DIGITAL SIMULATION OF AN AC/DC SYSTEM IN DIRECT-PHASE QUANTITIES

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ABSTRACT

The main purpose of this thesis is to simulate digitally a representative AC/DC system. This system consists of an AC/DC converter connected to a synchronous generator at its terminals as well as to an infinite bus-bar through a short transmission line. A group of filters is also connected at the converter AC bus-bar. The various components of the system have been analysed and represented mathematically in the direct-phase quantities. This is because the identity of the phase currents must be retained to define the details of the successive commutation processes of the AC/DC converter.

A digital computer program is used to simulate this AC/DC system. This program is developed such as to allow the accommodation of various configurations of the system. The program has been applied to study the behavior of the different components for various system arrangements. From this program, the waveforms of the different variables of the system are obtained. These waveforms are discussed to find out the effects of the generated harmonic currents of the converter station on the AC system. Particular attention has been given to their effects on the behavior of synchronous machines.

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$C_{f\ell 5}, C_{f\ell 7}, C_{f\ell 11}$	capacitance per phase of the 5th, 7th and 11th order harmonic filter respectively
f	frequency
$F(---)$	function of (---)
i_1, i_2, i_3	3-phase rectifier currents
i_a, i_b, i_c	3-phase armature currents
i_d, i_q	d- and q-axis component of the armature current respectively
i_{DC}	DC output current of the rectifier
i_{fd}	field winding current
$i_{f\ell_5 1}, i_{f\ell_5 2}, i_{f\ell_5 3}$	3-phase currents of the 5th order harmonic filter
$i_{f\ell_7 1}, i_{f\ell_7 2}, i_{f\ell_7 3}$	3-phase currents of the 7th order harmonic filter
$i_{f\ell_{11} 1}, i_{f\ell_{11} 2}, i_{f\ell_{11} 3}$	3-phase currents of the 11th order harmonic filter
i_{kd}, i_{kq}	d- and q-axis damper winding current respectively
i_{t1}, i_{t2}, i_{t3}	3-phase transmission line currents
I_{ACb}, I_{DCb}	AC and DC side base current respectively
I_{fdb}	field winding base current
I_{kdb}, I_{kqb}	d- and q-axis damper winding base current respectively
I_{rb}	rotor base current
I_{sb}	stator base current
L	inductance of both the DC transmission line and the smoothing inductor
L_1, L_2, L_3	3-phase converter transformer inductances
L_{aa}, L_{bb}, L_{cc}	phase a, phase b and phase c self-inductance respectively

L_{aao}	average value of the stator self-inductance
L_{aa2}	difference between maximum and average value of the stator self-inductance
$L_{ab} = L_{ba}$	mutual-inductance between phase a and phase b
$L_{ac} = L_{ca}$	mutual-inductance between phase a and phase c
$L_{bc} = L_{cb}$	mutual-inductance between phase b and phase c
L_{abo}	average value of the stator mutual-inductance
L_{bb2}	difference between maximum and average value of the stator mutual-inductance
L_{ad}, L_{aq}	d- and q-axis magnetizing inductance
$L_{akd} = L_{kda}$	mutual-inductance between d-axis damper winding and phase a
$L_{bkd} = L_{kdb}$	mutual-inductance between d-axis damper winding and phase b
$L_{ckd} = L_{kdc}$	mutual-inductance between d-axis damper winding and phase c
$L_{akq} = L_{kqa}$	mutual-inductance between q-axis damper winding and phase a
$L_{bkq} = L_{kqb}$	mutual-inductance between q-axis damper winding and phase b
$L_{ckq} = L_{kqc}$	mutual-inductance between q-axis damper winding and phase c
$L_{afd} = L_{fda}$	mutual-inductance between field winding and phase a
$L_{bfd} = L_{fdb}$	mutual-inductance between field winding and phase b
$L_{cfd} = L_{fdc}$	mutual-inductance between field winding and phase c
L_{afdo}	maximum mutual-inductance between field winding and phase a of the stator
L_{akdo}	maximum mutual-inductance between d-axis damper winding and phase a of the stator

L_{akqo}	maximum mutual-inductance between q-axis damper winding and phase a of the stator
L_d, L_q	d- and q-axis synchronous inductance respectively
L_e	transmission line equivalent inductance per phase
L_{fd}	field winding leakage-inductance
$L_{fdkd} = L_{kdfd}$	mutual-inductance between d-axis damper winding and field winding
$L_{fdkq} = L_{kqfd}$	mutual inductance between q-axis damper winding and field winding
L_{ffd}	self-inductance of the field winding
$L_{f\ell 5}, L_{f\ell 7}, L_{f\ell 11}$	inductance per phase of the 5th, 7th and 11th order harmonic filter respectively
L_{kd}, L_{kq}	d- and q-axis damper winding leakage-inductance respectively
$L_{kdkq} = L_{kqkd}$	mutual inductance between q-axis damper winding and d-axis damper winding
L_{kkd}, L_{kkq}	d- and q-axis damper winding self-inductance respectively
L_ℓ	armature leakage-inductance
n	order of harmonic
n_p	number of pole pairs
N_{fd}	field winding effective number of turns
N_{kd}, N_{kq}	d- and q-axis damper winding effective number of turns respectively
N_s	stator effective number of turns per phase
p	differential operator d/dt
P_{fdb}	field winding base power
P_{kdb}, P_{kqb}	d- and q-axis damper winding base power respectively

P_M	mechanical power input to the synchronous machine
Q	quality factor of filter
R	resistance of DC transmission line
R_1, R_2, R_3	3-phase converter transformer resistances
R_a	armature resistance
R_e	transmission line equivalent resistance per phase
R_{fd}	field winding resistance
$R_{f\ell 5}, R_{f\ell 7}, R_{f\ell 11}$	resistance per phase of the 5th, 7th and 11th order harmonic filter respectively
R_{kd}, R_{kq}	d- and q-axis damper winding resistance respectively
t	time
T_E	electrical torque
T_M	mechanical torque
v_1, v_2, v_3	3-phase voltages at the rectifier AC terminals
v_a, v_b, v_c	3-phase voltages at the infinite bus-bar
$v_{c_5 1}, v_{c_5 2}, v_{c_5 3}$	3-phase voltages across the capacitor of the 5th order harmonic filter
$v_{c_7 1}, v_{c_7 2}, v_{c_7 3}$	3-phase voltages across the capacitor of the 7th order harmonic filter
$v_{c_{11} 1}, v_{c_{11} 2}, v_{c_{11} 3}$	3-phase votlages across the capacitor of the 11th order harmonic filter
v_{DC-r}, v_{DC-s}	DC voltage at the receiving and sending end of the DC transmission line respectively
v_d, v_q	d- and q-axis component of synchronous machine terminal voltage
v_{fd}	field winding exciting voltage

v_{kd}, v_{kq}	d- and q-axis damper winding voltage respectively
V_{ACb}, V_{DCb}	AC and DC side base voltage respectively
V_{rb}	rotor base voltage
V_{sb}	stator base voltage
δ	load angle of the synchronous machine with respect to the infinite bus-bar
λ_d, λ_q	d- and q-axis equivalent permeance of the synchronous machine respectively
ψ_a, ψ_b, ψ_c	3-phase armature flux linkages
ψ_d, ψ_q	d- and q-axis armature flux linkage respectively
ψ_{fd}	field winding flux linkage
ψ_{kd}, ψ_{kq}	d- and q-axis damper winding flux linkage respectively
θ	rotor position with respect to phase a
$p\theta$	instantaneous speed of the rotor (elec. rad/sec)
$p\theta_m$	instantaneous speed of the rotor (mech. rad/sec)
$p\theta_o$	synchronous speed
H	inertia constant of the synchronous machine and its prime-mover in p.u.
J	inertia constant of the synchronous machine and its prime-mover in $kg.m^2$
H	inertia constant of the synchronous machine and its prime-mover in sec

1. INTRODUCTION

The idea of using direct current for the transmission of electrical power has attracted engineers for over half a century. At that time, in spite of the obvious advantages offered by AC, some engineers endowed with a pioneering spirit never forgot the savings in transmission costs resulting from the use of DC. Moreover, AC transmission has some limitations which led to the use of DC transmission in certain projects. In the period from 1954 to 1974, twelve direct current links have gone into commercial operation in various parts of the world [1,2]. From the first of these links to the last, the voltage has increased from 100 to 800 KV, the rated power from 20 to 1440 MW and the distance of transmission from 60 to 850 miles (96 to 1370 km). Several other DC links are now under construction or proposed.

The factor determining the choice of an AC or DC installation is mainly an economical one. However, in some cases, DC transmission is the only feasible solution.

The economical comparisons between DC and AC schemes are based mainly on two facts:

(1) DC converter stations cost considerably more than AC transformer stations

(2) For the same transmitted power, a DC line costs less than an AC line.

Thus, it is obvious that, beyond a certain length of line, DC transmission becomes less costly than AC. Estimates of the critical distances of overhead lines [2,3] range from 310 to 930 miles

(500 to 1500 km). For cables, the critical distance is much shorter than for overhead lines, lying between 15 and 30 miles (24 and 48 km) for submarine cables and, perhaps, twice as far for underground cables. According to an economical comparison [4] between AC and DC schemes for the cross-channel link and denoting the cost of AC scheme by 100%, the cost of the DC scheme is 80.7%. The respective terminal equipment costs are 21.5% and 49.3% and the costs of cables and installations are 53.5% and 20.5%.

Although such economical comparisons are interesting, the future of HVDC is more likely to rest on the inherent advantages of DC transmission than on the marginal economical benefits, which DC transmission could provide over AC for a particular scheme. The technical advantages of the DC transmission can be summarized as follows [2,3,5]:

(1) The security of a DC transmission is high because, in the event of a fault on one conductor, the other conductor can continue to operate with ground return during the period required for repairing the fault.

(2) The corona loss of a DC line is considerably less than that of the equivalent AC line.

(3) A DC link is asynchronous, thus the power transfer can be adjusted to any desired value irrespective of the magnitude and phase of the terminal AC voltages (under normal operating conditions).

(4) A DC link can be used for power transfer over national frontiers. This is because it ensures complete independence of the frequency of the linked AC systems.

(5) DC cables can be worked at a higher voltage gradient and there is practically no charging current.

(6) In DC systems, skin effect is completely absent. This results in a more uniform current distribution in the conductors and better utilization of the material.

(7) In DC transmission, reduced tower size is used due to the lower insulation levels and number of conductors used. This is because a DC line has only two conductors, while three conductors are used with an AC line.

On the other hand, DC transmission has some disadvantages. These are:

(1) A DC terminal station is far more complicated than the equivalent AC transformer station.

(2) Converters generate harmonics and, therefore, filters have to be used.

(3) Both rectifiers and inverters absorb lagging reactive power. These are usually supplied by static capacitors which form an integral part of the harmonic filters. In some cases, synchronous condensers have to be used also. Such filtering and VAR compensation equipment contribute appreciably to the cost of the converter station.

(4) Voltage transformation has to be carried out on the AC side.

(5) The lack of HVDC circuit breakers hampers multiterminal or network operation.

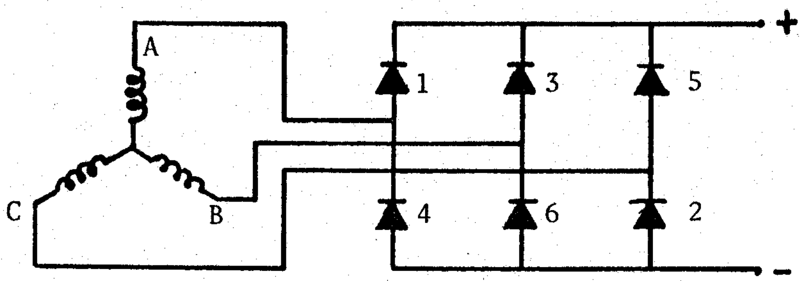
In HVDC transmission stations, the AC/DC converter is considered to be the basic unit. It could be described as the "heart" of the whole system. Until recently, all the HVDC systems in commercial operation have utilized mercury arc valves to perform the power conversion functions. The development of these valves initially made HVDC transmission possible, but as valves of higher voltage and current ratings were required, numerous technical and economical problems arose[6]. One well-known problem with mercury arc valves has been the frequency of arc-backs. In this case, the valve loses its grid control and continues to conduct in both directions until the current is interrupted. The frequent arc-backs severely stress the valves and the converter transformer and, if it is consequential in nature, it requires the interruption of the power transmission. As a result, most major electrical equipment manufacturers throughout the world undertook the development of thyristor valves in the mid 1960's. The thyristor valve[7-9] was considered to be a logical contender for the mercury arc valve for several reasons. First,there had been in this period a rapid growth in the voltage and current ratings of individual thyristors and diodes. Secondly,these devices had been extensively and successfully applied in industrial applications such as motor drives in steel mills,DC supplies for aluminum pot lines and other types of power conversion equipment. This success manifested itself in the high reliability and the low operating cost of this equipment.

The biggest difference between the concept of mercury arc and solid state valve designs is the fact that, if overstressed, the mercury arc valve will backfire but can be reconnected to the system after only a short interruption, i.e. the valve will not be damaged. On the other hand, if a thyristor valve is overstressed by voltage or by current, it completely loses its converting properties. For this reason, thyristor valves must be designed to be 100% reliable.

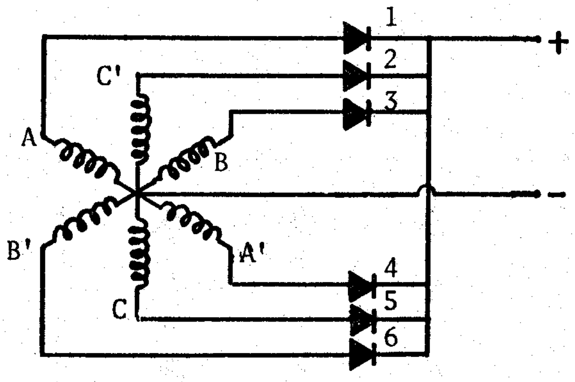
At the present time, mercury arc valves show a cost advantage over thyristor valves and exhibit lower losses [7-9]. However, with the improvements being made in thyristor technology, ratings and manufacturing techniques, and the optimization of thyristor valve design, this gap is closing.

In all the existing DC transmission schemes, the 3-phase bridge connection, Fig. 1.1a, is used because of its superiority over other possible connections, e.g. the diametral connection, Fig. 1.1b, and the double star connection, Fig. 1.1c.

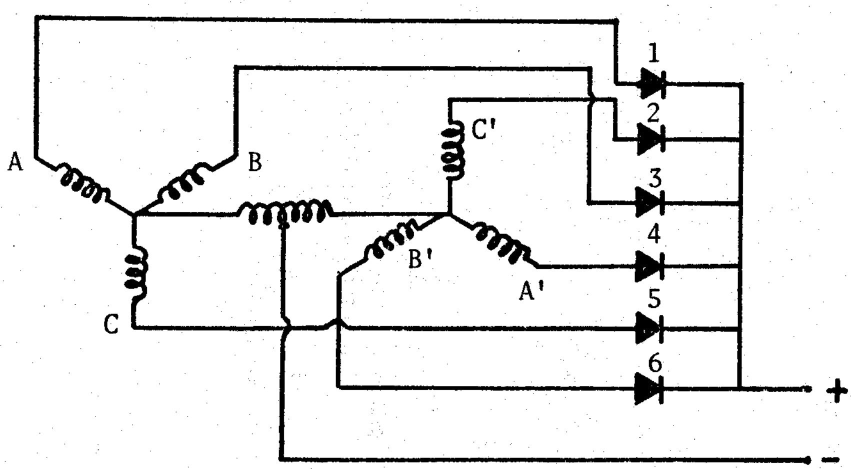
In fact, the bridge connection offers the minimum voltage stress across the valve and the best transformer utilization factor of all the known arrangements. On the other hand, the valve in the bridge connection carries the highest current. Since the voltage stresses are more troublesome than the current, the bridge connection is preferred. The best feature of the bridge circuit is the possibility of arc-back suppression, which it offers, if all the six valves are simultaneously blocked by making their grids negative for a short time. In the diametral and double-star connections, Figs. 1.1b and 1.1c, the only effect of blocking the grid is to prevent further short circuiting of the other two phases. In the faulty phase,



(a) Bridge Connection



(b) Diametral Connection



(c) Double-Star Connection

Fig. 1.1 Different Valve Arrangements

alternating current will still flow through the faulty valve, the load and the neutral. The only remedy for this is to trip the AC circuit breaker. In the bridge connection, there is no possibility of such an occurrence once the grid is made negative. This is because there are two valves which are conducting in series with the load all the time in the bridge connection. So, if one valve is faulty, the other valve, in most of the cases, is in a good condition to block the circuit.

As mentioned before [2,3], converters generate harmonic voltages and currents in both the DC and AC sides. In chapter 2 of the thesis, generated harmonics on the AC side and their effects are briefly discussed. It is noticed that most of the studies performed regarding the effect of these harmonics are conducted on simple systems, consisting of a converter station connected to an infinite bus-bar [10,11]. In practice, AC/DC systems cannot be represented by such a simple configuration.

It is the purpose of this thesis to deal with a more general representation of AC/DC systems. A representative example of such a system consists of an AC/DC converter station connected to a synchronous machine at its terminals, as well as to an infinite bus-bar through a transmission line. These synchronous machines, being close to the AC/DC converters, are considerably affected by the generated harmonics. Thus, it is of paramount importance to investigate fully the effects of these harmonics on the behavior of these machines.

In the simulation of such a system, the identity of the phase currents and voltages must be retained to define the details of the successive commutation processes of converters, and to take into consideration the various harmonics. Therefore, it is necessary to examine the system behavior in the direct-phase quantities.

In chapter 3, a general analysis and a mathematical representation suitable for the digital simulation of a bridge-connected rectifier are given. In addition, the equations of the AC transmission line and the filters are also derived.

The derivation of the non-linear differential equations of synchronous machines in direct-phase quantities is the main interest of chapter 4. These equations are arranged in such a way to allow the direct use of the machine parameters, which are usually given in the d-q-axis quantities.

The solution of the non-linear differential equations of the proposed system is carried out through the application of the 4th order Runge-Kutta method. For this, a digital computer program has been developed. This program, which is the main subject of chapter 5, has been developed in such a way to allow for the study of various system configurations. It can also be used as a subroutine in a computer program which simulates large power systems.

The developed program is used to perform some studies concerning the effects of the generated harmonics on the AC system in general and on the behavior of the synchronous machine in particular. The results of these studies are also reported in chapter 5.

2. GENERATED HARMONICS ON THE AC SIDE OF AN HVDC TRANSMISSION SYSTEM

2.1 General

If a 3-phase bridge-connected converter, Fig. 2.1a, is supplied from an infinite bus-bar and is operating under perfect conditions (no firing delay, no commutating reactance and smooth DC current), its 3-phase currents consist of two equally-spaced rectangular pulses per cycle. These pulses are of 120° duration, Fig. 2.1b. These currents consist of fundamental frequency and harmonic components[3,16]. The order of the current harmonics is given by[3]:

$$n = ks \pm 1$$

where

k is an integer

s is the number of pulses, s = 6 for one bridge,

s = 12 for two bridges whose supply voltages are 30° phase shifted from each other.

The root mean square values of these components in terms of the HVDC current I_d are given by:

$$I_n = \frac{\sqrt{6}}{n\pi} I_d \qquad 2.1$$

Taking into account the effect of the firing delay angle and the commutating reactance, but assuming smooth DC current, the current waveform is as shown in Fig. 2.2. This current may be divided into three parts[3]:

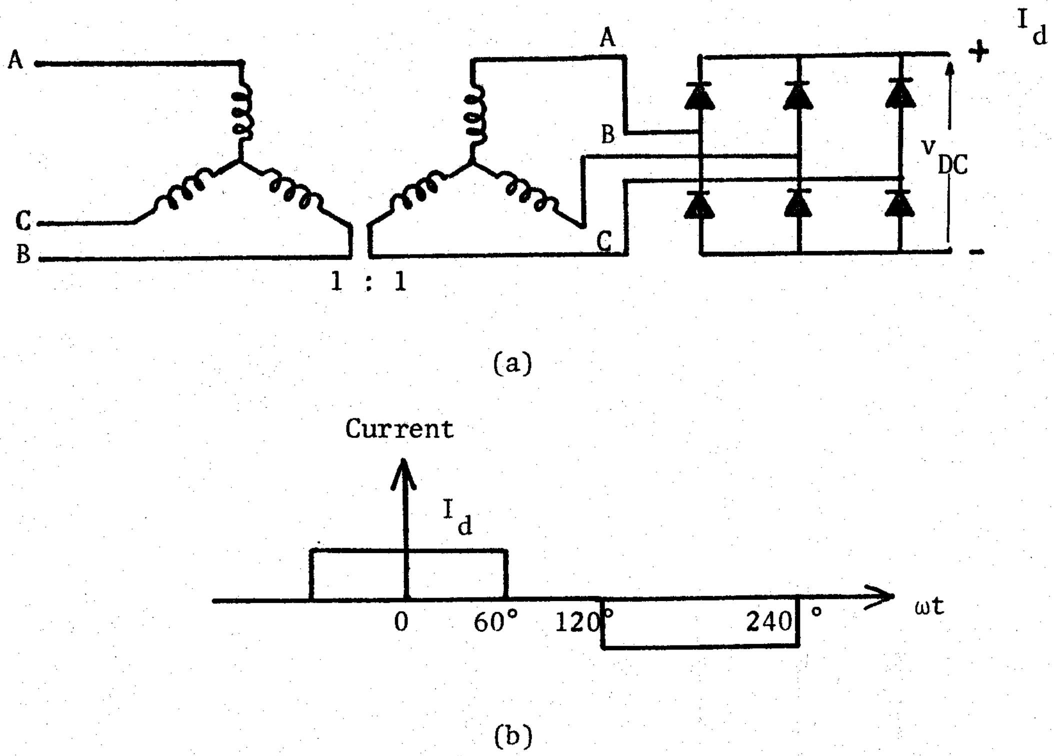


Fig. 2.1 (a) Bridge Connection
(b) Phase Current for Zero Delay Angle and No Commutating Reactance

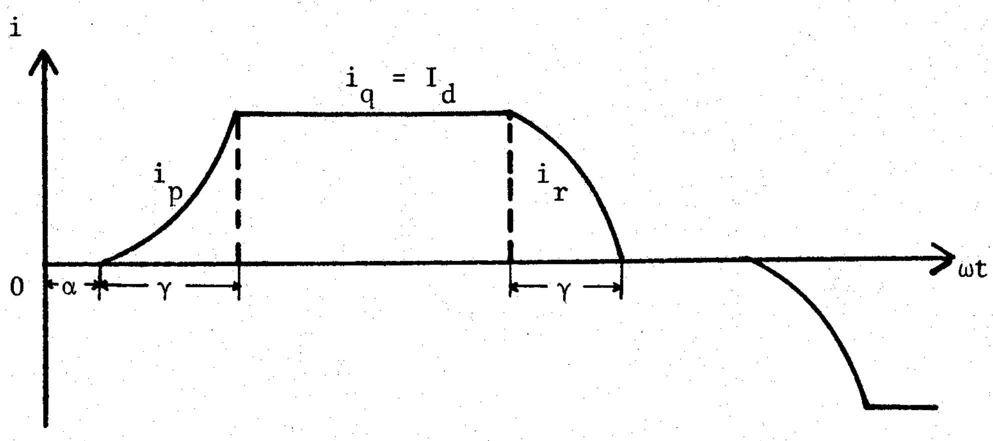


Fig. 2.2 Rectifier Phase Current Taking into Consideration the Effect of the Firing Delay and the Commutating Reactance

$$i_p = I_d \frac{(\cos\alpha - \cos\omega t)}{[\cos\alpha - \cos(\alpha+\gamma)]} \qquad \text{for } \alpha<\omega t<\alpha+\gamma \qquad 2.2$$

$$i_q = I_d \qquad \text{for } (\alpha+\gamma)<\omega t<(\alpha+\frac{2\pi}{3}) \qquad 2.3$$

$$i_r = I_d - I_d \frac{[\cos\alpha - \cos(\omega t - \frac{2\pi}{3})]}{[\cos\alpha - \cos(\alpha+\gamma)]} \quad \text{for } (\frac{2\pi}{3}+\alpha)<\omega t<(\frac{2\pi}{3}+\alpha+\gamma) \qquad 2.4$$

where

α is the firing delay angle

γ is the commutation angle

I_d is the DC current

The fundamental component of this current can be given as follows [3]:

$$I_{(1)} = I_{(1)o}\sqrt{K_{1a}^2 + K_{1r}^2} \qquad 2.5$$

where

$$K_{1a} = \frac{[\cos 2\alpha - \cos 2(\alpha+\gamma)]}{4[\cos\alpha - \cos(\alpha+\gamma)]} \qquad 2.6$$

$$K_{1r} = \frac{2\gamma + \sin 2\alpha - \sin 2(\alpha+\gamma)}{4[\cos\alpha - \cos(\alpha+\gamma)]} \qquad 2.7$$

$I_{(1)o}$ is the fundamental current with zero delay angle and no commutating reactance and is given by the equation:

$$I_{(1)o} = \frac{\sqrt{6}}{\pi} I_d \qquad 2.8$$

For the practical range of converter operation, Eqn. 2.5 can be approximated as follows [3]:

$$I_{(1)} \simeq I_{(1)o} \qquad 2.9$$

Thus the amplitude of the fundamental current component is practically independent of α and γ.

However, α and γ affect the various harmonic components. Reference 3 gives a family of curves which illustrates these effects on the 5th, 7th, 11th, 13th, 17th, 19th, 23rd and 25th harmonic currents. From these curves, the following can be concluded:

(i) As γ increases, the magnitudes of the harmonics decrease. The magnitude of the higher order harmonics decrease more rapidly than those of lower orders.

(ii) The rate of reduction of the magnitudes of the harmonics increases as γ increases up to a certain limit.

(iii) Each harmonic magnitude decreases to a minimum at an angle $\gamma \doteq 360^{\circ}/n$, and then rises slightly thereafter.

(iv) With a constant angle γ, the changes in the magnitudes of the various harmonics for different values of α are small.

(v) For a constant DC current, when the angle α is increased, the angle γ is reduced. In this case, the harmonics tend to increase and approach the highest values at $\gamma = 0$. In no case, however, the magnitudes of the harmonics shall exceed the values given by:

$$I_{(n)o} = \frac{I_{(1)o}}{n} = \frac{\sqrt{6}}{n\pi} I_d \qquad 2.10$$

where

$I_{(n)o}$ is the nth order harmonic current with zero delay angle and no commutating reactance.

These harmonics, which are generated in the AC system when the converter is operating with balanced voltages at its terminals and with symmetrical firing of the valves, are called the characteristic or the normal harmonics [15]. In practice, no supply is balanced and the control equipment perform within finite tolerance limits. These result in an inherent firing unbalance [17] and these imperfect conditions cause the converter to generate harmonics which are uncharacteristic to its theoretical behavior under normal balanced conditions [18].

A brief summary of the different types of unbalanced conditions and their general effects on the generation of the harmonics is given in the following table:

Unbalanced Condition	Causes	Harmonics generated
Unbalanced terminal voltages	System faults, untransposed lines, unsymmetrical loading of phases	All odd harmonics including the triplen
Unsymmetrical firing	Tolerances, effect of system voltage harmonics	All harmonics (i.e. both odd and even harmonics)and also a a DC component
Non-sinusoidal terminal voltages	Inherent property of large power system due to imperfections of the equipment of the system	Same as the second case

These "uncharacteristic" harmonics which are usually not provided for in the design of the system, affect it adversely and interfere with its working[17-24].

2.2 Effects of Harmonics on the AC System

The general effects of current harmonics on AC systems are well documented[3,18,23,28]. They are briefly discussed here with reference to HVDC systems.

2.2.1 Effects on rotating machines[23,26-28]

Any harmonic current flowing in the stator winding of a rotating machine increases the temperature rise of the stator winding and the surrounding parts[29]. Also, these harmonics can establish pairs of magnetomotive forces rotating in opposite directions, which produce rotating elliptical magnetic fields[30]. These fields induce eddy currents flowing in the surface of the rotor. Moreover, high frequncy currents will also be induced in the rotor windings. The flow of these currents in both the rotor surface and windings increases the rotor heating.

Due to the flow of these harmonic currents, a synchronous machine will develop an electric torque which is not constant. It contains high order harmonics. These high frequency components give rise to pulsating torques and thus to vibrations.

2.2.2 Effects on transformers[31-33]

The flow of the current harmonics in the transformer windings increases the copper loss of the winding. Also, it results in an eddy current heating in the ferromagnetic structure of the transformer as well as the windings. The hysteresis loss will be increased. Due to the commutation voltage surges of converters, there is also a possibility of insulation breakdown.

2.2.3 Interference with communication systems[3]

Harmonic currents in the AC system result in an unacceptable interference with communication circuits and cause disturbances in them. These disturbances may be through the medium of either electromagnetic or electric induction. The communication systems may be classified readily into:

(i) Telephone circuits

(ii) Telegraph circuits

Telegraph circuits usually carry currents whose main components have frequencies of less than 300 Hz, and hence they tend to be affected only by the fundamental frequency and lower order harmonics. Telephone circuits, on the other hand, carry currents of frequencies in the audio range lying between 100 Hz and 4 kHz. Telephone circuits are, thus, the most sensetive to outside interference from power lines since their small currents are directly representative of speech and induced harmonics may render it unintelligible.

For estimating the interference of the harmonics with telephone circuits, the C.C.I.T.T. (International Telegraph and Telephone Consultive Committee) Directives defines a quantity called the psophometric voltage given by:

$$v_{ps} = \sqrt{\Sigma(W_n V_n)^2} \qquad 2.11$$

where

V_n is the interfering voltage of frequency n

W_n is the psophometric weight at frequency n and which can be measured directly with suitable instruments.

However, different countries have different standerds in this respect.

2.2.4 Interference with ripple control systems[20]

Some electric power utilities sell electric energy at especially low rates for off-peak boads, such as water heaters. The periods, during which such loads can be connected, are controlled by transmitting audiofrequency tones in the range of 290 to 1650 Hz from substations over the power distribution circuits to customers' premises. These signals will control contactors in series with such loads. Similar control is also used for street lighting. The receiving devices for the control signals are broadly tuned and can accept harmonics from high power converters, which may cause undesired operation of the contactors or prevent desired ones. Such malfunction may, however, be avoided by decreasing the susceptibility of the ripple control system to harmonics.

2.2.5 Hamonic voltages in the system[34]

The harmonic currents generated by a converter cause harmonic voltages to appear in the system. These voltages distort the voltage waveform. This distortion may be unacceptable above a certain level.

2.2.6 Overvoltages from resonance[34]

If there is a large shunt capacitor bank in the power system, there is a possibility of a parallel resonance between it and the rest of the system at a harmonic frequency[19]. The order of the harmonic n, at which resonance may occur, is given by:

$$n = \sqrt{\frac{Q_s}{Q_c}} \qquad 2.12$$

where

Q_s is the short-circuit power of the power system at a point where the capacitor bank is connected.

Q_c is the rating of the capacitor bank.

Troubles from such a resonance occur most likely at a frequency close to a harmonic frequency, for which no filter is provided. Such resonance could have several undesirable effects:

(a) overheating of the capacitor banks

(b) overvoltage of the capacitor bank

2.2.7 Harmonic instability

Due to the various harmonic voltages, the voltages at the converter transformer terminals will not be sinusoidal. As a result, the AC timing voltages of the control system will be unequally spaced and distorted. This causes unequally spaced firing of valves which, in turn, generates uncharacteristic AC current harmonics. These current harmonics reinforce the existing ones causing magnification of the harmonic magnitudes. This process, under certain system conditions, can be cummulative and leads to harmonic instability[35].

2.3 Methods of Reducing Harmonics

2.3.1 Increased pulse number "s"

If two similar valve groups are connected in parallel on the transformer primary side and in series on the DC side, Fig. 2.3, and if the transformers have a relative phase shift of 30°, 12 pulse operation is obtained. This means that ripple voltage on the DC side has 12 equal pulses per fundamental frequency cycle. On the AC side, the harmonic currents of order 11, 13, 23, 25..... of both converters are in phase and add together. On the other hand, the harmonic currents of order 5, 7, 17, 19..... are in antiphase and cancel each other.

In low voltage high current rectifiers, high pulse numbers have sometimes been used varying from 24 to 108. This method of reducing harmonics is very effective as long as all valves are in service, but it requires complicated transformer connections. In high voltage, high current converters for DC transmission, problems of insulation of the converter transformers to withstand the high AC voltages in combination with the high DC voltage dictate simple transformer connections. A pulse number of 12 is easily obtained with simple connections of two six-pulse valve groups as mentioned before. Twenty-four pulses can be obtained with four six-pulse

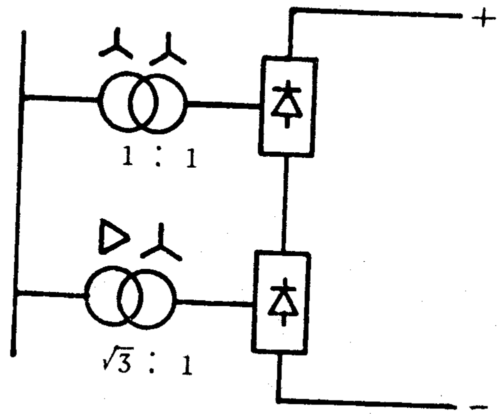


Fig. 2.3 Two Valve Groups Connected in Parallel on the AC Side and in Series on the DC Side

valve groups by use of a phase-shifting transformer bank in conjunction with two 12-pulse converters. In this case, the required phase shift is 15°. Operation with 48 pulses could be obtained from eight bridges by providing 7.5° phase shift between the voltages of the adjacent transformers. Even though the pulse number may be doubled, e.g. from 12 to 24, the system is still not designed to accept the harmonics of the twelve-phase operation if it has to operate with only one bridge under emergency conditions.

The extra expenses involved in increasing the pulse number must be weighed against that involved in other methods of reducing harmonics.

With regard to HVDC plants, it is Lamm's opinion and also that of the Soviet engineers[36], that it will always be technically and economically prohibitive to increase the pulse number beyond twelve, and that other methods must be adopted to reduce harmonics.

2.3.2 Using harmonic filters

The necessary reduction in the harmonics on the AC side of a converter -beyond that accomplished by the increase of pulse number- is usually achieved by the use of filters[2,3,37]. In most cases, filters are also needed on the DC side.

In addition to the AC harmonic filters at the converter station, such filters could also be placed at any point of the system,where harmonic interference with telephone lines is not acceptable. This is seldom done because it is usually cheaper to modify or relocate the telephone line.

AC filters serve to supply a part of the reactive power needed at fundamental frequency in addition to the reduction of the harmonics. Thus, the cost of the filter capacitors can be charged to the cost of the equipment needed to supply the reactive power. In this case, the part of the cost of filters chargeable to the need for reducing the harmonics is almost equal to the cost of the filter inductor only. The capital cost of AC filters is in the range of 5 to 15% of the cost of the converter station[2,22]. This is high enough to justify careful design from the standpoint of economy as well as adequacy. The cost of the losses in the filter elements is another factor which should also be taken into consideration.

2.3.3 Using synchronous capacitors or generators

It was considered technically feasible[2] to operate without AC filters and to absorb the harmonic currents from the converter in the synchronous generators near its terminals. It would, however, be necessary to have low reactance generators and transformers in this case. Converters without filters have been used at Volgograd U.S.S.R.[2]. Among the advantages claimed is that the provision of ample damper windings on the generators would cost less than the provision of AC harmonic filters. In this particular case, the generators are isolated from the rest of the AC system and, so, they are allowed to vary in speed with no concern about the de-tuning of the filters.

Studies of this possibility on the Nelson River project[37,38] showed, however, that filtering was more economical than building the generators to carry the harmonic currents continuously. An additional complication was that, because the project would be built in stages, the addition of the generators will be done also in stages. Thus, there is no assurance that the harmonics would divide among all these generators in any definite ratio without putting undue restrictions on the design of the future generators, AC lines, etc. Hence, it appears that the choice between using filters or specially designed synchronous machines is a matter which is dependent on the nature of the project itself.

2.4 Classifications of AC Harmonic Filters

The AC filters of converter stations may be classified according to their location, their manner of connection to the main circuit, their sharpness of tuning, the number of their resonant frequencies and their method of tuning.

2.4.1 Filter location

Filters on the AC side may be connected either on the primary (network) side of the converter transformer or to the tertiary winding if one is provided for this purpose.

Since the tertiary winding, if provided, has a lower voltage than the primary winding, the filters in this case are insulated for lower power-frequency and surge voltages and, therefore, cost less. The tertiary winding, however, adds to the cost of the transformer. This winding usually has a high leakage reactance, which inherently forms a common branch in series with all the shunt filters. This complicates the computation of the possible resonance between the filters and the AC network.

2.2.4 Filter connection

In this respect, filters can be classified into:

(a) Series, high impedance filters by which harmonics are impeded in passing from the converter to the power network or line.

(b) Shunt, low impedance filters in which harmonics are diverted.

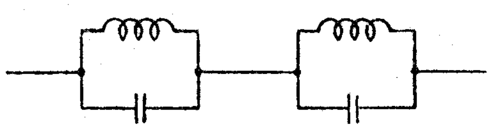
Fig. 2.4 illustrates the first two kinds. Each is a dual of the other.

The series filter must carry the full current of the main circuit and must be insulated throughout for the full voltage to ground. On the other hand, the shunt filter can be grounded at one end and carries the harmonic current for which it is tuned plus a fundamental current much smaller than that of the main circuit. Hence, a shunt filter is much cheaper than a series filter of equal effectiveness. At fundamental frequency, a shunt filter supplies reactive power while the series cne consumes it. For these reasons, shunt filters are used exclusively on the AC side.

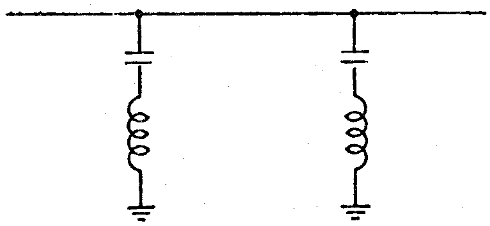
2.4.3 Sharpness of tuning of filters[2,16]

In this respect, two kinds of filters are used:

(a) Tuned filters (high Q filter), which are sharply tuned to one or two of the lower harmonic frequencies, e.g. the fifth and the seventh.



(a)



(b)

Fig. 2.4 (a) Series Filter
(b) Shunt Filter

(b) Damped filters (low Q filters), which offer a low impedance over a broad band of frequencies embracing, for example, the seventeenth and higher harmonics. **These** filters are called sometimes high-pass filters.

Figs. 2.5 and 2.6 show typical circuit diagrams of these two types and their characteristics.

2.4.4 The number of resonant frequencies of filters

Filters may be single tuned, double tuned, triple tuned ... etc. These filters can be designed using the single tuned filters. For example, the double tuned filter can be obtained by finding an equivalent filter to two resonant arms, and so on for other types.

