Device-independent verification of Einstein-Podolsky-Rosen steering

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If the presence of entanglement could be certified in a device-independent (DI) way, it is likely to provide various quantum information processing tasks with unconditional security. Recently, it was shown that a DI protocol, combining measurement-device-independent techniques with self-testing, is able to verify all entangled states, however, it imposes demanding requirements on its experimental implementation. In this work, we propose a much less-demanding protocol based on Einstein-Podolsky-Rosen (EPR) steering to certify entanglement. We establish a complete framework for DI verification of EPR steering in which all steerable states could be verified. We then analyze its robustness towards noise and imperfections of self-testing by considering the measurement scenario with three settings at each side. Finally, a four-photon experiment is implemented to demonstrate that even Bell local states can be device-independently verified. Our work may pave the way for realistic applications of secure quantum information tasks.

Entanglement is not only of fundamental importance to understand quantum theory, but also has found practical applications in information processing and computational tasks [1]. If its presence could be certified in a fully device-independent (DI) manner, such as violating some Bell inequality [2, 3], it is likely to offer quantum information processing tasks with unconditional security because it does not require any trust in both measurement and state-preparation devices. However, the conclusive violation of Bell inequalities usually demands the high efficiency of measurement apparatuses to close the detection loophole. Besides, it also needs the low transmission loss of prepared states since sufficiently lossy entangled states are unable to violate any Bell inequality [4]. Thus, although this trust-free verification of entanglement using Bell inequalities has promising applications in quantum tasks, its practical utility is limited due to noise.

Recently, Bowles *et. al.* proposed an alternate DI protocol able to verify all entangled states [5, 6]. Intrinsically, it is composed of two parts: The first part utilizes measurement-device-independent (MDI) techniques that injecting some well-prepared quantum states randomly into the characterized system relaxes the requirement of high measurement efficiency as introduced by Buscemi [7], allowing the detection loophole to be circumvented [8]; Then it employs self-testing [9, 10] to certify the above input states device-independently. However, its experimental test relies on the near-perfect self-testing of these states with average fidelity above 99.998% [6], making it unrealistic to implement within current technology.

As shown in Fig. 1, we present an experimental-friendly protocol based on Einstein-Podolsky-Rosen (EPR) steering beyond all above limitations. Specifically, EPR steering is a form of nonlocal correlations which lies intermediate between entanglement and Bell nonlocality [11–13], and has been operationally interpreted as a one-sided device-independent task to certify the presence of entanglement within quantum theory [11, 12]. Being confirmed in many experimental setups [14–17], it was also proven to be useful in one-sided secure key distribution [18] and randomness generation [19]. Furthermore, its verification was extended to the MDI scenario [20–25], and the corresponding experimental validations were reported in [21, 23, 26]. Here, inspired by results in [5, 6], we give a full analysis of DI verification of EPR steering.

More importantly, we find that our steering protocol is unconditionally secure and robust to the transmission loss, detection efficiency, and self-testing.

In this work, we first establish a complete framework for DI verification of EPR steering that the presence of entanglement in all steerable states can be certified device-independently. Then, we analyze the noise robustness of the three-measurement setting case and show that it is able to tolerant a self-testing fidelity lower than 98.5%, which is a significant reduction in comparison to entanglement verification for Werner states. Finally, two pairs of entangled photons are delicately generated to validate our DI steering protocol, and the results indicate that even Bell local states can be faithfully verified with an experimentally attainable self-testing fidelity of around 99.2%.

Results

Device-independent verification of EPR steering. Consider the quantum steering scenario in which two space-like separated observers, namely Alice and Bob, are involved. In principle, if Alice is able to steer Bob, then such steerability could be witnessed by violating a suitable linear steering inequality of the form [28]

$$W_S = \sum_j \langle a_j B_j \rangle \leq 0. \tag{1}$$

Here Alice obtains outcome a given the measurement j, and the operator B_j describes the corresponding observable chosen by Bob. Moreover, the above verification task could be accomplished in a measurement-device-independent way [20–25] where the trust in Bob as per Eq. (1) is completely transferred to a third observer, Charlie say, who randomly assigns some quantum states to Bob. Specifically, upon receiving these states described by density matrices $\{\tau_{b,j}^T\}$ where T denotes the transpose operation, Bob is required to perform an arbitrary binary measurement $\mathcal{B}$ with which the outcomes are modelled as either "Yes" or "No". Denote by $P(a, \text{Yes}\,|x, \mathcal{B}, \tau_{b,j}^T)$ the probability that Alice obtains a for the measurement x and Bob answers "Yes" when assigned to $\tau_{b,j}^T$. Thus, it follows from Eq. (1) that there is a MDI steering witness [21–23]

$$W_{\text{MDI}} = \sum_{a,b,j} g_{b,j} a_j P(a, \text{Yes}\,|x = j, \mathcal{B}, \tau_{b,j}^T) \leq 0 \tag{2}$$

with $g_{b,j}$ being some predetermined weights. In practice, Bob could perform a partial Bell state measurement (BSM) $\mathcal{B} \equiv \{\mathcal{B}_1, \mathbb{I} - \mathcal{B}_1\}$ where $\mathcal{B}_1 = |\Phi_d^+\rangle\langle\Phi_d^+|$ with $|\Phi_d^+\rangle = \sum_j |jj\rangle/\sqrt{d}$ models the answer "Yes" and d is the dimension of the Hilbert space of $\{\tau_{b,j}^T\}$ equal to that of Bob's local system. It has been shown in [21, 23] that each steering inequality (2) can be constructed from a corresponding witness (1), implying all steerable states could be detected in a MDI manner.

Within the above MDI framework, both Alice's and Bob's sides are already device-independent, except for the additional trust in the input states prepared by Charlie. Hence, eliminating this extra trust immediately gives rise to a fully DI steering verification. As illustrated in Fig. 1, this can be achieved via self-testing which aims to uniquely identify the state and the measurements for uncharacterized systems [9, 10, 29]. In particular, noting that the states $\{\tau_{b,j}^T\}$ for Bob can be generated by Charlie performing local measurements $\{\tau_{b,j}\}$ on the Bell state $|\Phi_d^+\rangle$ shared by Bob and Charlie, i.e., $\text{Tr}_C[I_B \otimes \tau_{b,j} |\Phi_d^+\rangle\langle\Phi_d^+|] = \tau_{b,j}^T/d$, we can employ self-testing to uniquely determine this state preparation process via certain Bell inequality because its maximal violation can only be achieved by a certain state and specific measurements up to some local isometry. Since all pure bipartite entangled states and the measurements could be self-tested [30, 31], confirmed in recent experiments [32, 33], it further follows from Eq. (2) that we are able to construct a DI steering inequality [34]

$$W_{\text{DI}} = \sum_{a,c,j} g_{c,j} a_j P(a, \text{Yes}, c\,|x = j, \mathcal{B}, z = j) \leq 0, \tag{3}$$

for every steerable state. Here each state $\tau_{b,j}^T$ in Eq. (2) corresponds to Charlie making a local measurement z on a Bell state and obtaining an outcome c in Eq. (3).

