existence will have to affect other groups and raise secondary questions which must be solved. A third and vital intangible asset is the wake of trained people that such a project produces.

3) The digital computer has been in operation over a year and all programs work in practice as planned.

4) Dispatcher–computer communication is practical and working well. The dispatchers who are responsible for its proper operation have shown good acceptance and are daily exhibiting imagination in the communication with and use of the digital computer.

5) The program has at no time produced a predispatch showing less dollar cost with a system constraint than the same system condition without the system constraint, indicating the effectiveness of the program in optimizing dollar dispatch cost.

6) The pumped hydro and stored hydro water balance algorithm has always worked as planned.

7) The various techniques described in the paper made it possible to limit the predispatch running time to well within acceptable limits.

8) The off-line predispatch study is used for operation planning and, in addition, it can also be used for studies not strictly pertaining to day by day dispatch, such as providing the necessary daily dispatch studies of hydro generation to produce a hydro rule curve.

9) Predispatch running time is generally between 2 and 30 minutes depending on the mode and on the stored hydro and pumped hydro storage constraints.

Summary

Considerable effort by both manufacturer and Union Electric personnel has led to the realization of the goal of a practical optimum predispatch program and on-line control system, including the classical elements of conventional hydro and steam dispatch with the added dimensions of pumped hydro storage and economy interchange dispatch.

Innovations, such as the development of the prorate cycle cost method of unit commitment, inclusion of optimum economy interchange evaluation and pricing as an integral part of system dispatch, and the development of a sophisticated water closure logic scheme led to an overall system which is valid and practical from the standpoints of operability, memory requirements, and program running time.

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Digital Simulation of Multimachine Power Systems for Stability Studies

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Abstract—**A digital simulation technique suitable for detailed analysis of both large and small disturbances on extensive power systems is presented. The analysis employs a hybrid reference frame for statement of the problem and for its solution. The equations of the interconnecting network are expressed with regard to a synchronously rotating common reference frame and are treated with the aid of matrix methods. Synchronous machine equations and equations of the voltage regulator and of the speed governor are solved in Park's reference frame fixed to the field of each individual machine. Provision is made for representing different machines in different degrees of detail. An efficient numerical technique for solution of the resulting complex nonlinear equations describing the behavior of the complete power system is introduced.**

IN VIEW OF the increasing complexity of present-day power systems, their design and operation requires a more detailed analysis of possible performance modes than may be achieved by available computer programs. In overcoming this difficulty, the present paper describes a digital simulation technique particularly applicable to stability studies of extensive systems.

Studies of power system stability to date have developed along two distinct paths. One approach, largely used in digital computer programs,[1],[2] is based on the well-established network-analyzer simulation. This approach is well suited for handling large power systems; however, its effectiveness is reduced by the inherent oversimplification in the representation of synchronous machines. The inadequacy is only partially offset by special techniques that approximately account for damping and braking torques. The second approach, widely used on analog computers,[3]–[6] is more closely based on Park's equations[7] and provides a more detailed representation of synchronous machines. Unfortunately, an analog computer is unable to handle any practical-size power system. A different approach is proposed by the present paper which combines the strong points of both approaches and, in consequence, is particularly suited for detailed stability analysis of large power systems.

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Development of Equations for Multimachine System

Equations describing the behavior of the entire system are developed on the basis of a hybrid reference frame. Each individual synchronous machine is described by Park's equations in a reference frame fixed to its own field and rotating with it. The complete set of equations includes governor and excitation systems. Provision is made to represent by simpler models, if desired, machines electrically remote from a disturbance.

The interconnecting network is stated in terms of a *mixed* network matrix composed of impedances, admittances, and dimensionless elements. The associated solution for currents and voltages at each specified node is expressed with regard to the synchronously rotating *common reference frame.*

Machines are *connected* to the network at the specified nodes, at which voltages and currents in the two reference frames are related to one another by *axis transformation.* During any disturbance, speeds of machines change and hence their individual reference frames *oscillate* with respect to the synchronously rotating *common reference frame.*

In order to distinguish between reference frames, subscripts d and q are used to denote components along the two axes of machine reference frames while subscripts D and Q refer to components of the *common reference frame.*

Description of Synchronous Machine

Complete description of the dynamic behavior of the synchronous machine requires consideration of its electrical and mechanical characteristics as well as those of associated control systems. The necessary mathematical statements are summarized in the following paragraphs. Only those modes of operation that do not require zero-axis variables are considered.

