Digital Postcompensation Using Volterra Series Transfer Function

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***Abstract*—We propose a noniterative digital backward propagation technique, based on an inverse modified Volterra series transfer function to postcompensate transmission linear and nonlinear impairments in the presence of optical noise. Using a single-channel 40-Gb/s nonreturn-to-zero quadrature phase-shift-keying optical signal propagated over 20 × 80 km of standard single-mode fiber, and performing digital postcompensation around the Nyquist rate, our compensation algorithm is able to surpass the maximum accuracy obtained with a symmetric split-step Fourier method, enabling us to increase the nonlinear tolerance by approximately 2 dB.**

***Index Terms*—Backward propagation, coherent detection, digital postcompensation, Volterra series transfer function.**

I. Introduction

In order to increase the spectral efficiency of optical communication systems, advanced modulation formats are used in conjunction with coherent detection. Coherent detection allows recovery of amplitude and phase information of the received field, opening the possibility to apply powerful digital postcompensation methods [1].

Recently, a digital postcompensation technique based on backward propagation (BP) has been proposed for the joint mitigation of both linear and nonlinear impairments in fiber [2], [3]. This technique requires high computational power in order to numerically solve the inverse nonlinear Schrödinger (NLS) equation. So far, digital BP has been preferably implemented using either a split-step Fourier (SSF) method [2], [3] or a split-step finite-impulse response filter (SS-FIR) [4]. While the first relies on an hybrid time and frequency-domain approach, the later is entirely implemented in the time-domain. Both are iterative numerical methods.

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Volterra-series expansion is a powerful tool for the analysis of time-invariant nonlinear systems. In [5], a frequency-domain Volterra series transfer function (VSTF) was derived from the NLS equation for single-mode fibers. In [6], a VSTF is used to analyze nonlinear effects in dense wavelength-division multiplex (WDM) systems. Nonlinear electrical equalizers based on time-domain Volterra series were proposed for coherent optical systems [7]. It was shown that its accuracy could be comparable to an SSF method with one step per span, if sufficient delay taps are used. This time-domain approach requires one order of magnitude less computations than an SS-FIR method, but still one order of magnitude more than an SSF method [7].

Being nonrecursive and applied in the frequency-domain, VSTF has the potential to be computationally more efficient than a backward propagation SSF (BP-SSF) method. In the scope of the design and analysis of optical links, VSTF accuracy was proved to be comparable with an SSF method [6]. However, its performance must be also studied for post-compensation purposes, where the sampling rates are much lower (around the Nyquist limit) and the amplified spontaneous emission (ASE) noise introduced by optical amplifiers may cause serious divergence problems.

We apply Volterra series theory to design a Volterra-based BP technique (BP-VSTF) in the frequency-domain. Using BP-VSTF with 2 samples per symbol to perform digital postcompensation of a 40 Gb/s nonreturn to zero quadrature phase-shift keying (NRZ-QPSK) signal transmitted over 20 × 80 km of standard single-mode fiber (SSMF), we observe a nonlinear tolerance improvement of approximately 2 dB over the performance limit attained by BP-SSF.

II. Theoretical Formulation

In order to implement digital BP using Volterra theory, we must find an inverse VSTF (IVSTF) capable of describing the reverse propagation of signal in fiber, which is modeled by the reverse NLS equation given by [2], [3],

$$-\frac{\partial A}{\partial z} = \frac{\alpha}{2}A + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - i\gamma|A|^2 A, \qquad (1)$$

where A is an abbreviation of $A(t,z)$ describing the slowly varying complex envelope of the optical field at time t and position z, α is the attenuation coefficient of the fiber, β_2 accounts for the group velocity dispersion and γ is the nonlinear coefficient accounting for the Kerr effect.

Starting from (1) and following a similar procedure to the one described in [5], we obtain a third-order truncated IVSTF, describing the input field spectrum $\tilde{A}(\omega,0)$ at the expense of the output field spectrum $\tilde{A}(\omega,z)$, which is analogous to the forward VSTF but with symmetric fiber parameters ($-\alpha$, $-\beta_2$ and $-\gamma$),

$$\tilde{A}(\omega,0) = H_1'(\omega,z)\tilde{A}(\omega,z) + \int\int H_3'(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z) \times \tilde{A}(\omega_1,z)\tilde{A}^*(\omega_2,z)\tilde{A}(\omega-\omega_1+\omega_2,z)d\omega_1 d\omega_2, \quad (2)$$

where $(.)^*$ represents the complex conjugate of $(.)$, $H_1'(\omega,z)$ is the inverse linear kernel,

$$H_1'(\omega,z) = \exp\left(\frac{\alpha}{2}z - i\frac{\beta_2}{2}\omega^2 z\right), \quad (3)$$

and $H_3'(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z)$ is the inverse third-order nonlinear kernel,

$$H_3'(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z) = -i\gamma H_1'(\omega,z) \times \frac{1-\exp(\alpha z - i\beta_2(\omega_1-\omega)(\omega_1-\omega_2)z)}{-\alpha + i\beta_2(\omega_1-\omega)(\omega_1-\omega_2)}. \quad (4)$$

When input power is high, the VSTF method has serious energy divergence problems, limiting its practical application. In order to solve this issue, a modified VSTF (MVSTF) method is presented in [8], based on a phase correction of the output optical field. Applying an analogous phase correction to our IVSTF we obtain an inverse MVSTF,

$$A(t,0) = \begin{cases} A_{LI}\exp\left(\frac{A_{NL}}{A_{LI}}\right), & \text{if } |A_{NL}| < |A_{LI}| \\ A_{LI} + A_{NL}, & \text{otherwise,} \end{cases} \quad (5)$$

where A_{LI} and A_{NL} are the linear and nonlinear contributions to the equalized optical field, given by the inverse Fourier transform of the first and second right-hand terms in (2), respectively.