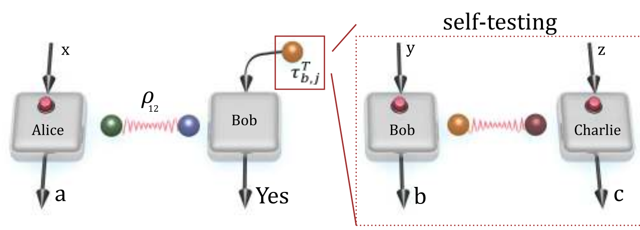

FIG. 1: **DI verification of EPR steering.** The whole protocol consists of two parts: The left describes MDI verification of states ρ_{12} to be tested: Alice takes a measurement x and obtains a, while Bob performs only one binary measurement on his local system and random input states $\{\tau_{b,j}^T\}$ and his answer "Yes" is collected; The right box represents the self-testing process where these states $\{\tau_{b,j}^T\}$ are device-independently certified via the test of some Bell inequalities, such as the simple Bell-CHSH test.

Hence, we have established a framework to device-independently certify all steerable states. It is worth noting that the self-testing process of $|\Phi_d^+\rangle$ and Charlie's measurements is not explicitly imposed in the DI steering witness as per Eq. (3), thus requiring a further detailed analysis case by case. For example, if Charlie is restricted to binary measurements, we can make use of Clauser-Horne-Shimony-Holt (CHSH) type inequalities [36] to do self-testing. In the following, we study the case in which three dichotomic measurement settings are chosen at each side and analyze its robustness towards imperfections of self-testing.

Three measurement settings. Suppose that Alice randomly takes three dichotomic measurements $x = 1,2,3$, and Bob randomly receives $\tau_{c,j} = (I + c\sigma_j)/2$ for $c = \pm 1$ and $j = x, y, z$ sent from Charlie where σ_js represent three Pauli observables. It follows from above discussions that we need to identify these states device-independently via self-testing. This could be accomplished if the following triple Bell-CHSH inequality [37]

$$\begin{aligned}\mathfrak{B} =& E_{1,1} + E_{2,1} + E_{1,2} - E_{2,2} \\ &+ E_{3,1} + E_{4,1} - E_{3,3} + E_{4,3} \\ &+ E_{5,2} + E_{6,2} - E_{5,3} + E_{6,3} \leq 6 \end{aligned} \quad (4)$$

is maximally violated by the measurement expectation $E_{y,z} = \sum_{b,c=\pm 1} b\, c\, p(b,c|y,z)$ where $y = 1,2,...,6$, refers to Bob's six dichotomic measurements and $z = 1,2,3$ Charlie's three measurements. Indeed, its maximal quantum violation is $\mathfrak{B} = 6\sqrt{2}$, which certificates an unique state $|\Phi_2^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and the corresponding measurements $\frac{\sigma_i \pm \sigma_j}{\sqrt{2}}$ for Bob with $(i,j) = \{(z,x),(z,y),(x,y)\}$ and $\sigma_x, \pm\sigma_y, \sigma_z$ for Charlie up to a local unitary. Note that the second measurement σ_2 for Charlie is not fully determined due to the sign problem. However, this issue does not affect its utility in entanglement certification [5].

Practically, it is unable to achieve the perfect self-testing, or equivalently, to obtain a maximal value $\mathfrak{B} = 6\sqrt{2}$. Thus, we need to modify the above DI steering inequality (3) to allow for imperfections. Here, we adopt the fidelity $f_0 = \langle\Phi_2^+|\,\rho^0_{\text{data}}\,|\Phi_2^+\rangle$ to evaluate the performance of self-testing, i.e., the overlap between estimated states self-tested from experimental data and the target state. Moreover, the corresponding fidelity for Charlie's measurements can be cast as the state fidelity in a similar form of $f_j = \langle\Phi_2^+|\,\sigma_j \rho^j_{\text{data}} \sigma_j\,|\Phi_2^+\rangle$ for $j = 1,2,3$. All these fidelity are calculated from data via a semi-definite program [38–40]. After considerable algebra [34], we are able to obtain the modified DI steering witness allowing for imperfections of self-testing [34]

$$W_{\text{DI}}^{\text{noisy}} = W_{\text{DI}} - \sum_{j=1}^{3} \sqrt{1 - f_j} \leq 0. \quad (5)$$

Here W_{DI} is given as per Eq. (3). It is remarked that the fidelity in [6] is defined in terms of trace distance, interchangeable with the one used in the above inequality [34]. Additionally, it differs from the one in [21] which is obtained via quantum state tomography.

Experimental setup

The experimental setup for the DI verification of EPR steering is displayed in Fig. 2. First, two pairs of entangled photons via the spontaneous parametric down-conversion (SPDC) process are generated. In particular, one pair is prepared as $|\Phi_2^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, labelled as ρ_{34}, and the other is a family of Werner states

$$\rho_{12} = v\,|\Psi_2^-\rangle\langle\Psi_2^-| + (1 - v)\frac{I}{4}, \quad v \in [0,1]. \quad (6)$$

Here, the white noise parameter $1 - v$ in Eq. (6) is simulated by flipping Alice's measurements with probability $(1 - v)/2$ [41]. The steerability of this class of states is to be verified via the steering witness (5).

Then, these photonic states are distributed to three observers. As shown in left side of Fig. 2, ρ_{12} is sent to Alice (photon 1: the green ball) and Bob (photon 2: the blue ball) through single-mode fibers while the entangled photonic pair encoding ρ_{34} is distributed to Charlie (photon 3: the red ball) and Bob (photon 4: the yellow ball) similarly. The detailed parameters for wave plates (WPs) to realize three Pauli measurements σ_j performed on the single photon (photon 4 for Bob) are given in Tab. I in Supplemental Materials [34]. Further, it is shown in the right side of Fig. 2 that Bob's partial BSM is implemented via three polarizing beam splitters, two 22.5° rotated HWPs, and four pseudo photon-number-resolving detectors (PPNRD). In each PPNRD, a balanced beam splitter splits the light into two fiber-coupled single photon detector, and thus we could detect two photons with probability 1/2. To improve the quality of this partial BSM, an interference filter of 3 nm is inserted in the o-light path for spectral selection so that a Hong-Ou-Mandel (HOM) interference visibility higher than 30 : 1 is observed in our experiment. Additionally, a standard quantum measurement tomography is performed to estimate the experimentally constructed BSM and a fidelity around $0:9831 \pm 0:0040$ is obtained [34].

