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Diffusion of Innovations Under Conditions of Uncertainty: A Stochastic Approach

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Published on: 01 Jan 1991

Topics: Diffusion of innovations, Sociological theory of diffusion, Technological change and Early adopter

Chapter 9

Diffusion of Innovations Under Conditions of Uncertainty: A Stochastic Approach

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9.1 Introduction

The diffusion of innovations is at the core of the pattern of technological change. Many attempts to explain and describe this process have been undertaken during the last decade and a vast bibliography of publications on this subject is presented in Rogers, 1962 and 1983; and Rogers and Shoemaker, 1971. The theory of innovation is an important part of economic and social science, and is both conceptual and formal. Their unity is a necessary premise for the success of any scientific theory.

Currently, researchers are aware of some mismatch between the conceptual and formal sides of innovation theory. The conceptual part draws increasing attention to the hidden mechanisms of technological change. The problems of uncertainty and unevenness of innovations are at the center of current conceptual discussions. Economists argue about the relationships between ruptures and continuity in long-term technological change, and instability and consistency of technological trajectories during the different phases of an innovation's life cycle. A classification of innovations and some important new concepts, which reflect the technological pattern of change [technological and techno-economic paradigms, technological trajectories (Dosi, 1984; Perez, 1983; Freeman, 1987), radical, basic, incremental, process, and product innovations, having different diffusion regularities] were recently introduced into economic theory. These *conceptual innovations* have not yet been adopted by the formal side of innovation theory.

The majority of the present mathematical models treat the diffusion of innovations in a traditional way as a deterministic process, which can be described by means of differential equations or logistic curves. This approach has been quite successful as many studies have shown. Without questioning the usefulness of this approach, we must emphasize that the hypothesis about the deterministic character of innovation diffusion is appropriate only for the growth and maturity phases of the innovation life cycle under stable conditions. In this chapter we present another approach to innovation diffusion modeling which considers uncertainty and random fluctuations within the process. We consider a simple model that enables us to trace the influence of innovators and imitators on the final market share. It is worth mentioning that this approach for describing competing technologies, based on the generalized urn scheme, was proposed by Brian Arthur (1983).

We concentrate our analysis here on the early stage of innovation diffusion, when the costs and benefits of a new technology are not clear and the trajectory is fluctuating. This phase is not considered by the traditional deterministic approach because of the uncertainty and instability.

The early phases of radical innovation diffusion are characterized by the two important features which are often missed in diffusion models: (1) the instability of the present development and the uncertainty of the future evolution trajectory, and (2) the existence of different alternative technologies, which compete for the potential adopters. The random fluctuations play an important role in this phase and must be taken into consideration.

9.2 Formulation of the Problem

According to the Schumpeterian theory of innovation, innovation diffusion is a process of cumulative growth of imitators, which introduces the innovation into the market (after its exposure by entrepreneurs) with expectations of high profits (Schumpeter, 1939). We assume that several alternative (from the point of view of their expected profitability and possibilities for adoption) technologies were simultaneously introduced into the market by various entrepreneurs. The relative advantages of these technologies are not clear for the imitators, who must make their choice in order to survive in the changing economic environment.

For the sake of simplicity, we consider two new technologies (say, A and B), introduced into the market by a corresponding number of innovators.

The difference between our approach and the traditional approach concerning the classification of the participants of the diffusion process is apparent. According to the latter, all of the participants can be divided into the following groups: innovators, early adopters, early majority, later majority, and laggards. All groups except the first are considered to be imitators (Rogers, 1983; Bass, 1980). The difference between innovators and imitators is based on the characteristic features of their behavior, "imitators unlike innovators are influenced in the timing of adoption by the decisions of other members of the social system" (Bass, 1980). From our point of view, we aggregate these groups into two wider ones.

