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DIFFEOMORPHISMS SATISFYING THE SPECIFICATION PROPERTY

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Abstract. Let f be a diffeomorphism of a closed C^∞ manifold M. In this paper, we introduce the notion of the C^1-stable specification property for a closed f-invariant set Λ of M, and we prove that $f_{|\Lambda}$ satisfies a C^1-stable specification property if and only if Λ is a hyperbolic elementary set. As a corollary, the C^1-interior of the set of diffeomorphisms of M satisfying the specification property is characterized as the set of transitive Anosov diffeomorphisms.

1. Introduction

The notion of the specification property due to Bowen has turned out to be a very important notion in the study of ergodic theory of dynamical systems on a compact metric space (see [3] and [5]). It is known that every dynamical system satisfying the specification property has positive topological entropy and that the set of all strongly mixing measures of the system is residual in the space of invariant measures of it. The definition of the specification property is quite complicated and seems to be very strong, but it is satisfied by many examples. Indeed, every elementary set of the so-called Bowen's decomposition of the basic sets of a diffeomorphism satisfying Axiom A satisfies the property, and dynamical systems satisfying the property are rather well investigated from the viewpoint of ergodic theory (see [2]).

However, it is not too much to say that the property has not been investigated from the viewpoint of geometric theory of dynamical systems. In this paper, we study the specification property from the viewpoint of geometric theory of dynamical systems and characterize diffeomorphisms satisfying the property under the C^1-stable assumption. Here C^1-stable means that the specification property under consideration is preserved by C^1-perturbation of the original map.

Let (X,d) be a compact metric space. A homeomorphism $f : X \to X$ satisfies the *specification property* (abbreviated *SP*) if for any $\epsilon > 0$ there is an integer $N = N(\epsilon) > 0$ such that for any $k \geq 2$, for any k points $x_1, x_2, \cdots, x_k \in X$, for any integers

$$a_1 \leq b_1 < a_2 \leq b_2 < \cdots < a_k \leq b_k \quad \text{with} \quad a_i - b_{i-1} \geq N$$

for $2 \leq i \leq k$, there exists a point $y \in X$ such that

$$d(f^j(y), f^j(x_i)) \leq \epsilon \quad \text{for} \quad a_i \leq j \leq b_i,\ 1 \leq i \leq k.$$

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If f satisfies the *SP*, then f is topologically mixing (see [3, Proposition 21.3]). Here f is *topologically mixing* if for any nonempty open sets $U, V \subset X$ there is an integer $N > 0$ such that for any $n \geq N$, $U \cap f^{-n}(V) \neq \emptyset$ (see [3, §6]). It is easy to see that if f is topologically mixing, then it is transitive, that is, there is a dense orbit.

Let us remark that our definition of the *SP* is really weaker than the original specification property introduced in [2] (see also [3, Definition 21.1]). In [5], the *SP* is called the weak specification property and group automorphisms of compact metric spaces possessing the property are intensively studied.

For $\delta > 0$, a sequence of points $\{x_i\}_{i\in\mathbb{Z}} \subset X$ is called a *δ-pseudo-orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $i \in \mathbb{Z}$. We say that f has the *shadowing property* if for every $\epsilon > 0$, there is $\delta > 0$ such that for any δ-pseudo-orbit $\{x_i\}_{i\in\mathbb{Z}}$ of f, there is $y \in X$ satisfying $d(f^i(y), x_i) < \epsilon$ for all $i \in \mathbb{Z}$. A homeomorphism f is *expansive* if there is a constant $c > 0$ such that for any $x, y \in X$, $d(f^n(x), f^n(y)) \leq c$ $(n \in \mathbb{Z})$ implies $x = y$. It is proved in [3, Proposition 23.20] that if an expansive homeomorphism f has the shadowing property and is topologically mixing, then f satisfies the stronger variant of the specification property. In "our case", it can be easily seen that if a homeomorphism g has the shadowing property and is topologically mixing, then g satisfies the *SP* by following the proof of [3, Proposition 23.20].

