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Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis

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Abstract. This paper introduces the concept of a single valued neutrosophic multiset (SVNM) as a generalization of an intuitionistic fuzzy multiset (IFM) and some basic operational relations of SVNMs, and then proposes the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigates their properties. Finally, the Dice similarity measure is applied to a medical diagnosis problem with SVNM information. This diagnosis method can deal with the medical diagnosis problem with indeterminate and inconsistent information which cannot be handled by the diagnosis method based on IFMs.

Keywords: Single valued neutrosophic set, multiset, single valued neutrosophic multiset, Dice similarity measure, medical diagnosis.

1 Introduction

In medical diagnosis problems, physicians can obtain a lot of information from modern medical technologies, which is often incomplete and indeterminate information due to the complexity of various diseases. Therefore, real medical diagnosis contains lots of incomplete and uncertainty information, which is a usual phenomenon of medical diagnosis problems. To represent incomplete and uncertainty information, Atanassov [1] introduced intuitionistic fuzzy sets (IFSs) as a generalization of fuzzy sets [2]. The prominent characteristic of IFS is that a membership degree and a non-membership degree are assigned to each element in the set. Then, various medical diagnosis methods have been presented under intuitionistic fuzzy environments [3, 4]. Recently, Ye [5] proposed a cosine similarity measure between IFSs and applied it to pattern recognition and medical diagnosis. Hung [6] introduced an intuitionistic fuzzy likelihood-based measurement and applied it to the medical diagnosis and bacteria classification problems. Further, Tian [7] developed the cotangent similarity measure of IFSs and applied it to medical diagnosis.

As a generalization of fuzzy sets and IFSs, Wang et al. [8] introduced a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set proposed by Smarandache [9]. SVNS consists of the three terms like the truth-membership, indeterminacy-membership and falsity-membership functions and can be better to express indeterminate and inconsistent information, but fuzzy sets and IFSs cannot handle indeterminate and inconsistent information. However, similarity measures play an important role in the analysis and research of medical diagnosis, pattern recognition, machine learning, decision making, and clustering analysis in uncertainty environment. Therefore, various similarity measures of SVNSs have been proposed and mainly applied them to decision making and clustering analysis. For instance, Majumdar and Samanta [10] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [11] proposed three vector similarity measures for simplified neutrosophic sets (SNSs), including the Jaccard, Dice, and cosine similarity measures for SVNSs and interval neutrosophic sets (INSs), and applied them to multicriteria decision-making problems with simplified neutrosophic information. Ye [12] and Ye and Zhang [13] further proposed the similarity measures of SVNSs for decision making problems. Furthermore, Ye [14] put forward distance-based similarity measures of SVNSs and applied them to clustering analysis.

In real medical diagnosis problems, however, by only taking one time inspection, we wonder whether one can obtain a conclusion from a particular person with a particular decease or not. Sometimes he/she may also show the symptoms of different diseases. Then, how can we give a proper conclusion? One solution is to examine the patient at different time intervals (e.g. two or three times a day). In this case, a fuzzy multiset concept introduced by Yager [15] is very suitable for expressing this information at different time intervals, which allows the repeated occurrences of any element. Thus, the fuzzy multiset can occur more than once with the possibility of the same or different membership values. Then, Shinoj and Sunil [16] extended the fuzzy multiset to the intuitionistic fuzzy multiset (IFM) and presented some basic operations and a distance measure for IFMs, and then applied the distance measure to medical diagnosis problem. Rajarajeswari and Uma [17] presented the Hamming distance-based similarity measure for IFMs and its application in medical diagnosis. However, existing IFMs cannot represent and deal with the indeterminacy and inconsistent information which exists in real situations (e.g. medicine diagnosis problems). To handle the medical diagnosis problems with indeterminacy and inconsistent information, the aims of this paper are: (1) to introduce a single valued neutrosophic multiset (SVNM) as a generalization of IFMs and some operational relations for SVNMs, (2) to propose the Dice similarity measure of SVNMs, (3) to apply the Dice similarity measure to medical diagnosis.

The rest of the article is organized as follows. Section 2 introduces some basic concepts of IFSs, IFMs, and SVNSs. Sections 3 introduces a concept of SVNM and some operational relations of SVNMs. In Section 4, we present the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigate their properties. In Section 5, we apply the proposed similarity measure to a medical diagnosis problem. Conclusions and further research are contained in Section 6.

2 Preliminaries

2.1 Some basic concepts of IFSs and IFMs

Atanassov [1] introduced IFSs as an extension of fuzzy sets [2] and gave the following definition.