Finally, we collect the measurement statistics to first do three Bell-CHSH tests $\mathfrak{B}_{i,j}$ to self-test Charlie's three Pauli measurements and thus quantum states assigned to Bob. And the corresponding fidelity is estimated from the expectation $\langle(\sigma_i^B \pm \sigma_j^B) \otimes \sigma_j^C\rangle/\sqrt{2}$ [34]. We then use them to test the DI steering inequality (5) in a more explicit form of

$$\begin{aligned} &4\sum_{j,a,c}\Big(a_j\, c_j\, P(a, \text{Yes}, c\,|x = z = j, \mathcal{B}) - P(a, c\,|j)/\sqrt{3}\Big) \\ &-\sum_j \sqrt{1 - f_j} \leq 0. \end{aligned} \quad (7)$$

For the family of Werner states (6), its theoretical prediction of the outcome statistics (7) should be $3v - (\sqrt{3} + \sum_{j=1}^{3}\sqrt{1 - f_j}) \leq 0$. In particular, the average fidelity around 98.5% of self-testing is allowed for Bell local states with $v = 0.7$, which is a significant reduction in comparison to entanglement verification with fidelity above 99.998% [6, 34].

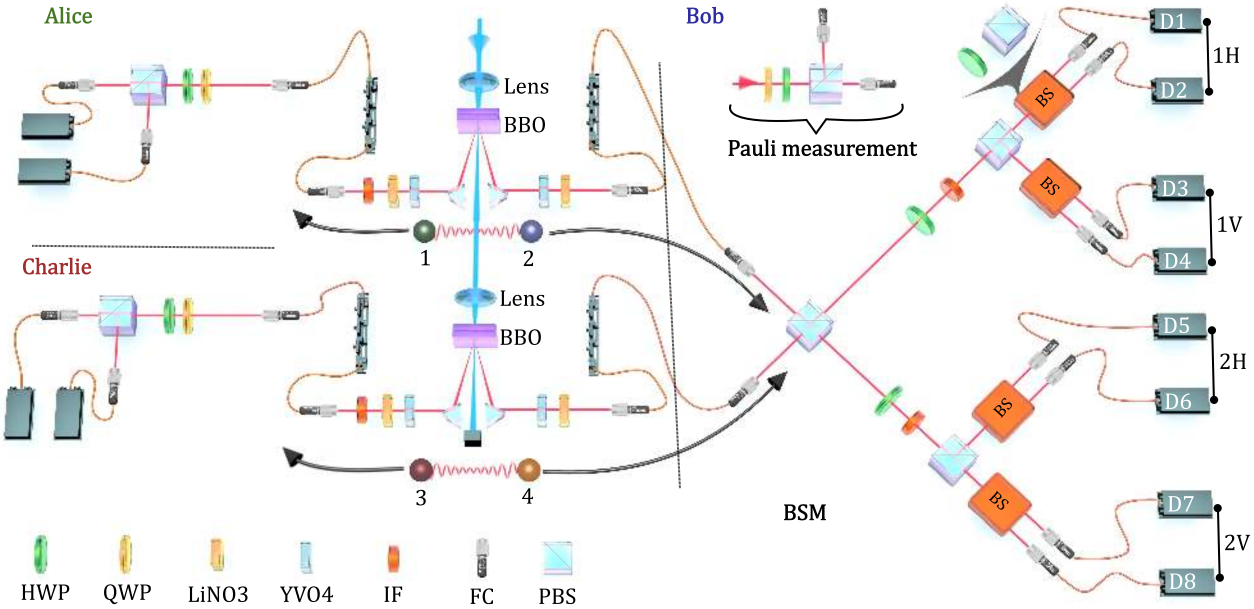

FIG. 2: **Experimental setup for the DI steering protocol.** Two pairs of entangled photons are generated via the spontaneous parametric down-conversion process, where the β-barium-borate (BBO) crystal is cut in a sandwich-like configuration to prepare desired states with high fidelity. One pair labeled as 1 and 2 is generated as a family of Werner states in Eq. (6) distributed to Alice and Bob, while the other labeled as 3 and 4 is produced as the Bell state $|\Phi_2^+\rangle$ sent to Charlie and Bob. A complete implementation of DI steering verification (7) requires three Bell-CHSH tests and one MDI steering test (2). Hence, Alice and Charlie perform three Pauli measurements σ_j on their respect photons, while Bob makes 6 measurements described by $(\sigma_i^B + \sigma_j^B)/\sqrt{2}$ and $(\sigma_i^B - \sigma_j^B)/\sqrt{2}$ on the photon 4 and an additional partial BSM $\mathcal{B}$ on his photons 2 and 4. Abbreviations of the components: HWP, half wave plate; QWP, quarter wave plate; PBS, polarizing beam splitter; IF, interference filter; FC, fiber coupler; BSM, the partial Bell state measurement $\mathcal{B}$; BD, beam splitter; D1-D8, single photon detector.

Experimental analysis

In this experiment, the entangled photon pairs encoding $|\Phi_2^+\rangle$ are collected up to 13000 per second with a pump power of 30 mW. We observe an extinction ratio over 500 : 1 in the H/V basis and the H+V/H-V basis, implying that it is generated with fidelity higher than 0.997. Charlie's three Pauli measurements are self-tested by means of three Bell-CHSH tests $\mathfrak{B}_{i,j}$ for $(i,j) = (1,2),(1,3),(2,3)$. With the fair-sampling assumption, we obtain $\mathfrak{B}_{12} = 2.8241$, $\mathfrak{B}_{13} = 2.8211$, and $\mathfrak{B}_{23} = 2.8189$, all close to the maximal value $2\sqrt{2} \approx 2.8284$. Correspondingly, the fidelity of three Pauli measurements self-tested from experimental data is obtained with $f_1 = 0.9931, f_2 = 0.9897$, and $f_3 = 0.9979$, respectively. The Poisson oscillation of the photon is counted with an uncertainty of 0.0009, and we test 100 groups of the Poisson oscillations to optimize the fidelity around which the standard deviation is 0.9936×10^{-5}.