2.4.5 Methods of tuning

1. Fixed tuned harmonic filter[39]

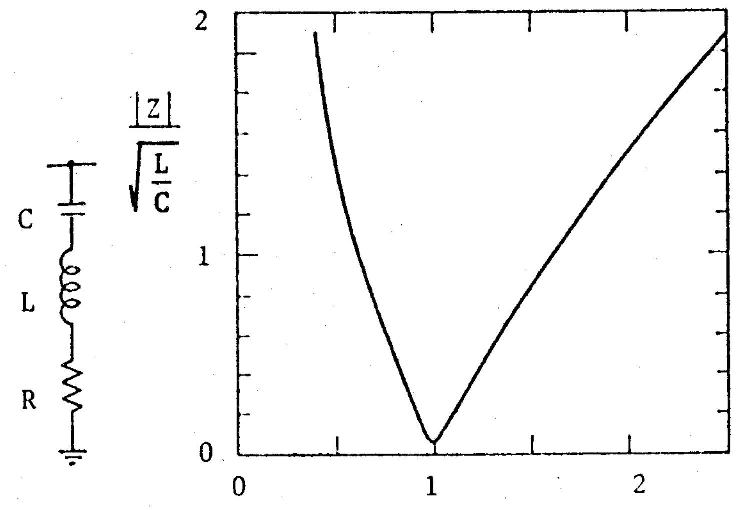
Such a filter consists of few arms sharply tuned to the dominant harmonic frequencies. Each tuned arm consists of a capacitor "C", reactor "L" and resistor "R" connected in series. A filter arm can go off tune due to the variation of the temperature of its components and/or due to the changes in the system frequency.

As a filter goes off tune, the impedance of the arm is given approximately($\sigma \ll 1$) by[2,16]:

$$Z_f = R(1 + jQ2\sigma) \qquad 2.13$$

where

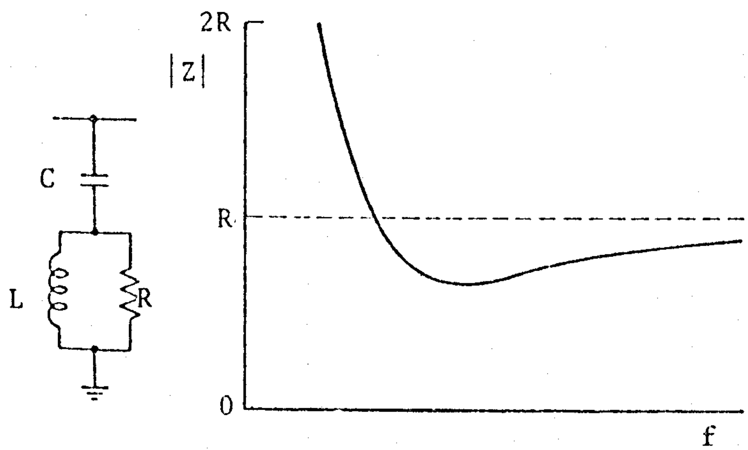
Q is the quality factor $(= \frac{1}{\omega_o CR})$



(a) (b)

Fig. 2.5 Single Tuned Shunt Filter

(a) Circuit
(b) Impedance Versus Frequency Characteristic



(a) (b)

Fig, 2.6 Second Order Damped Shunt Filter

(a) Circuit
(b) Impedance Versus Frequency Characteristic

σ is the per unit frequency deviation from the tuned frequency f_o due to all causes ($\sigma = \frac{\omega-\omega_o}{\omega_o}$)

$$\omega_o = 2\pi f_o = \frac{1}{\sqrt{LC}} \qquad 2.14$$

Fig. 2.7 shows three curves of filter impedance $|Z_f|$ versus frequency deviation σ. Curves A and B are for the same R, and, therefore, they have the same minimum impedance. Curves B and C are for the same X_o and, so, they have the same asymptotes "D" (corresponding to R=0). The equation of the asymptotes is $|X_f| = \pm 2X_o|\sigma|$. Curves A and C are for the same "Q"; they have, thus, the same passband "PB".

Theoretically a high value of "Q" is required so that the filter will be sharply tuned to the designed harmonic frequency and the filter losses will be low. However, if "Q" is high, the percentage de-tuning "σ" should be very small in order to keep the impedance very low and near to the value "R" (Eqn. 2.13). As shown from curve A (with high "Q"), Fig. 2.7, any small increase in "σ" will noticeably increase the value of "Z_f". Therefore, in practice, a compromise should be made in choosing "Q". The value of "Q" should be so chosen that the filter losses will be low and, at the same time, that any possible de-tuning will not change the filter impedance too much from its value at the harmonic frequency.

2. Self-tuned harmonic filter[39]

Filter arms can be maintained in tune by adjustment of

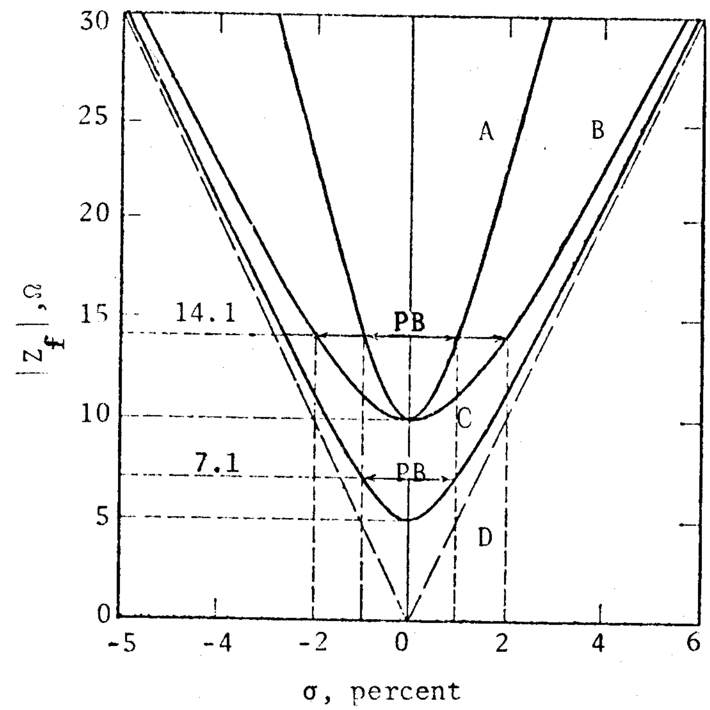


Fig. 2.7 Impedance of Tuned Filter as Function of Frequency Deviation. Curve D consists of the Asymptotes of Curves B and C.

Curve	R (Ω)	$X_o = \sqrt{\frac{L}{C}}$ (Ω)	Q	Passband (PB)
A	10	500	50	2%
B	10	250	25	4%
C	5	250	50	2%

the capacitor or, more usually, the inductor. This can result in a value of "σ" which is theoretically zero or, in practice, very small. This means that a high value of "Q" can be used. The consequent low value of resistance "R" means low loss and high filtering efficiency. Moreover, a capacitor of a kind that has a high temperature coefficient of capacitance but also has a high reactive power rating per unit of volume and per unit of cost may be used.

The additional cost of providing the inductor variation can be justified by savings in the capacitor cost and by the improved performance. Moreover, when a low reactive power is required, "X_o" will be large and the passband will be small. This results in that the impedance will be high over a frequency band bounded by the maximum deviation. Therefore, it is preferable to use a self-tuned filter in this case.

It should, also, be noted that a self-tuned filter may offer no cost advantage when the possible de-tuning is small, or when the required fundamental frequency reactive power of filter arm is high. In this latter case, "X_o" is small and the passband is large. Thus, the value of the impedance will be low over a wide range of frequency.

3. DIGITAL SIMULATION OF AN AC/DC CONVERTER STATION

3.1 Introduction

As mentioned in chapter 1, the 3-phase bridge connection is used in all the existing HVDC schemes. Therefore, in this chapter, a detailed analysis for this valve arrangement is carried out. The bridge operation is mathematically represented by a group of 1st order general differential equations, which is applicable to any conduction condition. The solution of these equations is obtained point by point to give the instantaneous values of the various quantities. This method of solution permits the study of the bridge circuit performance with any voltage wave-shape applied to its terminals.

3.2 Mathematical Representation of the Bridge Circuit

Fig. 3.1 shows the bridge connection where:

T_{11}, T_{21}, T_{31}, T_{12}, T_{22}, T_{32} are the valves of the bridge.

In the operation of this 3-phase bridge-connected recitifer, either two or three valves are conducting simultaneously [2,3]. Therefore, twelve different modes of operation exist per cycle as shown in Fig. 3.2. These possible modes of conduction are illustrated in Fig. 3.3.

Each mode of these twelve cases can be represented mathematically by its own differential equations. However, it is possible to represent them mathematically by general equations, since these twelve cases can, in fact, be divided into two main cases. As mentioned before, one case is when two valves are conducting, while

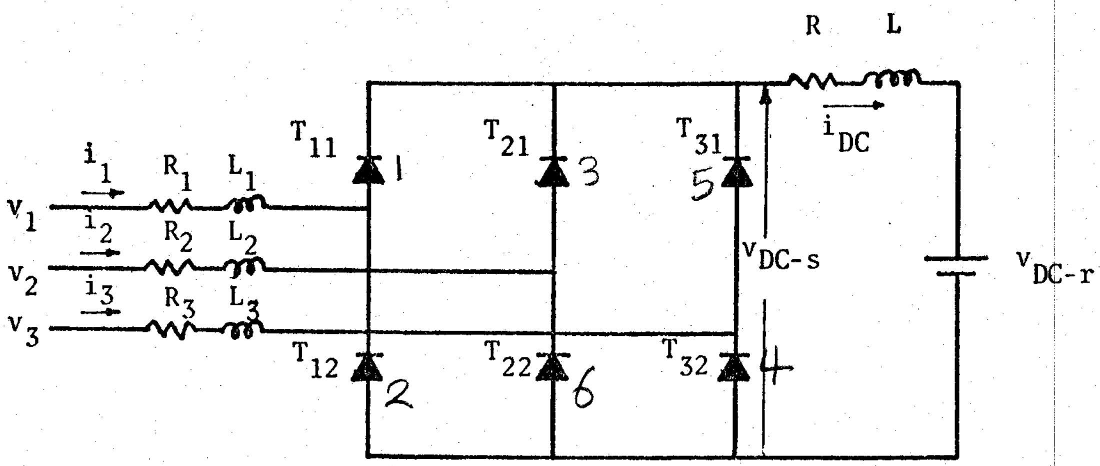


Fig. 3.1 Bridge-Connected Rectifier

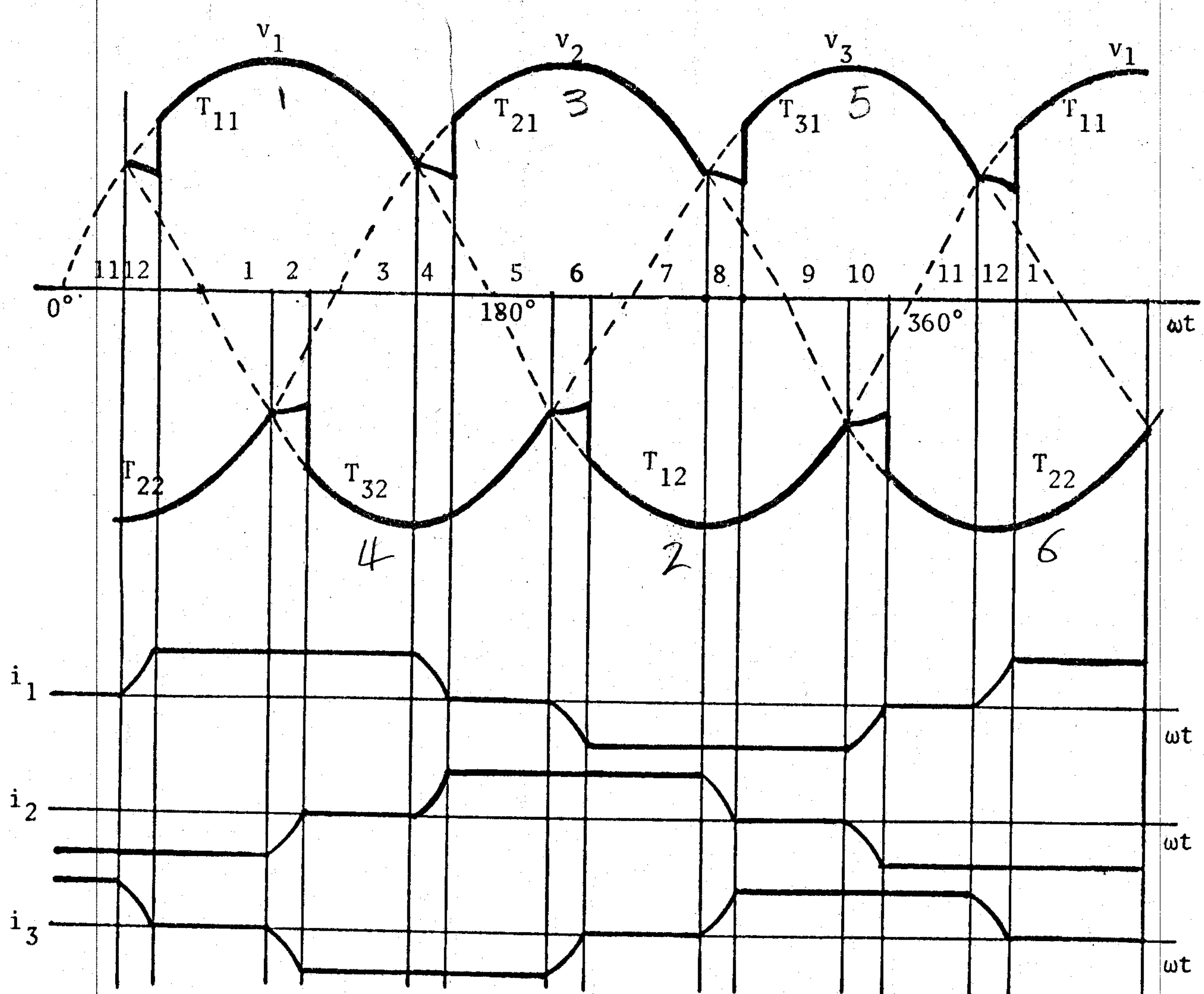
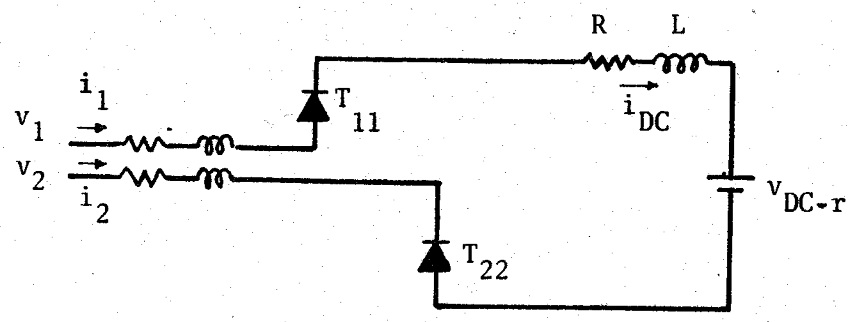
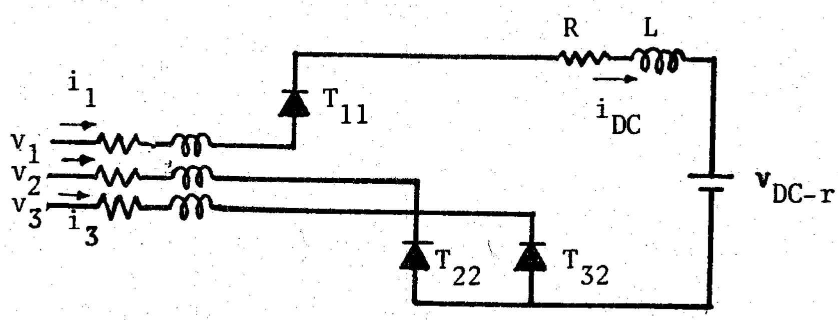


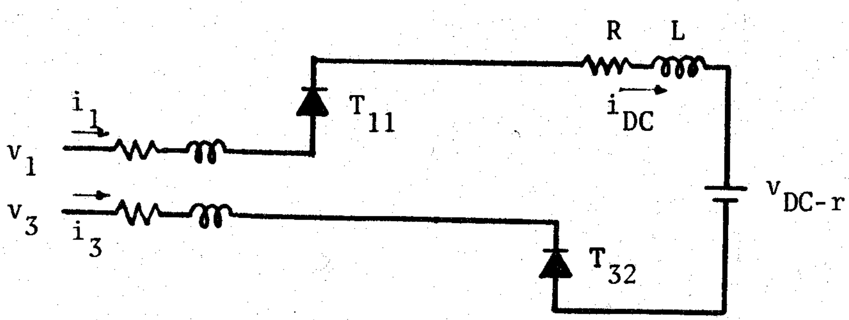
Fig. 3.2 Different modes of operation of the bridge connection during one cycle



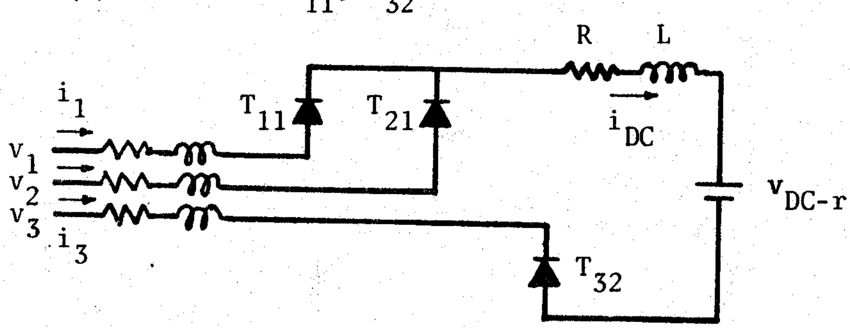
(a) Case 1: T_{11}, T_{22}



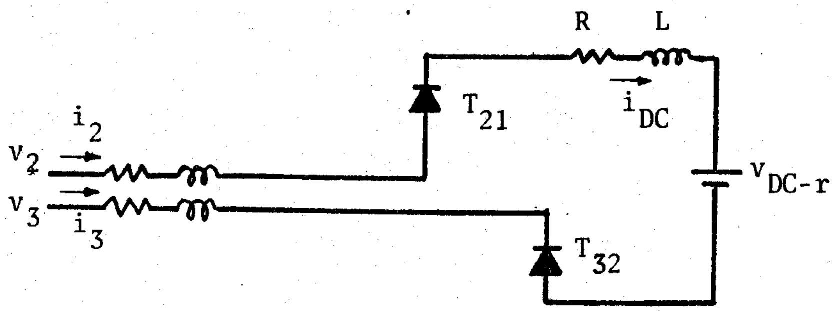
(b) Case 2: T_{11}, T_{22}-T_{32}



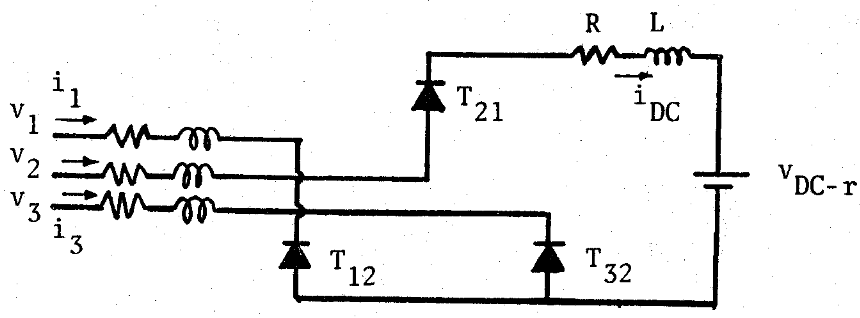
(c) Case 3: T_{11}, T_{32}



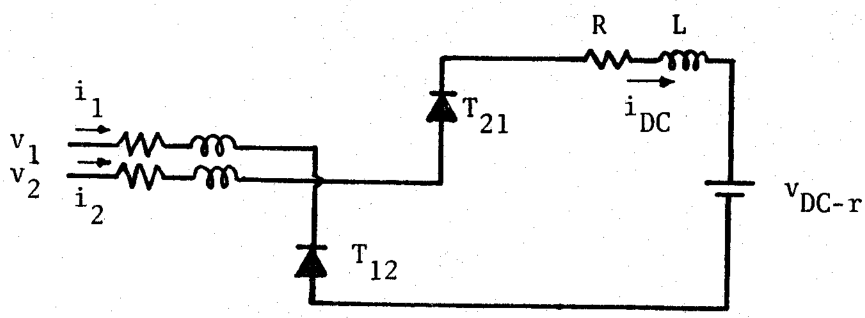
(d) Case 4: T_{11} - T_{21},T_{32}



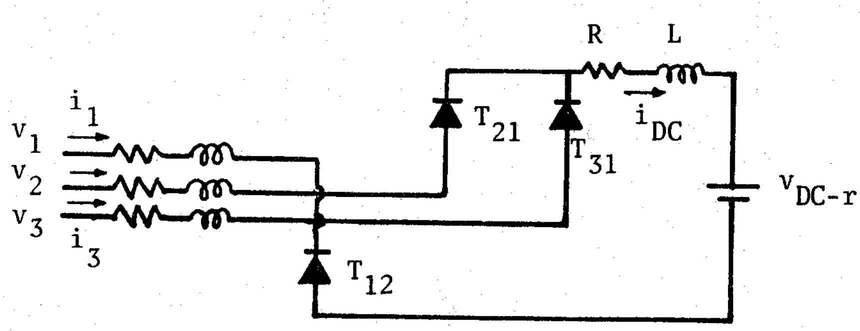
(e) Case 5: T_{21}, T_{32}



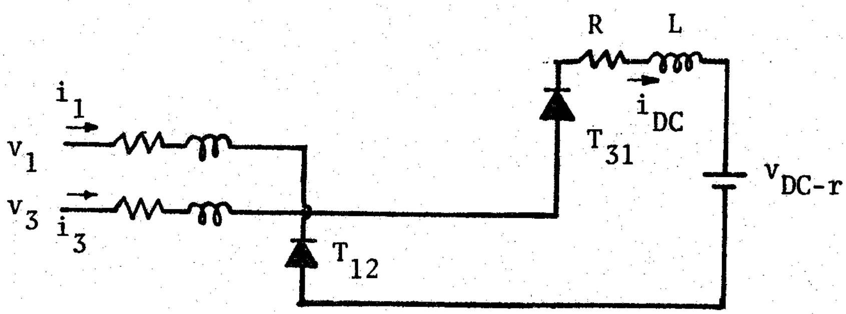
(f) Case 6: T_{21}, T_{32}-T_{12}



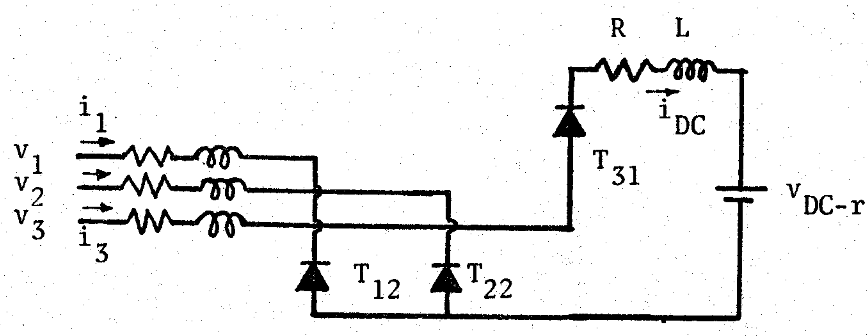
(g) Case 7: T_{21}, T_{12}



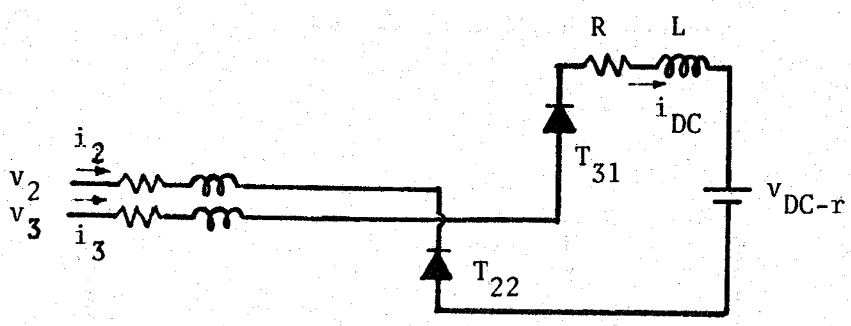
(h) Case 8: T_{21} - T_{31}, T_{12}



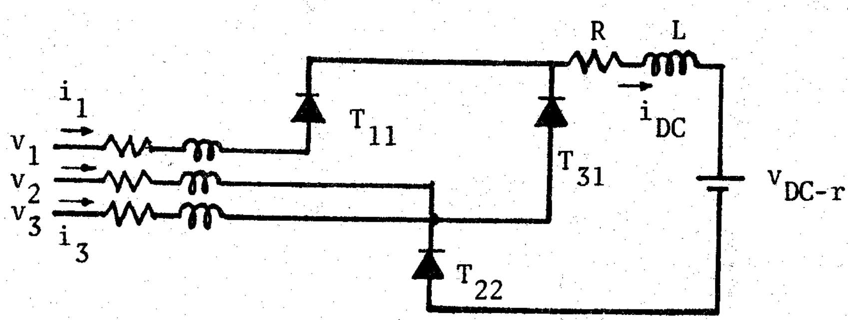
(i) Case 9: T_{31}, T_{12}



(j) Case 10: T_{31}, T_{12} - T_{22}



(k) Case 11: T_{31}, T_{22}



(l) Case 12: T_{31} - T_{11}, T_{22}

Fig. 3.3 Different Modes of Conduction of the Bridge-Connected Rectifier the other is when three valves are conducting. These general equations can be applied to all modes by simply defining the conducting valves in each case.

In order to obtain such general equations, the following two cases of operation are considered:

(a) Case 1: Valves T_{11} and T_{22} are conducting simultaneously

The mathematical equations representing this case can be written as follows:

$$i_2 = -i_1 \tag{3.1}$$

$$i_{DC} = i_1 \tag{3.2}$$

$$v_1 - v_2 = R_1 i_1 + L_1 \frac{di_1}{dt} + Ri_{DC} + L \frac{di_{DC}}{dt} + v_{DC-r} - R_2 i_2 - L_2 \frac{di_2}{dt} \tag{3.3}$$

Substituting Eqns. 3.1 & 3.2 in Eqn. 3.3, it follows:

$$v_1 - v_2 = (R_1 + R + R_2) i_1 + (L_1 + L + L_2) \frac{di_1}{dt} + v_{DC-r} \tag{3.4}$$

Thus,

$$\frac{di_1}{dt} = \{(v_1 - v_2) - (R_1 + R + R_2) i_1 - v_{DC-r}\} / (L_1 + L + L_2) \tag{3.5}$$

The DC voltage at the sending end of the DC transmission line can be obtained from the following equation:

$$v_{DC-s} = Ri_1 + L \frac{di_1}{dt} + v_{DC-r} \tag{3.6}$$

Eqns. 3.1, 3.2, 3.5 and 3.6 represent this case.

(b) Case 2: Valve T_{11} is conducting with both valves T_{22} and T_{32}

In this case the equations are:

$$i_1 = -(i_2+i_3) \tag{3.7}$$

$$i_{DC} = i_1 \tag{3.8}$$

$$v_1-v_2 = R_1 i_1 + L_1 \frac{di_1}{dt} + Ri_1 + L\frac{di_1}{dt} + v_{DC-r}$$
$$-R_2 i_2 - L_2 \frac{di_2}{dt}$$

$$= (R_1+R)i_1 + (L_1+L)\frac{di_1}{dt} + v_{DC-r} - R_2 i_2 - L_2 \frac{di_2}{dt} \tag{3.9}$$

$$v_2-v_3 = R_2 i_2 + L_2 \frac{di_2}{dt} - R_3 i_3 - L_3 \frac{di_3}{dt} \tag{3.10}$$

$$v_{DC-s} = R\, i_1 + L\frac{di_1}{dt} + v_{DC-r} \tag{3.11}$$

Eqn. 3.7 gives

$$i_3 = -(i_1+i_2) \tag{3.12}$$

Subsituting Eqn. 3.12 in Eqn. 3.10,

$$v_2-v_3 = R_2 i_2 + L_2 \frac{di_2}{dt} + R_3 i_1 + R_3 i_2 + L_3 \frac{di_1}{dt} + L_3 \frac{di_2}{dt}$$

$$= (R_2+R_3)i_2 + (L_2+L_3)\frac{di_2}{dt} + R_3 i_1 + L_3 \frac{di_1}{dt} \tag{3.13}$$

Therefore,

$$\frac{di_2}{dt} = \{(v_2-v_3)-(R_2+R_3)i_2-R_3 i_1 - L_3 \frac{di_1}{dt}\}/(L_2+L_3) \tag{3.14}$$

Eqn. 3.9 gives

$$\frac{di_1}{dt} = \{(v_1-v_2)-(R_1+R)i_1-v_{DC-r} + R_2 i_2 + L_2 \frac{di_2}{dt}\}/(L_1+L) \tag{3.15}$$

Substituting Eqn. 3.15 in Eqn. 3.14

$$\frac{di_2}{dt} = \frac{(v_2-v_3)}{(L_2+L_3)} - \frac{(R_2+R_3)}{(L_2+L_3)} i_2 - \frac{R_3}{(L_2+L_3)} i_1$$

$$- \frac{L_3}{(L_2+L_3)} [\frac{(v_1-v_2)}{(L_1+L)} - \frac{(R_1+R)}{(L_1+L)} i_1 - \frac{1}{(L_1+L)} v_{DC-r}$$

$$+ \frac{R_2}{(L_1+L)} i_2 + \frac{L_2}{(L_1+L)} \frac{di_2}{dt}] \qquad 3.16$$

$$\frac{di_2}{dt} = [-L_3 v_1 + (L_1+L+L_3)v_2 - (L_1+L)v_3 + \{(R_1+R)L_3$$

$$-(L_1+L)R_3\}i_1 - \{R_2L_3+R_2(L_1+L)$$

$$+ R_3(L_1+L)\}i_2 + L_3\, v_{DC-r}]/[(L_1+L)L_2$$

$$+ (L_1+L)L_3+L_2L_3] \qquad 3.17$$

$$\frac{di_2}{dt} = [-v_1+(1+ \frac{L_1+L}{L_3})v_2 - (\frac{L_1+L}{L_3})v_3 + v_{DC-r}$$

$$+ \{(R_1+R) - \frac{(L_1+L)R_3}{L_3} \}i_1 - \{R_2+ \frac{R_2\ (L_1+L)}{L_3}$$

$$+ \frac{R_3(L_1+L)}{L_3} \}i_2]/[L_2+(L_1+L) + \frac{(L_1+L)L_2}{L_3}] \qquad 3.18$$

and since $i_1 = -i_2-i_3$, then

$$\frac{di_2}{dt} = [-v_1+(1+ \frac{L_1+L}{L_3})v_2-(\frac{L_1+L}{L_3}) v_3 + v_{DC-r}-\{(R_1+R)$$

$$+R_2 + \frac{R_2(L_1+L)}{L_3} \}i_2-\{(R_1+R) - \frac{(L_1+L)R_3}{L_3} \}i_3]/$$

$$[L_2 + (L_1+L) + \frac{(L_1+L)L_2}{L_3}] \qquad 3.19$$

Substituting the value of $\frac{di_2}{dt}$ from Eqn. 3.19 in Eqn. 3.10, therefore,

$$\frac{di_3}{dt} = \frac{v_3}{L_3} - \frac{v_2}{L_3} + \frac{R_2}{L_3} i_2 - \frac{R_3}{L_3} i_3 + \frac{1}{L_3} \cdot \frac{1}{\{(L_1+L)+L_2+ \frac{(L_1+L)L_2}{L_3}\}}$$

$$[-v_1L_2+\{L_2+ \frac{(L_1+L)L_2}{L_3}\}v_2-\{\frac{(L_1+L)L_2}{L_3}\}v_3-\{(R_1+R)L_2$$

$$+ L_2R_2+ \frac{(L_1+L)L_2R_2}{L_3}\}i_2-\{(R_1+R)L_2- \frac{(L_1+L)L_2R_3}{L_3}\}i_3$$

$$+L_2v_{DC-r}\,] \qquad 3.20$$

$$\frac{di_3}{dt} = [-v_1- \frac{(L_1+L)}{L_3} v_2+\{1+ \frac{(L_1+L)}{L_2}\}v_3+v_{DC-r}-\{(R_1+R)- \frac{(L_1+L)R_2}{L_2}\}i_2$$

$$-\{(R_1+R)+R_3+ \frac{(L_1+L)R_3}{L_2}\}i_3]/[L_3+(L_1+L) + \frac{(L_1+L)L_3}{L_2}] \qquad 3.21$$

Therefore, for case 2 where valve T_{11} is conducting with both valves T_{22} and T_{32}, the differential equations representing the case are as follows:

$$\frac{di_2}{dt} = [-v_1+v_2(1+ \frac{L_1+L}{L_3})-v_3(\frac{L_1+L}{L_3})+v_{DC-r}-i_2\{(R_1+R)$$

$$+R_2+ \frac{R_2(L_1+L)}{L_3}\}-i_3\{(R_1+R)- \frac{R_3(L_1+L)}{L_3}\}]/$$

$$[(L_1+L)+L_2+ \frac{(L_1+L)L_2}{L_3}] \qquad 3.22$$

$$\frac{di_3}{dt} = [-v_1+v_3(1+ \frac{L_1+L}{L_2})-v_2(\frac{L_1+L}{L_2})+v_{DC-r}-i_3\{(R_1+R)$$

$$+R_3+ \frac{R_3(L_1+L)}{L_2}\}-i_2\{(R_1+R)- \frac{R_2(L_1+L)}{L_2}\}]/$$

$$[(L_1+L)+L_3+ \frac{(L_1+L)L_3}{L_2}] \qquad 3.23$$

$$\frac{di_1}{dt} = - \frac{di_2}{dt} - \frac{di_3}{dt} \qquad 3.24$$

$$v_{DC-s} = Ri_1 + L \frac{di_1}{dt} + v_{DC-r} \qquad 3.25$$

3.2.1 General equations representing the bridge operation

Eqns. 3.5, 3.22 and 3.23 can be written in an alternative, more general form which can be applied to any conduction condition of the bridge rectifier circuit as follows:

$$\frac{di_e}{dt} = [-v_d+v_e(1+ \frac{L_d+L}{L_g} K)-v_g \frac{L_d+L}{L_g} K-(-1)^m v_{DC-r}$$

$$-i_e\{(R_d+R)+R_e+ \frac{R_e(L_d+L)}{L_g} K\}-i_g\{(R_d+R)$$

$$- \frac{R_g(L_d+L)}{L_g}\}K]/[(L_d+L)+L_e+ \frac{(L_d+L)L_e}{L_g} K] \qquad 3.26$$

where,

subscripts e, d and g are for any of the three phases 1, 2 and 3.

K and m are constants.

R_e, R_d, R_g and L_e, L_d, L_g are the transformer resistances and inductances respectively.

Eqn. 3.24 may also be generalized as follows:

$$\frac{di_d}{dt} = - \frac{di_e}{dt} - K \frac{di_g}{dt} \qquad 3.27$$

By varying subscripts e, d and g and the values K and m according to the conduction condition, Eqns. 3.26 and 3.27 can be applied to any of the twelve modes of operation. To achieve this, the following rules are obtained empirically:

(i) If three valves are conducting as in Figs. 3.3b, 3.3d, 3.3f, 3.3h, 3.3j and 3.3ℓ, then K = 1; otherwise K = 0.

(ii) If two upper valves are conducting simultaneously, then m = 2; otherwise m = 1.

(iii) Suffix d always represents the phase number of a current that can be divided into the two other phases. It also represents the phase number having positive phase current with respect to the positive direction of i_{DC} when there are only two conducting phases.

(iv) If a phase is non-conducting the derivative of its current is zero.

