Received January 11, 2019, accepted January 17, 2019, date of publication February 18, 2019, date of current version February 27, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2896582

Device-to-Device Communications Underlaying an Uplink SCMA System

HYEON MIN KIM[1], HIEU V. NGUYEN[2], GIL-MO KANG[1], (Student Member, IEEE), YOAN SHIN[2], (Senior Member, IEEE), AND OH-SOON SHIN[1,2], (Member, IEEE)

[1]Department of ICMC Convergence Technology, Soongsil University, Seoul 06978, South Korea
[2]School of Electronic Engineering, Soongsil University, Seoul 06978, South Korea

Corresponding author: Oh-Soon Shin (osshin@ssu.ac.kr)

This work was supported by Institute for Information and Communications Technology Promotion (IITP) grant funded by the Korean Government (MSIT) (No. 2017-0-00724, Development of Beyond 5G Mobile Communication Technologies (Ultra-Reliable, Low-Latency, and Massive Connectivity) and Combined Access Technologies for Cellular-based Industrial Automation Systems).

ABSTRACT Device-to-device (D2D) communication has been a potential solution to improve spectral efficiency of cellular systems due to frequency resource sharing. This paper considers D2D communications underlaying an uplink cellular system enabling sparse code multiple access (SCMA) technology, where the base station (BS) can decode the signals of cellular users without mutual interference. The demands for high data rate as well as low latency in massive connectivity is the main challenging requirement in D2D communications, along with ensuring the quality of service (QoS) for the on-running cellular user equipment (CUEs). To tackle the problem, the BS is designed to first assigns the codewords in a codebook to CUEs based on the lower bound of the achievable CUE rates, so that the sum rate (SR) of CUEs is maximized. With the usage of mutual-interference suppression in the SCMA-enabled system, the BS then attempts to maximize the SR of D2Ds in a transmission block through a joint power and resource allocation subject to the QoS requirements for both CUEs and D2Ds. This task is formulated as a mixed-integer non-convex programming. We propose a low-complexity two-phase algorithm of joint heuristic and inner approximation method to efficiently solve the problem. The numerical results verify that the codebook assignment problem based on the lower bound of SR is easily solved, and the proposed algorithm to maximize the SR of D2Ds outperforms existing methods.

INDEX TERMS Device-to-device (D2D), heuristic search, inner approximation, power allocation, resource allocation, sparse code multiple access (SCMA), spectral efficiency.

I. INTRODUCTION

The explosive number of Internet of Things (IoT) devices and users with massive connectivity as well as the variety of applications, i.e., smart farms, smart factories, and autonomous vehicles have put pressure on future wireless communications [1]–[3]. In fact, the increase in IoT devices results in scarce radio resources, while the frequency reuse may cause dense interference among devices. In addition, the demand for high-speed data traffic is uninterruptedly growing day-by-day. As a result, it is necessary to develop new mobile communication technologies [4], [5].

In the last decade, the research on wireless communications has focused on a few perspectives, e.g., the use of small base stations (BSs) and the exploitation of various new frequencies. First, the miniaturization of communication networks could increase the data rate of many devices and distribute the network traffic load, due to a sufficient number of small cells, i.e., femtocells and picocells, which support the reuse of frequency resources [6]. However, the expensive cost for infrastructure and interference among small BSs is an obstacle to realize small cells. Second, the IEEE has supported the use of 5.8GHz frequency band as a solution for the near future to resolve the problem of saturation in 2.4GHz, on which Wi-Fi, Bluetooth and ZigBee operate [7]–[9]. Moreover, IEEE 802.11ac/ad with mmWave band is the forthcoming standard. However, Wi-Fi is difficult to support mobility, Bluetooth technology supports limited connection distance, and ZigBee provides low data rate. Meanwhile, the mmWave band suffers from strong attenuation when

The associate editor coordinating the review of this manuscript and approving it for publication was Mauro Fadda.

encountering pullbacks [10]. Therefore, substantial attention has recently turned into developing a new multiple access method.

The non-orthogonal multiple access (NOMA) technology, where the mutual interference is manageable and the non-orthogonal frequency resources are allocated to the users [11], has been proposed as a promising solution to efficiently utilize the limited frequency resources. Two main categories of NOMA, i.e., power domain NOMA (PD-NOMA) and code domain NOMA (CD-NOMA), have attracted a lot of attention [12], [13]. The PD-NOMA is more suitable for exploiting channel condition, while the CD-NOMA utilizes spreading gain for resource allocation [14]. In the CD-NOMA literature, many recent attempts have involved sparse code multiple access (SCMA), featuring the mixture of quadrature amplitude modulation (QAM) symbol mapping and spreading, which has been proposed to enhance the sum spectral efficiency (SE) [15]–[17]. With the SCMA technique, the transmitted information of a user mapped to a multidimensional complex codeword is decoded at the BS. Subsequently, SCMA allows improving spectral efficiency due to resource sharing among multiple users.

Device-to-device (D2D) communication, where two devices are allowed to directly communicate with each other, has recently attracted much interest, due to its widely prospective applications in next-generation wireless systems [18]–[21]. Many recent works have investigated resource sharing among D2Ds and cellular user equipment (CUEs), in which the same resources are allocated to D2Ds as well as CUEs. As an extension of D2D communications, a lot of vehicle-to-everything (V2X) categories, such as vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I), and vehicle-to-network (V2N), have been considered to improve traffic safety and SE [22], [23]. In Long-Term Evolution (LTE) and LTE-Advanced (LTE-A), the LTE-Vehicle (LTE-V) and LTE-Unlicensed (LTE-U) standards were developed in [24]–[26]. Although D2D communication is a promising candidate for IoT networks with numerous devices, the main challenge is the mutual interference among CUEs and D2Ds. Therefore, proper control of interference among CUEs and D2Ds is required to guarantee the QoS of both CUEs and D2Ds. Since interference management between D2Ds and CUEs in downlink transmission is difficult, most of the existing studies on D2D communication have focused on resource sharing in uplink transmission (spectrum in FDD, sub-frame in TDD). However, bandwidth bottleneck with dense devices is still a challenging problem [27], [28].

A. RELATED WORKS

In earlier works, D2D communications have been studied in the perspective of resource allocation [26], [29]–[34]. Nguyen *et al.* [30], [31] have proposed methods based on the bipartite graph matching problem for resource sharing among D2Ds and CUEs, to reduce the outage probability of D2D links. Zhong *et al.* [32] have considered sum rate (SR) maximization by using game-theoretic resource allocation. Reference [33] has considered the SR of D2Ds subject to rate requirements for CUEs. However, when the numbers of CUEs and D2Ds increase, the strong interference between D2Ds and CUEs degrades the system performance in terms of both the complexity and data rate. In [34], a weighted SR maximization has been proposed with consideration of the proportional fairness among D2D links. Wei *et al.* [26] considered the SR maximization for all CUEs and V2Xs, where power allocation and resource assignment are handled in separate steps. Although the algorithm of maximum weighted matching on bipartite graph provided low complexity, V2Xs were matched to resources by one-by-one assignments, which cannot be extended to an SCMA scheme with several orthogonal frequency division multiple access (OFDMA) tones per SCMA layer.

In very recent works, SCMA-assisted D2D communications, where D2Ds can share several OFDMA tones with CUEs served by an SCMA-enabled system, have been actively studied to further improve the SE [35]–[37]. Such model, also known as a hybrid network, can take advantage of SCMA as follows: *(i)* The mutual interference between CUEs can be completely eliminated; *(ii)* It allows to exploit multiple resources for D2Ds, i.e., codeword with several resources assigned. Accordingly, those relevant works have attracted attention to a further development of both CUEs and D2Ds. In [35], a graph-based method was proposed to improve the total rate of D2Ds and CUEs in the whole network. Nevertheless, this approach cannot simultaneously manage the power control, since the matching criteria require the transmit power to be predefined. To further enhance SR, the hybrid network was investigated in [36] and [37], with both resource and power allocation taken into account. However, these designs primarily focused on the resource assignment at first, while the transmit power for D2Ds was uniformly allocated in [36] or derived under the condition of optimized resource assignment in the prior stage [37]. Therefore, a joint optimization for both resource and power allocation is still open, and it will be a promising tool to enhance the system performance.

B. MOTIVATION AND CONTRIBUTION

Even when the preceding researches provide a continuous renovation for the evolution path towards next-generation network, there still remain some critical challenges in D2D communications underlaying an SCMA system. In particular, the network throughput, i.e., the summation of CUE and D2D rates, is usually considered as the objective function, which causes complicated computation for resource allocation. Moreover, various levels of QoS requirements should be satisfied for cellular links. In addition, a joint optimization of resource and power allocation for D2D communications underlaying an SCMA-enabled cellular network is not easy, since the problem belongs to the class of mixed-integer non-convex programs. Despite many challenges in D2D communications in the uplink SCMA system, its large benefit motivates us to further investigate the

SR enhancement for D2Ds subject to QoS constraints of CUEs.

In this paper, we exploit the model of resource sharing between D2Ds and CUEs, where the D2D pairs are allowed to share the same resources with CUEs in an uplink SCMA-enabled system. We first propose a new scheme to improve the SR of network, comprised of two stages. In the first stage, CUEs in different channel conditions are assigned to the codebook so that their SR is maximized. Then, the second stage is designed to maximize the SR of D2Ds under the QoS constraints for both CUEs and D2Ds. Accordingly, the D2D assignments are dynamically optimized along with the resource allocation for CUEs. Moreover, the power control is jointly optimized with resource allocation. This model offers a few advantages as follows. First, the SR maximization for D2Ds can be simply executed without the resource replacement for on-running cellular systems. The second payoff is to conveniently satisfy the QoS requirements of CUEs due to no interference among CUEs in the SCMA system, and to allow a joint optimization of CUE and D2D power allocation as well as D2D resource assignment. Another benefit of this scheme is that D2Ds can be assigned to several OFDMA tones, which assists D2Ds in exploiting SCMA codebook, when the number of assigned resources is equal to the number of nonzero entries in the codeword. Finally, the simple objective function allows D2Ds to be easily released from or admitted to the regarding network, which better supports the low-complexity methods as well as multiple concurrent users' requests.

The benefit of combining D2D communications with SCMA technique is paralleled with the difficulties in solving the design problem. Based on the proposed scheme, we first introduce an SR maximization problem for CUEs as the codebook assignments, which is a nonlinear assignment problem. To resolve this issue, we derive a lower bound for the SR of CUEs which is a linear assignment problem. Therefore, the bipartite-graph matching and greedy-based methods are efficiently applied to solve the problem. Meanwhile, an SR maximization problem for D2Ds is formulated through a joint CUE and D2D power allocation and resource assignments for D2Ds, subject to QoS constraints of CUEs and D2Ds with the predefined thresholds of target signal-to-interference-plus-noise ratio (SINR). This problem belongs to a class of mixed-integer non-convex programs, which is hard to solve. To address this problem, we propose a two-phase algorithm of a joint heuristic method and first-order inner approximation, where the association between transmit power and resource assignment variables is efficiently exploited. In the first phase, a heuristic search (HS)-based program expressed as an ℓ_1-norm problem under the cardinality constraints is derived to calculate the frequency resource assignments for D2Ds. Then, the power allocation for both CUEs and D2Ds is computed in the second phase by using the first-order inner approximation. To further relieve the complexity of the two-phase algorithm, we prove that the optimal solution obtained in the former phase can be simply applied, so that it is feasible for the program in the later phase. In addition, we use a binary search to efficiently handle the cardinality constraints. To further improve the SR of D2Ds, a greedy search (GS) based algorithm is proposed as an alternative to the HS-based program in the first phase. Numerical results demonstrate that the proposed scheme provides good performance and low complexity for both SCMA codebook assignment and D2D SR maximization.

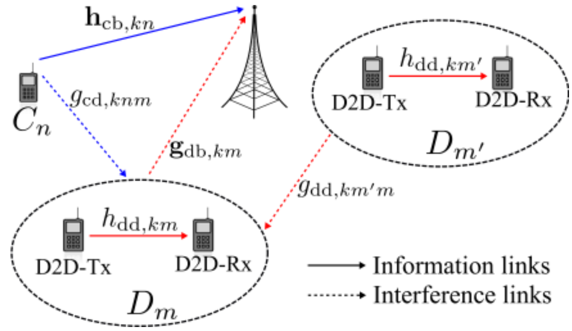


FIGURE 1. A multiuser system model at the k-th resource with M D2Ds underlaying a cellular network, where the BS serves N UL users.

