Differentially En/Decoded Orthogonal Space–Time Block Codes With APSK Signals

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***Abstract*—In this letter, we propose a differential en/decoding scheme for Alamouti's orthogonal space–time code using amplitude/phase-shift keying (STC-APSK) signals and two transmit antennas. It is compared with the differential en/decoding scheme using 16APSK and single transmit antenna. It is also compared with the differential en/decoding scheme for Alamouti's orthogonal space–time code using 16PSK (STC-16PSK) signals and two transmit antennas. We find that the performance of the differentially en/decoded STC-APSK with 4.5 b/s/Hz is significantly better than that of the differentially en/decoded 16APSK with 4 b/s/Hz, and is almost the same as that of the STC-16PSK with 4 b/s/Hz over Rayleigh flat fading channels.**

***Index Terms*—Differential en/decoding, differential amplitude/phase-shift keying (DAPSK), orthogonal space–time codes.**

I. Introduction

For SINGLE antenna wireless communications systems, the differentially en/decoded 16 amplitude/phase-shift keying (16APSK) has attracted considerable attention, see for example [1]–[3]. To resist fading in a wireless channel, multi-antenna systems and space–time coding have been shown promising to increase the capacity, see for example [4]–[8]. When the channel information is not available at the receiver, differential space–time coding has been proposed in Hochwald and Sweldens [11] and Hughes [12] independently. For space–time coding, Alamouti [9] has recently proposed an interesting scheme for two transmit antennas. Alamouti's scheme has been generalized to orthogonal space–time block codes (OSTBC) from orthogonal designs for multiple transmit antennas by Tarokh, Jafarkhani, and Calderbank [10]. The OSTBC has also been investigated in differential en/decoding in Tarokh and Jafarkhani [13] for phase-shift keying (PSK) signals.

In this letter, we propose a differentially en/decoded OSTBC with APSK signals for multi-antenna systems in wireless fading channels. In particular, we propose a differential en/decoding scheme for the Alamouti's orthogonal space–time code with two transmit antennas by using APSK signals (STC-APSK). Three APSK signals, 16APSK and 32APSK and 24APSK, are considered for the STC-APSK with bandwidth efficiencies 3.5 b/s/Hz, 4.5 b/s/Hz and 4 b/s/Hz, respectively. They are compared with the conventional single antenna 16DAPSK of bandwidth efficiency 4 b/s/Hz and the differential space–time coding using the Alamouti's two transmit antenna orthogonal space–time code and 16PSK signals (STC-16PSK). We find that the performance of the differentially en/decoded STC-32APSK with 4.5 b/s/Hz is significantly better than that of the differentially en/decoded 16APSK with 4 b/s/Hz, and is almost the same as that of the STC-16PSK with 4 b/s/Hz. In all the comparisons, single receive antenna is used and the channels are Rayleigh flat fading. Since the orthogonality of the Alamouti's STC, the decoding complexities of the STC-APSK and STC-PSK for muti-transmit antennas are the same as the ones of APSK and PSK for single transmit antennas.

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II. Differential STC-APSK

In what follows, script English letters denote sets, bold upper case letters denote matrices, bold lower case letters denote vectors, and lower case letters denote scalars.

A. 16APSK

For the 16DAPSK scheme, we use the notations in [3]. The 16DAPSK signal constellation is shown in Fig. 1(a), which consists of two independent 8 differential phase-shift keying (8DPSK) $\{\exp(2\pi jm/8) : 0 \leq m \leq 7\}$ and one 2 differential amplitude-shift keying (2DASK) $\{r_L, r_H\}$ with $a = r_H/r_L = 2$, $0.5(r_L^2 + r_H^2) = 1$ and $r_L = \sqrt{2/(a^2+1)}$. The four bit symbols (a_n, b_n, c_n, d_n) at the nth T seconds are differentially encoded and decoded, where the first three bits (a_n, b_n, c_n) are carried by the 8DPSK and the last bit d_n is carried by the 2DASK.