Starting with n_A A-technology innovators and n_B B-technology innovators we study how technologies are shared by the imitators in the market. We assume that for each time instant $t \geq 1$ one new imitator appears on the market (we consider a time scale connected with the appearance of new firms in the market). Technology A is chosen with probability $p_t(x_t)$ and technology B with probability $1 - p_t(x_t)$. Here x_t is the proportion (relative concentration) of the adopters that use technology A at time t:

$$x_t = \frac{n_t^A}{n_t^A + n_t^B} \quad ,$$

where n_t^A is the number of adopters that use technology A at time t and n_t^B is the number of adopters that use technology B at time $t \geq 1$. The probabilities of technological choice are considered to be a function of the relative concentration of the alternative technologies in the market. According to the premises of the model we assumed that the number (and share) of adopters of this or that technology is the indicator of the accumulated experience of its utilization. Also p_t is a function which maps $R(0,1)$ on [0,1], where $R(0,1)$ is the set of rational numbers from the interval (0,1). As far as $n_t^A + n_t^B = n_A + n_B + t - 1$ and $n_t^A = (n_A + n_B + t - 1)x_t$ our probability of additions of new adopters depends on both the total number of units of the technologies in the market $n_t^A + n_t^B$ at time t and the number of the adopters that use technology A (n_t^A) and technology B (n_t^B).

We are interested in finding the final ratio of the adopters that use technology A and technology B under the assumption that the market has an infinite capacity. Formally speaking we shall study the limit behavior of the value x_t as $t \to \infty$.

Let us consider $\beta_n(x), n \geq 1, x \in R(0,1)$, independent with respect to n random values which have Bernoulli distributions. Assume that

$$P\{\beta_t(x) = 1\} = p_t(x).$$

Then the process x_t, $t \geq 1$, follows the dynamics (Arthur *et al.*, 1987):

$$\begin{aligned} x_{t+1} &= x_t + \frac{1}{n_A + n_B + t}\{\beta_t(x_t) - x_t\} = x_t + \\ &+ \frac{1}{n_A + n_B + t}\{p_t(x_t) - x_t\} + \frac{1}{n_A + n_B + t} z_t(x_t), \\ &t \geq 1, x_1 = \frac{n_A}{n_A + n_B}. \end{aligned} \tag{9.1}$$

Consequently our process is driven on average by the term $p_t(x_t) - x_t$ (at time t).

The study of the asymptotic behavior of the process x_t, $t \geq 1$, may be done by means of the methods shown by Arthur *et al.* (1987 and 1988), but we will not consider it in detail. Here we are interested in the formation of probabilities p_t under different premises. We shall study the asymptotic behavior of the innovation diffusion process according to the different probability functions, inferred from the conceptual premises.

As was mentioned above participants of the real innovation diffusion process usually do not have sufficient information about the relative advantages of new technologies. According to the premises of the model, imitators when making their decisions, take into account the experience of earlier adopters. The information about this experience is not easily obtained because it is related to the competitive position of adopters (firms) in the market. As usual each firm can be acquainted with the experience of a limited sample, which is far less than the whole range. This is the main source of uncertainty in decision making and innovation diffusion that must be taken into consideration in a market economy. It can be eliminated only by accumulating experience about innovation adoption. But with decreasing uncertainty in the utilization of a new technology and the risk associated with its adoption, the profitability also decreases with the saturation of the market during innovation diffusion. The supernormal profitability of a successful innovation is temporal – it declines with the market shift towards a new equilibrium level while innovation diffuses according to well-known empirical laws.

We shall take into account both of the above-mentioned points. First, we shall consider the case of new technology uncertainty where imitators have no means to compare the expected profitabilities of the competing technologies, accompanied by information uncertainty about the real market situation (this case is typical of the early stages of radical innovation diffusion and for technological change during the turbulent phase of technological paradigm substitution). Second, we shall consider the case in which imitators have enough information to compare the expected profitability of competitive technologies, but they still do not have sufficient information about the market (it is typical for the growth phases of innovation diffusion and technological change within a consistent technological trajectory).

9.3 Diffusion of Innovations with Uncertain Probabilities (Imitative Behavior)

According to the above-described premises of the model, imitators make decisions to introduce a new technology according to the accumulated experience of its utilization by previous adopters. This is a traditional assumption made for diffusion innovation models (see Rogers, 1983). It is natural to suppose that among alternative, uncertain new technologies they will choose those that were successfully introduced by the majority of previous adopters from the known sample. In the case of two technologies this decision-making principle can be formulated strictly in the following way:

Rule 1. Ask an odd number r of the users of alternative technologies. If the majority of them use A, choose A. Otherwise choose B.

