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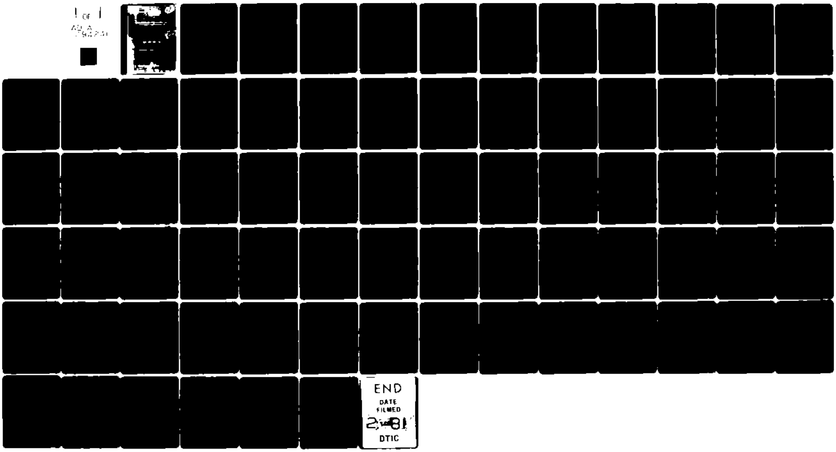

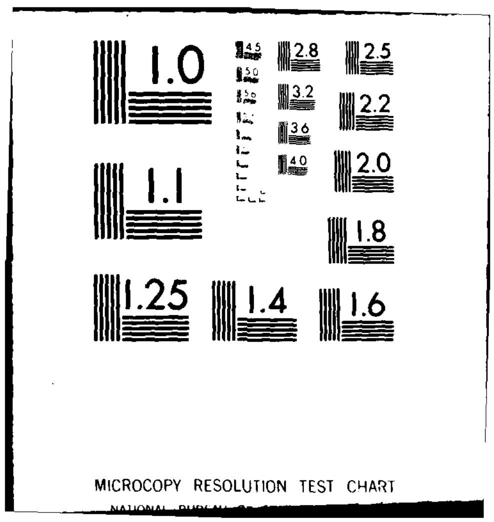
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DIGITAL REPRESENTATION OF THREE-DIMENSIONAL SURFACES BY TRIANGULATED IRREGULAR NETWORKS (TIN)

by

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WITH AN INTRODUCTION

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with 9 figures and 4 appendices

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consisting of

EXECUTIVE SUMMARY

If we have a good map, we can use it for many purposes. We get an overview of a landscape by just looking at it. We can use the contour lines to get an impression of the topography of a region (the relief). We can obtain a different view of the relief by deriving sets of vertical cross sections with height profiles. We can determine with some effort which connection between two points has the smallest maximum slope. We may calculate which parts of a terrain remain invisible from one or several look-out points. Considering each point of a ridge as the apex of an angle, one side points vertically downwards into the earth. The points where the other side touches ground can be derived. They line out a part of a terrain which will be in shadow or quasi-shadow for a given "source." If a map contains reliable sub-water bottom contours, we may use it to find out at which part of a long stretch of beach certain predetermined conditions of beach slope are maintained. We may derive skylines or horizons as seen not only from points at the surface but also from a pilot in an aircraft flying in some height over the terrain of the map. All these and many more procedures, which may have significance for a large host of civilian and military problems, are possible; the quality of the result depending on the resolution and the accuracy of the map. - If we do not have a map but can send an airplane out to investigate a certain terrain, the opposite task is given: we can transform what the pilot sees into a map; or we can get some answers to questions posed above directly from the pilot in some form other than a map.

In most cases, such derivations require much time, often very much time, if they are done by hand. In order to speed them up, machine processing is envisioned, and it is believed that as a base for it a digital presentation of the terrain will be necessary.

A similar digital presentation may, then, be advantageous also for some purposes applying relationships between three variables of any kind, e.g., laboratory results, environmental measuring results - all kinds of functions which cannot be expressed in a closed analytical form.

The basic question to be solved for the provision of such a digital presentation of three-dimensional surfaces is given by the necessity to select a system which requires a minimum amount of computer storage, machine time for the acquisition of the digital presentation, and minimum computer time for any procedures to which that presentation may then be applied - as indicated by the listing of possibilities above and including the actual generation of a map or some other form of display.

Apparently, a regular grid in Cartesian coordinates is the simplest approach. If we have the x, y, and z values of all points, we have the surface completely assessed. The problem is that we never shall have all points so that we have to decide how many we want to get. Trying to implement this, we quickly find out that with every possible selection, we shall have too many points in some cases and too few in others, and both cases will generally happen within the same study area. Consequently, we both use more computer than would actually be necessary and do not obtain the optimum accuracy and resolution. Why is this so? Because most of the relations between three variables which cannot well be expressed by a closed analytical form, are of an irregular nature. It follows that an irregular system is better fit than the Cartesian with its extreme regularity. While this is clear, it is by no means certain that such an irregular system will not pose unsurmountable difficulties of other kinds.

The content of the present document is dedicated to a demonstration that there are large difficulties, but that they can be overcome - with the result that the "Triangulated Irregular Network" and the higher-level data structures which are derived in connection with it, provide means to digitize three-dimensional surfaces in a significantly more economic form, called the "Geographic Data Structure."

To convey a simplified concept about a possibility to approach such a task, we may envision putting a horizontal triangle under a hilly or mountainous area of the earth's surface, and locate the highest point (e.g., hilltop) above it. We then get a triangular pyramid as the first rough approximation of the terrain. To refine it to a second approximation, we ask for the highest (or lowest) point above (below) each of the three sides of that pyramid and in this way construct three additional triangular pyramids on the original one. For a third approximation, the same procedure is applied to the 3 x 3 = 9 sides of the additional pyramids and so on. - The other approach leading to the higher-level data structures mentioned above, starts from the natural features of the terrain, such as peaks, passes, and pits, ridges and channels, breaks in the slopes, etc. and uses them for terrain presentation in a digital manner.

In section 5.3.2., it is stated that the structure developed by the authors is especially well suited to be applied to systems which involve more than three dimensions. It is predicted that the gain in economy by using this structure is even increasing with the number of dimensions involved.

The present document has been written with the educated layman in mind who has some knowledge about computers and about digital carthography, but its essential contents should be understandable also if that particular expertise is lacking. In its chapter 5, applications of the systems to the general purposes of digital modeling of three- or more dimensional surfaces are discussed. The authors do not consider in detail applications to problems in public life, civilian or military, which could be attacked by this method. That is largely left to the imagination of the reader. In chapter 6, however, one special problem of particular interest to the sponsoring agency of the U.S. government is briefly indicated: a Coastal Modeling System.

A study of the collection of abstracts of the individual chapters which is following this Executive Summary gives information to decide on a more careful investigation of the whole document.

Work is still in progress. It has been demonstrated, however, that the large potential of the new method is indeed existing as predicted, and no serious obstacles have been found. This new method, because of its potential to significantly shorten all types of computer requirements, opens up new avenues in the fabrication and application of maps, indeed opens up a new field - "carthographic technology," with practical applications even now hardly predictable. It may also find revolutionary ways in the treatment of other three- or more dimensional variations in physics, geophysics, chemistry, biology, and other fields.

For the expert who wants to read more on the new system, Appendix I provides a list of other reports. Appendix II lists the Programs and subroutines which have been developed so far for this system. Appendices III and IV give some more technical details.

ABSTRACTS FOR THE CHAPTERS 1 TO 6

CHAPTER 1: GENERAL FEATURES OF THE SYSTEM

In order to lead the reader to a description of the new system, we start out with a comparison of it with the familiar type of the rectangular, regular grid. In this way, specific traits of the new system are easily recognized. After a general comparison using the New York Bight as the study object in section 1.1, that comparison is quantified by applying a method originally developed for another study area in British Columbia (section 1.2). Precision, of course, plays an essential role in each quantification; thus the precision of the sources for the data used is discussed in section 1.3. Here, information is provided on the accuracy requirements of geographic maps used as sources for terrain digitizing; and the fact that digital terrain models often are needlessly redundant is pointed out quantitatively.

CHAPTER 2: SAMPLING CONSTRAINTS AND METHODS OF PREPARATION OF TRIANGULATED IRREGULAR NETWORKS

As pointed out in the preface, the "human approach" for terrain description provides the possibility to delineate the most important features quickly and with little effort; the effort will increase if more detail (higher resolution) is wanted. Basically, the same applies to the Triangulated Irregular Network method. This is to be discussed later in this document but will become clear only if we first negotiate the fact that the degree of sophistication of the techniques applied in sampling and triangulation will affect the accuracy of the Triangulated Irregular Network. That fact is discussed in this chapter 2. This involves a discussion on data sources which have been used in the context *of this and similar methods. At the end, the importance of filtering surface details with regard to the ultimate use of the model is explained.*

CHAPTER 3: MANUAL VERSUS AUTOMATED TECHNIQUES FOR THE CREATION OF TRIANGULATED IRREGULAR NETWORKS: INTUITION AND PATTERN RECOGNITION

In this chapter 3, the basic description of the Triangulated Irregular Network is concluded by a discussion of the various methods to construct the TIN from other sources. Manual, hybrid and some still naive automated techniques and their essential capabilities are described.

CHAPTER 4: HIGHER LEVEL DATA STRUCTURES AND TOPOLOGICAL CONSIDERATIONS

A "second" data structure is developed so-to-speak "on top" of the Triangulated Irregular Network. This is done because the TIN alone does not provide easy access to a data base - a problem if the data base is large. - The second structure, basically a ridge-and-channel structure, represents the surface in its general features, and serves as a kind of directory into the surface. It is discussed in this chapter 4 together with methods for the extraction of the surface features (ridges, channels; peaks, pits, passes; slopes, breaks, flats).

CHAPTER 5: APPLICATIONS OF THE TRIANGULATED IRREGULAR NETWORKS

In this chapter 5, application of the total Geographic Data Structure (GDS) and/or of its parts, the Triangulated Irregular Network (TIN) and the Higher Level Data Structures are discussed. Under "application" is understood an application for general purposes of digital modeling (including extraction, calculation tasks, display, etc.) of three-dimensional surfaces. - The authors do not deal with any detailed description of problems in public life, civilian or military, which can best be approached by digital methods of this kind; only some hints are given in this regard.

CHAPTER 6: FUTURE RESEARCH

In this document, tasks which are now being implemented are mentioned several times. Work in the near future will, of course, concentrate on these. The next step is intended to bring the GDS closer to actual tasks in the coastal environment if suitable bases for such tasks can be found. It is assumed that as a result of further discussions - in particular some based on the content of chapter 5 - further development towards specific applications will be required.

PREFACE

Abstract

Expression of relationships between 3 (or more) variables in form of numbers of other mathematical expressions is a modern requirement for adequate forms of transmission, storage, display and calculation of various types of information, e.g., on terrain. A man describing a landscape is using an approach which is different from that used by a machine transmitting a map of that landscape. A new system of digital representation of three-dimensional surfaces is being described which attempts to combine the advantages of those two approaches. It looks complicated at first glance but it is more economical than the systems now in use. In addition to providing a better method to digitize geographical topography, it can, in principle, be applied to any relation between three variables.

Simple (or complicated) mathematical relations between three (or more) variables are best stored in analytical form by writing down in an equation what the function f in the relation $z = f(x,y)$ exactly is. From that analytic storage graphical displays can be derived (manually or by computer) in several different forms; e.g. as a truly three-dimensional relief made of plaster, cardboard, etc., as a contour map, as a series of cross-section profiles, as a hologram, as a series of "pictures." Also, from the analytical form, calculations can be made to provide answers to specific questions, e.g. what are the maxima and minima of one variable (e.g. height in terms of other variables e.g. location); the steepest slopes; the largest areas x, y with no variation in z larger than a given threshold; the whole surface area or the volume under a certain z value within a given x and y range, and so on.

If relations between three (or more) variables cannot be represented with desired accuracy by any analytical expression, the storage, display and calculation problems become more severe. This is sometimes the case with results of laboratory experiments, and is mostly so with data from observations and measurements in the field. To give just one example for the latter case: the variation in time of a temperature profile in the atmosphere: $T = f(h,t)$.

