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DIELECTRIC CONSTANT MEASUREMENT FOR THIN MATERIAL AT MICROWAVE FREQUENCIES

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Abstract—A practical problem in the reflection method for dielectric constant measurement is the difficulty to ensure the sample is placed exactly at the waveguide flange. A small position offset of the dielectric sample will give rise to some errors in calculating the dielectric constant, especially when a thin sample is used. To circumvent this problem, a method to determine the dielectric constant by measuring the transmission coefficient of the thin slab placed in a waveguide has been developed. Slab position offset from the measurement reference plane has no effect on the measurement accuracy. An explicit expression for the dielectric constant is obtained in terms of the transmission coefficient by simplifying the exact solution for transmission through a thin dielectric slab. The method is verified with measurement on Teflon of 0.5-mm thickness. The measured dielectric constant of Teflon shows excellent agreement of both ε' and ε'' with published data. Subsequently, the dielectric constant of a vegetation leaf was measured.

1. INTRODUCTION

Microwave sensors are very important for remote sensing of earth resources in the tropics. Optical sensors are relatively less useful because the sky is covered by cloud most of the time. The capability of microwaves to penetrate through cloud and rain allows for monitoring of the earth in all weather, day and night.

The dielectric constant of vegetation has a direct effect on radar backscatter measured by airborne and space-borne microwave sensors. A good understanding of the dielectric properties of vegetation leaves is vital for extraction of useful information from the remotely sensed data for earth resources monitoring and management. Prompted by

the need to measure the microwave dielectric constant of vegetation leaves, a practical technique suitable for very thin substrate is sought.

There are many techniques for dielectric constant measurements. The lumped circuit techniques are only suitable for low frequencies and high loss materials [1]. In a cavity perturbation technique [2–6], the resonant cavity size may be in term of centimeters at microwave frequencies and the size of the material sample must be much smaller than the cavity size. The achievable measurement accuracy is limited. In the transmission line techniques, reflection coefficient and transmission coefficient are measured for a section of waveguide or coaxial line filled with the material sample under test [7–13]. Adequate sample thickness must be used in order to obtain an accurate measurement result. Open-ended coaxial probes have been investigated by many researchers [14–18]. Although the method is easy to use and capable of measuring dielectric properties over a wide frequency range, the accuracy is moderate. Furthermore, the material sample must be sufficiently thick (so that the fringing field at the end of the coaxial line is confined within the material sample) and the surface in contact with the probe must be flat and free of air gaps or bubbles. In the free-space techniques, a material sample is placed between a transmitting antenna and a receiving antenna, and the attenuation and phase shift of the signal are measured [19]. The width and height of the sample (perpendicular to the wave propagation direction) must be sufficiently large to avoid problems caused by diffraction effects at the edge of the sample. An attenuation of 10 dB through the sample layer must be maintained to avoid disturbances due to multiple reflections between the sample and the antennas.

Sarabandi and Ulaby [20] have proposed a technique for measuring the dielectric constant of thin materials. A rectangular waveguide with a matched termination is used as the sample holder. The thin dielectric slab under test is placed at the input flange of the waveguide. The magnitude and phase of the voltage reflection coefficient Γ is measured using a vector network analyzer. The relationship of Γ and the complex dielectric constant ε is obtained. Approximation is made to simplify the equation to a manageable level for inversion of ε from the measured Γ. One practical problem with this technique is to hold the thin slab at exactly the input flange of the waveguide. A small offset in the positioning of the thin slab will give rise to some errors in the inversion of ε. An improved method is proposed in this paper to circumvent the problem. The complex dielectric constant is determined from measurement of transmission coefficient.

