabsorption (3.24) or (3.33)-(3 34) Consequently, the role of relativistic particles in the calculation of $\tilde{n}$ can be ignored under the condition $(\tilde{n}_0 - \tilde{n}_e) \gg \frac{c}{2\omega}\mu$, which gives us

$$N_e \gg mc^2\left(\frac{3e}{2\pi m^3c^5}\right)^{\gamma/2}\frac{(\sin\theta)^{\frac{\gamma+2}{2}}}{\cos\theta}K_e H^{\frac{\gamma}{2}}\nu^{-\gamma/2} \sim$$
$$\sim 10^{-6}(3{,}5\cdot10^9)^{\gamma}\frac{(\sin\theta)^{\frac{\gamma+2}{2}}}{\cos\theta}K_e H^{\frac{\gamma}{2}}\nu^{-\gamma/2}\ \mathrm{cm}^{-3} \tag{3 41}$$

Under conditions of applicability of formulas (3.39), the problem of the transfer of radiation is greatly simplified. The tensors $R_{\alpha\beta\gamma\delta}$ and $K_{\alpha\beta\gamma\delta}$ take on a very simple form under these conditions, so that equation (3 3) can be written in the following form upon transition to Stokes parameters [12]

$$\left.\begin{aligned}
\frac{dI}{dz} &= \mathcal{E}_I - \frac{\mu_e + \mu_o}{2} I + \frac{\mu_e - \mu_o}{2} V \\
\frac{dV}{dz} &= \mathcal{E}_V - \frac{\mu_e + \mu_o}{2} V + \frac{\mu_e - \mu_o}{2} I \\
\frac{dQ}{dz} &= \mathcal{E}_Q - \frac{\mu_e + \mu_o}{2} Q + (k_e - k_o) U \\
\frac{dU}{dz} &= \mathcal{E}_U - \frac{\mu_e + \mu_o}{2} U - (k_e - k_o) Q
\end{aligned}\right\} \tag{3.42}$$

Here $k_{e,0} = \frac{\omega}{c}\tilde{n}_{e,0}$ and $\varepsilon_{I,V,Q,U}$ are combinations of $\varepsilon_{\alpha\beta}$ corresponding to the transition from tensor $I_{\alpha\beta}$ to Stokes parameters (see (3 1), for example, $\varepsilon_I = \varepsilon_{11} + \varepsilon_{22}$) The Faraday effect is defined by the difference $n_e - n_0 = \frac{c}{\omega}(k_e - k_0)$ and has no influence on equations for the intensity of I and the degree of circular polarization $\rho_c = V/I$, but influences the degree of linear polarization $\rho_e = \sqrt{Q^2 + U^2}/I$ and the orientation of the ellipse x (we recall that $\tan 2x = U/Q$) It is convenient to introduce the intensities of extraordinary and ordinary radiation

$$I_e = \frac{I - V}{2}, \quad I_o = \frac{I + V}{2} \tag{3.43}$$

According to (3 42) and (3 43)

$$\left.\begin{aligned} \frac{dI_{e,o}}{dz} &= \mathcal{E}_{e,o} - \mu_{e,o} I_{e,o} \\ \mathcal{E}_e = \frac{\mathcal{E}_I - \mathcal{E}_V}{2}, &\ \mathcal{E}_o = \frac{\mathcal{E}_I + \mathcal{E}_V}{2} \end{aligned}\right\} \tag{3.44}$$

This result (3 44) is rather obvious from the beginning in the linear medium being analyzed, the intensity (energy flux) in each of the normal waves is independent of the intensity of the other wave. This conclusion relates to any normal waves, but with arbitrary (elliptical) polarization of the waves, intensities I_e and I_0 are expressed in a complex manner through the Stokes parameters and the expediency of using them is not clear. At the same time, even with quasi-longitudinal propagation, complete characterization of the radiation requires that all four Stokes parameters be used (solution to equations (3 42), see [12])

Nevertheless, we will limit ourselves in the following to a disucssion only of the problem of a change in intensity of the waves e and 0, i e , we will base ourselves on equation (3 44) When waves of only one type are present, the polarization is fixed and equation (3 44) describes the radiation completely. This situation occurs in particular with negative reabsorption for a sufficiently thick layer. Actually, with negative reabsorption the intensity of waves increases exponentially upon passage through the layer Therefore, upon leaving the thick layer, radiation consisting of those normal waves for which the absolute value of the coefficient of reabsorption μ is greater will dominate.

As was indicated, under conditions (3.37) the influence of plasma on radiation is considered by formulas (3.19)-(3 21) In this case, with an accuracy to terms on the order of mc^2/E, one half of the total radiation power $q(\nu,E) \equiv p(\nu,E)$ defined by formula (3 21) goes over into each normal, circularly polarized wave Thus, $q_{e,0} = 1/2\, p(\nu,E)$ and, according to (3.17)

$$\left.\begin{aligned} \mu_e = \mu_0 &= -\frac{c^2}{8\pi\nu^2}\int_0^\infty E^2 \frac{d}{dE}\left(\frac{N(E)}{E^2}\right) p(\nu,E)\,dE = \\ &= \frac{c^2}{8\pi\nu^2}\int_0^\infty \frac{N(E)}{E^2}\frac{d}{dE}\left\{E^2 p(\nu,E)\right\}dE \\ p(\nu,E) &= \sqrt{3}\,\frac{e^3 H_\perp}{mc^2}\left[1+\frac{\nu_0^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2\right]^{-1/2}\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^\infty K_{5/3}(z)\,dz \\ \nu_c &= \frac{3eH_\perp}{4\pi mc}\left(E/mc^2\right)^2\left[1+\frac{\nu_0^2}{\nu^2}\left(E/mc^2\right)^2\right]^{-3/2}, \quad \nu_0^2 = \frac{\omega_0^2}{4\pi^2} = \frac{e^2 N_e}{\pi m} \end{aligned}\right\} \qquad (3.45)$$

For better understand of these formulas and their comparison with other expressions, we note that $1 + \frac{\nu_0^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2 =$

$$= \left(\frac{E}{mc^2}\right)^2\left(\left(\frac{mc^2}{E}\right)^2 + \frac{\omega_0^2}{\omega^2}\right) = \left(\frac{E}{mc^2}\right)^2 \eta^2 \simeq \left(\frac{E}{mc^2}\right)^2\left(1-\tilde{n}^2\beta^2\right).$$

It is clear from (3 45) that the influence of the plasma on synchrotron radiation and its reabsorption is not essential under the condition

$$\frac{\nu_0^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2 \ll 1 . \qquad (3.46a)$$

This condition leads us (see (1, 4 25)) to the inequality (3.35) already presented In area (3 46a), the contribution of integral (3.45) for $\mu_{e,0}$ is positive, from which it follows that in this case $\mu_{e,0} > 0$ always Since in a vacuum condition (3.46a) is always possible, in a vacuum $\mu > 0$ (see [7-9])[1]

