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DIMENSIONAL REDUCTION OF TYPE-II SUPERSTRINGS*)

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ABSTRACT

As a prototype of a type-II superstring compactified on a (2,2) superconformal field theory, we give the explicit expression of the SU(3)-invariant reduction of 10D, type-IIA supergravity to D = 4 dimensions. It is shown, as expected from general arguments, that this Lagrangian has a standard N = 2 supergravity form as dictated by the N = 2 tensor calculus.

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Recently, type-II superstrings [1], compactified on a (2,2) superconformal field theory [2], have been used to exploit some properties of the moduli space [3] (flat-directions) of (stringy) Calabi–Yau compactifications [4].

The additional information [5, 6] on the moduli space is due to the fact that (2,2) superconformal systems, when used for compactification of type-II superstrings, give rise to an enhanced N = 2 space–time supersymmetry [7] with respect to heterotic superstrings [8].

As in the case of heterotic superstrings, the natural question that arises is whether the effective 4D supergravity Lagrangian has a standard form [9], as predicted by the tensor calculus. Witten has shown [10] that for the case of heterotic superstrings, with N = 1 space-time supersymmetry, the effective Lagrangian for the massless fields has indeed a standard supergravity form [11].

The above result has been extended [12] to all toroidal-type compactifications, such as orbifolds, for the case of heterotic strings, also including the case of N = 2 space–time supersymmetry.

The main difference here is that the two supersymmetries come one from the left-moving sector and one from the right-moving sector, in contrast to the heterotic case where, for N = 2, the two supersymmetries come from the left-moving (world-sheet supersymmetric) sector*).

The result of this is, for example, that the universal sector of the superconformal field theory [the identity operator of the (2,2) system] that contains the 4D dilaton and antisymmetric tensor fields belongs to an N = 2 hypermultiplet rather than to an N = 2 vector multiplet [5, 6]. If the supergravity tensor calculus applies [9], then from this fact it follows that hypermultiplet scalars cannot be coupled to vector fields, in contrast to vector multiplet scalars. On the other hand, one is tempted to think [5] that since the dilaton is coupled to vectors, superstrings give rise to new couplings that cannot be explained by the usual tensor calculus for N = 2 supergravity.

In Ref. [6] it was shown that this is not the case. This was proven in two ways: by use of the hidden symmetries of the N = 8 supergravity Lagrangian [13], and by a direct string calculation. In both approaches it follows that the 4D dilaton (sometimes called the S field) does not couple to vectors. The point is that the 4D dilaton is not the same as the 10D dilaton (which indeed couples to 10D vectors), but is a mixture of the dilaton and the determinant of the internal metric.

It should be emphasized that the use of supergravity techniques [6] is adequate for this problem. Indeed, the universal hypermultiplet (containing the S field) is independent of the particular conformal field theory, so one can use any theory, e.g. a trivial toroidal compactification, to study its properties. Then one makes use of the fact that toroidal compactifications of 10D type-II superstrings reduce, in D = 4, to N = 8 superstrings, and the low-energy Lagrangian is uniquely fixed to be the Cremmer–Julia theory [13]. For example, all (scalar) untwisted configurations of $T_6/\mathbb{Z}_N$ orbifolds [14, 15] can be studied by making appropriate $\mathbb{Z}_N$ projections of the original $E_{7(7)}/SU(8)$ coset space [13]. In particular, all orbifolds with $\mathbb{Z}_N \subset SU(3)$ will have in common the SU(3) singlet states. Then, as an oversimplified example, one may use a toy model made up of the universal sector and the SU(3) singlet states. This was the sector considered in Ref. [10] for heterotic superstrings. We will consider the same sector for type-IIA superstrings.