Electrical Equations: Park's model describing the dynamic characteristics of a synchronous machine in per-unit[4],[8] form is given by the following equations:

Direct-axis flux linkage

$$\psi_{fd} = x_{ffd} i_{fd} + x_{ad} i_{kd} - x_{ad} i_d \tag{1}$$

$$\psi_d = x_{ad} i_{fd} + x_{ad} i_{kd} - x_d i_d \tag{2}$$

$$\psi_{kd} = x_{ad} i_{fd} + x_{kkd} i_{kd} - x_{ad} i_d. \tag{3}$$

Quadrature-axis flux linkage

$$\psi_q = x_{aq} i_{kq} - x_q i_q \tag{4}$$

$$\psi_{kq} = x_{kkq} i_{kq} - x_{aq} i_q. \tag{5}$$

Direct-axis voltages

$$v_{fd} = \frac{1}{\omega_0} p\psi_{fd} + r_{fd} i_{fd} \tag{6}$$

$$e_d = \frac{1}{\omega_0} p\psi_d - r i_d - \frac{\omega}{\omega_0} \psi_q \tag{7}$$

$$0 = \frac{1}{\omega_0} p\psi_{kd} + r_{kd} i_d. \tag{8}$$

Quadrature-axis voltages

$$e_q = \frac{1}{\omega_0} p\psi_q - r i_q + \frac{\omega}{\omega_0} \psi_d. \tag{9}$$

$$0 = \frac{1}{\omega_0} p\psi_{kq} + r_{kq} i_{kq}. \tag{10}$$

The per-unit system chosen is such that all per-unit mutual inductances between rotor and stator circuits in each axis are equal to one another. On this basis the following relations between self-, mutual, and leakage reactances pertain:

$$\begin{aligned} x_{ffd} &= x_{ad} + x_{f1} \\ x_d &= x_{ad} + x_{a1} \\ x_{kkd} &= x_{ad} + x_{kd1} \\ x_q &= x_{aq} + x_{a1} \\ x_{kkq} &= x_{aq} + x_{kq1}. \end{aligned} \tag{11}$$

Saturation: To provide for the variation of the machine reactances with iron saturation, it is assumed that as a consequence of saturation only the mutual flux linkage ψ_{ad} is reduced by a factor K_s and that leakage reactances are not significantly affected. Under this assumption, all that is needed is to replace x_{ad} by $K_s x_{ad0}$ in the developed machine equations. With knowledge of the open-circuit magnetization curve, the saturation factor K_s may be expressed as a function of ψ_{ad}.

For round rotor machines, x_{aq} also is replaced by $K_s x_{aq0}$ and the factor K_s is now expressed as a function of total mutual flux $\psi_t = (\psi^2_{ad} + \psi^2_{aq})^{1/2}$. The method used for the inclusion of saturation is by no means very rigorous. However, it provides a simple approach and leads to results that are reasonably well substantiated in practice.[4]

Control System Equations

Voltage Regulator: A widely used model for the continuously acting voltage regulator with derivative stabilizing transformer may be described by

$$E_{fd} = \frac{\mu}{1 + \tau_e p} (V_r - e_t - v_s) \tag{12}$$

$$v_s = \frac{\mu_s p}{1 + \tau_s p} E_{fd} \tag{13}$$

where

$$e_t = \sqrt{e^2_d + e^2_q}.$$

Speed Governor: The effect of the conventional speed governor may be represented by

$$\Delta T = \frac{\mu_g}{(1 + \tau_g p)(1 + \tau_h p)} \frac{p\alpha}{\omega_0}. \tag{14}$$

The foregoing equations are similar to those used in other papers.[3],[5],[9] If desired, any other model for the voltage regulator or speed governor can be easily introduced.

Mechanical Equations: In order to complete the description of the synchronous machine, the following equations of motion are necessary:

$$T_g = \psi_d i_q - \psi_q i_d \tag{15}$$

$$T_i = M(p^2\alpha) + T_g + K_d(p\alpha) + \Delta T. \tag{16}$$

Here α is the angle by which the d axis of the machine leads the D axis of the common reference frame.

Manipulation of Machine Equations

It is not desirable to solve (1)–(16) in their present form. First, the electrical equations (1)–(11) are manipulated to remove all variables other than the *integrable variables* and terminal voltages. Then the entire set of equations is rearranged in a form suitable for numerical integration:

$$p\psi_{fd} = \omega_0 \left[\frac{r_{fd}}{x_{ad}} E_{fd} + \frac{r_{fd}}{x_{f1}} (\psi_{ad} - \psi_{fd})\right] \tag{17}$$

$$p\psi_d = \omega_0 \left[e_d + \psi_q \frac{\omega}{\omega_0} + \frac{r}{x_{a1}} (\psi_{ad} - \psi_d)\right] \tag{18}$$

$$p\psi_{kd} = \omega_0 \frac{r_{kd}}{x_{kd1}} (\psi_{ad} - \psi_{kd}) \tag{19}$$

$$p\psi_q = \omega_0 \left[e_q - \psi_d \frac{\omega}{\omega_0} + \frac{r}{x_{a1}} (\psi_{aq} - \psi_q)\right] \tag{20}$$

$$p\psi_{kq} = \omega_0 \frac{r_{kq}}{x_{kq1}} (\psi_{aq} - \psi_{kq}). \tag{21}$$