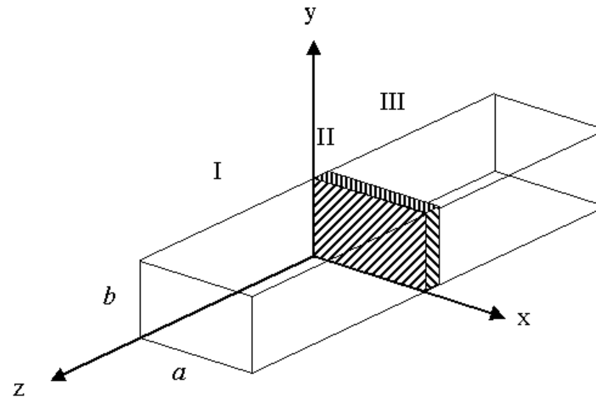


Figure 1. Rectangular waveguide with thin dielectric slab of thickness τ.

2. MODEL FOR A DIELECTRIC SLAB

Consider a rectangular waveguide of dimensions $a \times b$ containing a thin slab placed in a plane orthogonal to the propagation direction as shown in Fig. 1. The dielectric slab has a thickness τ, extending from $z = 0$ to $z = -\tau$. For TE_{10} propagation mode, the electric potentials in Regions I, II, and III are

$$\psi_I = \cos\left(\frac{\pi x}{a}\right)\left[C_1 e^{jk_z z} + C_2 e^{-jk_z z}\right] \qquad z \geq 0 \tag{1}$$

$$\psi_{II} = \cos\left(\frac{\pi x}{a}\right)\left[C_3 e^{jk_{z2} z} + C_4 e^{-jk_{z2} z}\right] \qquad 0 \geq z \geq -\tau \tag{2}$$

$$\psi_{III} = \cos\left(\frac{\pi x}{a}\right)\cdot C_5 e^{jk_z z} \qquad -\tau \geq z \tag{3}$$

The phase constant k_z in Regions I and III is given by

$$k_z = \frac{\pi}{\lambda a}\sqrt{4a^2 - \lambda^2} \tag{4}$$

where λ is the free-space wavelength. The phase constant k_{z2} in Region II is given by

$$k_{z2} = \frac{\pi}{\lambda a}\sqrt{4\varepsilon a^2 - \lambda^2} \tag{5}$$

The relative complex dielectric constant ε is

$$\varepsilon = \varepsilon' - j\varepsilon'' \tag{6}$$

The components of $\bar{E}$ and $\bar{H}$ can be obtained by applying the relations

$$\bar{E} = -\nabla \times (\psi \hat{z}) \tag{7}$$

$$\bar{H} = -j\omega\varepsilon (\psi \hat{z}) + \frac{1}{j\omega\mu} \nabla\nabla \cdot (\psi \hat{z}) \tag{8}$$

Applying the continuity conditions of the tangential $\bar{E}$ and $\bar{H}$ fields at the boundaries $z = 0$ to $z = -\tau$, we obtain the following expression for the transmission coefficient

$$T = \frac{C_5}{C_1} = \frac{2 (k_{z2}/k_z)}{2 (k_{z2}/k_z) \cos (k_{z2}\tau) + j \left[(k_{z2}/k_z)^2 + 1 \right] \sin (k_{z2}\tau)} \tag{9}$$

If $k_{z2}\tau$ is small, we can use the approximations

$$\sin (k_{z2}\tau) \cong k_{z2}\tau \tag{10}$$

$$\cos (k_{z2}\tau) \cong 1 - \frac{1}{2} (k_{z2}\tau)^2 \tag{11}$$

Equation (9) can be simplified to give

$$T = \frac{1}{\left\{ 1 + \frac{(k\tau)^2}{8} \left(\frac{\lambda}{a} \right)^2 + j \frac{(k\tau)^2}{2(k_z\tau)} \left[1 - \frac{1}{2} \left(\frac{\lambda}{a} \right)^2 \right] \right\} - \varepsilon \left[\frac{(k\tau)^2}{2} - j \frac{(k\tau)^2}{2(k_z\tau)} \right]} \tag{12}$$