We will show, in terms of the original 10D fields, the appropriate combinations that give rise to a standard supergravity form in four dimensions. The starting point is to consider N = 2, 10D supergravity, rather than N = 8 supergravity in D = 4. The reason for this is that in the Cremmer–Julia theory [13], all vectors have the same parity [as required by SO(8) invariance], whilst the N = 2 supergravity Lagrangian, as obtained by a SU(3) reduction from D = 10, has two vectors

*) The moduli space of N = 2 heterotic strings is related to (4,p) internal superconformal systems.

of opposite parities, in agreement with an N = 2 supergravity coupling of a matter vector multiplet specified by a holomorphic function which is a cubic monomial*) [6].

Type-IIA 10D supergravity can be obtained by dimensional reduction [16] from D = 11 to D = 10 of the maximal supergravity of Cremmer, Julia and Scherk [17].

The 11D supergravity (bosonic) fields**) are the vierbein $\hat{e}_{\hat{\mu}}^{\hat{a}}$ and the three-form $\hat{A}_{\hat{\mu}\hat{\nu}\hat{\varrho}}$. Dimensional reduction to D = 10 is obtained by fixing a triangular gauge for $\hat{e}_{\hat{\mu}}^{\hat{a}}$

$$\hat{e}_{\hat{\mu}}^{\hat{a}} = \begin{pmatrix} e_\mu^a & e_\mu^{11} \\ 0 & e_{11}^{11} \end{pmatrix} \quad \hat{a}, \hat{\mu} = 1 \ldots 11, \quad a, \mu = 1 \ldots 10 \tag{1}$$

with

$$e_{11}^{11} = \phi \quad , \quad e_\mu^{11} = \phi Z_\mu \tag{2}$$

Furthermore, from the three-form we define the two 10D gauge fields***):

$$B_{\mu\nu} = \hat{A}_{\mu\nu 11} \quad , \quad H_{\mu\nu\varrho} = 3\,\partial_{[\mu} B_{\nu\varrho]} \tag{3}$$

$$A_{\mu\nu\varrho} = \hat{A}_{\mu\nu\varrho} - 3 Z_{[\mu} B_{\nu\varrho]} \tag{4}$$

with $F_{a_1a_2a_3a_4} = \hat{F}_{a_1a_2a_3a_4} - 6\mathbb{Z}_{[a_1a_2}B_{a_3a_4]}$. It is trivial to show that $B_{\mu\nu}$, $\mathbb{Z}_\mu$, and $A_{\mu\nu\varrho}$ transform as 10D gauge fields. In the language of 10D, N = 1 supergravity, ϕ and $B_{\mu\nu}$ belong to the gravity multiplet, whilst $\mathbb{Z}_\mu$ and $A_{\mu\nu\varrho}$ belong to the multiplet of the second gravitino.

The D = 11 (bosonic part of the) Lagrangian is [17]

$$\mathcal{L}_{11} = -\frac{1}{2}\hat{e}\hat{R} - \frac{1}{48}\hat{e}\left(\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_4}\right)^2 + \frac{\sqrt{2}}{(12)^4}\varepsilon^{\hat{\mu}_1\ldots\hat{\mu}_{11}}\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_4}\hat{F}_{\hat{\mu}_5\ldots\hat{\mu}_8}\hat{A}_{\hat{\mu}_9\hat{\mu}_{10}\hat{\mu}_{11}} \tag{5}$$

Using the definition given for Eqs. (1) to (4), we get for the terms in Eq. (5) the following Lagrangian in D = 10:

$$-\frac{1}{2}\hat{e}\hat{R} \to -\frac{1}{2}\phi e R - \frac{1}{8} e \phi^3 Z_{\mu\nu}^2 \tag{6}$$

$$-\frac{1}{48}\hat{e}\left(F_{\hat{\mu}_1\ldots\hat{\mu}_4}\right)^2 \to -\frac{1}{48}\phi e\left\{\left(F_{\mu_1\ldots\mu_4} + 6 Z_{[\mu_1\mu_2} B_{\mu_3\mu_4]}\right)^2 + 4\phi^{-2}\left(H_{\mu_1\mu_2\mu_3}\right)^2\right\} \tag{7}$$

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*) Here we will work out only the bosonic sector of the Lagrangian, the rest being determined by supersymmetry.