Let M be a closed C^∞ manifold, and let $\mathrm{Diff}(M)$ be the space of diffeomorphisms of M endowed with the C^1-topology. Denote by d the distance on M induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle TM.

Hereafter let $f \in \mathrm{Diff}(M)$, and let $P(f)$ be the set of periodic points of f. Denote by $\mathcal{O}_f(p)$ the periodic f-orbit of $p \in P(f)$. If $p \in P(f)$ is a hyperbolic saddle with period $\pi(p) > 0$, then there are the local stable manifold $W^s_\epsilon(p)$ and the local unstable manifold $W^u_\epsilon(p)$ of p for some $\epsilon = \epsilon(p) > 0$. It is easy to see that if $d(f^n(x), f^n(p)) \leq \epsilon$ for any $n \geq 0$, then $x \in W^s_\epsilon(p)$ (a similar property also holds for $W^u_\epsilon(p)$ with respect to f^{-1}). The stable manifold $W^s(p)$ and the unstable manifold $W^u(p)$ of p are defined as usual. The dimension of the stable manifold $W^s(p)$ is sometimes called the *index of* p, and we denote it by $\mathrm{index}(p)$.

Let $\Lambda \subset M$ be a closed f-invariant set, and denote by $f_{|\Lambda}$ the restriction of f to the set Λ. Let $U \subset M$ be a compact neighborhood of Λ, and put

$$\Lambda_f(U) = \bigcap_{n\in\mathbb{Z}} f^n(U).$$

A set Λ is *locally maximal in* U if there is a compact neighborhood U of Λ such that $\Lambda = \Lambda_f(U)$. We say that $f_{|\Lambda_f(U)}$ satisfies the *C^1-stable specification property* (abbreviated *C^1-SSP*) if there are a compact neighborhood U of Λ and a C^1-neighborhood $\mathcal{U}(f)$ of f such that Λ is locally maximal in U and for any $g \in \mathcal{U}(f)$, $g_{|\Lambda_g(U)}$ satisfies the *SP*. Here

$$\Lambda_g(U) = \bigcap_{n\in\mathbb{Z}} g^n(U)$$

is called the *continuation of* $\Lambda_f(U) = \Lambda$. In the case $\Lambda = M$, we just say that f satisfies the *C^1-SSP*.

A set Λ is a *basic set* (resp. *elementary set*) if Λ is locally maximal and $f_{|\Lambda}$ is transitive (resp. topologically mixing). It is easy to see that if Λ is a hyperbolic basic set, then the periodic points are dense therein. Of course, every elementary set is a basic set.

In this paper, the following results are obtained.

Theorem 1.1. *Let Λ be a closed f-invariant set. Then $f_{|\Lambda_f(U)}$ satisfies the C^1-SSP if and only if Λ is a hyperbolic elementary set.*

Since M is connected, every transitive Anosov diffeomorphisms of M is topologically mixing. Thus we have the following corollary.

Corollary 1.2. *The set of diffeomorphisms of M satisfying the C^1-SSP is characterized as the set of transitive Anosov diffeomorphisms.*

Recently, it was announced in [9] that every Anosov diffeomorphism of M is transitive. Thus we may remove the transitivity condition from the above result.

In [8] the first author characterized the C^1-interior of the set of diffeomorphisms possessing the shadowing property as the set of diffeomorphisms satisfying both Axiom A and the strong transversality condition. Hence, if the *SP* is stronger than the shadowing property, then the above corollary is a direct consequence of the result. However, this assertion is not true. Indeed, we can construct an automorphism σ of the n-dimensional torus $\mathbb{T}^n$ which satisfies the *SP* but is not hyperbolic (see [5, Theorem (i) and (ii)]). It is well-known that an automorphism on $\mathbb{T}^n$ has the shadowing property if and only if it is hyperbolic. Thus, σ does not have the shadowing property.