Application of these rules to the twelve modes of operation gives the following:

Case 1: Valves T_{11} and T_{22} are firing

$e = 2 \qquad d = 1 \qquad g = 6-d-e = 3 \qquad K = 0 \qquad m = 1$

$$i_1 = -i_2 = i_{DC} \qquad \frac{di_3}{dt} = 0$$

Case 2: Valves T_{11} and T_{22}-T_{32} are firing

$e = 2 \qquad d = 1 \qquad g = 3 \qquad K = 1 \qquad m = 1$

$e = 3 \qquad d = 1 \qquad g = 2 \qquad K = 1 \qquad m = 1$

$$i_1 = -(i_2+i_3) = i_{DC} \qquad \frac{di_1}{dt} = -\left(\frac{di_2}{dt} + \frac{di_3}{dt}\right)$$

Case 3: Valves T_{11} and T_{32} are firing

$e = 3 \qquad d = 1 \qquad g = 2 \qquad K = 0 \qquad m = 1$

$$i_1 = -i_3 = i_{DC} \qquad \frac{di_2}{dt} = 0$$

Case 4: Valves T_{11}-T_{21} and T_{32} are firing

$e = 1 \qquad d = 3 \qquad g = 2 \qquad K = 1 \qquad m = 2$

$e = 2 \qquad d = 3 \qquad g = 1 \qquad K = 1 \qquad m = 2$

$$i_3 = -(i_1+i_2) = -i_{DC} \qquad \frac{di_3}{dt} = -\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$$

Case 5: Valves T_{21} and T_{32} are firing

$e = 3 \qquad d = 2 \qquad g = 1 \qquad K = 0 \qquad m = 1$

$$i_2 = -i_3 = i_{DC} \qquad \frac{di_1}{dt} = 0$$

Case 6: Valves T_{21} and T_{32}-T_{12} are firing

$e = 3 \quad d = 2 \quad g = 1 \quad K = 1 \quad m = 1$

$e = 1 \quad d = 2 \quad g = 3 \quad K = 1 \quad m = 1$

$$i_2 = -(i_1+i_3) = i_{DC} \qquad \frac{di_2}{dt} = -\left(\frac{di_1}{dt} + \frac{di_3}{dt}\right)$$

Case 7: Valves T_{21} and T_{12} are firing

$e = 1 \quad d = 2 \quad g = 3 \quad K = 1 \quad m = 1$

$$i_2 = -i_1 = i_{DC} \qquad \frac{di_3}{dt} = 0$$

Case 8: Valves T_{21}-T_{31} and T_{12} are firing

$e = 2 \quad d = 1 \quad g = 3 \quad K = 1 \quad m = 2$

$e = 3 \quad d = 1 \quad g = 2 \quad K = 1 \quad m = 2$

$$i_1 = -(i_2+i_3) = -i_{DC} \qquad \frac{di_1}{dt} = -\left(\frac{di_2}{dt} + \frac{di_3}{dt}\right)$$

Case 9: Valves T_{31} and T_{12} are firing

$e = 1 \quad d = 3 \quad g = 2 \quad K = 0 \quad m = 1$

$$i_3 = -i_1 = i_{DC} \qquad \frac{di_2}{dt} = 0$$

Case 10: Valves T_{31} and T_{12}-T_{22} are firing

$e = 1 \quad d = 3 \quad g = 2 \quad K = 1 \quad m = 1$

$e = 2 \quad d = 3 \quad g = 1 \quad K = 1 \quad m = 1$

$$i_3 = -(i_1+i_2) = i_{DC} \qquad \frac{di_3}{dt} = -\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$$

Case 11: Valves T_{31} and T_{22} are firing

$e = 2 \qquad d = 3 \qquad g = 1 \qquad K = 0 \qquad m = 1$

$$i_3 = -i_2 = i_{DC} \qquad \frac{di_1}{dt} = 0$$

Case 12: Valves T_{31}-T_{11} and T_{22} are firing

$e = 3 \qquad d = 2 \qquad g = 1 \qquad K = 1 \qquad m = 2$

$e = 1 \qquad d = 2 \qquad g = 3 \qquad K = 1 \qquad m = 2$

$$i_2 = -(i_1+i_3) = -i_{DC} \qquad \frac{di_2}{dt} = -\left(\frac{di_1}{dt} + \frac{di_3}{dt} \right)$$

For all the twelve modes of operation, the value of the DC current and its derivative can be generalized in the form:

$$i_{DC} = (-1)^{m+1}\, i_d \tag{3.28}$$

$$\frac{di_{DC}}{dt} = (-1)^{m+1} \frac{di_d}{dt} \tag{3.29}$$

Also, the DC terminal voltage of the rectifier can be obtained from the equation:

$$v_{DC-s} = v_{DC-r} + Ri_{DC} + L \frac{di_{DC}}{dt} \tag{3.30}$$

3.3 Mathematical Representation of a Short Transmission Line

Converter stations are usually connected to large AC systems through transmission lines. In the system under investigation in this thesis, a short transmission line, Fig. 3.4, is assumed to connect the AC/DC converter station to an infinite bus-bar. If the transmission line is very short, it is found (appendix A) that it can be represented by an equivalent resistance "R_e" and an

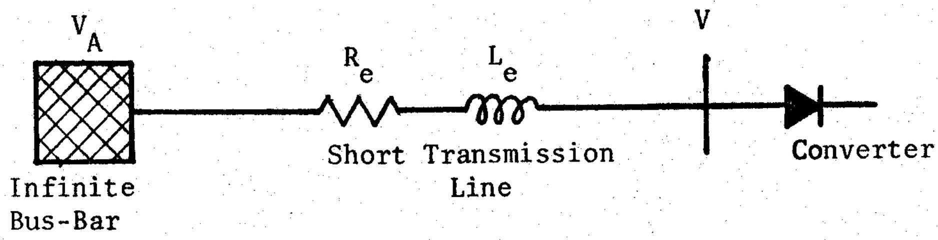


Fig. 3.4 Representation of the Short Transmission Line

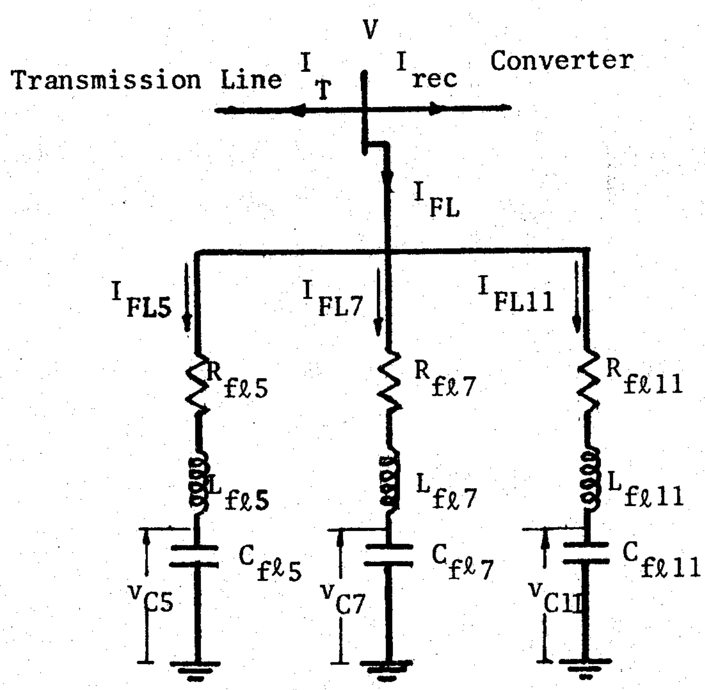


Fig. 3.5 Representation of 5th, 7th and 11th order Harmonic Filters

equivalent inductance "L_e". In such cases accurate results can be obtained even under the effect of the harmonics existing in the AC system due to the DC transmission.

In the matrix notation, the voltage at the converter bus-bar can be obtained from the following relation:

$$[V] = [V_A] + [R_E][I_T] + [L_E]p[I_T] \tag{3.31}$$

where

$[V] = [v_1 \quad v_2 \quad v_3]^t$

$[V_A] = [v_a \quad v_b \quad v_c]^t$

$[R_E] = \text{diag}[R_e \quad R_e \quad R_e]$

$[L_E] = \text{diag}[L_e \quad L_e \quad L_e]$

$[I_T] = [i_{t1} \quad i_{t2} \quad i_{t3}]^t$

3.4 Mathematical Model of Filters

It has been mentioned in chapter 2 that filters can be used to reduce harmonic currents and voltages in the AC system. They are R-L-C circuits which are designed to provide a low impedance path for harmonic currents at harmonic frequencies. In the system under investigation, a group of 5th, 7th and 11th order harmonic filters are connected at the converter AC terminals as shown in Fig. 3.5. The differential equations of these filters are:

$$p[I_{FL5}] = [\frac{1}{L_{FL5}}][V]-[\frac{1}{L_{FL5}}][V_{C5}]-[\frac{R_{FL5}}{L_{FL5}}][I_{FL5}] \tag{3.32}$$

$$p[V_{C5}] = [\frac{1}{C_{FL5}}][I_{FL5}] \tag{3.33}$$

$$p[I_{FL7}] = [\frac{1}{L_{FL7}}][V]-[\frac{1}{L_{FL7}}][V_{C7}]-[\frac{R_{FL7}}{L_{FL7}}][I_{FL7}] \quad 3.34$$

$$p[V_{C7}] = [\frac{1}{C_{FL7}}][I_{FL7}] \quad 3.35$$

$$p[I_{FL11}] = [\frac{1}{L_{FL11}}][V]-[\frac{1}{L_{FL11}}][V_{C11}]-[\frac{R_{FL11}}{L_{FL11}}][I_{FL11}] \quad 3.36$$

$$p[V_{C11}] = [\frac{1}{C_{FL11}}][I_{FL11}] \quad 3.37$$

where

$$[\frac{1}{L_{FL5}}] = \text{diag}\ [\frac{1}{L_{f\ell 5}}\ \ \frac{1}{L_{f\ell 5}}\ \ \frac{1}{L_{f\ell 5}}]$$

$$[\frac{1}{L_{FL7}}] = \text{diag}\ [\frac{1}{L_{f\ell 7}}\ \ \frac{1}{L_{f\ell 7}}\ \ \frac{1}{L_{f\ell 7}}]$$

$$[\frac{1}{L_{FL11}}] = \text{diag}\ [\frac{1}{L_{f\ell 11}}\ \ \frac{1}{L_{f\ell 11}}\ \ \frac{1}{L_{f\ell 11}}]$$

$$[\frac{R_{FL5}}{L_{FL5}}] = \text{diag}\ [\frac{R_{f\ell 5}}{L_{f\ell 5}}\ \ \frac{R_{f\ell 5}}{L_{f\ell 5}}\ \ \frac{R_{f\ell 5}}{L_{f\ell 5}}]$$

$$[\frac{R_{FL7}}{L_{FL7}}] = \text{diag}\ [\frac{R_{f\ell 7}}{L_{f\ell 7}}\ \ \frac{R_{f\ell 7}}{L_{f\ell 7}}\ \ \frac{R_{f\ell 7}}{L_{f\ell 7}}]$$

$$[\frac{R_{FL11}}{L_{FL11}}] = \text{diag}\ [\frac{R_{f\ell 11}}{L_{f\ell 11}}\ \ \frac{R_{f\ell 11}}{L_{f\ell 11}}\ \ \frac{R_{f\ell 11}}{L_{f\ell 11}}]$$

$$[\frac{1}{C_{FL5}}] = \text{diag}\ [\frac{1}{C_{f\ell 5}}\ \ \frac{1}{C_{f\ell 5}}\ \ \frac{1}{C_{f\ell 5}}]$$

$$[\frac{1}{C_{FL7}}] = \text{diag}\ [\frac{1}{C_{f\ell 7}}\ \ \frac{1}{C_{f\ell 7}}\ \ \frac{1}{C_{f\ell 7}}]$$

$$[\frac{1}{C_{FL11}}] = \text{diag}\ [\frac{1}{C_{f\ell 11}}\ \ \frac{1}{C_{f\ell 11}}\ \ \frac{1}{C_{f\ell 11}}]$$

$$[I_{FL5}] = [i_{f\ell_5 1}\ \ i_{f\ell_5 2}\ \ i_{f\ell_5 3}]^t$$

$$[I_{FL7}] = [i_{f\ell_7 1}\ \ i_{f\ell_7 2}\ \ i_{f\ell_7 3}]^t$$

$$[I_{FL11}] = [i_{f\ell_{11} 1}\ \ i_{f\ell_{11} 2}\ \ i_{f\ell_{11} 3}]^t$$

$$[V_{C5}] = [v_{c_5 1}\ \ v_{c_5 2}\ \ v_{c_5 3}]^t$$

$$[V_{C7}] = [v_{c_7 1}\ \ v_{c_7 2}\ \ v_{c_7 3}]^t$$

$$[V_{C11}] = [v_{c_{11} 1}\ \ v_{c_{11} 2}\ \ v_{c_{11} 3}]^t$$

4. THE MATHEMATICAL REPRESENTATION OF SYNCHRONOUS MACHINES IN DIRECT-PHASE QUANTITIES

4.1 General

In synchronous machines, all the mutual-inductances between the stator and rotor circuits are periodic functions of the rotor angular position. In addition, because of the rotor saliency, the self-inductances of the stator phases and the mutual-inductances between any two stator phases are also periodic functions of the rotor angular position. As a result, the characteristics of synchronous machines are expressed by a series of differential equations, most of whose coefficients are periodic functions of the rotor position. Such nonlinear equations are quite difficult to solve. For the cases of balanced conditions, the two-reaction theory [40, 41] was introduced to overcome this difficulty. This theory replaces the three phases of the armature winding by two equivalent fictitious windings, which are stationary with respect to the rotor. The first of these windings is attached to the pole-axis (direct-axis), while the other is attached to the inter-pole-axis (quadrature-axis). In Fig. 4.1, these two windings are denoted by "d" and "q" respectively. This d-q-axis [40-42] model yields differential equations with constant coefficients. These equations are linear provided that the speed is assumed to be constant. Once the speed variation is taken into account, they become nonlinear.

If the synchronous machine is connected to a rectifier load,

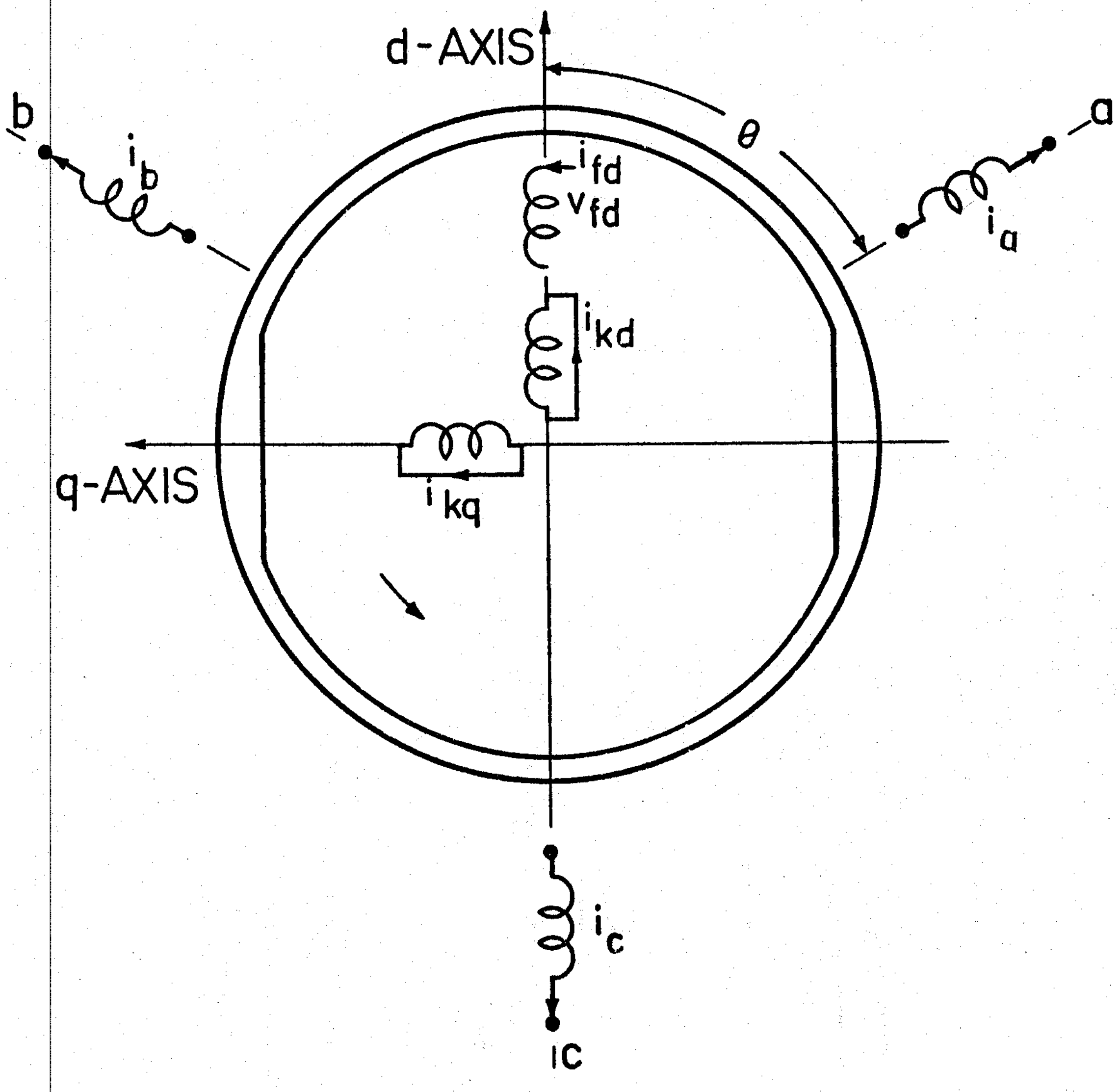


Fig. 4.1 Diagram of an Idealised Synchronous Machine

it will be necessary to examine the machine behavior in direct-phase quantities [44]. This is because the identity of the phase currents must be retained in order to define the details of the successive commutation processes. Moreover, with this type of loading, the voltages at the machine terminals are not sinusoidal due to the flow of the harmonic currents. A d-q-axis model, in this case, does not offer any significant advantage over a direct 3-phase model, especially if the speed variation is taken into consideration. Thus, it is more convenient for these types of studies to carry out the analysis in direct-phase quantities.

4.2 The Mathematical Model of 3-Phase Synchronous Machines in Direct-Phase Quantities

In developing the performance equations of synchronous machines, the following assumptions are usually made [40,41]:

(1) Saturation, hysteresis and eddy currents in all magnetic circuits are neglected.

(2) Each machine winding produces a sinusoidally space distributed magnetomotive force.

(3) The rotor magnetic and electric circuits are symmetrical about both the d- and q-axis. Hence, the flux in one axis will not interlink with the windings on the other axis. In other words, the mutual inductances between the rotor windings on both axes are zero.

(4) The stator slots cause no appreciable variation of any of the rotor inductances with the rotor position.

(5) The damper winding, if it exists, is replaced by two

equivalent damper circuits; one on the direct-axis and the other on the quadrature-axis.

Based on these assumptions, the performance of a synchronous machine may now be described by the following equations. In them, the convention adopted for the signs of voltages and currents are that v is the impressed voltage at the terminals and that the direction of positive current i corresponds to generation. The sign of the current in the damper winding is taken positive when it flows in a direction similar to that of a positive field current.

4.2.1 Flux linkage equations

Using the generator notation, the flux linkage equations in terms of the machine currents and inductances can be written in a matrix form as follows:

$$[\psi] = [L][I] \tag{4.1}$$

where

$$[\psi] = [\psi_a \quad \psi_b \quad \psi_c \quad \psi_{fd} \quad \psi_{kd} \quad \psi_{kq}]^t$$

$$[I] = [i_a \quad i_b \quad i_c \quad i_{fd} \quad i_{kd} \quad i_{kq}]^t$$

$$[L] = \begin{bmatrix} -L_{aa} & -L_{ab} & -L_{ac} & L_{afd} & L_{akd} & L_{akq} \\ -L_{ba} & -L_{bb} & -L_{bc} & L_{bfd} & L_{bkd} & L_{bkq} \\ -L_{ca} & -L_{cb} & -L_{cc} & L_{cfd} & L_{ckd} & L_{ckq} \\ -L_{fda} & -L_{fdb} & -L_{fdc} & L_{ffd} & L_{fdkd} & L_{fdkq} \\ -L_{kda} & -L_{kdb} & -L_{kdc} & L_{kdfd} & L_{kkd} & L_{kdkq} \\ -L_{kqa} & -L_{kqb} & -L_{kqc} & L_{kqfd} & L_{kqkd} & L_{kkq} \end{bmatrix} \qquad 4.2$$

4.2.2 Inductance equations[40,41]

Most of the inductances in Eqn. 4.2 are not constant but are functions of the rotor position θ, which is the angle between the pole-axis of the rotor and the axis of phase a as shown in Fig. 4.1.

(a) Stator self-inductances

The reluctance of the magnetic circuit of a synchronous machine at any section in the air-gap depends on the position of the pole structure. As iron has a very high permeability compared with air, the permeance of the magnetic circuit of any stator phase varies from a maximum (when its axis coincides with the direct-axis of the rotor) to a minimum (when its axis coincides with the quadrature-axis). This variation can be represented by a Fourier series expansion which contains even harmonics. Considering only the zero and the second order harmonic terms of this series, the self-inductance of the stator phases can then be expressed as follows:

$$\left.\begin{aligned} L_{aa} &= L_{aao} + L_{aa2} \cos 2\theta \\ L_{bb} &= L_{aao} + L_{aa2} \cos (2\theta + 120) \\ L_{cc} &= L_{aao} + L_{aa2} \cos (2\theta - 120) \end{aligned}\right\} \quad 4.3$$

As the leakage flux of any stator phase is independent of the rotor position, it is usually included in the constant L_{aao}.

(b) Stator mutual-inductances

The mutual-inductance between any two stator phases varies periodically from a maximum (when the quadrature-axis is midway between the axes of the two phases) to a minimum (when the quadrature-axis is 90° electrical from the maximum position). These mutual inductances can be expressed by:

$$\left.\begin{aligned} L_{ab} = L_{ba} &= -[L_{abo} + L_{bb2} \cos (2\theta + 60)] \\ L_{bc} = L_{cb} &= -[L_{abo} + L_{bb2} \cos (2\theta - 180)] \\ L_{ca} = L_{ac} &= -[L_{abo} + L_{bb2} \cos (2\theta + 300)] \end{aligned}\right\} \quad 4.4$$

Theoretical analysis shows that the difference between the maximum and the average values of the self-inductance is the same as that of the mutual inductance [40], i.e.

$$L_{bb2} = L_{aa2} \qquad 4.5$$

It has also been found that, apart from the leakage inductance, the average value of the self-inductance of a stator phase is double the average value of the mutual-inductance between any two stator phases. This can be expressed as follows:

$$L_{aao} - L_{\ell} = 2L_{abo} \qquad 4.6$$

(c) Mutual-inductances between stator and rotor circuits

The mutual-inductances between the stator and rotor circuits are maximum when the axis of the corresponding rotor winding coincides with the axis of the corresponding stator winding. It is zero when these two axes are electrically perpendicular. After 180° from the maximum position, the mutual inductance will be again maximum but negative. The variation is sinusoidal. Such a sinusoidal variation of the mutual inductances between the field winding and the phase windings results in the sinusoidal open circuit voltage, which is usually obtained in synchronous machines.

The mutual inductances between the field winding and the armature windings can thus be written as follows:

$$\left.\begin{aligned} L_{afd} &= L_{fda} = L_{afdo} \cos\theta \\ L_{bfd} &= L_{fdb} = L_{afdo} \cos(\theta - 120) \\ L_{cfd} &= L_{fdc} = L_{afdo} \cos(\theta - 240) \end{aligned}\right\} \quad 4.7$$

Similarly, the mutual inductances between the equivalent damper winding in the d-axis and the armature windings are:

$$\left.\begin{aligned} L_{akd} &= L_{kda} = L_{akdo} \cos\theta \\ L_{bkd} &= L_{kdb} = L_{akdo} \cos(\theta - 120) \\ L_{ckd} &= L_{kdc} = L_{akdo} \cos(\theta - 240) \end{aligned}\right\} \quad 4.8$$

Also, the mutual inductances between the equivalent damper winding in the q-axis and the armature windings are given by the following:

$$\left.\begin{aligned} L_{akq} &= L_{kqa} = -L_{akqo} \sin \theta \\ L_{bkq} &= L_{kqb} = -L_{akqo} \sin (\theta - 120) \\ L_{ckq} &= L_{kqc} = -L_{akqo} \sin (\theta - 240) \end{aligned}\right\} \quad 4.9$$

(d) Rotor self-inductances

All inductances of the rotor circuits, i.e. L_{ffd}, L_{kkd} and L_{kkq} do not depend on the rotor position and so they are considered constant.

(e) Rotor mutual-inductances

As there is no flux interlinkage between the windings on the d- and the q-axes, then:

$$L_{fdkq} = L_{kqfd} = L_{kdkq} = L_{kqkd} = 0 \qquad 4.10$$

The mutual inductance between the field winding and the equivalent damper winding in the d-axis L_{fdkd} is independent of the rotor position, i.e. it is constant.

4.2.3 Inductance matrix

Utilizing these inductance relations given in section 4.2.2, the inductance matrix can be written as follows:

$$
[L] = \begin{bmatrix}
-L_{aao} - L_{aa2}\cos 2\theta & L_{abo} + L_{aa2}\cos 2(\theta+30) & L_{abo} + L_{aa2}\cos 2(\theta+150) & L_{afdo}\cos\theta & L_{akdo}\cos\theta & -L_{akqo}\sin\theta \\
L_{abo} + L_{aa2}\cos 2(\theta+30) & -L_{aao} - L_{aa2}\cos 2(\theta-120) & L_{abo} + L_{aa2}\cos 2(\theta-90) & L_{afdo}\cos(\theta-120) & L_{akdo}\cos(\theta-120) & -L_{akqo}\sin(\theta-120) \\
L_{abo} + L_{aa2}\cos 2(\theta+150) & L_{abo} + L_{aa2}\cos 2(\theta-90) & -L_{aao} - L_{aa2}\cos 2(\theta-240) & L_{afdo}\cos(\theta-240) & L_{akdo}\cos(\theta-240) & -L_{akqo}\sin(\theta-240) \\
-L_{afdo}\cos\theta & -L_{afdo}\cos(\theta-120) & -L_{afdo}\cos(\theta-240) & L_{ffd} & L_{fdkd} & 0 \\
-L_{akdo}\cos\theta & -L_{akdo}\cos(\theta-120) & -L_{akdo}\cos(\theta-240) & L_{fdkd} & L_{kkd} & 0 \\
L_{akqo}\sin\theta & L_{akqo}\sin(\theta-120) & L_{akqo}\sin(\theta-240) & 0 & 0 & L_{kkq}
\end{bmatrix} \tag{4.11}
$$

4.2.4 Voltage equations

The voltage equations of the synchronous machine are given in a matrix notation form as follows:

$$[V] = p[\psi] - [R][I] \tag{4.12}$$

where

$$\left.\begin{aligned} [V] &= [v_1 \quad v_2 \quad v_3 \quad v_{fd} \quad v_{kd} \quad v_{kq}]^t \\ [R] &= \text{diag}\,[R_a \quad R_a \quad R_a \quad -R_{fd} \quad -R_{kd} \quad -R_{kq}] \end{aligned}\right\} \tag{4.13}$$

4.2.5 Torque equations

If the instantaneous electric torque of the synchronous machine differs from the prime-mover torque, its speed will vary. At any instant, the mechanical torque is equal to the electrical torque plus the accelerating torque. If P_m is the mechanical power input and $p\theta_m$ is the speed, the mechanical torque T_M is given by:

$$T_M = \frac{P_M}{p\theta_m} \tag{4.14}$$

The equation of motion can be expressed as:

$$p^2\theta_m = (T_M - T_E)/J \tag{4.15}$$

The electrical torque T_E is given in terms of the armature phase quantities as follows[40]:

$$T_E = \frac{1}{\sqrt{3}}\{\psi_a(i_b-i_c) + \psi_b(i_c-i_a) + \psi_c(i_a-i_b)\}.n_p \tag{4.16}$$

4.3 The d-q Model of the Synchronous Machine

Applying the d-q transformation [40-42] to Eqns. 4.1, 4.12 and 4.16, the following equations are obtained.

4.3.1 Flux linkage equations

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_{fd} \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix} = \begin{bmatrix} -L_d & 0 & L_{afdo} & L_{akdo} & 0 \\ 0 & -L_q & 0 & 0 & L_{akqo} \\ -\frac{3}{2} L_{afdo} & 0 & L_{ffd} & L_{fdkd} & 0 \\ -\frac{3}{2} L_{akdo} & 0 & L_{kdfd} & L_{kkd} & 0 \\ 0 & -\frac{3}{2} L_{akqo} & 0 & 0 & L_{kkq} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix} \qquad 4.17$$

where

$$\left.\begin{aligned} L_d &= L_{aao} + L_{abo} + \frac{3}{2} L_{aa2} = L_\ell + L_{ad} \\ L_q &= L_{aao} + L_{abo} - \frac{3}{2} L_{aa2} = L_\ell + L_{aq} \\ L_\ell &= L_{aao} - 2L_{abo} \end{aligned}\right\} \qquad 4.18$$

4.3.2 Voltage equations

$$\begin{bmatrix} v_d \\ v_q \\ v_{fd} \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} p & -p\theta & 0 & 0 & 0 \\ p\theta & p & 0 & 0 & 0 \\ 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 & p \end{bmatrix} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_{fd} \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix} - \begin{bmatrix} R_a & 0 & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 & 0 \\ 0 & 0 & -R_{fd} & 0 & 0 \\ 0 & 0 & 0 & -R_{kd} & 0 \\ 0 & 0 & 0 & 0 & -R_{kq} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

4.19

4.3.3 Torque equations

The electric torque may be expressed in terms of the d and q quantities as:

$$T_E = \frac{3}{2} (\psi_d . i_q - \psi_q . i_d) . n_p \tag{4.20}$$

The relation between the mechanical shaft torque and the electrical developed torque is still given by:

$$T_M = T_E + J.p^2 \theta_m \tag{4.21}$$

4.4 The Per-Unit System[45]

Per-unit systems are extensively used to simplify the mathematical representation of phenomena over a wide range of different physical problems. Some of the advantages,which can arise from the application of a well designed per-unit system to electrical power problems, are as follows:

(1) A simple inspection of per-unit parameters immediately reveals much more about the basic nature of a machine than may be observed from the ordinary parameters.

(2) The numerical range of per-unit parameters is small, in general being of the order of unity and less. This is valuable for solution by analogue or digital computers, since the variables are of a convenient order. Manual calculations are also simplified.

(3) Simplification occurs in the analysis of polyphase circuits under balanced conditions. By defining appropriate per-unit line quantities to correspond with chosen per-unit phase quantities, both line and phase parameters can be represented in one per-unit analysis and one equivalent circuit.

(4) In single and polyphase analysis, the turns ratios of transformers (and the manner of internal connection in the polyphase case) are removed from the analysis.

(5) In the two-reaction theory of the synchronous machine, a per-unit system is useful in removing those arbitrary numerical factors which can appear in the ordinary equations, having values depending on the transformation used.

(6) A basic set of dimensionless parameters can help to prevent errors in converting performance characteristics between different systemsof units.

4.4.1 Base values

As mentioned before, direct-phase quantities are used in the analysis of the synchronous machine. On the other hand, most of the data available of synchronous machines are given in per-unit in the d-q-axis. For these reasons, it has been found beneficial to use the d-q-axis base values to normalize the equations in the direct-phase quantities and, thus, to be able to use directly the d- and q-axis per-unit data.

(1) Stator base values

It is a common practice to choose the rated armature current and the rated phase voltage to be the stator current and voltage base values respectively. As the components of the armature current and phase voltage in both the d- and q-axis are instantaneous values, it is rather preferred to use their maximum values than the root mean square values. If I_n and V_n are considered to be the rated current and the rated phase voltage respectively the base values for the different parameters are as follows:

(i) The base value for all stator voltages (a, b, c, d, q) is the amplitude of the stator phase voltage. Hence:

$$V_{sb} = \sqrt{2}\, V_n \tag{4.22}$$

(ii) The base value for all stator currents (a, b, c, d, q) is the amplitude of the stator phase current. Hence:

$$I_{sb} = \sqrt{2}\, I_n \tag{4.23}$$

(iii) The base value of the power is:

$$P_{sb} = 3V_n I_n = \frac{3}{2} V_{sb}\, I_{sb} \tag{4.24}$$

This is the base for all active and reactive power and for the KVA.

(iv) The base value for the stator impedance is:

$$Z_b = \frac{V_n}{I_n} \tag{4.25}$$

(v) The base value for all the stator inductances is:

$$L_b = \frac{V_n}{2\pi f I_n} \tag{4.26}$$

(vi) The base value for all stator flux linkages is:

$$\psi_b = \frac{\sqrt{2}\, V_n}{2\pi f} \tag{4.27}$$

(2) Rotor base values

In order to define the base values of the rotor, the following points have to be considered.

(a) Power equality constraint

The power base of any rotor circuit is taken equal to the power base of the stator. Thus, equal reciprocal coupling between the stator and the rotor windings in the d-q representation can be achieved. Therefore,

$$I_{rb}V_{rb} = \frac{3}{2} I_{sb}V_{sb} = 3V_nI_n$$

or

$$P_{sb} = P_{fdb} = P_{kdb} = P_{kqb} \tag{4.28}$$

(b) Inductance relations

Before choosing the base current for each rotor circuit, it would be helpful to know the expressions of all machine mutual inductances in terms of the permeance and the number of turns. It is assumed that the mutual flux produced in one axis links equally all the circuits on this axis. This assumption is referred to as perfect mutual coupling. The following relations can then be written:

$$\left.\begin{aligned}
L_{ad} &= \frac{3}{2} \lambda_d N_s^2 \\
L_{aq} &= \frac{3}{2} \lambda_q N_s^2 \\
L_{afdo} &= \lambda_d N_s N_{fd} \\
L_{akdo} &= \lambda_d N_s N_{kd} \\
L_{akqo} &= \lambda_q N_s N_{kq} \\
L_{fdkd} &= \lambda_d N_{fd} N_{kd}
\end{aligned}\right\} \quad 4.29$$

(c) Rotor base currents

The choice of the base current for any rotor circuit is a problem which has been subject to several discussions. In general, such a choice can be made in an infinite number of ways. It has been found that it is more convenient to choose certain base values rather than the others. The chosen values were preferred on the basis of providing a representation which displays the physical picture of the machine and results in simplified equivalent circuits. Two of the most conveneint choices have resulted in the following per-unit systems [45]:

a. X_{ad} base system

b. equal mutual base system

It is worthwhile to mention that both systems are identical for the case in which the coupling between the machine circuits on each axis is perfect.

If the machine can be looked at as a multiwinding transformer on each axis, suitable rotor base quantities can be obtained by choosing the ideal turns ratio* between two windings to be the ratio between their base currents. Hence, the base rotor currents can be expressed as follows:

$$\begin{bmatrix} I_{fdb} \\ I_{kdb} \\ I_{kqb} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} N_s \end{bmatrix} \begin{bmatrix} 1/N_{fd} \\ 1/N_{kd} \\ 1/N_{kq} \end{bmatrix} \begin{bmatrix} I_{sb} \end{bmatrix} \qquad 4.30$$

---

*An ideal turns ratio between two windings is defined as follows:

$$\text{Ideal turns ratio} = \frac{\text{Total flux linkages of mutual flux with one winding}}{\text{Total flux linkages of mutual flux with the other winding}}$$

Substituting Eqn. 4.29 in Eqn. 4.30, it follows that the rotor base currents can be written in terms of the machine inductances as follows:

$$\begin{bmatrix} I_{fdb} \\ I_{kdb} \\ I_{kqb} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}\frac{N_s}{N_{fd}}\frac{N_s}{N_s}\frac{\lambda_d}{\lambda_d} \\ \frac{3}{2}\frac{N_s}{N_{kd}}\frac{N_s}{N_s}\frac{\lambda_d}{\lambda_d} \\ \frac{3}{2}\frac{N_s}{N_{kq}}\frac{N_s}{N_s}\frac{\lambda_q}{\lambda_q} \end{bmatrix} \begin{bmatrix} I_{sb} \end{bmatrix} = \begin{bmatrix} L_{ad}/L_{afdo} \\ L_{ad}/L_{akdo} \\ L_{aq}/L_{akqo} \end{bmatrix} \begin{bmatrix} I_{sb} \end{bmatrix} \qquad 4.31$$

From the power and current bases, the corresponding bases for voltage, impedance, inductance and flux linkage can be obtained as follows:

(i) The amplitude of the rotor base voltages are represented by

$$V_{rb} = \frac{3V_n I_n}{I_{rb}} \qquad 4.32$$

I_{rb} has different values for different rotor circuits:

(a) The base voltage for the field circuit is

$$V_{fdb} = \frac{3V_n I_n}{I_{fdb}} = \frac{3V_n I_n}{\sqrt{2} I_n \frac{L_{ad}}{L_{afdo}}} = \frac{3}{\sqrt{2}} V_n \frac{L_{afdo}}{L_{ad}} \qquad 4.33$$

(b) The base voltage for the d-axis damper winding is:

$$V_{kdb} = \frac{3V_n I_n}{I_{kdb}} = \frac{3}{\sqrt{2}} V_n \frac{L_{akdo}}{L_{ad}} \qquad 4.34$$

(c) The base voltage for the q-axis damper winding is:

$$V_{kqb} = \frac{3V_n I_n}{I_{kqb}} = \frac{3}{\sqrt{2}} V_n \frac{L_{akqo}}{L_{aq}} \qquad 4.35$$

(ii) The base value for the rotor flux linkage is:

$$\psi_{rb} = \frac{V_{rb}}{2\pi f} \qquad 4.36$$

This value differs from one circuit on the rotor to another as follows:

(a) The base value of the flux linkage for the field winding is:

$$\psi_{fdb} = \frac{V_{fdb}}{2\pi f} = \frac{3}{\sqrt{2}} \frac{1}{2\pi f} \frac{L_{afdo}}{L_{ad}} \qquad 4.37$$

(b) The base value of the flux linkage for the d-axis damper winding is:

$$\psi_{kdb} = \frac{V_{kdb}}{2\pi f} = \frac{3}{\sqrt{2}} \frac{1}{2\pi f} \frac{L_{akdo}}{L_{ad}} \qquad 4.38$$

(c) The base value of the flux linkage for the q-axis damper winding is:

$$\psi_{kqb} = \frac{V_{kqb}}{2\pi f} = \frac{3}{\sqrt{2}} \frac{1}{2\pi f} \frac{L_{akqo}}{L_{aq}} \qquad 4.39$$

(iii) The rotor base impedances are given by:

$$Z_{rb} = \frac{V_{rb}}{I_{rb}} \qquad 4.40$$

Its value also differs from one circuit on the rotor to another:

(a) The base imepdance of the field winding is:

$$Z_{fdb} = \frac{V_{fdb}}{I_{fdb}} = \frac{3V_n I_n}{I_{fdb}^2} = \frac{3V_n I_n}{\left(\frac{L_{ad}}{L_{afdo}}\right)^2 (\sqrt{2}\, I_n)^2} = \frac{3}{2} \frac{V_n}{I_n} \left(\frac{L_{afdo}}{L_{ad}} \right)^2 \qquad 4.41$$

(b) The base impedance of the d-axis damper winding is:

$$Z_{kdb} = \frac{V_{kdb}}{I_{kdb}} = \frac{3}{2} \quad \frac{V_n}{I_n} \left(\frac{L_{akdo}}{L_{ad}} \right)^2 \qquad 4.42$$

(c) The base impedance of the q-axis damper winding is:

$$Z_{kqb} = \frac{V_{kqb}}{I_{kqb}} = \frac{3}{2} \quad \frac{V_n}{I_n} \left(\frac{L_{akqo}}{L_{aq}} \right)^2 \qquad 4.43$$

(d) The base impedance of the mutual impedance between the field winding and the d-axis damper winding is:

$$Z_{fdkdb} = \frac{V_{fdb}}{I_{kdb}} = \frac{3}{2} \frac{V_n}{I_n} \left(\frac{L_{akdo} L_{afdo}}{L_{ad}^2} \right) \qquad 4.44$$

(iv) The rotor base inductance is:

$$L_{rb} = \frac{Z_{rb}}{2\pi f} \qquad 4.45$$

(a) The base inductance for the field winding is:

$$L_{fdb} = \frac{3}{2} \frac{1}{2\pi f} \left(\frac{L_{afdo}}{L_{ad}} \right)^2 \frac{V_n}{I_n} \qquad 4.46$$

(b) The base inductance for the d-axis damper winding is:

$$L_{kdb} = \frac{3}{2} \frac{1}{2\pi f} \left(\frac{L_{akdo}}{L_{ad}} \right)^2 \frac{V_n}{I_n} \qquad 4.47$$

(c) The base inductance for the q-axis damper winding is:

$$L_{kqb} = \frac{3}{2} \frac{1}{2\pi f} \left(\frac{L_{akqo}}{L_{aq}} \right)^2 \frac{V_n}{I_n} \qquad 4.48$$

(d) The base mutual inductance between the field winding and the d-axis damper winding is:

$$L_{fdkdb} = \frac{3}{2} \frac{1}{2\pi f} \left(\frac{L_{akdo} L_{afdo}}{L_{ad}^2} \right) \frac{V_n}{I_n} \qquad 4.49$$

(3) Time and speed bases

The normalized equations of the synchronous machines are further simplified if the electrical angular velocity $p\theta$ is also normalized. The synchronous electrical angular velocity $p\theta_o$ is conveniently chosen as the base value. Since $p\theta_o$'t is a dimensionless quantity, the selection of $p\theta_o$ as the base of $p\theta$ is equivalent to selecting $1/p\theta_o$ as a base time.