The probability of choosing technology A at time t under the above rule of decision making is given by the following formula:

$$\sum_{i=\frac{r+1}{2}}^{r} \frac{C^i_{n^A_t} C^{r-i}_{n^B_t}}{C^r_{n^A_t+n^B_t}}.$$

Here $C^p_q = \frac{q!}{p!(q-p)!}$ is the number of combinations from q to p. Also $q! = q(q-1)\ldots 1$. Let us designate this probability $p^I_t(x_t)$, where x_t is the proportion of technology A in the market. Then $p_t^I(x)$ equals to $p^I(x)$ with the accuracy of the order $o(1)$ as $t \to \infty$ (uniformly with respect to $x \in [0,1]$). Here

$$p^I(x) = \sum_{k=\frac{r+1}{2}}^{r} C_r^k x^k (1-x)^{r-k}.$$

When $r = 1$ we have $p_n^I(x) = p^I(x) = x$ for all $n \geq 1$. The graphics of the function p^I for different r are given in *Figure 9.1*. The function f whose zeros determine all possible limits for values of x_t (see Arthur *et al.*, 1987) is given now by the following formula:

$$f(x) = p^I(x) - x.$$

Let us consider the case when $r > 1$. The corresponding set $B^f([0,1])$ of the zeros consists of three points: 0, $\frac{1}{2}$, 1. It may be shown that both 0 and 1 are attainable, but $\frac{1}{2}$ is unattainable (see Arthur *et al.*, 1988). Consequently x_t converges as $t \to \infty$ to 0 or to 1 (and to both points with positive probability). This means that finally we shall have only one of the alternative technologies in the market. But each of them has the probability of being the winner.

Let us study the relationship of the probability of being the winner $p_{n_A,n_B}(1)$ (starting with n_A innovators of A and n_B innovators of B) of technology A to the proportion of initial adopters. Then

$$p_{n_A,n_B}(0) + p_{n_A,n_B}(1) = 1. \tag{9.2}$$

As far as $p_n^I(x) = 1 - p_n^I(1-x)$, we have that

$$p_{n_A,n_B}(0) = p_{n_B,n_A}(1)$$

and

$$p_{n_A,n_B}(1) = p_{n_B,n_A}(0).$$

With equality (9.2) we obtain

$$p_{n_A,n_B}(0) + p_{n_B,n_A}(0) = 1 \tag{9.3}$$

and

$$p_{n_A,n_B}(1) + p_{n_B,n_A}(1) = 1. \tag{9.4}$$

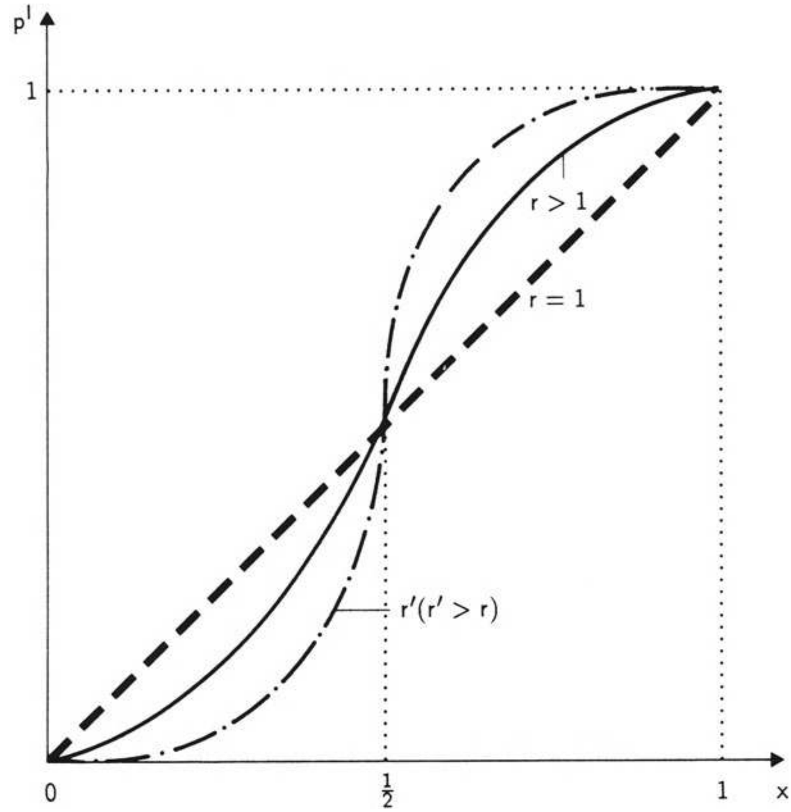

Figure 9.1. Probability of choosing (according to Rule 1) technology A as a function of its market proportion.