If

$$\frac{\nu_0^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2 \gg 1, \tag{3 46b}$$

the influence of the plasma is definitive In this case, with the proper selection of electron spectrum N(E), the coefficient $\mu_{e,0}$ may be negative [8, 9, 16, 25, 26, 28] For the power-law spectrum of electrons $N(E) = K_e E^{-\gamma}$ it is clear directly from (3 45) that negative value of $\mu_{e,0}$ is possible only where $\gamma < -2$, i e , for a spectrum which grows in a certain area more rapidly than E^2 Otherwise, the integrand in (3 45) is always positive (function $p(\nu,E)$ positive) The area where the function N(E) increases with increasing E usually cannot be very large and, in any case, with further increase in E it is replaced by the area where function N(E) decreases Therefore, in the case of negative reabsorption in question the power-law spectrus is not of particular interest (a spectrum of the form $N(E) = KE^{\gamma'}$, $\gamma' > 2$ where $E_1 < E < E_2$ and $N(E) = 0$ where $E > E_2$ and $E < E_1$ is analyzed in [8]) There is great significance in a spectrum with a rather sharp maximum at a certain energy E_1 (the width of the spectrum should satisfy the condition $\Delta E \ll 3eH_\perp \nu^2/4\pi me\nu_0^3$, this condition is quite compatible with inequality (3 12). For such a spectrum [8]

[1] This note is correct only for a rather smooth function N(E), when the expressions used for μ (see (3.17) and (3 45)) are correct For very "sharp" functions N(E) and anisotropic distributions of velocities, negative values of μ may be encountered in a vacuum.

$$\mu = \mu^{I} = \frac{4\pi}{3\sqrt{3}} \frac{e}{H_\perp} \left(\frac{mc^2}{E_i}\right)^5 N_e K_{5/3}(z_i), \quad z_i = \frac{4\pi mc\nu}{3eH_\perp}\left(\frac{mc^2}{E_i}\right)^2 \quad (3.47)$$

where $E_i^2 \ll E_*^2$, $E_* = mc^2\nu/\nu_0$ (see (3 45))

If $E_i^2 \gg E_*^2 = (mc^2\ \nu/\nu_0)^2$ (see (3 46), then

$$\mu = \mu^{II} = \frac{\sqrt{3}e^3 H_\perp}{8\pi m\nu\nu_0} \frac{mc^2}{E_i^2} N_i \Phi(z_i),$$
$$\Phi(z_i) = 2z\int_z^\infty K_{5/3}(u)\,du - z^2 K_{5/3}(u),\ z_i = \frac{4\pi mc\nu_0}{3eH_\perp \nu^2} \frac{E_i}{mc^2}. \quad (3.48)$$

In (3 47) and (3 48), N_i is the concentration of electrons with the energy in question $E_i \gg mc^2$

Expression (3 47) is always positive, in case there is no plasma, this expression is correct for all energies, which is in accordance to that stated above. Function $\Phi(z_i)$ may be negative, and in the corresponding area of values of z_i the coefficient $\mu^{II} < 0$ Coefficient μ^{II} is negative in an area on the order of (0 7-1 3) ν_{max}, where ν_{max} is the frequency at which the value of $|\mu^{II}|$ is maximal At this frequency

$$\mu^{II}_{max} \simeq -10^{-2} \frac{e^2}{mc} \frac{\nu_0^3}{\nu_{max}^4} N_i = -8{,}5\cdot 10^{-5} \frac{\nu_0^3}{\nu_{max}^4} N_i \text{ cm}^{-1}, \quad (3.49)$$
$$\nu_{max} \simeq \left(0{,}24 \frac{2\pi mc\nu_0^3}{eH_\perp} \cdot \frac{E_i}{mc^2}\right)^{1/2},$$

At the same time, coefficient μ^{I} at the maximum of the frequency spectrum (at frequency ν_m, see (1, 2 23)) is equal to

$$\mu^{I}(\nu_m) \simeq 2{,}4\cdot 10^{-8}\frac{N_e}{H_\perp}\left(\frac{mc^2}{E_b}\right)^{5}cm^{-1}; \quad \nu_m \simeq 0{,}07\,\frac{eH_\perp}{mc}\left(\frac{E_e}{mc^2}\right)^2 \quad (3.50)$$

A number of estimates of the negative coefficient of reabsorption as applicable to various space sources are presented in [8, 28].

In the preceding we have analyzed only the case of quasi-longitudinal propagation, in which the difference in coefficients $\mu_e - \mu_0$ was ignored In [25], transverse propagation (angle $\theta = \pi/2$) in a plasma is analyzed, and negative reabsorption is found possible. In [26], expressions are produced for μ_e and μ_0 with any angle θ between the field and the ray of vision The coefficients μ_j may be negative with any θ but, of course, only for spectra N(E) of a definite type and not through the entire frequency range Furthermore, an expression is produced in (26) for the difference $\mu_e - \mu_0$ with quasi-longitudinal propagation of waves This difference is slight, since

$$|\mu_e - \mu_0| \sim \left\{ a\frac{\omega_H^{(0)}\omega_0^2}{\omega^2(1-\tilde{n}^2\beta^2)} + b\frac{\omega_H^{(0)}}{\omega} + d\sqrt{1-\tilde{n}^2\beta^2} \right\} \mu_{e,0}, \quad (3\ 51)$$

where a, b and d are coefficients on the order of unity. At the radiation maximum $\omega \sim \omega_H^{(0)}\frac{mc^2}{E}(1-\tilde{n}^2\beta^2)^{-3/2} \simeq \omega_H^{(0)}\frac{mc^2}{E}\eta^{-3}$,

$$\eta = \sqrt{\left(\frac{mc^2}{E}\right)^2 + \frac{\omega_0^2}{\omega^2}} \simeq \sqrt{1-\tilde{n}^2\beta^2}$$

and, consequently, in this case

$$|\mu_e - \mu_0| \sim \left\{ a \frac{\omega_0^2 E}{\omega^2 mc^2} \eta + b \frac{E}{mc^2} \eta^3 + d\eta \right\} .$$

As is clear from conditions (3 46a) and (3 46b), in the area where the influence of the plasma is essential but still not too great $\frac{\omega_0^2}{\omega^2}\left(\frac{E}{mc^2}\right)^2 \sim 1$, $\eta \sim \frac{mc^2}{E}$ and, consequently, $|\mu_e - \mu_0| \sim mc^2/E$ In the broad and most important area of values of the parameters, where $\omega_0^2/\omega^2 \lesssim mc^2/E$, the difference $|\mu_e - \mu_0| \sim \eta = \sqrt{(mc^2/E)^2 + \omega_0^2/\omega^2}$ In most cases, the factor η is small, so that even with $|\mu_{e,0}|L \gg 1$ it is difficult to expect observation of the condition $|\mu_e - \mu_0|L \gtrsim 1$ If nevertheless this condition is fulfilled with negative $\mu_{e,0}$, one of the waves will predominate in the synchrotron radiation of the source, i.e., in this case total circular polarization should be observed (the general expression for the degree of circular polarization under condition $|\mu_{e,0}|L \gg 1$ is presented in [12])

In the approximation in which $\mu_e = \mu_0$ and the radiating capacities $\varepsilon_e = \varepsilon_0$, circular polarization cannot appear However, the linear polarization may also change in the case when the plasma has no influence on absorption and radiation of waves Namely, if condition (3 36) is not fulfilled, not only rotation of the plane of polarization, but also depolarization of radiation will be observed The problem is that under the influence of Faraday rotation alone, the degree of linear polarization is decreased by the factor $\left\{ \sin\left[\frac{1}{2}(k_e - k_0)L\right] \cdot \left[\frac{1}{2}(k_e - k_0)L\right]^{-1} \right\}$, where $k_{e,0} = \frac{\omega}{c}\tilde{n}_{e,0}$ and L is the dimension of the radiating area along the ray of vision (for example, see [11, 12]). The degree of circular polarization from a thick layer with $\mu > 0$ is (see [12])

$$\rho_c = \frac{V}{I} = \frac{\mu_e - \mu_o + (\mu_o + \mu_e)\rho_c^{(0)}}{\mu_e + \mu_o + (\mu_e - \mu_o)\rho_c^{(0)}} \approx \frac{\mu_e - \mu_o}{2\mu} + \rho_c^{(0)}, \quad (3.52)$$

where upon transition to this latter expression it is assumed that

$$|\mu_e - \mu_o| \ll \mu_{e,o} \simeq \mu \quad \text{и} \quad \rho_c^{(0)} = \frac{\mathcal{E}_o - \mathcal{E}_e}{\mathcal{E}_o + \mathcal{E}_e} \ll 1 .$$