Our experimental results for DI verification of Werner states in Eq. (6) are plotted in Fig. 3. We first do quantum state tomography to verify that each Werner state is generated with $v = 0.6469(4)$, $0.6742(4)$, $0.7015(4)$, $0.8090(4)$, $0.9239(3)$, and $0.9951(0.9)$ from about $9,800,000$ photon pairs [34]. For the ideal case, i.e., all $f_j = 1$, the theoretical prediction of the steering inequality (7) for Werner states should be $3v - \sqrt{3} \leq 0$ and it recovers the bound $v = 1/\sqrt{3} \approx 0.5774$ for steerability [28]. If self-testing is non-perfect, the steering inequality, incorporated with self-testing results obtained above, is shown as the red line in Fig. 3, while the experimental results are displayed in red dots. It is evident that we successfully witness steerability for $v \geq 0.7015(4)$, allowing for system errors, statistic errors, and imperfections of self-testing. Importantly, a violation of the steering inequality (7) up to 0.1189 ± 0.0714 is achieved at the point $v = 0.7015(4)$ which is lower than the Bell-CHSH bound $1/\sqrt{2} \approx 0.707$ [36] and even the Vértesi bound $\gtrsim 0.7056$ [42]. This implies that we are even able to faithfully verify Bell local states device-independently. However, the error bars for the steerable Werner states with $v = 0.6469(4), 0.6742(4)$ fall into the failure region and thus we cannot conclude that it is witnessed in our DI protocol. In contrast, we also

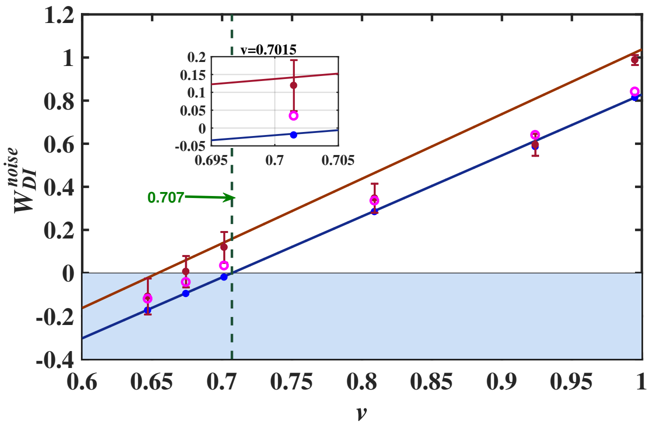

FIG. 3: **Experimental results for Werner states (6).** The theoretical prediction for DI steering verification (7) is plotted as the red line, while the corresponding experimental results are shown as red dots for $v = 0.6469(4), 0.6742(4), 0.7015(4), 0.8090(4), 0.9239(4), 0.9951(1)$. Especially, we observe a violation up to 0.1189 ± 0.0714 for the Bell local state with $v = 0.7015(4)$. In contrast, the corresponding theoretical and experimental results for using simple Bell-CHSH inequality with two measurements to certify steerability are also given in blue line and dots, and the error bars are about 0.001. The shaded blue region represents the failure of steering witnesses.

perform a simple Bell-CHSH test to verify steerability of these states device-independently. In Fig. 3, the blue line describes the theoretical result while blue dots are for the experimental results for these Werner states.

Method

In the fully DI verification of EPR steering for any quantum state ρ_{AB}, we need to collect the measurement statistics to check whether it violates either the DI steering inequality (3) for the ideal case or the one (5) accounting for imperfections. For example, suppose that Bob is input $\tau_{b,j}^T = \frac{1}{2}(I + b\sigma_j)$ with $b = \pm 1, j = 1, 2, 3$ randomly from Charlie. Alternate, Bob's input states could be generated by Charlie performing local measurements described by positive-operator-valued measures $\{E_{b,j} = \tau_{b,j}\}$ on the Bell state $|\Phi_2^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ shared by Bob and Charlie. As a consequence, the DI steering inequality (3) could be expressed in a more explicit form of

$$
\begin{aligned}
W_{\mathrm{DI}} &= \sum_{a,b,j} b_j a_j P(a, \mathrm{Yes}, c\,|x = j, \mathcal{B}, z = j) \\
&= \sum_{j,a,b} a_j\, b_j \mathrm{Tr}[I \otimes |\Phi_2^+\rangle \langle \Phi_2^+|_{BB_0} \otimes (I + b\sigma_j^C)/2 \\
&\quad \cdot \rho_{AB} \otimes |\Phi_2^+\rangle \langle \Phi_2^+|_{B_0C}] \\
&= \frac{1}{4} \sum_{j,a,b} a_j\, b_j \mathrm{Tr}\left[I \otimes (I + b\sigma_j^B)/2 \cdot \rho_{AB}\right] \\
&= \frac{1}{4} \sum_j \langle a_j B_j \rangle = \frac{1}{2} W_{\mathrm{MDI}} = \frac{1}{4} W_{\mathrm{S}}. \qquad (8)
\end{aligned}
$$

Here B_0 represents the Bob's subsystem to which the states $\tau_{b,j} = \frac{1}{2}(I + b\sigma_j)$ are assigned.

Further, those input states for Bob can be device-independently identified via self-testing. Consider that two noncommunicating parties, Bob and Charlie, are involved. Each has access to a black box with an underlying state $|\psi\rangle$. Since it is accomplished with three Bell-CHSH tests, Bob takes six dichotomic measurements $y = 1, 2, \ldots, 6$ and Charlie performs $z = 1, 2, 3$. Bob's inputs and outputs are denoted respectively by y and b, while Charlie's is by z and c. After rounds of experiments, the joint probability distribution $p(b, c|y, z)$ could be reconstructed. So we are able to test whether it violates the triple Bell operator as per Eq. (4).

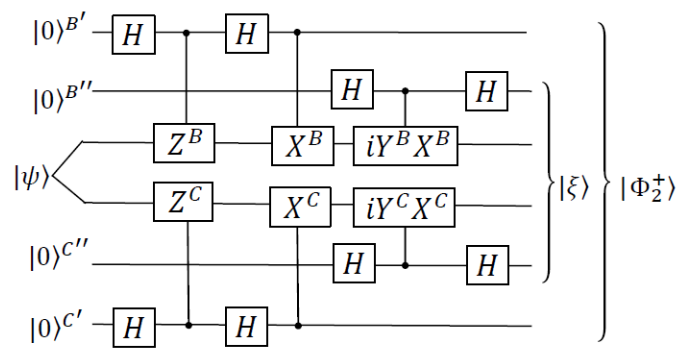

FIG. 4: The local isometry U is explicitly constructed to allow us to self-test the singlet state and Pauli operators.