Let Λ be as before. A set $\Lambda_f(U)$ is *robustly transitive* if Λ is locally maximal in U and there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, $g_{|\Lambda_g(U)}$ is transitive (see [1]). Recall that every dynamical system satisfying the *SP* is transitive. Let us remark at this point that there are no sinks and sources for every transitive system.

We will use the following result due to Mañé [6] in the proof of Theorem 1.1.

Theorem 1.3. *Let $\Lambda_f(U)$ be robustly transitive. Then the following conditions are equivalent:*

(1) *there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, any periodic point of $\Lambda_g(U)$ is hyperbolic and has the same index;*
(2) *there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, $\Lambda_g(U)$ is hyperbolic.*

Let us explain the result more precisely. Denote by $\Lambda_i(f)$ the closure of the set of hyperbolic periodic points of f with index i. Actually, it is proved in [6, Theorem B] that if there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, any periodic points of g are hyperbolic and $\Lambda_i(f) \cap \Lambda_j(f) = \emptyset$ for $0 \leq i \neq j \leq \dim M$, then f satisfies both Axiom A and the no-cycle condition. Since the proof is developed in a neighborhood of $\bigcup_{i=0}^{\dim M} \Lambda_i(f)$, we can see that the result also holds for our semi-local dynamical system $f_{|\Lambda_f(U)}$. Thus assertion (1) implies assertion (2) since $g_{|\Lambda_g(U)}$ is transitive for all g C^1-nearby f.

Observe that the proof of the 'if' part of Theorem 1.1 readily follows from the local stability of a hyperbolic set (see [7, Theorem 7.4]). Thus, to prove Theorem 1.1, it is enough to show the following proposition by Theorem 1.3.

Proposition 1.4. *If $f_{|\Lambda_f(U)}$ satisfies the C^1-SSP, then there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, any periodic point of $\Lambda_g(U)$ is hyperbolic and has the same index.*

2. Proof of Proposition 1.4

To prove the proposition, we prepare some lemmas that we need. In this section, let $f \in \mathrm{Diff}(M)$ and Λ be a closed f-invariant set.

Lemma 2.1. *Let $p, q \in \Lambda \cap P(f)$ be hyperbolic saddles. If $f_{|\Lambda}$ satisfies the SP, then $W^s(\mathcal{O}_f(p)) \cap W^u(\mathcal{O}_f(q)) \neq \emptyset$.*

Proof. Let $p, q \in \Lambda \cap P(f)$ be hyperbolic saddles, and let $\epsilon(p)$ and $\epsilon(q) > 0$ be as before with respect to p and q. Fix $\epsilon = \min\{\epsilon(p), \epsilon(q)\}$, and let $N = N(\epsilon) > 0$ be the number of the *SP* of $f_{|\Lambda}$. For any $n \geq N$ we set $x_1 = f^{-n}(p)$, $x_2 = f^{-N-n}(q)$, and put $a_1 = 0$, $b_1 = n$, $a_2 = N + n$ and $b_2 = N + 2n$. Clearly, $a_2 - b_1 = N$. Since $f_{|\Lambda}$ satisfies the *SP*, for any $n \geq N$ there is $z_n \in \Lambda$ such that

(*i*) $d(f^j(z_n), f^j(f^{-n}(p))) \leq \epsilon$ for $0 \leq j \leq n$,

(*ii*) $d(f^j(z_n), f^j(f^{-N-n}(q))) \leq \epsilon$ for $N + n \leq j \leq N + 2n$.