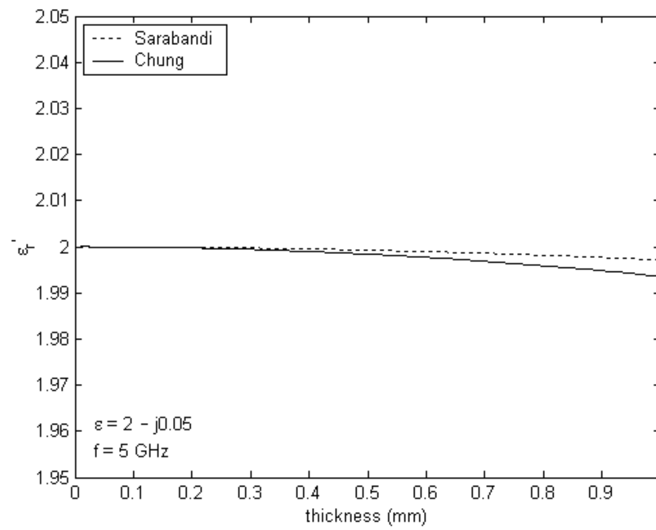
Hence, the relative dielectric constant is given by

$$\varepsilon = \frac{\left\{ 1 + \frac{(k\tau)^2}{8} \left(\frac{\lambda}{a} \right)^2 + j \frac{(k\tau)^2}{2(k_z\tau)} \left[1 - \frac{1}{2} \left(\frac{\lambda}{a} \right)^2 \right] \right\} - \frac{1}{T}}{\left[\frac{(k\tau)^2}{2} - j \frac{(k\tau)^2}{2(k_z\tau)} \right]} \tag{13}$$

In the above expressions, $k = 2\pi/\lambda$.

3. SENSITIVITY TO SLAB THICKNESS

The accuracy of the approximate expression given by (13) depends on the magnitude of $k_{z2}\tau$. To analyze the sensitivity, we compute the exact transmission coefficient T using Equation (9) and apply it in (13) to compute the approximate ε. Comparison is made with the second-order approximation of Sarabandi and Ulaby [20]. Fig. 2 shows the plots of ε at 5 GHz as a function of τ for a slab with actual dielectric constant $\varepsilon = 2 - j0.05$. A WR-187 waveguide with $a = 1.872$ inch is assumed. Fig. 3 shows the results for a material with $\varepsilon = 20 - j10$. We observe that the accuracy of the transmission method proposed in this paper is comparable to the reflection method of [20]. The error of ε' is within 0.1%, and the error of ε'' is within 0.2%, for slab thickness τ less than 1% of $\lambda/\sqrt{|\varepsilon|}$.



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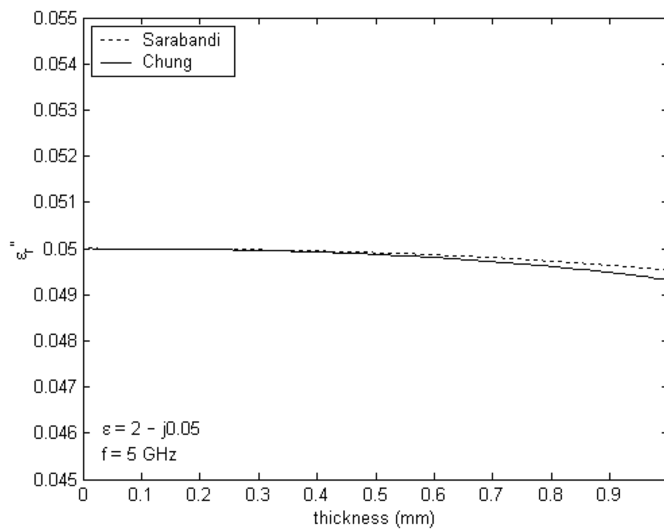

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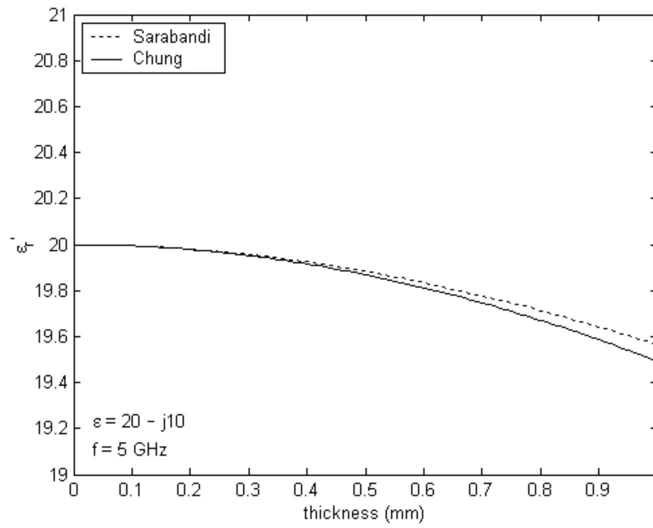
Figure 2. Relative dielectric constant (a) real part, and (b) imaginary part, of low-loss material.