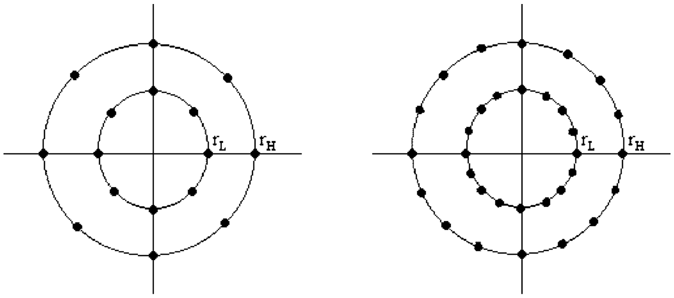
B. Alamouti's Space–Time Code

Let $\mathcal{S}_M$ be the MPSK signal set, i.e., $\mathcal{S}_M = \{\exp(2\pi jm/M) : 0 \leq m \leq M-1\}$. The Alamouti's space–time code [9] we use in this letter for two transmit antennas is

$$\mathcal{C} = \left\{ \mathbf{C}(x_1, x_2) = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} : \begin{matrix} x_1 \in \mathcal{S}_{M_1}, \\ x_2 \in \mathcal{S}_{M_2} \end{matrix} \right\} \quad \text{(II.1)}$$

where the first antenna transmits $x_1/\sqrt{2}$ and $-x_2^*/\sqrt{2}$, and the second antenna transmits $x_2/\sqrt{2}$ and $x_1^*/\sqrt{2}$ during the time intervals $[2(n-1)T, (2n-1)T]$ and $[(2n-1)T, 2nT]$, respectively. Clearly, for any x_1 and x_2, the space–time code matrix $\mathbf{C}(x_1, x_2)$ in $\mathcal{C}$ is unitary in the following sense:

$$\mathbf{C}(x_1, x_2)(\mathbf{C}(x_1, x_2))^\dagger = \frac{1}{2}(|x_1|^2 + |x_2|^2)\mathbf{I} \quad \text{(II.2)}$$



(a) (b)

Fig. 1. APSK signal constellations. (a) 16APSK. (b) 32APSK.

where † stands for the complex conjugate transpose and $\mathbf{I}$ is the 2×2 identity matrix. This space–time code has rate 1 and full diversity, where full diversity means that any difference codeword matrix $\mathbf{C}(x_1,x_2) - \mathbf{C}(y_1,y_2) = \mathbf{C}(x_1 - y_1, x_2 - y_2)$ is full rank for $(x_1,x_2) \neq (y_1,y_2)$. It also has a fast ML decoding as we will see later. Note that, the total mean transmission power of the above two transmit antenna system is the same as the one of a single transmit antenna system for transmitting symbols $x \in \mathcal{S}_M$.

C. Differential Space–Time Coding and STC-PSK

For a general space–time code $\mathcal{C} = \{\mathbf{C}_i, i = 0,1,2,\ldots,M-1\}$ of M different $N\times N$ unitary matrices with $\mathbf{C}_i^\dagger \mathbf{C}_i = \mathbf{I}$. The following differential space–time coding was proposed in [11], [12] for N transmit antennas:

$$\mathbf{S}_n = \mathbf{S}_{n-1}\mathbf{C}_{k_n}, \quad n = 1,2,\ldots \tag{II.3}$$

where $\mathbf{S}_0$ is an arbitrary fixed unitary matrix and k_n is encoded from the current $\log_2(M)$ bits of an information sequence over the time duration of NT seconds. The transmitted signal is $\sqrt{\rho}\mathbf{S}_n$ with the signal power ρ. In the existing differential space–time coding in [11]–[13], all the unitary matrices $\mathbf{C}_i$ have the same norm. If the orthogonal space–time block codes, such as the Alamouti's code in (II.1), are used, the two symbols x_1 and x_2 used in a time block have to satisfy that the sum of their powers, $|x_1|^2 + |x_2|^2$, has to be constant for any independent symbols x_1 and x_2 in a signal constellation. This implies that the signal constellation has to be a PSK, and may limit its performance when a high bandwidth efficient modulation scheme is preferred.