Notation: diag($\mathbf{x}$) is the diagonal matrix, where the elements on its main diagonal are formed by vector $\mathbf{x}$. $\|\cdot\|_1$, $\|\cdot\|$, and $|\cdot|$ denote the ℓ_1-norm, Euclidean norm of a vector, and absolute value of a complex scalar, respectively. $x \sim \mathcal{CN}(\mu, \sigma^2)$ indicates that the variable x follows the complex normal distribution with mean μ and variance σ^2.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. SIGNAL PROCESSING MODEL

We consider the D2D communication system shown in Fig. 1, with M pairs of D2Ds underlaying a cellular network. Each D2D pair consists of a transmit device (D2D-Tx) and receive device (D2D-Rx) in close proximity. In the cellular network, the BS equipped with N_{r} receive antennas serves N UL CUEs. The m-th D2D, with $m \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$, and n-th CUE, with $n \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$, are denoted by D_m and C_n, respectively. To decode the signals from CUEs, the BS applies SCMA technique, with the K-dimensional complex codewords of the codebook being sparse vectors with L ($L < K$) nonzero entries. Herein, the number of CUEs is assumed not to exceed the maximum number of codewords in the codebook, denoted by J, i.e., $N \leq J = \binom{K}{L}$, and then the number of codewords multiplexed per resource is $d_f = \binom{K-1}{L-1}$. Thus, one CUE is assigned to one codeword in the codebook, and each codeword is assigned to one CUE at most. For D2D communication, a D2D pair is assigned to some of the K frequency resources.

We consider the signal model under the k-th frequency resource, and all channels are assumed to be unchanged during a short time of transmission block. The signal-to-interference-plus-noise ratio (SINR) at the BS for decoding the information of the n-th CUE can be expressed as

$$\gamma^{\mathrm{c}}_{k,n}(\mathbf{p}_{\mathrm{c}}, \mathbf{p}_{\mathrm{d}}) = \frac{p^2_{\mathrm{c},kn}\|\mathbf{h}_{\mathrm{cb},kn}\|^2}{\sum_{m=1}^{M} p^2_{\mathrm{d},km}\|\mathbf{g}_{\mathrm{db},km}\|^2 + \sigma^2_{kn}}, \tag{1}$$

where $p^2_{\mathrm{c},kn}$ and $p^2_{\mathrm{d},km}$ are the transmit power of the n-th CUE and the m-th D2D-Tx, respectively. The channel vectors $\mathbf{h}_{\mathrm{cb},kn} \in \mathbb{C}^{N_r\times 1}$ and $\mathbf{g}_{\mathrm{db},km} \in \mathbb{C}^{N_r\times 1}$ are associated to the links from the n-th CUE and from the m-th D2D-Tx to the BS, respectively. Let $\mathbf{p}_\mathrm{c} \triangleq [p_{\mathrm{c},kn}]_{k\in\mathcal{K},n\in\mathcal{N}}$ and $\mathbf{p}_\mathrm{d} \triangleq [p_{\mathrm{d},km}]_{k\in\mathcal{K},m\in\mathcal{M}}$, while σ^2_{kn} denotes the power of the additive white Gaussian noise (AWGN) at the BS. Herein, the interference from other CUEs is completely removed thanks to the SCMA decoder at the BS. The SINR at the m-th D2D-Rx is given as

$$\gamma^{\mathrm{d}}_{k,m}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}) = \frac{p^2_{\mathrm{d},km}|h_{\mathrm{dd},km}|^2}{\phi^{\mathrm{d}}_{km}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d})}, \tag{2}$$

where $h_{\mathrm{dd},km}$ is the channel response between the m-th D2D-Tx and D2D-Rx. The interference-plus-noise is given as $\phi^{\mathrm{d}}_{km}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}) = \sum_{n=1}^{N}\beta_{kn}p^2_{\mathrm{c},kn}|g_{\mathrm{cd},knm}|^2 + \sum_{m'=1,m'\neq m}^{M} p^2_{\mathrm{d},km'}|g_{\mathrm{dd},km'm}|^2 + \sigma^2_{km}$, where the predefined constant $\beta_{kn} \in \{0,1\}$ indicates whether the k-th element of the codeword is assigned to the n-th CUE or not. It can be seen that the interference components in (2) contain the given assignment parameters β_{kn}, which are not in (1). Though $p_{\mathrm{c},kn}$ can be dynamically optimized, its aggregation with β_{kn} makes it faster to find $p_{\mathrm{c},kn}$ since the binary values of β_{kn} are aware in the cellular network. Meanwhile, the similar aggregation in (1) generates a complicated formulation, since the assignment between the m-th D2D and the k-th resource is undetermined. $g_{\mathrm{cd},knm}$ and $g_{\mathrm{dd},km'm}$ denote the channel responses from the n-th CUE and from the m'-th D2D-Tx ($m' \in \mathcal{M}$) to the m-th D2D-Rx, respectively. σ^2_{km} denotes the power of AWGN at the m-th D2D-Rx.

B. PROBLEM FORMULATION

We consider the SR maximization in two separate stages. The first stage is to maximize the SR of CUEs through an assignment problem without D2D pairs, where N CUEs are assigned to N codewords in the codebook. The second stage aims to maximize the SR of D2D pairs subject to the target SINRs for CUEs in the cellular system. In fact, the SR for all CUEs and D2Ds might be inconvenient and impractical. This is because the BS continuously serves CUEs while D2D communication underlaying cellular operation temporarily occurs between two devices in the proximity condition. Therefore, the SR optimization problems for CUEs and D2Ds are usually separate topics in the literature.

With SCMA technique, the BS can decode the information of all CUEs without mutual interference. This make the SR of CUEs depend only on the channels from CUEs to the BS, which are different from one another due to the CUEs' positions and occupied resources. Therefore, we can take advantage of the best assignment between codebook and CUEs, no matter what the power allocation for CUEs is. If we set the transmit power of CUEs to the maximum of the power budget, then the achievable rate of the k-th sub-carrier for the multiple access channel (MAC) is given as in [17] and [38], i.e.,

$$R^{\mathrm{c}}_k(\boldsymbol{\beta}) = \ln\Big(1+\frac{1}{L}\sum_{n=1}^{N}\beta_{kn}\gamma^{\mathrm{c}}_{k,n}(\mathbf{p}^{\max}_\mathrm{c},\mathbf{0})\Big) \quad (\text{nats/s/Hz}), \tag{3}$$

where $\mathbf{p}^{\max}_\mathrm{c}$ is the transmit power budget for CUEs, and $\boldsymbol{\beta} \triangleq [\beta_{kn}]_{k\in\mathcal{K},n\in\mathcal{N}}$. Let $\tilde{\boldsymbol{\beta}} \in \{0,1\}^{J\times N}$ be the assignment matrix, whose entry is $\tilde{\beta}_{jn} = 1$ ($j \in \mathcal{J} = \{0,1,\ldots,J\}$) if the n-th CUE is assigned to the j-th codeword, and $\tilde{\beta}_{jn} = 0$, otherwise. It is clear that $\boldsymbol{\beta}$ is a function of $\tilde{\boldsymbol{\beta}}$, i.e., $\boldsymbol{\beta} = \mathbf{J}_\mathrm{c}\tilde{\boldsymbol{\beta}}$ where the matrix $\mathbf{J}_\mathrm{c} \in \{0,1\}^{K\times J}$ stands for the codebook. Hence, the codebook assignment problem can be expressed as

$$\begin{aligned}
&\underset{\tilde{\boldsymbol{\beta}}}{\text{maximize}}\ \sum_{k=1}^{K} R^{\mathrm{c}}_k(\boldsymbol{\beta}) && (4a)\\
&\text{subject to}\ \tilde{\beta}_{jn} \in \{0,1\},\quad j\in\mathcal{J},\ n\in\mathcal{N}, && (4b)\\
&\qquad \sum_{n=1}^{N}\tilde{\beta}_{jn} \le 1,\quad j \in \mathcal{J}, && (4c)\\
&\qquad \sum_{j=1}^{J}\tilde{\beta}_{jn} = 1,\ n\in\mathcal{N}. && (4d)
\end{aligned}$$

It can be seen that the cost function $R^{\mathrm{c}}_k(\boldsymbol{\beta})$ in (4) corresponds to the total rate of all CUEs at the k-th resource. Meanwhile, the feasible region with respect to the assignment variables $\tilde{\beta}_{jn}$ represents a set of the matchings between the j-th codeword and the n-th CUE. This implies that $R^{\mathrm{c}}_k(\boldsymbol{\beta})$ is determined via the row of $\boldsymbol{\beta}$, only after collecting all assignment variables of $\tilde{\beta}_{jn}$. In other words, the problem (4) has an objective function with respect to $\boldsymbol{\beta}$ row-by-row, while $\boldsymbol{\beta}$ is derived from the permutations of the columns in $\mathbf{J}_\mathrm{c}$ through assignment variable $\tilde{\boldsymbol{\beta}}$. This is known as a non-linear assignment problem. In the literature, the total number of cases of $\tilde{\boldsymbol{\beta}}$ is given as $\binom{J}{N}$, and thus, the global optimization in (4) is NP-hard.

In the second stage, we aim to maximize the SR of D2D pairs under the power control, the target SINRs of CUEs and D2Ds, and D2D assignment constraints. From (2), the achievable rate for the m-th D2D using the k-th resource is given as

$$R^{\mathrm{d}}_{k,m} = \ln\big(1+\gamma^{\mathrm{d}}_{k,m}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d})\big) \quad (\text{nats/s/Hz}). \tag{5}$$

Then, the SR optimization problem can be formulated as

$$\begin{aligned}
&\underset{\boldsymbol{\alpha},\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}}{\text{maximize}}\ \sum_{k=1}^{K}\sum_{m=1}^{M}\alpha_{km}R^{\mathrm{d}}_{k,m}, && (6a)\\
&\text{subject to}\ p^2_{\mathrm{d},km} \le P^{\max}_{\mathrm{d},km},\quad k\in\mathcal{K},\ m\in\mathcal{M}, && (6b)\\
&\qquad p_{\mathrm{d},km} \ge 0,\quad k\in\mathcal{K},\ m\in\mathcal{M}, && (6c)\\
&\qquad p^2_{\mathrm{c},kn} \le P^{\max}_{\mathrm{c},kn},\quad k\in\mathcal{K},\ n\in\mathcal{N}, && (6d)\\
&\qquad p_{\mathrm{c},kn} \ge 0,\quad k\in\mathcal{K},\ n\in\mathcal{N}, && (6e)\\
&\qquad \|\boldsymbol{\alpha}_m\|_1 \le S,\quad m\in\mathcal{M}, && (6f)\\
&\qquad \alpha_{km} \in \{0,1\},\quad k\in\mathcal{K},\ m\in\mathcal{M}, && (6g)\\
&\qquad \gamma^{\mathrm{d}}_{k,m}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}) \ge \alpha_{km}\bar{\gamma}^{\mathrm{d}}_{k,m},\quad k\in\mathcal{K},\ m\in\mathcal{M}, && (6h)\\
&\qquad \gamma^{\mathrm{c}}_{k,n}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}) \ge \beta_{kn}\bar{\gamma}^{\mathrm{c}}_{k,n},\quad k\in\mathcal{K},\ n\in\mathcal{N}, && (6i)
\end{aligned}$$

where $P^{\max}_{\mathrm{d},km}$ and $P^{\max}_{\mathrm{c},kn}$ are the transmit power budgets of the m-th D2D and the n-th CUE for the k-th resource, respectively. The variable $\alpha_{km} \in \{0, 1\}$ indicates whether the m-th D2D occupies the k-th resource ($\alpha_{km} = 1$) or not ($\alpha_{km} = 0$). We define $\boldsymbol{\alpha}_m \triangleq [\alpha_{km}]_{k\in\mathcal{K}}$, $\boldsymbol{\alpha} \triangleq \left[\boldsymbol{\alpha}_m\right]_{m\in\mathcal{M}}$, while S ($S \le K$) denotes the maximum number of resources assigned to each D2D. $\bar{\gamma}^{\mathrm{d}}_{k,m}$ and $\bar{\gamma}^{\mathrm{c}}_{k,n}$ stand for the per-resource target SINR for the m-th D2D and the n-th CUE, respectively. Note that the right-hand sides of constraints (6h) and (6i) are equal to zero if α_{km} and β_{kn} are inactive, respectively. Since constraints (6f) and (6g) contain binary variables and the objective function is non-concave, (6) is a mixed-integer non-convex problem.