(4) Torque base

When writing the torque equation in M.K.S. system, it is inevitable that the number of pole pairs n_p appears. It is desirable in forming the per-unit equations of the machine to remove this parameter because it is not fundamental to the performance of the machine. As a consequence, the form of the per-unit rotor angular velocity is simplified, becoming the same whether expressed in mechanical or electrical form. This is done by defining the base mechanical angular velocity $p\theta_{mb}$ as that corresponding to the base electrical velocity. Thus

$$p\theta_{mb} = p\theta_o/n_p \qquad 4.50$$

Using the expressions for the base power and the base mechanical speed given by Eqns. 4.24 and 4.50 respectively, the base torque will be

$$T_{Eb} = \frac{3}{2}\,\frac{V_{sb}I_{sb}}{p\theta_o}\,n_p = \frac{3}{2}\,\psi_{sb}I_{sb}n_p \qquad 4.51$$

4.4.2 Normalized equations of synchronous machines

Having established the per-unit system, the normalized equations of synchronous machines in both the direct-phase and the d-q representations are obtained. In the following equations and hereafter, all the parameters are in per-unit values.

a) Normalized equations in the d-q representation

(1) Flux linkage equations

The normalized flux linkage equations can be arranged as follows:

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_{fd} \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix} = \begin{bmatrix} -L_d & 0 & L_{ad} & L_{ad} & 0 \\ 0 & -L_q & 0 & 0 & L_{aq} \\ -L_{ad} & 0 & L_{ffd} & L_{ad} & 0 \\ -L_{ad} & 0 & L_{ad} & L_{kkd} & 0 \\ 0 & -L_{aq} & 0 & 0 & L_{kkq} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix} \qquad 4.52$$

where

$$L_d = L_\ell + L_{ad} = L_{aao} + L_{abo} + \frac{3}{2} L_{aa2} \qquad 4.53$$

$$L_q = L_\ell + L_{aq} = L_{aao} + L_{abo} - \frac{3}{2} L_{aa2} \qquad 4.54$$

$$L_\ell = L_{aao} - 2L_{abo} \qquad 4.55$$

$$L_{ffd} = L_{fd} + L_{ad} \qquad 4.56$$

$$L_{kkd} = L_{kd} + L_{ad} \qquad 4.57$$

$$L_{kkq} = L_{kq} + L_{ad} \qquad 4.58$$

(2) Voltage equations

The normalized voltage equations in matrix form can be written as:

$$\begin{bmatrix} v_d \\ v_q \\ v_{fd} \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} p & -p\theta & 0 & 0 & 0 \\ p\theta & p & 0 & 0 & 0 \\ 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 & p \end{bmatrix} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_{fd} \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix} - \begin{bmatrix} R_a & 0 & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 & 0 \\ 0 & 0 & -R_{fd} & 0 & 0 \\ 0 & 0 & 0 & -R_{kd} & 0 \\ 0 & 0 & 0 & 0 & -R_{kq} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix} \qquad 4.59$$

(3) Torque equations

The developed electrical torque in per-unit values can be written in the following form

$$T_E = i_q \cdot \psi_d - i_d \cdot \psi_q \qquad 4.60$$

Also, the normalized equation of motion will be:

$$p^2\theta = (T_M - T_E)/H^* = p^2\delta \qquad 4.61$$

where, $\delta = \theta - p\theta_o \cdot t$

(b) Normalized equations in the direct-phase quantities

(1) Flux linkage equations

Using the base values mentioned before, the flux linkage equations (Eqn. 4.1) can be normalized to have the form:

$$[\psi] = [L][I] \qquad 4.62$$

where

$$[\psi] = [\psi_a \quad \psi_b \quad \psi_c \quad \psi_{fd} \quad \psi_{kd} \quad \psi_{kq}]^t$$

$$[I] = [i_a \quad i_b \quad i_c \quad i_{fd} \quad i_{kd} \quad i_{kq}]^t$$

---

* $H = 2.\ p\theta_o \cdot H$

where

$$H = \frac{1}{2} \frac{J \cdot (p\theta_{mb})^2}{\frac{3}{2} \cdot V_{sb} \cdot I_{sb}}$$

$$[L] = \begin{bmatrix} -L_{aao} - L_{aa2}\cos 2\theta & L_{abo} + L_{aa2}\cos 2(\theta+30) & L_{abo} + L_{aa2}\cos 2(\theta+150) & L_{ad}\cos\theta & L_{ad}\cos\theta & -L_{aq}\sin\theta \\ L_{abo} + L_{aa2}\cos 2(\theta+30) & -L_{aao} - L_{aa2}\cos 2(\theta-120) & L_{abo} + L_{aa2}\cos 2(\theta-90) & L_{ad}\cos(\theta-120) & L_{ad}\cos(\theta-120) & -L_{aq}\sin(\theta-120) \\ L_{abo} + L_{aa2}\cos 2(\theta+150) & L_{abo} + L_{aa2}\cos 2(\theta-90) & -L_{aao} - L_{aa2}\cos 2(\theta-240) & L_{ad}\cos(\theta-240) & L_{ad}\cos(\theta-240) & -L_{aq}\sin(\theta-240) \\ -\frac{2}{3} L_{ad}\cos\theta & -\frac{2}{3} L_{ad}\cos(\theta-120) & -\frac{2}{3} L_{ad}\cos(\theta-240) & L_{ffd} & L_{ad} & 0 \\ -\frac{2}{3} L_{ad}\cos\theta & -\frac{2}{3} L_{ad}\cos(\theta-120) & -\frac{2}{3} L_{ad}\cos(\theta-240) & L_{ad} & L_{kkd} & 0 \\ \frac{2}{3} L_{aq}\sin\theta & \frac{2}{3} L_{aq}\sin(\theta-120) & \frac{2}{3} L_{aq}\sin(\theta-240) & 0 & 0 & L_{kkq} \end{bmatrix} \quad 4.63$$

(2) Voltage equation

The normalized voltage equations in the matrix notation are

$$[V] = p[\psi] - [R][I] \qquad 4.64$$

where

$$[V] = [v_1 \quad v_2 \quad v_3 \quad v_{fd} \quad v_{kd} \quad v_{kq}]^t$$

$$[R] = \text{diag}\ [R_a \quad R_a \quad R_a \ - R_{fd} \ - R_{kd} \ - R_{kq}]$$

(3) Torque equations

The electrical torque equation in the normalized form is:

$$T_E = \frac{2}{3\sqrt{3}} \{\psi_a(i_b - i_c) + \psi_b(i_c - i_a) + \psi_c(i_a - i_b)\} \qquad 4.65$$

Also the normalized equation of motion is:

$$p^2\delta = p^2\theta = (T_M - T_E)/H \qquad 4.66$$

By comparing Eqns. 4.52 and 4.63, it is clear that the per-unit values of the machine inductances for the direct-phase representation are equal or correspond to those in the d-q-axis representation. While the rotor inductances for the two models are the same, the mutual-inductances between the stator and the rotor in the 3-phase model are expressed in terms of those of the d-q model (L_{ad} and L_{aq}). Also, the stator inductances in the 3-phase model are related to the corresponding inductances in the d-q model. They can be easily obtained from Eqns. 4.53, 4.54 and 4.55 as follows:

$$L_{aao} = \frac{1}{3}(L_\ell + L_d + L_q)$$

$$L_{aa2} = \frac{1}{3}(L_d - L_q)$$

$$L_{abo} = \frac{1}{6} (L_d + L_q - 2L_\ell) \qquad 4.67$$

Thus, the per-unit data of the machine in the d-q-axis, which are usually available, can be used directly in forming the normalized equations in the direct-phase quantities.

5. DIGITAL SIMULATION OF AN AC/DC SYSTEM

5.1 Introduction

In this chapter, a representative AC/DC system has been used to study the effects of the generated harmonics of the HVDC transmission on the AC system in general and on the performance of synchronous machines in particular. This system consists of an AC/DC converter connected to a synchronous generator at its terminals as well as to an infinite bus-bar through a short transmission line as shown in Fig. 5.1. A group of 5th, 7th and 11th order harmonic filters is also connected at the converter AC bus-bar.

To perform the various studies concerning the effects of the generated harmonics, this system has been digitally simulated using a generalized computer program. In this program, each part of the system has been represented by a set of first order differential equations (chapters 3 and 4). Thus, Eqns. 3.26-3.37, 4.62, 4.64 and 4.66 form the complete model of the system in direct-phase quantities. The program uses the 4th order Runge-Kutta method to solve these differential equations. This program is prepared to deal with the whole system or with any part of it, and a point by point solution for the instantaneous values of the different variables of the system is obtained. In all the cases considered, the rectifier and the filters are assumed to be initially disconnected from the system.

5.2 Method of Computation

The basic algorithm for the simulation of the 3-phase model on the digital computer is as follows:

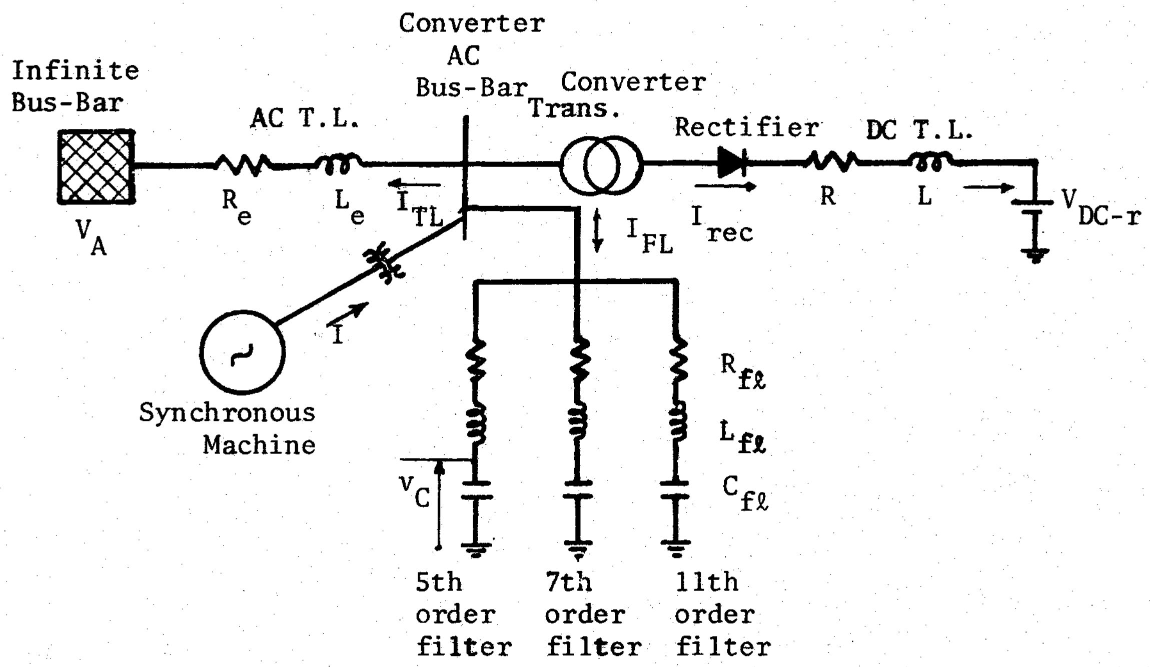


Fig. 5.1 Diagram of a Representative AC/DC System

(i) For a certain loading condition, the initial values of the various variables of the system and their derivatives are obtained.

(ii) At zero time, the rectifier and the group of filters are suddenly connected to the system.

(iii) The initial voltages, voltage derivatives, currents, current derivatives and rotor position and speed of the machine are used to start the computation.

(iv) The derivative vectors of the different variables are computed at the beginning of the nth step from the knowledge of the different variables at the end of the (n-1)th step using the following equations:

$$p[I_{rec}] = F(V,\ I_{rec},\ V_{DC-r}) \tag{5.1}$$

$$p[I_{FL}] = [\frac{1}{L_{FL}}][V] - [\frac{1}{L_{FL}}][V_C] - [\frac{R_{FL}}{L_{FL}}][I_{FL}] \tag{5.2}$$

$$p[V_C] = [\frac{1}{C_{FL}}][I_{FL}] \tag{5.3}$$

$$p[\psi] = [V] + [R][I] \tag{5.4}$$

$$p^2\delta = p^2\theta = (T_M - T_E)/ H \tag{5.5}$$

(v) Integrating these derivative vectors using the 4th order Runge-Kutta method, the rectifier currents, the filter currents, the voltages across the filter capacitors, the flux linkages of the synchronous machine, its rotor position and its rotor speed are obtained at the end of the nth step.

(vi) The rotor angle θ computed at the end of a step is used in forming the inductance matrix [L] of the synchronous machine (Eqn. 4.63).

(vii) At the end of the nth step, the machine currents and the DC voltage at the rectifier DC terminals are calculated respectively from the following two equations:

$$[I] = [L]^{-1}[\psi] \qquad 5.6$$

$$v_{DC-s} = v_{DC-r} + Ri_{DC} + Lpi_{DC} \qquad 5.7$$

Also, the transmission line current at the end of the step can be calculated from:

$$[I_T] = [I'] - [I_{rec}] - [I_{FL}] \qquad 5.8$$

where

[I'] is a vector which consists of the first three terms of the vector [I].

(viii) Making use of the currents and the voltages computed in the preceeding steps, the machine current derivative vector p[I] is obtained from the following equation:

$$p[I] = [L_m]^{-1}[V_{AR}] + [L_m]^{-1}[R_m][I] - [L_m]^{-1}[R_E][I_{rec} + I_{FL}]$$
$$-[L_m]^{-1}[L_E]p[I_{rec} + I_{FL}] - [L_m]^{-1}[pL][I] \qquad 5.9$$

The derivation of this equation is given in appendix B.

(ix) The transmission line current derivative vector $p[I_T]$ can then be obtained from:

$$p[I_T] = p[I'] - p[I_{rec}] - p[I_{FL}] \qquad 5.10$$

(x) Steps (vi)-(ix) are performed only once at the end of each step, and not four times as required by the rigorous Runge-Kutta method.

(xi) The currents and their derivatives computed above are utilized to calculate the converter AC bus-bar voltages given by:

$$[V] = [V_A] + [R_E][I_T] + [L_E]p[I_T] \qquad 5.11$$

(xii) If any part of the system does not exist, its currents and its current derivatives are replaced by zero.

(xiii) The new derivative vectors are computed as in step (iv) and used to start the (n + 1)th step of the computation.

(xiv) The calculations are thus advanced in a single-step fashion.

A simplified flow chart of the program is given in appendix C.

5.3 Data of the System under Study

Using the developed program, investigations are carried out for various operating conditions of the system shown in Fig. 5.1. The data of the AC/DC rectifier station, the filters and the synchronous machine are taken as those of the Nelson River at Radisson Station [37, 46]. The study is carried out for one valve group only. As this valve group represents one third of the total station, only four synchronous machines from the total twelve of the station are taken into consideration. Also, equivalent filters with one third of the rating of the station filters and with the same quality factor "Q" are used. The transmission line connecting the converter bus-bar with the infinite bus-bar is assumed to be 10 miles long. The inverter station is represented by a constant DC voltage source.

Using the relations given in appendix D, the base values for the different parameters of the AC/DC system will be as follows:

The base MVA = 250 MVA

The base AC voltage(instantaneous value) = 112.68 kV

The base AC impedance = 76 Ω

The base AC current(instantaneous value) = 1.48 kA

The base AC inductance = 0.202 H

The base AC capacitance = 0.35×10^{-4} F

The base DC voltage(instantaneous value) = 112.68 kV

The base DC current(instantaneous value) = 1.48 kA

The base DC resistance = 76 Ω

The base DC inductance = 0.202 H

As it will be mentioned later, a step up transformer(13.8kV/138kV) is connecting the synchronous machine to the 138kV AC bus-bar. Therefore, the base values of the stator parameters of the synchronous machine are as follows:

Base voltage(instantaneous value) = 11.268 kV

Base current(instantaneous value) = 14.8 kA

Base impedance = 0.76 Ω

Base inductance = 0.00202 H

Utilising the previous base values the data of the different parts of the system in absolute and p.u. values is as follows:

(1) The synchronous machines at Radisson station

(a) The machine data in the d-q axis:

Data	Individual Machine (13.8kV,120MVA rated)		Equivalent Machine (13.8kV,480MVA rated)	
	Actual value	P.U. value	Actual value	P.U. value
Volt-ampere rating	120 MVA	0.48	480 MVA	1.92
Peak value of the rated phase voltage	11.268 kV	1	11.268 kV	1

Rated power factor	0.85			0.85		
Rated reactive power	63.38	Mvar	0.2534	253.44	Mvar	1.0137
Rated power	102	MW	0.408	408		1.632
Armature resistance R_a	0.0058	Ω	0.00766	0.00145	Ω	0.001915
Field resistance R_{fd}	0.00177	Ω	0.00232	0.0004425	Ω	0.00058
Direct-axis damper winding resistance R_{kd}	0.0627	Ω	0.0823	0.015675	Ω	0.0206
Quadrature-axis damper winding resistance R_{kq}	0.0628	Ω	0.0825	0.0157	Ω	0.020625
Direct-axis magnetizing reactance x_{ad}	1.58	Ω	2.083	0.395	Ω	0.52075
Quadrature-axis magnetizing reactance x_{aq}	0.85	Ω	1.12	0.2125	Ω	0.28
Armature leakage reactance x_{ℓ}	0.168	Ω	0.2205	0.042	Ω	0.055125
Field leakage reactance x_{fd}	0.97	Ω	1.28	0.2425	Ω	0.32
Direct-axis damper winding leakage reactance x_{kd}	0.317	Ω	0.419	0.07925	Ω	0.10475
Quadrature-axis damper winding leakage reactance x_{kq}	0.288	Ω	0.38	0.072	Ω	0.095
Direct-axis synchronous reactance x_d	1.75	Ω	2.3	0.04375	Ω	0.575
Direct-axis transient reactance x'_d	0.475	Ω	0.626	0.11875	Ω	0.1565
Direct-axis subtransient reactance x''_d	0.369	Ω	0.45	0.09225	Ω	0.12125
Quadrature-axis synchronous reactance x_q	1.02	Ω	1.34	0.255	Ω	0.335
Quadrature-axis subtransient reactance x''_q	0.383	Ω	0.504	0.09575	Ω	0.126
Direct-axis transient open-circuit time constant T'_{do}	4.1	s	1540	4.1	s	1540
Direct-axis short-circuit time constant T'_d	1.123	s	424	1.123	s	424

Direct-axis open-circuit time constant T''_{do}	0.019 s	7.15	0.019 s	7.15
Direct-axis subtransient short-circuit time constant T''_{qo}	0.048 s	18.1	0.048 s	18.1
Armature short-circuit time constant T_a	0.172 s	64.8	0.172 s	64.8
Inertia constant of the machine H	3.42 s		3.42 s	
Inertia constant of the machine H		2578.68		10314.72
Speed	90 rpm		90 rpm	

The per-unit values of the reactances in the above table represent also the per-unit values of the corresponding inductances as they are equivalent in the per-unit system.

(b) The machine parameters in per-unit in the direct-phase representation

Data	Individual Machine	Equivalent Machine
L_{aao}	1.2868	0.3217
L_{abo}	0.533	0.13325
L_{aa2}	0.32	0.08
$L_{afdo} = L_{akdo} = L_{fdkd} = L_{ad}$	2.083	0.521
$L_{akqo} = L_{aq}$	1.12	0.28
$L_{ffd} = L_{fd} + L_{ad}$	3.363	0.841
$L_{kkd} = L_{kd} + L_{ad}$	2.502	0.6255
$L_{kkq} = L_{kq} + L_{aq}$	1.5	0.375

(2) The step-up transformer connecting each individual machine to the converter AC bus-bar has an inductance of 0.17 p.u. The effect of this inductance can be included by adding it to L_{aao} of the machine. Therefore, the equivalent L_{aao} is equal to 0.3642 p.u.

(3) The converter transformer at Radisson(134kV/141.5-134.5kV)

MVA rating = 341 MVA = 1.364 p.u.

Peak value of the rated phase voltage = 109.41 kV = 0.971 p.u.

Equivalent resistance = 0.0 Ω = 0.0 p.u.

Equivalent inductance = 0.0278 H = 0.138 p.u.

(4) The group of filters(138 kV rated)

Data	5th Order Harmonic Filter		7th Order Harmonic Filter		11th Order Harmonic Filter	
	Actual Values	Per-Unit Values	Actual Values	Per-Unit Values	Actual Values	Per-Unit Values
MVAR rated	11.43 Mvar	0.04572	5.7 Mvar	0.0228	9.27 Mvar	0.03708
Peak value of the rated phase voltage	112.68kV	1	112.68kV	1	112.68kV	1
Resistance	3.48Ω	0.0458	4.86Ω	0.064	1.884Ω	0.0245
Inductance	184.2 mH	0.91	184.2 mH	0.91	45.6 mH	0.226
Capacitance	1.52 μF	0.0435	0.78 μF	0.0223	1.28 μF	0.0365

(5) The DC transmission line

Resistance R = 3.8 Ω = 0.05 p.u.

Inductance L = 0.274 H = 1.355 p.u.

The constant DC voltage representing the inverter station is v_{DC-r} = 1.3 p.u.

(6) The AC transmission line

For a 10 miles transmission line [47]

Resistance R_e = 0.7 Ω = 0.0091 p.u.

Inductance L_e = 0.016968 H = 0.084 p.u.

Capacitance $C_e = 17.185 \times 10^{-10}$ F $= 49.1 \times 10^{-6}$ p.u.

As the line is short, C_e is very small and can be neglected as discussed in appendix A.

5.4 Application of the Computer Program to Different Cases

The computer program is applied to different cases. In all the curves presented in this chapter, the first part represents the transient solution directly after the connection of the rectifier, while the last part gives the steady state solution.

Case 1: Rectifier connected directly to an infinite bus-bar (Fig. 5.2)

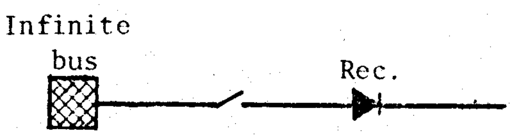


Fig. 5.2 Rectifier Connected to an Infinite Bus-Bar

In this case, the rectifier is suddenly connected to an infinite bus-bar at zero time. Fig. 5.3 shows the 3-phase rectifier currents versus the time. It can be noted that, at steady-state, the rectifier currents are not rectangular due to the effect of the commutating reactance. Also, the upper parts of the current waveforms are not leveled but contain dips. This can be explained by examining Fig. 5.4. For the period from b to c, valves 1 and 6 are conducting. If the inductance in the circuit is assumed to be very small, and if v_1 is the reference voltage ($v_1 = V_{max} \sin\omega t$), the current in phase 1 can be given approximately by:

$$i_1 = \frac{v_1 - v_2}{R_1 + R_2 + R} - \frac{v_{DC-r}}{R_1 + R_2 + R}$$

$$i_1 = \frac{\sqrt{3} V_{max}}{R_1 + R_2 + R} \sin(\omega t + 30) - \frac{v_{DC-r}}{R_1 + R_2 + R}$$

$$i_1 = I_{max1} \sin(\omega t + 30) - K_1 \qquad 5.12$$

For the period from c to d, valves 4 and 6 are conducting with valve 1 and the current in phase 1 is approximately:

$$i_1 = \frac{v_1 - \frac{(v_2 + v_3)}{2}}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R} - \frac{v_{DC-r}}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R}$$

$$i_1 = \frac{\frac{3}{2} V_{max} \sin\omega t}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R} - \frac{v_{DC-r}}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R}$$

$$i_1 = I_{max2} \sin \omega t - K_2 \qquad 5.13$$

From e to f, valves 1 and 4 are conducting and i_1 is given by:

$$i_1 = \frac{v_1 - v_3}{R_1 + R_3 + R} - \frac{v_{DC-r}}{R_1 + R_3 + R}$$

$$i_1 = \frac{\sqrt{3}\, V_{max} \sin(\omega t-30)}{R_1+R_3+R} - \frac{v_{DC-r}}{R_1+R_3+R}$$

$$i_1 = I_{max3} \sin(\omega t-30) - K_3 \qquad 5.14$$

If R_1, R_2 and R_3 are equal, I_{max3} will be equal to I_{max1} and K_3 will be equal to K_1. From Eqns. 5.12, 5.13 and 5.14, it can be noted that the current waveform from b to f follows three different sinusoidal waves. Also, the sudden transfer from d to e is due to the termination of the commutation period. If, however, the inductance in the circuit is considered, the current curve will be smoothed and this sudden change will disappear as given by the computer results. When this rectifier current is analysed, it is found that it contains fundamental current and harmonic frequency components of the order 6k±1, where k is an integer.

Fig. 5.5 shows the DC output current and the DC output voltage versus the time. Both consist of a DC component on which a number of harmonics of the order 6k are superimposed.

By comparing these results with the theoretical analysis of chapter 2, it is found that the discrepancy between them is very small.

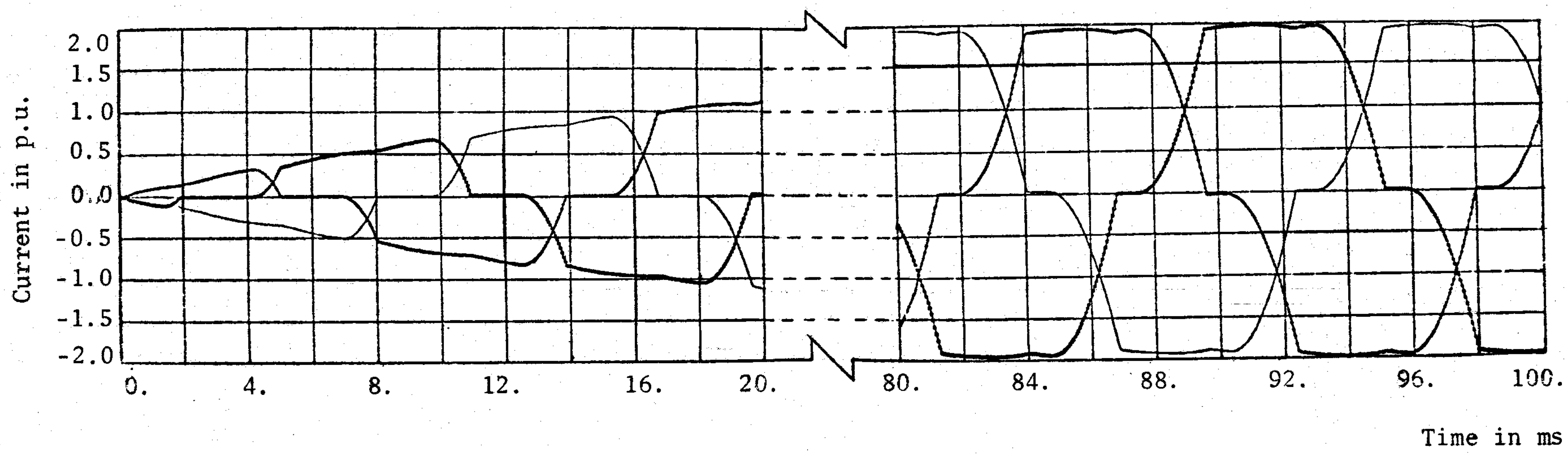
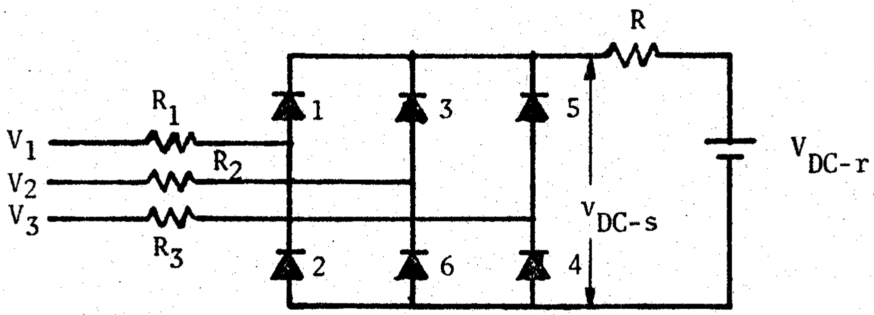
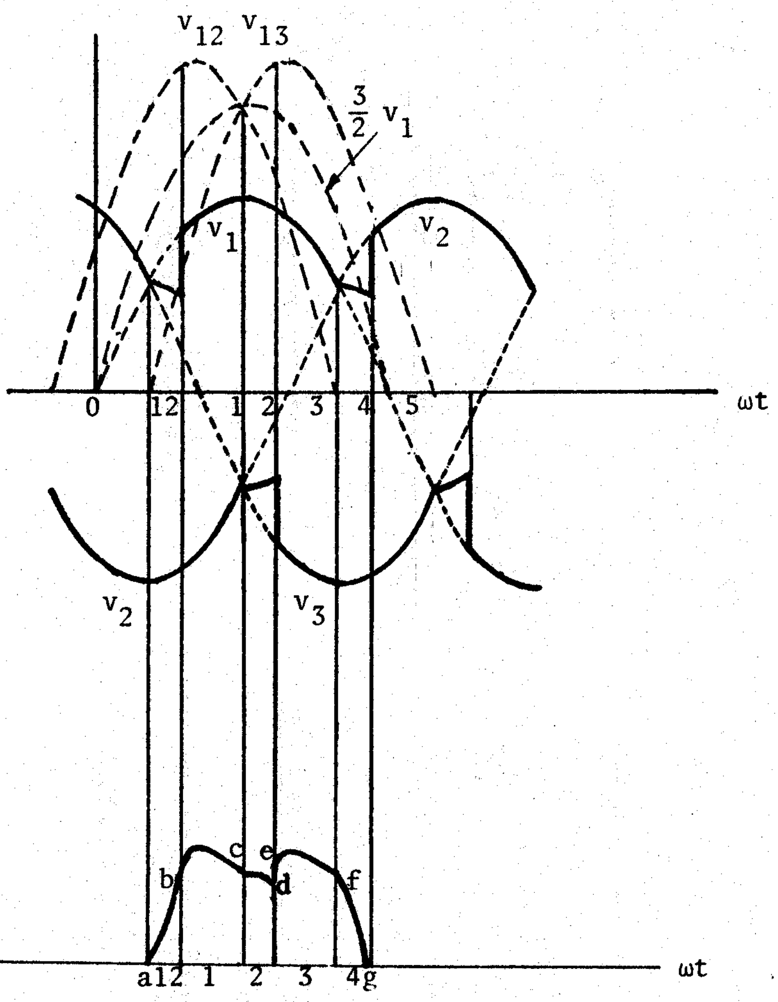


Fig. 5,3 The 3-phase Rectifier Currents Versus the time for the Case of Rectifier Only



(a)



(b)

Fig. 5.4 (a) Bridge Circuit
(b) Rectifier Current Waveform

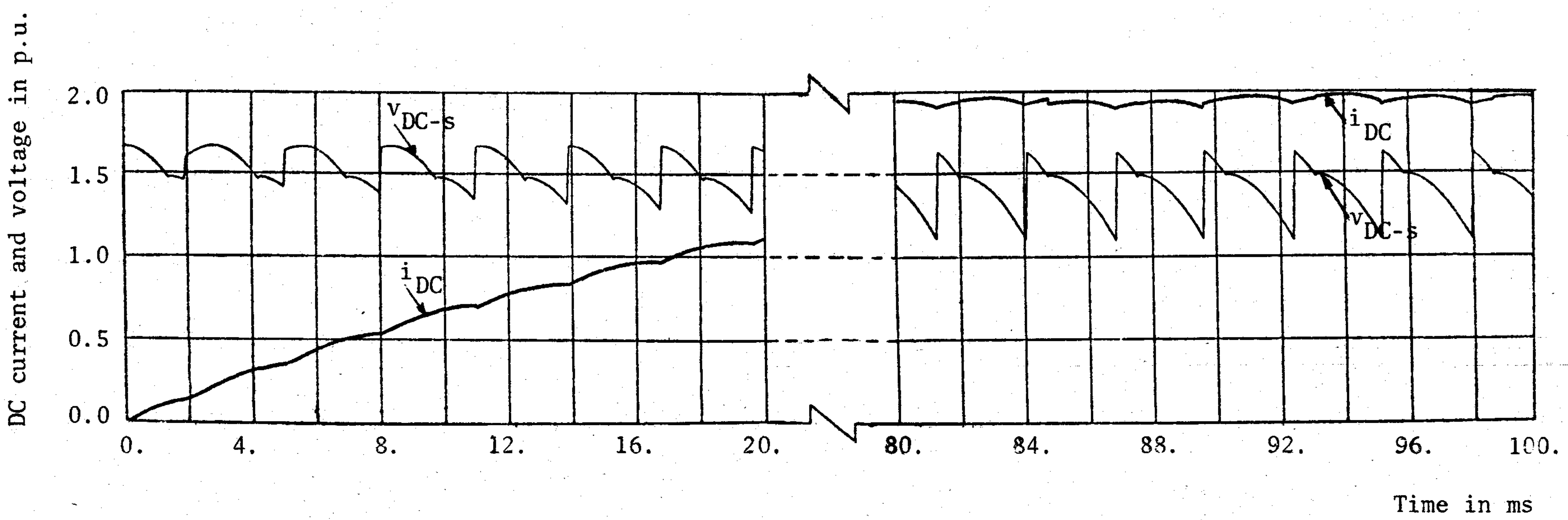


Fig. 5.5 The DC Output Current and Voltage Versus the Time for the Case of Rectifier Only

Case 2: Rectifier connected to an infinite bus-bar through a short transmission line (Fig. 5.6)

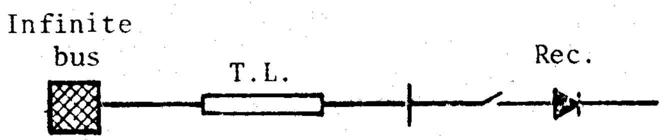


Fig. 5.6 Rectifier and Transmission Line Case

As shown in Fig. 5.7, the 3-phase rectifier currents have the same wave-shape as those of the previous case. As expected, the commutation period in this case has been increased due to the inductance of the transmission line.

The DC output voltage and current for this case are shown in Fig. 5.8.

Fig. 5.9 shows the 3-phase voltages at the converter AC terminals versus the time. These voltages are not sinusoidal due to the flow of the harmonic currents in the transmission line. The shown sudden jumps in the voltage wave-shape are due to the sudden change in the inductive voltage drop of the transmission line. This results from the sudden changes in the rate of change of the transmission line current at the end of the commutation periods. The equation of the voltage at the converter terminals is given by:

$$V = V_A - R_e i - L_e \frac{di}{dt} \qquad 5.15$$

Just before the beginning of the commutation process at point a, Fig. 5.10, both the current i and the current derivative $\frac{di}{dt}$ are zero and the voltage V will be the same as the voltage V_A. For the period between a and b, both the current i and the current derivative $\frac{di}{dt}$ increase gradually. Thus, the voltage V will differ from V_A due to the voltage drop in the transmission line resistance and inductance. At point b, the current i becomes constant and the current derivative $\frac{di}{dt}$ becomes suddenly zero. This sudden change in the current derivative results in a sudden change in the voltage V. For the period between b and c, the voltage V will be less than V_A by the resistive voltage drop in the transmission line. From c to d, the current i starts to decrease and the current derivative $\frac{di}{dt}$ is negative and decreasing. Therefore, the voltage V will differ from the voltage V_A due to the voltage drop in the transmission line resistance and inductance. At point d, the current is zero and the current derivative $\frac{di}{dt}$ changes suddenly to zero. Thus, from point d and after, the voltage V will follow V_A. These changes from a to d repeats itself twice every cycle.

