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DIAGNOSTICS WITH SCHOTTKY NOISE

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1. *INTRODUCTION*

In 1918, Schottky described the statistical current fluctuations caused by the finite number of charge carriers[1]. These fluctuations are often used in circular accelerators, and even more in storage rings, to study the machine's performance. They produce noise-like signals in pick-ups and thus allow non-destructive beam observation.

While coherent signals in pick-ups (e.g. from bunched beams) provide a signal power varying with the square of the number of particles N, Schottky noise is incoherent, i.e. each particle contributes independently to the output power, which is therefore proportional to N. As a consequence, Schottky signals are often relatively weak; they have to compete with the thermal noise from the pick-up amplifier. However, with present-day techniques they are easily observable and often provide a rich source of diagnostic information.

Schottky signals are also used for stochastic cooling. This subject will not be treated here.

Several excellent papers about Schottky diagnostics exist[2,3], on which this presentation does not claim to be an improvement. Some applications to the CERN Antiproton Accumulator (AA) will be mentioned after a repetition of the general principles.

2. *LONGITUDINAL SCHOTTKY SIGNALS*

A single particle with revolution frequency $f_0 = \omega_0/2\pi$, observed at a given point along the circumference of the machine, represents a current consisting of a delta function repeating at the revolution frequency. For a particle with charge e, passing the observation point at times 0, $1/f_0$, $2/f_0$, etc., this may be expressed by Fourier analysis as a series of cosine waves:

$$i(t) = ef_0 \sum_{n=-\infty}^{\infty} \exp(jn\omega_0 t) = ef_0 + 2ef_0 \sum_{n=1}^{\infty} \cos(n\omega_0 t) \ . \qquad (1)$$

Note that the contributions from negative frequencies add up to their positive counterparts.

This so-called longitudinal frequency spectrum thus consists of lines (called Schottky lines) at all harmonics of f_0.

For an unbunched beam of many particles with slightly different values of f_0, the spectrum will consist of bands, each band containing many lines. Within each band, the spectrum is indistinguishable from filtered random noise, with a total power proportional to N. It is worth while to understand in detail why this is so. In fact, the signal is the sum of a finite number of single-frequency components, which is definitely not the same as random noise. Two properties of the signal are important:

a) If N is large, the average frequency difference between adjacent lines Δf is extremely small. To observe this, a very long time ($>1/\Delta f$) would be needed. Thus, in practice we never see the fine structure.

b) Those particles that have the same frequency (within the measurement resolution) will have random phases. Therefore, their signals (1), with phase included, will add up quadratically and the total power will be proportional to N.

The width of each band is $n\Delta f_0$, where Δf_0 is the revolution frequency spread. Since, according to Eq. (1), the power in each band is the same, bands with higher n will have a lower power density. The width will become larger than the distance between bands above the harmonic number $n = f_0/\Delta f_0$, i.e. at frequencies higher than $f_0^2/\Delta f_0$. For diagnostic purposes it is usually imperative to stay below this frequency, to avoid overlapping bands.

The most obvious use of the observation of longitudinal Schottky bands is the determination of the revolution frequency distribution. As an illustration, Fig. 1 shows the longitudinal distribution of antiprotons in the AA ring at CERN, measured at n = 39, around 72 MHz, using a resonant pick-up for better sensitivity. Such distributions are simply obtained by connecting the amplified signal from a sum pick-up to a spectrum analyzer. In this example, the tails of the distribution may be seen in the presence of a peak power density that is more than 10^5 times higher. This is important for monitoring the addition of fresh particles to the stack, at low density.