**) The two formulations are related by a duality transformation [13], which is not needed here.

***) Under the B gauge transformation A is not inert, i.e. $\delta B = d\eta$, $\delta A = Z \wedge d\eta$.

$$\frac{\sqrt{2}}{(12)^4}\,\varepsilon^{\hat\mu_1\ldots\hat\mu_{11}}\,\hat F_{\hat\mu_1\ldots\hat\mu_4}\,\hat F_{\hat\mu_5\ldots\hat\mu_8}\,\hat A_{\hat\mu_9\ldots\hat\mu_{11}} \rightarrow \frac{\sqrt{2}}{(48)^2}\,\varepsilon^{\mu_1\ldots\mu_{10}}\Big\{\big(F_{\mu_1\ldots\mu_4} +$$

$$+6 Z_{\mu_1\mu_2} B_{\mu_3\mu_4}\big) F_{\mu_5\ldots\mu_8} B_{\mu_9\mu_{10}} + 12\, Z_{\mu_1\mu_2} Z_{\mu_3\mu_4} B_{\mu_5\mu_6} B_{\mu_7\mu_8} B_{\mu_9\mu_{10}}\Big\} \tag{8}$$

In order to have a canonically normalized Einstein action, we perform a Weyl rescaling by using the general formula, valid in any dimension D:

$$-\sqrt{g}\,\Omega^{D-2} R \rightarrow -\sqrt{g}\,R - \sqrt{g}\,(D-1)(D-2)\,(\partial_\mu \log \Omega)^2$$

$$g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu} \qquad (e^a_\mu \rightarrow \Omega^{-1} e^a_\mu) \tag{9}$$

For D = 10, then, $\Omega = \phi^{1/8}$, and we get from formulae (6) and (7),

$$-\frac{1}{2}\hat e\,\hat R \rightarrow e\left(-\frac{1}{2}R - \frac{9}{16}(\partial_\mu \log\phi)^2 - \frac{1}{8}\phi^{9/4} Z_{\mu\nu}^2\right) \tag{10}$$

$$-\frac{1}{48}\hat e\,(\hat F_{\hat\mu\hat\nu\hat\rho\hat\sigma})^2 \rightarrow -\frac{1}{48} e\,\phi^{3/4}(F_{\mu\nu\rho\sigma} + 6 Z_{[\mu\nu} B_{\rho\sigma]})^2 - \frac{1}{12} e\,\phi^{-3/2}(H_{\mu\nu\rho})^2 \tag{11}$$

the topological term being unaffected by the Weyl rescaling. Formulae (8), (10), and (11) give the bosonic sector of type-IIA supergravity in D dimensions.

To perform the dimensional reduction to D = 4, following ref. [10] we keep only the SU(3) singlets in the internal indices:

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu}, & g_{i\bar j} &= e^{\sigma}\delta_{i\bar j} \\ B_{\mu\nu} &\rightarrow B_{\mu\nu}, & B_{i\bar j} &= i a\,\delta_{i\bar j} \end{aligned} \tag{12}$$

$$\begin{aligned} Z_\mu &\rightarrow Z_\mu, & \phi &\rightarrow \phi \\ A_{\mu i\bar j} &= i A_\mu \delta_{i\bar j}, & A_{ijk} &= C\,\varepsilon_{ijk} \end{aligned} \tag{13}$$