III. PROPOSED ALGORITHM FOR CODEBOOK ASSIGNMENT PROBLEM

It is clear that (4) is an integer programing problem, and the composition function $R^{\mathrm{c}}_k(\boldsymbol{\beta})$ in (4a) is the natural logarithm of the summation of all users' SINR, corresponding to the entries on the k-th row of $\boldsymbol{\beta}$ as well as the k-th row of codebook $\mathbf{J}_{\mathrm{c}}$. Meanwhile, the assignment variable $\tilde{\boldsymbol{\beta}}$ indicates that one CUE is matched to a codebook, represented as a column of $\mathbf{J}_{\mathrm{c}}$. With brute-force search, it takes $\mathcal{O}(J!)$ which might be infeasible in real communication systems. In this section, we propose a method to efficiently solve the problem (4) with the polynomial complexity. We first consider the following lemma.

Lemma 1: By using Jensen's inequality, the lower bound of $R^{\mathrm{c}}_k(\boldsymbol{\beta})$ is expressed as

$$R^{\mathrm{c}}_k(\boldsymbol{\beta}) \ge \ln\frac{N}{L} + \frac{1}{N}\sum_{n=1}^{N}\ln\Big(\frac{L}{N} + \beta_{kn}\gamma^{\mathrm{c}}_{k,n}(\mathbf{p}^{\max}_{\mathrm{c}}, \mathbf{0})\Big). \quad (7)$$

■

Proof: See Appendix A. □

By using Lemma 1, the objective function (4a) is lower bounded as

$$\sum_{k=1}^{K} R^{\mathrm{c}}_k(\boldsymbol{\beta}) \ge \frac{1}{N}\sum_{k=1}^{K}\sum_{n=1}^{N}\ln\Big(\frac{L}{N} + \beta_{kn}\gamma^{\mathrm{c}}_{k,n}(\mathbf{p}^{\max}_{\mathrm{c}}, \mathbf{0})\Big) + \sum_{k=1}^{K}\ln\frac{N}{L}. \quad (8)$$

Based on the concavity of the logarithm function, the maximization for the right-hand side of (8) provides at least a local maximum for the left-hand side. Since $\frac{1}{N}$ and $\ln\frac{N}{L}$ are constant, we can omit them in the optimization problem without altering the optimal solution. Let $\boldsymbol{\beta}_n$ be the n-th column of $\boldsymbol{\beta}$, then the objective function of (4) is lower bounded by that of the following problem.

$$\underset{\tilde{\boldsymbol{\beta}}}{\text{maximize}} \ \sum_{n=1}^{N}\tilde{R}^{\mathrm{c}}_{n,\Sigma}(\boldsymbol{\beta}_n) \quad (9a)$$

$$\text{subject to } \tilde{\beta}_{jn} \in \{0, 1\}, \quad j \in \mathcal{J},\ n \in \mathcal{N}, \quad (9b)$$

$$\sum_{n=1}^{N}\tilde{\beta}_{jn} \le 1, \quad j \in \mathcal{J}, \quad (9c)$$

$$\sum_{j=1}^{J}\tilde{\beta}_{jn} = 1, \quad n \in \mathcal{N}, \quad (9d)$$

where $\tilde{R}^{\mathrm{c}}_{n,\Sigma}(\boldsymbol{\beta}_n) \triangleq \sum_{k=1}^{K}\ln\big(\frac{L}{N}+\beta_{kn}\gamma^{\mathrm{c}}_{k,n}(\mathbf{p}^{\max}_{\mathrm{c}}, \mathbf{0})\big)$ represents the per-user rate for the n-th CUE. Differently from (4), the cost function $\tilde{R}^{\mathrm{c}}_{n,\Sigma}(\boldsymbol{\beta}_n)$ in (9) corresponding to the rate of the n-th user is independently determined, no matter what the other users' assignments are chosen, and thus, we get a cost matrix, $\mathbf{C} \in \mathbb{R}^{J\times N}$, of which each entry is computed as

$$C_{jn} \triangleq \tilde{R}^{\mathrm{c}}_{n,\Sigma}(\mathbf{J}_{\mathrm{c}}\bar{\boldsymbol{\beta}}_j), \quad (10)$$

with $\bar{\boldsymbol{\beta}}_j = [\underbrace{0\ldots0}_{j-1}\ 1\ \underbrace{0\ldots0}_{J-j}]^T$ corresponding to $\tilde{\beta}_{jn} = 1$. Here, C_{jn} denotes the total rate of the n-th user to which the j-th codeword is assigned, and it can be computed in advance. It is obvious that if $\tilde{\beta}_{jn} = 1$, $\tilde{\beta}_{j'n} = 0$, $\forall j' \neq j$, due to constraints (9d). Therefore, $\tilde{R}^{\mathrm{c}}_{n,\Sigma}(\boldsymbol{\beta}_n)$ in (9a) can be expressed as $\sum_{j=1}^{J} C_{jn}\tilde{\beta}_{jn}$, leading to a standard linear program. Note that (4) and (9) have the same feasible region, and thus an arbitrary point is either feasible or infeasible to both problems. Accordingly, we can efficiently solve the linear problem (9) instead of the non-linear problem (4).

To solve (9), we consider the following problem. We define $\mathcal{G} \triangleq (\mathcal{J}, \mathcal{N}; \mathcal{E})$ to be a complete bipartite graph, in which $\mathcal{J}$ and $\mathcal{N}$ stand for two vertex sets, representing the codewords of the codebook and CUEs, respectively. $\mathcal{E}$ denotes the set of all edges with the weights given as the matrix $\mathbf{C}$. We consider a maximum weighted bipartite matching (MWBM) problem, where a matching $\mathcal{M} \subseteq \mathcal{E}$ needs to be found such that

1) The size of $\mathcal{M}$ is the largest.
2) $\mathcal{M}$ contains the edges without common vertices.
3) The sum of weights of the edges in $\mathcal{M}$ is a maximal value.

Let $e_{jn} \in \mathcal{E}$ be an assignment from the j-th codeword to the n-th CUE. Consider that $e_{jn} \in \mathcal{M}$ iff $\tilde{\beta}_{jn} = 1$ and $e_{jn} \notin \mathcal{M}$ iff $\tilde{\beta}_{jn} = 0$, as constraints (9b). In this regard, the first two conditions of the MWBM problem are equivalent to the constraints (9c) and (9d), while the last condition corresponds to the objective function (9a). Therefore, the problem (9) is considered as the MWBM problem. The Kuhn-Munkres algorithm is applied to efficiently find the optimal solution for this problem [39], which is summarized in Algorithm 1.

Remark 1: We can also apply an algorithm based on GS to solve problem (9), in which all codewords are successively matched to the first CUE, and thus the first CUE is simply assigned to the codeword that provides the best rate. Then, the codeword assigned to the second CUE is selected among the codewords in the rest of codebook, such that the rate of the second CUE is maximized, and so on. Although the GS method does not devise an optimal solution for problem (9) as compared to the MWBM method, it provides a suboptimal solution with a lower complexity computation. In particular, it takes $\mathcal{O}\big(N\binom{K}{L}\big)$, as compared to $\mathcal{O}\big(N^2\binom{K}{L}\big)$ in the MWBM problem. Note that the GS method cannot be directly applied to the original problem (4), since this scheme is to maximize

Algorithm 1 Proposed Algorithm Based on MWBM for the Codebook Assignment Problem (4)

1: Compute the cost matrix $\mathbf{C}$ as in (10).
2: Let $\mathcal{J}$ and $\mathcal{N}$ be the sets of codewords (columns in the codebook) and CUEs, respectively. Let $\mathcal{E}$ be the set of edges with the weights given by $\mathbf{C}$.
3: Establish the MWBM problem on a graph $\mathcal{G} = (\mathcal{J}, \mathcal{N}; \mathcal{E})$ as in (9).
4: Apply Kuhn-Munkres algorithm to find the optimal solution $\tilde{\boldsymbol{\beta}}^*$ for (9), which provides the lower bound of SR given by (9a).
5: The effective SR is obtained by applying $\tilde{\boldsymbol{\beta}}^*$ to the objective function (4a) as $\boldsymbol{\beta} = \mathbf{J}_{\mathrm{c}}\tilde{\boldsymbol{\beta}}^*$.

the per-user rate by selecting a vacant codeword. Meanwhile, the per-resource rate in (4a) is obtained only when all CUEs are assigned to codewords in priority.

IV. PROPOSED ALGORITHM FOR D2D SUM RATE MAXIMIZATION PROBLEM

A. TWO-PHASE ITERATIVE ALGORITHM BASED ON HEURISTIC SEARCH

It can be seen that (6) is a mixed-integer non-convex problem. This is because the objective function and constraints (6h)-(6i) are non-convex, while (6f) and (6g) are the integer constraints. In the rest of the paper, we introduce the approach of a joint heuristic algorithm and an inner approximation method to efficiently solve the problem.

Let us handle $R^{\mathrm{d}}_{k,m}$ in the objective function. We consider the logarithm function as $g(x) = \ln(1 + 1/x), x > 0$. As in [40], $g(x)$ is convex for $x > 0$, since its Hessian is a positive-define matrix. Then, the concave minorant of $g(x)$ is given as

$$\begin{aligned} g(x) &\geq g(\bar{x}) + g'(x)(x - \bar{x}) \\ &= \ln\left(1 + \frac{1}{\bar{x}}\right) + \frac{1}{\bar{x}+1} - \frac{x}{\bar{x}^2 + \bar{x}}. \end{aligned} \tag{11}$$

By applying (11) to (5), we obtain the concave minorant of $R^{\mathrm{d}}_{k,m}$ at the $(\kappa+1)$-th iteration as

$$R^{\mathrm{d}}_{k,m} \geq \lambda^{(\kappa)}_{k,m} - \xi^{(\kappa)}_{k,m} \frac{\phi^{\mathrm{d}}_{km}(\mathbf{p}_{\mathrm{c}}, \mathbf{p}_{\mathrm{d}})}{2p^{(\kappa)}_{\mathrm{d},km} p_{\mathrm{d},km} - (p^{(\kappa)}_{\mathrm{d},km})^2} := \tilde{R}^{\mathrm{d},(\kappa)}_{k,m}, \tag{12}$$

where

$$\lambda^{(\kappa)}_{k,m} \triangleq \ln\left(1 + \gamma^{\mathrm{d},(\kappa)}_{k,m}\right) + \frac{\gamma^{\mathrm{d},(\kappa)}_{k,m}}{\gamma^{\mathrm{d},(\kappa)}_{k,m} + 1}, \tag{13}$$

$$\xi^{(\kappa)}_{k,m} \triangleq \frac{(\gamma^{\mathrm{d},(\kappa)}_{k,m})^2}{(\gamma^{\mathrm{d},(\kappa)}_{k,m} + 1)|h_{\mathrm{dd},km}|^2}, \tag{14}$$

with $\gamma^{\mathrm{d},(\kappa)}_{k,m} \triangleq \gamma^{\mathrm{d}}_{k,m}(\mathbf{p}^{(\kappa)}_{\mathrm{c}}, \mathbf{p}^{(\kappa)}_{\mathrm{d}})$.

Then, the successive program at the $(\kappa+1)$-th iteration to solve (6) is expressed as

$$\underset{\boldsymbol{\alpha}, \mathbf{p}_{\mathrm{c}}, \mathbf{p}_{\mathrm{d}}}{\text{maximize}} \quad \tilde{R}^{(\kappa+1)}_{\Sigma} \triangleq \sum_{k=1}^{K}\sum_{m=1}^{M} \alpha_{km}\tilde{R}^{\mathrm{d},(\kappa)}_{k,m}, \tag{15a}$$

$$\text{subject to (6b)} - \text{(6i)}. \tag{15b}$$

It is clear that $\tilde{R}^{\mathrm{d},(\kappa)}_{k,m}$ is a concave function, and thus at the optimum, we obtain the equality as

$$R^{\mathrm{d}}_{k,m} = \tilde{R}^{\mathrm{d},(\kappa)}_{k,m}. \tag{16}$$

However, the problems in (15) are still mixed-integer non-convex since the binary variables $\boldsymbol{\alpha}$ are involved in the objective function and constraints. Here, the integer constraints are related to the assignment variable $\boldsymbol{\alpha}$, while the variables $\mathbf{p}_{\mathrm{c}}$ and $\mathbf{p}_{\mathrm{d}}$ are responsible for the continuous domain. Therefore, we next handle the integer variables by using the following lemma.