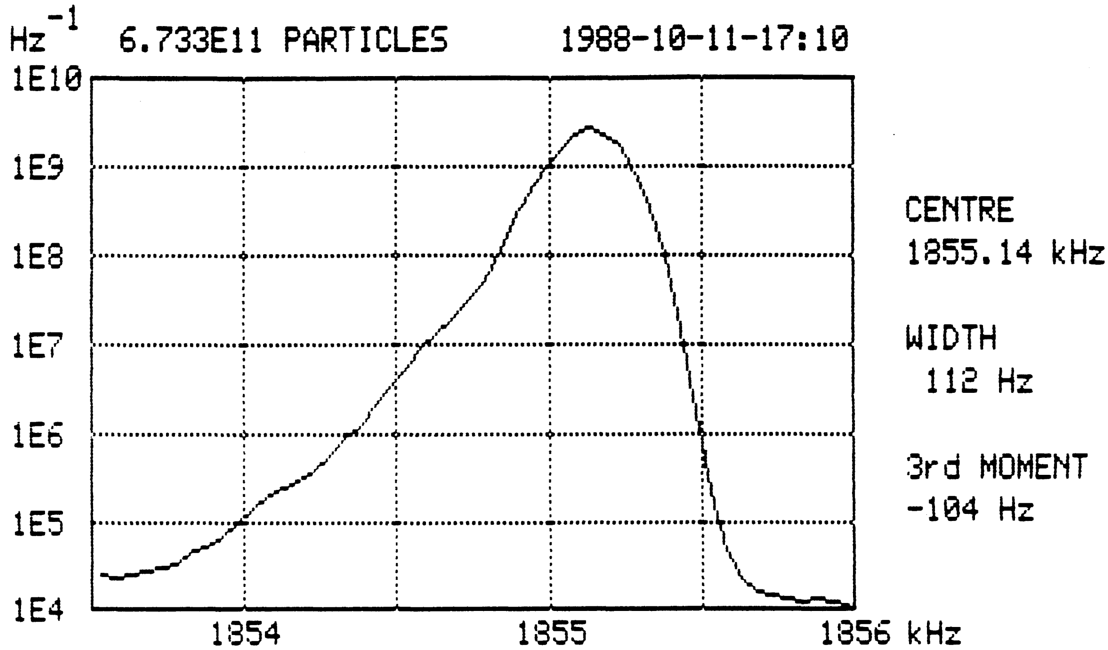

Fig. 1 - Typical longitudinal distribution of AA stack. Particle density is plotted vs revolution frequency.

Experience shows that in storage rings with extensive Schottky diagnostics the revolution frequency is a more convenient parameter than the particle momentum. However, if required, the momentum distribution may also easily be found if the parameter

$$\eta = \frac{df_0/f_0}{dp/p} = \frac{1}{\gamma^2} - \frac{1}{\gamma_T^2} \tag{2}$$

is known (γ = energy over rest mass, γ_T = γ at transition, p = momentum). Unfortunately, some authors define η with the opposite sign. Since the definition given here is more logical (and more frequent), the alternative should be avoided.

Longitudinal Schottky signal detectors may also be useful for measuring the total number of particles in a circulating beam, since they normally are far more sensitive than current transformers. As an example, in the ICE ring at CERN, beams of 80 antiprotons (beam current 50 pA) could easily be observed using a sensitive resonant pick-up[4]. Schnell[5] has pointed out that with a sufficiently sensitive pick-up, similar to multi-cavity accelerating structures, single particles could be observed. As far as I know, nobody has yet done this so far.

Schottky scans are sometimes a sensitive indicator for the occurrence of instabilities. For instance, if a beam is debunched, filamentation occurs and the filaments may in places have a very small local momentum spread, leading to microwave instability for small clusters of particles. This is seen as a slow explosion of the Schottky band, even at frequencies well below the unstable region; a small subset of particles produces a large (because coherent) signal that drifts away from the normal Schottky band because the unstable particles are decelerated. The phenomenon may be striking even if few particles are involved and the loss is hardly observable with a current transformer.

3. *TRANSVERSE SIGNALS*

If a difference pick-up is used instead of a sum pick-up, the signal (1) must be multiplied by the beam position, which varies with the betatron frequency Qf_0. An ideal difference pick-up will measure the dipole moment (current × displacement). For a single particle with amplitude A this is:

$$\begin{aligned} d(t) &= \left[ef_0 \sum_{n=-\infty}^{\infty} \exp(jn\omega_0 t) \right] A \cos(Q\omega_0 t) \\ &= Aef_0 \left[\sum_{n=0}^{\infty} \{(n+q)\omega_0 t\} + \sum_{n=1}^{\infty} \cos\{(n-q)\omega_0 t\} \right] \end{aligned} \qquad (2)$$

where q is the fractional part of Q. The two terms represent lines at frequencies $(n\pm q)f_0$, one on each side of each revolution harmonic. The n+q and n-q lines are usually called "fast wave" and "slow wave".

The lines again become bands for beams with frequency spread. However, the situation is slightly more complicated than for the longitudinal bands, because q may vary with f_0. If this dependence is linear, the relative width of the transverse and longitudinal bands is the absolute value of

$$\frac{d(n\pm q)f_0}{d(nf_0)} = \frac{n\pm q}{n} \pm \frac{f_0}{n}\frac{dq}{df_0} = 1 \pm \frac{1}{n}\left(q + \frac{\xi Q}{\eta}\right) \qquad (3)$$

where $\xi = (p/Q)(dq/dp)$ is the chromaticity. Depending on the sign and amplitude of ξ and η, Eq. (3) may be larger or smaller than unity and even become zero or negative. At high frequencies (large n) the second term is negligible. Usually (for not too small n) one sideband is wider, the other narrower than the longitudinal band.