Estimate $|\mu_e - \mu_0|$ has already been produced (see (3 51)), as is clear from formulas (3.17), (3.18) and (3.28), in the area of applicability of these formulas $\frac{\mu_e - \mu_0}{2\mu} \sim \rho_c^{(0)}$ On the other hand, formula (3 17) for μ_j was produced on the assumption of acicular radiation, i e., by ignoring terms on the order of mc^2/E It is known that in a vacuum $\rho_c^{(0)} \sim mc^2/E$ (see [1, 2, 29]) Combining the various estimates, we come to the conclusion that usually (where $\mu > 0$), the degree of circular polarization $\rho_c^{(0)}$ or ρ_c is slight and on the order of

$$mc^2/E \quad \text{or} \quad \eta = \sqrt{\left(\frac{mc^2}{E}\right)^2 + \left(\frac{\omega_0^2}{\omega^2}\right)} .$$

Thus, the appearance of circular or elliptical polarization of the synchrotron radiation is significant, since in the simplest cases this radiation is always linearly polarized The circular or elliptical polarization of synchrotron radiation for the set of quasi-isotropic radiating electrons can arise only upon transition to relativistic energies which are not too high, or upon consideration of the influence of anisotropy of the plasma (consideration of terms on the order of $\eta = \sqrt{(mc^2/E)^2 + \omega_0^2/\omega^2}$) Under conditions of negative reabsorption, in addition to changes in polarization, the dependence of the coefficients of reabsorption $\mu_{\perp,\parallel}$ or $\mu_{e,0}$ on angle θ between the field and the ray of vision may be significant As a result, if the field at the source is heterogeneous but not completely chaotic, with $\mu < 0$ radiation will be preferentially amplified in directions with maximal $|\mu|$ Therefore, where $|\mu|L > 1$, and especially where $|\mu|L \gg 1$, individual areas of the heterogeneous source will appear anomalously brightly

We have discussed only a small portion of the problem of the influence of a "cold" plasma on synchrotron radiation and its reabsorption In this area, we must analyze a number of additional problems and possibilities (primarily we must be concerned with the negative reabsorption and polarization relationships under various conditions and as applicable to sources of various types).

4. Some Problems Related to the Theory of Synchrotron Radiation

4 1. The Radiation of Sources Moving at Relativistic Velocities

Until recently, it was considered that under space conditions we must deal with relativistic velocities of macroscopic radiation sources (galaxies, stars, gas clouds and streams) only for very remote sources participating in the expansion of the universe. In other words, it was assumed that in the areas with red shift parameter $z = \frac{\lambda - \lambda_0}{\lambda_0}, \ll 1$, all velocities of macroscopic radiation sources are nonrelativistic This statement is actually correct in most cases, in particular for such sources of synchrotron radiation as galactic clouds of supernova stars, the speed of the center of gravity of these clouds and the rate of their expansion is quite small in comparison to the speed of light c (incidentally, clouds are known which expand at speeds $v \approx 10^9$ cm/sec, so that $v/c \approx 3 \cdot 10^{-2}$). For sources moving at nonrelativistic speeds, the intensity of radiation is practically the same as the intensity of the same, nonmoving source (see formula (2.38))

Some observations of radio galaxies and quasars give us reason to believe, however, that in these cases the radiation producing "clouds" and shells may have relativistic speeds [30-34] This conclusion does not seem particularly strange if we are speaking of sources of synchrotron radiation formed as a result of powerful explosions If as a result of an explosion (concretely an explosion in a galactic nucleus or quasar nucleus) ultrarelativistic particles are formed with tremendous total energies (apparently this energy for powerful radiogalaxies reaches values on the order of 10^{62} erg), the expansion of a cloud of such particles might quite possibly occur at relativistic speeds. This expansion could be contained only by a rather powerful magnetic field or by the presence of a large quantity of gas surrounding the area of the explosion or coexisting with the relativistic particles (having in mind the presence of a rather dense "cold" plasma in an area filled with relativistic particles, i.e., cosmic rays) As was stated above, data are available which indicate that in the radiogalaxies and quasars at least in some cases the braking factors are insufficiently effective and the expansion actually does occur at rather high velocities $v \sim c$.

In section 2 of this article we saw that for a cloud moving at relativistic velocity V_r in the direction of the observer, the intensity of radiation increases by $(1 - v_r/c)^{-1}$ times (see (2.38)) But this is not the extent of the matter For a rapidly moving cloud, the estimates concerning magnetic field intensity, concentration of relativistic electrons, reabsorption and kinematic characteristics of sources all change [32, 33]

Let us discuss first of all the basically elementary problem of the change in angular dimensions of the source

Let us assume that a certain source is being observed (the nature of its radiation is insignificant in this case), the angular dimension of which ϑ changes with velocity $\omega = d\vartheta/dt$ Using ordinary "nonrelativistic" considerations, it could be concluded that the distance to such a source R cannot exceed the value c/ω Obviously, this conclusion is based on the assumption that the velocity of the surface of the source perpendicular to the ray of vision (assuming, let us say, that the source is a cloud) $v_\perp = \omega R$ and cannot exceed the speed of light c (from this, $R \leq c/\omega$) However, in the case of relativistic speeds, this estimate of R is quite erroneous The source of the error is actually the failure to consider the finite nature of the speed of propagation of light Actually, let us analyze a certain spherical shell (product of explosion), whose surface moves at constant velocity v. The explosion occurred at point 0 (Figure 5) at moment t_e and the signal concerning this explosion reached the point of observation P at moment $t_r = 0$ Obviously, $t_e = -R/c$, where R is distance OP and the influence of the medium on the propagation of the signal (light, radio waves) is ignored Let us now find the location of the points (the "visible" shell), radiation from which reaches the observer at moment $t_r = t$. The points on the "visible" surface will be characterized by distance r from point 0 and angle θ between vector $\vec{r}$ and line OP (Figure 5) The time of emission t'_e corresponding to point (r,θ) and the time of observation t is $t'_e = t - R'/c \approx t - \frac{R}{c} + \frac{r \cos \theta}{c}$, where $R' \approx R - r \cos \theta$, on the basis of the assumption $r \ll R$. On the other hand, obviously $t'_e - t_e = t'_e + \frac{R}{c} = r/v$, since path r is traveled at speed v Combining these two expressions for t'_e, we produce

$$r = \frac{vt}{1 - \frac{v}{c}\cos\theta} \tag{4.1}$$

The factor $(1 - \frac{v}{c}\cos\theta)^{-1}$ corresponds here to the factor which appears in the formula for the Doppler effect. This is understandable, since in both cases the essence of the matter is consideration of the delay or, which amounts to the same thing, consideration of the finite nature of the speed of propagation of the radiation. It is curious that the difference between the true form of a rapidly moving object and its form visible from some one fixed point, remained unnoticed for some time, and, in any case, has not been emphasized in the literature. In recent years, however, this fact has been noted several times (see review [35]) and was discussed as applicable to quasars in [32]. The speed of the "visible" envelope perpendicular to the ray of vision $U_\perp = \frac{dr}{dt}\sin\theta = \frac{v\sin\theta}{1 - v/c\cos\theta}$ and $\omega = \frac{d\vartheta}{dt} = \frac{U_\perp}{R}$. Velocity $U_\perp$ is maximum where $dU_\perp/d\theta = 0$ for a certain angle θ_{max}, where $\cos\theta_{max} = v/c$. From this

$$u_{\perp,max} = \frac{v}{\sqrt{1 - v^2/c^2}}, \quad \omega_{max} = \frac{v}{R\sqrt{1 - v^2/c^2}} \tag{4.2}$$

The speed $u = dr/dt = \dfrac{v}{1 - \frac{v}{c}\cos\theta}$ is maximum where $\theta = 0$ and in this point is equal to $u_{max} = \dfrac{v}{1 - v/c}$.