In our simulations, we are only interested in the comparison with the differential STC using the Alamouti's code in (II.1) and the MPSK, which is named STC-MPSK for short. Its detailed encoding is as follows.

Let $K = \log_2(M)$. For the $2K$ bits of information $(I_1(n),\ldots,I_K(n),I_{K+1}(n),\ldots,I_{2K}(n))$ at the time $2nT$, the first K bits, $I_1(n),\ldots,I_K(n)$, are mapped to $x_1 \in \mathcal{S}_M$ and the second K bits, $I_{K+1}(n),\ldots,I_{2K}(n)$, are mapped to $x_2 \in \mathcal{S}_M$. Then, a space–time codeword matrix $\mathbf{C}(x_1,x_2)$ is formed as shown in (II.1). Then, the transmitted signals for two antennas are the two rows of matrix $\sqrt{\rho}\mathbf{S}_n$ during the time intervals $[2(n-1)T,(2n-1)T]$ and $[(2n-1)T,2nT]$, respectively, where

$$\mathbf{S}_0 = \mathbf{I} \quad \text{and} \quad \mathbf{S}_n = \mathbf{S}_{n-1}\cdot \mathbf{C}(x_1,x_2), \qquad \text{for } n = 1,2,\ldots.$$

D. STC-APSK: Differential Encoding

We now describe STC-(M_1+M_2)DAPSK scheme. Its signal constellation consists of two independent M_1PSK and M_2PSK and one 2ASK as shown in Fig. 1(a) and (b) for $M_1 = M_2 = 8$ and $M_1 = M_2 = 16$, respectively, where $r_H = a\, r_L$ and a is the ratio between high and lower magnitudes of the 2ASK signals as in the 16APSK we described before. In each $2T$ seconds, two symbols x_1 and x_2 are transmitted and their amplitudes may be different in the next $2T$ seconds, and totally $\log_2(M_1) + \log_2(M_2) + 1$ bits are carried in $2T$ seconds. Therefore, the bandwidth efficiency is $(\log_2(M_1)+\log_2(M_2)+1)/2$ b/s/Hz. Note that the bandwidth efficiencies for the MAPSK and the STC-MPSK are $\log_2(M)$ b/s/Hz. The detailed encoding is as follows.

Let $K_i = \log_2(M_i)$ for $i = 1,2$. For each K_1+K_2+1 bits of information

$$(I_1(n),\ldots,I_{K_1}(n),I_{K_1+1}(n),\ldots,I_{K_1+K_2}(n),I_{K_1+K_2+1}(n))$$

during the time interval $[2(n-1)T,2nT]$, the first K_1 bits, $I_1(n),\ldots,I_{K_1}(n)$, are mapped to $x_1 \in \mathcal{S}_{M_1}$, the second K_2 bits, $I_{K_1+1}(n),\ldots,I_{K_1+K_2}(n)$, are mapped to $x_2 \in \mathcal{S}_{M_2}$, and the last bit $I_{K_1+K_2+1}(n)$ determines whether the mean signal power of $\mathbf{S}_n$ stands the same as the previous one $\mathbf{S}_{n-1}$ or change between r_L and r_H. The algorithm is

$$\mathbf{S}_0 = a_0\mathbf{P}_0, \quad \text{with } a_0 = r_L \quad \text{and} \quad \mathbf{P}_0 = \mathbf{I} \tag{II.4}$$

$$\begin{aligned}
&a_n = a_{n-1}b_n, \quad \text{where}\\
&b_n = \begin{cases} 1, & \text{if } I_{K_1+K_2+1}(n) = 0\\ r_H/r_L, & \text{if } I_{K_1+K_2+1}(n) = 1 \quad \text{and} \quad a_{n-1} = r_L\\ r_L/r_H, & \text{if } I_{K_1+K_2+1}(n) = 1 \quad \text{and} \quad a_{n-1} = r_H \end{cases}\\
&\mathbf{P}_n = \mathbf{P}_{n-1}\cdot \mathbf{C}(x_1,x_2)\\
&\mathbf{S}_n = a_n\mathbf{P}_n
\end{aligned} \tag{II.5}$$

for $n = 1,2,\ldots$, where $\mathbf{C}(x_1,x_2)$ is the 2×2 space–time codeword matrix formed as in (II.1).