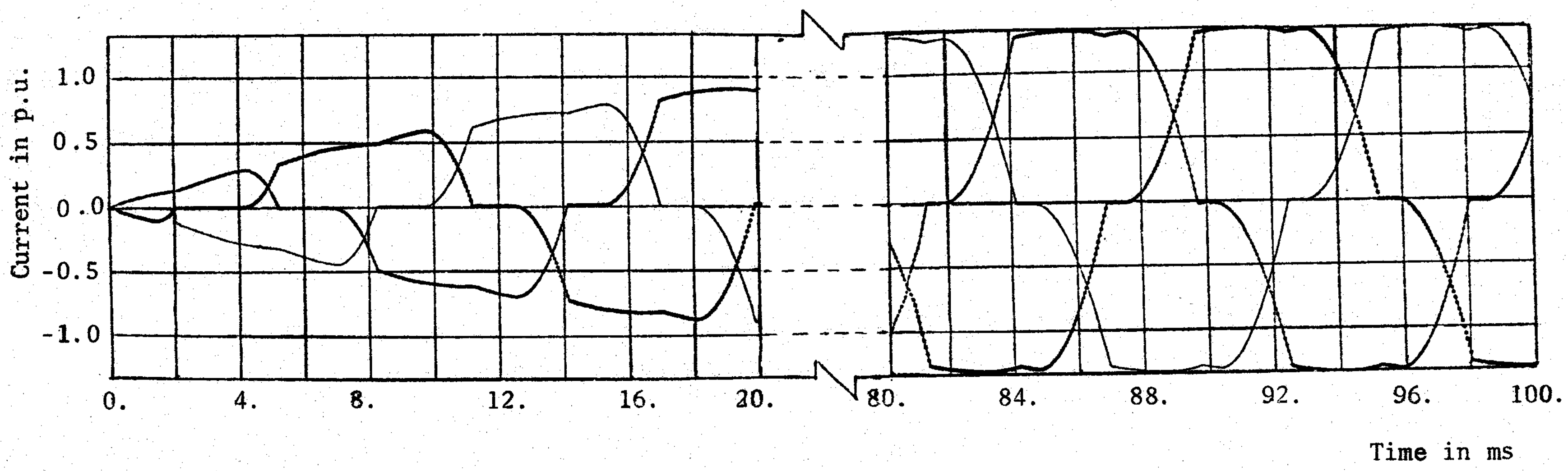


Fig. 5.7 The 3-phase Rectifier Currents Versus the Time for the Case of Rectifier and Transmission Line

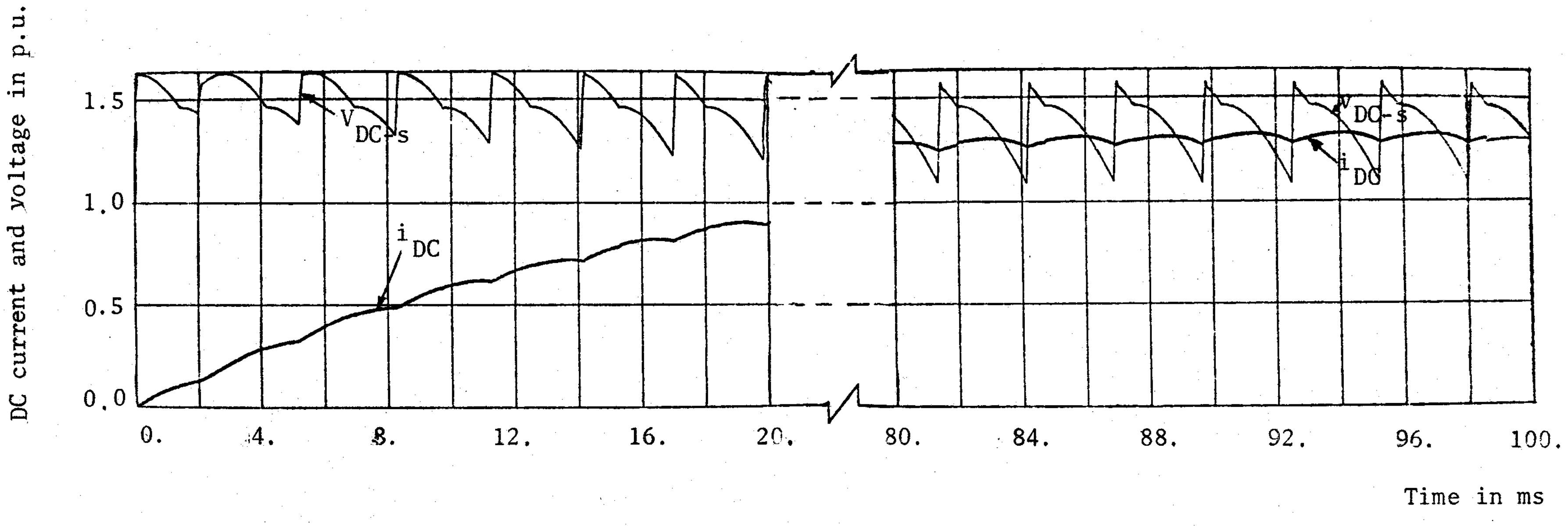


Fig. 5.8 The DC Output Current and Voltage Versus the Time for the Case of Recitifer and Transmission Line

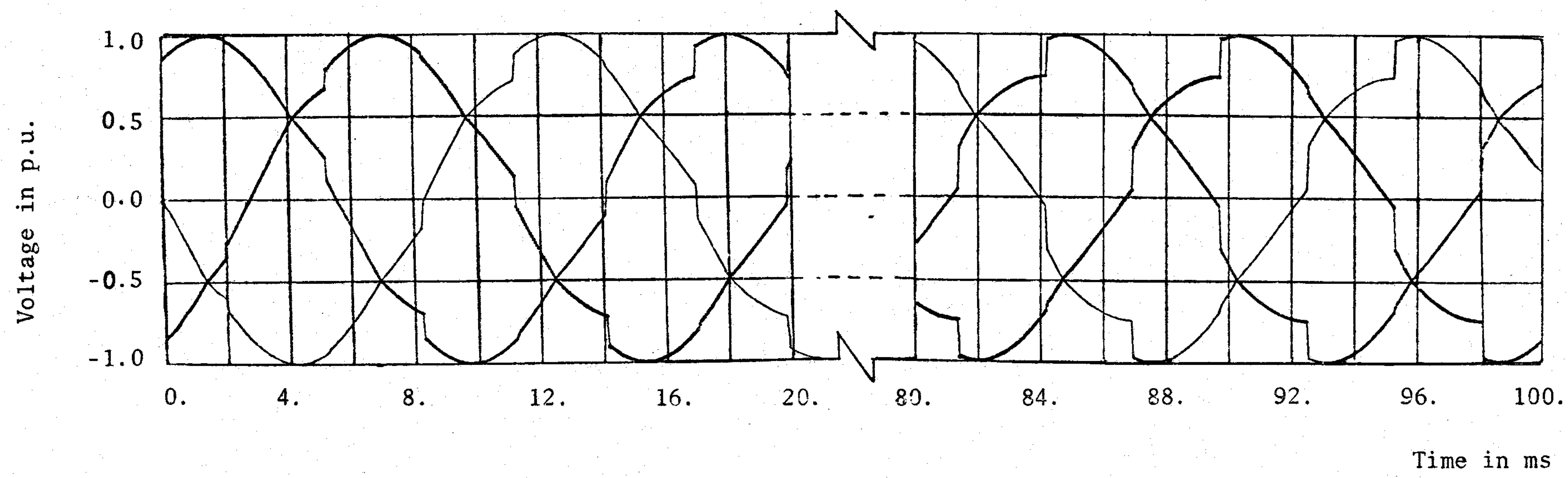


Fig. 5.9 The 3-phase Voltages at the AC Terminals of the Converter Versus the Time for the Case of Rectifier and Transmission Line

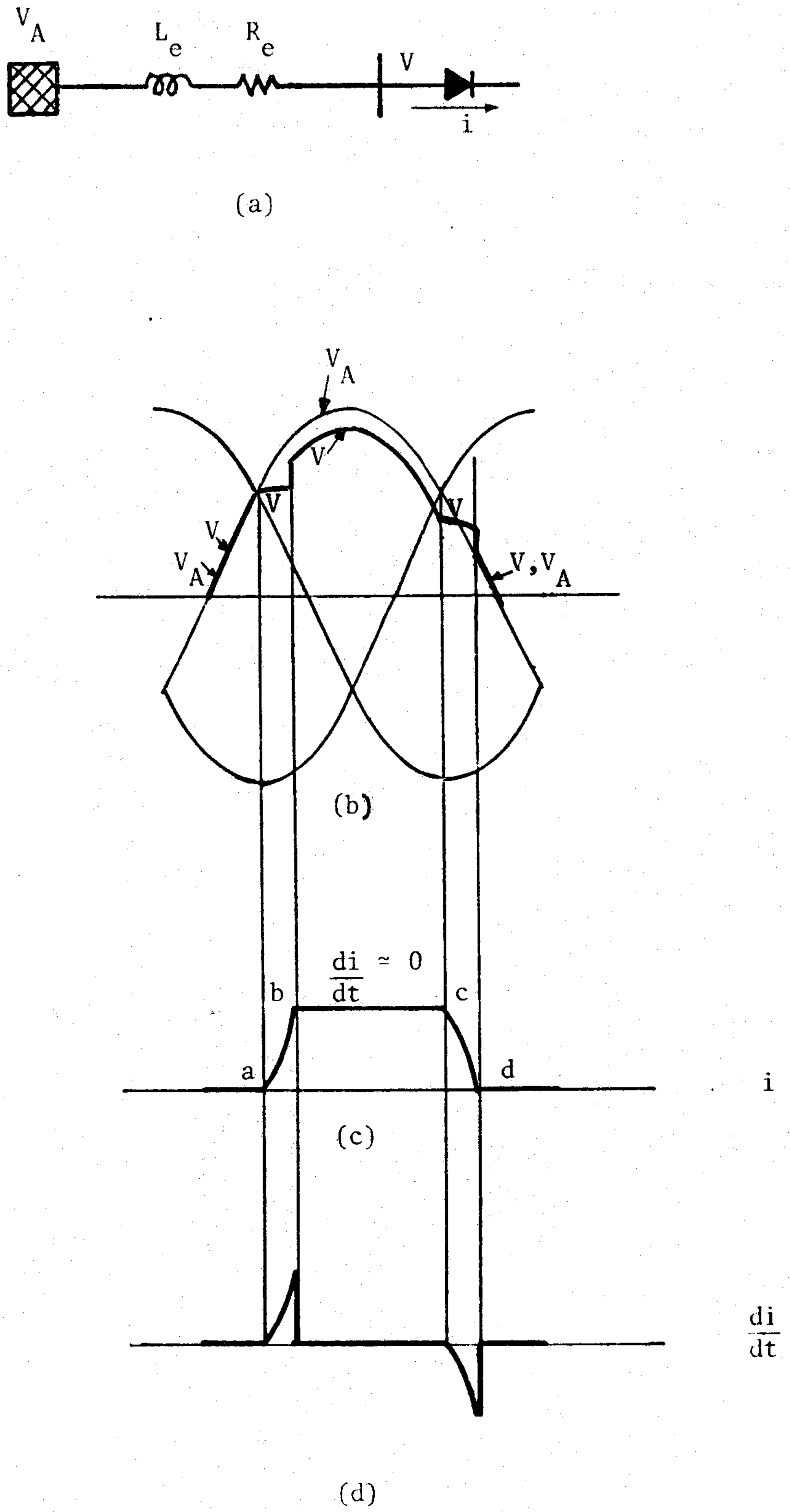


Fig. 5.10 (a) The Circuit Diagram (b) The Voltage Wave Form (c) The Transmission Line Current i (d) The Transmission Line Current Derivative $\frac{di}{dt}$

Case 3: Rectifier connected to an infinite bus-bar through a short transmission line with a group of filters connected to the rectifier AC terminals (Fig. 5.11)

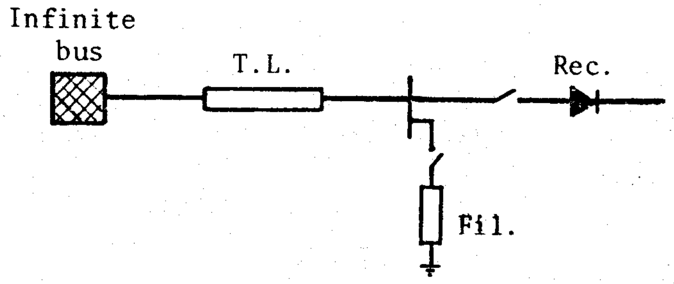


Fig. 5.11 Rectifier, Transmission Line and Filters Case

In this case, both the rectifier and the filters are suddenly connected at zero time to the infinite bus-bar through a short transmission line. The 3-phase rectifier currents, Fig. 5.12, have the same wave-shape as that of the previous cases. However, the periods of commutation in this case are less than that of case 2 due to the existence of the filters. The filters in the circuit moves the commutation point, at which the commutating e.m.f. is sinusoidal, towards the converter AC bus [2,3,46]. Therefore, the effective commutating reactance becomes approximately the converter transformer leakage reactance only. Fig. 5.13 shows the 3-phase transmission line currents which, in this case, have approximately a sinusoidal wave-shape. The most prominent harmonic currents generated by the rectifier, namely the 5th, the 7th and the 11th, have been diverted from the transmission line to pass in the corresponding filters connected at the terminals of the converter. Fig. 5.14 shows the 3-phase voltages at the

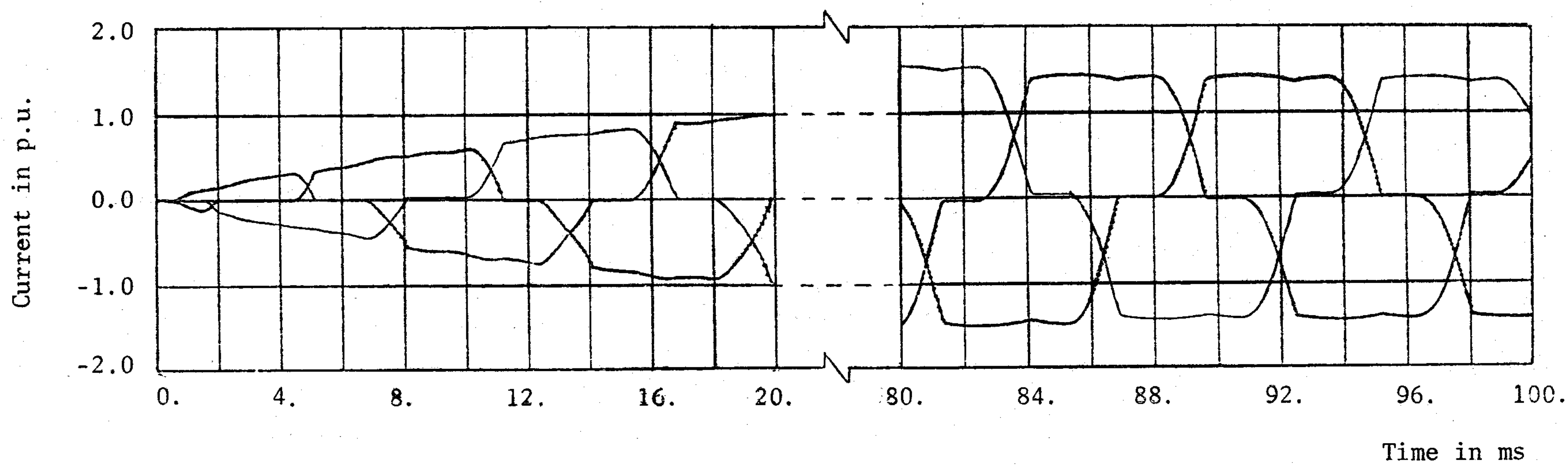


Fig. 5.12 The 3-phase Rectifier Currents Versus the Time for the Case of Rectifier, Transmission Line and Filters

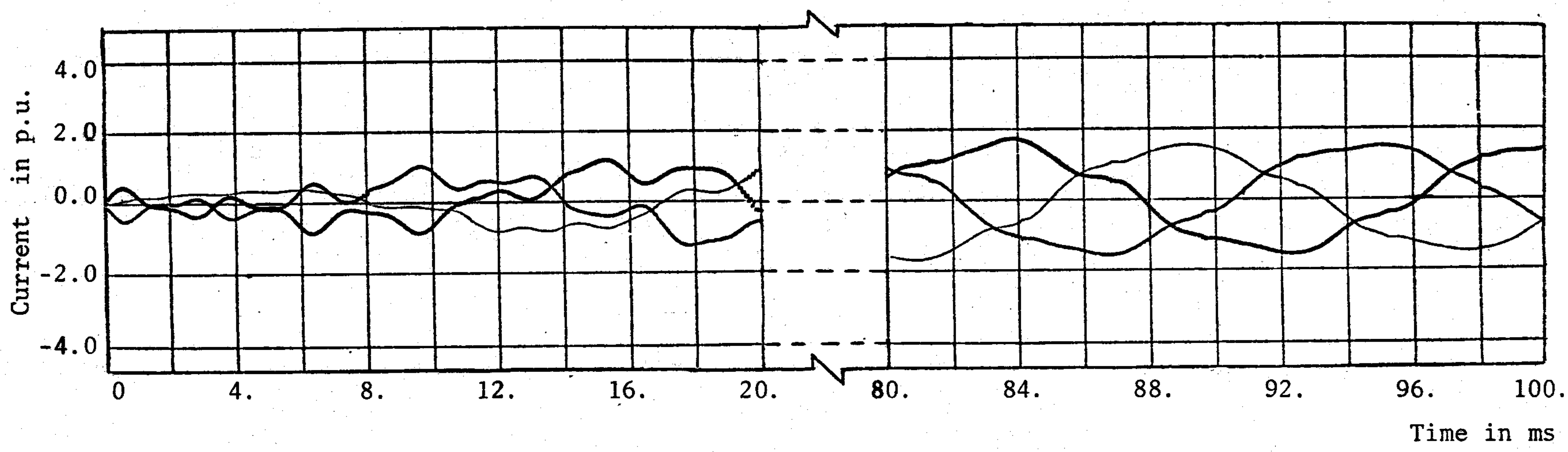


Fig. 5.13 The 3-phase Transmission Line Currents Versus the Time for the Case of Rectifier, Transmission Line and Filters

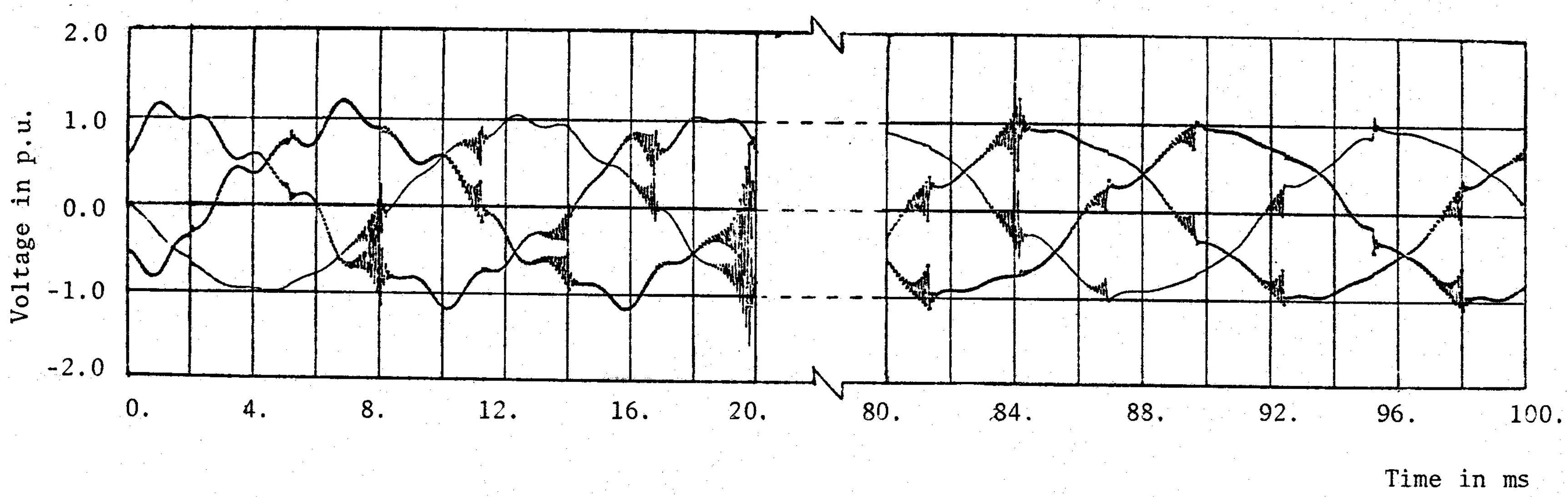


Fig. 5.14 The 3-phase Voltages at the AC terminals of the Converter Versus the Time for the Case of Rectifier, Transmission Line and Filters

AC terminals of the converter. The waveforms of these voltages, in this case, are not distorted as in case 2, since the transmission line currents are almost sinusoidal. It can also be noted that high frequency oscillations appear in the voltage waves during the periods of commutation. These oscillations [3,49] can be attributed to the effect of the commutation processes on the filters which are oscillatory circuits.

The 5th, 7th and 11th order harmonic filters currents are shown in Figs. 5.15, 5.16 and 5.17 respectively. It can be noted from the three figures that the amplitude of the current harmonics decreases as the order of the harmonic icnreases. Also, it can be observed that the midpoint of the two envelops of each current changes sinusoidaly with the fundamental frequency. This is due to the flow of a small portion of the fundamental frequency current through the filters. The effect of this fundamental component becomes more prominent as the order of the current harmonics increases. For example, it is relatively larger in the 11th filter than in the 7th filter and in the 7th filter than in the 5th one. Moreover, Fig. 5.17 shows that the 11th order harmonic current is greatly distorted because of the flow of the higher order harmonic currents, which are not filtered. The effect of these higher order harmonicsappears here significantly, since the value of the 11th order harmonic current is relatively small and, also, the resonant frequency of this filter is the nearest resonant frequency to these higher order harmonics.

Fig. 5.18 shows the DC output voltage and current in this case.

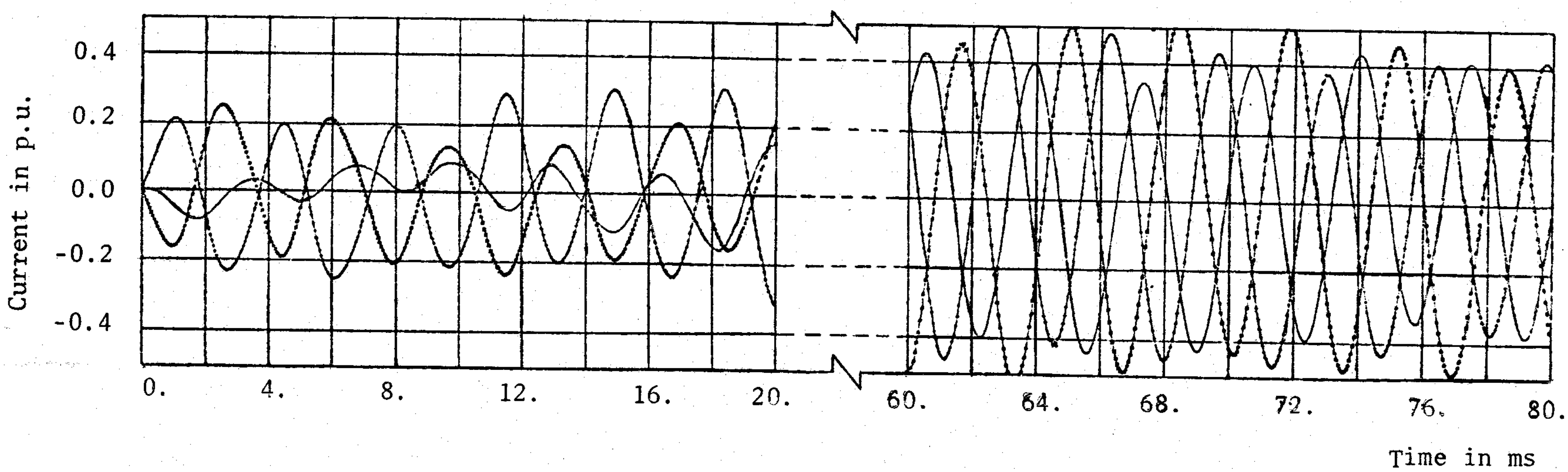


Fig. 5.15 The 3-phase Currents of the 5th order Harmonic Filter versus the Time for the Case of Rectifier, Transmission Line and Filters

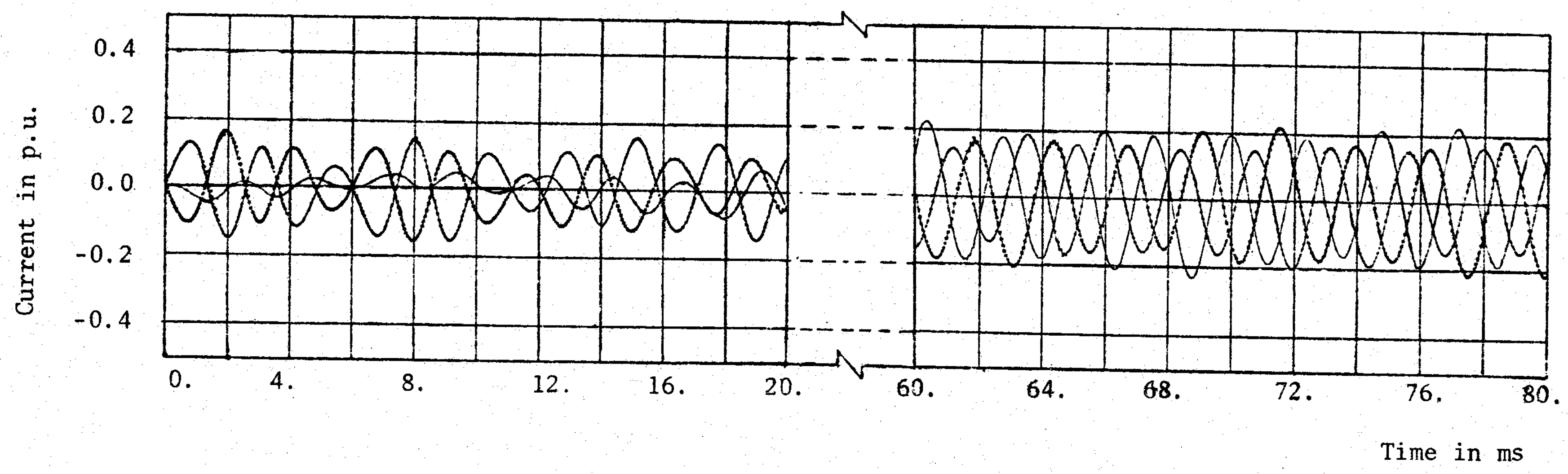


Fig. 5.16 The 3-phase Currents of the 7th Order Harmonic Filter Versus the Time for the Case of Rectifier, Transmission Line and Filters

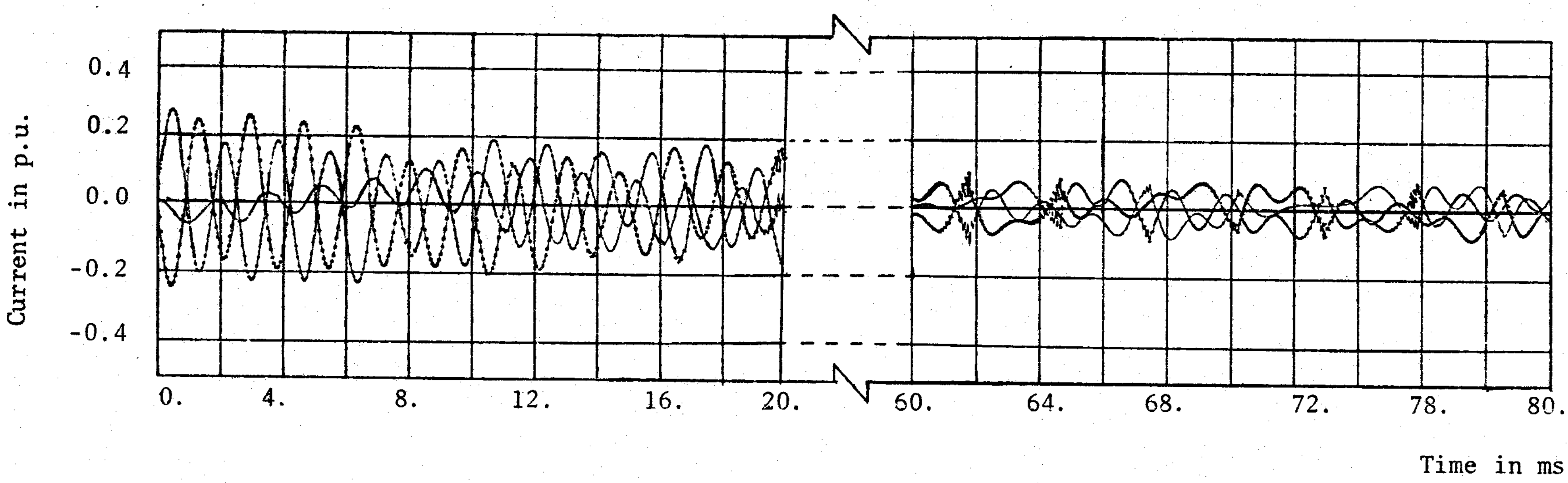


Fig. 5.17 The 3-phase Currents of the 11th Order Harmonic Filter Versus the Time for the Case of Rectifier, Transmission Line and Filters

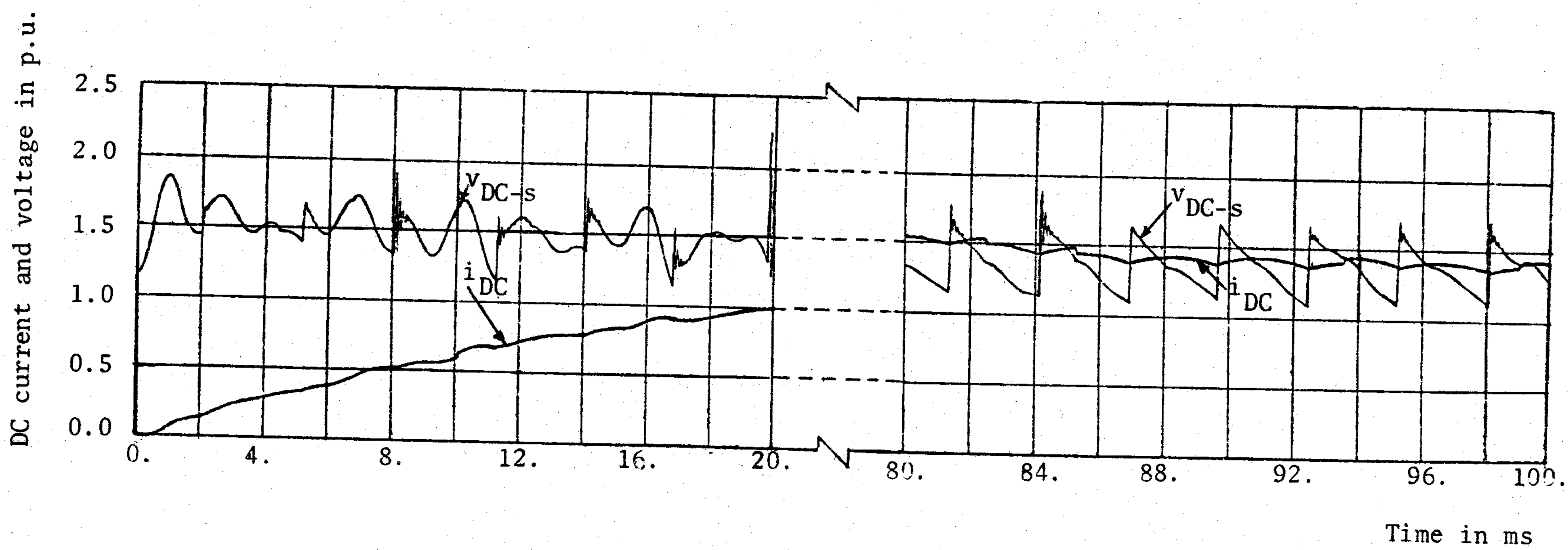


Fig. 5.18 The DC Output Current and Voltage Versus the Time for the Case of Rectifier, Transmission Line and Filters

The high frequency oscillations due to the presence of the filters appear also on the waveform of the DC voltage. This is expected as this voltage is nothing but a combination of the AC line voltages at the converter terminals. However, the high frequency oscillations on the DC voltage decay faster than those of the AC voltage waveforms.

Case 4: Rectifier connected to a synchronous machine as well as to an infinite bus-bar through a short transmission line (Fig. 5.19)

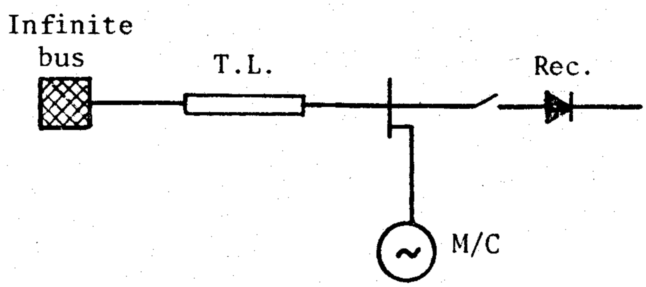


Fig. 5.19 Rectifier, Transmission Line and Synchronous Machine Case

In this case, it is assumed that the synchronous machine is operating at steady-state as a synchronous generator for a certain loading condition (v_a=-1 sin($p\theta_o$.t) p.u., v_1=-0.99996 sin($p\theta_o$.t+7.9°)p.u., i_a=-1.633 sin($p\theta_o$.t+10°) p.u., P_M=1.637 p.u.,δ =40.08°, v_{fd}=0.0015064p.u., i_{fd}=2.597 p.u.). At zero time, the rectifier is suddenly connected to the machine terminals. Fig. 5.20 shows the 3-phase rectifier currents which do not differ in shape from those of the previous cases. Also, the waveforms of the DC output voltage and current are similar to those of cases 1 and 2 as shown in Fig. 5.21.

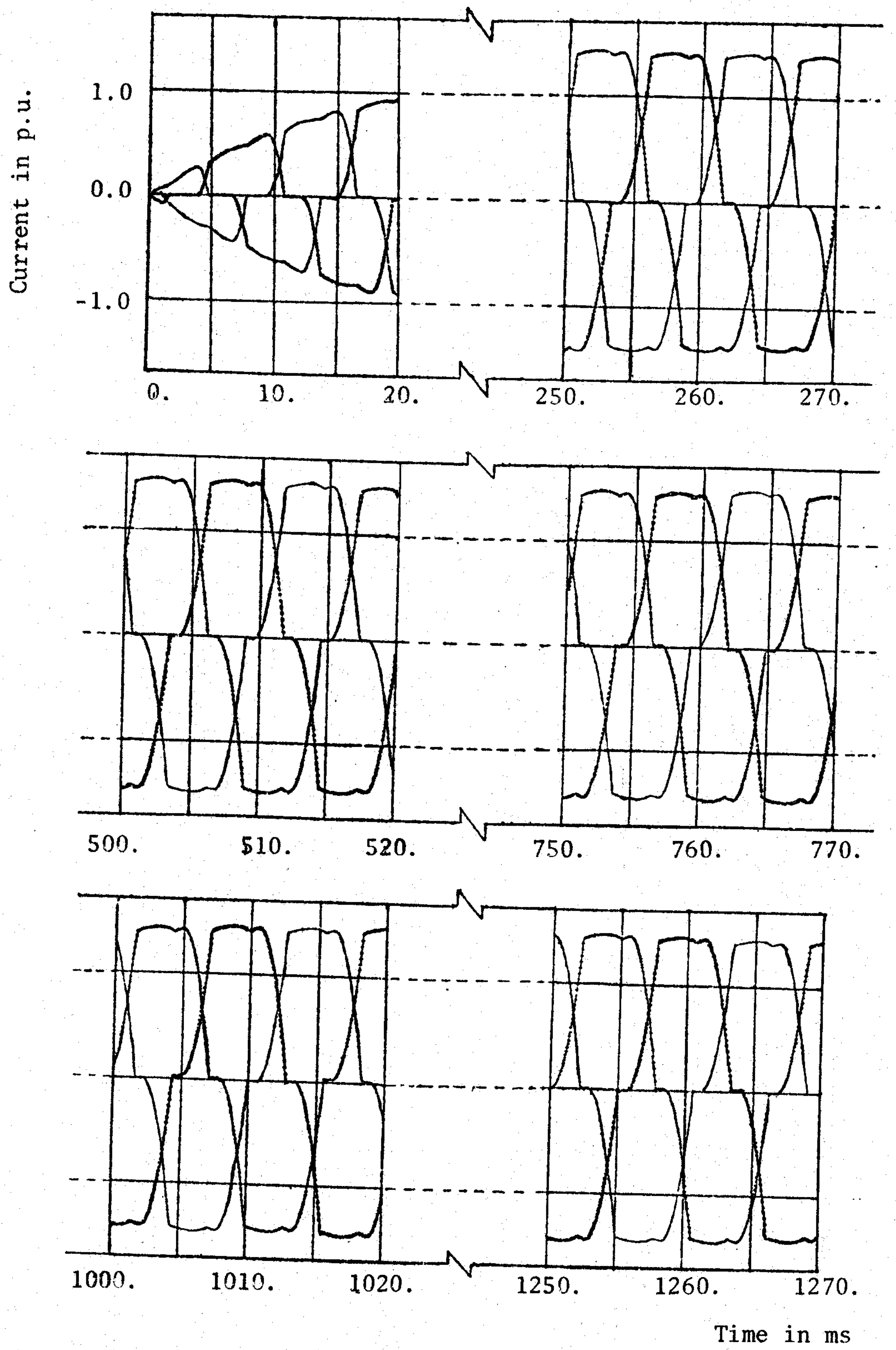


Fig. 5.20 The 3-phase Rectifier Currents Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

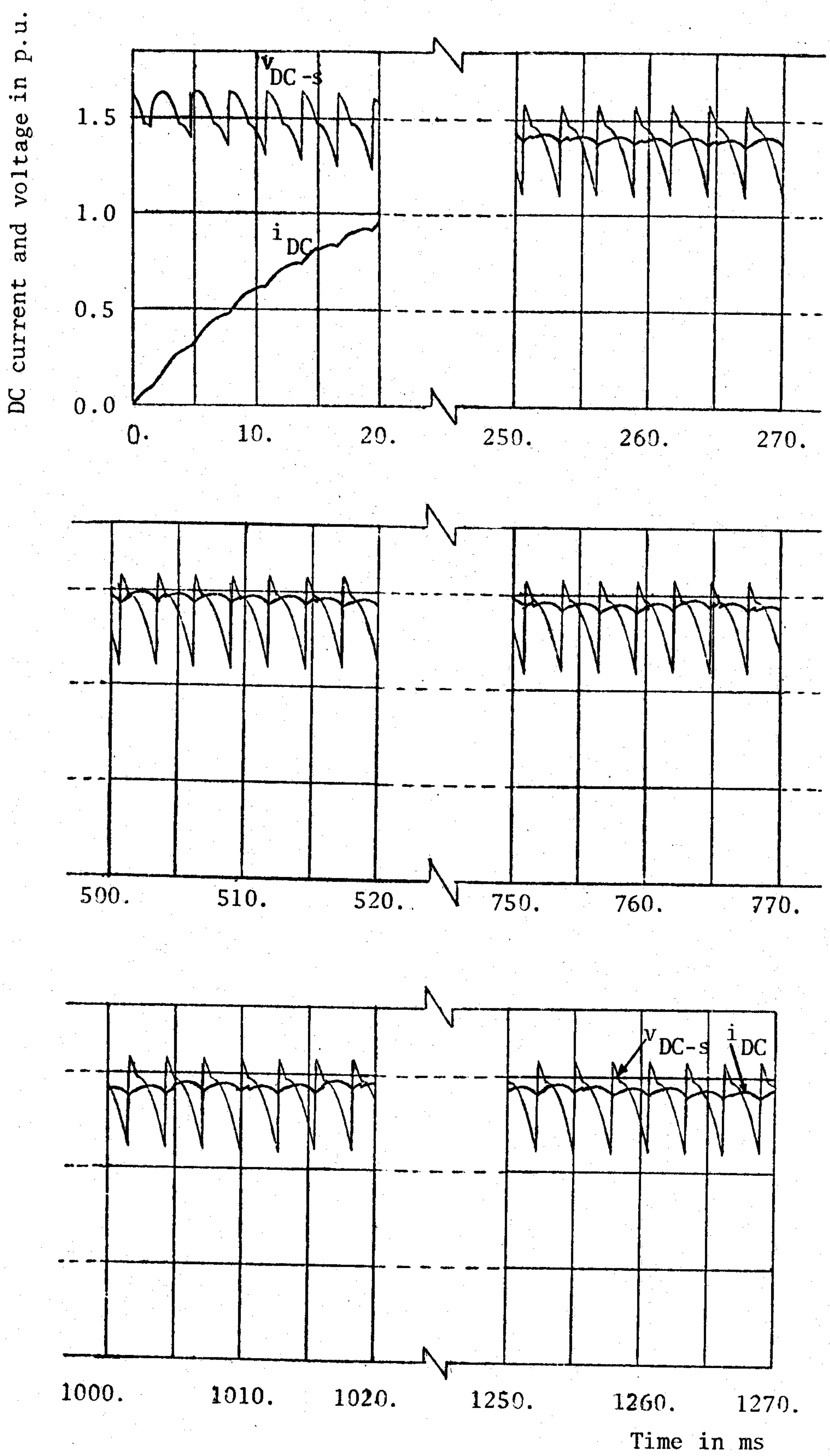


Fig. 5.21 The DC Output Current and Voltage Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

Regarding the periods of commutations, it can be noted that they are less than in case 2. This can be attributed to the fact that the connection of the machine to the converter AC bus-bar adds an inductance in parallel to the AC system inductance. Therefore, the equivalent commutating reactance becomes less than in case 2.

Fig. 5.22 shows the AC converter terminal voltages. It can be noted that the heights of the jumps in these curves are less, in this case, than those of case 2. This is because the rate of change of the transmission line current at the end of the commutation period is relatively less. The synchronous machine absorbs some of the harmonics of the rectifier current and smooths the transmission line current.

As shown in Fig. 5.23, the 3-phase armature currents are not sinusoidal. They contain harmonic current components, the most prominent of which are of the order $6k \pm 1$ (k is an integer). These harmonic currents are flowing from the rectifier to the machine.