Combining equalities (9.3) and (9.4) with $n_A = n_B = n$ one has

$$p_{n,n}(0) = p_{n,n}(1) = \tfrac{1}{2}. \qquad (9.5)$$

Let $n_A < n_B$. Then all trajectories $x_t, t \geq 1$, that lead to 1 should at least once exceed the value $\frac{1}{2}$. Let us show that the process $x_t, t \geq 1$, takes this value. Indeed, suppose that for some $k, m (1 \leq k < m)$ there will be

$$k/m < \tfrac{1}{2} \text{ and } \tfrac{k+1}{m+1} > \tfrac{1}{2} \qquad (9.6)$$

(this means that the process does not take this value). If m is an odd number, i.e., $m = 2p + 1$ for some $p \geq 1$, then the smallest k that ensures the second one of the inequalities (9.6) is $k = p + 1$. So we have

$$k/m = \tfrac{p+1}{2p+1} > \tfrac{1}{2},$$

what contradicts inequalities (9.6). Similarly we obtain a contradiction in (9.6) when m is an even number. Consequently one of the inequalities (9.6) is indeed an equality. It means that the process $x_t, t \geq 1$, when crossing the middle of the segment [0,1] takes the value $\frac{1}{2}$. Taking into account the above and (9.5) we obtain

$$\begin{aligned} p_{n_A,n_B}(1) &= \sum_{k=n_B-n_A}^{\infty} P\left\{x_t \to 1 | x_1 = \tfrac{k}{2k}\right\} \\ &\quad P\left\{x_1 = \frac{n_A}{n_A+n_B}, x_2 < \tfrac{1}{2}, \ldots, x_{k-1} < \tfrac{1}{2}, x_k = \tfrac{1}{2}\right\} \\ &\stackrel{\text{def}}{=} \tfrac{1}{2}\,\gamma_{n_A,n_B}. \end{aligned} \tag{9.7}$$

As far as

$$P_{n_A,n_B}\left\{x_t < \tfrac{1}{2}, t \geq 1\right\} = 1 - \gamma_{n_A,n_B}$$

and

$$\begin{aligned} P_{n_A,n_B}\{x_t < \tfrac{1}{2},\ \ t \geq 1\} &\geq P_{n_A,n_B}\{x_t = n_A(n_A+n_B+t-1)^{-1}, \\ t \geq 1\} &\geq \prod_{k\geq 1}^{\infty}\left\{1 - p_k^I\left(\frac{n_A}{n_A+n_B+t-1}\right)\right\} > 0 \end{aligned}$$

we have that $\gamma_{n_A,n_B} < 1$. Consequently [because of (9.7)] $p_{n_A,n_B}(1) < \frac{1}{2}$ and [because of (9.2)] $p_{n_A,n_B}(0) > \frac{1}{2}$ for $n_A < n_B$.

This implies that the probability of being the winner is greater for the technology with the larger number of innovators.

For $r = 1$ we can use the results of Polya (1931) and Athreya (1969) to find that the limit of x_t has a Beta distribution with parameters n_A and n_B. We designate this limit random variable $\bar{x}$. Then $\bar{x}$ has a density (with respect to the Lebesgue measure in R^1) of the following form

$$f_{\bar{x}}(y) = \begin{cases} \frac{(n_A+n_B-1)!}{(n_A-1)!(n_B-1)!} y^{n_A-1}(1-y)^{n_B-1} & \text{for } y \in [0,1], \\ 0 & \text{for } y \notin [0,1]. \end{cases}$$

This means that at the limit both of the technologies can exist in the market in all possible combinations. However, each individual combination has zero probability. We have for the following events non-zero probability "the final combination belongs to the interval (α, β)", where $(\alpha, \beta) \subseteq (0, 1)$ is arbitrary. The case $r = 1$ corresponds to the situation in which each imitator only has sufficient information about a single previous introduction of an alternative technology. This reflects a situation of extreme high uncertainty in the market. A negligible share only is known by the followers. According to the results of this model, one can expect here instability in the final sharing of the market by alternative technologies.