Definition 1 [1]. An IFS A in the universe of discourse X is defined as $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$, where $\mu_A(x)$: $X \to [0, 1]$ and $\nu_A(x)$: $X \to [0, 1]$ are the membership degree and non-membership degree of the element x to the set A with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$ for $x \in X$.

Then, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called Atanassov's intuitionistic index or a hesitancy degree of the element x in the set A. obviously there is $0 \le \pi_A(x) \le 1$ for $x \in X$.

Further, Shinoj and Sunil [16] introduced an IFM concept by combining the two concepts for IFSs and fuzzy multisets together and gave the following definition.

Definition 2 [16]. Let X be a nonempty set. Then, an IFM drawn from X is characterized by the two functions: count membership of CM_A and count non-membership of CN_A such that $CM_A(x)$: $X \to R$ and $CN_A(x)$: $X \to R$ for $x \in X$, where R is the set of all real number multisets drawn from the unit interval [0, 1]. Thus, an IFM A is denoted by

$$A=\left\{\left\langle x,(\mu_A^1(x),\mu_A^2(x),...,\mu_A^q(x)),(\nu_A^1(x),\nu_A^2(x),...,\nu_A^q(x))\right\rangle \mid x\in X\right\},$$

where the membership sequence $(\mu_A^1(x),\mu_A^2(x),...,\mu_A^q(x))$ is a decreasingly ordered sequence $\mu_A^1(x)\ge\mu_A^2(x)\ge,...\ge,\mu_A^q(x)$, the corresponding non-membership sequence $(\nu_A^1(x),\nu_A^2(x),...,\nu_A^q(x))$ may not be in decreasing or increasing order, and the sum of $\mu_A^i(x)$ and $\nu_A^i(x)$ satisfies the condition $0 \le \mu_A^i(x) + \nu_A^i(x) \le 1$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, an IFM A can be denoted by the following simplified form:

$$A=\left\{\left\langle x,\mu_A^i(x),\nu_A^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\}.$$

Let $A=\left\{\left\langle x,\mu_A^i(x),\nu_A^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\}$ and $B=\left\{\left\langle x,\mu_B^i(x),\nu_B^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\}$ be two IFMs. Then there are the following relations [16]:

(1) Complement: $A^c=\left\{\left\langle x,\nu_A^i(x),\mu_A^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\}$;

(2) Inclusion: $A \subseteq B$ if and only if $\mu_A^i(x) \le \mu_B^i(x)$, $\nu_A^i(x) \ge \nu_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:
$$A\cup B=\left\{\left\langle x,\mu_A^i(x)\vee\mu_B^i(x),\nu_A^i(x)\wedge\nu_B^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\};$$

(5) Intersection:
$$A\cap B=\left\{\left\langle x,\mu_A^i(x)\wedge\mu_B^i(x),\nu_A^i(x)\vee\nu_B^i(x)\right\rangle \mid x\in X, i=1,2,...,q\right\};$$

(6) Addition:
$$A+B=\left\{\begin{matrix}\left\langle x,\mu_A^i(x)+\mu_B^i(x)-\mu_A^i(x)\mu_B^i(x),\nu_A^i(x)\nu_B^i(x)\right\rangle\\ \mid x\in X, i=1,2,...,q\end{matrix}\right\};$$

(7) Multiplication:
$$A\times B=\left\{\begin{matrix}\left\langle x,\mu_A^i(x)\mu_B^i(x),\nu_A^i(x)+\nu_B^i(x)-\nu_A^i(x)\nu_B^i(x),\right\rangle\\ \mid x\in X, i=1,2,...,q\end{matrix}\right\}.$$

2.2 Some concepts of SVNSs

Smarandache [9] originally presented the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

However, the neutrosophic set introduced from philosophical point of view is difficult to apply it to practical applications. Thus, Wang et al. [8] introduced a SVNS as a subclass of the neutrosophic set and the following definition of SVNS.

Definition 3 [8]. Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted as

$$A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \right\},$$

where the sum of $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ satisfies $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for each x in X.

For two SVNSs $A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \right\}$ and $B = \left\{ \langle x, T_B(x), I_B(x), F_B(x) \rangle \mid x \in X \right\}$, there are the following relations [8]:

(1) Complement: $A^c = \left\{ \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle \mid x \in X \right\}$;

(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:
$$A \cup B = \left\{ \langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) \rangle \mid x \in X \right\};$$

(5) Intersection:
$$A \cap B = \left\{ \langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) \rangle \mid x \in X \right\}.$$

3 Single valued neutrosophic multisets

This section introduces SVNMs as a generalization of SVNSs and IFMs and some operational relations for SVNMs.