If the dependence of q on f_0 is non-linear, the sign of Eq. (3) may vary inside a band. In this case, the band will seem to fold over upon itself.

The width of the transverse bands may also be increased if Q depends on amplitude.

As may be seen from (2), the power in each transverse band is the same for uncorrelated particles with random phase. The power is proportional to A^2, i.e. to emittance. Thus, the beam emittance may be found by measuring the total power in a transverse band and dividing it by the beam current.

In the presence of coupling between the horizontal and vertical plane, each plane's Schottky scan will be contaminated with a small fraction of the other plane's signal. If the tunes are different, this may be used as a sensitive criterion for adjusting skew quadrupole strength to cancel the coupling.

4. *MEASUREMENT OF* q

By measuring the frequency of two sidebands $f_+ = (n+q)f_0$ and $f_- = (n-q)f_0$, the q value may be found:

$$q = n \frac{f_+ - f_-}{f_+ + f_-} \tag{4}$$

What is found is the incoherent q value. This is in contrast to the method where the entire beam is kicked and the coherent signals are observed; this results in the coherent tune.

For a precise measurement, a narrow and symmetric distribution is preferable; otherwise the exact shape of each sideband depends on chromaticity and the frequency is not well defined. Since the lower n values give narrower lines, these would be preferable from this point of view. They also give a better signal-to-noise power density ratio. However, the lower frequencies are more sensitive to influence from feedback via the beam environment (transverse impedance), which may distort the results under conditions near transverse instability. In fact, this feedback also depends on the spectral density. In practice, lines with n above 20 are usually perfectly satisfactory.

If a bunched beam is used, the sidebands will become symmetric (as will be discussed in section 7) and the average revolution frequency will be precisely determined. This facilitates the measurement. Also, both the rf sytem and the spectrum analyzer may be computer-controlled, so that a measurement of Q vs revolution frequency may be automatically performed in a short time (Fig. 2).

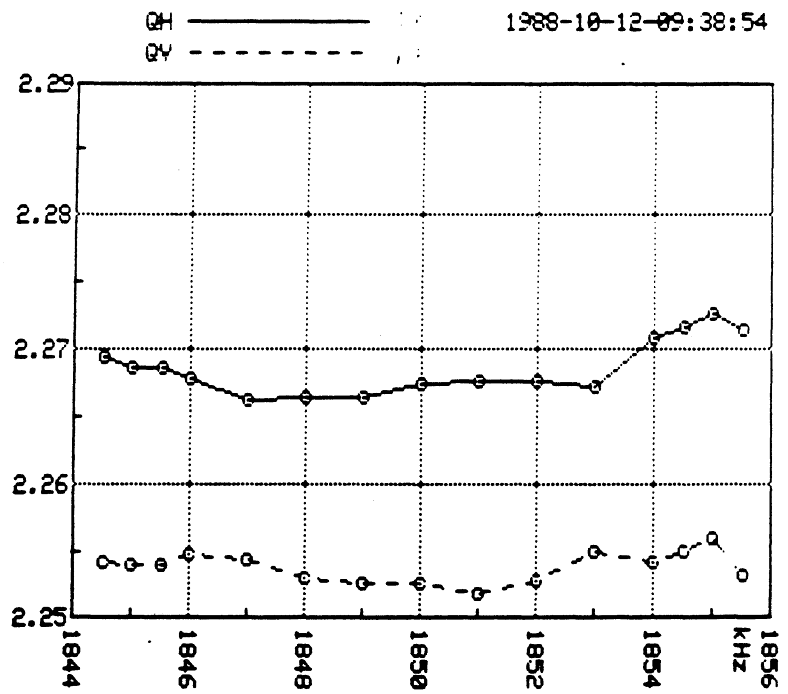

Fig. 2 - Plot of Q values vs revolution frequency (kHz) in the AA. The precision of these measurements is ±3 × 10^{-4}.

5. MEASUREMENT OF TUNE AND EMITTANCE VS REVOLUTION FREQUENCY IN WIDE DEBUNCHED STACKS

In storage rings with wide debunched stacks such as the ISR or the AA, it is very important to know the exact tune distribution vs frequency throughout the stack. The reason for this is that for a good beam lifetime non-linear resonances, even of high order, must be avoided.