The best known case is, of course, that of the geographic surface, the topography or orography of the Earth. Without forgetting about the wider aspects indicated in the preceding paragraph, we shall concentrate in this document on these aspects of digital cartography. Naively, we may envision two possible approaches to this problem. If a man stands on a mountain and is asked to describe the topography of what he sees through a telephone, he probably will start saying that he sees - for example - three large mountains in such and such directions and so and so distances, which have the height a, b, and c. If that information is sufficient for the listener at the other end of the telephone line, our man has done his job. If it is not, he will continue by saying, for example, that between him and mountain A there is a hill and he will say where it is and how high and, if that is wanted, which form it has. And so it goes on, until the listener has all the information he needs. Let's call this, for the moment, the "human approach."

The other approach we call the "grid approach." It will probably be used by somebody who has a contour map in front of him and has been given a task to devise a machine which would transmit all the information or a carefully defined part of the information contained in the map via a telegraph line to another machine which reproduces the map at the distant location. Most probably, this man would look at the map and then, depending on the detail shown and the resolution deemed necessary, define basic distances $x_n - x_{n+1}$ and $y_m - y_{m+1}$, and then have the z- values of each of these grid points telegraphed to the other machine. Once the mesh-width of this grid has been determined and the resolution for the z- values resolved, there is only one set of information, independent of what the man using the distant machine actually wants.

Obviously, both approaches have their advantages and disadvantages. It is the purpose of this document to give a general description of a new system which tries to keep the advantages of both the "human" and the "grid" and to avoid their disadvantages. As might be expected, this is not an easy task, and the description of the work performed to achieve it is everything but naive. It is, however, worthwhile because it turns out that with the new system the actual operations of acquiring the data, of performing calculations with them, and of displaying the information in various forms, are significantly less demanding in computer time and storage space - and that is what counts.

In the following paragraph, the new system is summarily described. The text of the document then reports on the system, discusses applications, ponders future research in a language which should be easily understandable to the geographer with some experience in digital mapping. Four appendices, finally, list previous reports and enlarge on various aspects.

The Geographic Data Structure (or GDS) is a geographical information system which differs from most existing systems by the structure of its data-base and of most of its manipulation routines. The basis of the structure is a Triangulated Irregular Network (TIN) of irregularly-distributed points with their neighborhood relations explicitly stored with the point coordinates.

This arrangement allows for a highly flexible adaptation to surfaces of varying scale, size, and complexity. A second structure, based on the structural points and lines of the surface (peaks, pits, passes, ridge lines and channel lines), is developed as a generalization of and directory into the first data base.

1. GENERAL FEATURES OF THE SYSTEM

Abstract for Chapter 1

In order to lead the reader to a description of the new system, we start out with a comparison of it with the familiar type of the rectangular, regular grid. In this way, specific traits of the new system are easily recognized. After a general comparison using the New York Bight as the study object in section 1.1, that comparison is quantified by applying a method originally developed for another study area in British Columbia (section 1.2). Precision, of course, plays an essential role in each quantification; thus, the precision of the sources for the data used is discussed in section 1.3. Here, information is provided on the accuracy requirements of geographic maps used as sources for terrain digitizing; and the fact that digital terrain models often are needlessly redundant is pointed out quantitatively.

1: GENERAL FEATURES OF THE SYSTEM

1.1: A Comparison of the TIN and Regular Grid via an Example

In order to visually test the "feature" representation and recognition capabilities of the GDS, we have taken as an example the topography of the sea floor near New York City (the "New York Bight" area). We have represented this area by a regular square grid of 22 by 23 points (506 points, Figure 1), and by a TIN of only 31 points (a 16.3 : 1 ratio; Figure 2). It has been found (see section 1.2) that for some geomorphological measurements 14 points of a regular grid are needed instead of one point in the Triangulated Irregular Network (TIN) to achieve the same accuracy (Mark, 1975). However, for an accurate representation of terrain features, the ratio has to be higher: The principal "feature" of this surface, the Hudson submarine canyon, is clearly represented more adequately in the display based on the TIN. In a regular grid, a much higher density of points than the 22 x 23, would be needed to "capture" the canyon's smoothly curving form. A comparison of the contour map of the triangulated irregular network (TIN) (figure 4) and the contours of the regular grid (figure 3) shows that the regular sampling regime of the grid creates pits, closed contours, along the floor of the canyon. These do not exist on the original map from which the data were taken. Also the regular grid misses some of the shoreline features such as Sandy Hook and Breezy Point. Furthermore, a regular grid requires that the density be increased throughout the data area to the level required at the smallest feature of interest. In the GDS (Geographical Data Structure), the sampling density is adapted to the local complexity of the surface. In the TIN system, each recorded point requires more storage space and pre-processing time, because the TIN has to store the x and y coordinates of the point as well as the numbers (labels) of its neighbors, whereas the regular grid only stores the z coordinate. Thus, the storage requirements of the TIN in this example are about 5/8 rather than 1/16 those of the grid. Still, the TIN is clearly superior while requiring less storage.

Within the Geographic Data Structure, the sample points are organized into triangular facets which cover the study region. This constitutes the TIN, the Triangulated Irregular Network. The surface-specific points, peaks, pits, passes, and points along significant breaks of slope form the core of the data points. Additional points are chosen so that the planar facets of the resulting triangles closely match the terrain that is digitized (see section 3.1). The triangles are not explicitly stored, rather the links between points on edges are stored. Storage by triangle requires for each

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GDS: Geographic Data Structure (see last paragraph of the Preface).
TIN: Triangulated Irregular Network

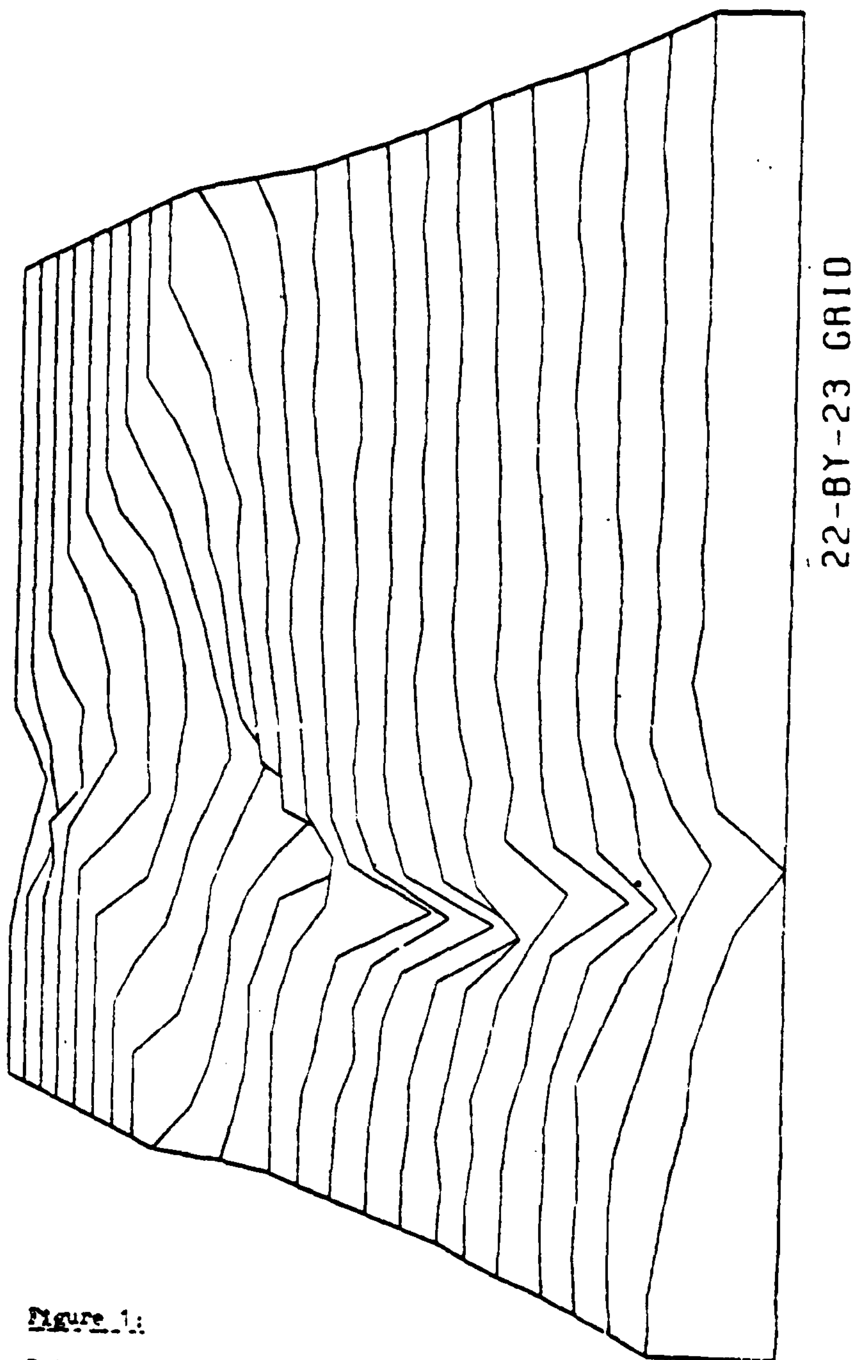


Figure 1:

Bottom topography of New York Bight displayed by a regular, rectangular grid of 22 x 23 grid points.

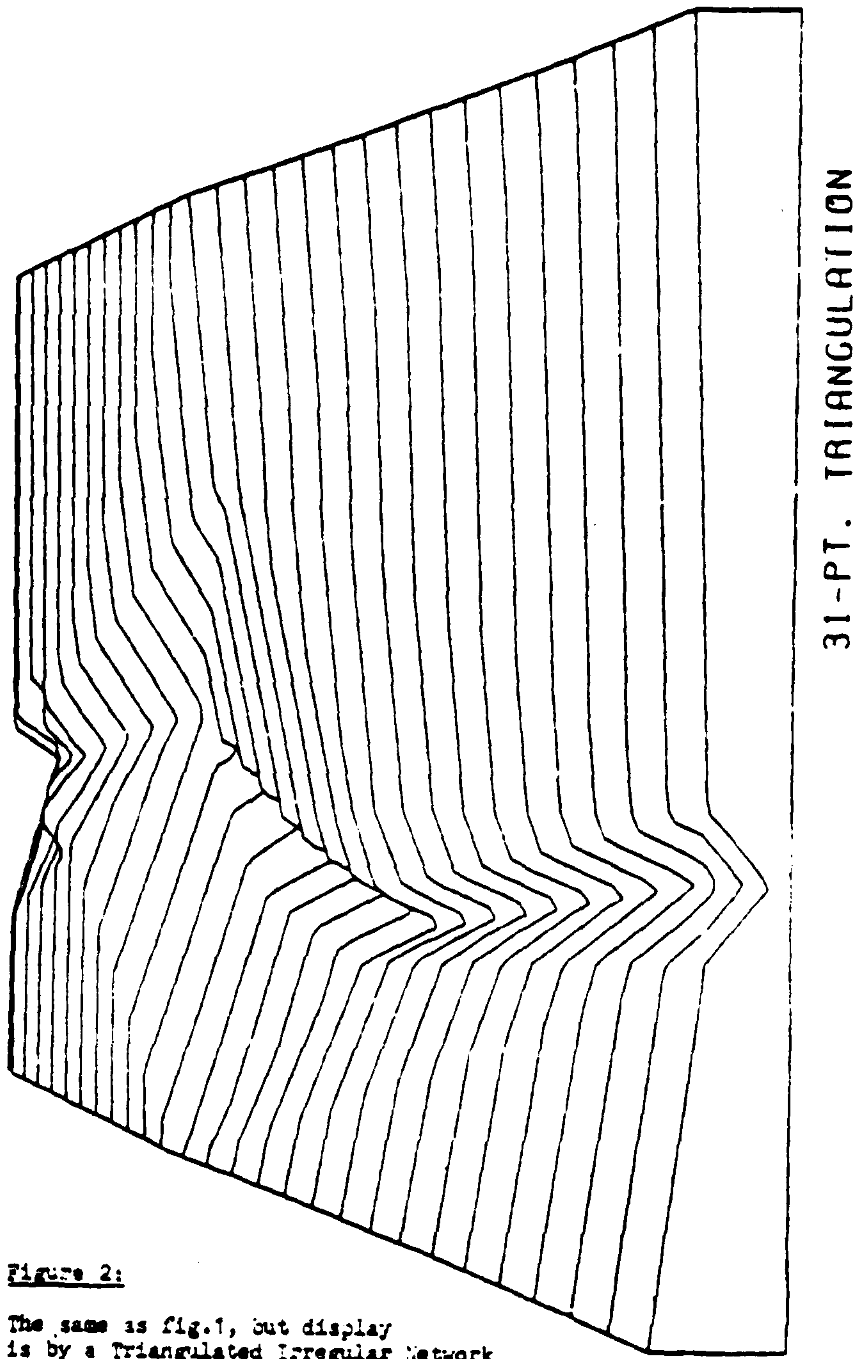


Figure 2:

The same as fig.1, but display is by a Triangulated Irregular Network of 31 points.