In (17)–(21), ψ_{ad} and ψ_{aq} are mutual flux linkages given by

$$\psi_{ad} = \frac{1}{K_1}\left(\frac{\psi_d}{x_{a1}} + \frac{\psi_{fd}}{x_{f1}} + \frac{\psi_{kd}}{x_{kd1}}\right) \quad (22)$$

$$\psi_{aq} = \frac{1}{K_2}\left(\frac{\psi_q}{x_{a1}} + \frac{\psi_{kq}}{x_{kq1}}\right). \quad (23)$$

Also, for (17)–(23),

$$K_1 = \frac{1}{x_{ad}} + \frac{1}{x_{f1}} + \frac{1}{x_{a1}} + \frac{1}{x_{kd1}}$$

$$K_2 = \frac{1}{x_{aq}} + \frac{1}{x_{a1}} + \frac{1}{x_{kq1}}$$

$$\omega = \omega_0 + p\alpha$$

$$x_{ad} = K_s x_{ad0}$$

$$x_{aq} = K_s x_{aq0} (K_s = 1 \text{ for salient-pole machines})$$

$$K_s = f(\psi)$$

where

$$\psi = \psi_{ad} \text{ for salient-pole machines}$$

$$\psi = (\psi^2_{ad} + \psi^2_{aq})^{1/2} \text{ for round rotor machines.}$$

Similarly, mechanical and control system equations written as a set of first order differential equations become

$$p(p\alpha) = \frac{1}{M}[T_i - T_g - K_d(p\alpha) - \Delta T] \quad (24)$$

$$p(\alpha) = (p\alpha) \quad (25)$$

$$p(E_{fd}) = \frac{1}{\tau_e}[\mu(V_r - e_t - v_s) - E_{fd}] \quad (26)$$

$$p(v_s) = \frac{1}{\tau_s}[\mu_s(pE_{fd}) - v_s] \quad (27)$$

$$p(p\Delta T) = \frac{1}{\tau_g \tau_h}\left[\frac{\mu_g(p\alpha)}{\omega_0} - (\tau_g + \tau_h)(p\Delta T) - \Delta T\right] \quad (28)$$

$$p(\Delta T) = (p\Delta T). \quad (29)$$

The terminal currents required for *connection* with the network are

$$i_d = \frac{1}{x_{a1}}(\psi_{ad} - \psi_d) \quad (30)$$

$$i_q = \frac{1}{x_{a1}}(\psi_{aq} - \psi_q). \quad (31)$$

Equations (17)–(31) describe the operation of a synchronous machine in the reference frame rotating with its field. The behavior of the entire power system is expressed by one such set of equations for each machine together with the terminal constraints imposed by the interconnecting network. In the process of computation, voltages e_d and e_q, which result from network constraints and appear as *nonintegrable variables* in machine equations, are considered as the input quantities for the solution of the machine differential equations, whereas currents i_d and i_q are looked upon as their output quantities.

Provision for Simpler Models

In the analysis of a large power system, it is often not necessary to describe all the machines to the same degree of detail. The actual machine being investigated or machines close to the disturbance may be represented in as much detail as desired, and the degree of detail may be decreased as one moves away from the point of interest.[4]

As a first order simplification, the amortisseur effects are neglected and an equivalent damping coefficient is used in the torque equations. As a further simplification, the effects of voltage regulator and speed governor are neglected, in which case the machine is represented by a fixed voltage behind its transient reactance.

Combination of Machines

If two or more similar machines are connected to the same node, they may be represented by an equivalent machine whose resistance and reactance parameters are obtained by treating them as if the corresponding resistances or reactances of the individual machines were connected in parallel. The equivalent inertia constant is the sum of the inertia constants of individual machines.

If two unlike machines are connected to the same node, it is necessary to represent them separately.

Network Equations

In stability studies it has been found adequate to represent the network as a collection of lumped resistances, inductances, and capacitances, and to neglect the short-lived electrical transients in the transmission system.[3],[5],[9],[10] As a consequence of this fact, the terminal constraints imposed by the network appear as a set of algebraic equations which may be conveniently solved by matrix methods.

It is shown in Appendix I that the interconnecting network may be suitably represented by a *mixed matrix* composed of submatrices of impedance, admittance, and dimensionless elements, respectively. The network equations may then be written in real matrix form:

$$[e_D] = [R][i_D] - [x][i_Q] - [RG][E_{MD}] + [xB][E_{MQ}]$$

$$[e_Q] = [R][i_Q] + [x][i_D] - [RG][E_{MQ}] - [xB][E_{MD}] \quad (32)$$

$$[I_{MD}] = [RG]^T[i_D] - [xB]^T[i_Q] + [G][E_{MD}] - [B][E_{MQ}]$$

$$[I_{MQ}] = [RG]^T[i_Q] + [xB]^T[i_D] + [B][E_{MD}] + [G][E_{MQ}] \quad (33)$$

where

$[e_D]$ and $[i_D]$	vectors representing D axis components of voltages and currents at the n nodes to which machines represented in detail by Park's equations are connected
$[e_Q]$ and $[i_Q]$	vectors representing Q axis components of the foregoing quantities
$[E_{MD}]$ and $[I_{MD}]$	vectors representing D axis components of voltages and currents at the m nodes behind the reactances of the machines represented by constant voltages and transient reactances
$[E_{MQ}]$ and $[I_{MQ}]$	vectors representing Q axis components of the foregoing quantities.