The problem is to find which points in n+q and n-q bands correspond to the same revolution frequency. In the ISR, this was first done by recognizing salient points in the distribution (such as the stack edges, or peaks that could be ascribed to the excitation of resonances)[2]. For these points, the tunes could then be found as above. However, for stacks with a flat distribution this method is unsatisfactory, especially since the sidebands may be distorted by the chromaticity effect.

One solution to this problem was to make small dips (markers) in the longitudinal stack distribution by moving small empty buckets into the stack up to regularly spaced frequencies. The dips served to find corresponding points in both transverse bands.

In the meantime, Hereward had proposed a better method, which is nowadays used routinely in the AA where, in fact, not only the tunes, but also the emittances are measured vs revolution frequency. This method is based on the fact that each particle contributes the same power to all bands. We now consider a plot of power density vs frequency for a fast-wave and slow-wave sideband (Fig. 3). We shall assume that we work in a region where expression (3) is positive. We may then find corresponding points in the two bands by noting that the total power below this point should be the same for both bands. We may then determine q as before.

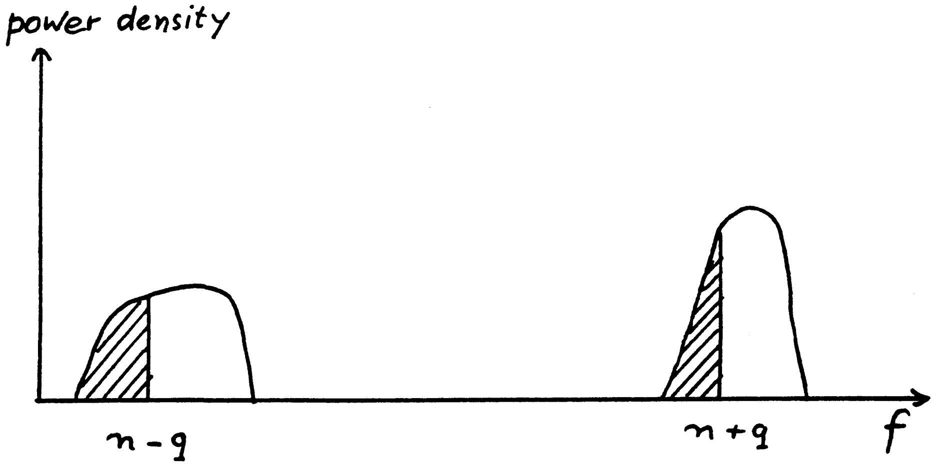

Fig. 3 - Corresponding points in n-q and n+q bands found by equal subdivision of areas.

To obtain the emittance, we use the fact that the spectral power density ψ in each point is proportional to

$$\varepsilon \frac{dN}{d(n \pm q) f_0} \tag{5}$$

where ε is the local emittance. Using expression (3) we find

$$\psi \sim \frac{\varepsilon}{n} \frac{dN}{df_0} \frac{1}{1 \pm (1/n)(q + \xi Q/\eta)} \quad . \tag{6}$$

Of course, ξ, Q, η all depend on f_0. However, if we call the density in corresponding points of the fast and slow bands ψ_+ and ψ_-, we find from Eq. (6)

$$\frac{\psi_+ \psi_-}{\psi_+ + \psi_-} \sim \frac{\varepsilon}{n} \frac{dN}{df_0} \tag{7}$$

independent of chromaticity, etc. Since we can obtain dN/df_0 from the longitudinal Schottky scan, this gives us the emittance vs f_0, at least in relative units. Calibration is in principle possible if the characteristics of pick-ups and amplifiers are measured. In practice, it is easier to do the calibration with a destructive measurement, using scrapers.

What is measured is the rms emittance. For gaussian distributions this is proportional to the "95%" emittance which is more often used in proton machines.

Figure 4 illustrates the results for a typical AA antiproton stack. The dotted curves in the background indicate the longitudinal distribution on a logarithmic scale covering, in this example, about 5 decades. Here, the tunes are nearly equal for both planes, and coupling effects make the emittances equal as well. The emittance measurement becomes less precise in the extreme tails where the density is less than 10^{-3} times the peak density. Errors are caused in these regions by coupling between both planes and by the limited resolution of the spectrum analyzer (necessary to obtain a reasonable measurement time). The increasing emittances at each edge of the stack are real. They are caused by the neighbourhood of high-order non-linear resonances. Monitoring of the emittances and the tunes (lying between 11th order and 26th order resonances) turns out to be quite important for efficient operation of the machine.