It has been proven by Bowles *et al.* [6] that if the maximal violation $\mathfrak{B} = 6\sqrt{2}$ is observed, then there exists a local auxiliary state $|00\rangle \in [\mathcal{H}_{B'} \otimes \mathcal{H}_{B''}] \otimes [\mathcal{H}_{C'} \otimes \mathcal{H}_{C''}]$ ($|00\rangle$ is short for $|0000\rangle_{B'B''C'C''}$) and a local isometry U (see Fig. 4) such that

$$
\begin{aligned}
U[|\psi\rangle \otimes |00\rangle] &= |\xi\rangle \otimes |\Phi_2^+\rangle^{B'C'}, \\
U[X^C |\psi\rangle \otimes |00\rangle] &= |\xi\rangle \otimes \sigma_{\mathrm{x}}^{C'} |\Phi_2^+\rangle^{B'C'}, \\
U[Z^C |\psi\rangle \otimes |00\rangle] &= |\xi\rangle \otimes \sigma_{\mathrm{z}}^{C'} |\Phi_2^+\rangle^{B'C'}, \\
U[Y^C |\psi\rangle \otimes |00\rangle] &= \sigma_{\mathrm{z}}^{C''} |\xi\rangle \otimes \sigma_{\mathrm{y}}^{C'} |\Phi_2^+\rangle^{B'C'}, \qquad (9)
\end{aligned}
$$

where $|\xi\rangle$ is the junk state left in systems $[\mathcal{H}_B \otimes \mathcal{H}''_B] \otimes [\mathcal{H}_C \otimes \mathcal{H}''_C]$, in the form of

$$|\xi\rangle = |\xi_0\rangle^{BC} \otimes |00\rangle^{B''C''} + |\xi_1\rangle^{BC} \otimes |11\rangle^{B''C''} \quad (10)$$

with $\langle\xi_0|\xi_0\rangle + \langle\xi_1|\xi_1\rangle = 1$. It means that we can extract the exact information of the maximally entangled state of two-qubit $|\Phi_2^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and Charlie's three measurements

$$X^C = \sigma_{\mathrm{x}},\ \ Y^C = \pm\sigma_{\mathrm{y}}\ \ Z^C = \sigma_{\mathrm{z}} \quad (11)$$

Although there exists the sign problem of σ_y to be distinguished, it does not pose any constraint to verify entanglement [5] and EPR steering similarly.

Finally, noting that $\sigma_y^T = -\sigma_y$, the measurement set $\{\sigma_x, -\sigma_y, \sigma_z\}$ could be transformed from the set $\{\sigma_x, \sigma_y, \sigma_z\}$ on which is acted the transpose operation T, or vice verse. It is easy to verify that the state ρ_{AB} has a LHS model with one measurement set if and only if it does also for the other set because the partial operation preserves the nonlocal property of EPR steering. Thus, we obtain the DI steering test

$$W_{\mathrm{DI}} = \frac{1}{4}(3\,v - \sqrt{3}\,). \quad (12)$$

for Werner states (6), and

$$W_{\mathrm{DI}}^{\mathrm{noisy}} = \frac{1}{4}(3\,v - \sqrt{3}\,) - \sum_i \sqrt{1 - f_j}\,. \quad (13)$$

which allows for the imperfections of self-testing.

Conclusions

We have studied the DI verification of EPR steering and implemented an optical experiment to validate our DI protocol. First, we prove that all steerable states, including Bell local states, can be verified device-independently. Furthermore, we analyze its robustness against noise in the implementation process, such as imperfections of self-testing, and derive a suitable steering inequality as per Eq. (5) for the three-measurement setting case. Finally, we give a proof of principle experiment to successfully demonstrate our DI steering protocol. We believe that our results may pave the way for realistic implementations of secure quantum information processing tasks involving EPR steeirng or entanglement.

Finally, it is pointed out that there exist possible ways to improve the performance of our DI steering protocol and avoid the potential loopholes. For example, the delicate methods proposed in [15, 25, 43] may help to tolerate more worse transmission loss and measurement efficiency. Moreover, the resource efficient method used in [44] could improve the success probability of the partial BSM, and the self-testing could be more noise robust by adopting other techniques [10]. Finally, it is interesting to follow an alternate framework [45] to verify quantum steering.

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[1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).
[2] J. S. Bell, On the Einstein Podolsky Rosen paradox, Physics **1**, 195 (1964).
[3] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, Rev. Mod. Phys. **86**, 419 (2014).
[4] R. F. Werner, Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model, Phys. Rev. A **40**, 4277 (1989).
[5] J. Bowles, I. Šupić, D. Cavalcanti, and A. Acín, Device-Independent entanglement certification of all entangled states, Phys. Rev. Lett. **121**, 180503 (2018).
[6] J. Bowles, I. Šupić, D. Cavalcanti, and A. Acín, Self-testing of Pauli observables for device-independent entanglement certification, Phys. Rev. A **98**, 042336 (2018).
[7] F. Buscemi, All entangled quantum states are nonlocal, Phys. Rev. Lett. **108**, 200401 (2012).
[8] C. Branciard, D. Rosset, Y.-C. Liang, and N. Gisin, Measurement-Device-Independent entanglement witnesses for all entangled quantum states, Phys. Rev. Lett. **110**, 060405 (2013).
[9] D. Mayers, A. Yao, Self testing quantum apparatus, arXiv:quant-ph/0307205 (2004).
[10] I. Šupić and J. Bowles, Self-testing of quantum systems: a review, Quantum **4**, 337 (2020).
[11] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Steering, entanglement, nonlocality, and the Einstein-Podolsky-Rosen paradox, Phys. Rev. Lett. **98**, 140402 (2007).
[12] S. J. Jones, H. M. Wiseman, and A. C. Doherty, Entanglement, Einstein-Podolsky-Rosen correlations, Bell nonlocality, and steering, Phys. Rev. A **76**, 052116 (2007).
[13] M. T. Quintino, T. Vértesi, D. Cavalcanti, R. Augusiak, M. Demianowicz, A. Acín, and N. Brunner, Inequivalence of entanglement, steering, and Bell nonlocality for general measurements, Phys. Rev. A **92**, 032107 (2015).
[14] D. J. Saunders, S. J. Jones, H. M. Wiseman, and G. J. Pryde, Experimental EPR-steering using Bell-local states, Nature Phys. **6**, 845 (2010).
[15] A. J. Bennet, D. A. Evans, D. J. Saunders, C. Branciard, E. G. Cavalcanti, H. M. Wiseman, and G. J. Pryde, Arbitrarily loss-tolerant Einstein-Podolsky-Rosen steering allowing a demonstration over 1 km of optical fiber with no detection loophole, Phys. Rev. X **2**, 031003 (2012).
[16] D. H. Smith, G. Gillett, M. de Almeida, C. Branciard, A. Fedrizzi, T. J. Weinhold, A. Lita, B. Calkins, T. Gerrits, H. M. Wiseman, S. W. Nam, and A. G. White, Conclusive quantum steering with superconducting transition-edge sensors, Nat. Commun. **3**, 625 (2012).