E. STC-APSK: Differential Decoding

Consider single receive antenna at the receiver. The received signals in the time intervals $[2(n-1)T,(2n-1)T]$ and $[(2n-1)T,2nT]$ form the following 1×2 vector:

$$\mathbf{r}_n = \sqrt{\rho}\mathbf{h}\mathbf{S}_n + \mathbf{w}_n \tag{II.6}$$

where $\mathbf{h} = [h_1,h_2]$, h_1 and h_2 are the channel gains or coefficients from the first and the second transmit antennas to the receive antenna, respectively, and they are independent complex Gaussian random variables of mean 0 and variance 1, and $\mathbf{w}_n = [w_{n,1},w_{n,2}]$ is the additive complex Gaussian noise vector with two independent identically distributed components of mean 0 and variance N_0. In our simulations, this variance is determined by the total energy per bit, E_b, at the transmitter over N_0, i.e., E_b/N_0, since the total transmit signal mean power is normalized to 1 for the convenience. From (II.4) to (II.5), the received signal $\mathbf{r}_n$ can be rewritten as

$$\mathbf{r}_n = \mathbf{r}_{n-1}b_n\mathbf{C}(x_1,x_2) + \hat{\mathbf{w}}_n \tag{II.7}$$

where $\hat{\mathbf{w}}_n = \mathbf{w}_n - \mathbf{w}_{n-1}b_n\mathbf{C}(x_1,x_2)$.

The differential decoding has two steps. The first step is to detect the $(K_1 + K_2 + 1)$th bit by detecting whether b_n is 1 or not: by noticing that $b_n \in \{1, a, 1/a\}$,

$$\bar{b}_n = \arg \min_{b \in \{1,a,1/a\}} |||\mathbf{r}_n|| - b||\mathbf{r}_{n-1}||| \tag{II.8}$$

where $||\mathbf{v}|| = \sqrt{\sum_i |v_i|^2}$ if $\mathbf{v} = (v_i)$. If $\bar{b}_n = 1$, then $\bar{I}_{K_1+K_2+1}(n) = 0$, otherwise $\bar{I}_{K_1+K_2+1}(n) = 1$.

After b_n is detected, the second step is to detect the PSK symbols x_1 and x_2 for the first K_1 bits and the second K_2 bits, respectively, as follows.

$$\begin{aligned}(\bar{x}_1, \bar{x}_2) &= \arg \min_{x_1 \in \mathcal{S}_{M_1}, x_2 \in \mathcal{S}_{M_2}} ||\mathbf{r}_n - \bar{b}_n \mathbf{r}_{n-1} \mathbf{C}(x_1, x_2)||^2 \\ &= \arg \max_{x_1 \in \mathcal{S}_{M_1}, x_2 \in \mathcal{S}_{M_2}} (f_1(x_1) + f_2(x_2)) \\ &= \arg \left(\max_{x_1 \in \mathcal{S}_{M_1}} f_1(x_1) + \max_{x_2 \in \mathcal{S}_{M_2}} f_2(x_2) \right) \\ &= \left(\arg \max_{x_1 \in \mathcal{S}_{M_1}} f_1(x_1), \arg \max_{x_2 \in \mathcal{S}_{M_2}} f_2(x_2) \right)\end{aligned}$$

where

$$f_1(x_1) = \mathrm{Re}\{(\mathbf{r}_{n,1}^* \mathbf{r}_{n-1,1} + \mathbf{r}_{n,2} \mathbf{r}_{n-1,2}^*) x_1\} \tag{II.9}$$