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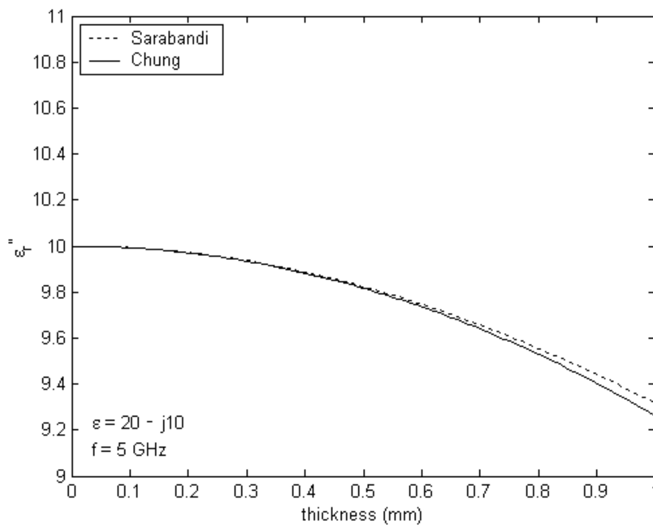

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Figure 3. Relative dielectric constant (a) real part, and (b) imaginary part, of lossy material.

4. SENSITIVITY TO SLAB POSITION OFFSET

A vector network analyzer can be used for accurate measurement of s-parameters of the waveguide structure. Two-port calibration is performed without the thin slab using a standard Thru-Reflect-Line (TRL) method [21]. The measurement reference plane is thereafter set at the mating flange of the two waveguides at $z = 0$.

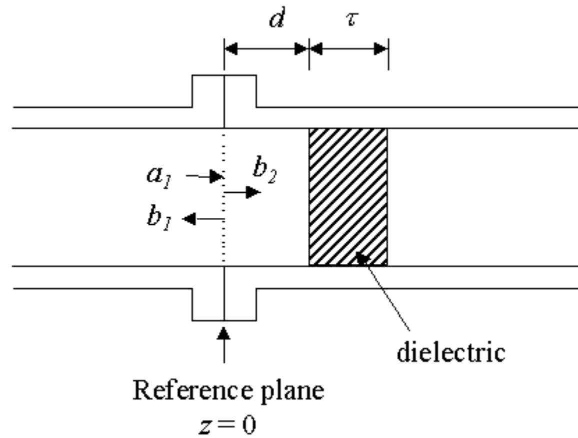


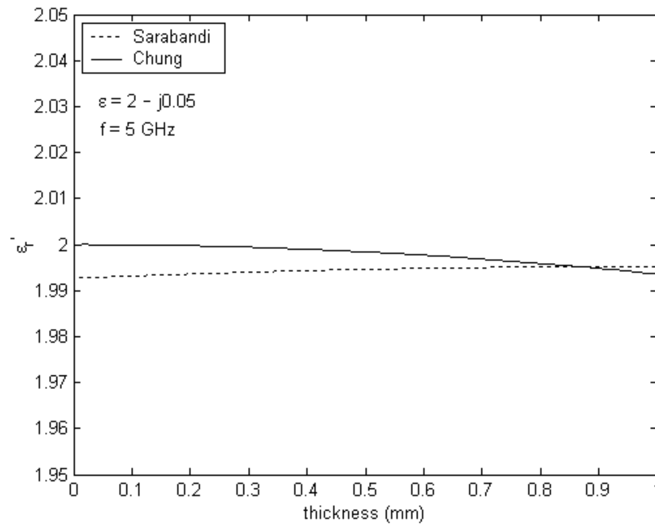
Figure 4. Waveguide with a dielectric slab placed at an offset d from the waveguide flange.