It may be noted that the network solution described involves only matrix multiplication. This is a very desirable feature increasing the speed of computation, since the network must be solved completely once during each step of integration.

Comments on $p\psi_d$ and $p\psi_q$ Terms

A remark on the effect of including $p\psi_d$ and $p\psi_q$ terms in (7) and (9) is appropriate at this stage. In most stability analyses,[3],[5],[9] these terms are equated to zero. In the proposed simulation they are included mainly to represent accurately the braking torques which exist in machines during a fault. The effect of these torques is considered very important[6] and has to be accurately accounted for in any detailed analysis.

Incidentally, there is an added advantage to the necessity of solving network equations separately when $p\psi_d$ and $p\psi_q$ terms are present. The solution in that case, as explained earlier, involves only matrix multiplication. If, however, $p\psi_d$ and $p\psi_q$ are neglected, the resulting algebraic equations describing machines have to be solved simultaneously with those of the network.[10], [11] In consequence, solution involving only matrix multiplication will not be feasible and it may then be necessary to resort to interative or other more complicated numerical techniques.

Axis Transformation

Equations (17)–(31) describe an individual machine with respect to its own reference frame. In general the reference frame of each machine is different from that of any other machine as well as from the common reference frame rotating at synchronous speed. Consequently, it is necessary to perform axis transformation at each *connection* node in order to relate the components of voltages and currents expressed in the d, q reference axes of each machine to the synchronously rotating reference axes D, Q of the network.

Phasor relations between the two reference frames are shown in Fig. 1. On its basis the transformation of e_D, e_Q to e_d, e_q and of i_d, i_q to i_D, i_Q may be stated as

$$e_d = e_D \cos \alpha + e_Q \sin \alpha$$

$$e_q = e_Q \cos \alpha - e_D \sin \alpha \tag{34}$$

$$i_D = i_d \cos \alpha - i_q \sin \alpha$$

$$i_Q = i_d \sin \alpha + i_q \cos \alpha. \tag{35}$$

For a synchronous machine represented by a fixed voltage behind the transient reactance, no axes transformation is needed. It is necessary only to find the components of the fixed voltage along the D, Q axes of the common reference frame corresponding to each position of the machine's rotor. The values of current components obtained from the network solution together with these voltage components give the electrical torque output to be used in (16).

If it is desired to represent one or more nodes as infinite buses, they can be handled in the same way as explained in the previous paragraph, with the only difference that in this case the components of the voltage along D, Q axes remain fixed. Consequently, any multiplication involving infinite bus voltage components need be performed only once and stored, thereby saving on computer time.

Initial Conditions

Before the differential equations representing the machines can be solved, it is necessary to find the initial values of pertinent variables. Prior to a disturbance the power output, power factor, terminal voltage, and current are known for each machine. Also, under steady-state conditions, i_{kd}, i_{kq}, and all the derivative terms are zero.

The situation is represented in the phasor diagram of Fig. 2, from which the load angle δ results as

$$\delta = \arctan \left(\frac{i_t x_q \cos \theta - i_t R \sin \theta}{e_t + i_t R \cos \theta + i_t x_q \sin \theta} \right) \tag{36}$$

where

δ angle between q axis and e_t
$\cos \theta$ power factor
i_t machine terminal current.

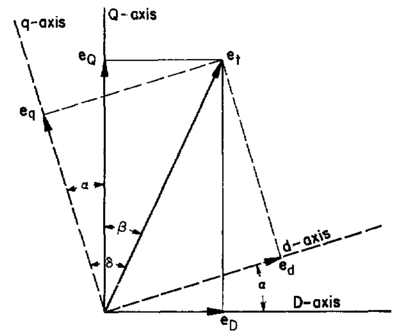

Fig. 1. Axis-transformation phasor diagram.

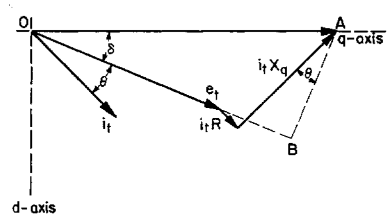

Fig. 2. Phasor diagram for computation of initial load angle.