To summarize the above argument we can make the following conclusion. When there is uncertainty about innovations, sharing the market depends on the strategies of the imitators (dependence of the final market share on the number r and numbers of innovators). In the case where followers make their choice according to the knowledge of their predecessors (if they know more than one case of previous innovation adoption) innovators can essentially affect the final sharing of the market. In particular the probability of dominating the market is greater for the technology with the larger number of innovators at the beginning of its life cycle. It is necessary also to mention that according to the premises and results of the model, innovators can only influence the market sharing tendency, but cannot predetermine (in a deterministic way) the domination of one of the alternative technologies. In addition, this model illustrates a very important regularity in the formation of new technological trajectories: the earlier phases of innovation diffusion play a relatively more important role in this process than later ones. As a result, the structure of the innovation diffusion process is formed in the very early stages.

9.4 Diffusion of Innovations with Expected Profitability and Uncertainty of Current Market Sharing

Now we shall consider the case in which the decisions of imitators are determined by the expected probability of alternative technologies. We assume that imitators have enough information about alternative innovations to estimate the expected dynamics of their relative profitabilities. This case corresponds to the diffusion of new technologies in stable environments in which the trajectories of evolution have already been formed and are stable. At the same time, as in the previous case, the main source of information for the followers of new technologies is the experience of earlier adopters. They make decisions based on their estimation of the dynamics of innovation profitability and the market situation. We assume that imitators interpret the apparent sharing of the market sample between alternative technologies as the sharing of the whole market. We must emphasize once more that according to the premises of our approach, imitators make their decisions based on limited information about market structure – it is an important source of uncertainty in the decision making and in the randomness of innovation diffusion.

As was mentioned above, the profitability of a new technology usually decreases with the increase in the number of adopters and later saturation of the market. Therefore it is natural to suppose that imitators will recognize the decreasing profitability of a technology as the number of its adopters increases.

Let us introduce positive functions g_A and g_B which describe the dependence of the proportions of the A and B technologies in the market sample and the imitators expected profitability. These principles of decision making can be formalized in the following way:

Rule 2. Ask an odd number r of the previous adopters. Let N of them use technology A. (Consequently $r - N$ use technology B.) Calculate the values $g_A(\frac{N}{r})$ and $g_B(1 - \frac{N}{r})$. If the first of these values is greater, choose A. Otherwise choose B.

According to the premise that a decrease in the proportion of a technology corresponds to an increase in the expected profitability, functions g_A and g_B should be nonincreasing. If they are decreasing there can be only one solution of the equation

$$g_A(x) = g_B(x), x \in [0,1]. \tag{9.8}$$

To ensure that the solution exists, one requires continuity of the functions, and that $g_A([0,1]) = g_B([0,1])$ for example. (This last condition means that values of the expected profitabilities change in the same interval.) Now we suppose that the solution of the equation (9.8) exists. *Figure 9.2* demonstrates one of the possible situations.

Here x corresponds to the proportion of technology A in the market. Functions g_A and $\tilde{g}_A$ demonstrate two possible ways in which the expected profitability can decrease. One can see that if $\tilde{g}_A(x) \leq g_A(x)$ for every x, then $\tilde{x}^* < x^*$.