Definition 4. Let X be a nonempty set with generic elements in X denoted by x. A SVNM A drawn from X is characterized by the three functions: count truth-membership of CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x)$: $X \to R$, $CI_A(x)$: $X \to R$, $CF_A(x)$: $X \to R$ for $x \in X$, where R is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by

$$A = \left\{ \left\langle \begin{array}{l} x, (T_A^1(x), T_A^2(x), \ldots, T_A^q(x)), (I_A^1(x), I_A^2(x), \ldots, \\ I_A^q(x)), (F_A^1(x), F_A^2(x), F_A^q(x)) \end{array} \right\rangle \mid x \in X \right\},$$

where the truth-membership sequence $(T_A^1(x), T_A^2(x), \ldots, T_A^q(x))$, the indeterminacy-membership sequence $(I_A^1(x), I_A^2(x), \ldots, I_A^q(x))$, and the falsity-membership sequence $(F_A^1(x), F_A^2(x), \ldots, F_A^q(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x)$, $I_A^i(x)$, $F_A^i(x) \in [0, 1]$ satisfies the condition $0 \le \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \le 3$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, a SVNM A can be denoted by the simplified form:

$$A = \left\{ \langle x, T_A^i(x), I_A^i(x), F_A^i(x) \rangle \mid x \in X, i = 1,2,\ldots,q \right\}.$$

Definition 5. The length of an element x in a SVNM is defined as the cardinality of $CT_A(x)$ or $CI_A(x)$, or $CF_A(x)$ and is denoted by $L(x\text{: } A)$. Then $L(x\text{: } A) = |CT_A(x)| = |CI_A(x)| = |CF_A(x)|$.

Definition 6. Let A and B be two SVNMs in X, then the length of an element x in A and B is denoted by $l_x = L(x\text{: } A, B) = \max\{L(x\text{: } A), L(x\text{: } B)\}$.

For example, we consider SVNMs in the set $X = \{x_1, x_2, x_3\}$ as

$A = \{<x_1, (0.1, 0.2), (0.2, 0.3), (0.6, 0.8)>, < x_2, (0.3, 0.4, 0.5), (0.2, 0.3, 0.4), (0.5, 0.6, 0.7)>\}$,

$B = \{<x_1, (0.2), (0.2), (0.4) >, < x_3, (0.3, 0.4, 0.5, 0.6), (0.1, 0.2, 0.3, 0.4), (0.1, 0.2, 0.3, 0.5)>\}$.

Thus, there are $L(x_1\text{: } A) = 2$, $L(x_2\text{: } A) = 3$, $L(x_3\text{: } A) = 0$; $L(x_1\text{: } B) = 1$, $L(x_2\text{: } B) = 0$, $L(x_3\text{: } B) = 4$, $l_{x1} = L(x_1\text{: } A, B) = 2$, $l_{x2} = L(x_2\text{: } A, B) = 3$, and $l_{x3} = L(x_3\text{: } A, B) = 4$.

For convenient operation between SVNMs A and B in X, one can make $L(x\text{: } A) = L(x\text{: } B)$ by appending sufficient minimal numbers for the truth-membership degree and sufficient maximum numbers for the indeterminacy-membership and falsity-membership degrees as pessimists or sufficient maximum numbers for the truth-membership value and sufficient minimal numbers for the indeterminacy-membership and falsity-membership values as optimists.

Definition 7. Let $A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x, T_B^i(x), I_B^i(x), F_B^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ be two SVNMs in X. Then, there are the following relations:

(1) Inclusion: $A \subseteq B$ if and only if $T_A^i(x) \le T_B^i(x)$, $I_A^i(x) \ge I_B^i(x)$, $F_A^i(x) \ge F_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

(2) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(3) Complement:
$$A^c = \left\{ \langle x, F_A^i(x), 1 - I_A^i(x), T_A^i(x) \rangle \mid x \in X, i = 1,2,\ldots,q \right\};$$

(4) Union:
$$A \cup B = \left\{ \begin{array}{l} \langle x, T_A^i(x) \vee T_B^i(x), I_A^i(x) \wedge I_B^i(x), F_A^i(x) \wedge F_B^i(x) \rangle \\ \mid x \in X, i = 1,2,\ldots,q \end{array} \right\};$$

(5) Intersection:
$$A \cap B = \left\{ \begin{array}{l} \langle x, T_A^i(x) \wedge T_B^i(x), I_A^i(x) \vee I_B^i(x), F_A^i(x) \vee F_B^i(x) \rangle \\ \mid x \in X, i = 1,2,\ldots,q \end{array} \right\}.$$

4 Dice similarity measure of SVNMs

In this section, we propose the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigate their properties.