Once the load angle δ is determined, other variables may be computed from

$$\begin{aligned}
e_d &= e_t \sin \delta \\
e_q &= e_t \cos \delta \\
i_d &= i_t \sin (\delta + \theta) \\
i_q &= i_t \cos (\delta + \theta) \\
\psi_q &= - e_d - r i_d \\
\psi_d &= e_q + r i_q \\
i_{fd} &= \frac{e_q + r i_q + x_d i_d}{x_{ad}} \\
\psi_{kd} &= x_{ad}(i_{fd} - i_d) \\
\psi_{kq} &= -x_{aq} i_q \\
\psi_f &= x_{ffd} i_{fd} - x_{ad} i_d \\
T_i &= \psi_d i_q - \psi_q i_d \\
e_{fd} &= i_{fd}\, r_{fd}.
\end{aligned} \tag{37}$$

The value of reactances x_{ad} and x_{aq} entering (37) are the saturated values and are actually given by

$$x_{ad} = K_s x_{ad0}$$

$$x_{aq} = K_s x_{aq0} \; (K_s = 1 \text{ for salient-pole machine}).$$

In the absence of a prior knowledge of K_s it may be assumed equal to unity and (37) may be solved iteratively along with the equation relating K_s to the mutual flux linkage.

Initially the *common reference frame* may be chosen arbitrarily. Once this is done, however, the values of β, the angle between the terminal voltage of each machine and the chosen Q axis, become fixed. With knowledge of β and δ, the angle between the reference frame of the individual machine and the *common reference frame* may be computed from

$$\alpha = \delta - \beta. \tag{38}$$

Computation Procedure

Stability analysis involves the solution of the set of differential equations (17)–(29) together with that of the algebraic equations (30)–(35).

Choice of Numerical Integration Scheme

The literature on numerical integration for solution of ordinary differential equations contains many techniques. In order to establish an efficient scheme of computation, Runge–Kutta and predictor–corrector methods are used alternatively.

Runge–Kutta–Gill Method (RKG): Among the family of Runge–Kutta methods, the most widely used version is that due to Gill. This method is readily applicable to digital computers requiring a minimum number of storage registers and controlling growth of round-off errors.[12] In common with any of the Runge–Kutta family of methods, it is self-starting and stable. As indicated by the description of the RKG method in Appendix II, the solution of a set of differential equations involves the evaluation of the rate of change of variables four times during each step of integration. The present case involves a set of differential equations which are to be solved simultaneously with a set of algebraic equations, and hence the latter have to be solved four times during each step.

Hamming's Predictor–Corrector Method: The reason for the choice of Hamming's method is the fact that in addition to being stable it is noniterative. The method is summarized in Appendix II and requires two evaluations of the rate of change of each variable per step. To find the solution at any instant, knowledge of four previous values of each variable is necessary and hence, besides requiring more storage registers, it is not self-starting.

Integration Step Length: As the total computing time is largely dependent on the integration interval, maximum permissible step length is a very important factor in the choice of the integration method. For accurate solution of the differential equations under consideration, tests revealed that, for both RKG and Hamming's methods, the largest step length permitted is in the neighborhood of 0.001 second.

This observation is in contrast with the results presented by Humpage and Scott,[13] where, working with a simplified model, it was found that predictor–corrector methods permitted larger step lengths than the RKG method. This observation is apparently the result of the fact that, when using the RKG method, Humpage and Scott solved algebraic equations only once at the end of each integration interval, whereby simultaneous solution of differential and algebraic equations was not achieved. Should simultaneous solution be performed, the permissible step length in the RKG method would be much larger, leading to an overall reduction of computation time in spite of the requirement that the network be solved a larger number of times during each interval.

Auxiliary Prediction of Nonintegrable Variables

It was mentioned earlier that Hamming's method requires the solution of algebraic equations twice during each integration interval, one of them being (53) of this paper. An alternative to the solution of the network at this point is the possibility of extrapolating the nonintegrable variables e_d and e_q. A suitable formula for extrapolation using four previous values is given by (39), which is derived on the basis of Lagrange's equations in Appendix III.

$$e_{K+1} = -e_{K-3} + 4e_{K-2} - 6e_{K-1}, + 4e_K. \tag{39}$$

Equation (39) was tested by comparing the resulting values of e_d and e_q with those obtained by the solution of the network. The agreement between the two sets was found to be very good.

Final Form of Integration Scheme

As a result of the auxiliary prediction of e_d and e_q, Hamming's method requires the network solution, for each integration step, only once in addition to two evaluations of the rate of change of integrable variables. With this scheme, total computing time required is about half that required by the RKG method. Therefore, it was decided to adopt Hamming's method throughout the computation except for starting, which is achieved by the RKG method. When there is an abrupt change in the function being integrated, for example, after a switching operation, the integration is restarted by the RKG method.

With a step length equal to 0.001 second, a 3-machine system with all the machines represented in complete detail requires, for a study time duration of 3 seconds, 1.3 minutes of computing time on an IBM 7094-II computer.

Conclusion

The digital simulation technique described in the paper is suitable for detailed stability analysis of a power system to any disturbance either large or small. The synchronous generator model used provides for an accurate account of effects related to slip and to various damping and braking torques. These are important factors to be considered in any detailed analysis. The approach presented here can be extended to include other situations such as unbalanced faults and dynamic loads.

