

DIFFERENTIAL EQUATIONS, DYNAMICAL SYSTEMS, AND AN INTRODUCTION TO CHAOS

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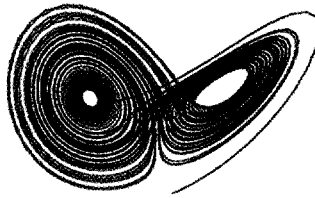
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Contents

Preface x

CHAPTER 1 First-Order Equations 1

- 1.1 The Simplest Example 1
- 1.2 The Logistic Population Model 4
- 1.3 Constant Harvesting and Bifurcations 7
- 1.4 Periodic Harvesting and Periodic Solutions 9
- 1.5 Computing the Poincaré Map 12
- 1.6 Exploration: A Two-Parameter Family 15

CHAPTER 2 Planar Linear Systems 21

- 2.1 Second-Order Differential Equations 23
- 2.2 Planar Systems 24
- 2.3 Preliminaries from Algebra 26
- 2.4 Planar Linear Systems 29
- 2.5 Eigenvalues and Eigenvectors 30
- 2.6 Solving Linear Systems 33
- 2.7 The Linearity Principle 36

CHAPTER 3 Phase Portraits for Planar Systems 39

- 3.1 Real Distinct Eigenvalues 39
- 3.2 Complex Eigenvalues 44

3.3	Repeated Eigenvalues	47
3.4	Changing Coordinates	49
CHAPTER 4 Classification of Planar Systems 61		
4.1	The Trace-Determinant Plane	61
4.2	Dynamical Classification	64
4.3	Exploration: A 3D Parameter Space	71
CHAPTER 5 Higher Dimensional Linear Algebra 75		
5.1	Preliminaries from Linear Algebra	75
5.2	Eigenvalues and Eigenvectors	83
5.3	Complex Eigenvalues	86
5.4	Bases and Subspaces	89
5.5	Repeated Eigenvalues	95
5.6	Genericity	101
CHAPTER 6 Higher Dimensional Linear Systems 107		
6.1	Distinct Eigenvalues	107
6.2	Harmonic Oscillators	114
6.3	Repeated Eigenvalues	119
6.4	The Exponential of a Matrix	123
6.5	Nonautonomous Linear Systems	130
CHAPTER 7 Nonlinear Systems 139		
7.1	Dynamical Systems	140
7.2	The Existence and Uniqueness Theorem	142
7.3	Continuous Dependence of Solutions	147
7.4	The Variational Equation	149
7.5	Exploration: Numerical Methods	153
CHAPTER 8 Equilibria in Nonlinear Systems 159		
8.1	Some Illustrative Examples	159
8.2	Nonlinear Sinks and Sources	165
8.3	Saddles	168
8.4	Stability	174
8.5	Bifurcations	176
8.6	Exploration: Complex Vector Fields	182

CHAPTER 9 Global Nonlinear Techniques 189

- 9.1 Nullclines 189
- 9.2 Stability of Equilibria 194
- 9.3 Gradient Systems 203
- 9.4 Hamiltonian Systems 207
- 9.5 Exploration: The Pendulum with Constant Forcing 210

CHAPTER 10 Closed Orbits and Limit Sets 215

- 10.1 Limit Sets 215
- 10.2 Local Sections and Flow Boxes 218
- 10.3 The Poincaré Map 220
- 10.4 Monotone Sequences in Planar Dynamical Systems 222
- 10.5 The Poincaré-Bendixson Theorem 225
- 10.6 Applications of Poincaré-Bendixson 227
- 10.7 Exploration: Chemical Reactions That Oscillate 230

CHAPTER 11 Applications in Biology 235

- 11.1 Infectious Diseases 235
- 11.2 Predator/Prey Systems 239
- 11.3 Competitive Species 246
- 11.4 Exploration: Competition and Harvesting 252

CHAPTER 12 Applications in Circuit Theory 257

- 12.1 An *RLC* Circuit 257
- 12.2 The Lienard Equation 261
- 12.3 The van der Pol Equation 262
- 12.4 A Hopf Bifurcation 270
- 12.5 Exploration: Neurodynamics 272

CHAPTER 13 Applications in Mechanics 277

- 13.1 Newton's Second Law 277
- 13.2 Conservative Systems 280
- 13.3 Central Force Fields 281
- 13.4 The Newtonian Central Force System 285

- 13.5 Kepler's First Law 289
- 13.6 The Two-Body Problem 292
- 13.7 Blowing Up the Singularity 293
- 13.8 Exploration: Other Central Force Problems 297
- 13.9 Exploration: Classical Limits of Quantum Mechanical Systems 298

CHAPTER 14 The Lorenz System 303

- 14.1 Introduction to the Lorenz System 304
- 14.2 Elementary Properties of the Lorenz System 306
- 14.3 The Lorenz Attractor 310
- 14.4 A Model for the Lorenz Attractor 314
- 14.5 The Chaotic Attractor 319
- 14.6 Exploration: The Rössler Attractor 324

CHAPTER 15 Discrete Dynamical Systems 327

- 15.1 Introduction to Discrete Dynamical Systems 327
- 15.2 Bifurcations 332
- 15.3 The Discrete Logistic Model 335
- 15.4 Chaos 337
- 15.5 Symbolic Dynamics 342
- 15.6 The Shift Map 347
- 15.7 The Cantor Middle-Thirds Set 349
- 15.8 Exploration: Cubic Chaos 352
- 15.9 Exploration: The Orbit Diagram 353

CHAPTER 16 Homoclinic Phenomena 359

- 16.1 The Shil'nikov System 359
- 16.2 The Horseshoe Map 366
- 16.3 The Double Scroll Attractor 372
- 16.4 Homoclinic Bifurcations 375
- 16.5 Exploration: The Chua Circuit 379

CHAPTER 17 Existence and Uniqueness Revisited 383

- 17.1 The Existence and Uniqueness Theorem 383
- 17.2 Proof of Existence and Uniqueness 385

17.3	Continuous Dependence on Initial Conditions	392
17.4	Extending Solutions	395
17.5	Nonautonomous Systems	398
17.6	Differentiability of the Flow	400

Bibliography	407
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Index	411
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