Item (i) implies that

$$d(f^{-i}(f^n(z_n)), f^{-i}(p)) \leq \epsilon \quad \text{for} \quad 0 \leq i \leq n,$$

and (ii) implies that

$$d(f^i(f^N(f^n(z_n))), f^i(q)) \leq \epsilon \quad \text{for} \quad 0 \leq i \leq n.$$

Put $w_n = f^n(z_n)$ and let $w = \lim_{n\to\infty} w_n$ by taking a subsequence if necessary. Then, since $d(f^{-i}(w_n), f^{-i}(p)) \leq \epsilon \leq \epsilon(p)$ and $d(f^i(f^N(w_n)), f^i(q)) \leq \epsilon \leq \epsilon(q)$ for $0 \leq i \leq n$, we have that $w \in W^u_{\epsilon(p)}(p) \subset W^u(p)$ and $f^N(w) \in W^s_{\epsilon(q)}(q)$; that is, $w \in W^s(f^{-N}(q))$. Hence $w \in W^u(p) \cap W^s(f^{-N}(q)) \subset W^u(\mathcal{O}_f(p)) \cap W^s(\mathcal{O}_f(q))$. □

If $p \in P(f)$ is hyperbolic, then for any $g \in \mathrm{Diff}(M)$ C^1-nearby f, there exists a unique hyperbolic periodic point $p_g \in P(g)$ nearby p such that $\pi(p_g) = \pi(p)$ and $\mathrm{index}(p_g) = \mathrm{index}(p)$. Such a p_g is called the *continuation of* p.

A diffeomorphism f is said to be *Kupka-Smale* if the periodic points of f are hyperbolic, and if $p, q \in P(f)$, then $W^s(p)$ is transversal to $W^u(q)$. It is well-known that the set of Kupka-Smale diffeomorphisms is C^1-residual in $\mathrm{Diff}(M)$ (see [7]).

Lemma 2.2. *Let $f_{|\Lambda_f(U)}$ satisfy the C^1-SSP, and let $\mathcal{U}(f)$ be as in the property. Then for any hyperbolic saddles $p, q \in \Lambda_g(U) \cap P(g)$ $(g \in \mathcal{U}(f))$,* $\mathrm{index}(p) = \mathrm{index}(q)$.

Proof. Let $f_{|\Lambda_f(U)}$ satisfy the C^1-*SSP*, and let $\mathcal{U}(f)$ be as in the property. Fix any $g \in \mathcal{U}(f)$, and let $p, q \in \Lambda_g(U) \cap P(g)$ be hyperbolic saddles. Then there is a C^1-neighborhood $\mathcal{V}(g) \subset \mathcal{U}(f)$ of g such that for any $\varphi \in \mathcal{V}(g)$, there are the continuations p_φ and q_φ (of p and q) in $\Lambda_\varphi(U)$, respectively (recall that since $\Lambda_f(U) = \Lambda \subset \mathrm{int}\, U$, we may assume that $\Lambda_g(U) \subset \mathrm{int}\, U$ for any $g \in \mathcal{U}(f)$ reducing $\mathcal{U}(f)$ if necessary).

The proof is by contradiction. Suppose that $\mathrm{index}(p) < \mathrm{index}(q)$, and thus $\dim W^s(p, g) + \dim W^u(q, g) < \dim M$ (the other case is similar). Here $W^s(p, g)$ and $W^u(q, g)$ are the stable and the unstable manifolds of p and q with respect to g. Take a Kupka-Smale diffeomorphism $\varphi \in \mathcal{V}(g)$. Then

$$W^s(p_\varphi, \varphi) \cap W^u(q_\varphi, \varphi) = \emptyset$$

since $\dim W^s(p, g) = \dim W^s(p_\varphi, \varphi)$ and $\dim W^u(q, g) = \dim W^u(q_\varphi, \varphi)$. On the other hand, since $\varphi \in \mathcal{U}(f)$, $\varphi_{|\Lambda_\varphi(U)}$ satisfies the *SP* so that $W^s(p_\varphi, \varphi) \cap W^u(q_\varphi, \varphi) \neq \emptyset$ by Lemma 2.1. This is a contradiction. □

The existence of the nonhyperbolic periodic point of f easily gives two hyperbolic periodic points with different indices for some g C^1-neaby f (see Lemma 2.4 below). To show this fact we use the next lemma several times.