Lemma 2: At the convergence of the program in (15), we suppose that the optimal point for problem (6) is given as $\mathbf{X}^* = \{\boldsymbol{\alpha}^*, \mathbf{p}^*_{\mathrm{c}}, \mathbf{p}^*_{\mathrm{d}}\}$. If we obtain an arbitrary assignment variable $\alpha^*_{k'm'} = 0$, $\forall k' \in \mathcal{K}$, $\forall m' \in \mathcal{M}$, then $p^*_{\mathrm{d},k'm'} = 0$. ■

Proof: See Appendix B. □

Remark 2: From Lemma 2, it is realized that $p^*_{\mathrm{d},km} > 0$, α^*_{km} must be active to obtain the optimum. On the other hand, when $p^*_{\mathrm{d},km} = 0$, $R^{\mathrm{d}}_{k,m} = 0$ regardless of α^*_{km}. However, α^*_{km} should be set to 0, representing that the m-th D2D is inactive as $p^*_{\mathrm{d},km} = 0$. In other words, the optimal values of $\boldsymbol{\alpha}$ are strongly associated with those of $\mathbf{p}_{\mathrm{d}}$, so that $\boldsymbol{\alpha}^*$ can be derived via $\mathbf{p}^*_{\mathrm{d}}$. ■

Now, the program (15) can be executed by omitting $\boldsymbol{\alpha}$ as follows. By exploiting the relationship between $\boldsymbol{\alpha}$ and $\mathbf{p}_{\mathrm{d}}$ from Lemma 2 and Remark 2, we fix the values of $\boldsymbol{\alpha}$ to $\mathbf{1}$, and then the program (15) becomes

$$\underset{\mathbf{p}_{\mathrm{c}}, \mathbf{p}_{\mathrm{d}}}{\text{maximize}} \quad \sum_{k=1}^{K}\sum_{m=1}^{M} \tilde{R}^{\mathrm{d},(\kappa)}_{k,m}, \tag{17a}$$

$$\text{subject to (6b)} - \text{(6e)},\ \text{(6i)}, \tag{17b}$$

$$\mathbf{card}(\mathbf{p}_{\mathrm{d},m}) \leq S, \quad m \in \mathcal{M}, \tag{17c}$$

$$\gamma^{\mathrm{d}}_{k,m}(\mathbf{p}_{\mathrm{c}}, \mathbf{p}_{\mathrm{d}}) \geq \bar{\gamma}^{\mathrm{d}}_{k,m}\mathbf{card}(p_{\mathrm{d},km}), \quad k \in \mathcal{K}, \quad m \in \mathcal{M}, \tag{17d}$$

where $\mathbf{card}(\mathbf{x})$ represents the number of nonzero elements of vector $\mathbf{x}$, and $\mathbf{p}_{\mathrm{d},m} \triangleq [p_{\mathrm{d},km}]_{k\in\mathcal{K}}$. It can be seen that problem (17) only contains the continuous variables, and at the optimum, the binary variable $\boldsymbol{\alpha}$ is determined by the function $\mathbf{card}(p_{\mathrm{d},km})$. However, the globally optimal solution for (17) is still hard to obtain, since it takes high complexity to tackle the quasi-concave cardinality constraints in (17c) and (17d). Instead of directly solving (17), we propose a two-phase iterative algorithm to effectively derive the solution as the following steps.

In the first phase, we apply the HS method to (17), and at the $(\kappa+1)$-th iteration, the ℓ_1-regularized form is expressed as

$$\underset{\mathbf{p}_c,\mathbf{p}_d}{\text{maximize}}\ \tilde{R}_{\Sigma,1}^{(\kappa+1)} \triangleq \sum_{k=1}^{K}\sum_{m=1}^{M}\tilde{R}_{k,m}^{\mathrm{d},(\kappa)} - \sum_{m=1}^{M} f_m^{p,(\kappa)}(\mathbf{p}_\mathrm{d}), \quad (18a)$$

$$\text{subject to (6b)} - \text{(6e)},\ \text{(6i)}, \quad (18b)$$

$$\gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_\mathrm{c},\mathbf{p}_\mathrm{d}) \geq \bar{\gamma}_{k,m}^{\mathrm{d}}, \quad k\in\mathcal{K},\ m\in\mathcal{M}, \quad (18c)$$

where $f_m^{p,(\kappa)}(\mathbf{p}_\mathrm{d}) = \left|\delta_m^{(\kappa)}\|(\mathbf{D}_m^{(\kappa)}+\epsilon\mathbf{I})^{-1}\mathbf{p}_{\mathrm{d},m}\|_1 - S\right|$, $\forall m\in\mathcal{M}$, are known as the penalty functions, with $\mathbf{p}_{\mathrm{d},m}^{(\kappa)} \triangleq [p_{\mathrm{d},km}^{(\kappa)}]_{k\in\mathcal{K}}$ being the m-th partitioned vector of $\mathbf{p}_\mathrm{d}^{(\kappa)}$ and $\mathbf{D}_m^{(\kappa)} \triangleq \mathrm{diag}(\mathbf{p}_{\mathrm{d},m}^{(\kappa)})$. In each iteration, $\delta_m^{(\kappa)}$ is the smallest value that gives $\mathbf{card}(\tilde{\mathbf{p}}_{\mathrm{d},m}^{(\kappa)}) = S$, in which a heuristic pattern solution for the D2D transmit power, $\tilde{\mathbf{p}}_{\mathrm{d},m}^{(\kappa)}$, is derived from $\mathbf{p}_{\mathrm{d},m}^{(\kappa)}$. In particular, the problem (18) is initially solved with $\delta_m^{(0)} = 1$, $\forall m$, while the constant ϵ is always kept at $\epsilon = 0.1$ [41]. Then, $\delta_m^{(\kappa)}$, which is used for the next iteration, is minimized in the range [0, 1], such that $\mathbf{card}(\tilde{\mathbf{p}}_{\mathrm{d},m}^{(\kappa)}) = S$, where $\tilde{\mathbf{p}}_{\mathrm{d},m}^{(\kappa)}$ is determined by

$$\tilde{p}_{\mathrm{d},km}^{(\kappa)} = \begin{cases} 0, & \text{if } p_{\mathrm{d},km}^{(\kappa)} < \Delta_m^{(\kappa)}, \\ p_{\mathrm{d},km}^{(\kappa)}, & \text{if otherwise}, \end{cases}$$

with $\Delta_m^{(\kappa)} \triangleq \delta_m^{(\kappa)} \max_{k\in\mathcal{K}}\{p_{\mathrm{d},km}^{(\kappa)}\}$. Therefore, $\tilde{\mathbf{p}}_{\mathrm{d},m}^{(\kappa)}$ is considered as the heuristic pattern for $\mathbf{p}_{\mathrm{d},m}$, at which constraints (17b)-(17d) hold.

Remark 3: Note that the D2D association variables in (15) are replaced by the cardinality functions with respect to the D2D transmit power in (17). Accordingly, we just handle a simpler problem without integer variables. To solve (17) with cardinality function, an HS method is applied through the penalty functions $f_m^{p,(\kappa)}$ as in (18), in which δ_m is updated per iteration to attempt splitting the K-length vector $\mathbf{p}_\mathrm{d}$ into two parts, such that the values of the S elements in one part are significantly higher than those of the $(K-S)$ remaining elements in the other one. ■

At the convergence of problem (18), the sets of $(K-S)$ that include some indices of $\mathcal{K}$ are formulated as

$$\mathcal{K}'_m \triangleq \{k\in\mathcal{K}|p_{\mathrm{d},km}^{(\kappa)} < \Delta_m^{(\kappa)}\}, \quad \forall m\in\mathcal{M}, \quad (19)$$

such that the number of elements $|\mathcal{K}'_m| = (K-S)$. Using Remark 2, we can easily determine the value of assignment variable $\boldsymbol{\alpha}$ as

$$\alpha_{km} = \begin{cases} 0, & \text{if } k\in\mathcal{K}'_m, \\ 1, & \text{if otherwise}. \end{cases} \quad (20)$$

By substituting $\boldsymbol{\alpha}$ into problem (15), the second phase derives the solution for (6) by solving the convex successive

[1]Note that $(\boldsymbol{\alpha}^*, \mathbf{p}_\mathrm{c}^*, \mathbf{p}_\mathrm{d}^*)$ represents the optimal solution for (6), while $(\mathbf{p}_\mathrm{c}^\star, \mathbf{p}_\mathrm{d}^\star)$ denotes a per-iteration optimal solution for (18) or (21).

Algorithm 2 Proposed Iterative Algorithm Based on HS for the D2D SR Maximization Problem (6)

1: **Initialization:** Set $\kappa := 0$, $\varepsilon := 10^{-3}$, and $\tilde{R}_{\Sigma,1}^{(0)} := -\infty$.
2: **Generating an initial point:** Randomly generate $(\mathbf{p}_\mathrm{c}^{(0)}, \mathbf{p}_\mathrm{d}^{(0)})$.
3: **repeat**
4: Solve (18) to obtain the objective value $\tilde{R}_{\Sigma,1}^{(\kappa+1)}$ and solution $(\mathbf{p}_\mathrm{c}^\star, \mathbf{p}_\mathrm{d}^\star)$.
5: Update $\mathbf{p}_\mathrm{c}^{(\kappa+1)} := \mathbf{p}_\mathrm{c}^\star$ and $\mathbf{p}_\mathrm{d}^{(\kappa+1)} := \mathbf{p}_\mathrm{d}^\star$.
6: Find $\delta_m^{(\kappa+1)}$ and $\Delta_m^{(\kappa+1)}$.
7: Set $\kappa := \kappa + 1$.
8: **until** $\tilde{R}_{\Sigma,1}^{(\kappa)} - \tilde{R}_{\Sigma,1}^{(\kappa-1)} < \varepsilon$.
9: Generate the sets $\mathcal{K}'_m$, $\forall m$, as in (19).
10: Calculate $\boldsymbol{\alpha}^*$ as in (20).
11: Set $\kappa := 0$, $\tilde{R}_{\Sigma,2}^{(0)} := -\infty$, and $(\mathbf{p}_\mathrm{c}^*, \mathbf{p}_\mathrm{d}^*)^1 := \mathbf{0}$; Randomly generate $(\boldsymbol{\alpha}^{(0)}, \mathbf{p}_\mathrm{c}^{(0)}, \mathbf{p}_\mathrm{d}^{(0)})$.
12: **repeat**
13: Solve (21) to obtain the objective value $\tilde{R}_{\Sigma,2}^{(\kappa+1)}$ and solution $(\mathbf{p}_\mathrm{c}^\star, \mathbf{p}_\mathrm{d}^\star)$.
14: Update $\mathbf{p}_\mathrm{c}^{(\kappa+1)} := \mathbf{p}_\mathrm{c}^\star$ and $\mathbf{p}_\mathrm{d}^{(\kappa+1)} := \mathbf{p}_\mathrm{d}^\star$.
15: Set $\kappa := \kappa + 1$.
16: **until** $\tilde{R}_{\Sigma,2}^{(\kappa)} - \tilde{R}_{\Sigma,2}^{(\kappa-1)} < \varepsilon$.
17: **Output:** The solution collects $\boldsymbol{\alpha}^*$ in Step 10, and $(\mathbf{p}_\mathrm{c}^*, \mathbf{p}_\mathrm{d}^*) := (\mathbf{p}_\mathrm{c}^{(\kappa)}, \mathbf{p}_\mathrm{d}^{(\kappa)})$; The achievable rate is given by $\tilde{R}_{\Sigma,2}^{(\kappa)}$.

problems, in which the problem at the $(\kappa+1)$-th iteration is formulated as

$$\underset{\mathbf{p}_c,\mathbf{p}_d}{\text{maximize}}\ \tilde{R}_{\Sigma,2}^{(\kappa+1)} \triangleq \sum_{k=1}^{K}\sum_{m=1}^{M}\tilde{R}_{k,m}^{\mathrm{d},(\kappa)}, \quad (21a)$$

$$\text{subject to (6d)} - \text{(6e)}, \text{(6h)},\ \text{(6i)}, \quad (21b)$$

$$p_{\mathrm{d},km}^2 \leq P_{\mathrm{d},km}^{\max}, \quad m\in\mathcal{M},\ k\in\mathcal{K}\setminus\mathcal{K}'_m, \quad (21c)$$

$$p_{\mathrm{d},km} \geq 0, \quad m\in\mathcal{M},\ k\in\mathcal{K}\setminus\mathcal{K}'_m, \quad (21d)$$

$$p_{\mathrm{d},km} = 0, \quad m\in\mathcal{M},\ k\in\mathcal{K}'_m. \quad (21e)$$

In summary, the first phase is used to address the cardinality constraints in (17) and to find the value of $\boldsymbol{\alpha}$. To do these, the ℓ_1-form in (18) is solved to produce the heuristic patterns $\tilde{\mathbf{p}}_{\mathrm{d},m}$, such that (17c) holds. By using this pattern, the variables of vector $\mathbf{p}_{\mathrm{d},m}$ corresponding to the nonzero elements of $\tilde{\mathbf{p}}_{\mathrm{d},m}$ are optimized, while the others are forced to be zero during the later phase in (21). At the convergence, we obtain a heuristic solution $(\boldsymbol{\alpha}^*, \mathbf{p}_\mathrm{c}^*, \mathbf{p}_\mathrm{d}^*)$ for (6). The proposed algorithm is summarized in Algorithm 2.