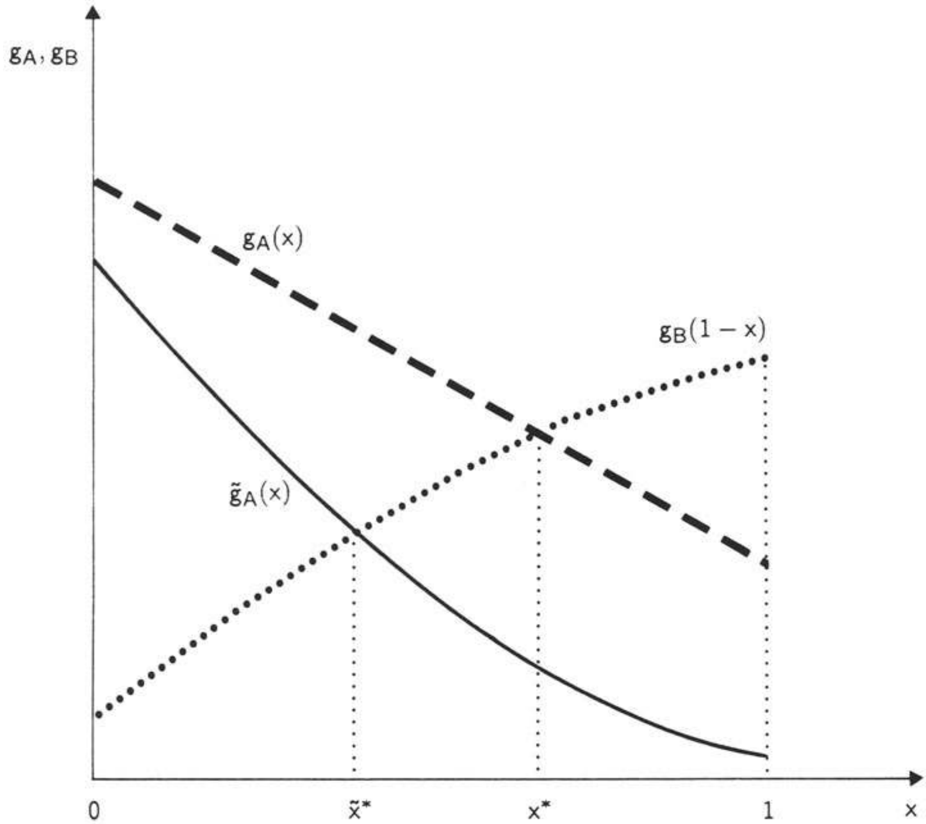

Figure 9.2. Expected profitabilities of the technologies as functions of a proportion of technology A.

Consider $r > 1$. Let $N(r)$ be the smallest N such that $N/r \geq x^*$. Then in the same manner as in the previous case, the probability of choosing technology A at time t is given by the formula

$$\sum_{i=0}^{N(r)-1} \frac{C^i_{n^A_t} C^{r-i}_{n^B_t}}{C^r_{n^A_t+n^B_t}}$$

and corresponding function $p^{II}_t(x)$ equals $p^{II}(x)$ with the accuracy of the order $o(1)$ as $t \to \infty$ (uniformly with respect to $x \in [0,1]$). Here

$$p^{II}(x) = \sum_{i=0}^{N(r)-1} C^i_r \, x^i (1-x)^{r-i}.$$

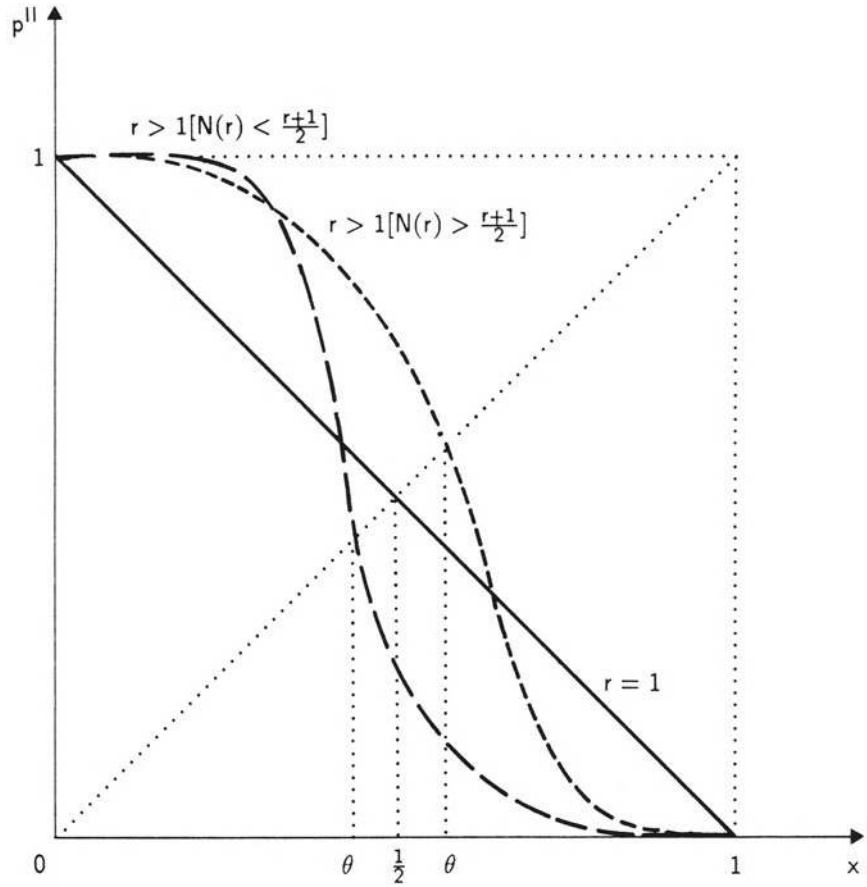

Figure 9.3. Probability of choosing (according to Rule 2) technology A as a function of its proportion on the market.