Appendix I

Matrix Representation of Network

The network admittance matrix Y may be written in partitioned form as

$$Y = \begin{bmatrix} y_1 & y_{12} \\ y_{21} & y_2 \end{bmatrix}. \tag{40}$$

In the partioned matrix, the subscript 1 is associated with nodes to which controlled sources are connected and subscript 2 refers to those not connected to controlled sources. For the analysis no information about nodes associated with subscript 2 is necessary, and for this reason they are eliminated by a series of single row and column reductions in accordance with

$$Y_R = y_1 - y_{12}y_2^{-1}y_{21}. \tag{41}$$

The resultant network equations in terms of the reduced admittance matrix are

$$I = Y_R E.$$

These may be written in expanded form as

$$\begin{bmatrix} I_N \\ I_M \end{bmatrix} = \begin{bmatrix} Y_{NN} & Y_{NM} \\ Y_{MN} & Y_{MM} \end{bmatrix} \begin{bmatrix} E_N \\ E_M \end{bmatrix} \tag{42}$$

where the subscripts denote

- N nodes connected to synchronous machines represented in detail by Park's equations
- M nodes behind transient reactances of machines represented by fixed voltages.

In the analysis proposed, complete solution of the network is required once for each step of integration. Currents at the nodes associated with the subscript N are known after the solution of machine differential equations; voltages E_M, being fixed in magnitude, are also known. Therefore, the purpose of the network solution is to find voltages E_N and currents I_M for use in the next step of integration. For this purpose (42) is rearranged into

$$\begin{bmatrix} E_N \\ \hline I_M \end{bmatrix} = \left[\begin{array}{c|c} Z_{NN} & -Z_{NN}Y_{NM} \\ \hline Y_{MN}Z_{NN} & Y_{MM} - Y_{MN}Z_{NN}Y_{NM} \end{array}\right] \begin{bmatrix} I_N \\ \hline E_M \end{bmatrix} \tag{43}$$

where Z_{NN} is the inverse of Y_{NN} and can be found by any standard procedure.

If there are n nodes connected to machines associated with subscript N, and m nodes associated with subscript M, then the submatrices of (43) can be expressed in expanded form as

$$Z_{NN} = [R] + j[x] \tag{44}$$

where $[R]$ and $[x]$ are matrices of resistance and reactance elements, respectively, each of size $n \times n$;

$$(Y_{MM} - Y_{MN}Z_{NN}Y_{NM}) = [G] + j[B] \tag{45}$$

where $[G]$ and $[B]$ are matrices of conductance and susceptance elements, respectively, each of size $m \times m$; and

$$Z_{NN}Y_{NM} = [RG] + j[xB] \tag{46}$$

$$Y_{MN}Z_{NN} = [GR] + j[Bx]$$

where $[RG]$, $[GR]$, $[xB]$, and $[Bx]$ are matrices of dimensionless elements of size $(n \times m)$ or $(m \times n)$.

It may be noted here that matrices $[R]$, $[x]$, $[G]$, and $[B]$ are symmetric and therefore only about half of each has to be stored for computation. Storage requirements are further reduced by the fact that $[GR]$ and $[Bx]$ are transposes of $[RG]$ and $[xB]$, respectively.

Appendix II

Numerical Methods for Solution of Differential Equations

Runge–Kutta–Gill Method

A system of N first order differential equations is considered:

$$y_i'(t) = f_i(t, y, y_2, \ldots, y_N) \tag{47}$$

with the initial values $y_{i,0}$ for the N dependent variables and with $i = 1, 2, \ldots N$.

Using $j = 1, 2, 3, 4$ to denote the four stages in each step of integration, Gill's method of solving the given system of differential equations may be described as follows:[12]

$$y_{i,j}' = f_i(t, y_{1,j-1}, \ldots, y_{N,j-1}) \tag{48}$$

$$y_{i,j} = y_{i,j-1} + h[a_j(y_{i,j}' - b_j q_{i,j-1})] \tag{49}$$

$$q_{i,j} = q_{i,j-1} + 3[a_j(y_{i,j}' - b_j q_{i,j-1})] - C_j y_{i,j}' \tag{50}$$

where

h = integration step length

$$\begin{array}{lll} a_1 = 1/2 & b_1 = 2 & C_1 = a_1 \\ a_2 = 1 - \sqrt{1/2} & b_2 = 1 & C_2 = a_2 \\ a_3 = 1 + \sqrt{1/2} & b_3 = 1 & C_3 = 1 + \sqrt{1/2} \\ a_4 = 1/6 & b_4 = 2 & C_4 = 1/2. \end{array}$$

The solution for the dependent variables at the end of the step is given by $y_{i,4}$. Initially $q_{i,0} = 0$ and thereafter, in advancing the solution, $q_{i,0}$ for the next step is equated to $q_{i,4}$ of the preceding step.