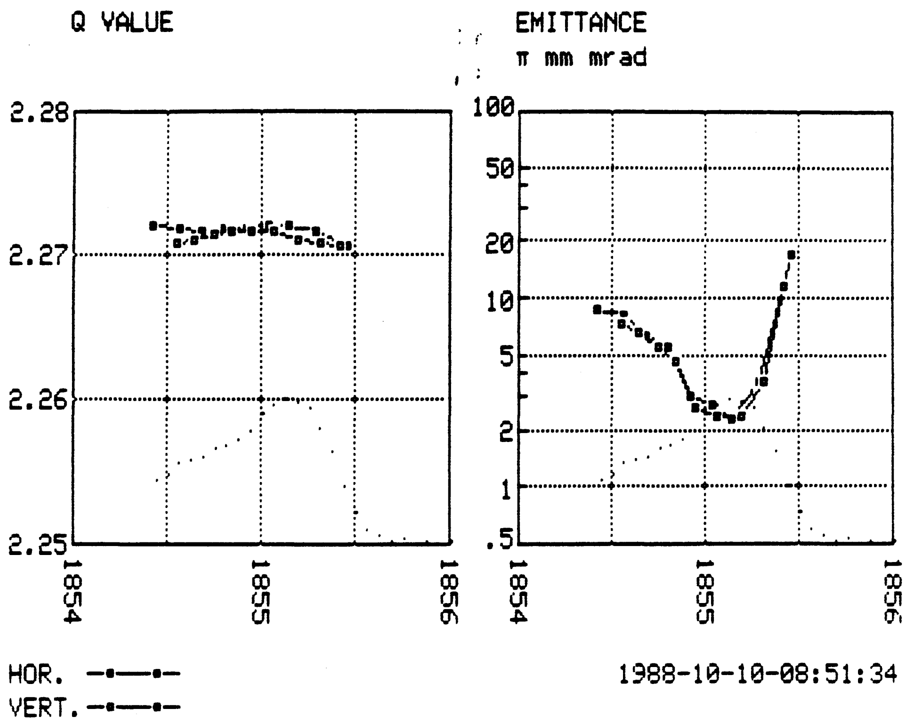

Fig. 4 - *Q values and emittance vs revolution frequency measured for a typical AA antiproton stack.*

6. SCHOTTKY SIGNALS FROM BUNCHED BEAMS (LONGITUDINAL)

If the beam is bunched, the Schottky bands will have a more complicated structure. Particles will execute synchrotron oscillations, so that for a single particle and a sum pick-up, the time t in Eq. (1) will be replaced by $t+\tau \sin(\Omega t+\varphi)$, where Ω is the synchrotron frequency. We may obtain the frequency spectrum by using the expansion[6]

$$\cos[at + b\sin(ct+\varphi)] = \sum_{k=-\infty}^{\infty} J_k(b) \cos[(a+kc)t + k\varphi] \tag{8}$$

where J_k is the Bessel function of order k and the sum is taken over all integer values of k. The result is for the longitudinal signal

$$i(t) = ef_0 + 2ef_0 \sum_{n=1}^{\infty} \sum_{k=-\infty}^{\infty} J_k(n\omega_0\tau) \cos[(n\omega_0+k\Omega)t + k\varphi] . \tag{9}$$

Each line is now split up into an infinite number of satellites spaced at the synchrotron frequency (Fig. 5). Their relative amplitudes are given by $J_k(n\omega_0\tau)$. The shape of the envelope is irregular and depends on the exact value of $n\omega_0\tau$. For many particles, the envelope will become bell-shaped because of the quadratic averaging over τ. Also, the signals will again add up incoherently because of the random synchrotron phase φ. However, this is not true for the central line (k = 0) at each n. Because ω_0 is now the synchronous revolution frequency, i.e. the same for all particles, the central line will be coherent and its power will vary with N^2. Therefore, the longitudinal Schottky bands are sometimes difficult to observe with a bunched beam; the strong central line may saturate the sensitive pick-up amplifier. Suppressing the low frequencies before the amplifier may help; at large $n\omega_0\tau$ the average of $J_0(n\omega_0\tau)$ over τ becomes small.