Lemma 2.3. *Let $f \in \mathrm{Diff}(M)$ and let $\mathcal{U}(f)$ be given. Then there is $\delta > 0$ such that for a finite set $\{x_1, x_2, \cdots, x_N\}$, a neighborhood U of $\{x_1, x_2, \cdots, x_N\}$ and linear maps $L_i : T_{x_i}M \to T_{f(x_i)}M$ satisfying $\|L_i - D_{x_i}f\| \le \delta$ for all $1 \le i \le N$, there are $\epsilon_0 > 0$ and $g \in \mathcal{U}(f)$ such that*

(*a*) *$g(x) = f(x)$ if $x \in M \setminus U$, and*
(*b*) *$g(x) = \exp_{f(x_i)} \circ L_i \circ \exp_{x_i}^{-1}(x)$ if $x \in B_{\epsilon_0}(x_i)$ for all $1 \le i \le N$.*

Observe that assertion (*b*) implies that $g(x) = f(x)$ if $x \in \{x_1, x_2, \cdots, x_N\}$ and that $D_{x_i}g = L_i$ for all $1 \le i \le N$. The proof is essentially contained in the proof of [4, Lemma 1.1].

Lemma 2.4. *Let Λ be locally maximal in U, and let $\mathcal{U}(f)$ be given. If $p \in \Lambda_g(U) \cap P(g)$ ($g \in \mathcal{U}(f)$) is not hyperbolic, then there is $\varphi \in \mathcal{U}(f)$ possessing hyperbolic periodic points q_1 and q_2 in $\Lambda_\varphi(U)$ with different indices.*

Proof. Let Λ be locally maximal in U, and let $\mathcal{U}(f)$ be given. Suppose that $p \in \Lambda_g(U) \cap P(g)$ ($g \in \mathcal{U}(f)$) is not hyperbolic. Fix $\mathcal{V}(g) \subset \mathcal{U}(f)$; then we show that there is $\varphi \in \mathcal{V}(g)$ possessing a φ^k-invariant C^1-curve in U (for some $k > 0$) whose endpoints are both hyperbolic with different indices.

At first, by Lemma 2.3, with a small modification of the map g with respect to the C^1-topology, we may assume that $D_pg^{\pi(p)}$ has only one eigenvalue λ with modulus equal to 1 (and hence other eigenvalues of $D_pg^{\pi(p)}$ are with modulus less than 1 or greater than 1). Denote by E^s_p the eigenspace corresponding to the eigenvalues with modulus less than 1, by E^c_p the eigenspace corresponding to λ, and by E^u_p the eigenspace corresponding to the eigenvalues with modulus greater than 1. Thus, $T_pM = E^s_p \oplus E^c_p \oplus E^u_p$.

We divide the proof into two cases: $\dim E^c_p = 1$, that is, the eigenvalue λ is real; or $\dim E^c_p = 2$, that is, the eigenvalue λ is complex.

Case 1. $\dim E^c_p = 1$; that is, the eigenvalue λ is real with modulus equal to 1.

In this case, we suppose further that $\lambda = 1$ for simplicity (the other case is similar). Then, by Lemma 2.3, there are $\epsilon_0 > 0$ and $\varphi \in \mathcal{V}(g)$ such that $\varphi^{\pi(p)}(p) = g^{\pi(p)}(p) = p$ and

$$\varphi(x) = \exp_{g^{i+1}(p)} \circ D_{g^i(p)}g \circ \exp^{-1}_{g^i(p)}(x)$$

if $x \in B_{\epsilon_0}(g^i(p))$ for $0 \le i \le \pi(p) - 2$, and

$$\varphi(x) = \exp_p \circ D_{g^{\pi(p)-1}(p)}g \circ \exp^{-1}_{g^{\pi(p)-1}(p)}(x)$$

if $x \in B_{\epsilon_0}(g^{\pi(p)-1}(p))$. Since the eigenvalue λ of $D_pg^{\pi(p)}_{|E^c_p}$ is 1, there is a small arc $\mathcal{I}_p \subset B_{\epsilon_0}(p) \cap \exp_p E^c_p(\epsilon_0)$ with its center at p such that $\varphi^{\pi(p)}(\mathcal{I}_p) = \mathcal{I}_p$. Here $E^c_p(\epsilon_0)$ is the ϵ_0-ball in E^c_p with its center at the origin O_p.