Thus, the "apparent" rate of change of the dimensions of the envelope $u_{\perp,max}$ may be greater than the speed of light c. It is therefore clear that observation of the change in angular dimensions of an object can lead to an estimate of the distance to the object only if we assume nonrelativistic speeds of expansion of the object. If we know the distance to the object, measurement of the rate of expansion $u_{1,max}$ allows us to find the velocity of its surface v. Incidentally, it is assumed here that we are concerned with the movement of an envelope, for example as formed by an explosion If we are observing only the expansion of some luminous area, other possibilities also exist in principle First of all, material transfer may not be taking place at all Let us assume, for example, that the role of the explosion at point 0 is played by a burst of radiation The radiation propagating through the medium (for example a gas cloud) at velocity v, which may reach c, can cause secondary radiation (luminescence, scattering) [34] The envelope which we record at point p in rays of secondary radiation is described by the same equation as in the case of an explosion (4 1) Secondly, the expansion of a luminous area may correspond to the evolution of the object itself, not to its expansion Let us assume as an example that we have a large cloud of gas (protogalaxy), in which stars have not yet been formed The cloud evolves, and a situation is possible in which rapid star formation might begin almost simultaneously throughout the entire cloud The cloud will therefore become visible, or more precisely speaking, its brightness will change essentially However, this does not represent the propagation of explosion products or of any "signal" (in other words, the change in brightness of various areas in the cloud is not causally related) Therefore, the changes in angular dimensions of the source provide no legitimate estimate of the distance to the source This example is probably quite unrealistic if we are speaking of changes in dimensions of a remote object (galaxy, quasar) over a period of several years We wish however to emphasize that when changes of angular dimensions of a source are observed, the distance to which is unknown , its distance or the upper limit of possible distance can be estimated only on the basis of far-reaching assumptions. In practice, for quasars when rapid changes in the form of the luminescent formations surrounding them are observed, the most probable assumption is that of the movement of these formations at relativistic velocities

Macroscopic sources moving at relativistic velocities relative to the observer (relative to the earth) will be referred to for brevity as simply relativistic sources The simplest model of a relativistic source is some formation ("cloud") moving as a whole at constant velocity v, forming angle θ with the direction of the observer (x axis is line OP on Figure 5) If the velocity distribution function of the radiating particles is known, calculation of the intensity of radiation I and of tensor $I_{\alpha\beta}$ in general can be performed using formula (2 36) However, this formula does not consider reabsorption and furthermore the problem of the selection of distribution function N requires special analysis This is also true of the selection of all other parameters of the "cloud," such as magnetic field intensity, density of "cold" plasma, etc In this connection, another approach is more efficient for a "cloud" moving as a unit whole the introduction of a collocated system of coordinates in which the "cloud" is not moving Calculation is then preformed in this system, then the intensities and other quantities are converted to the coordinate system of the observer (laboratory system) Here, which is the essential feature, in the collocated system the parameters of the "cloud" are naturally selected as is done for nonrelativistic objects (for example, in the collocated system the distribution function of particles and the magnetic field can be considered isotropic on the average, the "cold plasma" can also be considered isotropic and homogeneous, etc.) This analysis was performed as applicable to a number of relativistic sources in [33] (see also [32]). Let us represent by

$\varepsilon'_{\nu'}(\vec{r},t)$ and $\mu'_{\nu'}(\vec{r},t)$ the radiating capacity and coefficient of absorption (including reabsorption) of radiation at frequency ν' respectively in the collocated coordinate system (for simplicity we will consider the radiation nonpolarized, $\bar{r}$ and t are the coordinates and time in the system of the observer) Then, the values of ε_ν and μ_ν related to the system of the observer are expressed as follows

$$\varepsilon_\nu(\vec{r},t) = \frac{1-v^2/c^2}{\left(1-\frac{v}{c}\cos\theta\right)^2}\,\varepsilon'_{\nu'}(\vec{r},t)$$
$$\mu_\nu(\vec{r},t) = \frac{1-\frac{v}{c}\cos\theta}{\sqrt{1-v^2/c^2}}\,\mu'_{\nu'}(\vec{r},t),\quad \nu' = \frac{\nu\left(1-\frac{v}{c}\cos\theta\right)}{\sqrt{1-v^2/c^2}} \tag{4.3}$$

These relationships are simplest to establish by considering the relativistic invariance of the number of protons in an element of phase volume As was stated, in relation to $\varepsilon'_{\nu'}$ and $\mu'_{\nu'}$ is natural to make the same assumptions as for "ordinary" nonmoving sources Furthermore, for intensities of nonpolarized radiation in any inertial coordinate system (i e., in the collocated system and in the system of the observer), the following transfer equation is correct

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right) I_\nu = c\left(\varepsilon_\nu - \mu_\nu I_\nu\right) \tag{4.4}$$

Integration of this equation for certain simple models of relativistic sources is performed in [33] Here in the expression for the radiation flux $F_\nu = \int I_\nu d\Omega$, quite large additional factors sometimes appear (in comparison with the expression for the flux of radiation from analogous nonmoving sources)

Thus, for a nontransparent (optical thickness $\tau \gg 1$) nonexpanding cylinder moving toward the observer at velocity v_r, the factor $\zeta \sim (1 - v_r^2/c^2)^{-1/4}$ appears If expansion of the cylinder is also important, in some conditions $\zeta \sim (1 - v_r^2/c^2)^{-13/4}$ For a nontransparent sphere, the center of which is nonmoving, but the surface of which expands at speed v, $\zeta \sim (1 - v^2/c^2)^{-9/4}$. The total energy of the relativistic electrons in the sources with fixed flux F_ν decreases simultaneously by ζ^3 times Also, the change in intensity of nonstationary sources with time varies, differently for different frequencies New possibilities also appear with respect to the polarization of the radiation of the sources Briefly speaking, analysis of relativistic sources opens a completely new chapter in astronomy (of course this problem was partially analyzed long ago for the case of remote sources participating in the expansion of the universe) It has been our purpose simply to emphasize that for relativistic sources the ordinary (see 1) estimates of energy of radiating particles, field intensity, influence of reabsorption and other factors are generally not correct A more detailed analysis of the problem of relativistic sources might be the theme of a special article, and at the same time would be possible at the present time only to a very limited extent It might be thought that in the near future, a great deal of new progress might be expected in this area both as concerns the theory and as concerns observations

4.2 Synchrotron Radiation of Protons

Usually, when we speak of synchrotron radiation, we have in mind the radiation of electrons (and positrons) The existence of this radiation for protons and other charged particles is, of course, beyond doubt. However, the very simplest estimates indicate that in the overwhelming majority of realistic cases, synchrotron radiation of protons is not of interest