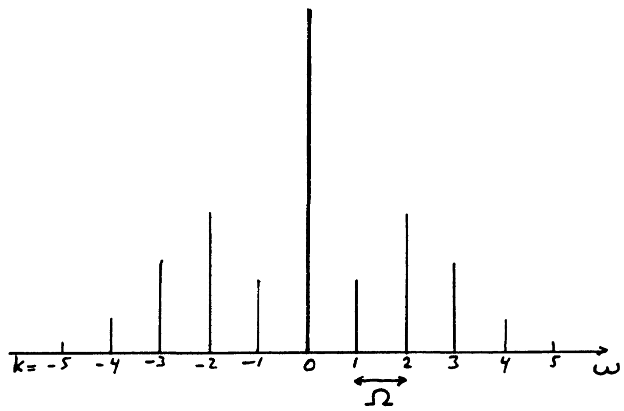

Fig. 5 - Schottky lines for a single particle and a single n value. The lines are spaced by the synchrotron frequency.

The envelope over the satellites has about the same width as the band that would be observed with a debunched beam of the same frequency spread. In fact, such a beam would have Schottky bands of half-width $n\omega_0\tau\Omega$. This would correspond to $K = n\omega_0\tau$ and for k larger than its argument $n\omega_0\tau$ the Bessel function J_k becomes negligible.

At high values of k, the satellites broaden because of the spread of Ω. They may even overlap; this especially happens at large n, where the maximum k is large as well.

The longitudinal Schottky scan of a bunched beam may be used to measure the synchrotron frequency. Otherwise, it is of little diagnostic interest.

7. TRANSVERSE SIGNALS FROM BUNCHED BEAMS

For the transverse signals, we may treat Eq. (2) in exactly the same way. Each sideband will again be split into satellite bands. The only difference is that the signals from different particles now add up incoherently even for the central line with k = 0. The reason is that the particles have random betatron phase (whereas longitudinally they are kept together by the bunching).

Since the power from all particles now again adds up linearly, the total power in each sideband is the same as for an unbunched beam. In fact, if the synchrotron frequency is low compared to the width of a band, the measurement resolution may be chosen so that separate satellites are not resolved. The bands then look much like the bands for an unbunched beam, except that their distribution is now symmetrical. This is advantageous for Q measurement; we obtain a well-defined Q value, in first order independent of chromaticity and valid for the average revolution frequency, now determined by the rf frequency.

8. SIGNAL TREATMENT

The easiest way to observe Schottky spectra is by means of a spectrum analyzer. These instruments have reached a high degree of perfection and may be controlled and read by computer. Automatic sequences may be programmed, e.g. for subtracting amplifier noise from the Schottky noise, or for combining the data from longitudinal and transverse bands as described in section 5.

One problem with spectrum analyzers is that the measurement sometimes is slow. This is because these instruments scan through a given frequency span Δf_s with a given resolution Δf_r. For observing the noise power density within Δf_r with a good precision, a time large compared to $1/\Delta f_r$ is needed. (In fact, for a measurement time T the rms error is $1/\Delta f_r T$). Thus, the total scan time must be large compared to $\Delta f_s/(\Delta f_r)^2$.

This time may be minimized by choosing a high harmonic, since both Δf_s and and Δf_r are proportional to n. However, at high harmonics the signal-to-noise ratio decreases. Also, increasing the frequency is useless in the case of Q measurements because the frequency error required does not depend on n.

For fast measurements and high resolution it is possible to use a so-called FFT (fast Fourier transform) signal analyzer. This works by continuously sampling and digitizing the signal at a fixed frequency. The time record so obtained is transformed into the frequency domain by a built-in processor that computes the Fourier transform. Since the apparatus now continuously monitors the signal over the full frequency span, the observation time must be long compared to $1/\Delta f_r$ only and a factor $\Delta f_s/\Delta f_r$ is gained. A condition is that the computation does not add much to the total measuring time; this depends on the equipment used and is not always guaranteed. However, a considerable gain usually still remains, either in speed or in resolution.

The FFT analyzers now available cover a band from 0 to 50 or 100 kHz (i.e. half the sampling frequency), which is often well-matched to the width of Schottky bands. However, the signal must first be converted to the low-frequency region by mixing it with a carrier. A single-sideband mixer is useful to avoid doubling the noise background and to eliminate disturbing signals at mirror frequencies.

With such an equipment it is now possible to observe the longitudinal cooling in the Antiproton Collector in real time. The width of the longitudinal bands decreases by a large factor within a few seconds. It is also possible to make near-instantaneous measurements of a distribution to monitor the loss of particles during the various cooling processes without interrupting normal operation.

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