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# DIFFERENTIAL SENSITIVITY THEORY APPLIED TO MOVEMENT OF MAXIMA RESPONSES*



P. J. Maudlin, C. V. Parks, and D. G. Cacuci†

Computer Sciences Division
at Oak Ridge National Laboratory
Union Carbide Corporation, Nuclear Division



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†Engineering Physics Division.

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Differential sensitivity theory (DST) is a recently developed methodology to evaluate response derivatives $dR/d\alpha$ by using adjoint functions which correspond to the differentiated (with respect to an arbitrary parameter $\alpha$) linear or nonlinear physical system of equations.[1,2] However, for many problems, where responses of importance are local maxima such as peak temperature, power, or heat flux, changes in the phase space location of the peak itself are of interest. This summary will present the DST procedure for predicting phase space shifts of maxima responses as applied to the MELT-III fast reactor safety code.[3]

An adjoint version of the MELT-III code has been developed to allow evaluation of $dR/d\alpha$ via DST methodology.[4] The adjoint system solved is of the form

$$\underline{L}^{*} \vec{u}^{*} = \vec{s}^{*}, \tag{1}$$

where $\underline{L}^{*}$, $\vec{u}^{*}$, and $\vec{s}^{*}$ are the adjoint operator, function, and source, respectively. The general sensitivity expression is of the form

$$\frac{dR}{d\alpha} = \int_t \int_V \vec{s} \cdot \vec{u}^{*} \, dVdt + BT, \tag{2}$$

where $\vec{s}$ is the source term for the differentiated physical system and BT denotes boundary terms.

Consider a peak fuel response

$$R = \int_t \int_V T(r,z,t) \; \delta(r-r_o) \; \delta(z-z_o) \; \delta(t-t_o) \; dVdt, \tag{3}$$

where the phase space location $(r_o, z_o, t_o)$ is dependent on $\alpha$. For brevity, consider only response shifts in the time domain. As shown in Ref. 5, an expression for $dt_o/d\alpha$ can be derived:

$$\frac{dt_o}{d\alpha} = -\left[\frac{\frac{\partial}{\partial t_o}\left(\frac{dR}{d\alpha}\right)}{\frac{\partial^2 T}{\partial t^2}}\right]_{(r_o, z_o, t_o)} . \tag{4}$$

Note that a similar expression involving other maxima responses and phase-space variables can be derived.

The denominator of Eq. (4) is easily evaluated from the solution of the physical problem. The numerator of Eq. (4) is best obtained by differentiating Eq. (3):

$$\frac{\partial}{\partial t_o}\left(\frac{dR}{d\alpha}\right) = \int\!\!\int_{t\ V} \vec{s} \cdot \left(\frac{\partial \vec{u}^*}{\partial t_o}\right) dVdt + \frac{\partial}{\partial t_o}(BT), \tag{5}$$

where $\partial \vec{u}^*/\partial t_o$ satisfies

$$L^*\left(\frac{\partial \vec{u}^*}{\partial t_o}\right) = \left(\frac{\partial \vec{s}^*}{\partial t_o}\right) . \tag{6}$$

Note that for the response of Eq. (3), the only non-zero term of $\vec{s}^*$ is $\delta(r-r_o)\ \delta(z-z_o)\ \delta(t-t_o)$. Thus, the only non-zero term of $\partial \vec{s}^*/\partial t_o$ is $-\delta(r-r_o)\ \delta(z-z_o)\ \delta'(t-t_o)$, and so Eq. (6) can be solved with the adjoint MELT code by correctly specifying the adjoint source. The $\delta'(t-t_o)$ term used in the adjoint source was numerically applied in a manner consistent with the definition of Ref. 6.

An FFTF protected transient involving a \$.23/s ramp reactivity insertion with scram on high power was selected for investigation. The peak fuel temperature occurred at $t_o = .87s$. Adjoint calculations were performed for solution of Eqs. (1) and (6) from which $dR/d\alpha$ and $dt_o/d\alpha$ were obtained via Eqs. (2) and (4)-(5). Figure 1 shows a profile of the portion of the adjoint solution of Eq. (6) associated with the coolant energy conservation equation. The ripple at $t \sim .87s$ is caused by the adjoint source while the contour changes at $t \sim .5s$ are related to a source connected with the reactor trip.[4] Table I shows results for the parameters which cause the greatest time shift in the response. The second and third columns show the first-order DST predictions for the response magnitude change and the time shift. The fourth and fifth columns indicate magnitude changes and time shifts obtained by direct recalculation with $\alpha + \Delta\alpha$ as input. The results provide adequate validation of the time shifts predicted with DST methodology.

In conclusion, it should be noted that only two adjoint calculations were necessary to calculate the response magnitude change and time shift for all the MELT parameters. This summary has shown that once the adjoint code is available, only a simple source modification is needed to allow prediction of the phase space movement of maxima responses.