Using expressions (2)–(5) we are able to describe the reverse propagation of signal in fiber, in a similar way as BP-SSF. Therefore, henceforward we will denominate this technique as BP-VSTF. However, whereas BP-SSF is a recursive method, the BP-VSTF here derived presents the advantage of being an approximated closed-form solution of the reverse NLS equation.

III. Compensation Results

A general model of an optical coherent system is shown in Fig. 1. This system model will be considered throughout this work to obtain our simulation results. The main parameters regarding signal transmission and propagation are shown in Table I, where B is the signal bit rate, N_{bits} is the total number of transmitted bits, L_{span} is the span length, G is the optical amplifier gain and F_n is the amplifier noise figure. Fiber parameters are set in accordance to typical SSMF (Table I). Both transmitter and local oscillator lasers intensity and phase noise are neglected. The 90 degree optical hybrid and the pair of balanced photodiodes are assumed to perform optical-to-electrical down-conversion without distorting the received signal. We emulate signal propagation in fiber in the forward direction using the symmetric version of the SSF method with very high temporal and spatial resolution.

As a figure of merit for compensation performance we use the error vector magnitude (EVM) percentage relatively to optimal constellation, defined as $\text{EVM} = \sqrt{|A_c - A_{tx}|^2/|A_{tx}|^2}$, where A_{tx} and A_c are respectively the transmitted and compensated optical fields.

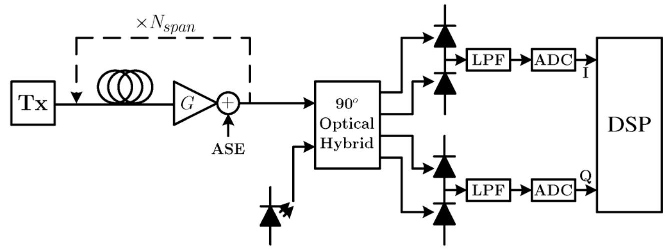

Fig. 1. Coherent NRZ-QPSK optical system model adopted in this work. Digital postcompensation is implemented in the digital signal processing (DSP) block. LPF: Low-pass filter. ADC: Analog-to-digital converter.

TABLE I
Set of Simulation Parameters

Input Signal	Fiber	Optical Amplifier
Format: NRZ-QPSK	$\alpha = 0.2$ dB/km	$G = \exp\left(\frac{\alpha}{2}L_{span}\right)$
$B = 40$ Gb/s	$\beta_2 = -20.4$ ps^2/km	$F_n = 5$ dB
$N_{bits} = 2048$	$\gamma = 1.3$ W^{-1}km^{-1}	
	$L_{span} = 80$ km	

We performed digital postcompensation using both BP-VSTF and BP-SSF with different number of steps per span, N_{steps}, and samples per symbol, N_{sp}. Third order low-pass Butterworth filters (LPFs in Fig. 1) with cutoff frequency at 80% of the symbol rate are used to filter the out-of-band ASE noise and to reduce the aliasing effects due to downsampling in the ADCs. The results presented in Figs. 2(a) and 2(b) allow to draw two main conclusions: (i) BP-SSF reaches its performance limit at approximately 8 steps per span and (ii) BP-VSTF approaches BP-SSF maximum performance for 3 and 4 samples per symbol. The reason for (i) is that temporal resolution sets an upper limit for the compensation performance, above which increasing the spatial resolution is useless. On its turn, despite of being a third-order approximation, BP-VSTF only depends on temporal resolution, and at 3 and 4 samples per symbol its performance is similar to an heavily iterative BP-SSF method. Furthermore, in Fig. 2(c), where we kept only 2 samples per symbol, we can see that BP-VSTF is able to largely surpass BP-SSF maximum performance. In fact, BP-SSF has been reported to require at least 3 samples per symbol in order to produce reliable results [9]. The degradation of performance experienced by the BP-SSF method at 2 samples per symbol is due to the aliasing components generated by the time-domain implementation of the nonlinear operator, which is subsequently transposed to frequency-domain. In turn, since BP-VSTF is entirely implemented in the frequency-domain, this aliasing phenomenon does not take place. Using BP-VSTF with 2 samples per symbol we have only experienced a marginal loss of precision due to the use of such low temporal resolution. Considering the 10% limit in EVM values for which the system bit error rate (BER) is kept bellow 10^{-9} [10], we observe that there is approximately a 2 dB improvement in the nonlinear tolerance when compensation is performed by BP-VSTF. In Fig. 3 we observe