[17] V. Händchen, T. Eberle, S. Steinlechner, A. Samblowski, T. Franz, R. F. Werner, and R. Schnabel, Observation of one-way Einstein–Podolsky–Rosen steering, Nat. Photonics **6**, 596 (2012).

[18] C. Branciard, E. G. Cavalcanti, S. P. Walborn, V. Scarani, and H. M. Wiseman, One-sided device-independent quantum key distribution: Security, feasibility, and the connection with steering, Phys. Rev. A **85**, 010301(R) (2012).

[19] P. Skrzypczyk and D. Cavalcanti, Maximal randomness generation from steering inequality violations using qudits, Phys. Rev. Lett. **120**, 260401 (2018).

[20] E. G. Cavalcanti, M. J. W. Hall, and H. M. Wiseman, Entanglement verification and steering when Alice and Bob cannot be trusted, Phys. Rev. A **87**, 032306 (2013).

[21] S. Kocsis, M. J. W. Hall, A. J. Bennet, D. J. Saunders, and G. J. Pryde, Experimental measurement-device-independent verification of quantum steering, Nat. Commun. **6** 5886 (2015).

[22] M. J. W. Hall, Trust-free verification of steering: why you can't cheat a quantum referee, in: Reality and Measurement in Algebraic Quantum Theory, ed. M. Ozawa et al., Springer Proceedings in Mathematics & Statistics, vol 261, ch. 7. Springer, Singapore. (2018), arXiv:1606.00196.

[23] Y. Guo, S. Cheng, X. Hu, B.-H. Liu, E.-M. Huang, Y.-F. Huang, C.-F. Li, G.-C. Guo, and E. G. Cavalcanti, Experimental Measurement-Device-Independent Quantum Steering and Randomness Generation Beyond Qubits, Phys. Rev. Lett. **123**, 170402 (2019).

[24] H.-Y. Ku, S.-L. Chen, H.-B. Chen, F. Nori, and Y.-N. Chen, Measurement-device-independent measure of steerability and witnesses for all steerable resources, arXiv:1807.08901 (2018).

[25] I. Jeon, and H. Jeong, Measurement-device-independent and arbitrarily loss-tolerant verification of quantum steering, Phys. Rev. A **99**, 012318 (2019).

[26] Y.-Y. Zhao, H.-Y. Ku, S.-L. Chen, F. Nori, Y.-N. Chen, G.-Y. Xiang, C.-F. Li, and G.-C. Guo, Experimental measurement-device-independent measure of quantum steering, npj Quantum Inf. **6**, 77 (2020).

[27] E. Schrödinger, Discussion of probability relations between separated systems. Proc. Cambridge. Philos. Soc. **31**, 555 (1935).

[28] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox, Phys. Rev. A **80**, 032112 (2009).

[29] V. Scarani, The device-independent outlook on quantum physics, arXiv:1303.3081

[30] T. H. Yang and M. Navascuès, Robust self-testing of unknown quantum systems into any entangled two-qubit states, Phys. Rev. A **87**, 050102(R) (2013).

[31] A. Coladangelo, K. T. Goh, V. Scarani, All pure bipartite entangled states can be self-tested, Nat. Commun. **8**, 15485 (2017).

[32] W.-H. Zhang, G. Chen, X.-X. Peng, X.-J. Ye, P. Yin, X.-Y. Xu, J.-S. Xu, C.-F. Li, and G.-C. Guo, Experimental realization of robust self-testing of Bell state measurements, Phys. Rev. Lett. **122**, 090402 (2019).

[33] W.-H. Zhang, G. Chen, P. Yin, X.-X. Peng, X.-M. Hu, Z.-B. Hou, Z.-Y. Zhou, S. Yu, X.-J. Ye, Z.-Q. Zhou, X.-Y. Xu, J.-S. Tang, J.-S. Xu, Y.-J. Han, B.-H. Liu, C.-F. Li, and G.-C. Guo, Experimental demonstration of robust self-testing for bipartite entangled states, npj Quantum Inf. **5**, 4 (2019).

[34] See Supplemental Material for details about the derivation of the DI steering inequality (3) for the framework for the DI verification, how to obtain the self-testing fidelity f_j, the nontrivial derivation of the noisy DI steering inequality (5), and the experimental details to implement our steering protocol,which also includes Ref. [35].

[35] C. Zhang, Y.-F. Huang, Z. Wang, B.-H Liu, C.-F. Li, and G.-C. Guo, Experimental Greenberger-Horne-Zeilinger-type six-photon quantum nonlocality, Phys. Rev. Lett. **115**, 260402 (2015).

[36] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, Phys. Rev. Lett. **24**, 549 (1970).

[37] A. Acín, S. Pironio, T. Vértesi, and P. Wittek, Optimal randomness certification from one entangled bit, Phys. Rev. A **93**, 040102 (2016).

[38] T. H. Yang, T. Vértesi, J.-D Bancal, V. Scarani, M. Navascués, Robust and versatile black-box certification of quantum devices, Phys. Rev. Lett. **113**, 040401 (2014).

[39] J.-D Bancal, M. Navascués, V. Scarani, T. Vértesi, and T. H. Yang, Physical characterization of quantum devices from nonlocal correlations, Phys. Rev. A **91**, 022115 (2015).

[40] M. Navascués, S. Pironio and A. Acín, A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations, New J. Phys., **10**, 073013 (2008).

[41] D. J. Saunders, A. J. Bennet, C. Branciard and G. J. Pryde, Experimental demonstration of nonbilocal quantum correlations, Sci. Adv. **3**, e1602743 (2017).

[42] T. Vértesi, More efficient Bell inequalities for Werner states, Phys. Rev. A **78**, 032112 (2008).

[43] A. Rutkowski, A. Buraczewski, P. Horodecki, and M. Stobińska, Quantum Steering Inequality with Tolerance for Measurement-Setting Errors: Experimentally Feasible Signature of Unbounded Violation, Phys. Rev. Lett. **118**, 020402 (2017).

[44] E. Verbanis, A. Martin, D. Rosset, C. C. W. Lim, R. T. Thew, and H. Zbinden, Resource-efficient measurement-device-independent entanglement witness, Phys. Rev. Lett. **116**, 190501 (2016).

[45] M. J. W. Hall and A. Rivas, Geometry of joint reality: device-independent steering and operational completeness, Phys. Rev. A **100**, 062105 (2019).