Consider the waveguide in Fig. 4 where the thin slab is placed at an offset distance d from the waveguide flange. The measured reflection coefficient s_{11} and transmission coefficient s_{21} are

$$s_{11} = \frac{b_1}{a_1} = \Gamma \cdot e^{-j2k_z d} \tag{14}$$

$$s_{21} = \frac{b_2}{a_1} = T \cdot e^{jk_z \tau} \tag{15}$$

The value of T can be easily obtained by multiplying s_{21} with $e^{-jk_z\tau}$. The slab position offset d has no effect on s_{21}. Hence, no effect on ε determined using the transmission method. However, the unknown value of d that can only be neglected will give rise to some errors on ε determined using the reflection method of [20]. Fig. 5 and 6 show the sensitivity analysis for $\varepsilon = 2 - j0.05$ and $\varepsilon = 20 - j10$, respectively, when there is a slab position offset of 0.5 mm.



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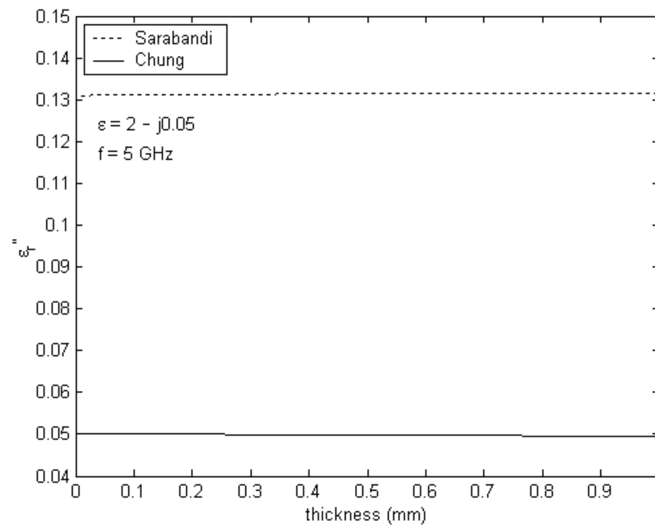

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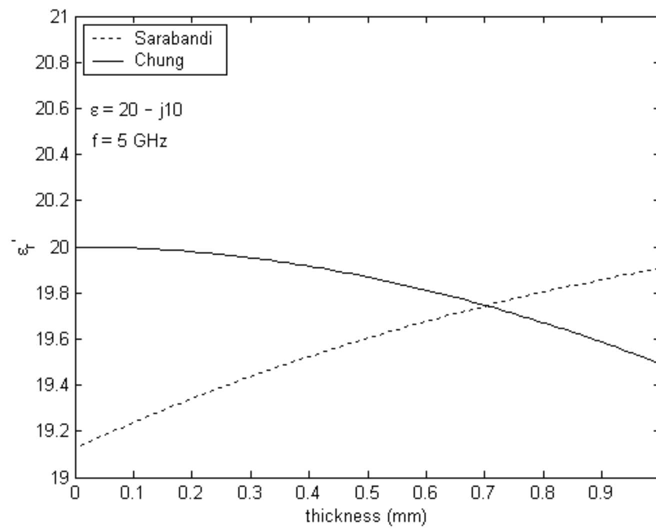
Figure 5. Relative dielectric constant (a) real part, and (b) imaginary part, of low-loss material when there is an offset of 0.5 mm.