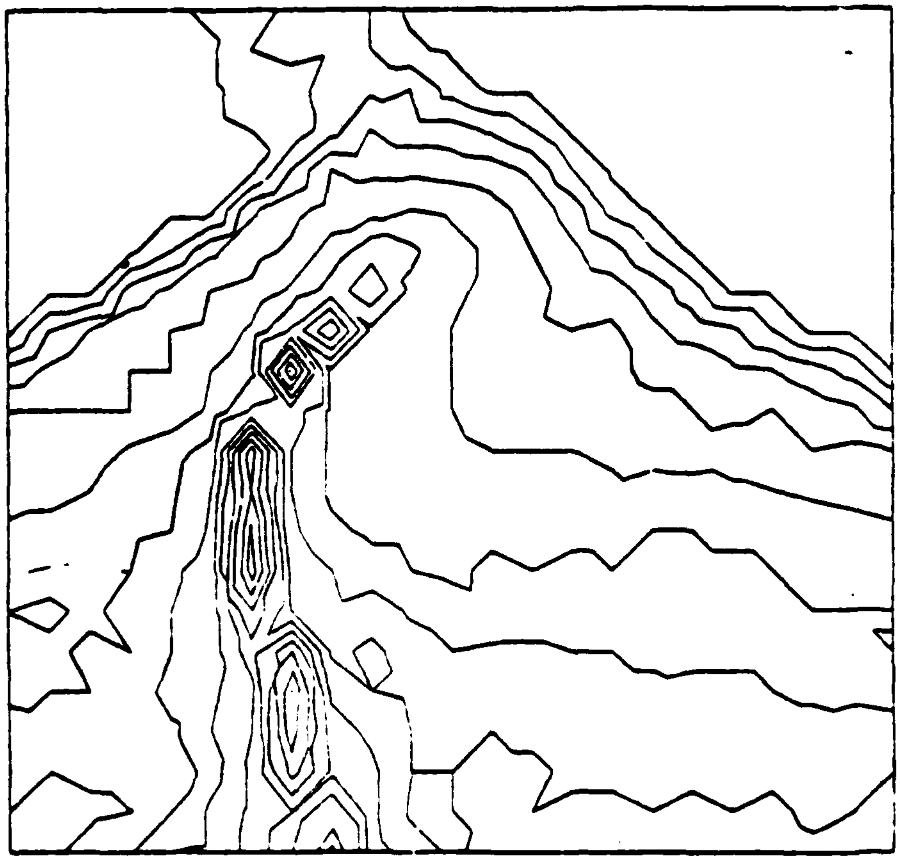


Figure 3:

Contour map of the Hudson Submarine Canyon in New York Bight, from a regular, rectangular grid.

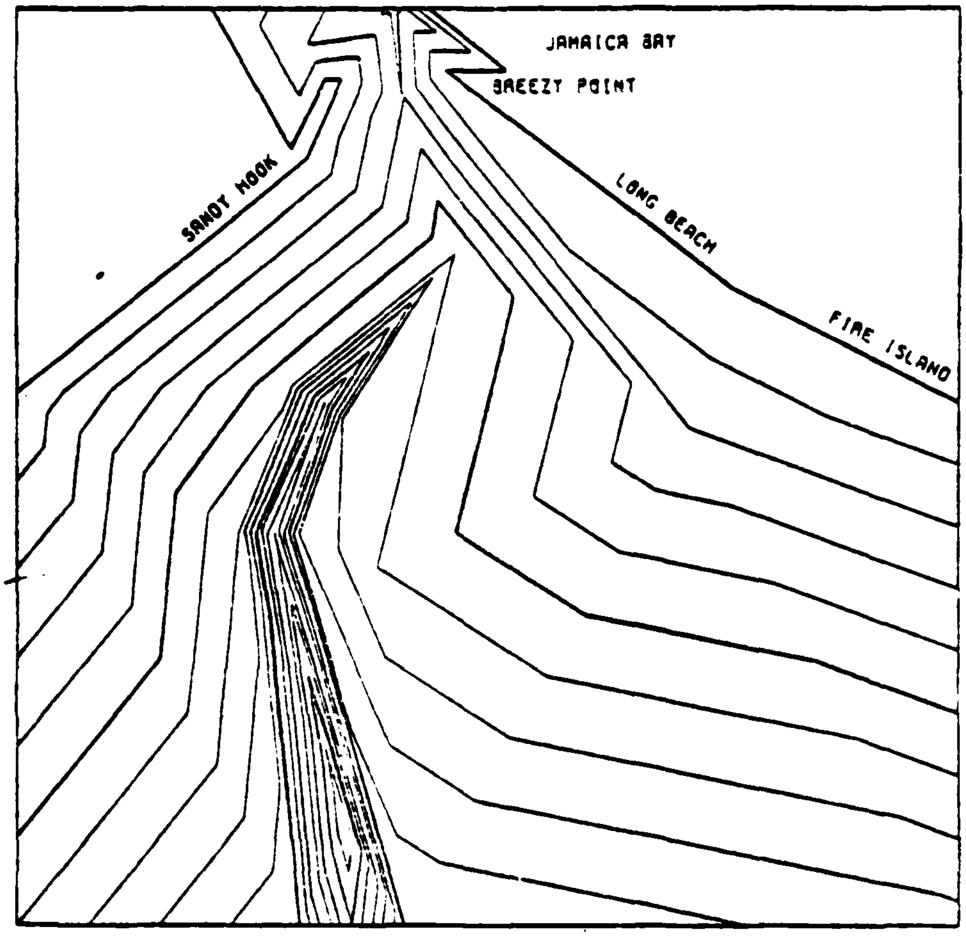


Figure 4:

Contour map of the Hudson Submarine Canyon in New York Bight from a Triangulated Irregular Network triangle 3 pointers to adjacent triangles and 3 pointers to the component nodes, which is a total of 6 pointers per triangle. For a triangulated surface composed of N points, of which B are on the boundary of the region, there are 2N - (B + 2) triangles, from Euler's relation. Therefore, storage by triangle requires approximately 12N pointers for a data set with N points. In a TIN, the data structure stores for each point the labels of all points which are linked with the point by an edge of a triangle, clockwise from the north. This organization defines the relative locations of the sample points and the triangular facets. Specifically, the set of points which are linked to a given point are termed its "neighbors" (Peucker and Chrisman, 1975). Since the average number of neighbors per point in a TIN cannot exceed 6 (Peucker, 1973), the expected number of pointers in a TIN of N points is at most 6N. Thus the representation method we have chosen, by links to neighbors, is twice as space-efficient as storage by triangles.

Within the familiar format of the regular, rectangular grid, the neighborhood relationship is expressed implicitly in the structure of the grid itself. A point at position (I,J) in the grid has orthogonal neighbors at (I-1,J), (I,J-1), (I,J+1), and (I+1,J), and diagonal neighbors at (I-1,J-1), (I-1,J+1), (I+1,J-1) and (I+1,J+1). This is a topological relation that captures the connections between the elements of the digital terrain model (DTM). - Since the relative position of the points in the sample are not determined by a given spacing as in the rectangular grid, the Triangulated Irregular Network (TIN) does not suffer from a directional bias (see section 5.0).

1.2: Quantification of the Comparison

As Boehm (1967, p 414) so correctly pointed out, "one cannot discuss the relative efficiencies of tabular representation methods without reference to the problem being solved". The only detailed study which has quantitatively compared the performance of a TIN with other representation methods (specifically, regular grids) used as the "problem" the estimation of a selected set of geomorphometric measures. This work is reported in detail in Mark (1975), and will be outlined briefly here. The research involved the estimation of topographic local relief (H), which is the difference in elevation between the highest and lowest points in a terrain sample; average land slope (g), the mean tangent of surface gradient (the vector of steepest ascent): and the "hypsometric integral" (I), a measure of the relative land-mass volume enclosed by the land surface (related to average elevation). Six 7 by 7 km study areas from differing terrain types in southern British Columbia were selected and digitized from topographic maps (1:50,000 scale; 100 foot contour interval) using both 500 m grids (15 by 15; 225 points) and TIN's (81 to 142 points: mean=114). The number of points in the TIN's was adapted according to terrain variability or "roughness". Each of the parameters mentioned above was estimated for each area from each data base, and also manually from the maps using "standard" analysis procedures. Table 1 (Appendix III) summarizes the

DTM : Digital Terrain Model
TIN : Triangulated Irregular Network

results. Slope (g), and to some extent the hypsometric integral (I), represent "average surface properties", and the point ratios for these (14:1 and 8.4:1, respectively) may represent the overall average surface precision. Local relief (H), however, depends on points more or less coinciding with the true extreme points of the terrain, and as such represents the problem of recognizing valleys, ridges, and other specific features. The 287:1 point ratio for this parameter agrees roughly with the visual impression of surface feature recognition in the New York Bight example, since a 287:1 ratio would involve grids roughly 4 times as dense (16 times as many points) as those used. It was our visual impression that such a density increase would be needed to properly portray the Hudson submarine canyon.

1.3: Precision of the Regular Grid (especially DMA DTM's)

The source from which height data are obtained is critical, since no DTM can have a higher level of precision than its source. Since most DTM's are currently compiled from topographic contour maps, it is in order to consider briefly the precision of such sources.

The accuracy standard for U.S. topographic maps requires that plotted points (including contours) must appear within 1/50" (0.5 mm) of their true positions. If this represents a 90% confidence limit (and such a limit is explicitly stated for vertical accuracy of these maps), the root-mean-square (rms) horizontal error of a 1:250,000 map may be up to about 76 m (250 ft). Taking an average ground slope of 10% (see Wood and Snell, 1959), this horizontal shift could result in a height rms error of 7.6 m (25 ft). This is the vertical accuracy standard for a point which appears to be actually on a contour, regardless of the contour interval.

When estimating arbitrarily-located points from a contour map, interpolation error must also be considered. Here, the U.S. standard calls for 90% of points to be within 1/2 the contour interval (i) of their true heights (after the above horizontal shift). Taking a 100 ft (30.5 m) contour interval (a rather conservative figure for the map series in question), the rms contribution of interpolation error would be 0.30 i, or 9.1 m (30 ft). The total map precision is thus the root-mean-square of these two values, or 11.9 m (39 ft). No DTM derived from the U.S. 1:250,000 series could correctly claim an average height precision less than this value (except for those derived from the very few maps with a contour interval less than 100 ft).

The height precision of a regular grid DTM depends upon the relation between the grid spacing and the characteristic wavelengths of the terrain (see Makarovic, 1972). This procedure breaks down the terrain into a series of

DTM: Digital Terrain Model, see near end of section 1.1, above
DMA : Defense Mapping Agency, Washington, DC, USA

Fourier components, and relates these to the grid spacing (d). It is an over-simplification, but nevertheless a useful approximation, to propose a linear relation between d and the error introduced by the grid spacing (see the sampling theorem, below; see also Mark, 1975). A study which interpolated regular grids from digitized contours (Gottschalk and Neubauer, 1974) [as does the Defence Mapping Agency (DMA)] claimed that a 50 m grid introduced a rms height error of 3.5 m (presumably assuming the contour heights to be "accurate"). This, coupled with the linearity assumption above, assumes that grid rms error is approximately 7% of the grid spacing (for an interpolation algorithm similar to the DMA's 1:250,000 DTM series), and to get the total DTM precision, this must be combined with the contour height accuracy (see above). If we equate total grid precision to total map precision, we find that the grid spacing must be 4.29 times the contour interval. Any grid spacing less than this would clearly imply redundancy, since it would appear of being capable of giving a precision higher than that of the source map (which is impossible). The DMA grids have a grid spacing of 63.5 m; this would be sufficiently dense to capture all the theoretical precision of a map with a contour interval of 50 ft or about 17 m (rare in the 1:250,000 series) have half the required spacing (hence 4 times as many points) for a 100 ft interval, and about one quarter the spacing (16 times as many points) as for the more typical 200 ft interval. It may therefore be concluded that most of the DMA grids are highly redundant in terms of apparent average height precision. Mark (1975) found that one surface-specific point in a TIN could "represent" about 14 grid points for estimating measures such as average slope (see above); after removing grid redundancy, our TIN's should be able to represent 1:250,000 map sheets with about 20,000 points, compared with some 4 1/2 million points in the DMA grids.