Definition 8. Let $A = \{\langle x_j, T_A^i(x_j), I_A^i(x_j), F_A^i(x_j) \mid x_j \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x_j, T_B^i(x_j), I_B^i(x_j), F_B^i(x_j) \mid x_j \in X, i = 1, 2, \ldots, q\}$ be any two SVNMs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, we define the following Dice similarity measure between A and B:

$$S_D(A,B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\frac{2}{l_j}\sum_{i=1}^{l_j}\left[\begin{array}{l} T_A^i(x_j)T_B^i(x_j) + I_A^i(x_j)I_B^i(x_j) \\ + F_A^i(x_j)F_B^i(x_j) \end{array}\right]}{\left(\begin{array}{l} \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_A^i(x_j)\right)^2 + \left(I_A^i(x_j)\right)^2 + \left(F_A^i(x_j)\right)^2\right] \\ + \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_B^i(x_j)\right)^2 + \left(I_B^i(x_j)\right)^2 + \left(F_B^i(x_j)\right)^2\right] \end{array}\right)}, \quad (1)$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$.

Then, the Dice similarity measure has the following Proposition 1:

Proposition 1. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the Dice similarity measure $S_D(A, B)$ should satisfy the following properties (P1)-(P3):

(P1) $0 \le S_D(A, B) \le 1$;

(P2) $S_D(A, B) = S_D(B, A)$;

(P3) $S_D(A, B) = 1$ if $A = B$, i.e., $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, q$.

Proof:

(P1) It is obvious that the property is true according to the inequality $a^2 + b^2 \ge 2ab$ for Eq. (1).

(P2) It is straightforward.

(P3) If $A = B$, then there are $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, q$. Hence there is $S_D(A, B) = 1$. □

Taking the weight w_j of each element x_j ($j = 1, 2, \ldots, n$) into account with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, we introduce the following weighted Dice similarity measure between SVNMs A and B:

$$W_D(A,B) = \sum_{j=1}^{n} w_j \frac{\frac{2}{l_j}\sum_{i=1}^{l_j}\left[\begin{array}{l} T_A^i(x_j)T_B^i(x_j) + I_A^i(x_j)I_B^i(x_j) \\ + F_A^i(x_j)F_B^i(x_j) \end{array}\right]}{\left(\begin{array}{l} \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_A^i(x_j)\right)^2 + \left(I_A^i(x_j)\right)^2 + \left(F_A^i(x_j)\right)^2\right] \\ + \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_B^i(x_j)\right)^2 + \left(I_B^i(x_j)\right)^2 + \left(F_B^i(x_j)\right)^2\right] \end{array}\right)}, \quad (2)$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$. If $W = (1/n, 1/n, \ldots, 1/n)^T$, then Eq. (2) reduces to Eq. (1).

Then, the weighted Dice similarity measure has the following Proposition 2:

Proposition 2. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the weighted Dice similarity measure $W_D(A, B)$ should satisfy the following properties (P1)-(P3):

(P1) $0 \le W_D(A, B) \le 1$;

(P2) $W_D(A, B) = W_D(B, A)$;

(P3) $W_D(A, B) = 1$ if $A = B$, i.e., $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, q$.

By a similar proof method of Proposition 1, we can prove that the properties (P1)–(P3).

5 Medical diagnosis using the Dice similarity measure

In this section, we apply the Dice similarity measure to the medical diagnosis problem with SVNM information. The details of a typical example adapted from [16] are given below.

Let $P = \{P_1, P_2, P_3, P_4\}$ be a set of four patients, $D = \{D_1, D_2, D_3, D_4\}$ = {Viral fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases, and $S = \{S_1, S_2, S_3, S_4, S_5\}$ = {Temperature, Cough, Throat pain, Headache, Body pain} be a set of symptoms. In the medical diagnosis problem, when we have to take three different samples in three different times in a day (e.g. morning, noon and night), the characteristic values between patients and the indicated symptoms are represented by the following SVNMs:

P_1 ={<S_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>, <S_2, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.4)>, <S_3, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>, <S_4, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>, <S_5, (0.4, 0.3, 0.2), (0.6, 0.5, 0.5), (0.6, 0.4, 0.4)>};