Convergence Analysis: As in [40] and [41], the programs in (18) and (21) generate non-decreasing objective function values and satisfy the convergence condition at the Karush-Kuhn-Tucker point. In implementation, the iterative algorithms terminate when the difference between two successive objective values is smaller than $\varepsilon = 10^{-3}$.

Complexity Analysis: It is realized that the program (18) and (21) contain linear and second-order cone (SOC)

Algorithm 3 Proposed Algorithm Based on Binary Search to Find $\{\delta_m^{(\kappa+1)}\}$ and $\{\Delta_m^{(\kappa+1)}\}$

Input: $\mathbf{p}_{\mathrm{d}}^{(\kappa+1)}$, S, and $\varepsilon := 10^{-3}$.
for $m \in \mathcal{M}$ **do**
  Set $\delta_{\max} := 1$, and $\delta_{\min} := 0$.
  while $(\delta_{\max} - \delta_{\min} > \varepsilon)$ **do**
    Set $\delta := \frac{\delta_{\max}+\delta_{\min}}{2}$ and $\Delta := \delta \max_{k\in\mathcal{K}}\{p_{\mathrm{d},km}^{(\kappa+1)}\}$.
    Calculate $\mathcal{K}'_m$ by replacing $\Delta_m^{(\kappa)}$ with Δ in (19).
    if $|\mathcal{K}'_m| < (K - S)$ **then**
      $\delta_{\min} := \delta$.
    else
      $\delta_{\max} := \delta$.
    end if
  end while
  Set $\delta_m^{(\kappa+1)} := \delta$ and $\Delta_m^{(\kappa+1)} := \Delta$.
end for
Output: $\{\delta_m^{(\kappa+1)}\}$ and $\{\Delta_m^{(\kappa+1)}\}$.

---

constraints. The number of constraints in (18) and (21) are determined by $c_1 = 3K(N+M)$ and $c_2 = 3KN + (2K+S)M$, respectively. Two programs have the same number of variables, denoted by $\upsilon = K(N+M)$. As in [42], the complexities of (18) and (21) are given as $\mathcal{O}(c_1^{2.5}(\upsilon^2 + c_1))$ and $\mathcal{O}(c_2^{2.5}(\upsilon^2 + c_2))$, respectively.

B. CONVERGENCE RATE AND COMPLEXITY IMPROVEMENT

To relieve the complexity, at Step 6 of the first phase in Algorithm 2, we apply a binary search to find the values of $\delta_m^{(\kappa)}$ as well as $\Delta_m^{(\kappa)}$; and then the sets of $\mathcal{K}'_m$ are determined to solve problem (21) in the next phase. Algorithm 3 briefly summarizes the binary search method. The convergence rate of Algorithm 2 is further boosted by considering the following lemma.

Lemma 3: Suppose that $\mathbf{s}_1^* = (\mathbf{p}_{\mathrm{c}}^{(\kappa)}, \mathbf{p}_{\mathrm{d}}^{(\kappa)})$ is the optimal solution obtained in (18). Then, $\mathbf{s}_2^* = (\mathbf{p}_{\mathrm{c}}^{(\kappa)}, \tilde{\mathbf{p}}_{\mathrm{d}}^{(\kappa)})$, where $\tilde{\mathbf{p}}_{\mathrm{d}}^{(\kappa)}$ is generated by forcing some elements of $\mathbf{p}_{\mathrm{d}}^{(\kappa)}$ to zeros, is called the heuristic pattern solution of $\mathbf{s}_1^*$. $\mathbf{s}_2^*$ is also a feasible point of (21). ■

Proof: See Appendix C. □

By applying Lemma 3, at Step 11 in Algorithm 2, we do not have to reset κ and randomly generate the initial point $(\mathbf{p}_{\mathrm{c}}^{(0)}, \mathbf{p}_{\mathrm{d}}^{(0)})$. Instead, $(\mathbf{p}_{\mathrm{c}}^{(\kappa)}, \mathbf{p}_{\mathrm{d}}^{(\kappa)})$ obtained by solving (18) is used as the initial feasible point to solve (21).

C. ACHIEVABLE RATE IMPROVEMENT

Although the HS-based algorithm provides a low-complexity, there exists rate loss as compared to the optimal value. Therefore, we further propose a method based on GS for resource allocation, while using the same power allocation algorithm. To do this, we successively consider the D2Ds one-by-one. For the m-th additional D2D to the system, the algorithm executes a loop to pick a vector

Algorithm 4 Proposed Iterative Algorithm Based on GS for the D2D SR Maximization Problem (6)

Initialization: Set $\varepsilon := 10^{-3}$, $\boldsymbol{\alpha}^* = \mathbf{0}$ and $\tilde{R}_\Sigma := -\infty$.
for $m \in \mathcal{M}$ **do**
  Set *isAdded* $:= 0$ and $\boldsymbol{\alpha} := \boldsymbol{\alpha}^*$.
  for $\mathbf{v} \in \mathcal{S}_\alpha$ **do**
    Set $\boldsymbol{\alpha}_m := \mathbf{v}$, $\kappa := 0$, $\tilde{R}_{\Sigma,2}^{(0)} := -\infty$ and randomly choose an initial point $(\mathbf{p}_{\mathrm{c}}^{(0)}, \mathbf{p}_{\mathrm{d}}^{(0)})$.
    Update the sets $\mathcal{K}'_m \triangleq \{k \in \mathcal{K} | \alpha_{km} = 0\}$ instead of (19).
    repeat
      Solve (21) to obtain $\tilde{R}_{\Sigma,2}^{(\kappa+1)}$, and $(\mathbf{p}_{\mathrm{c}}^\star, \mathbf{p}_{\mathrm{d}}^\star)$.
      Update $\mathbf{p}_{\mathrm{c}}^{(\kappa+1)} := \mathbf{p}_{\mathrm{c}}^\star$ and $\mathbf{p}_{\mathrm{d}}^{(\kappa+1)} := \mathbf{p}_{\mathrm{d}}^\star$.
      Set $\kappa := \kappa + 1$.
    until $\tilde{R}_{\Sigma,2}^{(\kappa)} - \tilde{R}_{\Sigma,2}^{(\kappa-1)} < \varepsilon$.
    if $(\tilde{R}_{\Sigma,2}^{(\kappa)} > \tilde{R}_\Sigma)$ or $(\textit{isAdded} = 0)$ **then**
      Set *isAdded* $:= 1$ and $\tilde{R}_\Sigma := \tilde{R}_{\Sigma,2}^{(\kappa)}$.
      Update $\boldsymbol{\alpha}^* := \boldsymbol{\alpha}$ and $(\mathbf{p}_{\mathrm{c}}^*, \mathbf{p}_{\mathrm{d}}^*) := (\mathbf{p}_{\mathrm{c}}^{(\kappa)}, \mathbf{p}_{\mathrm{d}}^{(\kappa)})$.
    end if
  end for
end for
Output: The solution $(\boldsymbol{\alpha}^*, \mathbf{p}_{\mathrm{c}}^*, \mathbf{p}_{\mathrm{d}}^*)$; The corresponding achievable rate is given by $\tilde{R}_\Sigma$.

$\mathbf{v} \in \mathcal{S}_\alpha \triangleq \{\mathbf{v} \in \{0,1\}^{K\times 1} | \mathbf{card}(\mathbf{v}) = S\}$ and assign $\mathbf{v}$ to $\boldsymbol{\alpha}_m$. Then, $\boldsymbol{\alpha}_m$ is used to maximize the total rate of added D2Ds by solving the program (21) as in Algorithm 2. The pattern of resource allocation $\boldsymbol{\alpha}_m$ that provides the best total rate is assigned to the corresponding D2D, and the algorithm considers the next D2D pair. Algorithm 4 summarizes the proposed method based on GS.

V. NUMERICAL RESULTS

To evaluate the performance of the proposed algorithm, we consider an SCMA-enabled cellular network with 6 uplink CUEs randomly placed within a 100-meter radius from the BS at the center of the cell. An underlaid D2D system consists of M D2Ds, sharing $K = 4$ resources with CUEs. Each D2Ds can be assigned to $S = L = 2$ resources by mapping to a codeword in the codebook used for SCMA scheme. The information channel vectors for the link from CUEs and D2D to BS are generated as $\mathbf{h} = \sqrt{10^{-\mathtt{PL}_{\mathtt{U,BS}}/10}}\hat{\mathbf{h}}$, where $\mathbf{h} \in \{\mathbf{h}_{\mathrm{cb},kn}, \mathbf{g}_{\mathrm{db},km}\}$ corresponds to the communication links from $\mathtt{U} \in \{C_n, D_m\}$ to the BS at the distance $d_{\mathtt{U,BS}}$ in kilometer. Here, the path loss (PL) model, $\mathtt{PL}_{\mathtt{U,BS}}$, represents the large-scale fading, and the entries of $\hat{\mathbf{h}} \sim \mathcal{CN}(0,1)$ stand for the small-scale fading. Similarly, the channel responses from D2D-Tx to D2D-Rx and from CUEs to D2D-Rx are set as $g = \sqrt{10^{-\mathtt{PL}_{\mathtt{U,U}}/10}}\hat{g}$, where $g \in \{h_{\mathrm{dd},km}, g_{\mathrm{cd},knm}, g_{\mathrm{dd},km'm}\}$, and $\mathtt{PL}_{\mathtt{U,U}}$ is the PL with the distance $d_{\mathtt{U,U}}$ (in kilometer) and the entries $\hat{g} \sim \mathcal{CN}(0,1)$. According to [43], the other parameters are tabulated in Table 1. In all figures, the achievable rates of CUEs and D2Ds in the whole K resources are divided by ln 2, to be presented in bps/Hz.

TABLE 1. Simulation parameters.

Parameters	Value
Radius of small cell	100 m
Bandwidth	10 MHz
Noise power spectral density	-174 dBm/Hz
PL from user to BS, $\mathrm{PL}_{\mathrm{U,BS}}$	$103.8 + 20.9\log_{10}(d_{\mathrm{U,BS}})$ dB
PL between users, $\mathrm{PL}_{\mathrm{U,U}}$	$145.4 + 37.5\log_{10}(d_{\mathrm{U,U}})$ dB
Power budgets, $P^{\max}_{\mathrm{c},kn}, \forall k, n$	10 dBm
Number of (No.) antennas at BS	4
No. CUEs, N	6
No. resources, K	4
No. non-zero entries per codeword, L	2
No. assigned resources for D2Ds, S	2

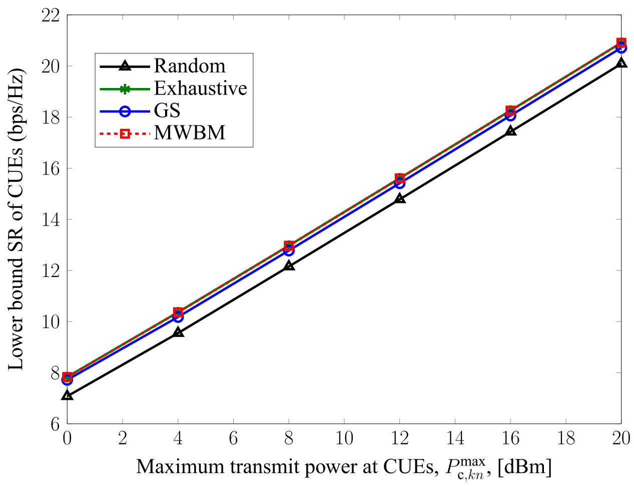


FIGURE 2. The average lower bound SR of CUEs, when $N = 6$.