With TIN DTM's, precision depends upon how closely the triangles approximate the surface (see Mark, 1975, p. 184; also Mark and Peucker, 1975). This in turn depends upon the height accuracy of the points (see above), the spacing of the points (i.e., the "size" of the triangles), the selection of the points (i.e. the sampling of the surface), and the way in which they are connected (the triangulation). Of these, the last two seem to be the more important. More work is needed to quantify these relationships, which appear to vary with the triangulation procedure used (see below).

DMA: Defense Mapping Agency (Washington, D.C. USA)
DTM: Digital Terrain Model
TIN: Triangulated Irregular Network

2. SAMPLING CONSTRAINTS AND METHODS OF PREPARATION OF TRIANGULATED IRREGULAR NETWORKS (TIN)

Abstract for Chapter 2

As pointed out in the Preface, the "human approach" for terrain description provides the possibility to delineate the most important features quickly and with little effort; the effort will then increase if more detail (higher resolution) is wanted. Basically, the same applies to the Triangulated Irregular Network method. This is to be discussed later in this document but will become clear only if we first negotiate the fact that the degree of sophistication of the techniques applied in sampling and triangulation will affect the accuracy of the Triangular Irregular Network. That fact is discussed in this chapter 2. This involves a discussion on data sources which have been used in the context of this and similar methods. At the end the importance of filtering surface details with regard to the ultimate use of the model is explained.

2.: SAMPLING CONSTRAINTS AND METHODS OF PREPARATION OF TIN'S

Digital terrain models based on triangulated irregular networks (TIN's) are not restricted to any particular source, scale, or method for establishing the triangulated net. As will be outlined later, the local sampling density for data points is dependent upon the desired resolution for the digital terrain model and the local surface roughness. The sophistication of the sampling and triangulating techniques in recognizing the important features on the surface, however, has a direct effect upon the efficiency of a triangulated irregular network in accurately representing the surface. We shall attempt to review the issues involved in the capture and structuring of data for Digital Terrain Models in general and discuss the various data sources and techniques used by the authors of this report and the methods used by other researchers in the context of similar systems.

2.1: Properties of Representational Methods

There are several properties which any technique for the representation of surfaces should have. These become critical when evaluating the performance of methods which purport to have advantages with respect to data compression and manipulative ease.

First, the method must, to within a specified accuracy, preserve any analytic measures of surface behavior required by the purpose for which the model is created. Typical analytic measures might be average slope, average height, and various other surface measures such as those used in the quantitative example above (section 1.2). Also in this category are the conventional measures of surface roughness.

Second, the technique should capture all of the relevant features of the surface. That is, no relevant behavior of the surface should be lost due either to smoothing them away or to sampling errors. (This is equivalent to the "surface feature recognition" problem alluded to in section 1.1, above).

Third, and perhaps of most relevance for data compression schemes, is the requirement that the representation technique be able to differentiate between the noise and the information in its data source. That is, the technique should be immune to input noise but at the same time be able to eliminate internal redundancy in the representation.

The first two criteria are not equivalent. For illustrative purposes, let us examine these problems in more detail.

2.2: The Sampling Theorem

According to Shannon's sampling theorem (Shannon and Weaver, 1949) in order to detect the Fourier component with wavelength Lambda of a phenomenon distributed along some dimension, a sample spacing less than or equal to one half Lambda is necessary. This theorem places an absolute lower bound on the number of observations necessary to represent a surface whose details are not already known. Once the modeler has decided on the upper bound on the size of phenomena which he is willing to let escape, he has also determined the maximum allowable size of his sample spacing. This is a general result which cannot be avoided.

By making assumptions as to the continuity of the surface and the continuity of some of its derivatives, the surface may be smoothed between the sample points by any of a large selection of fitting techniques. While these smoothing procedures may, by the theoretical validity of the assumptions behind them, refine the overall precision of some of the analytic measures of the surface, they will nonetheless be unable to detect features of size less than twice the characteristic sampling distance of the original data -- they do not add any new information to the model. Thus, these methods are not really very suitable for compressing the representation of arbitrary data surfaces. By way of reiteration, let us repeat that the sampling theorem dictates the density necessary for regular sampling, and that polygon interpolation routines cannot add any information about the surface. This is not to say that the fitting of sampled data points to theoretical models of surface behavior will not be able to predict features between data points; rather, in the absence of such a theory, curve fitting alone will not yield additional information.

The sampling theorem implies that any efficient model of a surface must start from a sampling dense enough to detect the shortest wavelength phenomena needed by the purposes of the model; any data compression must start from this basis.

2.3: Surface Storage by Polynomial Patch Systems

One approach to data compression in the context of a polynomial patch system is that of Jancaitis (1975) and his associates. Their Weighting Function Interpolation Technique (WIT) and its contouring program CONSAC have been used for data compression, smoothing, and display of data digitized by the UNAMACE Scanner of the U.S. Army. They use a least squares procedure to fit a polynomial approximation of predetermined degree to a set of neighbors which are centered on a set of predetermined sample points regularly distributed over the model. They then interpolate these patches using a polynomial weighting scheme. Because of the problems in doing least squares fits to higher order polynomials, the original patch approximations which are fit are "planars" (that is, their equations have terms in XY as well as X and Y, but none of higher order). To produce continuity in both elevation and its first derivatives, the weighting polynomial is of third degree. The plane is covered by the product of the first degree approximations and the third degree weighting functions. This results in a fourth degree polynomial patch as the final approximation.

This technique has more than a null response to phenomena of wavelength less than twice the patch size, but it still does not adequately represent them. The cause of this difficulty lies in the fact that the least squares procedure is only fitting a planar to the local surface. Any phenomenon deviating from this form in the region of approximation is regarded by the least squares process as an error or as noise, the effects of which should be minimized. This will be covered in more detail in a paper now being prepared by Fowler. The authors of that system (Jancaitis, 1975) have responded to this problem by designing an "editor", a program which superimposes manually detected and defined features on the output of the polynomial patch model.

One must therefore take one of three courses: one can surrender to the aforementioned problem globally and thus have to choose a grid or patch size which will capture the shortest wavelengths desired. Alternatively, one could try to fit higher order functions to patches (e.g. the Jancaitis technique of fitting cubic polynomials, or localized Fourier transforms). Perhaps the most viable technique, though, would be the use of a data compression technique which adaptively and locally increases the sample density in regions of high frequency. The triangulated irregular network is one such approach.

2.4: What are Relevant Features?

The definitions of "relevant feature of the surface" and of "adequate preservation of the surface's measurements" depend on the ultimate use of the model and must be determined under consideration of that use.

There are lessons to be learned from the world of conventional cartography. For instance, aeronautical charts are typically made at relatively small scale because aircraft navigation purposes have little need for detailed topography. They do include, however, enhancements of the topography in the detailed attention paid to the location of prominent towers, cable spans, and radio beacons. At the other extreme are the site plans drawn for civil engineering purposes in which every detail of the site may be located to within a few centimeters. Nautical charts for coastal regions may contain very detailed bathymetric detail but only crude representations of the landforms inland. The coastline itself, though, is enhanced through the inclusion of the land-based aids to navigation and extra detail at particularly distinguishable features such as cliffs, towers, rocks, and wrecks. The manual cartographer, knowing the scale and the purposes for which his map is intended, thus enhances those details important for that use and deletes non-pertinent detail. Likewise, the designer of an automated terrain model should consider the purposes of the model and should consider the use of filters which will treat insignificant detail as noise.

3. MANUAL VS. AUTOMATED TECHNIQUES FOR THE CREATION OF TIN'S: INTUITION AND PATTERN RECOGNITION

Abstract to Chapter 3

In this Chapter, the basic description of the Triangulated Irregular Network method is concluded by a discussion on the various methods to obtain the TIN from other sources. Manual, hybrid and some still naive automated techniques and their essential capabilities are described.

3.: MANUAL VS. AUTOMATED TECHNIQUES FOR THE CREATION OF TIN'S: INTUITION AND PATTERN RECOGNITION

3.1 Manual Methods

Manual selection of the points and links which constitute the triangular network is one of the most obvious and accessible methods of creating a TIN. Besides the surface specific points and lines, other points and edges must be chosen so that the resulting triangular facets in the network fit planar or nearly planar regions. Once the surface has been covered with these triangles, the TIN is converted to machine readable form. This method relies upon the intelligence and image processing abilities of the manual digitizer to decide what constitutes a feature of interest and what constitutes an adequate representation of the form of the surface for analytic purposes. Because of the extremely high performance of human operators in image processing tasks, our manual digitizations remain the standard by which to evaluate other more automated methods.

Manual sampling and triangulation techniques are currently used in a production oriented terrain modeling system: W.E. Gates and Associates (1974), in the ADAPT system, use a manual method in which the triangulation of the topographic surface is compared with the original surface as well as the surfaces of other data items for the same area. It is recursively tuned until each triangle represents a sufficiently planar region in each dimension of a multivariate data space. This data base is then used as the basis for a model of hydrologic systems. Because of the feedback between the model and the user, areas which have been found to be inadequately digitized to begin with may be refined to a more detailed triangular net without affecting the digitization of the remainder of the model. Thus, this process insures that phenomena of high spatial frequency will be captured while minimizing redundancy in areas of low frequency.

It should be noted that existing contour maps and their machine readable equivalents are secondary data sources. As such, the digitization of terrain models from these sources will necessarily propagate the errors in these sources (described above, section 1.3). To minimize this problem, it would be preferable to use primary data sources in constructing a system. Since most of the recent topographic data is in the form of aerial stereo imagery and is compiled by human operators, manual sampling and triangulation is conceptually feasible by outfitting a stereo plotter with a real time stereo display of the triangulated net superimposed upon the stereo model.

TIN : Triangulated Irregular Network

3.2 A Naive Approach to Complete Automation of Triangulation

At the opposite extreme from the use of the human image processor is the creation of the triangulated surface from a set of arbitrarily sampled points on the surface. In fact, the points may not be really sampled in a truly random fashion, but as far as the triangulation algorithm is concerned, they appear to be. Methods of this type have only the coordinates of the points from which to work. This paradigm for triangulation is conceptually simple and has been used in various forms by us and by other researchers.

Because the only information needed for each data point is its coordinates, the input data can come from a very wide variety of sources with relatively little data preparation. This rather blind approach is the only available method when data sources such as well logs, bathymetric soundings, or truly random samples contain only the coordinates of the points. In other cases, it is simply convenient to ignore at this state any additional information one may have.

The concept of this approach is that some triangulation procedure is to digest the collection of unconnected sample points and by one of several algorithms connect them into a "good" triangulation. In this approach, the triangulation produces no further compaction of the data. All data points are used. Furthermore, there is no reference to the differing information content of the data points, nor is there any consideration of the fact that some subsets of the points are representations of features on the surface, and therefore have an implied connectivity. In that sense this approach is naive.

We must define what we mean by a "good" triangulation. Ultimately, the evaluation of a triangulation as "good" must be that it best represents the surface using some fixed amount of resources, or that it adequately represents the surface with a minimal amount of storage. Both of these criteria rely on a comparison of the model with the original surface. Because this approach to triangulation does not have the means to make this comparison, since the only data available are the sample points, some other, internal definition of a "good" triangulation is necessary to evaluate these algorithms. By the same reasoning, it is improbable that this method will produce a "good" triangulation in the wider sense of fidelity to the surface, especially when compared to the more sophisticated techniques.

The minimum weight triangulation chooses the triangles so that the total length of the edges in the graph is minimized. In this calculation, only the projections of the points on the x-y plane are considered, so the resulting triangles do not necessarily correspond to the surface. This approach has been proposed by some authors to produce an "optimal" triangulation (Shamos and Hoey, 1975). This optimality appears to refer only to the minimization of edge length, for none of these authors has proposed any reason why this triangulation is good for the representation of surfaces.

Although some algorithms have purported to efficiently solve the minimum weight triangulation problem, it has been demonstrated that none of the proposed algorithms actually produce minimum total edge length (Mark and others, 1976). Because the only algorithm known to work is the total enumeration of the triangulations and because there is no good argument for the "optimality" of the minimum weight triangulation, we have abandoned the minimum weight criterion as a viable alternative.