The synchrotron losses of energy for ultrarelativistic particles with total energy E, mass M and charge eZ are equal to

$$P = -\frac{dE}{dt} = \frac{2(eZ)^4}{3M^2c^3} H_\perp^2 \left(\frac{E}{Mc^2}\right)^2, \tag{4 5}$$

i.e , they differ by a factor of $\frac{m^4Z^4}{M^4}$ from the losses for electrons (mass m, charge e) For protons, consequently, the losses are $(M/m)^4 \sim 10^{13}$ times less than for electrons The radiation of protons will be maximal at the following frequency (see (1, 2 23))

$$\nu_m^{(p)} = 0.07 \frac{eH_\perp}{Mc}\left(\frac{E}{Mc^2}\right)^2 = 7.5 \cdot 10^{-16} H_\perp (E_{ev})^2, \tag{4.6}$$

and the spectral power of the radiation at the maximum will be M/m = 1836 times less than for electrons (see (1, 2 24))

For the envelopes of supernova stars and in radiogalaxies, $H_\perp \lesssim 10^{-3}$, consequently, for protons with $E \lesssim 10^{12}$ ev, frequency $\nu_m^{(p)} \lesssim 10^6$ and $\lambda_m^{(p)} = \frac{c}{\nu_m^{(p)}} \gtrsim 300$ m, i e , the radiation lies beyond the bounds of the radio astronomy range The power of the radiation, as is clear from the above, is also relatively low All of this can be confirmed with respect to the sun True, the field on the sun can be great, but usually protons are not accelerated with $E > 10^9$-10^{10} ev With $H_\perp \sim 10^2$ and $E \lesssim 10^{10}$ ev, frequency $\nu_m^{(p)} \lesssim 10^7$ and $\lambda_m^{(p)} \gtrsim 30$ m These estimates, performed 15-20 years ago, led to a cessation of the discussion of the synchrotron radiation of protons (see [36a]) However, in recent times, in connection with the problem of the study of quasars, this question has once more attracted attention [36] The reason is that for quasars in the area of their "nuclei," responsible for the short wave radiation (infrared invisible portion of the spectrum), the magnetic field may be quite strong (this fact was noted some time ago, for example, see [37]) With $H \sim 2 \cdot 10^4$ oe and $E \sim 2 \cdot 10^{11}$ ev, as is assumed in [36], $\nu_m \sim 10^{12}$ and $\lambda'_m = c/\nu_m \sim 3 \cdot 10^{-2}$ cm Permissible values are produced for the total energy of relativistic protons at the source in this case and, in general, the corresponding model is noncontradictory Incidentally, if the radiation actually does come from an area with a strong field, the usage of the ordinary "electron" synchrotron mechanism involves difficulties resulting from the necessity of extremely rapid replacement of energy lost by the electrons. From this point of view it is only natural that protons should be considered resposible for the radiation, since the losses for protons are considerably less, relativistic electrons "do not survive" in the strong field More precisely, they could survive only under conditions of very effective acceleration or rapid diffusion from an area with a weak field This need not be understood as a conclusion in favor of the proton synchrotron mechanism of radiation of quasars Quasar models are not being discussed here, and in most of them the particles responsible for the radiation are electrons, and various difficulties can be avoided to some extent by the selection of the required configurations and intensities of the magnetic field, as well as by considering the relativistic velocity of the envelopes (see section 4 1) The purpose of this section is only to recall the possibility, in the case of strong fields, of looking upon proton synchrotron radiation as a realistic mechanism for radiation at high frequencies All of the general formulas produced in 1 and above can be converted to this case if the mass of the particle m is taken as the proton mass M (a degree of caution is required in consideration of the influence of the "cold" plasma, the Langmuir frequency of the plasma $\omega_0 = \sqrt{4\pi e^2 Ne/m}$ contains the mass of the electron m, naturally, regardless of the rest mass of the radiating or absorbing relativistic particle)

4.3 The Change in Magnetic Field Related to Deceleration (Energy Losses) of Particles Moving in a Field

In analyzing the radiation of a charged particle moving in a magnetic field, and also in considering losses or gains in energy by this particle due to any other mechanisms, the magnetic field itself is usually considered given It is quite obvious that this statement of the problem has a limited area of applicability Actually, a particle moving in the magnetic field creates its own magnetic field $\vec{H}_1$, which weakens the external field $\vec{H}_0$ (diamagnetic effect) Field $\vec{H}_1$ depends on the energy of the particle E and, concretely, is decreased as this energy is decreased $E = mc^2/\sqrt{1 - v^2/c^2}$. Therefore, in considering energy losses field $\vec{H}_1$ is decreased, which can lead to a change not only of the total field $\vec{H} = \vec{H}_0 + \vec{H}_1$, but also of field $\vec{H}_0$ (consideration of mutual induction, see below) As a result of the change in the magnetic field, the induction electric field $\vec{\varepsilon}$ arises, which may in turn change the energy of the particle In this connection, the question has been raised as to whether the particle can "scoop" energy from the field and thereby lose not only its kinetic energy $E_k = E - mc^2$, but also high energy [38]. As will be shown below, this conclusion would be incorrect, but still the energetic relationships involved in the movement of a particle in a magnetic field considering losses (or gains) of energy are doubtless interesting, and so we will now discuss them

When a particle moves in a homogeneous magnetic field with intensity[1] $\vec{H}_0$, the particle with charge e and mass m has magnetic moment

$$\vec{\mu} = -\frac{m v_\perp^2 \vec{H}_0}{2H_0^2\sqrt{1-v^2/c^2}} = -\frac{|e| r_{\parallel} v_\perp}{2c}\left(\frac{\vec{H}}{H_0}\right) = -\frac{v_\perp^2 E}{2c^2 H_0}\left(\frac{\vec{H}}{H_0}\right) \quad (4\ 7)$$

Actually, the rotational frequency of the particle in the magnetic field $|\omega_H = \frac{|e| H_0}{mc}\frac{mc^2}{E} = \frac{|e|H_0}{mc}\sqrt{1-v^2/c^2}$ and the radius of the projection of the orbit on the plane perpendicular to $\vec{H}_0$, $r_H = v_\perp/\omega_H = mv_\perp/|e|H_0\sqrt{1-v^2/c^2}$ Finally, magnetic moment $\vec{\mu} = \frac{e}{2c}[\vec{r}\vec{v}]$, from which we arrive at (4 7), where $v_\perp$ is the projection of the velocity v in the plane perpendicular to the field $\vec{H}_0$ The sign in (4 7) can be selected from general considerations, since we know that the gas of charged particles is diamagnetic (without considering the spin). If there are many particles and they move independently, their moments are simply added In this case the natural field of all particles is small in comparison with the external field $\vec{H}_0$ (this field is created by sources located outside the area in consideration) under the condition that $4\pi N\mu \ll H_0$, where $\mu = |\vec{\mu}|$ and N is the concentration of particles (moments) More precisely, if we are concerned with particles with various values of μ, the role of $N\mu$ is played by the total moment of a unit volume, i e , magnetization M The inequality $4\pi M \ll H_0$, in terms of the theory of magnets, obviously means that $B = H_0 + 4\pi M \approx H_0$ (from which the appearance in

[1] We are essentially concerned with magnetic induction $\vec{B}_0$ In the following, however, fields $\vec{H}$ and $\vec{B}$ will not be distinguished, although this may be useful in the macroscopic approach to the problem being discussed (see the end of article [2])

this inequality of the factor 4π is also clear).