If $r = 1$ then $p_t^{II}(x) = p^{II}(x) = 1 - x$ for all $t \geq 1$. Function p^{II} is given in *Figure 9.3*.

The function f whose zeros determine all possible limit values for x_t (Arthur *et al.*, 1987) is given by the following formula:

$$f(x) = p^{II}(x) - x.$$

The corresponding set $B^f([0,1])$ of zeros is singleton. As it follows from Arthur *et al.* (1987) x_t goes to θ with probability 1 as $t \to \infty$. It is easy to see that $\theta = \frac{1}{2}$ for $r = 1$ and for $r > 1$ there will be

$$\begin{array}{ll} \theta > \frac{1}{2} & \text{when } N(r) > \frac{r+1}{2} \\ \theta = \frac{1}{2} & \text{when } N(r) = \frac{r+1}{2} \\ \theta < \frac{1}{2} & \text{when } N(r) < \frac{r+1}{2}. \end{array}$$

These results then have the following conceptual interpretation:

(1) When imitators make their decisions according to the expected probability dynamics, final sharing (namely, the position of the point θ) of the market by the new alternative technologies is determined by the strategies of the imitators. In contrast with the previous case, imitators' strategies influence the domination of one of the alternative technologies in the market in a deterministic way.

(2) Under the given circumstances, both of the technologies will exist when the market reaches the limit.

(3) For the given function of the reduction in the expected profitability for technology B (say g_B) and two given functions (say g_A and $\tilde{g}_A$) of the reduction in the expected profitability for technology A, the limit share of technology A will be smaller for the faster decreasing function (this means that $\tilde{g}_A(x) \leq g_A(x)$ for all $x \in [0,1]$).

The results show that the final sharing of the market depends upon changes in the imitators' expectations of the profitability of new technologies. *Figures 9.2* and *9.3* illustrate the dependence between changes in the expectations of the imitators and shifts in the final structure of the market. The decrease in the rate of technology-expected profitability means that imitators estimate their chances of gaining profits by the introduction of technology A as greater than by the introduction of technology B. Therefore the changes in the expectations of imitators lead to a corresponding change in the limit structure of the market. Within this framework we can deal with a situation where one of the technologies (say B) is a conventional one. This means that $g_B(x) = \text{const}$ for all $x \in [0,1]$.

To illustrate this let us consider the simplest examples. Assume that $g_A(x) = a + b(1-x), g_B(1-x) = c + dx$. Here $g_A(0) = a + d$ is the expected profitability of technology A when nobody in the market sample uses it. Also $g(1) = a$ is the expected profitability of technology A when all adopters in the sample us it. Consequently $a \geq 0, b \geq 0$. Similarly $c \geq 0$ and $d \geq 0$. These functions are given in *Figure 9.4*.

If $a + b > c$ and $d + b > 0$ there is only one solution x^* of the equation $g_A(x) = g_B(x)$. It is easy to check that