$$f_2(x_2) = \mathrm{Re}\{(\mathbf{r}_{n,1}^* \mathbf{r}_{n-1,2} - \mathbf{r}_{n,2} \mathbf{r}_{n-1,1}^*) x_2\} \tag{II.10}$$

where Re stands for the real part, and $r_{i,1}$ and $r_{i,2}$ are the two components of the received signal vector $\mathbf{r}_i = (r_{i,1}, r_{i,2})$. Thus, symbols x_1 and x_2 can be detected from

$$\bar{x}_i = \arg \max_{x_i \in \mathcal{S}_{M_i}} f_i(x_i), \quad i = 1, 2 \tag{II.11}$$

whose complexity is similar to that in single antenna systems.

III. Simulations

In simulations, E_b/N_0 is used as the channel SNR, where E_b is the total energy per bit used in the transmission and is the summation of energies in all transmit antennas in a multiple antenna system. In all simulations, 100 information symbols are used in each trial and 10 000 trials are used. The fading channels are flat, i.e., constant, in each trial and are complex Gaussian random variables over 10 000 trials.

Three different kinds of differential en/decoding schemes are compared. The first one is the 16-DAPSK with single transmit and receive antenna and its bandwidth efficiency is 4 b/s/Hz. The second kind is the STC-16DPSK with two transmit antennas and one receive antenna. As we explained before, in STC-16DPSK, the Alamouti's STC and 16PSK signals are used and its bandwidth efficiency is 4 b/s/Hz. The third kind is the STC-DAPSK proposed in Section II-D–II-E. Three different STC-DAPSK schemes are simulated. They are STC-32DAPSK with $M_1 = M_2 = 16$, i.e., two independent 16PSK and its bandwidth efficiency 4.5 b/s/Hz; STC-16DAPSK with $M_1 = M_2 = 8$, i.e., two independent 8PSK and its bandwidth efficiency 3.5 b/s/Hz; and STC-24DAPSK with $M_1 = 16$ and $M_2 = 8$, i.e., two independent 16PSK and 8PSK and its bandwidth efficiency 4 b/s/Hz. In STC-32DAPSK, STC-24DAPSK, and STC-16DAPSK, the magnitude ratios a are 1.5, 1.5 and 2, respectively, which we find optimal in the SNR region in our simulations. Their bit error rates (BER) versus E_b/N_0 are compared in Fig. 2. From Fig. 2, we find that the STC-32DAPSK with 4.5 b/s/Hz performs similarly to the STC-16DPSK with 4 b/s/Hz, and the STC-24DAPSK with 4 b/s/Hz slightly outperforms the STC-16DPSK with 4 b/s/Hz.

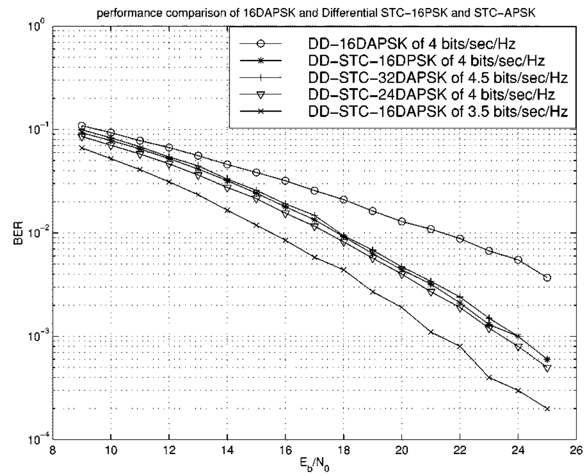


Fig. 2. Performance comparison of different differential en/decoding schemes with one or two transmit antennas and one receive antenna.

IV. Conclusion

In this letter, we proposed differential en/decoded space–time modulation using APSK signals and the Alamouti's orthogonal space–time code for two transmit antennas.

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