Considering (4 7), consequently, we come to the condition of weakness of the diamagnetic effect in the form

$$\frac{\overline{v_\perp^2 E}\mathcal{N}}{4c^2} \ll \frac{H_0^2}{8\pi}, \qquad (4.8)$$

where the overline indicates averaging with respect to the energy spectrum With isotropic distribution of ultrarelativistic particles (for definition we will keep in mind cosmic rays) as to the directions of their velocities $\frac{v_\perp^2}{c^2} = 2/3$, and condition (4 8) can be written in the form

$$\overline{\overline{w}}_{cr} \ll 6\, w_m, \quad w_{cr} = \overline{E}\mathcal{N}, \quad w_m = \frac{H_0^2}{8\pi}, \quad E \gg mc^2, \qquad (4\ 9)$$

where the value of particle mass m is insignificant

Thus, in order for the influence of the relativistic particles themselves on the magnetic field to be weak, their energy density must be small in comparison to the magnetic energy density However, under space conditions in many cases

$$\overline{w}_{cr} \sim w_m \qquad (4\ 10)$$

Under these conditions, the relativistic particles obviously influence the field, but generally speaking, the field may still be rather strong (in the sense that the field in the medium is on the order of the external field H_0)

If as sometimes occurs

$$W_{cr} \gg W_m = \frac{H_0^2}{8\pi},$$

the dynamic effect could lead to total screening of the field, instabilities, etc. The development of these considerations allows us, as might be expected, to produce additional information concerning the relationship between w_r and w_m under various conditions.

Without discussing this interesting problem in greater detail (see [39] for some notes on the subject), let us analyze the case of a single particle, the properties and states of which are described by values of e, m, $E = mc^2/\sqrt{1 - v^2/c^2}$, and $v_\perp$, for definition, we will consider the external field $\vec{H}_0$ to be homogeneous, created in a long solenoid (Figure 6) The current flowing through the "winding" of the solenoid per unit length of the solenoid $i = \int j dr = \frac{c}{4\pi} H_0$, where j is the density of the current in the "winding" (without considering screening $i = jd$, where d is the thickness of the "winding") Let us consider that the trajectory of the particle is located completely in the solenoid, but rather far from its walls The volume of the solenoid

$$V = \pi r_0^2 L, \quad L \gg r_0$$

The equation of the movement of the particle has the form

$$\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-v^2/c^2}} = e\left\{\vec{\mathcal{E}} + \frac{1}{c}[\vec{v}\vec{H}]\right\} - \vec{f}, \qquad (4.11)$$

where $\vec{f}$ is the "force of friction" leading to the energy loss. From this, after multiplying by velocity $\vec{v}$, we produce

$$\frac{dE}{dt} = e\vec{\mathcal{E}}\vec{v} - P, \quad E = \frac{mc^2}{\sqrt{1-v^2/c^2}}, \quad P = \vec{f}\,\vec{v} \qquad (4\ 12)$$

Of course, if acceleration occurs rather than losses, then $P < 0$, the force of radiation friction obviously is included in the expression for $\vec{f}$

Let us represent the density of the currents creating the field as $\vec{j}$, the current related to the particle being analyzed will not be included here, its density is $e\vec{v}\delta(\vec{r} - \vec{r}_e(t))$ Then, the pointing theorem, which follows from the field equations, should be written in the form

$$\frac{\partial}{\partial t}\left\{\frac{\mathcal{E}^2 + H^2}{8\pi}\right\} + e\vec{\mathcal{E}}\vec{v}\delta(\vec{r} - \vec{r}_e) = -\frac{c}{4\pi}\operatorname{div}[\vec{\mathcal{E}}\vec{H}] - \vec{j}\vec{\mathcal{E}} \qquad (4\ 13)$$

or, after integration with respect to a certain volume V and consideration of equation (4 12), in the form

$$\frac{d}{dt}\left\{\int\frac{\mathcal{E}^2 + H^2}{8\pi}dv + E\right\} = -\oint\vec{S}\vec{n}\,d\sigma - \int\vec{j}\vec{\mathcal{E}}\,dv - P, \qquad (4.14)$$

where the pointing vector $\vec{S} = \frac{c}{4\pi}[\vec{\varepsilon}\vec{H}]$ is integrated on the surface limiting volume V (obviously, $\vec{n}$ is the external normal in this surface)

Expressions (4.11)-(4 14) are, of course, general in nature, but we will apply them to the case of the field in the solenoid (with no particle, field $\vec{H} = \vec{H}_0 = \text{const}$) Within the solenoid, the total field $\vec{H} = \vec{H}_0 + \vec{H}_1$, where $\vec{H}_1$ is the field created by the particle itself For simplicity, we will consider it to move in a circle At a sufficiently great distance $r \gg r_H$ from the particle trajectory, its field averaged over the period is equivalent to the field of the magnetic moment (4.7) with $v_\perp = v$ Consequently, far from the particle

$$\vec{H}_1 = \mathrm{rot}\,\mathrm{rot}\,\frac{\vec{\mu}}{r}, \quad \vec{\mathcal{E}}_1 = -\frac{1}{c}\,\mathrm{rot}\,\frac{\dot{\vec{\mu}}}{r}, \tag{4.15}$$

where the value of $\vec{\mu}(t')$ should be taken at moment $t' = t - r/c$ (see [4], paragraph 72)

Figure 6

Let us now consider that the winding of the solenoid is located at distance r from the particle, much less than the wavelength $\lambda = c/\tau$, where τ is the characteristic time of change of the moment due to losses ($d\mu/dt \equiv \dot{\mu} \approx \mu/\tau$) In this case, i e , ignoring delay,

$$\vec{\mu}(t') = \vec{\mu}(t) \quad \text{and}$$

$$\begin{aligned} &\vec{H}_1 = \operatorname{rot} \vec{A}_1 = \frac{3\vec{r}(\vec{r}\vec{\mu}) - \vec{\mu} r^2}{r^5}, \quad \vec{A}_1 = \left[\nabla \frac{1}{r}, \vec{\mu}\right], \\ &\vec{\mathcal{E}}_1 = -\frac{1}{c}\dot{\vec{A}} = -\frac{1}{c}\left[\nabla \frac{1}{r}, \dot{\vec{\mu}}\right] = -\frac{1}{c}\left[\dot{\vec{\mu}}, \frac{\vec{r}}{r^3}\right] \end{aligned} \tag{4.16}$$

Let us now apply relationship (4.14), selecting the internal surface of a cylindrical "winding" as the integration surface Fields H_1 and ε_1 are small quantities in comparison with H_0 and therefore it can be shown that (for more detail see [2])

$$\left.\begin{aligned} &\int \frac{\mathcal{E}^2 + H^2}{8\pi} dV \simeq \frac{H_0^2}{8\pi} V + \frac{H_0}{4\pi} \int \vec{H}_1 dv, \quad \oint \vec{S}\vec{n} d\sigma \simeq \\ &\simeq \frac{c}{4\pi} \int [\vec{\mathcal{E}}_1 \vec{H}_0]_n d\sigma = -\dot{\vec{\mu}} \cdot \vec{H}_0, \\ &\frac{\vec{H}_0}{4\pi} \int \vec{H}_1 dv = \vec{\mu} \vec{H}_0 \end{aligned}\right\} \tag{4 17}$$