$$x^* = \frac{a+b-c}{d+b}$$

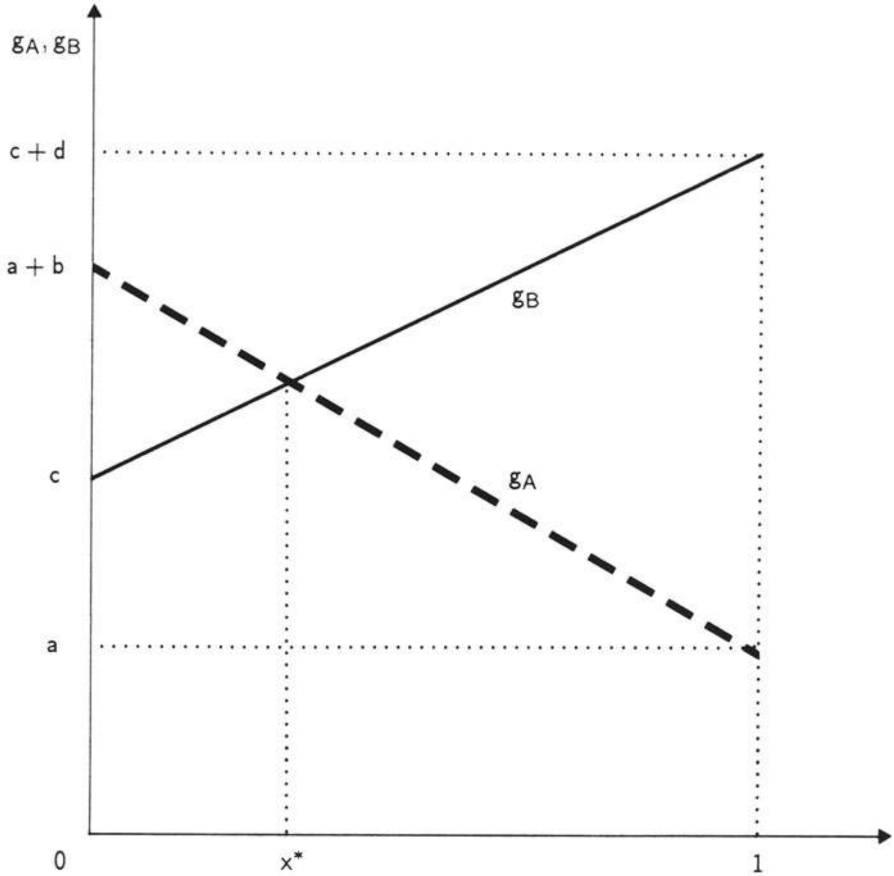

Figure 9.4. Linear (with respect to the proportions on the market) expected profitabilities of the technologies.

and x^* belongs to (0,1) if and only if $a + b > c$ and $a < c + d$. As one can see in *Figures 9.2* and *9.3*, if $\frac{a+b-c}{d+b} > \frac{1}{2}$ then technology A will dominate the market.

Thus, under this dependence of the expected probability and share of alternative technologies, the final structure of the market is determined by both the initial and final expected profitabilities of the alternative technologies or by the relation between initial expected profitabilities and rates of change. In the case when $g_A(0) = g_B(0)$, i.e., $a + b = c + d$, technology A will dominate in the market if, and only if, the rate at which its expected profitability will decrease is smaller (i.e., $b < d$). If the rates coincide (i.e.,

$b = d$) then $x^* = \frac{1}{2} + \frac{a-c}{2b} = \frac{1}{2} + \frac{(a+b)-(c+d)}{2b}$. Consequently technology A will dominate in the market in this case if and only if its initial profitability is larger (i.e., $a + b > c + d$ or $a > c$).

9.5 Conclusions

With the help of this model we can simulate the diffusion of alternative innovations under different assumptions about the influence of predecessors on the technological choice of followers so-called path-dependent processes of innovation diffusion. In this chapter we have considered the case of the diffusion of two new alternative technologies under different assumptions. It is not difficult to consider the general case with n alternative technologies. But some conceptual conclusions about the innovations diffusion can already be inferred from the results of this 2-technology model.

The interesting results concern the role of innovators (entrepreneurs) and imitators in innovation diffusion at the stage when the market share is decided. The model showed that imitators determine the trajectory of this process and the results of innovation competition. Entrepreneurs open up new technological possibilities, but their realization is determined by the imitators' choice of technologies. With uncertainty of technological choice, the probability of dominating the market is greater for the technology with the larger number of innovators. Of course, newcomers can change the situation. The result of technological competition is determined by the choice of all actors in the market. But the influence of earlier adopters on the formation of a technological trajectory is higher than those who adopt later.

These results describe important features of the alternative innovation diffusion. Both in market and centrally planned economies it is difficult to estimate the relative advantages of alternative innovations in the early phase of their diffusion or in the periods of technological paradigm substitution. In this case followers make their choice according to information about predecessor choices, and the trajectory of innovation diffusion is determined by the innovators. The technological trajectory is formed during the early phase of innovation diffusion.

The role of innovators become less important when imitators have enough information to estimate the dynamics of the expected profitability of new technologies. In this case followers make their choice according to their own estimations of future profits. These expectations determine the trajectory of innovation diffusion and the final share of alternative technologies. With the help of this approach one can simulate the formation of new technological trajectories under different types of imitator behavior.

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