Hamming's Method

The expression $y(t)$ is an N-dimensional vector, associated with N dependent variables of a system of differential equations

$$\frac{dy}{dt} = f(t,y).$$

using the symbols

$$y_K = y(t_K) \text{ and } y_K' = \left.\frac{dy}{dt}\right|_{t = tK}$$

$$K = 0, 1 \ldots$$

With t_K as the value of t at the Kth step of integration, and with h equal to the interval between steps, Hamming's method may be summarized by the following equations:[12]

Predictor:

$$P_{K+1} = Y_{K-3} + \frac{4h}{3}(2y_K' - y_{K-1}' + 2y_{K-2}') \tag{51}$$

Modifier:

$$m_{K+1} = P_{K+1} - \frac{112}{121}(P_K - C_K) \tag{52}$$

$$m_{K+1}' = f(t_{K+1}, m_{K+1}) \tag{53}$$

Corrector:

$$C_{K+1} = 1/8[9\, y_K - y_{K-2} + 3h(m_{K+1}' + 2y_K' - y_{K-1}')] \tag{54}$$

Final value:

$$y_{K+1} = C_{K+1} + \frac{9}{121}(P_{K+1} - C_{K+1}). \tag{55}$$

Appendix III

Formula for Auxiliary Prediction of Nonintegrable Variables e_d and e_q

Lagrange's polynomial $y(x)$ can be written in the form

$$y(x) = L_0(x)y_0 + L_1(x)y_1 + \ldots + L_n(x)y_n \tag{56}$$

where each of the expressions $L_i(x)$ is a polynomial of degree n given by

$$L_i(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_{i-1})(x - x_{i+1})\ldots(x - x_n)}{(x_i - x_0)(x_i - x_1)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n)}. \tag{57}$$

The value of the variable y is extrapolated using four previous values at equal intervals h in x

$$L_0(x) = \frac{(3h)(2h)(h)}{(-h)(-2h)(-3h)} = -1$$

$$L_1(x) = \frac{(4h)(2h)(h)}{(h)(-h)(-2h)} = 4$$

$$L_2(x) = \frac{(4h)(3h)(h)}{(2h)(h)(-h)} = -6$$

$$L_3(x) = \frac{(4h)(3h)(2h)}{(3h)(2h)(h)} = 4.$$

Substituting for different values of $L_i(x)$ in (56)

$$y(x_5) = -y(x_1) + 4y(x_2) - 6y(x_3) + 4y(x_4). \tag{58}$$

Rewriting (58) in a general form to give the value of voltage e at the instant $k + 1$ yields

$$e_{k+1} = -e_{k-3} + 4e_{k-2} - 6e_{k-1} + 4e_k. \tag{59}$$

Nomenclature

d, q	direct and quadrature axis of reference frame of individual synchronous machines
D, Q	direct and quadrature axis of common reference frame
ψ_d, ψ_q	direct and quadrature axis armature flux linkages
ψ_{kd}, ψ_{kq}	direct and quadrature axis amortisseur flux linkages
ψ_{fd}	field flux linkage
ψ_{ad}, ψ_{aq}	direct and quadrature axis mutual flux linkages
e_d, e_q, i_d, i_q	direct and quadrature axis voltages and currents
e_t, i_t	machine terminal voltage and current
v_{fd}, i_{fd}	field circuit voltage and current
$E_{fd} = v_{fd}\, x_{ad0}/r_{fd}$	air-gap line, open-circuit excitation voltage
$x_{ffd}, x_{kkd}, x_{kkq}$	rotor-circuit self-reactances on d and q axes
x_{ad}, x_{aq}	mutual reactances on d and q axes
x_{ad0}, x_{aqd0}	unsaturated values of mutual reactances
r_{fd}, r_{kd}, r_{kq}	rotor-circuit resistances on d and q axes
r	armature resistance
ω_0	rated angular frequency, electrical rad/s
ω	instantaneous angular frequency, electrical rad/s
α	angular displacement between d, q axes of machine and D, Q axes of common reference frame, rad
δ	angle between q axis of machine and its terminal voltage, rad
K_d	damping coefficient
K_s	saturation factor
H	inertia constant, kWs/kVA
M	angular momentum
T_i	initial torque input to rotor
ΔT	change in torque input to rotor owing to governor action
T_g	air-gap torque
V_r	voltage regulator reference voltage
v_s	derivative stabilizing signal
μ	voltage regulator open-loop gain
μ_g	loop gain of prime-mover governing system
μ_s	gain of derivative stabilizing loop
τ_e	time constant of main exciter, seconds
τ_s	time constant of derivative stabilizing loop, seconds
τ_g	relay (governor) time constant, seconds
τ_h	time constant representing turbine delay, seconds
t	time, seconds
p	differential operator, d/dt.

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Discussion

Friedrich Heilbronner (Institut für Hochspannungs- und Anlagentechnik, Technische Hochschule, Munich, Germany): In this paper the authors present a rigorous and very clear treatment of the complex situation in synchronous machines and associated networks during transients. The joint use of typical analog computer equations (17)–(31) and typical digital computer equations (32)–(33) and their connection by the transformations (34)–(35) and by the auxiliary prediction (56)–(57) is so excellent and lucid that only a few details perhaps require further clarification.