Let us assume now that field $\vec{H}_0$ is maintained constant in spite of the change in the moment of the particle $\vec{\mu}$ resulting from losses This can be done, of course, only by the work of external sources of emf ("batteries"), included into the circuit of the winding Under these conditions, considering (4 17) and the assumption that $\vec{H}_0$ = const, equation (4 14) takes on the form

$$\frac{dE}{dt} = -P(E), \quad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{4 18}$$

i e., the commonly used equation for particle energy in the presence of losses is produced. Of course, this result is immediately clear from (4.12), since where $\vec{H}_0$ = const to the electrical field $\vec{\varepsilon} = 0$. However, this analysis allows us to see what occurs with the magnetic field and magnetic energy. The total energy of the field in volume V (in the solenoid), according to (4 7) and (4.17), is equal to

$$\int \frac{H^2}{8\pi} dv \simeq \frac{H_0^2}{8\pi} V + \frac{\vec{H}_0}{4\pi} \int \vec{H}_1 dv = \frac{H_0^2}{8\pi} V + \vec{\mu} \vec{H}_0 = \frac{H_0^2}{8\pi} V - \frac{v^2 E}{2c^2}, \tag{4.19}$$

where we assume $v = v_\perp$ (movement in a circle). As the particle loses energy, moment $\mu = |\vec{\mu}|$ is decreased and the total magnetic energy increases, since $\vec{\mu}\vec{H}_0 < 0$. This increase occurs as a result of the energy flux flowing inside the solenoid. At the end of the process (the particle has lost energy and its moment $\mu = 0$) field $\vec{H}_0$, according to the assumption, remains unchanged, and the "batteries" have expended energy

$$-\vec{\mu}(0)\vec{H}_0 = \frac{v_\perp^2(0)\, E(0)}{2c^2}, \tag{4.20}$$

where argument t = 0 indicates the initial values of $\vec{\mu}$, v and E. A somewhat more interesting statement of the problem is that in which field $\vec{H}_0$ is not considered fixed, but the "winding" of the solenoid is closed and formed by a flux of electrons experiencing no impedance (i e , the electrons describe circles with radius r_0, filling a thin layer with thickness d, see Figure 6) Since the conductivity of the medium in a cosmic plasma is very great, this case has certain features near those encountered in reality. The degree of this similarity should not be overestimated, however, since under space conditions the entire medium within the solenoid would also have to be considered conducting Furthermore, for simplicity we will consider that the "winding" does not distort the field of the particle, i e., the field of moment $\vec{\mu}$ This means that the "winding" must be rather thin ($d \ll \delta$, where $\delta = \sqrt{mc^2/4\pi e^2 N_0}$ is the depth of penetration of the field into the "winding"[1]) under these conditions, we place the surface limiting the volume analyzed in (4 14) outside the winding Here $H_0 = 0$, on assumption (4 17) the energy flux $\vec{S} = 0$ and if screening is ignored as before $\frac{H_0}{4\pi}\int \vec{H}_1 dv = \vec{\mu}\vec{H}_0$ (see (4 17)) As a result, equation (4 14) takes on the form

$$\frac{d}{dt}\left\{\frac{H_0^2}{8\pi}V + \vec{\mu}\vec{H}_0 + E\right\} = -P - \int \vec{j}\vec{\mathcal{E}}\,dv, \tag{4 21}$$

where $\int \vec{j}\vec{\varepsilon}dv$ is taken with respect to the volume of the "winding " Obviously

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[1] For a free electron gas $\varepsilon = 1 - 4\pi e^2 N_e/m\omega^2$ and where $\varepsilon < 0$, $|\varepsilon| \gg 1$, the field attenuates according to the law

$$e^{-\frac{\omega}{c}\sqrt{|\varepsilon|}\,z} = e^{-z/\delta}, \quad \delta^2 = \frac{mc^2}{4\pi e^2 N_e}$$

integral $\int \vec{j}\vec{\varepsilon}dv = \frac{d}{dt}\mathcal{K}$, where $\mathcal{K} = \frac{mu}{2} \cdot 2\pi r_0 dL N_e$ is the kinetic energy of ordered movement of electrons in the "winding," responsible for creation of field H_0 ($2\pi r_0 dL$ is the volume of the "winding," N_e is the concentration of electrons considered nonrelativistic) As was already stated, the current density $i/d = cH_0/4\pi d$ and, on the other hand, $j = eN_e u$ It is easy to show that under condition $2\delta^2/r_0 d \ll 1$, consideration of the term $\int \vec{j}\vec{\varepsilon}dv$ in (4 21) would only mean the introduction of a small correction to the term $\frac{d}{dt}(\frac{H_0^2}{8\pi}V)$ In addition to Equation (4 21), solution of the problem requires that we use equation (4 12), expressing $\vec{\varepsilon}$ through $d\vec{H}_0/dt$ As a result (see [2])

$$\vec{\mathcal{E}} = \frac{m\vec{v}_\perp \frac{dH_0}{dt}}{2e\sqrt{1 - v^2/c^2}\, H_0}, \quad \frac{dE}{dt} = \frac{mv^2}{2\sqrt{1 - v^2/c^2}} \frac{d}{dt} \ln H_0 - P, \tag{4.22}$$

where in the second expression we assume $v = v_\perp$ (circular movement)

Using equation (4 21) without the last term and equation (4.22), we can establish the relationship between the field $H_0(0)$ at moment $t = 0$ (here $E = E(0)$, $v = v(0)$ and $\mu = \mu(0)$) and the field $H_0(\infty)$ at time $t \to \infty$, when the particle has lost all of its energy ($\mu(\infty) = 0$, $v(\infty) = 0$). This relationship is as follows [38, 2]

$$\frac{H_0^2}{8\pi}V - \frac{H_0^2(\infty)}{8\pi}V = \frac{v_0^2 E(0)}{2c^2} = -\mu(0)H_0(0) \tag{4.23}$$

The sense of relationship (4.23), in which the equality $\frac{v^2(0)E(0)}{2c^2} = -\mu(0)H_0(0)$ follows from (4.7), where $v_\perp = v$, it is quite clear if we recall the discussion related to formulas (4 19) and (4 20) Namely, the total energy of the magnetic field in the solenoid (see (4 19)) is

$$\int \frac{H^2}{8\pi} dv \simeq \frac{H_0^2}{8\pi} V + \mu H_0$$

Further, on the assumption $\mu(\infty)H_0(\infty) = 0$, relationship (4 23) is a simple condition for retention of the full magnetic energy. In this case, however, the field $\vec{H} = \vec{H}_0 + \vec{H}_1$ changes and is redistributed as the absolute value of moment μ is decreased, field H_1 also decreases, therefore, in order to retain the total magnetic energy we must also decrease the homogeneous field H_0, since field $\vec{H}_1'$ is directed opposite to field $\vec{H}_0$ (diamagnetic effect)

Thus, the situation finally turned out to be rather trivial everything is reduced to consideration of the diamagnetic effect which occurs as charged particles move through a magnetic field, as well as the usage of the law of conservation of energy (pointing theorem). In both of the problems here analyzed (constant field $\vec{H}_0$, and solenoid with "short circuited winding") the particle loses only its energy and cannot "scoop" energy from the magnetic field.

In order to make the picture complete and, more importantly, having in mind the possibility of generalization to a more complex case of a set of radiating particles, the problem discussed in this section is analyzed in [2] by a macroscopic method as well.

The results, of course, agree completely with those outlined above.