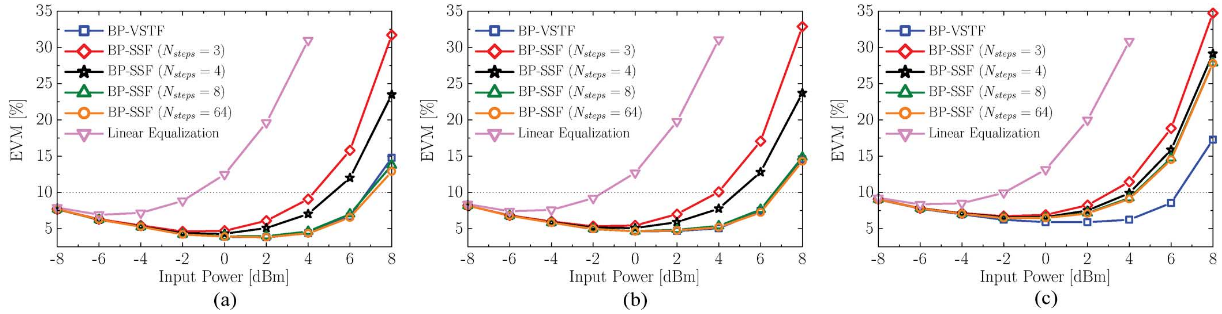

Fig. 2. Error vector magnitudes of BP-VSTF and BP-SSF after propagation over 20 × 80 km of SSMF. The oversampling factors are (a) 4; (b) 3; and (c) 2.

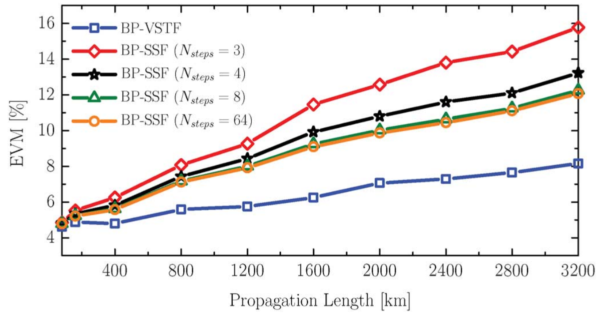

Fig. 3. EVM of BP-VSTF and BP-SSF as a function of propagation length, for a fixed input power of 4 dBm and using two samples per symbol.

that, after 3200 km, BP-VSTF performance becomes comparable to the maximum BP-SSF performance at 1200 km. This represents an improvement of approximately 2.6 fold in propagation distance attained by the BP-SSF method.

IV. Computational Requirements

In order to compare the number of computations required by BP-SSF and BP-VSTF we can use the upper bounds on computational requirements provided in [5]. BP-VSTF complexity evolves as $O(N_{FFT}^2)$, where N_{FFT} is the fast-Fourier transform (FFT) block-size. BP-SSF complexity is of the order of $O(N_{step} N_{FFT} \log_2(N_{FFT}))$. We must also consider that $N_{FFT} = N_{sym} N_{sp}$, where N_{sym} is the total number of symbols fed to each FFT block. If the FFT block-size is kept small enough, the computational load of BP-VSTF tends to be smaller than that of BP-SSF. However, since the number of computations in BP-VSTF increases quadratically with N_{FFT}, the FFT block-size becomes critical. Overlap-and-save and overlap-and-add algorithms are possible solutions to reduce the FFT block-size. Another key advantage of BP-VSTF lies on its high modularity, since linear and nonlinear equalization can be separately implemented. Thus, nonlinear compensation becomes much more flexible than that of BP-SSF, enabling the receiver to trade off performance for computational load.

To employ the BP-VSTF method in multichannel optical systems we may consider two different approaches: (i) apply a BP-VSTF per channel, disregarding interchannel effects; (ii) apply the BP-VSTF to the entire signal spectrum, enabling to compensate interchannel nonlinearities. Considering a WDM system with N_{ch} channels with N_{FFT} block-size per channel, the complexity of (i) is $O(N_{FFT}^2)$ per channel, whereas the overall complexity of (ii) is $O(N_{ch}^2 N_{FFT}^2)$. The modified version of the 3rd order BP-VSTF enables a high tolerance to input power levels, which is critical for analyzing WDM systems. However, if the launched power is further increased, higher order kernels must be considered, at the cost of increased computational complexity.

V. Conclusion

We present a noniterative numerical method based on an inverse modified Volterra series transfer function of a single-mode optical fiber, which is capable of performing digital BP in optical coherent communication systems. Using a 40 Gb/s NRZ-QPSK test signal and taking only 2 samples per symbol, we have performed digital postcompensation of a 20 × 80 km optical link in the presence of ASE noise. We have observed that BP-VSTF allows to extend the quasi-linear regime in fiber by approximately 2 dB, comparing with BP-SSF. Being a noniterative and highly modular BP approach, BP-VSTF has the potential to decrease the computational effort required for postcompensation, maintaining high performance.

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