Since, in most of the cited references, the changes of the flux linkages were neglected, it would be of value to comment on the results of including these changes in the calculations. The mathematical procedure differs also from the usual step-by-step-method with trapezoid integration. Could the authors comment on the degree of exactitude obtained by using this refined procedure?

A third point gives rise to a more general discussion. The use of the per-unit system makes (1)–(10) and the equations which follow very comprehensive. With the digital computers available, per-unit values could of course be replaced by variables of true physical dimensions. But it is evident that the clarity of any presentation would suffer accordingly.

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K. Prabhashankar and **W. Janischewskyj:** The authors would like to thank Mr. Heilbronner for his kind remarks and wish to provide the following explanations to his questions.

The paper constitutes only the first phase of investigations commenced at the University of Toronto: the description of a digital computer program suitable for detailed analysis of the dynamic

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performance of power systems. The next step is the use of the program for study of computed stability performance resulting from omission of specific details in the system. Partial results of this study were given in the presentation; a separate paper is planned for future presentation.

The changes in the flux linkages included in (18) and (20) represent the electrical transients in the stator circuits. When these effects are considered, the air-gap torque includes high-frequency components as well as additional unidirectional terms. Of these, the effects of unidirectional components are very important for machines very close to a short circuit. The braking effect of these torque components is often so large that the machine actually slows down for the first few cycles before accelerating. Inclusion of $p\psi_d$ and $p\psi_q$ terms, as discussed in the paper, offers an additional advantage, since the solution of the network algebraic equations becomes noniterative. A disadvantage in this case, however, is that the maximum integration step length for numerically stable solution is limited to about 0.001 second rather than the 0.01 to 0.05 second permissible in the case when the stator transients are neglected. This fact may more than offset the advantage gained by the noniterative nature of the network solution. We are at present considering the advantages to be gained by the reduction of computer time when $p\psi_d$ and $p\psi_q$ terms are neglected and the resulting additional algebraic equations are solved by iterative techniques simultaneously with network equations. In this case the fundamental frequency components of i_d and i_q, which correspond to the dc and second harmonic components of the phase currents, do not appear in the solution. The stator and rotor resistance losses due to these components of currents may be computed to account for the initial braking torque.

As regards the numerical method used, in view of the relatively large number of differential equations involved, it was considered appropriate to use higher order integration methods which are more efficient than the simple step-by-step first-order method.

The third point raised by Mr. Heilbronner may be discussed in great detail; however, at this time it should suffice to state that the presented computer program could be easily modified to use all system variables in their true physical dimensions. On the other hand, the employment of the per-unit system is justifiable on the basis that system data are presently entered in the logs of power companies in per-unit values.

Formation of the Coefficient Matrix of a Large Dynamic System

JAMES E. VAN NESS, SENIOR MEMBER, IEEE, AND WILLIAM F. GODDARD

Abstract—**The coefficient matrix for the set of first-order differential equations that describe a dynamic system is formed from the block diagram or equations that describe the system. The method for accomplishing this is described in terms of matrix equations. Then the derivative of this coefficient matrix with respect to one of the system parameters is found in terms of the matrices used to form it. The coefficient matrix and its derivative can be used to find the eigenvalues of the system and the sensitivities of the eigenvalues to the system's parameters.**

Introduction

A WELL-KNOWN approach in control systems analysis is to represent the system by a set of first-order, linear differential equations and to examine the eigenvalues of these equations. If the equations describe the operation of the system around some equilibrium point, then the eigenvalues will characterize the dynamic response of the system. And, even though the equations are based upon a linear description of the system, the eigenvalues will give much information as to the response of the system to large disturbances where nonlinearities come into effect. Recently this approach has been successfully applied to the study of large power systems.[1]–[3] A difficulty is that the original description of the dynamic systems is generally in the form of block diagrams and not a set of differential equations.

A method will be described here of forming a set of equations of the form

$$\dot{Y} = AY \tag{1}$$

from the block diagram and other equations that describe a system. In this equation, Y is a vector of the state variables for the system. $\dot{Y}$ is its time derivative, and A is the coefficient matrix. It is from the coefficient matrix that the eigenvalues can be found. The method has been made very general so that many different configurations can be studied. Thus, with this method, voltage regulators, devices to control the power flow on dc transmission lines, different types of governors, and other control equipment can be easily added to the model, and their effect on systems dynamics studied. Using this method a computer program can be written which will take parameters of a block diagram (gains, time constants, and signal flow) and supplementary equations as describing the system and form the A matrix in the computer. Other programs which find the eigenvalues and their sensitivities are presently available.

Since the sensitivities of the eigenvalues with respect to the system's parameters have proved to be very valuable in analyzing large systems,[1],[2] it is necessary to be able to form the derivative of the matrix with respect to various system parameters. The method developed here gives a general formula for so doing.

Describing the System to be Studied

Dynamic systems of the type to be studied by these methods are most often described in terms of a block diagram with blocks of the type shown in Fig. 1. The method of forming the A matrix to be developed here will use these blocks as one of the major forms of input data. In addition, equations such as the linearized

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