One of the methods which was thought by some authors to be a minimum weight triangulation is known as the "Thiessen", "Voronoi", or proximal polygon method (Peucker, 1973). As in the minimum weight method, the "Thiessen" technique considers only the x-y coordinates of the sample points. This method constructs the graph of Thiessen (or Voronoi or proximal) polygons so that all points within a polygon centered on a data point are closer to that point than to any other data point. These polygons uniquely tile the plane and are of special importance in the field of spatial analysis. The dual of the graph of proximal polygons is proven to be a triangulation, except for "degenerate" cases which can be removed by arbitrary infinitesimal perturbations. Because of the regularity of the proximal polygons, the corresponding triangulation is also very regular.

In addition to producing regular triangles, the Thiessen method is computationally attractive. The "Proximal Polygon" criterion is a completely local test. It is geometrically equivalent to tests as to whether a given point is within a specified circle and to the test of whether or not the sum of the opposite angles of a convex quadrilateral is greater or less than 180 degrees. Such computational simplicity means that the Thiessen Triangulation can be done extremely efficiently. We (Brassel and Fowler, in prep.) have developed a program whose running time increases with N to the three halves power, where N is the number of data points. Shamos (1975) has recently demonstrated an algorithm which can triangulate in order of N log(N) time and which can join pre-triangulated regions in time proportional to the number of points on the common boundary. Also, the Messerschmidt-Bölkow-Blohm corporation in West-Germany has a production digital terrain model system based upon a Thiessen triangulation of arbitrary data points (Bauhuber et al, 1975).

There have been other digital terrain systems and contouring programs which apply naive triangulations (ARCON, 1968; Brunker, 1972). The triangulation algorithms used in these systems, however, do not seem to be as well conceived or as well behaved as the Thiessen method.

The Thiessen triangulation seems to be the best of those algorithms which use only the coordinates of the data points. When there is additional information available, either in the form of a linguuistic description of the surface, e.g. that a certain line of points define a "stream" or "shoreline," or in the form of redundant information which is not to be included into the triangulation, then other techniques, among them semi-automatic ones, may better serve the purpose.

3.3: Semi-Automated Techniques

Semi-automated methods of triangulation are those in which there is some manual feedback into the triangulation process. In most cases this means the manual identification of features and pseudo-features on the surface.

Our currently preferred technique for triangulation is just such a semi-automated method. In the manual digitization phase the features of interest are tagged as they are digitized. The points are then passed to a Thiessen triangulation routine and are triangulated without regard for the features. The resulting triangulation is then edited to restore the desired features. The reason for splitting the feature restoration from the triangulation is that the triangulation without constraints is so well behaved. Methods for including the features in the triangulation are being investigated.

This family of techniques can use mechanized sources of data acquisition, particularly along the lines of photogrametric instrumentation. Rather than track along contour lines, the operator of a stereo plotter can create a linguistic description of the surface. He can track along ridge lines, channel lines, breaks of slope and can then fill in intermediary points.

A related technique to the digitization of high information content features which should be preserved under a triangulation operation is the digitization of relatively low-information content pseudo- features which should also be preserved. An extreme variation on this technique is the digitization of contours and subsequent triangulation (Keppel, 1975). Because contour lines have the property that they bisect the plane into inside and outside regions, the resulting triangulation routine can be extremely fast. Unfortunately, because contour lines are not a very efficient method of surface representation, the resulting triangulation is similarly inefficient. This technique appears to be more suited to interpolation when one already has contours.

3.4: Second Generation Automated Triangulation Techniques

Hybrid techniques of triangulation obtain absolute performance comparable with the manual techniques. That is, they produce good fidelity reproductions of the surface and are therefore superior to the naive techniques of automated triangulation. They do, however, need the presence of a human operator. This need for manual labor may eventually handicap the hybrid techniques on large-scale production applications, and some sort of automated triangulation should be used in those cases. If we use the naive techniques, then we will probably have to sacrifice some of the data compression efficiency of TIN's by including redundant data points into the triangulation to better define features of interest. There is therefore a need for the development of automated triangulation methods which recognize and preserve the surface-specific lines. The first step, the recognition, is described in the following chapter.

4. HIGHER-LEVEL DATA STRUCTURES AND TOPOLOGICAL CONSIDERATIONS

Abstract to Chapter 4

A "second" data structure is developed so-to-speak "on top" of the Triangulated Irregular Network. This is done because the TIN alone does not provide easy access to a data base - a problem if the base is large.

The second structure, basically a ridge-and-channel structure, represents the surface in its general features, and serves as a kind of directory into the surface. It is discussed in this Chapter 4 together with methods for the extraction of the surface features (ridges, channels; peaks, pits, passes; slopes, breaks, flats).

4.: HIGHER-LEVEL DATA STRUCTURES AND TOPOLOGICAL CONSIDERATIONS

Although the simple TIN data structure described above seems to be efficient in terms of storage capacity and other considerations, it does not provide easy access to the data base, which can be very large. It is for this reason that we are developing the "second" data structure both to represent the general structure of the surface and to serve as a directory into the surface (see Pfaltz, 1976). The first step in the creation of this second data structure is to find the ridges, channels and breaks of slope; i.e., the "surface-specific" lines.

4.1: The Ridge and Channel Structure

Peaks, pits and passes form the major portion of the set of surface specific points. Peaks are points that are relative maxima, i.e., higher than all surrounding neighbors. Pits, likewise, are relative minima. Between any two adjoining peaks we can draw a line on the surface that connects the two peaks, following the path at the highest altitude. This is a ridge. Similarly, channels connect adjoining pits along paths of lowest altitude. Passes are points at the junction of ridge and channel lines. These ridges and channels connect the peaks and pits into a network. Warntz (1966) has described these networks, finding that each pass has exactly two ridges (and two channels) emanating from it. In his interpretation, the ridges bound watersheds, and the channels bound hills. However, for the purposes of topographic generalization, one cannot determine the path of these lines from the digital terrain model alone. Warntz's surface specific lines can cross areas of little or no relief. On the other hand, we can determine the lines of topographic significance which will characterize the surface. These may not fully connect into a network but give instead a graph structure that forms the "backbone" of the surface.

This graph of maxima and minima characterizes the variation of the surface in a highly generalized manner. The resulting information serves as a skeleton of vital points for use in automatic triangulation or semi-automated methods, as described in section 3.2, and is useful in applications such as in hill-shading (Brassel, 1974), contouring, and as a general directory into the surface.

TIN : Triangulated Irregular Network

4.2: Extracting the Ridges and Channels from a Regular Grid

To create a TIN from a gridded DTM, it is vital to retain the points along this graph among the points and lines extracted to form the triangular facets. Because of the prevalence of the regular gridded DTM, we have designed a method to extract these surface-specific features automatically from the grids. The technique marks all those points in the grid which are possible ridge (channel) points. Then using pattern recognition techniques, the marked points are linked together to form the ridge and channel graph, and the peaks, pits and passes are recognized as the connection points in the graph.

Peucker and Douglas (1975) outlined automatic methods extracting ridge and channel information. One of their two approaches has its origin in the work of Greysukh (1966), see for this section 4.3, below. In their other method, they process the grid using the observation that the points on a ridge (channel) are never reached as the lowest (highest) neighbor of points in the DTM. Since this method is "local," i.e., only the three neighbors to the right of and below (south of) each point are examined, this process is relatively cheap. We have extended this technique to search the grid and extract the total network.

To explain this method, we shall concentrate, in the following, on the extraction of the ridges. Basically, the same methods are applied for the channels but then the calculation proceeds upwards instead of downwards, and vice versa. Initially every point and its three neighbors to the right and below are examined. The point among them at the steepest slope downward is marked. When scanning is completed, all points that are not marked are ridge candidates. Then a separate procedure is invoked to connect these points into ridges and discover the connections among them.

Although ridges are lines at maximum height from peak to pass and then to another peak, the technique we use starts at passes and climbs to a peak. (Starting at a peak and traversing the unmarked points at maximum height causes the line to circle the peak). All unmarked points are examined in order to find one which is lower than any unmarked points among its eight neighbors and not already assigned to a ridge. This is marked as a pass. From this point we search to find its highest unmarked neighbor, continuing until we find a point higher than all its unmarked neighbors, a peak, or a point already in a ridge. In the latter case, the ridge being constructed is connected to the existing ridge, and the two ridges are merged if the new ridge connects at the start of the existing ridge.

DTM: Digital Terrain Model
TIN: Triangular Irregular Network

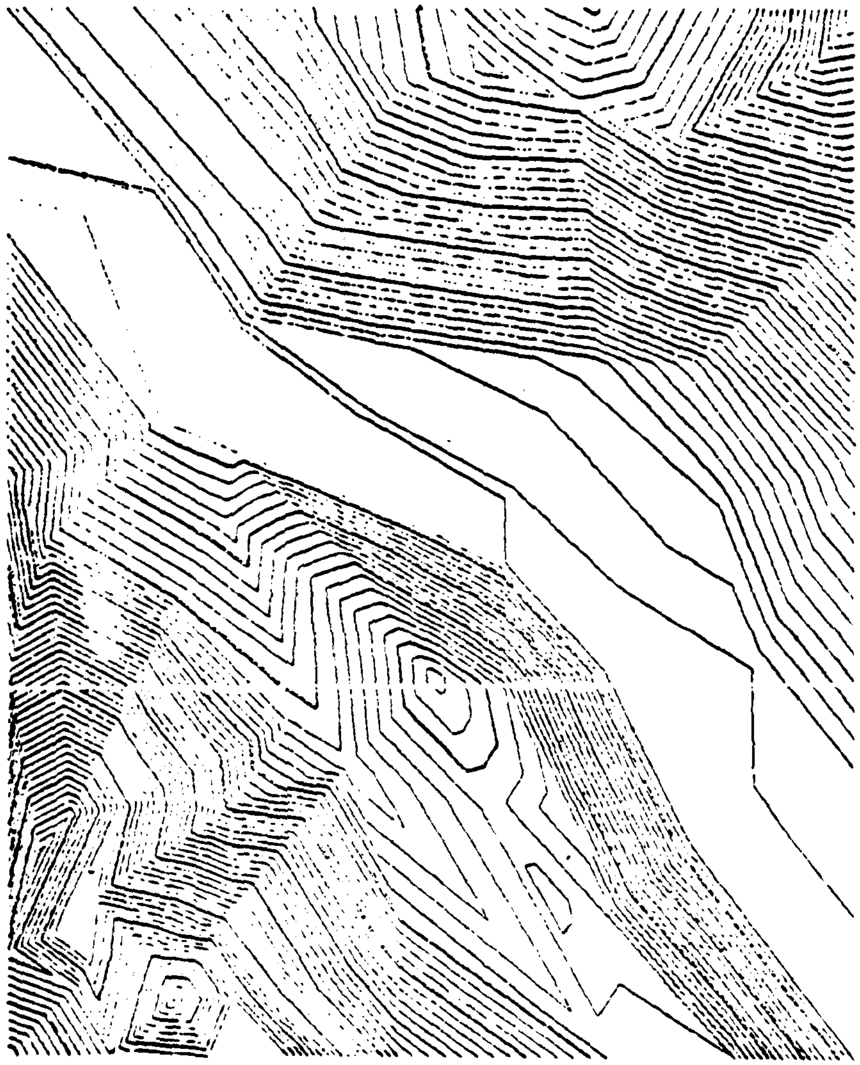
Proceeding in this manner, every unmarked point is eventually added to a ridge line and the peaks and passes are discovered. Short ridges connecting to a long ridge are removed since the marking process leaves a cloud of points about the actual ridges. As indicated above, the channels are found in a similar manner. Then the passes found in each sub-network are tested and joined if they are the same or a neighboring point. In this fashion the majority of the lines are interconnected.

Figure 5 shows a contour map (map by routine TRIKONT) of the Canoe River Valley, a mountainous region in British Columbia. The surface was modeled by a TIN. For testing of the ridge finding algorithm a 36 by 36 regular grid was manually digitized from the source contour map, the same map from which the TIN was digitized. The ridge graph derived from this grid is shown in figure 6; the passes are symbolized by circles and the peaks by x's. Figure 7 shows the channel graph for the area; here x's denote pits. Figure 8 shows the ridge and channel network, with peaks as x's, pits as circles, and passes as paired triangles.