The new terms, with respect to the N = 1, D = 10 supergravity, are those coming from the $\mathbb{Z}_\mu$ vector field and the three-form $A_{\mu\nu\varrho}$. In D = 4 they reduce to

$$-\frac{1}{8} e\,\phi^{9/4} Z_{\mu\nu}^2 \rightarrow -\frac{1}{8} e\,\phi^{9/4} e^{3\sigma} Z_{\mu\nu}^2 \tag{14}$$

which is Weyl-invariant,

$$-\frac{1}{48} e\, \phi^{3/4} \left(F_{\mu\nu\rho\sigma} + 6 Z_{[\mu\nu} B_{\rho\sigma]} \right)^2 \to$$

$$-\frac{3}{4} e\, \phi^{3/4} e^{\sigma} \left(F_{\mu\nu} + a Z_{\mu\nu} \right)^2 - e\, \phi^{3/4} e^{-3\sigma} \left| \partial_\mu C \right|^2 - \frac{1}{48} e\, \phi^{3/4} e^{9\sigma} \left(F_{\mu\nu\rho\sigma} + 6 Z_{[\mu\nu} B_{\rho\sigma]} \right)^2 \tag{15}$$

after a Weyl rescaling $e \to \Omega^{-1} e$, with $\Omega = e^{3\sigma/2}$, which brings the Einstein term to the canonical normalization $-1/2\, e\, R$.

Finally, the topological term (which is Weyl-invariant) reduces to

$$-\frac{i}{\sqrt{2}\,6} \varepsilon^{\mu\nu\rho\sigma} \bar{C} \overleftrightarrow{\partial}_\mu C H_{\nu\rho\sigma} + \frac{1}{2\sqrt{2}} \varepsilon^{\mu\nu\rho\sigma} \left(a^3 Z_{\mu\nu} Z_{\rho\sigma} + 3a^2 Z_{\mu\nu} F_{\rho\sigma} + 3a F_{\mu\nu} F_{\rho\sigma} \right) \tag{16}$$

Formulae (14), (15), and (16) must be added to the part of the $N = 1$, $D = 10$ bosonic reduction (worked out in Ref. [10]), which is

$$-\frac{1}{2} e R - 3e \left(\partial_\mu \sigma \right)^2 - \frac{9}{16} e \left(\partial_\mu \log \phi \right)^2 - \frac{3}{2} e\, \phi^{-3/2} e^{-2\sigma} \left(\partial_\mu a \right)^2$$

$$- \frac{1}{12} e\, \phi^{-3/2} e^{6\sigma} \left(H_{\mu\nu\rho} \right)^2 \tag{17}$$

with $H_{\mu\nu\rho} = 3\, f_{\mu\nu\rho}$ (ref. 10), $\quad B_{\mu\nu} = a_{\mu\nu}$ (ref. 10)

If we now perform a duality transformation on the $B_{\mu\nu}$ field, this amounts to adding a Lagrange multiplier

$$\frac{1}{3} \phi^{-3/2} e^{6\sigma} H_{\mu\nu\rho} = e\, \varepsilon_{\mu\nu\rho\lambda} \left(\partial^\lambda D + \frac{i}{3\sqrt{2}} \bar{C} \overleftrightarrow{\partial}^\lambda C \right) \tag{18}$$

and to integrating over $H^{\mu\nu\rho}$ treated as a Lagrange multiplier. Owing to the topological term [Eq. (16)], this gives an interference between the D and C fields.

If we now define the complex field

$$S = \phi^{-3/4} e^{3\sigma} + i\, 3\sqrt{2} D + \bar{C} C\,, \qquad A^0_\mu = \sqrt{2}\, Z_\mu$$

$$i T = \phi^{3/4} e^{\sigma} - i \sqrt{2}\, a \qquad\qquad A^1_\mu = 2 A_\mu\,, \quad \frac{X^1}{X^0} = T \tag{19}$$

we get, for the scalar kinetic terms, a Kähler manifold with the Kähler potential given by

$$K = -3 \log i (T - \bar{T}) - \log (S + \bar{S} - 2 \bar{C} C) \tag{20}$$

In agreement with Ref. [6], this is a product of an N = 2 Kähler manifold [18] SU(1,1)/U(1) and a quaternionic manifold*) [19, 20] SU(2,1)/SU(2) × U(1), which corresponds to the 'universal sector' of any (2,2) conformal system in type-II superstrings. The vector kinetic terms depend only on the T field, and they are in a standard supergravity form if, in the language of the tensor calculus [9–18], one defines the holomorphic F function to be