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Author contributions

Y.-Y Zhao and C. Zhang contributed equally to this work. Y.-Y. Zhao and C. Zhang designed and conducted the experiment with the help from Y. Guo, H. Y. Ku and S. L. Chen. S. Cheng and X. Li developed the theory. Y.-Y. Zhao and C. Zhang analysis the data with the help from B. Liu, G. Y. Xiang, C. F. Li and G. C. Guo. S. Cheng, and B. Liu supervised the whole project. All the authors jointly wrote the manuscript.

Additional information

The authors declare that they have no competing interests.

Figures

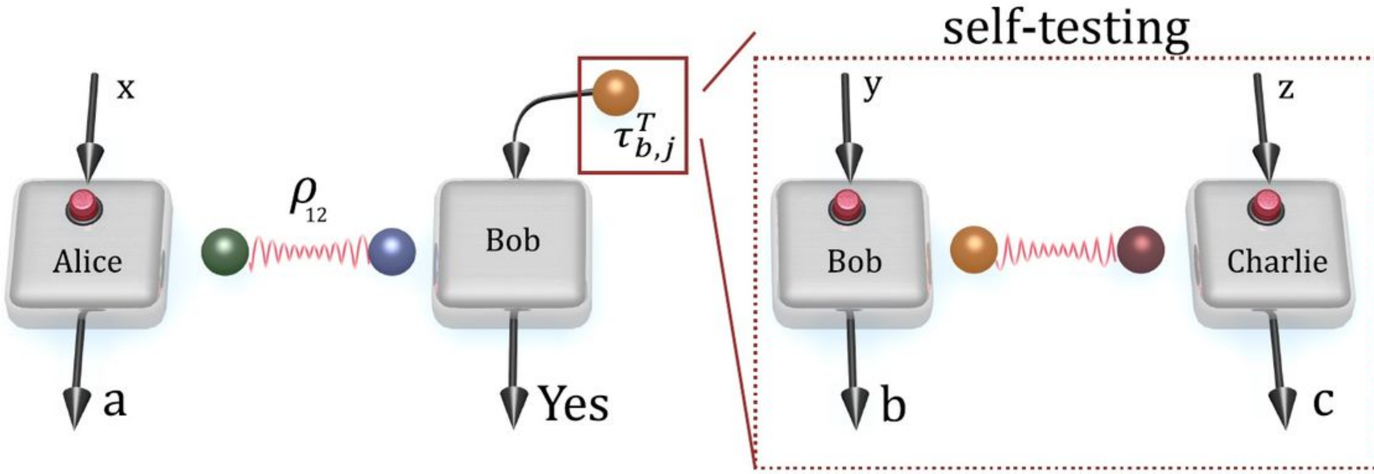

Figure 1

DI verification of EPR steering. (see Manuscript file for full figure caption)

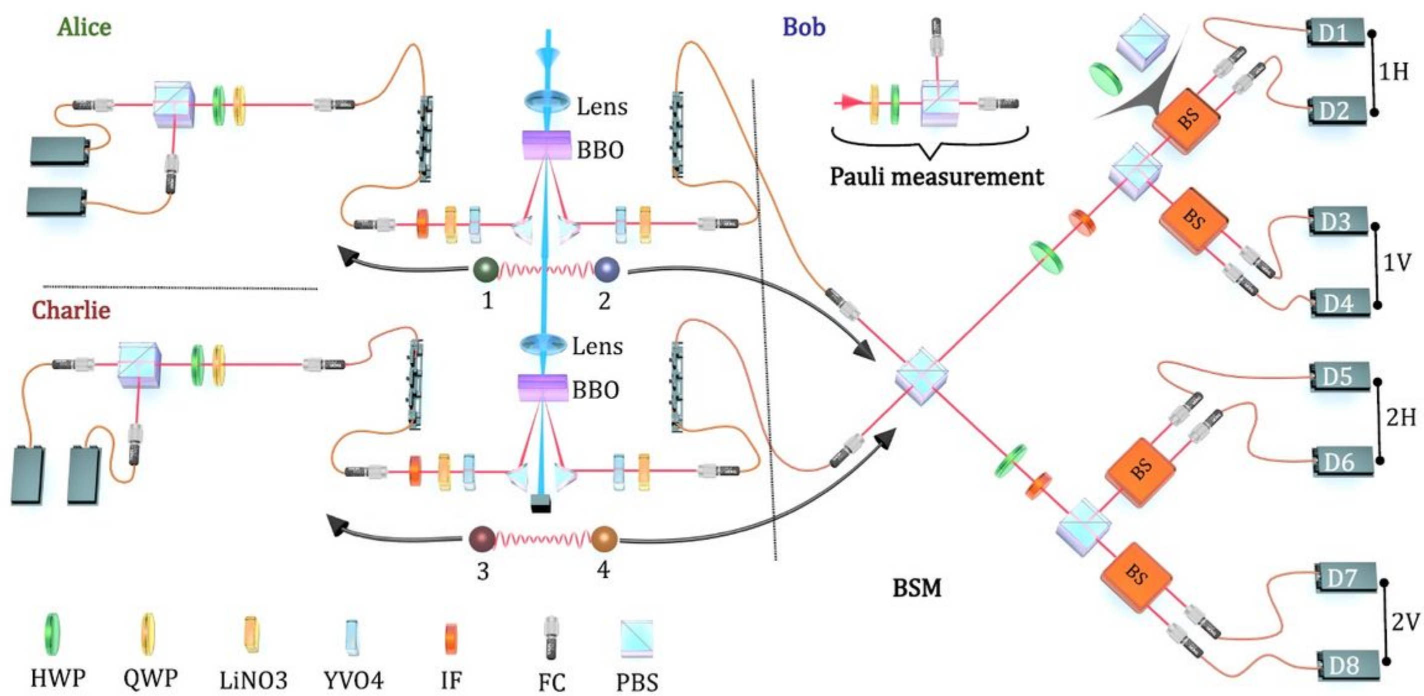

Figure 2

Experimental setup for the DI steering protocol. (see Manuscript file for full figure caption)