4.3: Greysukh's Method for Characterizing Surface Geometry

Another method for characterizing surface geometry at a point is to examine the configuration of the terrain around that point; Greysukh (1966) described a method for achieving this. Briefly, the heights of the land surface at its intersections with a vertical cylinder centered at the point are graphed against the angle from an (arbitrary) origin. This forms the "characteristic" curve of the landform. Figure 9 shows the contours associated with several landforms and the characteristic curves derived from them. The configuration of the characteristic, particularly the number of extrema and their positions relative to the height of the central point, differs with varying surface geometry, and can be used to describe the geomorphometry of the land centered at the point in question. There are theoretically an infinite number of distinct characteristics, but those with more than 6 extrema are extremely unlikely to occur, provided the radius of the cylinder is not too large. This still leaves three possible curves with one maximum, 5 with two, and 8 with three maxima. Of all of these, Greysukh considered only 6 of the 8 simpler ones and none of the 8 more complex ones. This latter group contains such commonly-occuring forms as channel (ravine, course) junctions and ridge (cape, divide) junctions. In the only work we know of which has applied Greysukh's ideas to surface feature recognition (Peucker and Douglas, 1975), consideration was restricted to Greysukh's original six curves plus three others (representing flat, and concave and convex breaks of slope). In spite of the fact that they do not have topologically-distinct characteristic curves, these landforms are important surface features.

TIN : Triangulated Irregular Network



<u>Figure 5:</u>

Canoe River in British Columbia; example of the "Trikont" output (for the Trikont, see section 5.1, below).

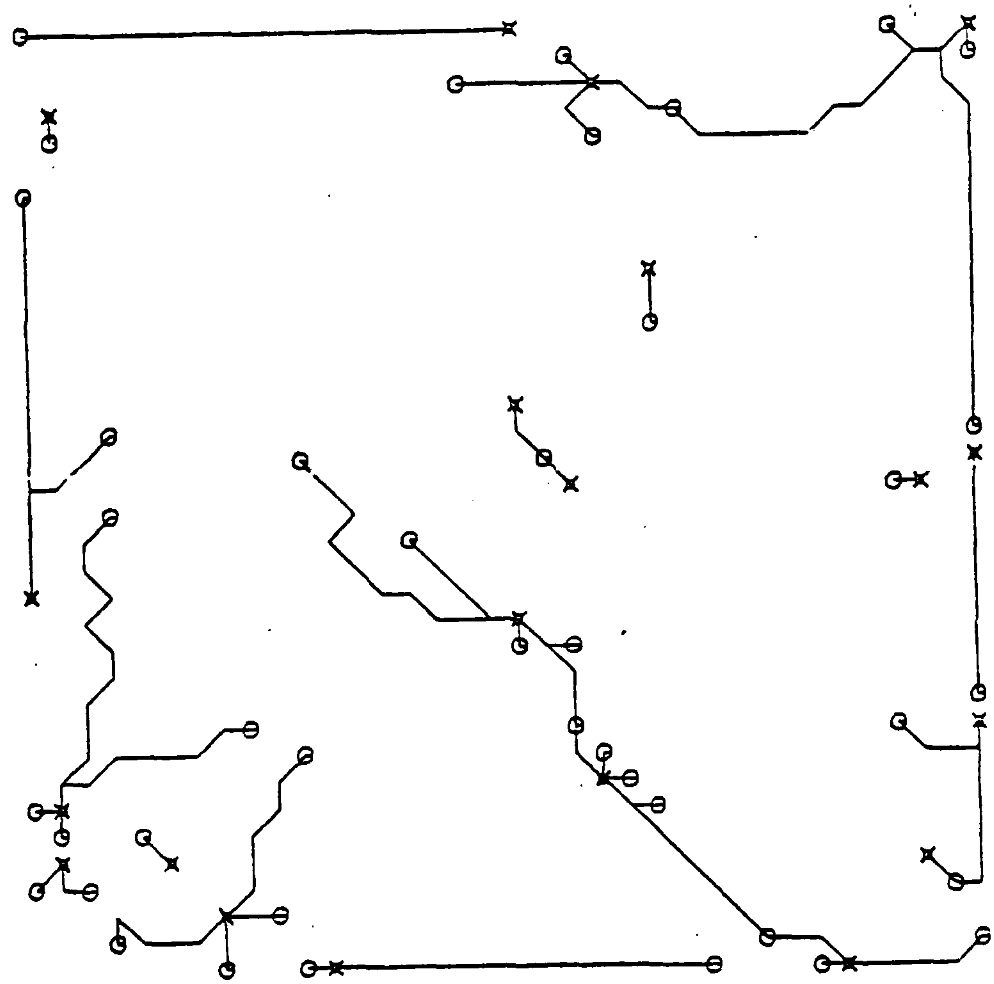
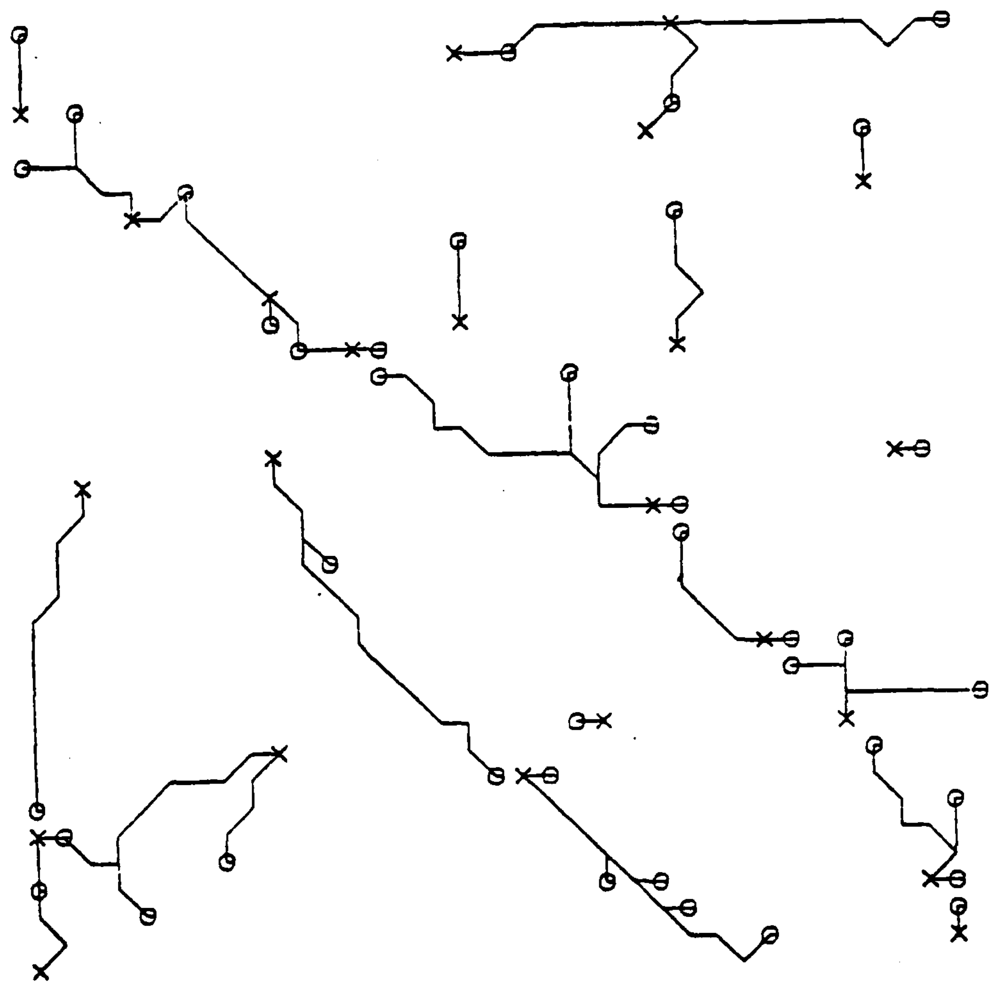


Figure 6:

The ridges extracted from contour map (figure 5)



<u>Figure 7:</u>

The channels extracted from contour map (figure 5)

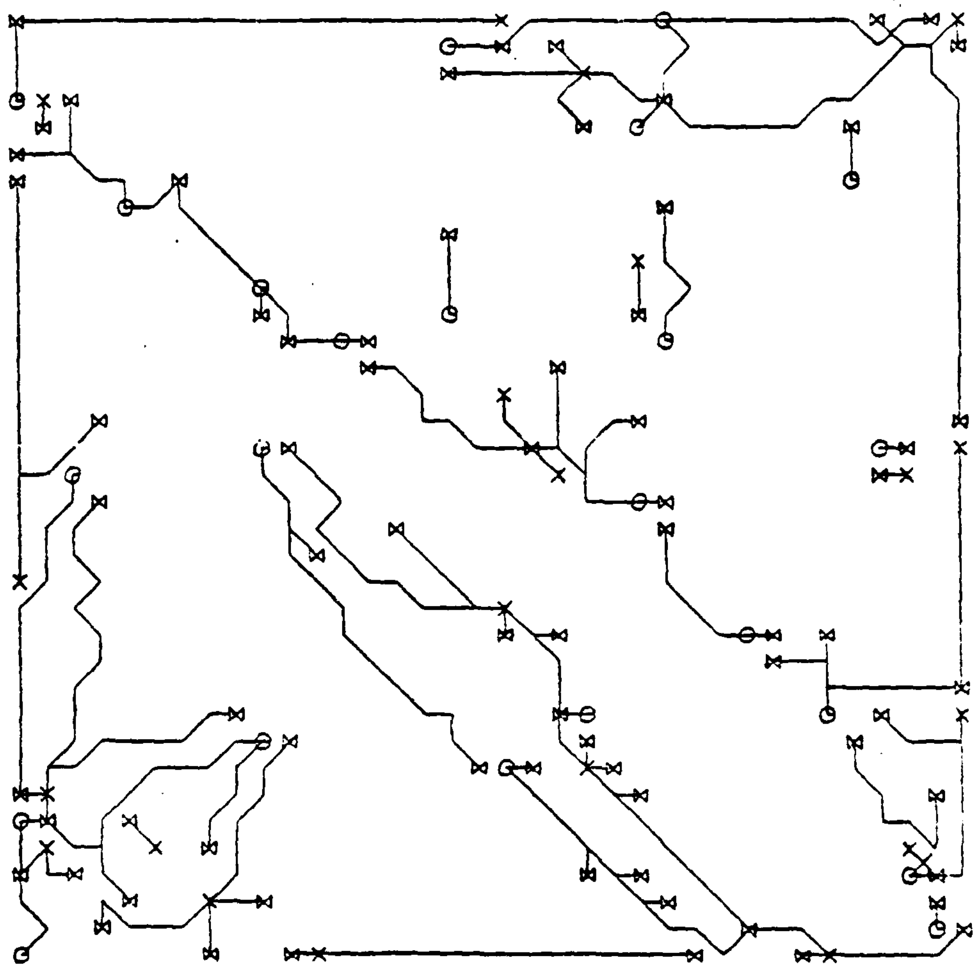


Figure 8:

Ridges and channels as extracted from contour map (figure 5), combined

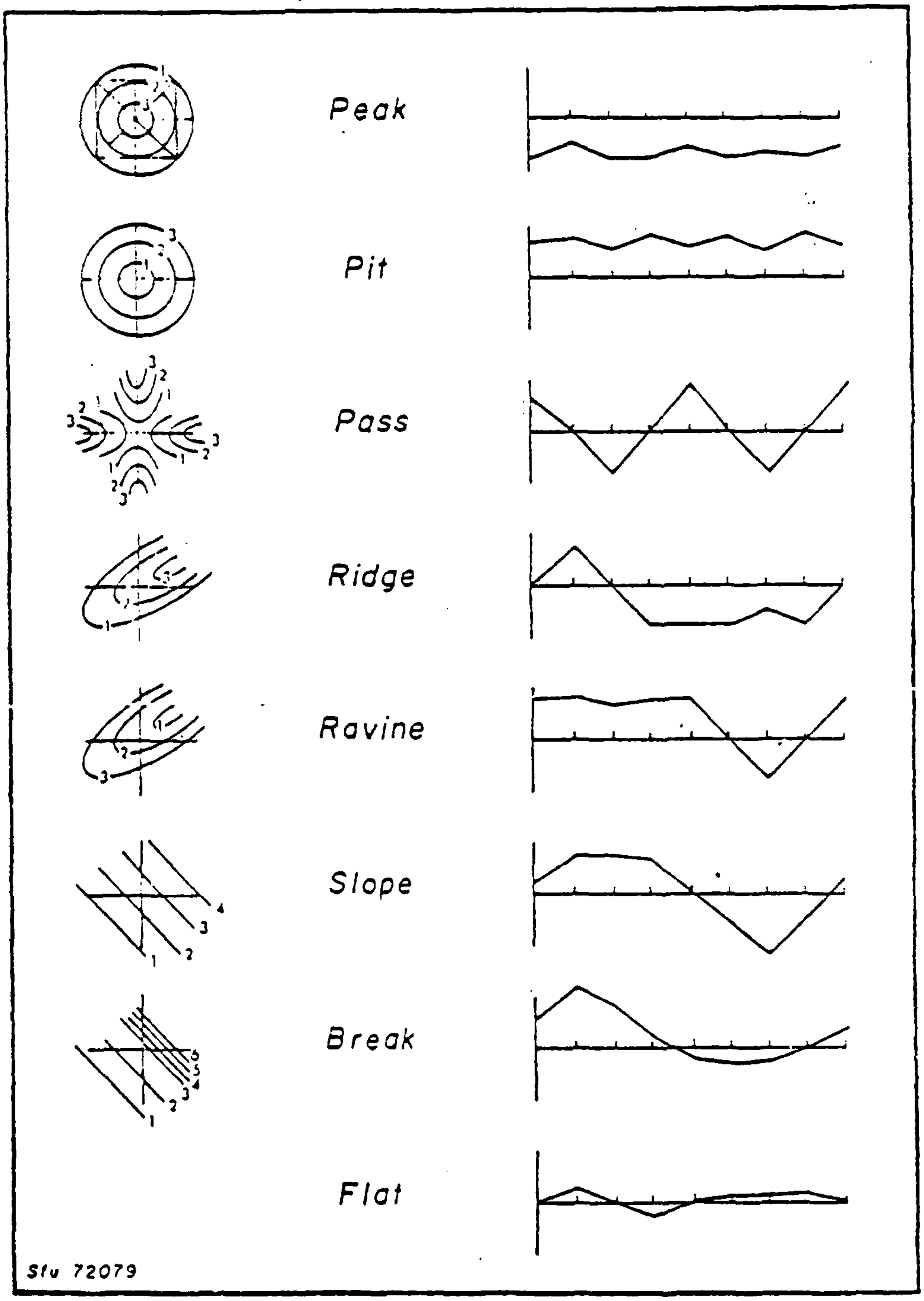


Figure 9:

Certain land forms and their contours as used in Greysukh's Method.

Greysukh (1967, p 1-6) recognized an important problem in the application of his method to terrain represented by points which are not surface-specific (for example, points of regular grids). If the "cylinder" overlaps a ridge or channel without being centered on its axis, a misleading characteristic may result which Greysukh called "distorted." This effect is probably one of the reasons why Peucker and Douglas's grid-based approach was largely "not satisfying" (1975, p 383).

A subroutine (GRYSKH) has been written to classify a point into the appropriate one of the 16 Greysukh-type codes, the 3 additions of Peucker and Douglas, or 5 other classes for more complex points. When GRYSKH was applied to regular grids, problems similar to Peucker and Douglas's arose. In a "properly-triangulated" TIN based on surface-specific points, however, distortion should not occur, and the characteristics should faithfully reflect surface geometry. (Indeed, we have found GRYSKH to be useful in testing triangulations, since distorted characteristics due to incorrect triangulation can usually be readily detected by visual comparison with the contours).

Briefly, then, if surface features are adequately represented in a DTM, Greysukh's method will be useful for identifying them for further processing. If the points do not correctly represent the features (and this will generally be the case for surface-random points, including regular grids), Greysukh's method will be of little use in surface feature recognition.

TIN : Triangulated Irregular Network
DTM : Digital Terrain Model

5. APPLICATIONS OF THE TRIANGULATED IRREGULAR NETWORK

Abstract to Chapter 5

In this chapter 5, application of the total Geographic Data Structure (GDS) and/or of its parts, the Triangulated Irregular Network (TIN) and the Higher Level Data Structures are discussed. Under "application" is understood an application for general purposes of digital modeling (including extraction, calculation tasks, display, etc.) of three-dimensional surfaces.

The authors do not deal with any detailed description of problems in public life, civilian or military, which can best be approached by digital methods of this kind; only some hints are given in this regard.

5. APPLICATIONS OF THE TRIANGULATED IRREGULAR NETWORK

Because the relative number of elements (points and regions) is low, and the topological connections among them are easily accessible, many of the routine tasks of a digital terrain model can be performed efficiently on a TIN. At present several component modules for a surface modeling system have been developed. Contouring and rapid creation of profiles are completed, and several others described below are now being implemented. Fundamental to many of these applications is the process of tracking, following a path described by a set of parameters on the coordinate axes.

5.1: Coordinate Independence and Generalized Tracking

The class of tracking processes is one of the most common operations performed on digital terrain models and in other computer graphic systems for representing 3 dimensional surfaces. Scanning along the rows, columns, or either set of diagonals in a regular grid shows up as a subproblem of the production of block diagrams (SYMVU, ASPEX, VIEWBLOK), terrain shading, and video display. Because of the coordinate dependence of the regular grid, these four directions are usually the only ones for which most systems can produce scans cheaply. The coordinate independence of the TIN allows one to produce scans in any arbitrary direction. It is that coordinate independence which allows general three-dimensional computer graphics systems with data structures based upon planar facet polyhedra (of which the TIN is a special case) to routinely perform rapid scan conversion for the video display of the object being modeled. Using methods borrowed from such systems, a scan line converter (Appendix IV) has been implemented to construct parallel relief lines for block diagrams, profiling and conversion of TIN's to regular gridded format.

In a coordinate-dependent system, the production of contours is a distinctly different process from the production of scans. In a coordinate-independent system, contouring and scanning are just special instances of a more general tracking process.

The problem of the general tracking process can be stated as follows. Given a metric space (e.g. the model space) and some parameterized function, produce the set of intersections of the model with the function for a set of discrete values of the parameter. For instance, if the function is $X - k \times S = 0$, k = the integers, then the generalized scanning process will produce a set of scans parallel to the x axis at spacing S. Similarly, if the function is $Z - k \times S = 0$ then the result will be the set of elevation contours of the model. As is usual with special cases, optimized programs have been written within the TIN system to produce rectilinear

TIN : Triangulated Irregular Network

scans and contour maps. Because of the coordinate independence, it is possible to perform transformations into coordinate systems in which a special scanning problem becomes equivalent to one of these optimized special cases. An example is the production of 360 degree horizon profiles by transforming into a spherical coordinate system centered on the observer. Other problems, however, such as the simulation of an aircraft borne scanning device producing a set of parabolic profiles along the track of the aircraft are not so amenable to that process and require the production of a specialized tracking system. We have written such a generalized tracking program for the TIN.

5.2: Data Display Routines

A contouring program, TRIKONT, has been developed using the tracking methods facilitated by the TIN's structure. This routine was utilized to produce the diagrams shown in Figures 4 and 5. Its capabilities include rotation of the map coordinates, independent scaling along the x and y axes, darkening of the index contours, and labeling of specific map features. Currently, the possibility of smoothing the contours by using higher order interpolation methods on the surface is under investigation (see section 5.3 (e)), as is the potential of labeling the contours. Research on the structure of surfaces which we have done, has led to an automatic method of determining the nesting of contours from the ridge and channel network. This information can speed the construction of contour maps by eliminating both the need to search for starting points along contours and the need to check each element as to whether or not it has been crossed already. In its present state, TRIKONT produces maps rapidly by taking advantage of the explicit topology of the TIN to minimize the amount of searching performed. Because the surface represented by the TIN has much less elements than a regular grid of the same precision, TRIKONT also is much faster in comparison to a grid based contouring program. The three-dimensional block diagrams of TIN's in this report were produced by using profiles extracted by the scan line converter (see Appendix IV) and then plotted by the program ASPEX (Laboratory for Computer Graphics, Harvard University, 1976). A three-dimensional representation program which uses the triangular facets to depict the surface is now in the implementation stage (see section 5.3 (d)).

5.3: Classes of Applications

Beyond the present use in data display, the GDS has considerable applicability to various problems involving analysis of three-dimensional data. The problems, as we see them, fall into several classes:

a) Representation of Several Surfaces on the Same Sample Points. The

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TIN : Triangulated Irregular Network
GDS : Geographic Data Structure, combined by TIN and secondary structures

relative storage efficiency of the TIN increases if several z-values are stored at each point. When two sets of z-values are used in a regular grid system, the required storage doubles. In the GDS, the storage-space increase is only 10 per cent for the first additional set of values and the relative amount of increase declines as more values are added at each sample point. Thus, the data structure is advantageous for handling stratigraphic data, as obtained from well-logs, and oceanographic data gathered at different depths, such as water temperature, salinity readings or measures of turbidity. Much work on such subsurface data has, to date, interpolated from irregular points to regular grids; the GDS can handle such data directly.

b) Generalization and Analysis of Topography. Surface data in the form of a TIN can be processed in a simple manner to extract the ridge and channel network (see section 4.1) much more easily than data from regular grids because the surface definition is unambiguous. The resulting information is useful in analytic hill-shading (Brassel, 1974) and watershed analysis for basin hydrology. Current research is directed toward utilizing the surface network in generalization of surface data, as in change of scale for cartographic applications.

c) Mathematical Manipulations of the Surface. Civil engineering applications involving volumetric calculation, such as cut and fill, benefit from the reduction of the number of data elements processed when using the GDS. In determining the profile along a roadbed, or finding cross-sections along a given track, both the relatively small number of data items in the GDS and the lack of directional bias in the TIN promote efficient processing.

d) Three-dimensional Representation, Radiation Input and Visibility. The techniques developed in computer graphics (Newman and Sproull, 1973) for depicting objects constructed of planar facets are applicable also to surfaces represented by TIN's. In particular, the problem of drawing the surface as seen from a given viewing point is susceptible to solution by such techniques. In all methods for producing a three-dimensional drawing with hidden lines removed, the crucial step is handling the objects in a certain order, those elements closest to the viewer first. Because the triangular facets of the TIN are contiguous and non-overlapping, a simple sorting of the nodes in the dataset suffices to produce the proper order for processing. The decision as to whether a given edge is visible is then straightforward, involving only a test of that edge against the upper edge of the region already hidden, termed the horizon. The process finds the outlines of both the visible and invisible regions. Lineal features, such as roads and railways, and areal features, such as land-use types, can be easily constructed, by intersecting the plan layout of the visible regions with the layouts of the features. The sections of the lineal and

GDS : Geographic Data Structure, consisting of the Triangulated Irregular Network plus the Higher Level Structures

TIN : Triangulated Irregular Network

areal features within can be drawn with no further testing.

By transforming the coordinates of the elements of the GDS to the appropriate polar coordinates, the same technique can be utilized to find the regions of the surface visible from a given point. Besides the possibility of fast production of radar maps, this procedure finds use in the areas of the proper location of fire lookout towers and the situation of facilities which are eyesores, such as powerline towers. The application module to produce representations of the surface is now being implemented, and its extension to visibility is soon to be done. This method also yields "skyline" information, for determining radiation input, which bears directly on problems of glacial melt and basin hydrology. Young (1973) describes a set of routines to produce these data from a regular grid. We believe the method described above can obtain these data more efficiently from the TIN.

<u>e) Modeling of Dynamic Behavior on Surfaces.</u> The Triangular Irregular Network, or as it is sometimes called, the Irregular Triangle Grid (Dahlquist and Bjoerck, 1974, p. 322), has been successfully used in the field of continuum mechanics. Finite element methods for the solution of differential equations are often formulated using triangulated networks of irregularly spaced points very similar to the TIN. In some applications it may be advantageous for us to incorporate the higher order interpolation techniques used in finite element methods. In the modeling of systems with a dynamic component, such as coastal regions, the compatibility of the TIN with the finite element formulation of the dynamic problem allows this very powerful and efficient technique to be employed. Among the areas in which these formulations would be very advantageous in oceanography and modeling of coastal regions, are prediction of tides, changes in salinity, turbidity or algal concentration, and representation of the behavior of continuous variates on the surface.

TIN : Triangulated Irregular Network

GDS : Geographic Data Structure, consisting of TIN plus higher order structures

6. FUTURE RESEARCH

Abstract to Chapter 6

In this document, tasks which are now being implemented, are mentioned several times. Work in the near future will, of course, concentrate on these. The next step is intended to bring the GDS closer to actual tasks in the coastal environment if suitable bases for such tasks can be found. It is assumed that as a result of further discussions - in particular some based on the content of chapter 5 - further development towards specific applications will be required.