The rotor currents shown in Figs. 5.24a and 5.24b consist of AC harmonic components superimposed on a DC component which changes with time. This DC component starts with the initial steady-state value of the rotor current before the connection of the rectifier, rises after its connection and then starts to decay after a certain time until it reaches finally its original steady-state value. For the field winding, the sustained component is equal to the DC field current before the connection of the rectifier plus AC harmonic currents, most of which are of the order 6k. For the damper windings, the sustained component is AC

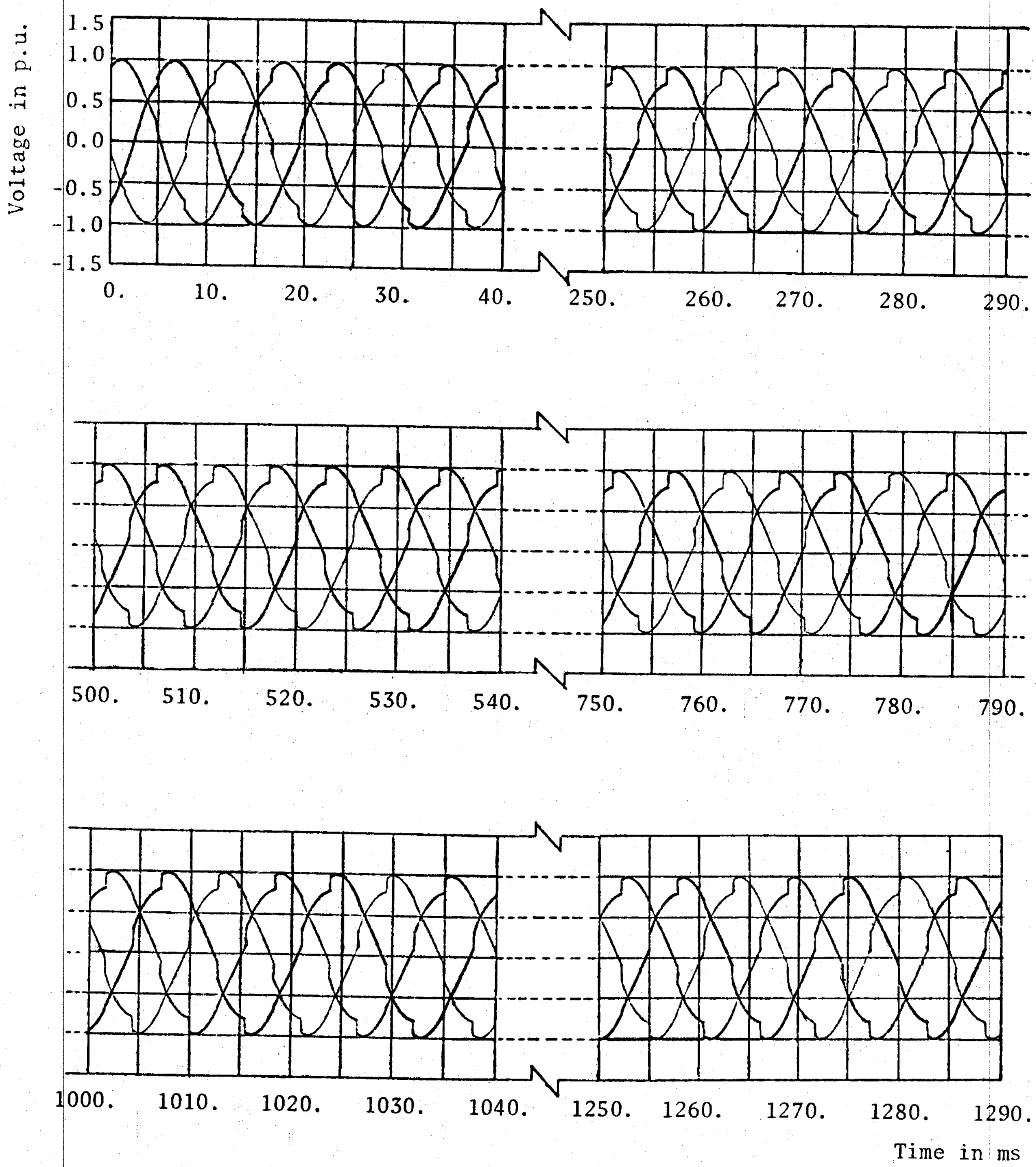


Fig. 5.22 The 3-phase Voltages at the AC Terminals of the Converter Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

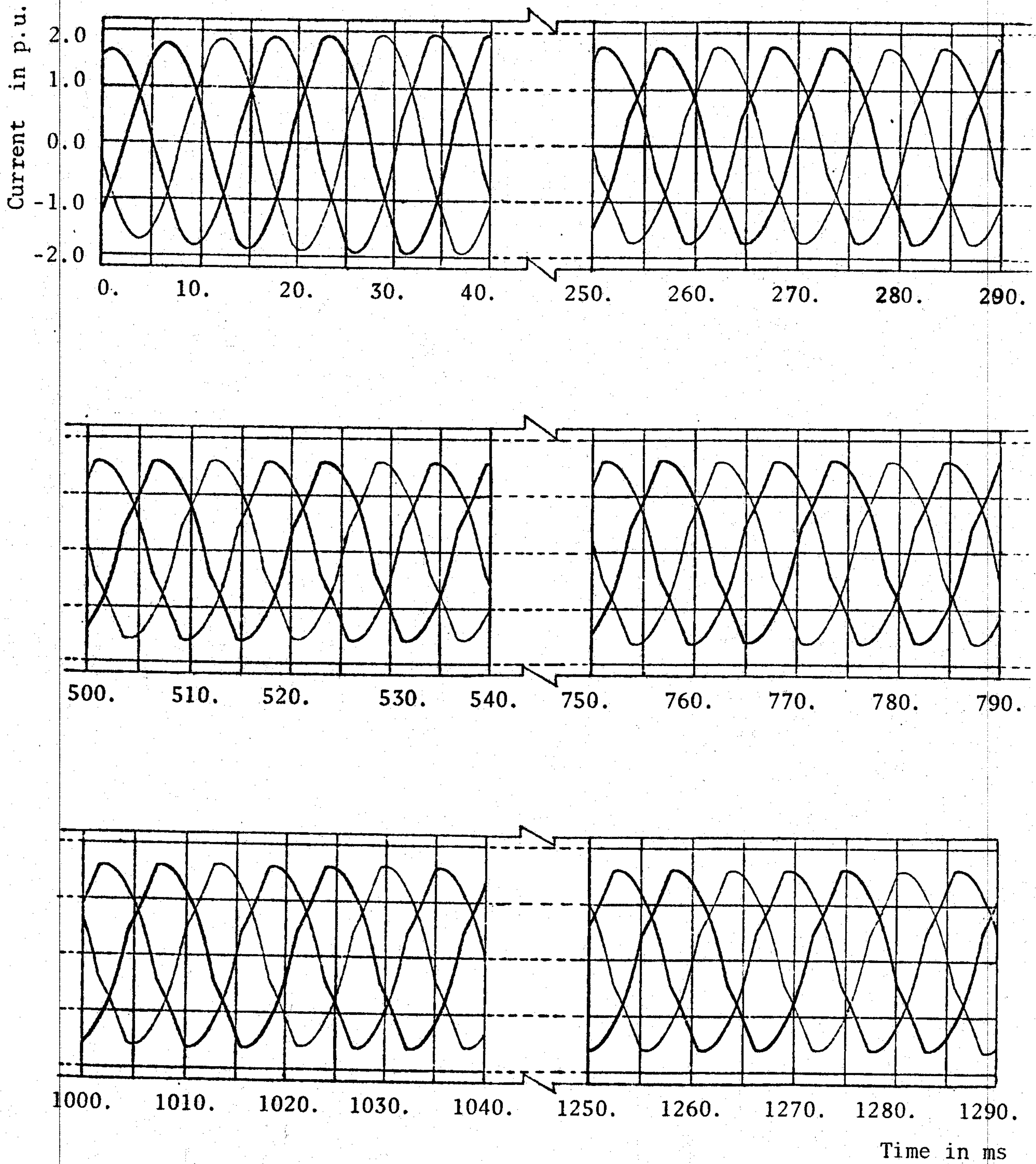


Fig. 5.23 The 3-phase Armature Currents of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

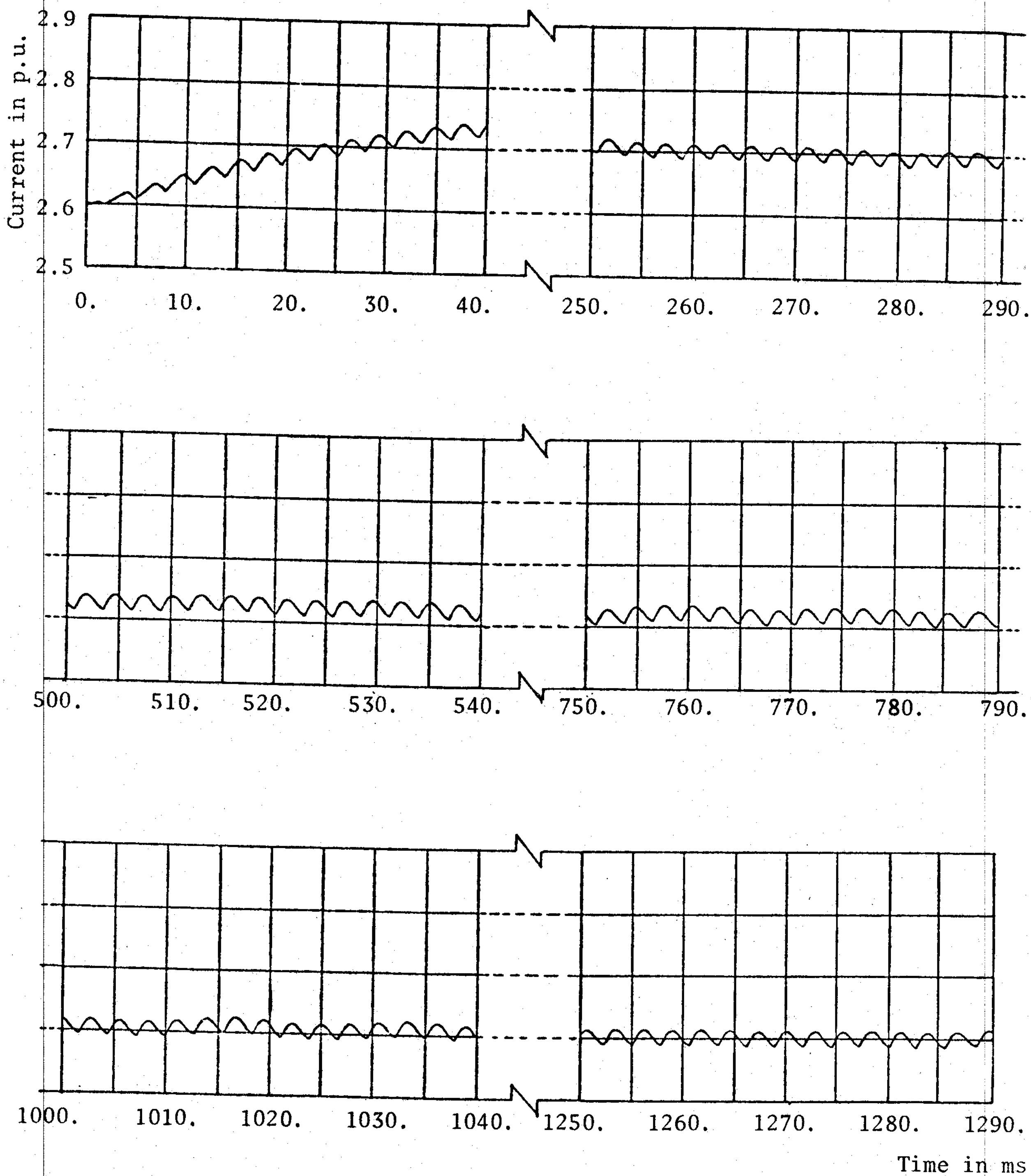


Fig. 5.24 a The Field Winding Current of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

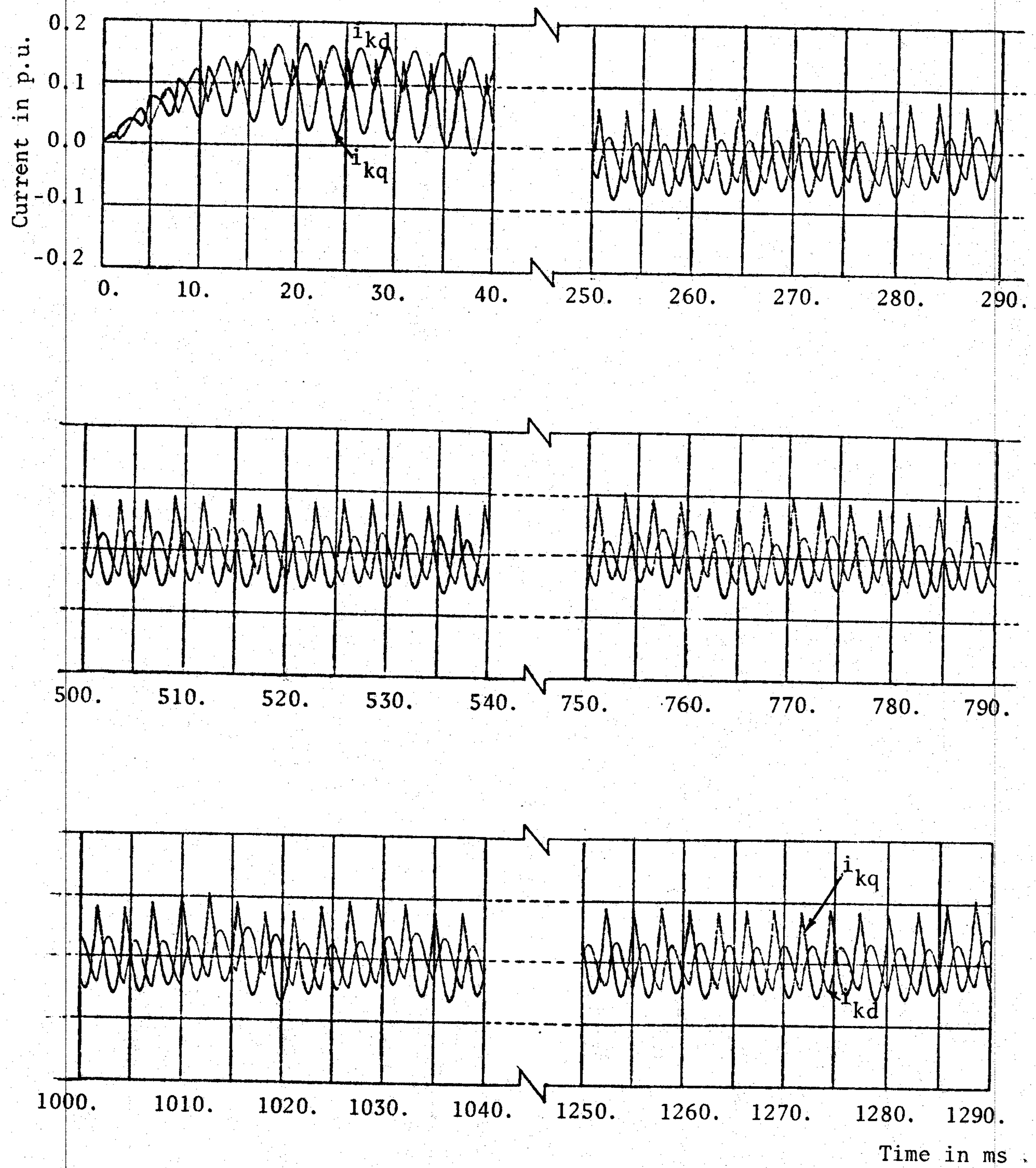


Fig. 5.24 b The d- and q-axis Damper Winding Currents of the Synchronous Machine Versus the Time for the Case of Rectifier Transmission Line and Synchronous Machine

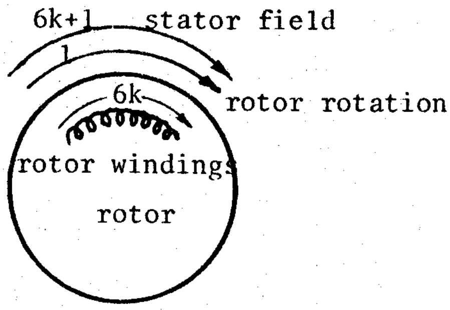
harmonic currents, most of which are of the order 6k. It can be noted that a small fundamental frequency component appears also in the rotor currents.

The existance of the harmonic currents of the order 6k in the rotor windings is due to the presence of the harmonic current components in the stator windings. These stator harmonics of the order 6k+1 create an mmf with the same sense of rotation as that of the rotor, Fig. 5.25a. Therefore, the relative velocity of such an mmf with respect to the rotor is 6k times that of the rotor. On the other hand, the harmonic components of the stator phase currents of the order 6k-1 create an mmf which has also a relative velocity with respect to the rotor of the same value as those of the 6k+1 harmonics but of opposite direction as shown in Fig. 5.25b. These two mmfs rotating in opposite directions result in an elliptical field [27,30], as shown in Fig. 5.25c, and they induce harmonic currents of the order 6k in the rotor circuits.

Fig. 5.26 shows the electrical torque and the applied mechanical torque of the synchronous machine. It is assumed that the machine is supplied by a constant power input. However, the change in the mechanical torque due to the change of speed is unnoticeable. This is because the change of the speed is very small as shown in Fig. 5.27. Without the presence of the rectifier in the circuit, the electrical torque of the synchronous machine under steady-state conditions is constant and equal to the mechanical torque. When the rectifier is connected, there are harmonic currents flowing in the stator and the rotor windings as explained before. As a result, these high frequency components give rise to pulsating electric torque as shown in Fig. 5.26. These pulsations may produce vibrations during the machine operation.

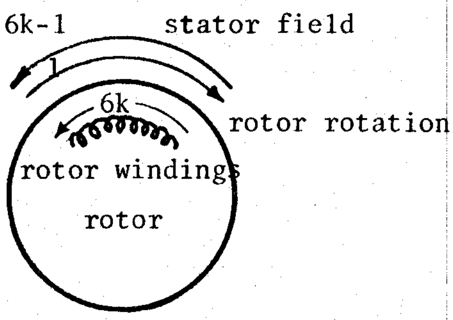
When the rectifier is connected to the machine bus-bar, the rotor angle of the synchronous machine changes its initial value to another steady-state value in an oscillatory manner. Corresponding to this variation of the rotor angle ($f<1Hz$), Fig. 5.27, there is a slow variation in the different quantities of the machine, particularly in the electrical torque, Fig. 5.26, and the rotor currents, Figs. 5.24a and 5.24b.

Fig. 5.28 shows the 3-phase transmission line currents. When they are compared with those of case 2, it is noticed that their harmonic contents are less. They now approach the sinusoidal form in comparison with the rectangular form of case 2. This is because the synchronous machine absorbs some of the harmonic currents due to its low subtransient reactance.



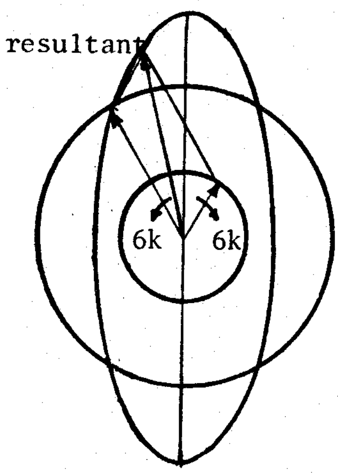
(a)

Generation of Harmonic Currents of Order 6k in the Rotor due to Harmonic Currents of Order 6k+1 in the Stator.



(b)

Generation of Harmonic Currents of Order 6k in the Rotor due to Harmonic Currents of Order 6k-1 in the Stator.



(c)

The Resultant Elliptical Field.

Fig. 5.25

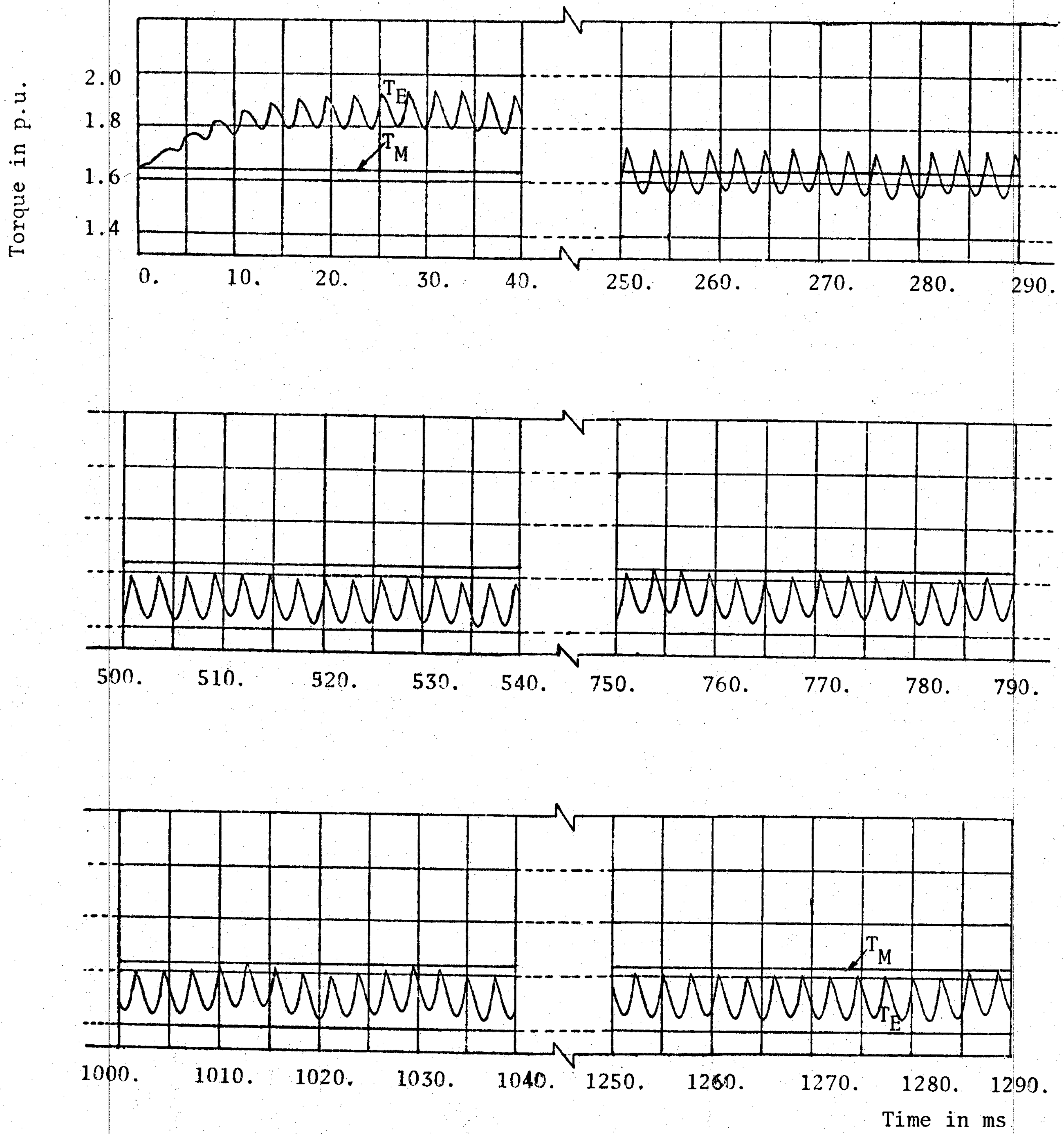


Fig. 5.26 The Electrical Torque and the Applied Mechanical Torque of the Synchronous Machine versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

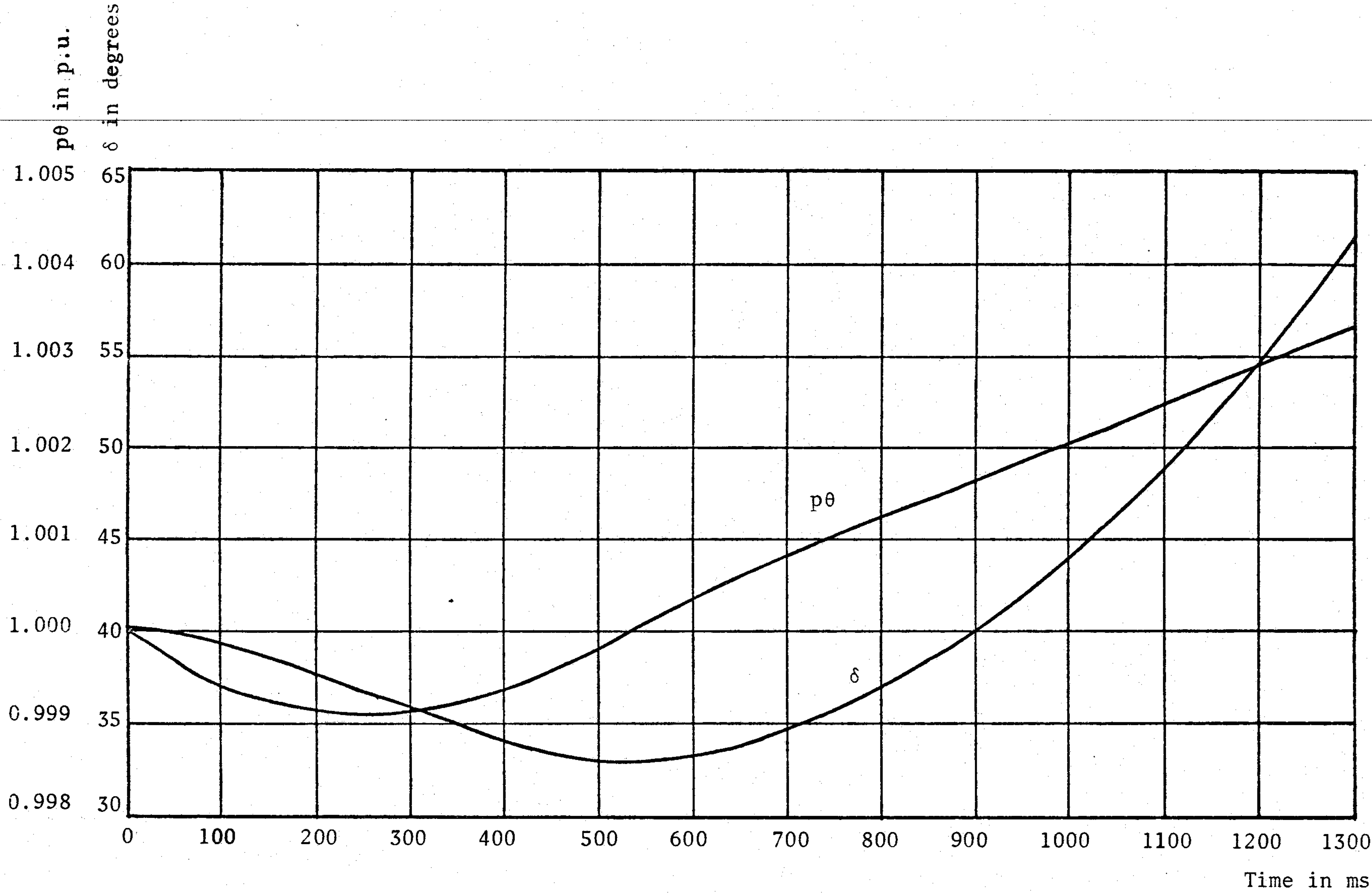


Fig. 5.27 The Rotor Angle and the Rotor Speed of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

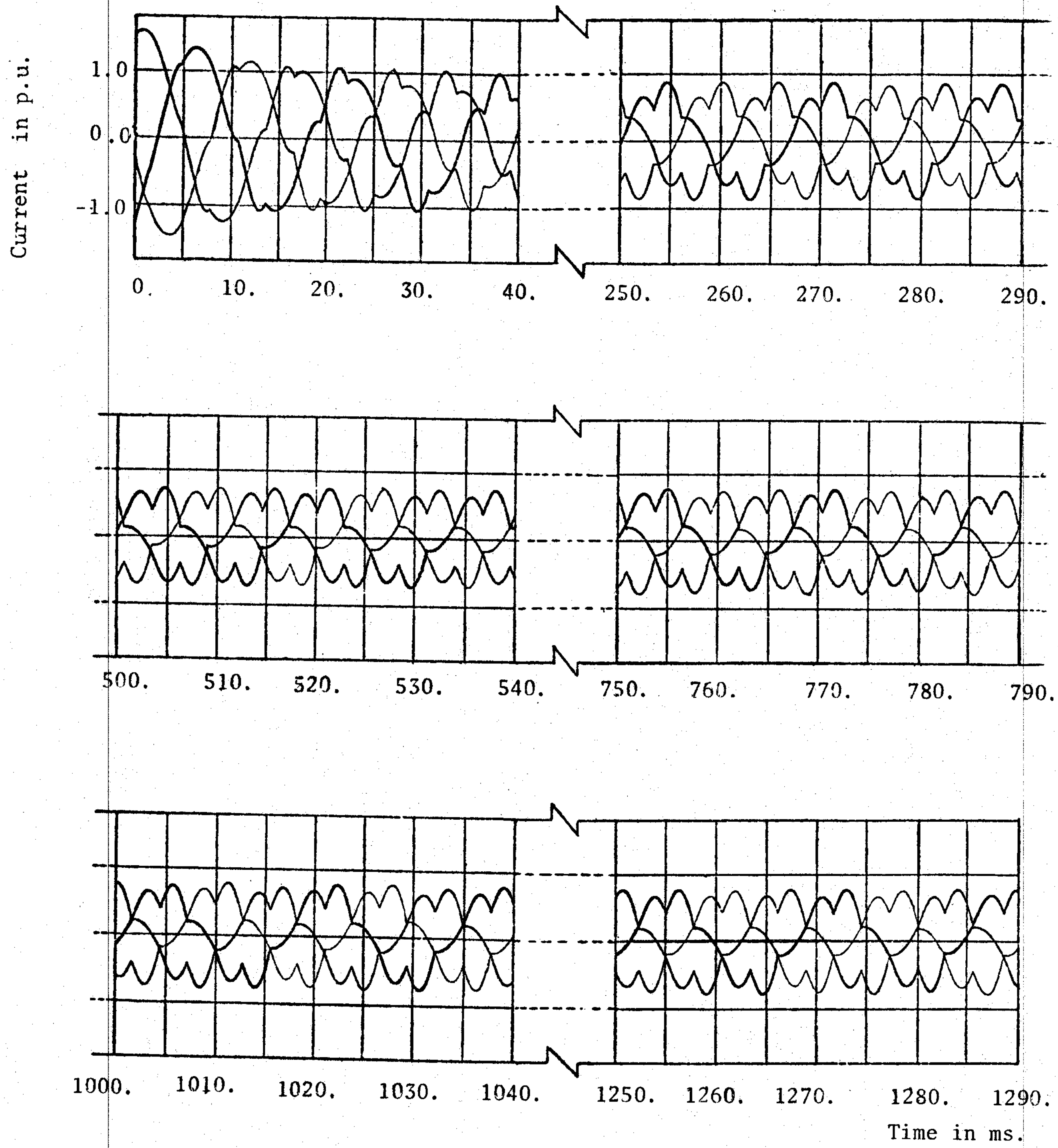


Fig. 5.28 The 3-phase Transmission Line Currents Versus the Time for the Case of Rectifier, Transmission Line and Synchronous Machine

Case 5: A rectifier and a group of filters are connected to a synchronous machine which is connected to an infinite bus-bar through a short transmission line (Fig. 5.29)

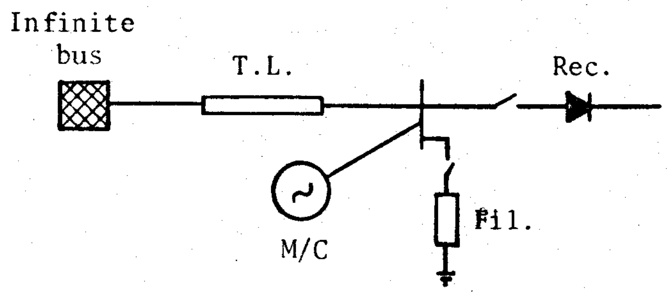


Fig. 5.29 Rectifier, Transmission Line, Synchronous Machine and Filters Case

In this case, it is also assumed that the synchronous machine is operating at steady-state as a synchronous generator for the same loading condition as in case 4. At zero time, the rectifier and its group of filters are suddenly connected to the bus-bar at which the machine is connected.

As shown in Fig. 5.30, the shape of the 3-phase rectifier currents do not differ considerably from those of the previous cases. However, it can be noted that the periods of commutation in this case are less than those of case 4. The presence of the filters in the system results in moving the commutation point towards the converter AC bus-bar and thus reducing the commutating reactance.

Fig. 5.31 shows the 3-phase voltages at the converter AC terminals. In comparison with case 3, these voltages approach the sinusoidal shape due to the presence of the filters. These

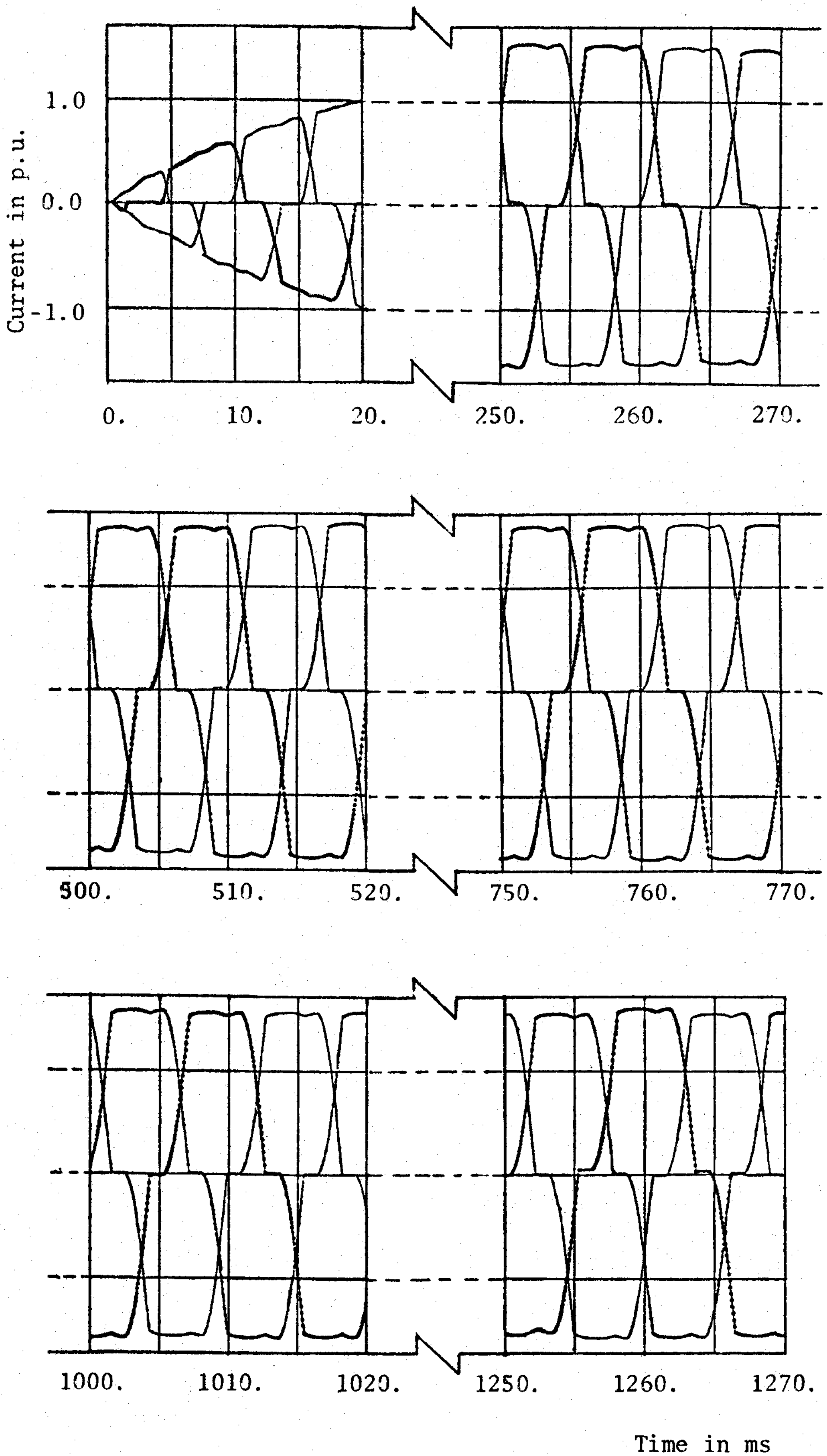


Fig. 5.30 The 3-phase Rectifier Currents Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

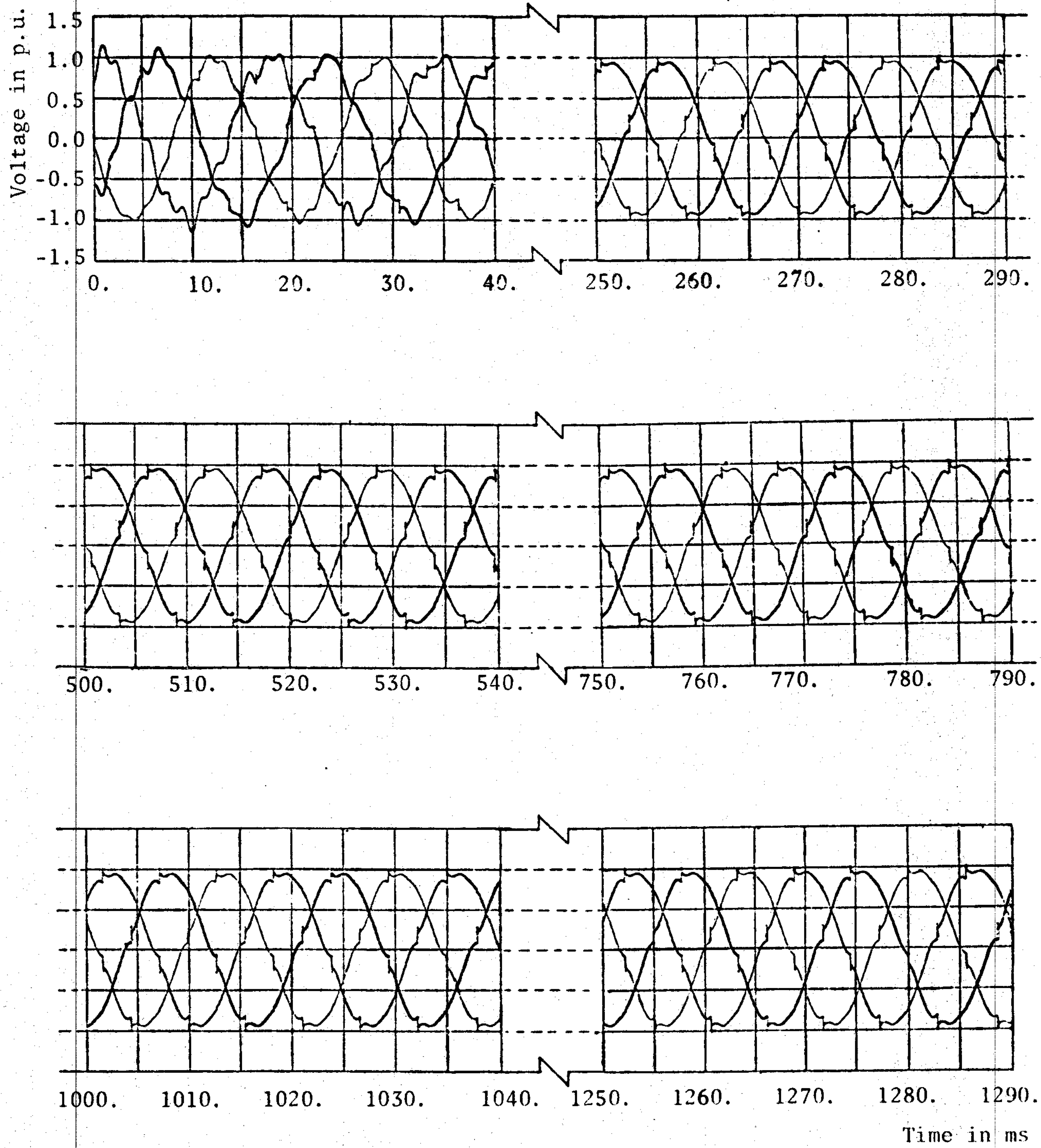


Fig. 5.31 The 3-phase Voltages at the AC Terminals of the Converter Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

filters absorb most of the harmonic components of the 5th, 7th and 11th order of the rectifier currents. Fig. 5.32 shows the DC output voltage and current. It can be noted from Figs. 5.31 and 5.32 that high frequency oscillations [3, 49] appear in both the AC and DC voltages as in case 3. The high frequency oscillations in the present case are less than those of case 3. This can be attributed to the effect of the machine inductance.