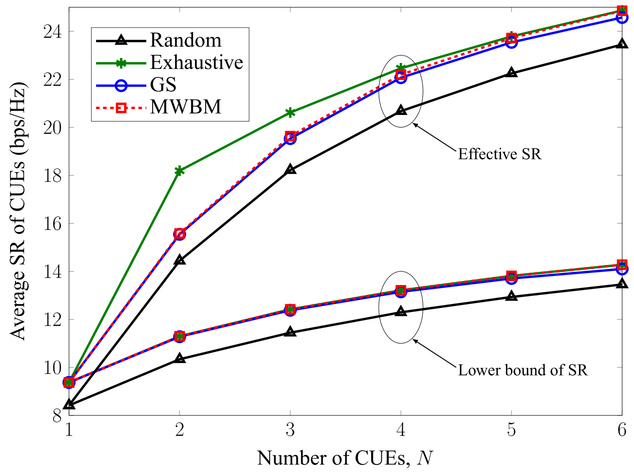


FIGURE 3. The average SR of CUEs.

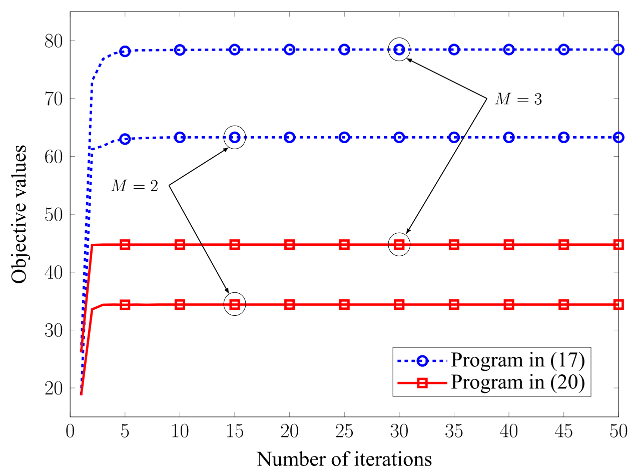


FIGURE 4. The convergence rates of the programs in (18a) and (21a) with different values of M and $P^{\max}_{\mathrm{d},km} = 10$ dBm.

A. CODEBOOK ASSIGNMENT FOR CUES

We consider the first stage of our scheme, where the resources are allocated to CUEs. Fig. 2 shows the lower bound of SR versus the maximum transmit power of CUEs. By using the MWBM method, the SR obtained through the lower bound form is exactly equal to the exhaustive search method. This is because the Kuhn-Munkres algorithm used for solving the MWBM problem is proven to find the optimal solution. It is also shown that the GS-based method with lower complexity gives lower SR, while the random scheme provides the worst performance. As compared to the GS-based and random scheme, the MWBM method always provides better performance when the maximum transmit power increases. This verifies that the codebook assignment plays an important role for performance improvement in the SCMA-enabled system.

Fig. 3 further examines the lower bound of SR and effective SR as the number of CUEs increases. By using (3), the effective SRs when using random and exhaustive search methods are obtained by directly applying the solutions to (3). For the MWBM and GS methods, we have to find their solutions via the lower bound as in (9), and then apply the optimized solutions to (3). The MWBM provides better performance than GS in terms of both the lower bound and effective SR; in particular, the gap between MWBM and GS rises along with the number of CUEs. This is because the GS method finds the best solution for one CUE at once, while MWBM considers the assignments across all CUEs. Interestingly, as compared to exhaustive search, the rate loss of MWBM decreases when the number of CUEs increases. This truly reflects the property of the logarithm function that when the number of components in summation the lower bound in (7) becomes tight.

B. D2D SUM RATE MAXIMIZATION

Without loss of generality, the per-resource target SINRs at D2D-Rx and BS for all CUEs are set as $\bar{\gamma}^{\mathrm{d}}_{k,m} = 5$ dB and $\bar{\gamma}^{\mathrm{c}}_{k,n} = 10$ dB, respectively. To assess the performance, we compare the proposed HS- and GS-based methods (as in Algorithms 2 and 4, respectively) with the other two schemes for the resource allocation as follows.

- Random scheme, where $\boldsymbol{\alpha}_m, \forall m \in \mathcal{M}$ are randomly assigned to a vector $\mathbf{v} \in \mathcal{S}_\alpha$.
- Exhaustive search scheme, where all combinations generated by choosing $\boldsymbol{\alpha}_m \in \mathcal{S}_\alpha, \forall m \in \mathcal{M}$, are considered.

TABLE 2. Per-iteration complexities.

Method	Complexity
Random	$\mathcal{O}(c_2^{2.5}(\upsilon^2 + c_2))$
Exhaustive search	$\binom{K}{S}^M \times \mathcal{O}(c_2^{2.5}(\upsilon^2 + c_2))$
Algorithm 2	$\mathcal{O}(c_1^{2.5}(\upsilon^2 + c_1) + c_2^{2.5}(\upsilon^2 + c_2))$
Algorithm 4	$\binom{K}{S}.M \times \mathcal{O}(c_2^{2.5}(\upsilon^2 + c_2))$

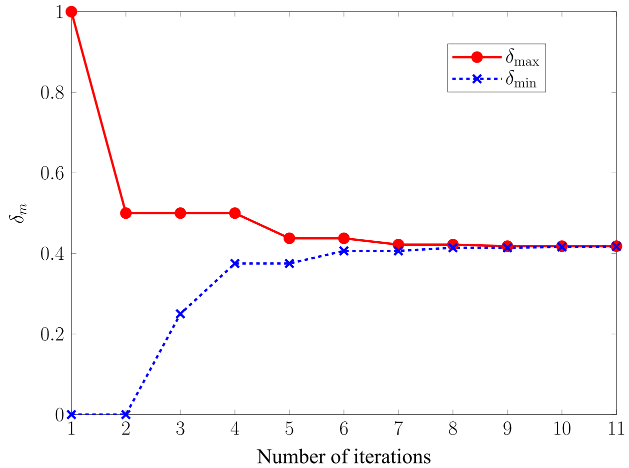


FIGURE 5. **The convergence rate of binary search to find δ_m in Algorithm 3.**

Then, based on the proposed program in (21), these basic schemes can manage the power allocation efficiently.

Fig. 4 shows the convergence behaviors of the proposed algorithms for resource and power allocation programs in (18) and (21), respectively. It can be seen that both of the proposed algorithms need only few iterations to reach the convergence condition for the various number of D2Ds. Intuitively, the program (18) allows D2Ds to use all K resources, and thus its objective value is higher than that of (21). Although four methods can use the same program (21) for power allocation with the same number of iterations to convergence, their complexities are totally different, depending on how to derive the resource allocation variables. By using the complexity analysis in Section IV, Table 2 provides the per-iteration complexities of the four methods. It is clear that the complexity of the HS-based method is nearly double that of the random method, implying that they can be merely approximate in big-O notation when c_1, c_2 and υ become large. The complexity of the GS-based method increases as the product of M and the number of SCMA layers rises, while the complexity of exhaustive search follows the exponential function. To further observe the complexity, Fig. 5 describes the typical convergence rate of Algorithm 3 to find δ_m, which is updated in the first phase of Algorithm 2. At the beginning, the search region of δ_m is $[\delta_{\min}, \delta_{\max}]$, with $\delta_{\min} = 0$ and $\delta_{\max} = 1$. The value of δ_m is obtained within only around 9 iterations, at which the gap between $\delta_{\min}$ and $\delta_{\max}$ is very small. Therefore, the total complexity of Algorithm 3 approximates to $\mathcal{O}(KM)$. By connecting to the complexity of Algorithm 2 in Table 2, the computational time for binary search is negligible, which confirms the of the HS-based method.

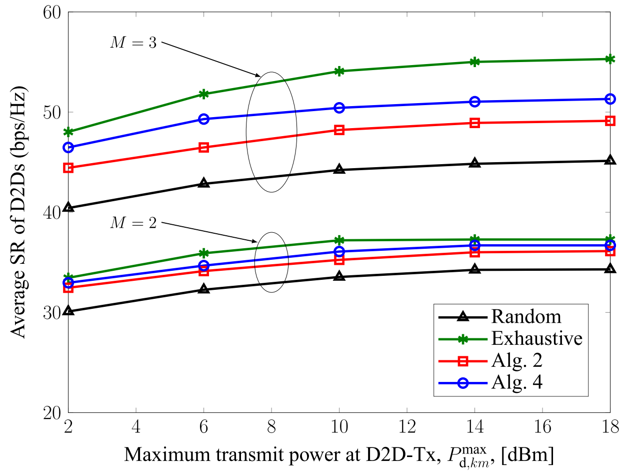


FIGURE 6. **The SR of D2Ds versus the maximum transmit power at D2D-Tx.**

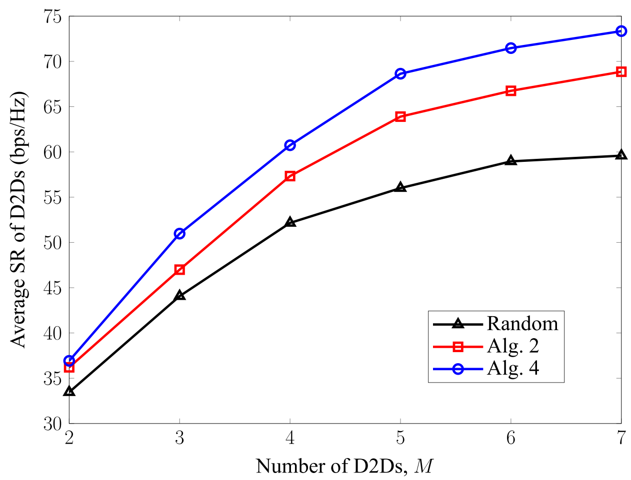


FIGURE 7. **The SR of D2Ds versus the number of D2Ds.**

Fig. 6 plots the SR of D2Ds as a function of the maximum power budget at D2D-Tx, $P_{\mathrm{d},km}^{\max}$, with various number of D2Ds, $M = \{2, 3\}$. Clearly, the exhaustive search outperforms all three other methods, while the random scheme gives the worst performance. For the two proposed algorithms, the gap between the GS-based and HS-based methods is higher in the case of $M = 3$ than in the case of $M = 2$. This is because the penalty function in the HS-based scheme is linear, which supports the better approximation for the smaller number of degrees of freedom. However, the HS-based method provides much better performance than random selection, and it is well-adapted to the D2D system due to its low complexity. To gain further insights into the algorithms, Fig. 7 shows the SR of D2Ds when the number of D2Ds increases. Here, we omit the case of exhaustive search due to its very high complexity, i.e., exponential growth along with the number of D2Ds. The two proposed algorithms give the higher gain with the greater number of D2Ds as compared to the random scheme, indicating that they are able to adapt to a dense network of devices.

VI. CONCLUSION

We have studied a D2D system underlaying an uplink SCMA-enabled cellular network. A joint SR optimization of CUEs and D2Ds under resource and power allocation requires high complexity. Therefore, a two-stage scheme has been considered, where the first stage is to maximize the SR for CUEs, and the second stage focuses on the SR optimization for D2Ds. First, the resource allocation problem to maximize SR for CUEs has been introduced as a non-linear program, which is hard to solve. Accordingly, a lower bound for the CUE rate is derived, so that the problem is effectively solved by applying MWBM and GS-based methods. In the second stage, a new design of the SR maximization problem for D2Ds has been considered under the resource and power allocation subject to QoS requirements of CUEs, which is formulated as a mixed-integer non-convex programming. To tackle the problem, we have proposed a low-complexity two-stage iterative algorithm based on heuristic and inner approximation methods. To further improve the convergence, we have investigated the binary search algorithm and exploited the feasible point in the heuristic program for the later step. In addition, the SR of D2Ds has been further improved by using the GS-based method. Numerical results with realistic parameters have demonstrated that both the proposed algorithms provide substantial performance enhancement. Interestingly, the complexity of the heuristic-based method has been proven to be comparable to that of the random approach.