Table I. Sensitivity Comparison Featuring a Peak

Fuel Temperature Response

| Input Parameter, $\alpha$ | DST $\frac{\Delta R}{\Delta\alpha/\alpha}$ | DST $\frac{\Delta t_o}{\Delta\alpha/\alpha}$ | Recalculation $\frac{\Delta R}{\Delta\alpha/\alpha}$ | Recalculation $\frac{\Delta t_o}{\Delta\alpha/\alpha}$ |
|---|---|---|---|---|
| Initial Fuel Temperature | $10.2 \frac{K}{.5\%}$ | $0.021 \frac{s}{.5\%}$ | $10.0 \frac{K}{.5\%}$ | $0.02 \frac{s}{.5\%}$ |
| Scram Power | $1.27 \frac{K}{.3\%}$ | $0.012 \frac{s}{.3\%}$ | $1.3 \frac{K}{.3\%}$ | $0.01 \frac{s}{.3\%}$ |
| Initial Power | $-0.08 \frac{K}{.4\%}$ | $-0.012 \frac{s}{.4\%}$ | $-.1 \frac{K}{.4\%}$ | $-0.01 \frac{s}{.4\%}$ |
| Fuel Conductivity | $-17.75 \frac{K}{5\%}$ | $-0.024 \frac{s}{5\%}$ | $-18.3 \frac{K}{5\%}$ | $-0.025 \frac{s}{5\%}$ |

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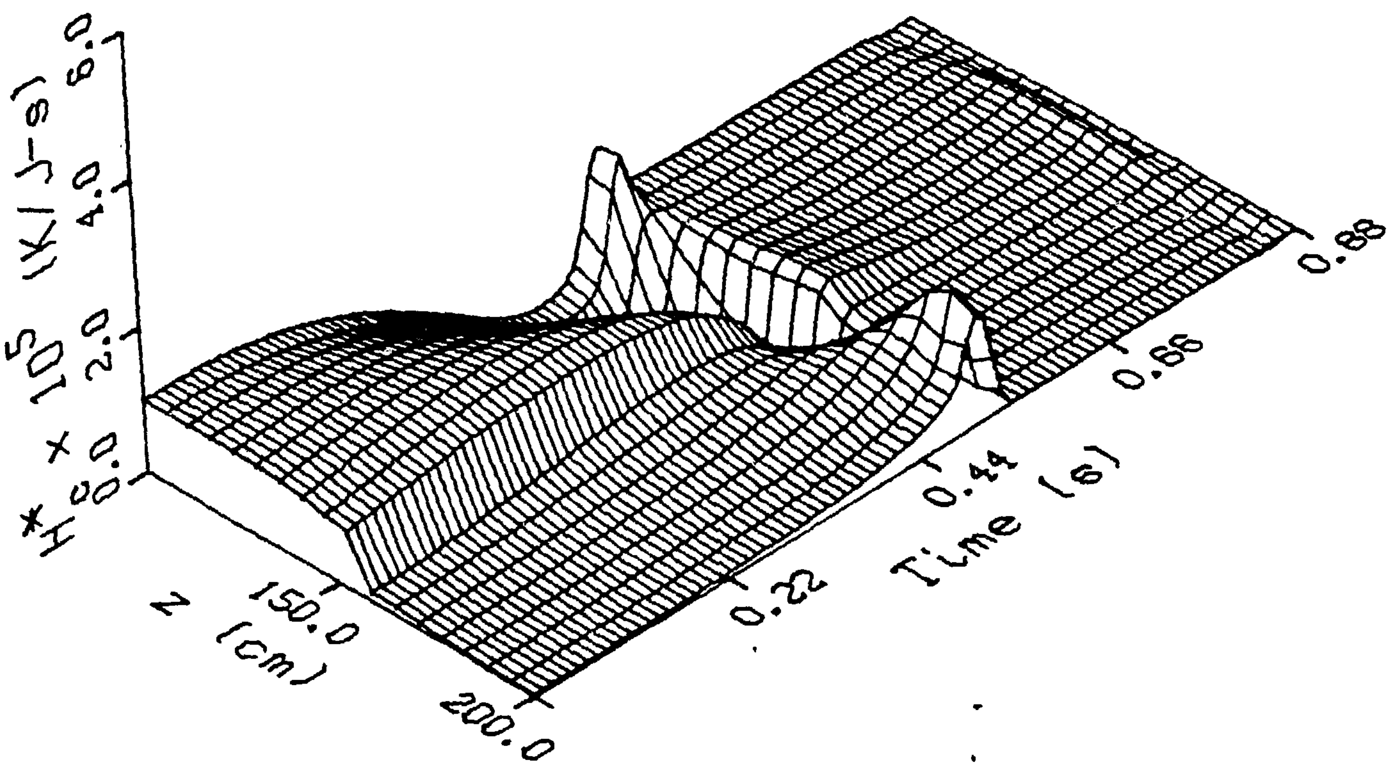


Fig. 1. Adjoint profile corresponding to the coolant energy conservation equation.