The 3-phase armature currents, Fig. 5.33, are approaching the sinusoidal shape in comparison with those of case 4, Fig. 5.23. Regarding the rotor currents, Figs. 5.34a and 5.34b, it can be noted that their harmonic contents are much less than in case 4. This is because the filters in the system offer a low impedance path to the harmonic currents of the rectifier. Therefore, the harmonics in the stator windings and,as a consequence, those induced in the rotor windings are reduced.

The 3-phase currents of the 5th, 7th and 11th order harmonic filters are shown in Figs. 5.35, 5.36 and 5.37 respectively. It is clear that they have the same nature as the filters currents of case 3. The presence of the synchronous machine in the system, however, has reduced their magnitudes as some of the harmonic currents of the rectifier will flow in the machine.

The electrical torque and the applied mechanical torque of the synchronous machine are shown in Fig. 5.38. The mechanical torque is almost constant as the change of the speed, Fig. 5.39, is very small. The electrical torque still contains some pulsating components due to the flow of some harmonic currents in the stator and rotor windings of the synchronous machine. These pulsations are much less in this case if compared with those of case 4.

When the rectifier and the filters are connected to the machine bus-bar, the rotor angle of the synchronous machine changes in an oscillatory manner as in case 4. However, in the present case, the deviation of the rotor angle, Fig. 5.39, from its initial value for the first swing is larger than its deviation in case 4. This is because the addition of the rectifier and the filters to the system requires more power from the synchronous machine than the power required when the rectifier is connected alone . As in case 4, corresponding to the slow variation of the rotor angle ($f<1Hz$), there is a slow variation in the electrical torque, Fig. 5.38, and the rotor currents, Figs. 5.34a and 5.34b.

Fig. 5.40 shows the 3-phase transmission line currents. As expected, their harmonic contents are appreciably reduced if compared with case 4 due to the presence of the filters.

Despite the presence of the filters in cases 3 and 5, it is noted that the waveforms of the various electrical quantities are not perfectly sinusoidal as it should be expected. This is due to the fact that these filters are only of the 5th, 7th and 11th order and their impedances at their resonant frequencies still have a definite value. Thus, the corresponding harmonic currents will not be completely short-circuited by these filters. Therefore, some of these harmonic currents in addition to the higher order harmonics, which are not filtered, will flow in the AC system.

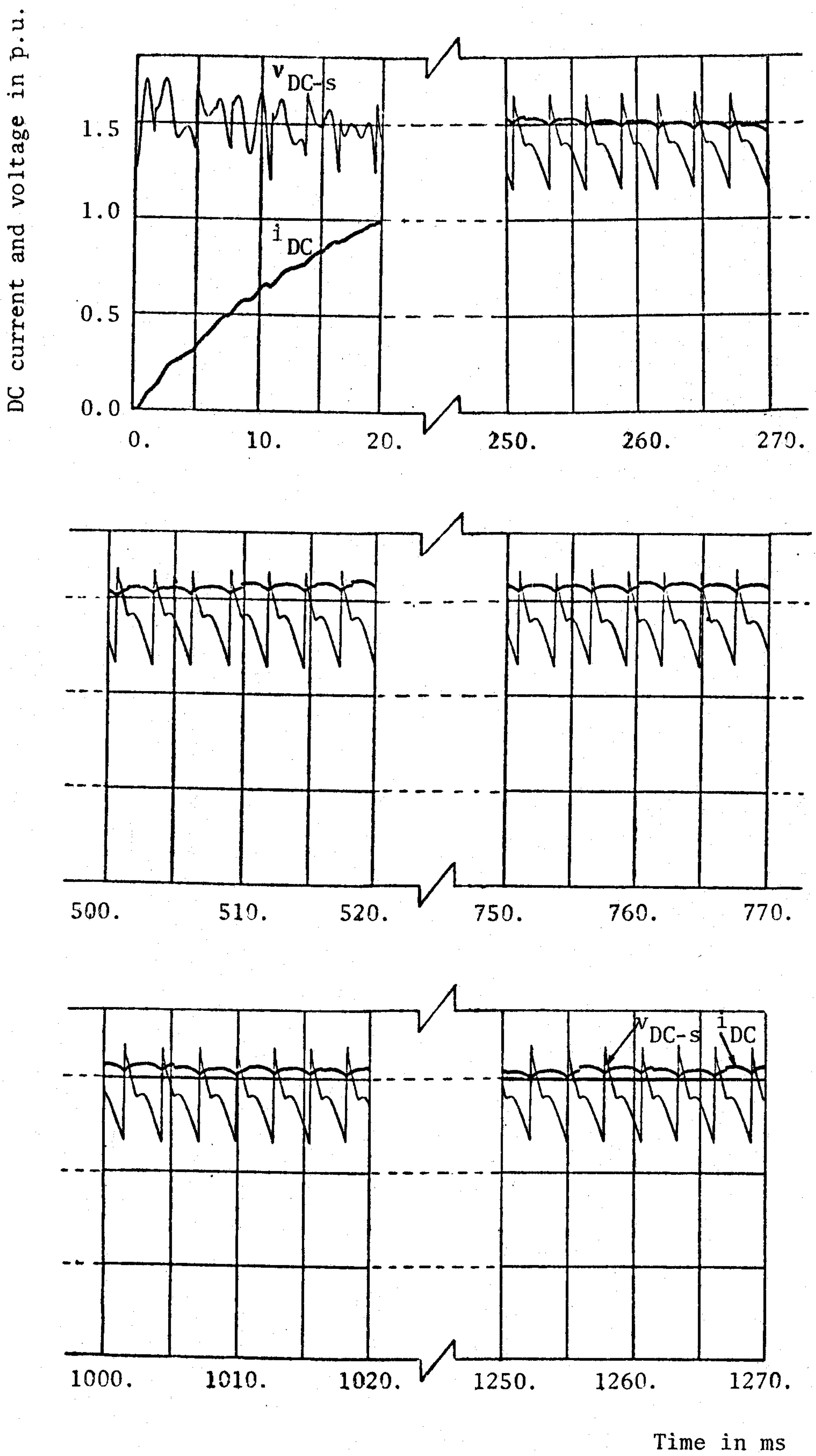


Fig. 5.32 The DC Output Current and Voltage Versus the Time for the Case of Recitifier, Transmission Line, Synchronous Machine and Filters

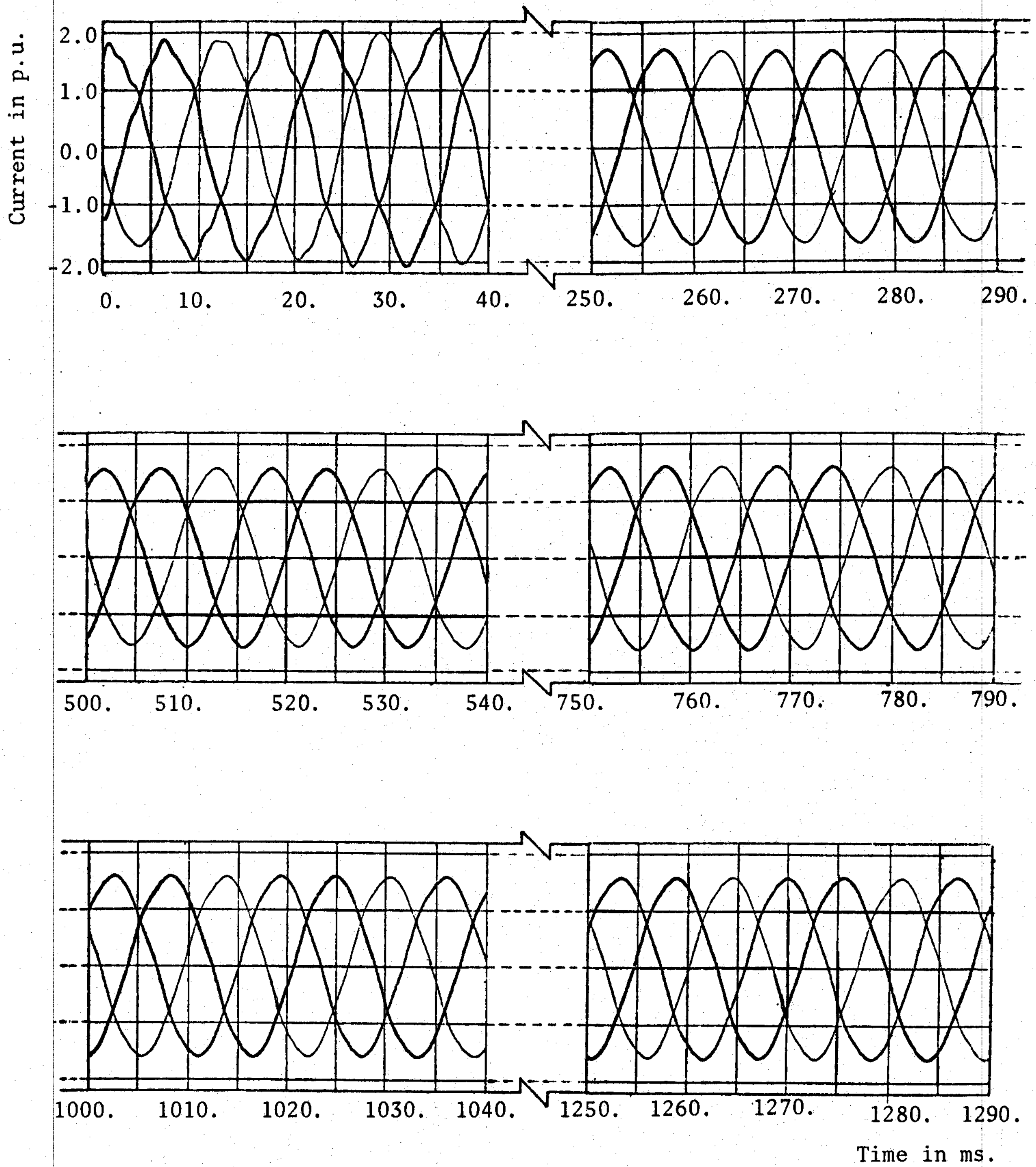


Fig. 5.33 The 3-phase Armature Currents of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

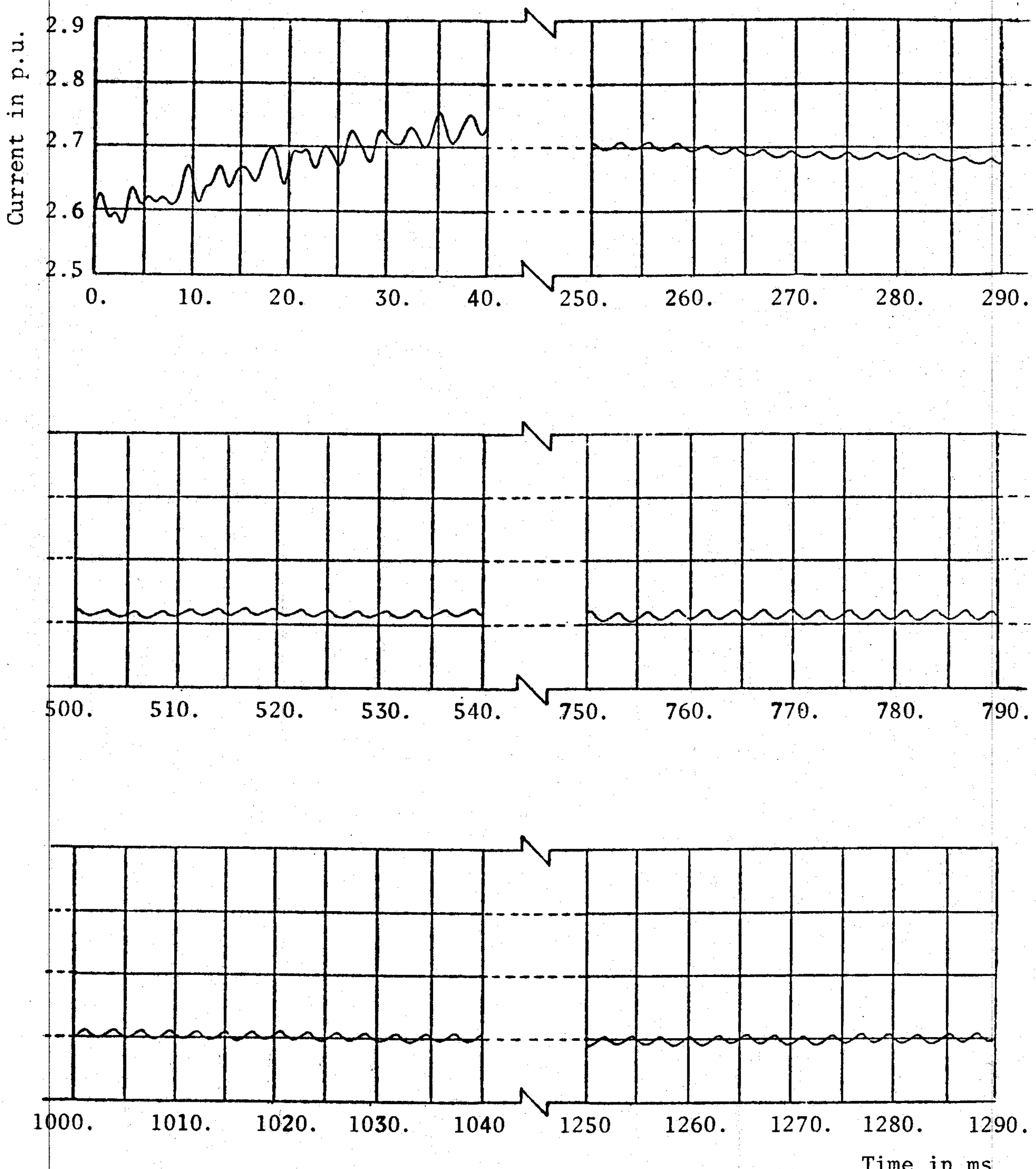


Fig. 5.34 a The Field Winding Current of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

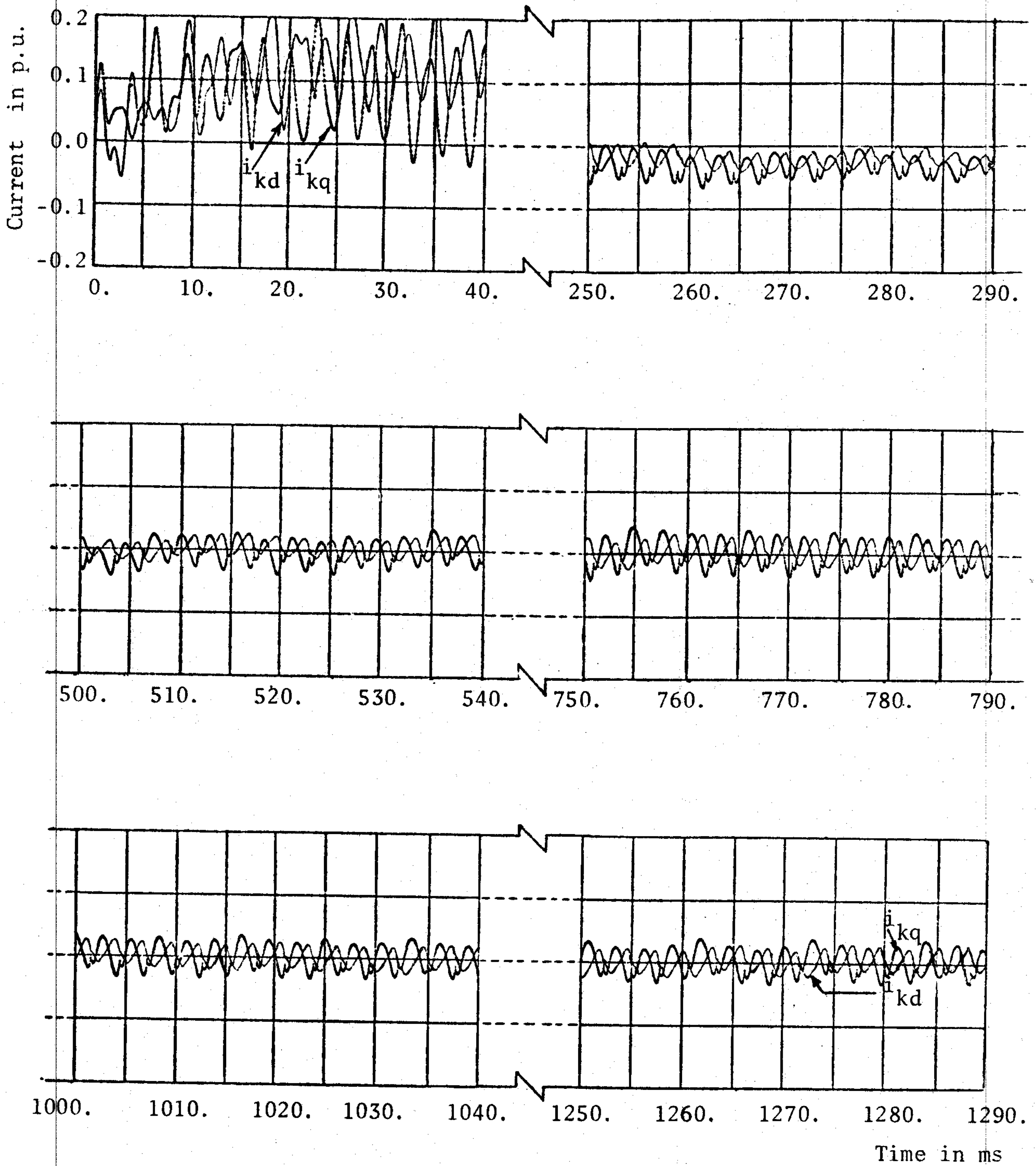


Fig. 5.34 b The d- and q-axis Damper Winding Currents of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

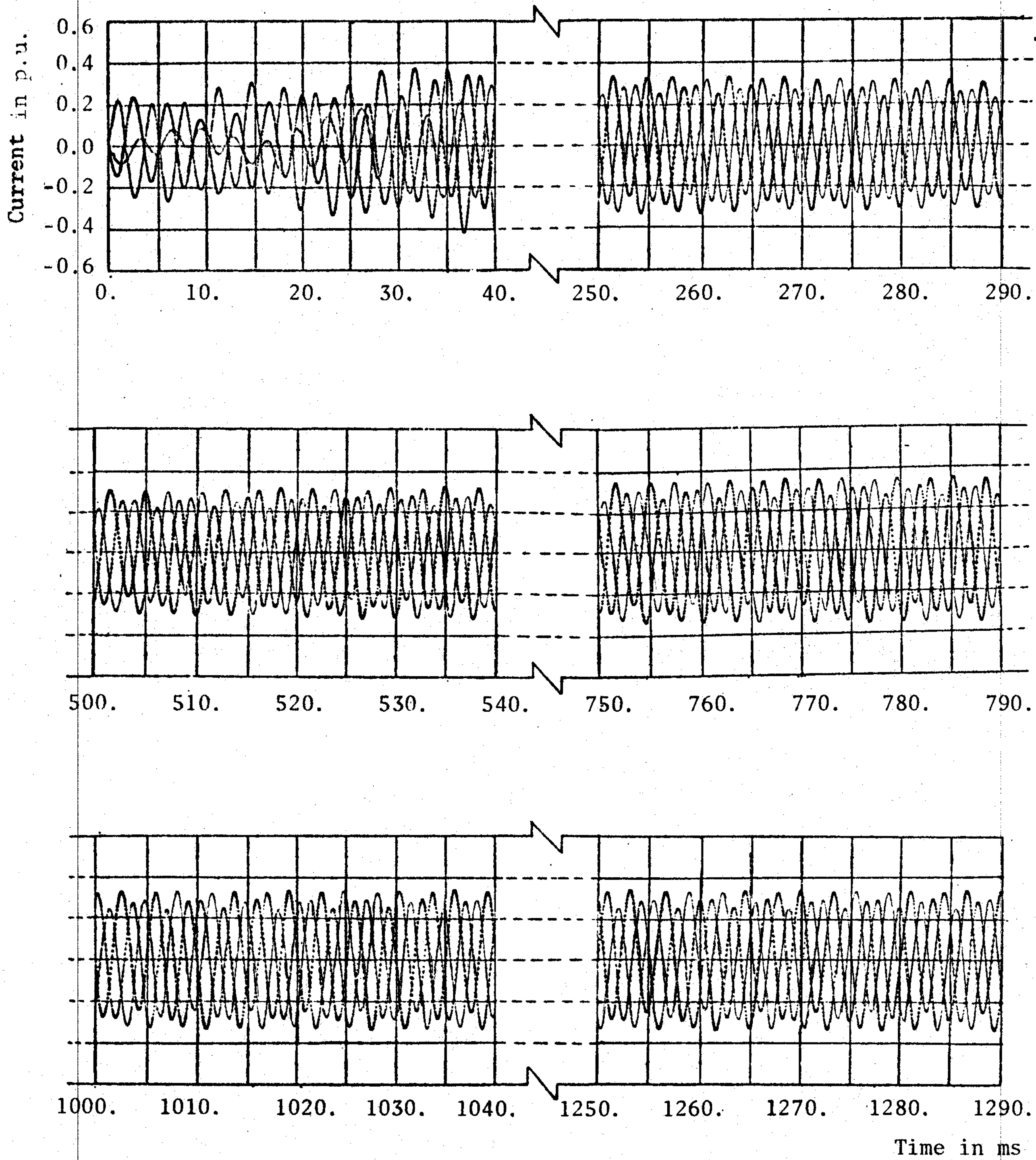


Fig. 5.35 The 3-phase Currents of the 5th Order Harmonic Filter Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

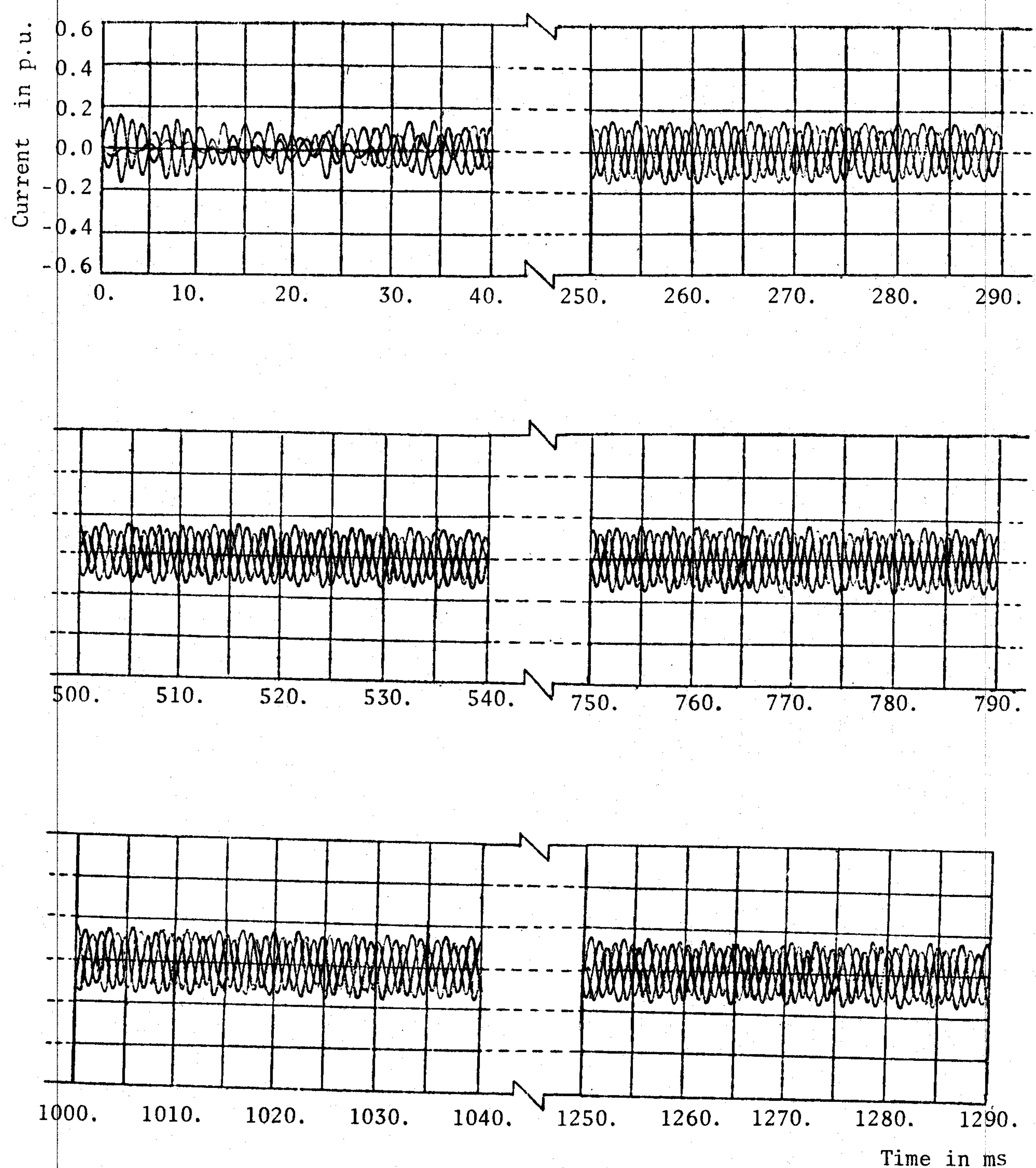


Fig. 5.36 The 3-phase Currents of the 7th Order Harmonic Filter Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filter

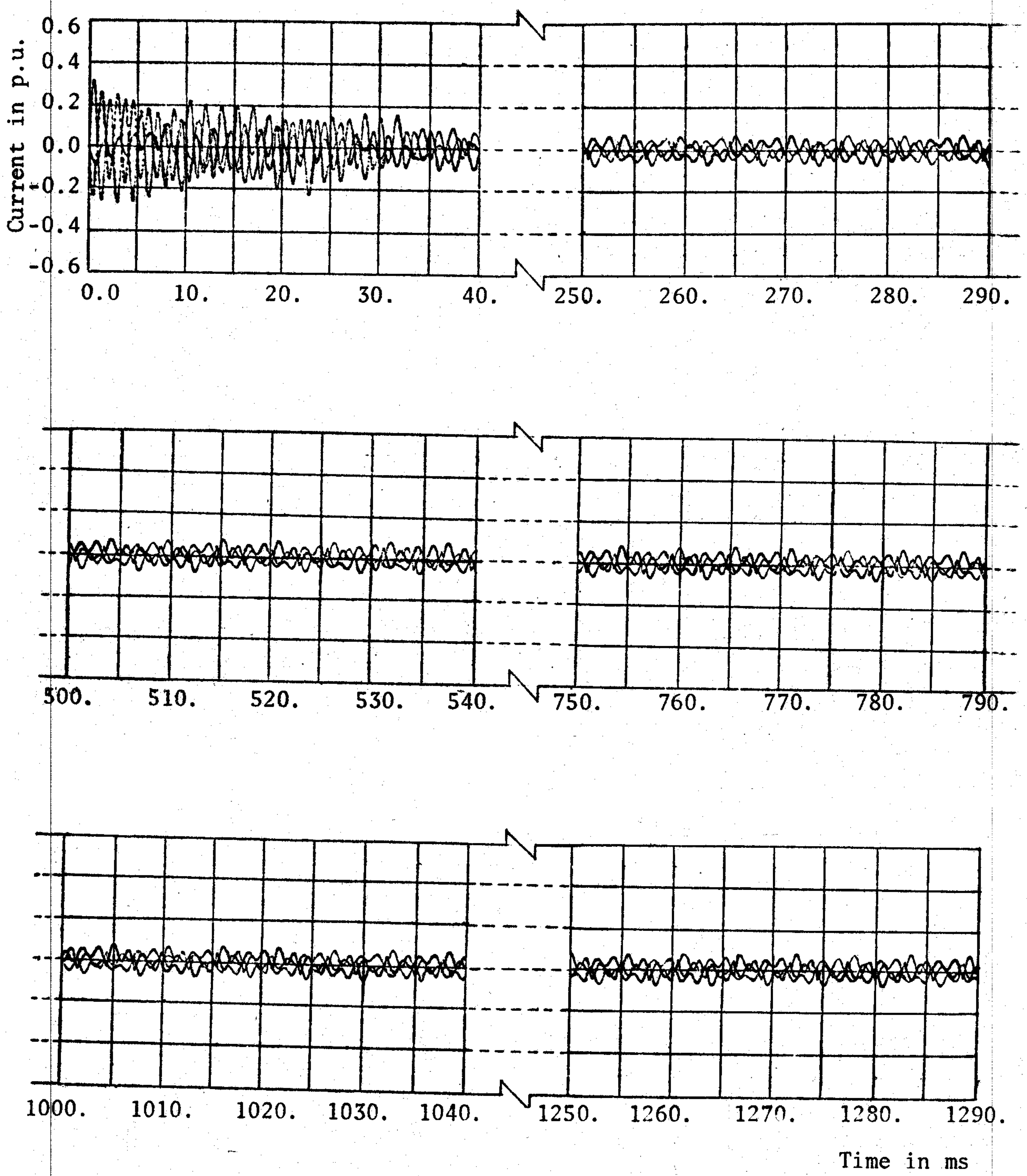


Fig. 5.37 The 3-phase Currents of the 11th Order Harmonic Filter Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

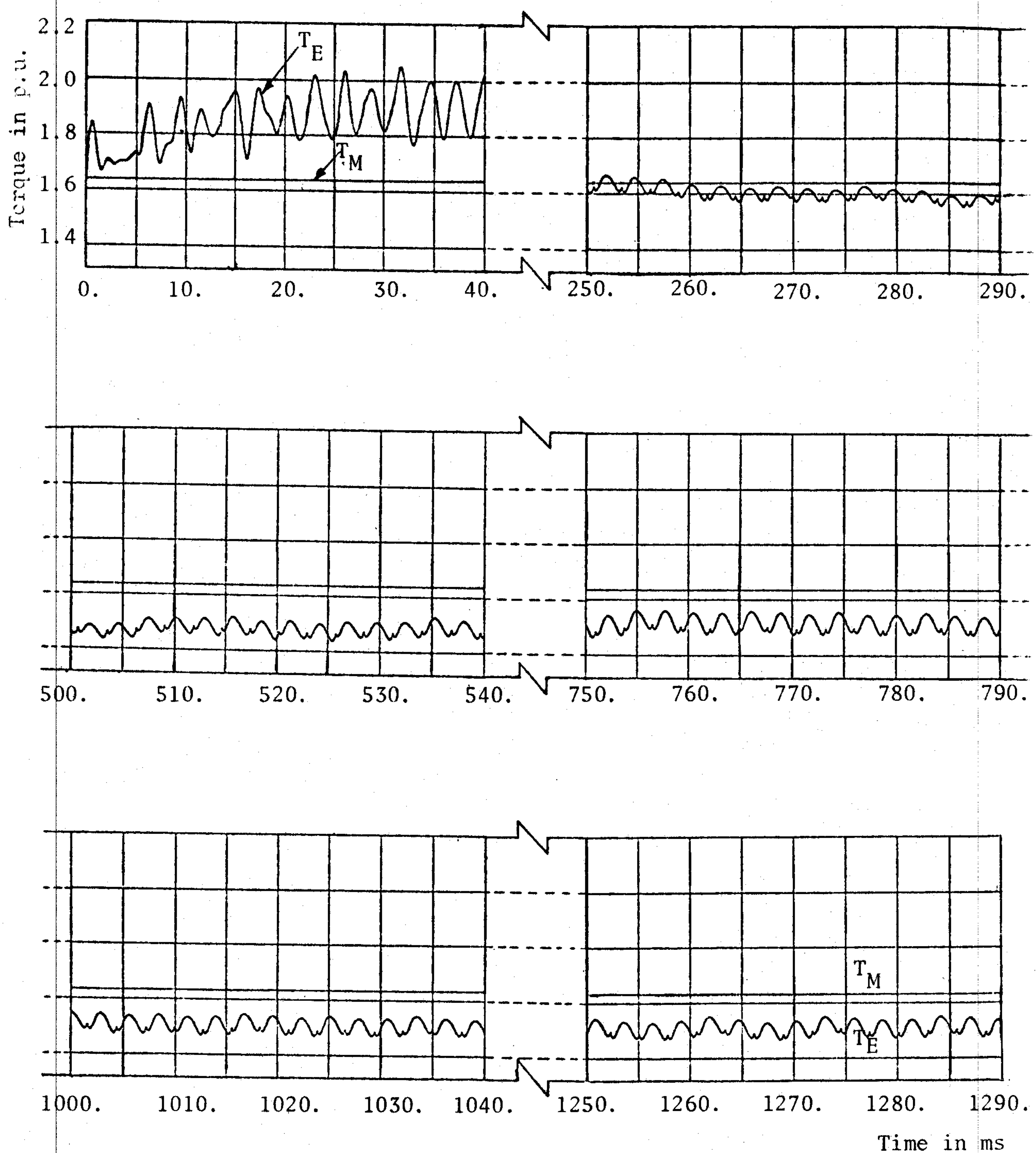


Fig. 5.38 The Electrical Torque and the Applied Mechanical Torque of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

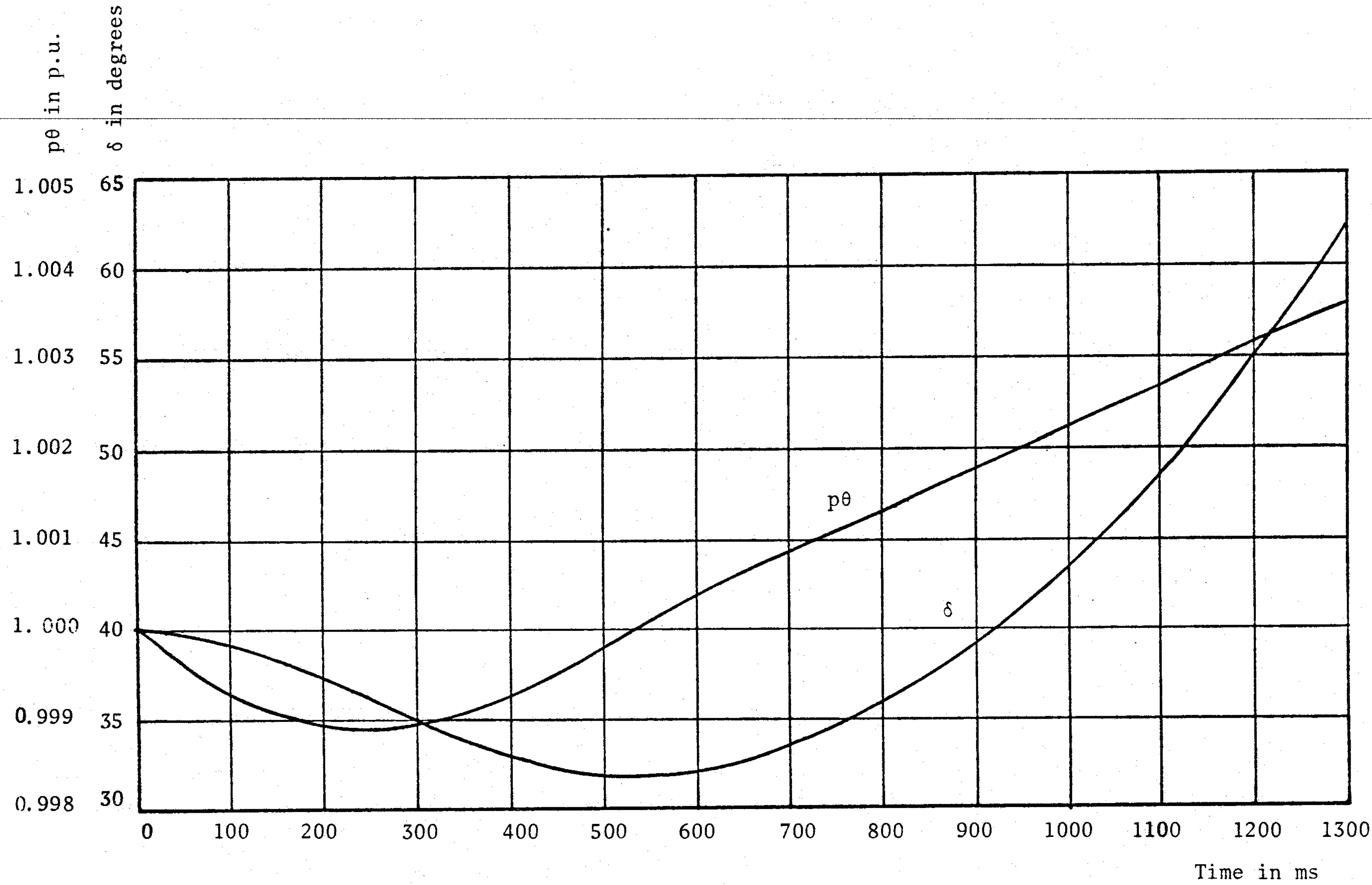


Fig. 5.39 The Rotor Angle and the Rotor Speed of the Synchronous Machine Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

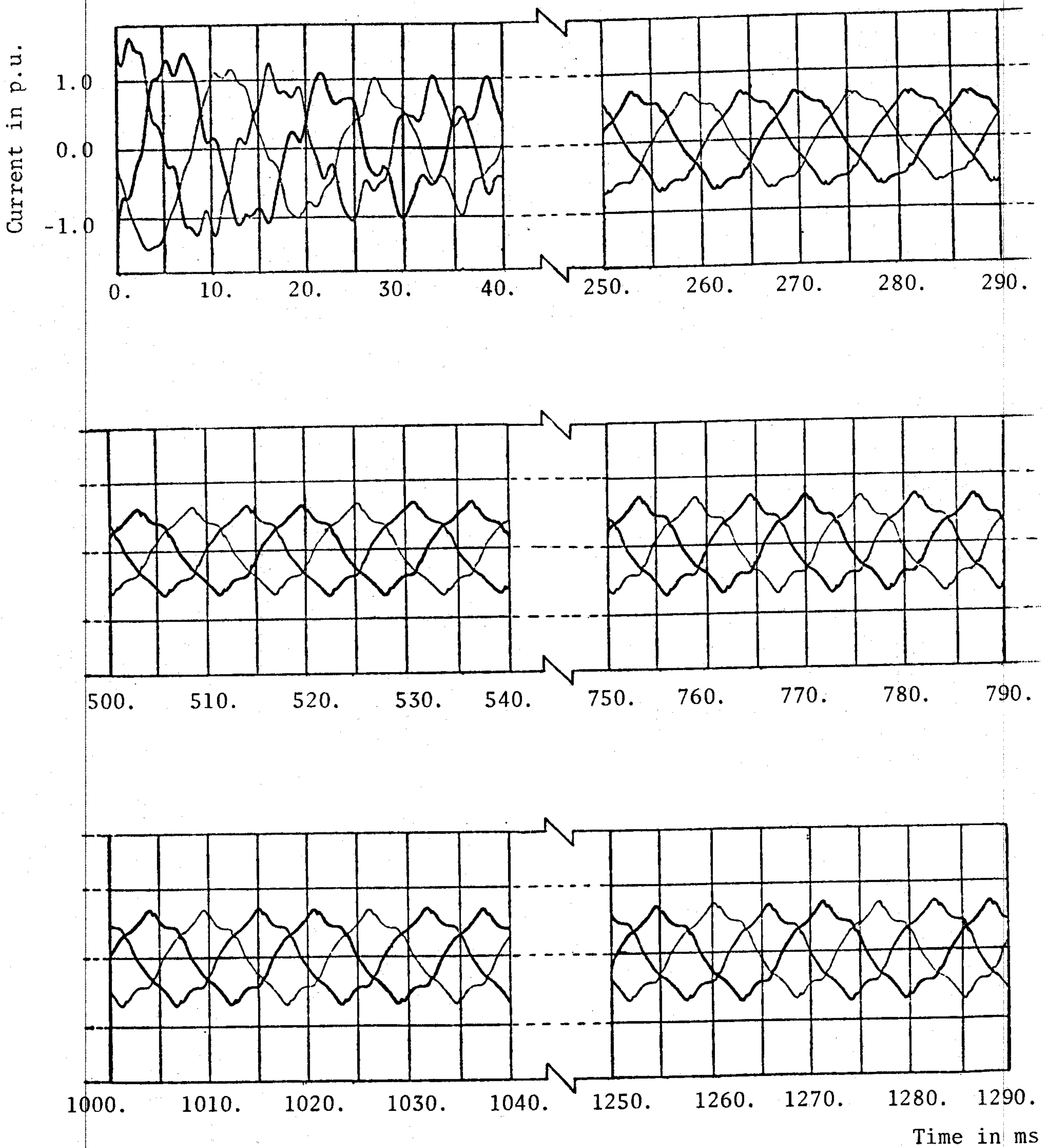


Fig. 5.40 The 3-phase Transmission Line Currents Versus the Time for the Case of Rectifier, Transmission Line, Synchronous Machine and Filters

6. CONCLUSIONS

In this thesis, a digital computer program is used to simulate the dynamic behavior of a representative AC/DC system. This system is composed of a synchronous generator supplying power to an AC/DC converter which is connected to an infinite bus-bar through a short transmission line. Also, a group of 5th, 7th and 11th order harmonic filters is used to reduce the harmonic contents in the AC system. This program is developed such as to allow the accommodation of various alternative configurations of the AC/DC system. Moreover, it can be used as a subroutine in a larger computer program to investigate a large interconnected AC/DC system.