$$F(T) = i\,T^3 = i\,\frac{X'^3}{X^{o3}} = (X^o)^{-2} F(X', X^o) \tag{21}$$

In terms of Eq. (21), the normalization of the vector kinetic term and of the topological $F\tilde{F}$ terms is given by the real and the imaginary part of the kinetic matrix [9] (I,J = 0,1):

$$\mathcal{N}_{IJ} = \frac{1}{4}\bar{F}_{IJ} - \frac{(NX)_I (NX)_J}{(X^L N_{LK} X^K)} \tag{22}$$

with $F_{IJ} = \partial_I \partial_J F$, $N_{IJ} = \frac{1}{4}(F_{IJ} + \bar{F}_{IJ})$

The entries 0,1 refer to the $\mathbb{Z}_\mu$ and A_μ fields, respectively. The three entries of the $\mathcal{N}$ matrix are given by

$$\begin{aligned} \mathcal{N}_{00} &= \frac{i}{32}(T-\bar{T})^3 - \frac{3i}{32}(T-\bar{T})(T+\bar{T})^2 - \frac{i}{16}(T+\bar{T})^3 \\ \mathcal{N}_{01} &= \frac{3i}{16}(T-\bar{T})(T+\bar{T}) + \frac{3i}{16}(T+\bar{T})^2 \\ \mathcal{N}_{11} &= -\frac{3i}{8}(T-\bar{T}) - \frac{3i}{4}(T+\bar{T}) \end{aligned} \tag{23}$$

By using the formula

$$\frac{1}{4} F^{+\,I}_{\mu\nu} \mathcal{N}_{IJ} F^{+\,J}_{\mu\nu} + h.c. \quad (F^{+}_{\mu\nu} = \text{self-dual combination})$$

one easily reproduces the terms given in formulae (15) and (16). We note, in particular, the Peccei–Quinn symmetry [6],

$$a \to a + c \quad , \quad A_\mu \to A_\mu - c\,Z_\mu \tag{24}$$

under which the Lagrangian is invariant, up to a total derivative induced by the topological term in Eq. (16). Formulae (24) tell us that the Peccei–Quinn symmetry is a rotation between the graviphoton and the matter vector. In the language of the N = 2 superconformal tensor calculus, it is a rotation between matter vector multiplet and the compensating vector multiplet.

*) The hypermultiplet manifold can be written in a standard form by introducing a U(1) auxiliary vector multiplet [19].

In conclusion, we see that no coupling is present between the four-dimensional (dilaton) S and the vector fields. Therefore no unusual couplings are generated in $N = 2$ supergravity theories, even when the two supersymmetries come one from the left-movers and one from the right-movers.

From the general analysis given in Ref. [6] it is rather obvious that these results are model-independent and are properties of the effective Lagrangian of type-II superstrings compactified on an arbitrary (2,2) superconformal system.

We explicitly worked out the case of type-IIA superstrings. In type-IIB superstrings [1] the same truncation gives rise to a theory with no matter vector multiplet and a quaternionic manifold [6] $G_2/SO(4)$ with quaternionic dimension 2. The graviphoton is in this case the SU(3)-invariant vector $A_{\mu ijk} = A_\mu \, \epsilon_{ijk}$, where $A_{\mu\nu\varrho\sigma}$ is the four-form of chiral 10D supergravity, and the two hypermultiplets correspond to the fields $g_{i\bar{\imath}}$, $b^c_{i\bar{\imath}}$, $A_{\mu\nu i\bar{\imath}}$, ϕ^c, $b^c_{\mu\nu}$, where $b^c_{\mu\nu}$ and ϕ^c are the complex two-form and the complex dilaton of chiral 10D supergravity.

Using the SU(1,1) covariance [1] of the equations of motion and the self-duality of the field strength of $A_{\mu\nu\varrho\sigma}$, it can immediately be seen that, also in this case, no scalar couples to the graviphoton, according to $N = 2$ tensor calculus rules.

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