6. FUTURE RESEARCH

In our future research, our first priority will be to complete those aspects of the work outlined above which are not yet fully refined. This will include methods for automatically extracting TIN's from more "conventional" data sets such as regular grids. The new thrust of our work will be to combine this three-dimensional structure with a planar polygon overlay system in order to produce a comprehensive Coastal Modeling System for modeling and monitoring various characteristics of near-shore areas. Our approach is to assume that a geographic information system for polygonal data exists and can be acquired relatively easily. By taking this tack we can concentrate on constructing those components of the integrated system which involve the TIN, and the TIN's relationship with polygonal systems. Others have in the past and will in the future concern themselves with the polygon data bases, for example the Urban Geometric Heurisitic, UGH, (Schumacher, 1972), the Natural Resource Inventory System, NRIS (Raytheon Company, 1973), POLYVRT (Chrisman and Little, 1974), and regional land used systems from the Australian organization CSIRO (Cook, 1975). Common to all of the polygon based analyses is a topologically oriented method of data storage which stores the boundaries of the regions. We have chosen to use the POLYVRT structure as a working model of the data structure for polygons since it can be converted to many of the others. The interface between the TIN and polygon structures consists of several modules to create polygons from the TIN (contouring), aggregate triangular facets to form polygons (clustering of faces of similar slope azimuth), and produce the intersection of a polygon network with the TIN. This list is by no means exhaustive, for there are as many interfaces as there are subsystems in each model. The modules described in section 5.3 form a core of routines for development of the Geographic Data Structure to a full-blown coastal modeling system. We feel that the development of an integrated geographic information system based on the concepts of the TIN and a topologically structured polygonal system will offer many practical and theoretical advantages unavailable from present systems.

TIN : Triangulated Irregular Network

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APPENDICES

A P P E N D I X 1

TECHNICAL REPORTS SUBMITTED UNDER THIS "GEOGRAPHIC DATA STRUCTURES" PROJECT

APPENDIX I: Technical Reports, "Geographic Data Structures" Project

1. Peucker, T.K., 1973, Geographic data structures. Progress Rept. after Year One (Tech. Rept. #1), ONR Contract #N00014-73-C-0109, Dept. Geogr., Simon Fraser U., Burnaby, B.C., Canada.

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APPENDIX II

COMPUTER PROGRAMS AND SUB-PROGRAMS

APPENDIX II: Computer Programs and Subprograms.

Note: All programs are written in PL 1, except where indicated.

DATA STRUCTURE PRIMITIVES

ALNFLG - Allocates and initializes the facility for flagging directed edges in the triangulated network.

RSNFLG - Reset the neighbor flag vector.

SETNFLG - Set a given edge flag to a given value.

TSTNFLG - Test an edge flag.

COORDS - Returns the x, y, and z coordinates of a node in the network.

CONEXT - Finds the next clockwise neighbor of a given node relative to a specified direction.

NEXT - Finds the next clockwise neighbor of a given node relative to another neighbor of that node.

LAST - Returns the next counterclockwise neighbor of a given node relative to another neighbor of that node.

SETUP - Initializes the data base facilities.

LSETUP - Initializes the data base facilities and produces a structured listing of the network.

DATA BASE MAINTENANCE ROUTINES

BRUTE - the brute force batch editor.

TRIFIX - Completes the border of a triangulated net and creates a data base file containing that network.

GRAPH - Converts the data base to a format readable by FORTRAN programs.

TRINGB - Produces a TIN from a set of triangles. FORTRAN.

SLEEZ - Reads a triangulated net from a card file and initializes an internal representation of that net.

TRIANGULATION PROGRAMS

TRIAP - Produces a triangulation of irregularly sampled points by successively adding points to the triangulation.

THIESSN - Produces a triangulation using the Thiessen criterion.

DISPLAY AND ANALYSIS PROGRAMS

TRIKONT - Produces contour maps.

ISOPLOT - Produces isoline maps of arbitrary functions of the coordinates of the nodes.

PROMAP - Produces block diagrams directly from the triangulated net.

TRIPLT - Plots the triangulated net.

SURFER - Displays the TIN three-dimensionally as triangular facets, removing hidden lines.

SHADE - Depicts the relief on the TIN using shading in the Swiss manner. The input consists of the TIN, the normals, and the second data structure.

SCANNER - Finds the intersections of a set of parallel lines with the TIN. These intersections can be passed to GRIDDR.

GRIDDR - Takes as input a list of irregularly spaced values along a line and linearly interpolates regularly spaced values, used with the SCANNER.

RMS - Calculates the root mean square error between two gridded data sets, using their point by point difference.

BANDR - Set of FORTRAN routines which rapidly calculate the intersections of two lines.

TRINORM - Calculates the normals to all the triangles in the TIN. Output vectors can be either normalized or unnormalized.

SLOPE - Calculates the maximum slope for every triangle from the normal vectors.

DIANG - Calculates the dihedral angle between adjacent triangles across the area, using the normals as input.

VEXER - Determines at an edge whether slopes are divergent, convergent, or neither.

GREY - Produces a list of slopes to neighboring points for each point in the TIN.

RIDGE - Uses the methods described in section 4.1 to extract the ridges and channels from regularly gridded topographic data.

TRIDGE - Uses the methods described in section 4.1 to extract the ridges and channels for the TIN. Output is the set of edges for each of the ridges and channels.

CLSTPT - Finds the node closest to a given point.

RNORTH - Produces a computationally cheap surrogate for the compass bearing angle between two points.

AREA2 and AREA3 - Return the area of a triangle in space and the area projected or the x, y-plane.

PSLICE - Finds the intersection of a line segment with an infinite line, if such an intersection exists.

RTLFT - Determines whether a point lies in the left or right halfplanes defined by a given directed line segment.

ELRANG - Sets the parameters (the Euler angles) for rotations in three dimensions.

ELRRØT - Perform the Euler rotation on a point.

QUIKST - A much modified variation on the Quicksort algorithm.

TIMER - Returns the processor time left in the current job step.

A P P E N D I X   III

COMPARISON OF TRIANGULATED IRREGULAR NETWORK'S AND REGULAR GRIDS FOR ESTIMATING GEOMORPHOMETRIC PARAMETERS

APPENDIX III: Comparison of TIN's and Grids for Estimating Geomorphometric Parameters

Empirical comparison after Mark (1975), for significance of letters see sect.1.2

	Parameter		
root-mean-square errors:	H	G	I
15 x 15 grids	42.4	0.053	0.015
TIN's	3.3	0.019	0.007
ratio	12.9:1	2.8:1	2.1:1

Characteristics of regular grids theoretically required* to produce the the same precision as the TIN's

d (metres)	39	179	234
size	181x181	40x40	31x31
number of points	32,761	1,600	961

Ratios to TIN's (114 points):

number of points	287:1	14:1	8.4:1
digitization time	55:1	2.7:1	1.6:1
storage space	29:1	1.4:1	0.9:1

* assuming a linear relation between grid spacing and grid precision.

A P P E N D I X IV

TRIANGULATED SCAN LINE COHERENCE

APPENDIX IV: Triangulated Scan Line Coherence

The power of the TIN data scheme is evident in analysis of the structure of terrain, especially where the proximity or connectedness of features is important. The triangular structure is also efficient for simple tasks. For display, as for video, it is important to be able to produce a set of parallel scans through the model as efficiently as possible. These scans, each of which is a set of intersections with the edges of the triangular facets, also can be interpolated to produce a regular grid of points. Creating these scan lines is called scan conversion. The efficiency of a scan converter is directly related to how efficiently one can trace the path of a line through the TIN.

For a large class of problems, one wants to be able to determine which triangles a given line intersects. The line may consist of a group of connected segments, such as a roadway, or a straight line, as in scan lines. The case for scan lines is simpler, since the line can be imagined as infinite, so that simple evaluation of the equation of the line determines intersection. Once it has been determined that the scan intersects a side of a triangle then it is obvious that it must intersect one of the other two sides or the point shared by both sides. Using this observation a scheme has been devised to follow the path of a line through the triangulated surface. This process, SEARCHNET, is described more fully in the year one report (Peucker et al., 1973; Cochrane, 1974). This method is fast, checking less than twice as many links as the number of links found to intersect.

The SEARCHNET tracking process has itself been implemented by members of the project for a scan converter; however, by using the property of scan line coherence the process can be speeded up. Scan line coherence is the property that "adjacent scan lines appear very similar" (Newman and Sproull, 1973). In our context this means that if the distance between scan lines is less than the average length of the links then it can be expected that several links intersected by a scan line will also intersect the next. Upon finding that a given link is intersected by a scan line it is possible to predict how many further scans will intercept that link. Watkins (1970) implemented a hidden-line algorithm for video display using a scheme whereby each intersection of a line with an edge is stored as an x, y, z value, together with the increment in x, y, z by which it is expected to change by the next line, and the number of scans that the link will further intersect. The scheme for scanning the triangular irregular grid is quite similar.

The links which intercept the first scan must be found by the SEARCHNET procedure. For each intersection a record is stored as follows:

XYZ - the x, y, z coordinates of the intersection;
DELX, DELY, DELZ - the increments in x, y, z which will be added to XYZ to calculate the intersection for the following scan line:
NUMSC - the number of further scans which the link will cross;
TAIL - the node at the far end of the link.

These are stored in a singly linked list. On succeeding lines, the list of intersections is examined and NUMSC for each link is tested. If it is greater than or equal to 1 then DELX, DELY, DELZ are added to the XYZ values and these new values replace XYZ. NUMSC is then decremented by 1. The values for XYZ are then placed in the list of intersections for that scan.

If NUMSC is less than 1 then TAIL for that entry is added to a list of failed links and the entry removed from the intersection list. Following the discovery of a failed link other links for which NUMSC is less than 1 are added to the fail list until a link which crosses the scan is found or the scan exits from the model area. Then these nodes are used by the SEARCHNET procedure to find links emanating from them which cross the scan. SEARCHNET will start at the last link which crossed the present line and find links crossing until it either encounters the edge of the area or a link already found. These intersections are added to the scan list and to the list of future intersections. If the edge of the model was not encountered then processing continues on the link list. The method thus repairs the intersection list on the go as gaps appear when a scan line passes beyond a link. The method combines a fast searching technique with a bookkeeping technique that eliminates rediscovering information which can easily be predicted.

The speed of the algorithm is unaffected by the direction of the scans relative to the coordinate axes of the TIN, since there is no implied directionality. A link once crossed need never be searched for again. In a TIN of N points there are $3(N-1)-B$ edges, where B is the number of points on the boundary of the model. Let E be $3(N-1)-B$. Letting R be the cost of discovering a crossing edge, then the cost for all the links is $R \times E$. Let L be the average length of the links in the TIN. The expected length of the links projected perpendicular to the scan lines is: PIXL/4. If the distance between scans is M then the expected number of times a link will be crossed is $PIXL/(4 \times M) = T$. We can now write the total cost for the scan conversion, letting CO stand for startup cost and I be the cost per intersection:

$$Cost = CO + R \times E + I \times E \times T$$

or

$$Cost = CO + R \times E + I \times E(PIXL)/(4 \times M)$$

For a particular TIN where the length of the links in the direction perpendicular to scanning is D, M can be expressed as a function of the number of scans X:

$$M = D/X$$

So cost can be rewritten:

$$\text{Cost} = CO + R*E + I*E*(PI*L)*X/(4*D)$$

Thus cost per scan line decreases linearly with the number of scans. Some test results follow:

Time to Produce X Scans in Seconds

Points	X = 36	71	141	281	561
31	.07	.11	.145	.25	.45
85	.19	.26	.35	.52	.83
279	.53	.64	.82	1.00	1.43

(For references quoted in this appendix see the general list of references on page 47 - 50)

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13. ABSTRACT

A "Geographic Data Structure" is developed which permits the digital acquisition, storage and application of data sets of three or more dimensions, in particular topographic terrains. The Structure consists of a "Triangulated Irregular Network" and higher-level structures. Comparisons with systems based on regular grids reveal that the Geographic Data Structure is advantageous in terms of storage space needed, computer time, and accuracy, in the acquisition, storage and application of digital terrain. The presentation in this document is aiming at the educated layman, references to more detailed descriptions are provided.

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Digital presentation of three-and more dimensional data sets of all kinds						
Geographic Information Systems						
Data Structures						


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