APPENDIX A
PROOF OF LEMMA 1

From (3), the CUEs' rate can be rewritten as

$$\begin{aligned} R_k^{\mathrm{c}}(\boldsymbol{\beta}) &= \ln\Bigg(\frac{N + \frac{N}{L}\sum_{n=1}^{N}\beta_{kn}\gamma_{k,n}^{\mathrm{c}}(\mathbf{p}_{\mathrm{c}}^{\max},\mathbf{0})}{N}\Bigg) \\ &= \ln\Bigg(\frac{\frac{N}{L}(L + \sum_{n=1}^{N}\beta_{kn}\gamma_{k,n}^{\mathrm{c}}(\mathbf{p}_{\mathrm{c}}^{\max},\mathbf{0}))}{N}\Bigg) \\ &= \ln\Bigg(\frac{\frac{N}{L}\sum_{n=1}^{N}\big(\frac{L}{N} + \beta_{kn}\gamma_{k,n}^{\mathrm{c}}(\mathbf{p}_{\mathrm{c}}^{\max},\mathbf{0})\big)}{N}\Bigg), \end{aligned} \quad (22)$$

By applying Jensen's inequality to the concave function $\ln(x)$, the logarithm of the arithmetic mean-geometric mean inequality is given as

$$\ln\Big(\frac{\sum_{n=1}^{N} x_n}{N}\Big) \geq \frac{1}{N}\sum_{n=1}^{N}\ln(x_n). \quad (23)$$

From (22) and (23), we obtain

$$R_k^{\mathrm{c}}(\boldsymbol{\beta}) \geq \ln\frac{N}{L} + \frac{1}{N}\sum_{n=1}^{N}\ln\Big(\frac{L}{N} + \beta_{kn}\gamma_{k,n}^{\mathrm{c}}(\mathbf{p}_{\mathrm{c}}^{\max},\mathbf{0})\Big), \quad (24)$$

which verifies the result in (7).

APPENDIX B
PROOF OF LEMMA 2

Equation (16) implies that the optimal solution for (6) is achieved at the convergence of program (15). We first let $f_{\mathrm{d}}(\mathbf{X}^*) = \sum_{k=1}^{K}\sum_{m=1}^{M}\alpha_{km}^* R_{k,m}^{\mathrm{d},(*)}$ be the optimal value of (6), where $R_{k,m}^{\mathrm{d},(*)}$ is determined as in (5) through $\gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_{\mathrm{c}}^*,\mathbf{p}_{\mathrm{d}}^*)$. We need to prove that whenever $\alpha_{k'm'}^* = 0,\ \forall k' \in \mathcal{K},\ \forall m' \in \mathcal{M}$ and $p_{\mathrm{d},k'm'} > 0$, the corresponding primal value is not larger than $f_{\mathrm{d}}(\mathbf{X}^*)$.

Suppose that an arbitrary feasible point $\tilde{\mathbf{X}} = \{\boldsymbol{\alpha}^*, \mathbf{p}_{\mathrm{c}}^*, \tilde{\mathbf{p}}_{\mathrm{d}}\}$, such that $\exists\ \alpha_{k'm'}^* = 0$ and $\tilde{p}_{\mathrm{d},k'm'} \in (0,\varepsilon)$ for any $k' \in \mathcal{K},\ m' \in \mathcal{M}$, and ε is very small. Then, the primal value of (6) is expressed as

$$\begin{aligned} f_{\mathrm{d}}(\tilde{\mathbf{X}}) &= \alpha_{k'm'}^*\tilde{R}_{k',m'}^{\mathrm{d}} + \sum_{\substack{k=1\\(k,m)\neq(k',m')}}^{K}\sum_{m=1}^{M}\alpha_{km}^*\tilde{R}_{k,m}^{\mathrm{d}} \\ &= \sum_{\substack{k=1\\(k,m)\neq(k',m')}}^{K}\sum_{m=1}^{M}\alpha_{km}^*\tilde{R}_{k,m}^{\mathrm{d}}, \end{aligned} \quad (25)$$

where $\tilde{R}_{k,m}^{\mathrm{d}}$ is determined by (5) via $\gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_{\mathrm{c}}^*,\tilde{\mathbf{p}}_{\mathrm{d}})$. Since $\alpha_{k'm'}^* = 0$, $\alpha_{k'm'}^*\tilde{R}_{k',m'}^{\mathrm{d}} = 0$ no matter how $\tilde{R}_{k',m'}^{\mathrm{d}}$ is calculated.

As in (2), we obtain $\gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_{\mathrm{c}}^*,\tilde{\mathbf{p}}_{\mathrm{d}}) < \gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_{\mathrm{c}}^*,\mathbf{p}_{\mathrm{d}}^*)$ for $(k,m) \neq (k',m')$. In fact, since $\tilde{p}_{\mathrm{d},k'm'} > p_{\mathrm{d},k'm'}^* = 0$, the interference terms are given as $\phi_{km}^{\mathrm{d}}\big(\mathbf{p}_{\mathrm{c}}^*,\tilde{\mathbf{p}}_{\mathrm{d}}\big) > \phi_{km}^{\mathrm{d}}\big(\mathbf{p}_{\mathrm{c}}^*,\mathbf{p}_{\mathrm{d}}^*\big)$, while the function $\phi_{km}^{\mathrm{d}}\big(\mathbf{p}_{\mathrm{c}},\mathbf{p}_{\mathrm{d}}\big)$ is inversely proportional to $\gamma_{k,m}^{\mathrm{d}}(\mathbf{p}_{\mathrm{c}},\mathbf{p}_{\mathrm{d}})$. This leads to $0 \leq \tilde{R}_{k,m}^{\mathrm{d}} < R_{k,m}^{\mathrm{d},(*)}$, and thus $\alpha_{km}^*\tilde{R}_{k,m}^{\mathrm{d}} \leq \alpha_{km}^* R_{k,m}^{\mathrm{d},(*)}$, where the equality holds when $\alpha_{km}^* = 0$. From (25), we always obtain $f_{\mathrm{d}}(\tilde{\mathbf{X}}) \leq f_{\mathrm{d}}(\mathbf{X}^*)$. Moreover, k' and m' are the arbitrary values in $\mathcal{K}$ and $\mathcal{M}$, respectively. Therefore, at the optimum, if $\alpha_{k'm'}^* = 0,\ \forall k' \in \mathcal{K},\ \forall m' \in \mathcal{M}$, then $p_{\mathrm{d},k'm'}^* = 0$, which completes the proof.

APPENDIX C
PROOF OF LEMMA 3

We denote the feasible sets for (18) and (21) as $\mathcal{F}_1$ and $\mathcal{F}_2$, respectively. Accordingly, $\mathcal{F}_1$ and $\mathcal{F}_2$ can be expressed as

$$\mathcal{F}_1 \triangleq \Big\{\big(\mathbf{p}_{\mathrm{c}},\mathbf{p}_{\mathrm{d}}\big)|(18\mathrm{b}) \text{ and } (18\mathrm{c}) \text{ hold}\Big\}, \quad (26)$$

$$\mathcal{F}_2 \triangleq \Big\{\big(\mathbf{p}_{\mathrm{c}},\mathbf{p}_{\mathrm{d}}\big)|(21\mathrm{b}) - (21\mathrm{e}) \text{ hold}\Big\}. \quad (27)$$

We have an assumption that $\mathbf{s}_1^* = (\mathbf{p}_{\mathrm{c}}^{(\kappa)},\mathbf{p}_{\mathrm{d}}^{(\kappa)}) \in \mathcal{F}_1$ is the optimal solution for (18). It is realized that constraints (6b) and (6c) in (18b) are decomposed into (21c)-(21e). In addition, all elements of $\mathbf{p}_{\mathrm{c}}^{(\kappa)}$ in $\mathbf{s}_1^*$ always satisfy constraints (6d)-(6e) since the domains of these constraints in the two problems are the same. To prove $\mathbf{s}_2^* = (\mathbf{p}_{\mathrm{c}}^{(\kappa)},\tilde{\mathbf{p}}_{\mathrm{d}}^{(\kappa)}) \in \mathcal{F}_2$, we need to show that $\mathbf{s}_2^*$ satisfies the constraints (6h), (6i), (21c)-(21e), which are associated with the variables $p_{\mathrm{d},km},\ k \in \mathcal{K},\ m \in \mathcal{M}$. Therefore, we examine the feasibility of $\mathbf{s}_2^*$ in problem (21), through the feasibility of $p_{\mathrm{d},km}$ in both cases of $\alpha_{km} = 1$ and $\alpha_{km} = 0,\ \forall k, m$.

For $\alpha_{km} = 1,\ k \in \mathcal{K}\backslash\mathcal{K}_m',\ \mathbf{s}_2^* \in \mathcal{F}_2$ only when $p_{\mathrm{d},km}$ satisfies constraints (6h), (6i), (21c) and (21d). With $\alpha_{km} = 1$, constraints (6h), (6i), (21c), and (21d) in problem (21) are equivalent to (18c), (6i), (6b), and (6c) in problem (18), respectively. This means that all values of $p_{\mathrm{d},km}$ with $\alpha_{km} = 1$ make (21b)-(21d) hold.

When $\alpha_{km} = 0$, $k \in \mathcal{K}'_m$, $\mathbf{s}_2^* \in \mathcal{F}_2$ only if $p_{\mathrm{d},km}$ satisfies constraints (6h), (6i), and (21e). First, Lemma 2 indicates that $p_{\mathrm{d},km} = 0$, so that the constraint (21e) holds. We need to show that constraints (6h) and (6i) also hold. Obviously, constraints (6h) and (6i) related to $\alpha_{km} = 0$ become

$$\gamma^{\mathrm{d}}_{k,m}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right) \geq 0, \quad m \in \mathcal{M},\ k \in \mathcal{K}'_m, \tag{28}$$

$$\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right) \geq \beta_{kn}\bar{\gamma}^{\mathrm{c}}_{k,n},\ k \in \mathcal{K},\ n \in \mathcal{N}. \tag{29}$$

From (2), we have $\gamma^{\mathrm{d}}_{k,m}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right) = 0$ as constraint (28) hold. Since $\mathbf{p}_{\mathrm{d}}^{(\kappa)} \succeq \tilde{\mathbf{p}}_{\mathrm{d}}^{(\kappa)}$, the denominator of $\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_1^*\right)$ resulting from (2) is larger than that of $\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right)$, leading to $\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right) \geq \gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_1^*\right)$. Moreover, $\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_1^*\right) \geq \beta_{kn}\bar{\gamma}^{\mathrm{c}}_{k,n}$ holds due to $\mathbf{s}_1^* \in \mathcal{F}_1$. They demonstrate that $\gamma^{\mathrm{c}}_{k,n}\left(\mathbf{s}_2^* | p_{\mathrm{d},km} = 0\right) \geq \beta_{kn}\bar{\gamma}^{\mathrm{c}}_{k,n}$, as constraint (29) hold.

In summary, for both cases of α_{km}, $\tilde{\mathbf{p}}_{\mathrm{d}}^{(\kappa)}$ satisfy all corresponding constraints and $\mathbf{s}_2^* \in \mathcal{F}_2$.

REFERENCES

[1] M. R. Palattella *et al.*, "Internet of Things in the 5G era: Enablers, architecture, and business models," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 3, pp. 510–527, Mar. 2016.

[2] A. Al-Fuqaha, M. Guizani, M. Mohammadi, M. Aledhari, and M. Ayyash, "Internet of Things: A survey on enabling technologies, protocols, and applications," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 2347–2376, 4th Quart., 2015.

[3] C. Perera, C. H. Liu, and S. Jayawardena, "The emerging Internet of Things marketplace from an industrial perspective: A survey," *IEEE Trans. Emerg. Topics Comput.*, vol. 3, no. 4, pp. 585–598, Dec. 2015.