4.4 Synchrotron and Other Mechanisms of Cosmic Radiation

One of the clearly expressed tendencies appearing in modern astronomy is an ever broader consideration of relativistic phenomena and effects In particular, it has been determined that relativistic particles (cosmic rays) have a primary dynamic and energetic role to play in the universe [39]. Ever greater attention is being turned to macroscopic relativistic objects (clouds, surges, see section 4 1) The synchrotron mechanism of radiation is essentially relativistic and its utilization for explanation of an ever broader range of observational data is quite natural, due to the "relativization" of astrophysics just mentioned. The most important thing, of course, is that the synchrotron mechanism is effective in a vacuum and, consequently, in the most rarefied areas of outer space At the same time, the magnetic field intensity H may also be comparatively low.

Let us explain this statement by comparing synchrotron radiation with Bremsstrahlung and the "plasma" mechanisms of radiation

The intensity of Bremsstrahlung (for example, in a hydrogen plasma) is proportional to N_e^2T, where N_e is the electron concentration and T is the temperature (assuming that $h\nu \ll kT$) Obviously, this braking mechanism is effective only in a rather dense plasma and, furthermore, the effective temperature of radiation will not exceed the plasma temperature T. Here, it is true, we have in mind an equilibrium plasma But absorption in general changes little for a nonequilibrium plasma containing an increased number of higher speed particles.

The "plasma" mechanisms of radiation involve the excitation of various "normal" electromagnetic waves in the plasma considering subsequent transformation of these waves into the radiation observed Waves may be excited by beams, shock waves and in general as a result of most perturbations of the equilibrium state of the plasma. However the influence of the plasma on the propagation of high frequency waves is determined primarily by the ratio of the carrier frequency $\omega = 2\pi\nu$ to the plasma and gyrofrequencies respectively

$$\omega_0 = \sqrt{\frac{4\pi e^2 N_e}{m}} = 5{,}64 \cdot 10^4 \sqrt{N_e}, \quad \omega_H^{(0)} = \frac{eH}{mc} = 1{,}76 \cdot 10^7 H \qquad (4.24)$$

True, the characteristic frequency ω_c defining the influence of the plasma may be more complex, but for a plasma at rest usually $\omega_c \sim \omega_0$, $\omega_c \sim \omega_H^{(0)}$ or $\omega_c \sim \sqrt{\omega_0^2 + [\omega_H^{(0)}]^2}$. For a moving plasma ω_c depends also on $2\pi u/\lambda$, where u is the velocity of the plasma and $\lambda = \lambda_0/\tilde{n}$ is the wavelength in the medium (in a vacuum, of course, $\lambda = \lambda_0 = 2\pi c/\omega$). For nonrelativistic objects in most cases $2\pi v/\lambda \sim \omega u/c \ll \omega$ and consideration of plasma movement introduces nothing new in principle. It can be affirmed in this case that the influence of the plasma is generally slight under the condition

$$\omega \gg \omega_c \sim \sqrt{\omega_0^2 + [\omega_H^{(0)}]^2} \qquad (4.25)$$

In interstellar space, in the envelopes of supernova stars, in the galaxies and radiogalaxies (with the exception of their nuclei) according to well known estimates $N_e \lesssim 10^4$ cm^{-3}, $H \lesssim 10^{-2}$ oe and, consequently, $\omega_0 \lesssim 5 \cdot 10^6$, $\omega_H^{(0)} \lesssim 10^5$ and $\omega_c \lesssim 5 \cdot 10^6$, $\lambda_c = \dfrac{2\pi c}{\omega_c} \gtrsim 300\,\text{m}$ (for the area of the galactic disk, $N_e \lesssim 1$, $H \lesssim 10^{-5}$ and $\lambda_c \gtrsim 30$ km). These estimates demonstrate that for the galaxies and many galactic nebulae, the "plasma" mechanism of radio radiation is ineffective or, more precisely, has no role to play in the typical radio astronomical wavelength range At the same time, well known estimates of frequencies and intensities of synchrotron radiation indicate the effectiveness of this mechanism in weak fields ($H \prec 10^{-2}$ and even $H \lesssim 10^{-4}$) with permissible concentrations of radiating relativistic electrons

All of the above is elementary and well known, but we have recalled the situation in order to emphasize with the greatest possible clarity the nonuniversality of the synchrotron mechanism Whereas in the beginning of the 1950's, the usage of the synchrotron mechanism in astronomy encountered difficulties (in the most part apparently of a psychological nature), after the successes achieved by applying the synchrotron mechanism, an attraction of the opposite sort was observed Specifically, some of the limitations which arise when the synchrotron mechanism is applied to quasars came to be looked upon as indications of the possible closeness of the quasars, etc Actually, whereas the more or less ordinary synchrotron model of the source encounters difficulties (suffice it to say that with fixed dimensions and consideration of reabsorption, the luminosity of the synchrotron source is limited), a number of other possibilities arise without even changing the assumed distance to the source Thus, all estimates can be essentially varied for synchrotron, but relativistic sources (see [32, 33] and section 4 1 above) Furthermore, for sufficiently dense sources the synchrotron mechanism loses its exclusive position, since inequality (4 25) may not be observed In the case of the radio radiation of the sun, the possibility and necessity of analyzing non-synchrotron mechanisms has long been well known (for example, see [17, 18]) The same thing is true of quasars and the compact source of long wave radio radiation in the Crab nebula in the sense that the "plasma" mechanisms can be rather effective for these objects in principle [40] In all probability, the mechanism of radiation of the pulsing radio sources (pulsars) is also a "plasma" mechanism [43]

It should be added to the above that synchrotron radiation, which is analyzed in astronomy under conditions in which reabsorption is nonessential, is coherent -- the intensity of the radiation of the sources proportional to the number of radiating particles or in homogeneous conditions to their concentration N Reabsorption (both positive and negative) makes up a definite class of coherent phenomena Suffice it to say that the reabsorption coefficient μ depends on N and, consequently, the intensity of the source including consideration of reabsorption is a nonlinear function of N (in the simplest case $I \sim Ne^{-\mu(N)L}$) In a sufficiently dense source, particularly in the presence of various movements, the formation of instabilities and turbulence, a coherent radiation of a slightly different type appears -- the grouping of radiating particles in areas comparable to a wavelength The intensity of the radiation of a cloud of particles with characteristic dimensions $l \ll \lambda$ is proportional to N^2 (for a quasi-spherical cloud, $I \sim N^2 l^3$) Obviously, the nonlinear dependence of I on N is retained in a much more general case as well. The generation, propagation and mutual transformation of various waves in a medium (isotropic or magnetoactive plasma) are also coherent (collective) processes, involving many particles in the medium (concretely, a "cold" plasma) In dense sources, when relativistic electrons and a "cold" plasma are present, it is generally impossible to distinguish various coherent processes, and the boundaries between the synchrotron and "plasma" mechanisms disappear Of course, this is also true of relativistic sources (for relativistic "plasma" sources parameter $\frac{2\pi}{\lambda}u = \frac{\omega\tilde{n}u}{c}$ is comparable with ω and the possibilities for disruption of condition $\omega \gg \omega_c(\omega_0, \omega_H^{(0)}, \frac{2\pi}{\lambda}u)$ are even more widely expanded)

It is therefore clear that the theory of dense sources can be based on conceptions concerning the noncoherent synchrotron mechanism of radiation, supplemented by a consideration of reabsorption, only in certain particular cases One of the most important problems for further investigation is a more detailed and comprehensive analysis of coherent processes and effects in dense cosmic radiation sources (in this connection, see [7, 17, 18, 27, 40, 42, 43]).