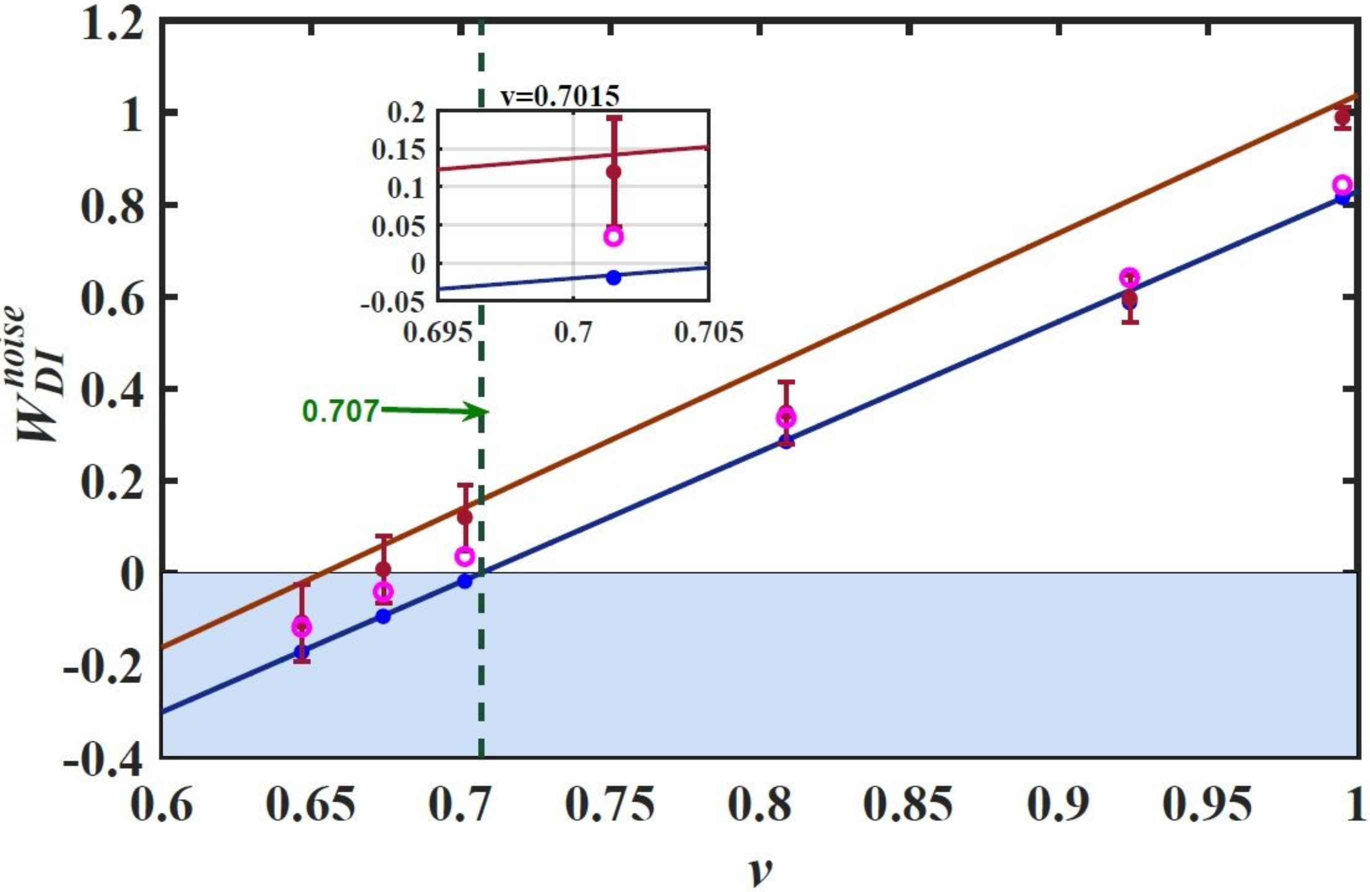

Figure 3

Experimental results for Werner states (see Manuscript file for full figure caption)

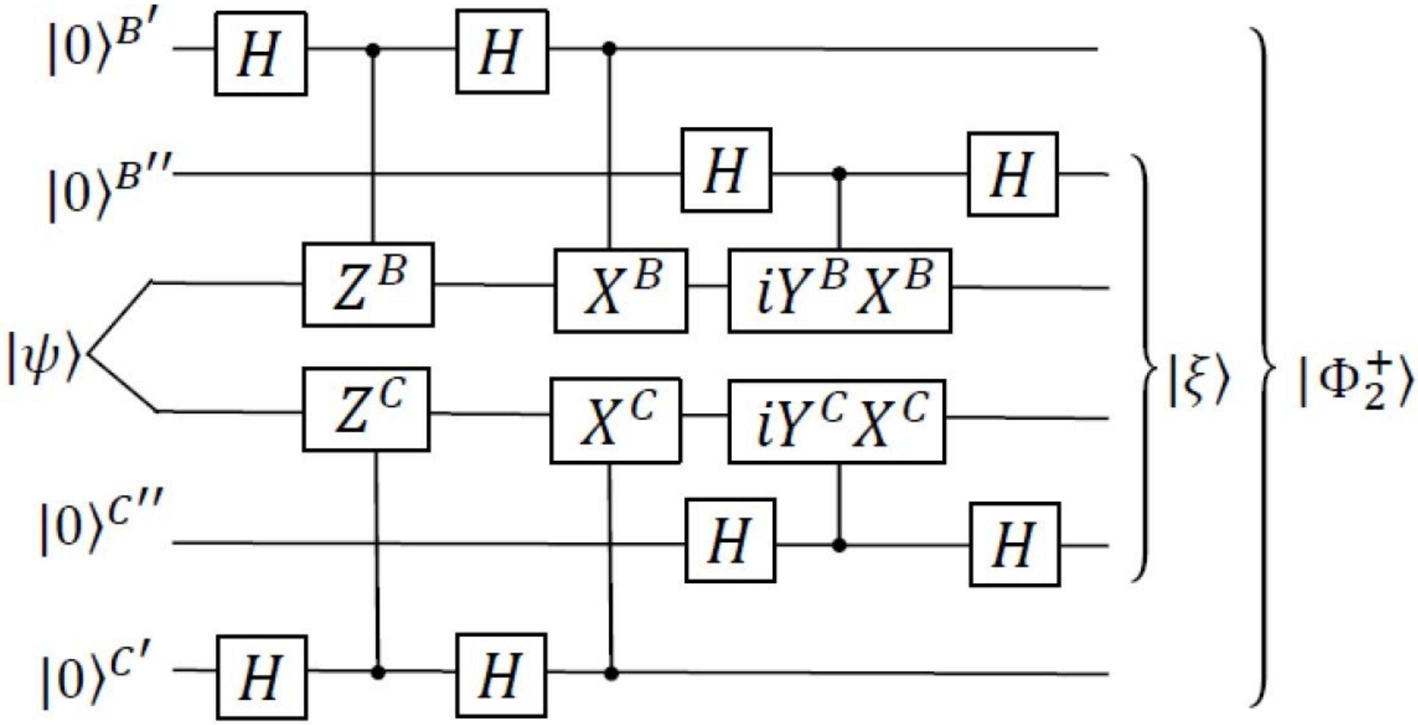

Figure 4

The local isometry U is explicitly constructed to allow us to self-test the singlet state and Pauli operators.

Supplementary Files

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