[4] E. Hossain and M. Hasan, "5G cellular: Key enabling technologies and research challenges," *IEEE Instrum. Meas. Mag.*, vol. 18, no. 3, pp. 11–21, Jun. 2015.

[5] H. S. Dhillon, H. Huang, and H. Viswanathan, "Wide-area wireless communication challenges for the Internet of Things," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 168–174, Feb. 2017.

[6] A. Damnjanovic *et al.*, "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, Jun. 2011.

[7] M. Siekkinen, M. Hiienkari, J. K. Nurminen, and J. Nieminen, "How low energy is Bluetooth low energy? Comparative measurements with ZigBee/802.15.4," in *Proc. IEEE Wireless Commun. Netw. Conf. Workshops (WCNCW)*, Apr. 2012, pp. 232–237.

[8] C. Gomez, J. Oller, and J. Paradells, "Overview and evaluation of bluetooth low energy: An emerging low-power wireless technology," *Sensors*, vol. 12, no. 9, pp. 11734–11753, 2012.

[9] "The need for low cost, high reach, wide area connectivity for the Internet of Things: A mobile network operator's perspective," Machina Res., London, U.K., White Paper, 2014.

[10] S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter-wave cellular wireless networks: Potentials and challenges," *Proc. IEEE*, vol. 102, no. 3, pp. 366–385, Mar. 2014.

[11] L. Dai, B. Wang, Y. Yuan, S. Han, C.-L. I, and Z. Wang, "Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74–81, Sep. 2015.

[12] S. M. R. Islam, N. Avazov, O. A. Dobre, and K.-S. Kwak, "Power-domain non-orthogonal multiple access (NOMA) in 5G systems: Potentials and challenges," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 721–742, 2nd Quart., 2017.

[13] L. Song, Y. Li, Z. Ding, and H. V. Poor, "Resource management in non-orthogonal multiple access networks for 5G and beyond," *IEEE Netw.*, vol. 31, no. 4, pp. 8–14, Jul. 2017.

[14] Y. Cai, Z. Qin, F. Cui, G. Y. Li, and J. A. McCann, "Modulation and multiple access for 5G networks," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 1, pp. 629–646, 1st Quart., 2018.

[15] H. Nikopour and H. Baligh, "Sparse code multiple access," in *Proc. IEEE Annu. Int. Symp. Pers. Indoor Mobile Radio Commun. (PIMRC)*, Sep. 2013, pp. 332–336.

[16] F. Wei and W. Chen, "Low complexity iterative receiver design for sparse code multiple access," *IEEE Trans. Commun.*, vol. 65, no. 2, pp. 621–634, Feb. 2017.

[17] Z. Yang, J. Cui, X. Lei, Z. Ding, P. Fan, and D. Chen, "Impact of factor graph on average sum rate for uplink sparse code multiple access systems," *IEEE Access*, vol. 4, pp. 6585–6590, 2016.

[18] G. Araniti, C. Campolo, M. Condoluci, A. Iera, and A. Molinaro, "LTE for vehicular networking: A survey," *IEEE Commun. Mag.*, vol. 51, no. 5, pp. 148–157, May 2013.

[19] D. Feng, L. Lu, Y. Yuan-Wu, G. Li, S. Li, and G. Feng, "Device-to-device communications in cellular networks," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 49–55, Apr. 2014.

[20] *Technical Specification Group SA; Study on Architecture Enhancements to Support Proximity Services (ProSe) (Release 12)*, document 23.703 V2.0.0, 3GPP, Mar. 2014.

[21] K. Zheng, Q. Zheng, P. Chatzimisios, W. Xiang, and Y. Zhou, "Heterogeneous vehicular networking: A survey on architecture, challenges, and solutions," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 2377–2396, 4th Quart., 2015.

[22] *IEEE Guide for Wireless Access in Vehicular Environments (WAVE)—Architecture*, IEEE Standard 1609.0-2013, 2016.

[23] K. Abboud, H. A. Omar, and W. Zhuang, "Interworking of DSRC and cellular network technologies for V2X communications: A survey," *IEEE Trans. Veh. Technol.*, vol. 65, no. 12, pp. 9457–9470, Dec. 2016.

[24] *Evolved Universal Terrestrial Radio Access (e-Utra) and Evolved Universal Terrestrial Radio Access Network (e-Utran); Overall Description; Stage 2 (v14.3.0, Release 14)*, document 3GPP 36.300, Jun. 2016.

[25] R. Molina-Masegosa and J. Gozalvez, "LTE-V for sidelink 5G V2X vehicular communications: A new 5G technology for short-range vehicle-to-everything communications," *IEEE Veh. Technol. Mag.*, vol. 12, no. 4, pp. 30–39, Dec. 2017.

[26] Q. Wei, L. Wang, Z. Feng, and Z. Ding, "Wireless resource management in LTE-U driven heterogeneous V2X communication networks," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7508–7522, Aug. 2018.

[27] M. N. Tehrani, M. Uysal, and H. Yanikomeroglu, "Device-to-device communication in 5G cellular networks: Challenges, solutions, and future directions," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 86–92, May 2014.

[28] B. Antonescu, M. T. Moayyed, and S. Basagni, "mmWave channel propagation modeling for V2X communication systems," in *Proc. IEEE Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Oct. 2017, pp. 1–6. doi: 10.1109/PIMRC.2017.8292718.

[29] P. Phunchongharn, E. Hossain, and D. I. Kim, "Resource allocation for device-to-device communications underlaying LTE-advanced networks," *IEEE Wireless Commun.*, vol. 20, no. 4, pp. 91–100, Aug. 2013.

[30] H. V. Nguyen, Q. Duong, V.-D. Nguyen, Y. Shin, and O.-S. Shin, "Optimization of resource allocation for underlay device-to-device communications in cellular networks," *Peer-Peer Netw. Appl.*, vol. 9, no. 5, pp. 965–977, Sep. 2016.

[31] H. V. Nguyen, Q. Duong, and O. S. Shin, "Resource allocation optimization for device-to-device communications in cellular networks," in *Proc. Int. Conf. Inf. Commun. Technol. Converg. (ICTC)*, Oct. 2014, pp. 377–378.

[32] W. Zhong, Y. Fang, S. Jin, K.-K. Wong, S. Zhong, and Z. Qian, "Joint resource allocation for device-to-device communications underlaying uplink MIMO cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 1, pp. 41–54, Jan. 2015.

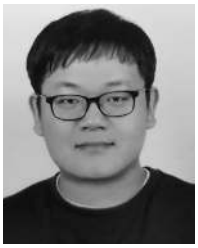
[33] W. Zhao and S. Wang, "Resource sharing scheme for device-to-device communication underlaying cellular networks," *IEEE Trans. Commun.*, vol. 63, no. 12, pp. 4838–4848, Oct. 2015.

[34] T. D. Hoang, L. B. Le, and T. Le-Ngoc, "Resource allocation for D2D communications under proportional fairness," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2014, pp. 1–6.

[35] Y. Dai, M. Sheng, K. Zhao, L. Liu, J. Liu, and J. Li, "Interference-aware resource allocation for D2D underlaid cellular network using SCMA: A hypergraph approach," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2016, pp. 1–6.

[36] J. Liu, M. Sheng, L. Liu, Y. Shi, and J. Li, "Modeling and analysis of SCMA enhanced D2D and cellular hybrid network," *IEEE Trans. Commun.*, vol. 65, no. 1, pp. 173–185, Jan. 2017.

[37] Y. Li, M. Sheng, Y. Zhu, T. Jiang, and J. Li, "Sum rate maximization in underlay SCMA device-to-device networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2016, pp. 1–6. doi: 10.1109/GLOCOM.2016.7842305.
[38] M. Al-Imari, M. A. Imran, R. Tafazolli, and D. Chen, "Subcarrier and power allocation for LDS-OFDM system," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Budapest, Hungary, May 2011, pp. 1–5.
[39] H. W. Kuhn, *The Hungarian Method for the Assignment Problem*. Berlin, Germany: Springer, 2010.
[40] H. V. Nguyen, V.-D. Nguyen, O. A. Dobre, and O.-S. Shin, "Sum rate maximization based on sub-array antenna selection in a full-duplex system," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2017, pp. 1–6. doi: 10.1109/GLOCOM.2017.8254806.
[41] E. J. Candès, M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted ℓ_1 minimization," *J. Fourier Anal. Appl.*, vol. 14, nos. 5–6, pp. 877–905, 2008.
[42] D. Peaucelle, D. Henrion, Y. Labit, and K. Taitz, "User's guide for SeDuMi interface 1.04," in *Proc. LAAS-CNRS*, Toulouse, France, Jun. 2002, pp. 1–36.
[43] *Evolved Universal Terrestrial Radio Access (E-UTRA): Further Advancements for E-UTRA Physical Layer Aspects (Release 9)*, document 3GPP TS 36.814 V9.0.0, 3GPP Technical Specification Group Radio Access Network, 2010.



HYEON MIN KIM received the B.S. degree in electronic engineering from Soongsil University, Seoul, South Korea, in 2012, where he is currently pursuing the joint master's and Ph.D. degree with the Department of ICMC Convergence Technology.

Since 2012, he has been with the Wireless Communications Laboratory, Soongsil University. His research interests include wireless communications, with particular focus on device-to-device communications, 5G systems, and signal processing.

HIEU V. NGUYEN received the B.S. degree in electronics and telecommunications from the Danang University of Science and Technology, Danang, Vietnam, in 2011, and the M.S. degree in electronic engineering from Soongsil University, South Korea, in 2016, where he is currently pursuing the Ph.D. degree with the Department of Information and Telecommunication Engineering.

From 2011 to 2013, he was with the Danang University of Science and Technology, as an Assistant Researcher and a Lecturer. Since 2014, he has been with the Wireless Communications Laboratory, Soongsil University. His research interests include wireless communications, with particular focus on optimization techniques, device-to-device communications, 5G systems, energy harvesting, beamforming design, and cognitive radio.

GIL-MO KANG received the B.S. and M.S. degrees in electronic engineering from Soongsil University, Seoul, South Korea, in 2013 and 2015, respectively, where he is currently pursuing the Ph.D. degree with the Department of ICMC Convergence Technology.

Since 2013, he has been with the Wireless Communication Laboratory, Soongsil University. In 2016, he received the Global Ph.D. Fellows Scholarship from the National Research Foundation of Korea, during the period of 2016–2019. His research interests include wireless communications, with particular focus on device-to-device communications, beamforming, 5G systems, and V2X communications.

YOAN SHIN received the B.S. and M.S. degrees in electronics engineering from Seoul National University, Seoul, South Korea, in 1987 and 1989, respectively, and the Ph.D. degree in electrical and computer engineering from The University of Texas at Austin, in 1992.

From 1992 to 1994, he was with Microelectronics and Computer Technology Corporation, Austin, TX, USA, as a Member of Technical Staff. Since 1994, he has been with the School of Electronic Engineering, Soongsil University, Seoul, where he is currently a Professor. From 2009 to 2010, he was a Visiting Professor with the Department of Electrical and Computer Engineering, The University of British Columbia, Vancouver, BC, Canada. His research interests include magnetic induction communication, localization, and compressive sensing. He has been serving as an Organizing/Technical Committee Member for various prominent international conferences, including VTC 2003, ISITA 2006, ISPLC 2008, APCC 2008, ISIT 2009, APWCS 2009, APWCS 2010, APCC 2010, ICTC 2010, APWCS 2011, ICTC 2011, ISAP 2011, APWCS 2012, and APCC 2012. In particular, he was the Technical Program Committee Chair of APWCS 2013 and the General Co-Chair of APWCS 2014 and APWCS 2015.

OH-SOON SHIN received the B.S., M.S., and Ph.D. degrees in electrical engineering and computer science from Seoul National University, Seoul, South Korea, in 1998, 2000, and 2004, respectively.

From 2004 to 2005, he was with the Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA, USA, as a Postdoctoral Fellow. From 2006 to 2007, he was a Senior Engineer with Samsung Electronics, Suwon, South Korea. In 2007, he joined the School of Electronic Engineering, Soongsil University, Seoul, where he is currently a Professor. His research interests include communication theory, wireless communication systems, and signal processing for communication.