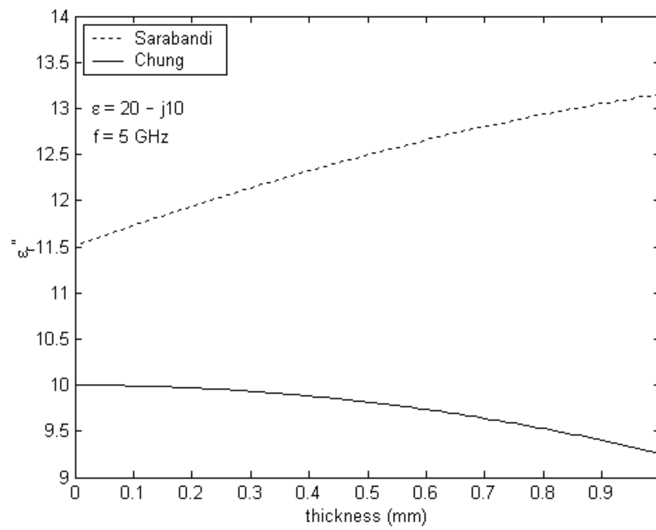


Figure 6. Relative dielectric constant (a) real part, and (b) imaginary part, of lossy material when there is an offset of 0.5 mm.

5. SAMPLE MEASUREMENTS

An Agilent 8720D vector network analyzer (VNA) was used to measure the magnitude and phase of transmission coefficient s_{21} of dielectric slabs placed in a waveguide sample holder. Two coax-to-waveguide adapters are used to connect an empty waveguide section to Port 1 and the sample holder to Port 2 of the VNA. The measurement system is shown in Fig. 7. A thin piece of styrofoam is placed in the empty waveguide flush with the waveguide flange. Another thin piece of styrofoam is placed in the sample holder section to keep the sample in place.

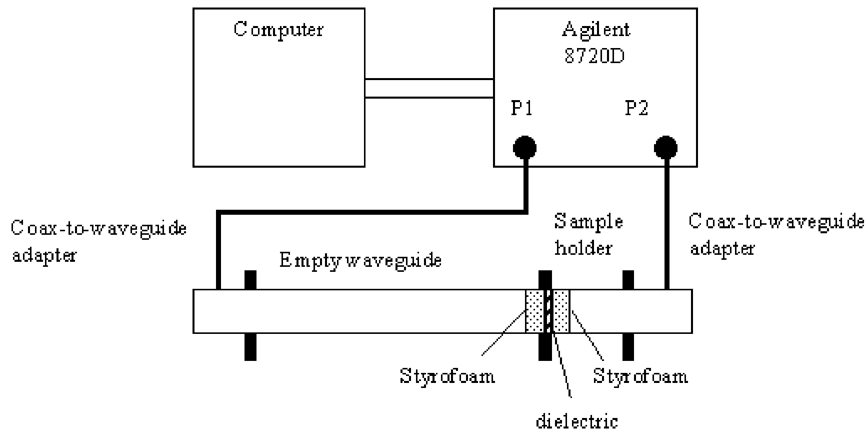
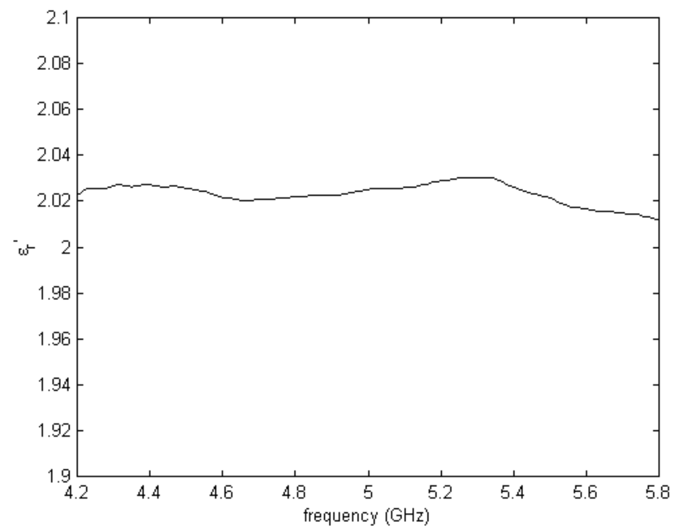


Figure 7. Measurement system setup.