All the components of this AC/DC system are dynamically represented by a group of equations in the state-space form. Direct-phase quantities are used throughout the representation so that the identity of the phase currents is retained to define the details of the successive commutation processes. Such a representation takes into consideration the dynamics of the alternating voltages, all the current harmonics and the synchronous machine dynamic performance.

The program has been applied to various cases in order to study the performance of the different components of the system. The cases considered are:

(1) A rectifier directly connected to an infinite bus-bar.

(2) A rectifier connected to an infinite bus-bar through a short transmission line.

(3) A rectifier connected to an infinite bus-bar through a

short transmission line and a group of 5th, 7th and 11th order harmonic filters is connected at the rectifier AC bus-bar.

(4) A synchronous generator is supplying power to a rectifier which is connected to an infinite bus-bar through a short transmission line.

(5) A group of 5th, 7th and 11th order harmonic filters is connected to the system of case 4.

For these cases, the program generates the waveforms of all the variables of the system directly after the sudden connection of the converter station to the AC system. As it is expected, the derived waveforms start with a transient period and it has been found that they reach the electrical steady-state condition after about 5 cycles. However, in the cases where the synchronous machine is present, a damped slow oscillation exists due to the electromechanical oscillation of the machine.

From the studies carried out on the model, several conclusions can be drawn. These are:

(1) It has been found that the addition of the transmission line to the AC system increases the commutation period due to the increase in the commutating reactance. The addition of filters to the AC system, which consists of a rectifier and a transmission line, reduces the commutation period. This is because the filters in the circuit moves the commutation point, at which the commutating emf is assumed sinusoidal, towards the bus-bar at which they are connected. Thus, the commutating reactance is reduced to approximately the converter transformer leakage reactance. Also,

if a synchronous machine is connected to a system consisting of a rectifier and a transmission line, the period of commutation is reduced. This can be attributed to the fact that the connection of the machine to this system adds a reactance in parallel to the AC system and reduces the effective commutating reactance. Moreover, if filters are added to a system which consists of a rectifier, a transmission line and a synchronous machine, the commutation period is reduced. In this case, the effective commutating reactance is also reduced. It will be approximately the converter transformer leakage reactance.

(2) The harmonic currents flowing in the AC system due to the AC/DC converter distort the waveforms of the different variables of the AC side. However, the presence of the 5th, 7th and 11th order harmonic filters reduces this distortion.

(3) When the synchronous machine is connected without filters to the terminals of the converter it introduces a filtering action to the harmonics. It reduces the distortion of the different AC quantities. However, this filtering effect is not as efficient as using filters. This may be improved if the synchronous machine is designed to have a low subtransient reactance. This can be achieved by providing low resistance damper windings in the rotor of such machines.

(4) Harmonic currents affect the operation of synchronous machines. The flow of these harmonic components of the order $6k \pm 1$ (k is an integer) in the stator windings creates harmonic components of the order $6k$ in the rotor circuits. The flow of these currents in

the stator and the rotor windings increases the losses and the temperature rise of both windings. Moreover, the presence of high frequency currents in the rotor windings gives rise to skin effect. Due to this skin effect, the rotor parameters will be dependent on the frequency.

(5) The addition of filters to the converter AC terminals reduces the harmonic contents in the currents of the rotor and the stator windings. Therefore, the machine performance is improved and the pulsating electrical torques are reduced.

(6) Even with the presence of the filters in the system, some harmonic currents are still flowing in the AC system and the stator windings of the synchronous machine. This is because these filters are of the 5th, 7th and 11th order only and their impedances still have definite values at their resonant frequencies. Thus, the corresponding harmonic currents will not be completely short-circuited by these filters. Therefore, some of these harmonic currents in addition to the higher order harmonics, which are not filtered, will flow in the AC system.

(7) As mentioned above, some harmonics still flow in the synchronous machine windings despite the presence of the filters. Therefore, careful consideration should be taken in the design of synchronous machines operating near AC/DC converter stations so that they can withstand the extra heatings due to the flow of these harmonics in their windings. Also, with the presence of synchronous machines in the system less filters ratings can be chosen.

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8. APPENDICES

Appendix A: Transmission Lines Representation for Harmonic Frequencies

Transmission lines are usually classified according to their length. This classification[48] depends upon what approximations are justified in treating the parameters of the line. Resistance, inductance and capacitance are uniformly distributed along the line, and exact calculations of long lines must recognize this fact. Lines less than 50 miles long are short lines and are represented by a lumped equivalent impedance. The shunt capacitance is very small and usually neglected. Medium length lines are roughly between 50 and 150 miles long. For these lines, the shunt admittance, generally pure capacitance, is included in the calculations. If all the shunt admittance is lumped at the middle of the circuit representing the line, the circuit is called a nominal-T. Such a circuit is shown in Fig. 8.1 where:

$Z = z\ell$ = total series impedance per phase

$Y = y\ell$ = total shunt admittance per phase to neutral

ℓ = length of line

z = series impedance per unit length per phase

y = shunt admittance per unit length per phase to neutral

In the nominal-π circuit, the total shunt admittance is divided into two equal parts placed at the sending and receiving ends of the line as shown in Fig. 8.2.

The nominal-T and the nominal-π circuits are not equivalent to each other. Moreover, neither the nominal-T circuit nor the nominal-π

circuit represents exactly the line because they do not account for the parameters of the line being uniformly distributed. Therefore, in the case of long transmission lines (more than 150 miles), both circuits are inaccurate. In this case, the general equation of transmission lines, which takes into consideration that the line parameters are uniformly distributed, should be used. Moreover, if the solution of the transmission line is required only at the receiving and sending ends, an equivalent circuit can be used. This equivalent circuit, which represents the line accurately with respect to the ends, can be obtained from the measurements at the ends of the line. For the equivalent-π circuit, Fig. 8.3, the series impedance Z' and the shunt admittance $Y'/2$ can be derived from the general equation of the transmission line. These expressions in terms of Z and Y are:

$$Z' = Z\,\frac{\operatorname{sh}\gamma\ell}{\gamma\ell} = ZF_Z \qquad 8.1$$

$$\frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh(\gamma\ell/2)}{(\gamma\ell/2)} = \frac{Y}{2}\,F_Y \qquad 8.2$$

where

$\gamma = \sqrt{zy}$ and is called the propagation constant. The terms F_Z and F_Y are factors by which the series impedance and the shunt admittance of the nominal-π can be multiplied to convert the nominal-π to the equivalent-π. For small values of $\gamma\ell$, the factors F_Z and F_Y approach unity. This fact shows that the nominal-π is accurate enough to represent the medium length transmission line [48]. An equivalent-T circuit may also be found for a transmission line in a similar manner.

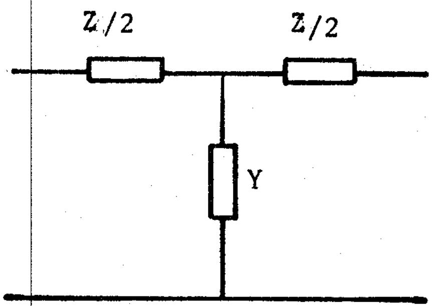


Fig. 8.1 Nominal-T Circuit of a Transmission Line

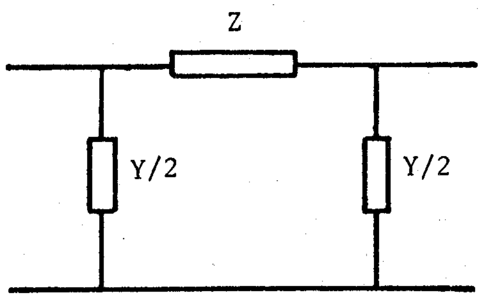


Fig. 8.2 Nominal-π Circuit of a Transmission Line

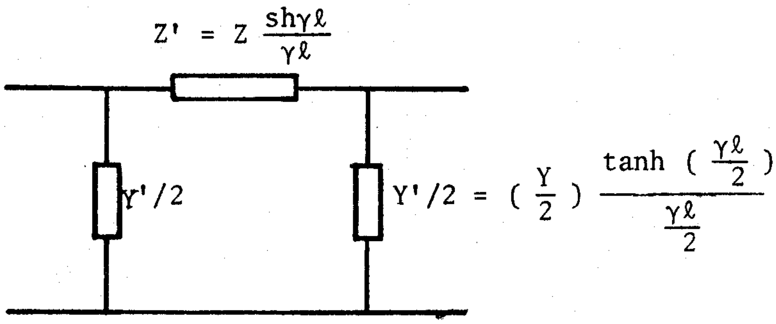


Fig. 8.3 Equivalent-π Circuit of a Transmission Line

If the line is split into two or more sections, each section can be represented by a nominal-T or -π with a reasonable accuracy. Such representation of the line will approach the equivalent circuit representation but the resulting work is more cumbersome than the use of the equivalent circuit in the first place.

The previous classifications of transmission lines according to length is valid only as far as fundamental frequency currents are concerned. To demonstrate this, a study is carried out to find out how the correction factors between the nominal-π and the equivalent-π circuits change with the length of the line for the different harmonic frequencies. The line investigated has the same parameters per unit length as the one of the AC/DC system under investigation in the thesis.

Fig. 8.4 shows the correction factor F_Z versus the length of the line for different harmonics. It can be noticed that, for the fundamental frequency, the factor F_Z differs from unity when the length of the line increases. However, for short and medium lines, it can be considered almost unity. For other harmonics, it is found that, even for short lines (less than 50 miles),the factor F_Z differs appreciably from unity. As the order of the harmonics increases, the deviation of F_Z from unity increases.

The correction factor for the shunt admittance F_Y is shown in Fig. 8.5. By examining this figure, it is also noticed that, for the fundamental frequency, this factor is almost unity for the short and medium lines. However, it differs from unity as the length increases. It is also found that F_Y is far away from unity

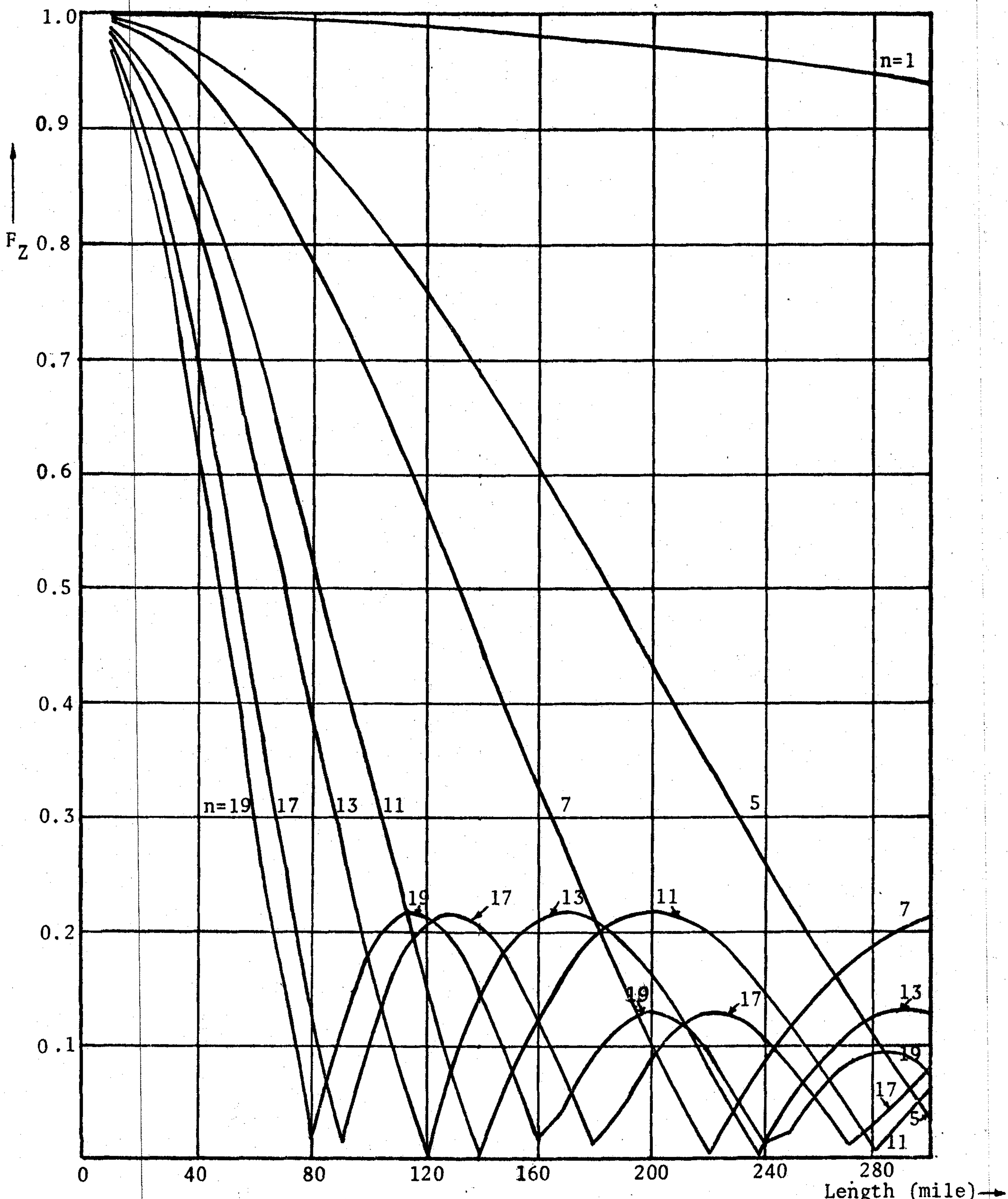


Fig. 8.4 Variation of the Factor F_Z with the Length of the Transmission Line for Different Harmonic Frequencies

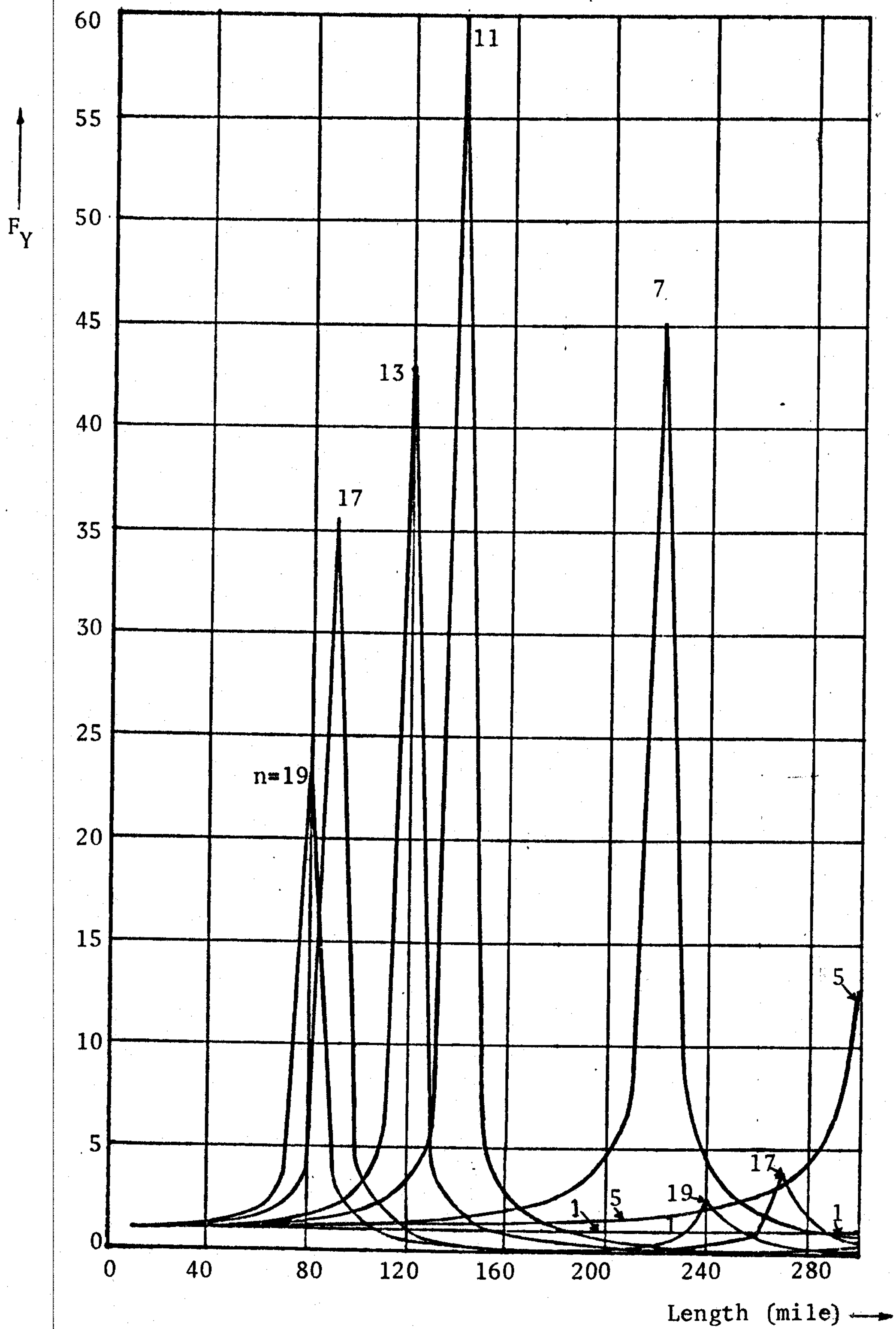


Fig. 8.5 Variation of the Factor F_Y with the Length of the Transmission Line for Different Harmonic Frequencies

as long as harmonic frequencies are concerned even for short lines.

It follows that, at fundamental frequency, the representation of a medium line by a nominal-π can be considered accurate enough. As far as harmonics are concerned, a nominal-π network cannot represent the line accurately even for medium or short lines. Thus, for the case of transient problems, e.g. travelling waves on the lines, sudden changes in the electrical quantities like those present in the system, the general differential equations of the line should be used or the line should be split to as many sections as needed and which can be represented by nominal-π circuits for all frequencies. However, for a very short transmission line (10 miles) like the one used in the AC/DC system under investigation in the thesis, the factors F_Z and F_Y approach unity for the practical range of the harmonics generated from the converter. In this case, a nominal-π can be considered accurate enough to represent this line. Since this line is very short, the admittance branch can be omitted and the line can be represented by an equivalent resistance in series with an equivalent inductance.

Appendix B: The Calculation of the Synchronous Machine Current Derivatives

In the matrix notation, the machine flux-linkages relationships in terms of the machine currents are:

$$[\psi] = [L][I] \tag{8.3}$$

therefore,

$$p[\psi] = [L]p[I]+[pL][I] \tag{8.4}$$

where

$$[\psi] = [\psi_a \quad \psi_b \quad \psi_c \quad \psi_{fd} \quad \psi_{kd} \quad \psi_{kq}]^t$$

$$[I] = [i_a \quad i_b \quad i_c \quad i_{fd} \quad i_{kd} \quad i_{kq}]^t$$

[L] is the inductance matrix of the machine which is given by expression 4.63.

[pL] is the derivative of the [L] matrix and can be defined as follows:

$$[pL] = \begin{bmatrix} 2L_{aa2}\sin\theta \; p\theta & -2L_{aa2}\sin2(\theta+30) \; p\theta & -2L_{aa2}\sin2(\theta+150) \; p\theta & -L_{afdo}\sin\theta \; p\theta & -L_{akdo}\sin\theta \; p\theta & -L_{akqo}\cos\theta \; p\theta \\ -2L_{aa2}\sin2(\theta+30) \; p\theta & 2L_{aa2}\sin2(\theta-120) \; p\theta & -2L_{aa2}\sin2(\theta-90) \; p\theta & -L_{afdo}\sin(\theta-120) \; p\theta & -L_{akdo}\sin(\theta-120) \; p\theta & -L_{akqo}\cos(\theta-120) \; p\theta \\ -2L_{aa2}\sin2(\theta+150) \; p\theta & -2L_{aa2}\sin2(\theta-90) \; p\theta & 2L_{aa2}\sin2(\theta-240) \; p\theta & -L_{afdo}\sin(\theta-240) \; p\theta & -L_{akdo}\sin(\theta-240) \; p\theta & -L_{akqo}\cos(\theta-240) \; p\theta \\ L_{afdo}\sin\theta \; p\theta & L_{afdo}\sin(\theta-120) \; p\theta & L_{afdo}\sin(\theta-240) \; p\theta & 0 & 0 & 0 \\ L_{akdo}\sin\theta \; p\theta & L_{akdo}\sin(\theta-120) \; p\theta & L_{akdo}\sin(\theta-240) \; p\theta & 0 & 0 & 0 \\ L_{akqo}\cos\theta \; p\theta & L_{akqo}\cos(\theta-120) \; p\theta & L_{akqo}\cos(\theta-240) \; p\theta & 0 & 0 & 0 \end{bmatrix} \qquad 8.5$$

From Eqn. 4.64

$$p[\psi] = [V] + [R][I] \tag{8.6}$$

where

$$[V] = [v_1 \quad v_2 \quad v_3 \quad v_{fd} \quad v_{kd} \quad v_{kq}]^t$$

$$[R] = \text{diag}\,[R_a \quad R_a \quad R_a \quad -R_{fd} \quad -R_{kd} \quad -R_{kq}]$$

The machine terminal voltages [V] can be obtained from the following equation:

$$[V] = [V_{AR}] + [R_E][I_{TR}] + [L_E]p[I_{TR}] \tag{8.7}$$

where

$$[V_{AR}] = [v_a \quad v_b \quad v_c \quad v_{fd} \quad v_{kd} \quad v_{kq}]^t$$

$$[I_{TR}] = [i_{t1} \quad i_{t2} \quad i_{t3} \quad i_{fd} \quad i_{kd} \quad i_{kq}]^t$$

$$[R_E] = \text{diag}\,[R_e \quad R_e \quad R_e \quad 0 \quad 0 \quad 0]$$

$$[L_E] = \text{diag}\,[L_e \quad L_e \quad L_e \quad 0 \quad 0 \quad 0]$$

By substituting Eqn. 8.7 in Eqn. 8.6, it follows

$$p[\psi] = [V_{AR}] + [R_E][I_{TR}] + [L_E]p[I_{TR}] + [R][I] \tag{8.8}$$

Equating Eqns. 8.4 and 8.8, therefore

$$[V_{AR}] + [R_E][I_{TR}] + [L_E]p[I_{TR}] + [R][I] =$$

$$[L]p[I] + [pL][I] \tag{8.9}$$

From Fig. 5.1, it can be noticed that:

$$[I_{TR}] = [I] - [I_{rec}] - [I_{FL}] \tag{8.10}$$

where

$$[I_{rec}] = [i_1 \quad i_2 \quad i_3 \quad 0 \quad 0 \quad 0]^t$$

$$[I_{FL}] = [i_{f\ell 1} \quad i_{f\ell 2} \quad i_{f\ell 3} \quad 0 \quad 0 \quad 0]^t$$

$$i_{f\ell 1} = i_{f\ell_5 1} + i_{f\ell_7 1} + i_{f\ell_{11} 1}$$

$$i_{f\ell 2} = i_{f\ell_5 2} + i_{f\ell_7 2} + i_{f\ell_{11} 2}$$

$$i_{f\ell 3} = i_{f\ell_5 3} + i_{f\ell_7 3} + i_{f\ell_{11} 3}$$

Substituting Eqn. 8.10 in Eqn. 8.9, therefore

$$[V_{AR}] + [R_E][I-I_{rec}-I_{FL}] + [L_E]p[I-I_{rec}-I_{FL}] + [R][I] =$$

$$[L]p[I] + [pL][I]$$

$$[V_{AR}] + [R_E+R][I] - [R_E][I_{rec}] - [R_E][I_{FL}] - [L_E]p[I_{rec}] -$$

$$[L_E]p[I_{FL}] - [pL][I] = [L-L_E]p[I]$$

$$[L_m]p[I] = [V_{AR}] + [R_m][I] - [R_E][I_{rec}] - [R_E][I_{FL}] -$$

$$[L_E]p[I_{rec}] - [L_E]p[I_{FL}] - [pL][I] \qquad 8.11$$

Therefore, the machine current derivatives can be obtained from the following relations:

$$\text{A} \quad p[I] = [L_m]^{-1}[V_{AR}] + \overset{C}{[L_m]^{-1}[R_m][I]} - \overset{E}{[L_m]^{-1}[R_E][I_{rec}+I_{FL}]}$$

$$-[L_m]^{-1}[L_E]p[I_{rec}+I_{FL}] - [L_m]^{-1}[pL][I] \qquad 8.12$$

where

$$[L_m] = [L] - [L_E]$$

$[L_m]$ is the modified inductance matrix having the same elements as the L matrix defined before except for the following:

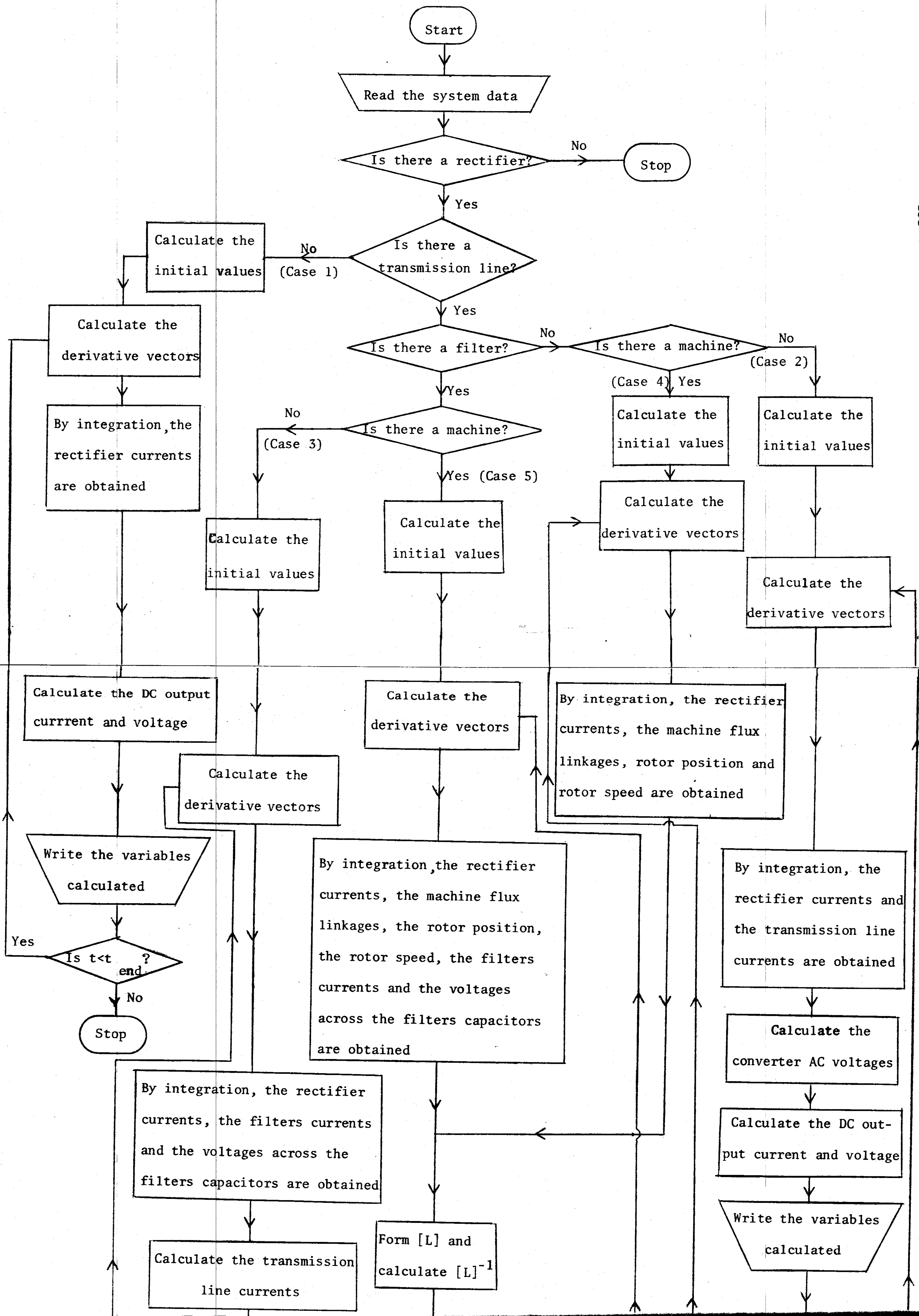
$$L_{m1,1} = -L_{aao} - L_e - L_{aa2} \cos 2\theta$$

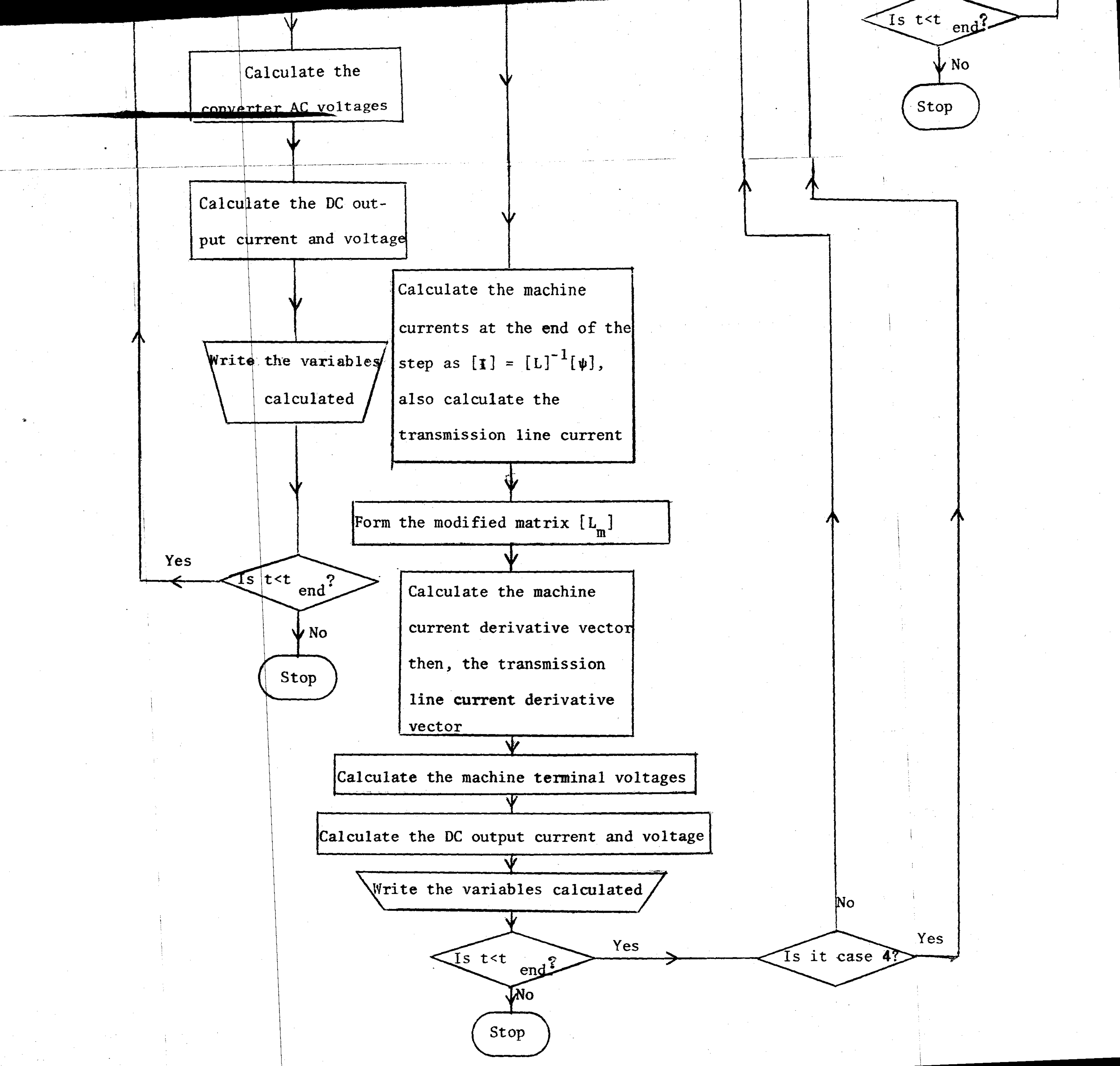
$$L_{m2,2} = -L_{aao} - L_e - L_{aa2} \cos 2(\theta-120)$$

$$L_{m3,3} = -L_{aao} - L_e - L_{aa2} \cos 2(\theta-240)$$

$$[R_m] = \text{diag}\,[R_a+R_e \quad R_a+R_e \quad R_a+R_e \quad -R_{fd} \quad -R_{kd} \quad -R_{kq}]$$

Appendix C: A Simplified Flow Chart for the Computer Program





Calculate the converter AC voltages
Calculate the DC output current and voltage
Write the variables calculated
Yes
Is t<t end?
No
Stop
Calculate the machine currents at the end of the step as [I] = [L]^-1[ψ], also calculate the transmission line current
Form the modified matrix [Lm]
Calculate the machine current derivative vector then, the transmission line current derivative vector
Calculate the machine terminal voltages
Calculate the DC output current and voltage
Write the variables calculated
Is t<t end?
Yes
No
Stop
Is it case 4?
No
Yes
Is t<t end?
No
Stop

Appendix D: Per Unit System of the AC/DC System

D.1. Base values:

It is a common practice to choose the rated current and the rated phase voltage (rms values) of the AC system to be the AC current and voltage base values for the steady-state operation respectively. In such a case, AC/DC converters can be treated as voltage and frequency transformers and the base values[50] of the DC side variables can be defined according to their relation with the AC side variables.

As the problem in this thesis is of a transient nature and the solution is obtained in instantaneous values, it is rather preferred to use the maximum values of the current and phase voltage than their root mean square values as base values of the AC side. For the DC side, the converter valves can be looked at as switches. This means that the instantaneous AC line voltage will appear at the bridge DC terminals as instantaneous DC voltage. Also, the instantaneous DC current will be the same as the instantaneous AC current. For this reason, the base values of the DC side can be chosen the same as the base values of the AC side. If I_n and V_n are considered to be the rated current and the rated phase voltage of the AC side respectively, the base values for the different parameters are as follows:

(i) The base value for all AC and DC voltages is the amplitude of the AC side phase voltage. Hence:

$$V_{ACb} = V_{DCb} = \sqrt{2}\, V_n \qquad 8.13$$

(ii) The base value for all AC and DC currents is the amplitude of the AC side phase current. Hence:

$$I_{ACb} = I_{DCb} = \sqrt{2}\, I_n \qquad 8.14$$

(iii) The base value of the power is:

$$P_b = 3V_nI_n = \frac{3}{2} V_{ACb}I_{ACb} = \frac{3}{2} V_{DCb}I_{DCb} \qquad 8.15$$

This is the base for all active and reactive power and for the kVA. It is also the base for the DC power.

(iv) The base value of the impedance for both the AC and the DC sides is:

$$Z_b = \frac{V_n}{I_n} \qquad 8.16$$

(v) The base value of the inductance for both the AC and the DC sides is:

$$L_b = \frac{Z_b}{2\pi f} \qquad 8.17$$

(vi) The base value of the capacitance for both the AC and the DC sides is:

$$C_b = \frac{1}{2\pi f Z_b} \qquad 8.18$$

D.2. Normalized equation of the AC power

The instantaneous AC power in absolute quantities is given by:

$$P_{AC} = v_1i_1 + v_2i_2 + v_3i_3 \qquad 8.19$$

Therefore, for a base power $P_b = 3V_nI_n$, it follows:

$$\frac{P_{AC}}{P_b} = \frac{v_1i_1+v_2i_2+v_3i_3}{3V_nI_n} = \frac{v_1i_1+v_2i_2+v_3i_3}{\frac{3}{2} V_{ACb}I_{ACb}}$$

Thus

$$P_{ACp.u.} = \frac{2}{3}(v_{1p.u.}i_{1p.u.} + v_{2p.u.}i_{2p.u.} + v_{3p.u.}i_{3p.u.}) \qquad 8.20$$

D.3. Normalized equation of the DC power

The instantaneous DC power is given by the following equation;

$$P_{DC} = v_{DC}\, i_{DC} \qquad 8.21$$

Since the base power P_b is the same for both the AC and DC sides and is given by:

$$P_b = 3V_nI_n \ ,$$

the normalized equation of the DC power can be written as follows:

$$P_{DCp.u.} = \frac{v_{DC} \ i_{DC}}{3V_nI_n} = \frac{v_{DC} \ i_{DC}}{\frac{3}{2} \ V_{DCb}I_{DCb}}$$

$$= \frac{2}{3} \ (v_{DCp.u.} \cdot i_{DCp.u.}) \qquad 8.22$$