P_2 ={<S_1, (0.5, 0.4, 0.3), (0.3, 0.3, 0.2),(0.5, 0.4, 0.4)>, <S_2, (0.9, 0.8, 0.7), (0.2, 0.1, 0.1), (0.2, 0.1, 0.0)>, <S_3, (0.6, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.3, 0.3)>, <S_4, (0.6, 0.4, 0.3), (0.3, 0.1, 0.1), (0.7, 0.7, 0.3)>, <S_5, (0.8, 0.7, 0.5), (0.4, 0.3, 0.1), (0.3, 0.2, 0.1)>;

P_3 ={<S_1, (0.2, 0.1, 0.1), (0.3, 0.2, 0.2), (0.8, 0.7, 0.6)>, <S_2, (0.3, 0.2, 0.2), (0.4, 0.2, 0.2), (0.7, 0.6, 0.5)>, <S_3, (0.8, 0.8, 0.7), (0.2, 0.2, 0.2), (0.1, 0.1, 0.0)>, <S_4, (0.3, 0.2, 0.2), (0.3, 0.3, 0.3), (0.7, 0.6, 0.6)>, <S_5, (0.4, 0.4, 0.3), (0.4, 0.3, 0.2), (0.7, 0.7, 0.5)>;

P_4 ={<S_1, (0.5, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.4, 0.3)>, <S_2, (0.4, 0.3, 0.1), (0.4, 0.3, 0.2), (0.7, 0.5, 0.3)>, <S_3, (0.7, 0.1, 0.0), (0.4, 0.3, 0.3), (0.7, 0.7, 0.6)>, <S_4, (0.6, 0.5, 0.3), (0.6, 0.2, 0.1), (0.6, 0.4, 0.3)>, <S_5, (0.5, 0.1, 0.1), (0.3, 0.3, 0.2), (0.6, 0.5, 0.4)>.

Then, the characteristic values between symptoms and the considered diseases are represented by the form of SVNSs:

D_1 (Viral fever) = {<S_1, 0.8, 0.1, 0.1>, <S_2, 0.2, 0.7, 0.1>, <S_3, 0.3, 0.5, 0.2>, <S_4, 0.5, 0.3, 0.2>, <S_5, 0.5, 0.4, 0.1>};

D_2 (Tuberculosis) = {<S_1, 0.2, 0.7, 0.1>, <S_2, 0.9, 0.0, 0.1>, <S_3, 0.7, 0.2, 0.1>, <S_4, 0.6, 0.3, 0.1>, <S_5, 0.7, 0.2, 0.1>};

D_3 (Typhoid) = {<S_1, 0.5, 0.3, 0.2>, <S_2, 0.3, 0.5, 0.2>, <S_3, 0.2, 0.7, 0.1>, <S_4, 0.2, 0.6, 0.2>, <S_5, 0.4, 0.4, 0.2>};

D_4 (Throat disease) = {<S_1, 0.1, 0.7, 0.2>, <S_2, 0.3, 0.6, 0.1>, <S_3, 0.8, 0.1, 0.1>, <S_4, 0.1, 0.8, 0.1>, <S_5, 0.1, 0.8, 0.1>}.

Then, by using Eq. (1), we can obtain the Dice similarity measure between each patient P_i (i = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in Table 1.

Table 1 Measure values of $S_D(P_i, D_j)$

	D_1 (Viral fever)	D_2 (Tuberculosis)	D_3 (Typhoid)	D_4 (Throat disease)
P_1	0.7810	0.7753	**0.8007**	0.6946
P_2	**0.7978**	0.7656	0.7969	0.6826
P_3	0.7576	0.7063	**0.7807**	0.6492
P_4	0.8188	**0.8278**	0.8266	0.7139

In Tables 1, the largest similarity measure indicates the proper diagnosis. Hence, Patient P_1 suffers from typhoid, Patient P_2 suffers from viral fever, Patient P_3 also suffers from typhoid, and Patient P_4 suffers from tuberculosis.

6 Conclusion

This paper introduced a concept of SVNM and some basic operational relations of SVNMs, and then proposed the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigated their properties. Finally, the Dice similarity measure of SVNMs was applied to medicine diagnosis under the SVNM environment. The Dice similarity measure of SVNMs is effective in handling the medical diagnosis problems with indeterminate and inconsistent information which the similarity measures of IFMSs cannot handle, because IFMSs cannot express and deal with indeterminate and inconsistent information.

In further work, it is necessary and meaningful to extend SVNMs to interval neutrosophic multisets and their operations and measures and to investigate their applications such as decision making, pattern recognition, and medical diagnosis.

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