We may suppose that $\mathcal{I}_p \subset \Lambda_\varphi(U)$, reducing both $\mathcal{U}(f)$ and ϵ_0 if necessary (observe that Λ is locally maximal). Denote by q_1 and q_2 the two endpoints of $\mathcal{I}_p$. Observe that

$$D_{q_i}\varphi^{\pi(p)}_{|E^c_p} = D_pg^{\pi(p)}_{|E^c_p} = 1$$

for $i = 1, 2$. Hence, by Lemma 2.3, with a C^1-modification of the map φ at the endpoints, we may have that both the points are hyperbolic with different indices; that is, $\mathrm{index}(q_1) \neq \mathrm{index}(q_2)$ with respect to φ.

Case 2. $\dim E^c_p = 2$, and the corresponding eigenvalues λ are complex conjugate with modulus equal to 1.

In the proof of the second case, to avoid notational complexity, we consider only the case $g(p) = p$. As in the first case, by Lemma 2.3, there are $\epsilon_0 > 0$ and $\varphi \in \mathcal{V}(g)$ such that $\varphi(p) = g(p) = p$ and

$$\varphi(x) = \exp_{g(p)} \circ D_p g \circ \exp_p^{-1}(x)$$

if $x \in B_{\epsilon_0}(p)$. With a small modification of the map $D_p g$, we may suppose that there is $l > 0$ (the minimum number) such that $D_p g^l(v) = v$ for any $v \in \exp_p^{-1}(E^c_p(\epsilon_0))$ by Lemma 2.3.

Take $v_0 \in \exp_p^{-1}(E^c_p(\epsilon_0))$ such that $\|v_0\| = \epsilon_0/4$, and set

$$\mathcal{J}_p = \exp_p(\{t \cdot v_0 : 1 \leq t \leq 1 + \epsilon_0/4\}).$$

Then $\mathcal{J}_p \subset \Lambda_\varphi(U)$ is an arc such that

· $\varphi^i(\mathcal{J}_p) \cap \varphi^j(\mathcal{J}_p) = \emptyset$ if $0 \leq i \neq j \leq l-1$,
· $\varphi^l(\mathcal{J}_p) = \mathcal{J}_p$ and $\varphi^l_{|\mathcal{J}_p}$ is the identity map.

As in the first case, with a C^1-modification of the map at the endpoints q_1 and q_2 of $\mathcal{J}_p$, we have that both points are hyperbolic with different indices. □

2.1. **End of the proof of Proposition 1.4.** Let $f_{|\Lambda_f(U)}$ satisfy the C^1-*SSP*, and let $\mathcal{U}(f)$ be as in the property. To get the conclusion, it is enough to show that every $p \in \Lambda_g(U) \cap P(g)$ ($g \in \mathcal{U}(f)$) is hyperbolic by Lemma 2.2. By contradiction, suppose that $p \in \Lambda_g(U) \cap P(g)$ ($g \in \mathcal{U}(f)$) is not hyperbolic. Then by Lemma 2.4, there is $\varphi \in \mathcal{U}(f)$ possessing hyperbolic periodic points q_1 and q_2 in $\Lambda_\varphi(U)$ with different indices; that is, $\mathrm{index}(q_1) \neq \mathrm{index}(q_2)$. This is a contradiction again by Lemma 2.2 since $f_{|\Lambda_f(U)}$ satisfies the C^1-*SSP*.

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