First, the TRL calibration is performed without the dielectric sample. Next, the sample is placed in the waveguide and the complex transmission coefficient is measured over the frequency range from 4.2 to 5.8 GHz. Measurement was performed on Teflon slab with 0.5-mm thickness. Fig. 8 shows the measured dielectric spectrum of the material. Excellent agreement of both ε' and $\varepsilon''$with published data is obtained.

A vegetation leaf was selected for dielectric constant measurement. The thickness was 0.275 mm and the gravimetric moisture content was 65.3%. The measured dielectric constant is shown in Fig. 9. A dispersive behavior similar to that predicted by the dual-dispersion model of Ulaby and El-Rayes [22] is observed.



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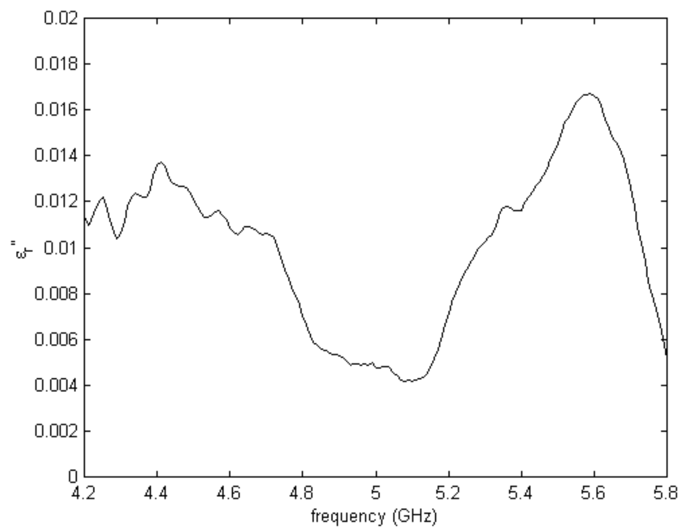

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Figure 8. Measured dielectric constant (a) real part, and (b) imaginary part, of 0.5-mm thick Teflon.

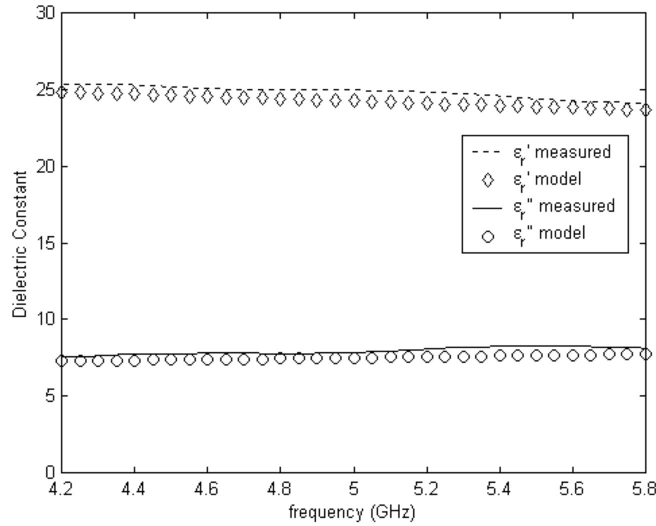

Figure 9. Measured dielectric constant of the leaf.

6. CONCLUSION

A method to determine the dielectric constant by measuring the transmission coefficient of thin dielectric slab placed in a waveguide has been developed. It overcomes the practical problem in the reflection method where the sample must be placed exactly at the waveguide flange. With the transmission method presented in this paper, position offset of the dielectric sample from the measurement reference plane has no effect on the measurement accuracy. Hence, it does not require meticulous control of the sample position, and the offset length does not need to be accurately measured using sophisticated length measuring instrument. This feature is very useful practically when a number of thin samples with different thicknesses are to be measured. The method does not require machining of the sample to a suitable thickness. This is very useful for measurement of naturally thin samples. The use of soft styrofoam plug makes sure that the sample is not adversely compressed. The method is verified with measurement on Teflon of 0.5-mm thickness. The measured dielectric constant of Teflon shows excellent agreement of both ε' and ε'' with published data. Subsequently, the dielectric constant of a vegetation leaf was measured. Dielectric constant of tropical crops such as oil palm, rubber, and rice will be measured in the near future.

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