Diquark-antidiquark states with hidden or open charm and the nature of $X(3872)$

L. Maiani*

Università di Roma "La Sapienza" and I.N.F.N., Roma, Italy

F. Piccinini†

I.N.F.N. Sezione di Pavia and Dipartimento di Fisica Nucleare e Teorica, Pavia, Italy

A. D. Polosa‡

Dipartimento di Fisica, Università di Bari and I.N.F.N., Bari, Italy

V. Riquer§

CERN Theory Department, Geneva, CH-1211, Switzerland

Heavy-light diquarks can be the building blocks of a rich spectrum of states which can accommodate some of the newly observed charmoniumlike resonances not fitting a pure $c\bar{c}$ assignment. We examine this possibility for hidden and open charm diquark-antidiquark states deducing spectra from constituent quark masses and spin-spin interactions. Taking the $X(3872)$ as input we predict the existence of a 2^{++} state that can be associated to the $X(3940)$ observed by Belle and reexamine the state claimed by SELEX, $X(2632)$. The possible assignment of the previously discovered states $D_s(2317)$ and $D_s(2457)$ is discussed. We predict $X(3872)$ to be made of two components with a mass difference related to $m_u - m_d$ and discuss the production of $X(3872)$ and of its charged partner $X^\pm$ in the weak decays of $B^{+,0}$.

I. INTRODUCTION

It is an old idea that the light scalar mesons $a(980)$ and $f(980)$ may be 4-quark bound states [1]. The idea was more or less accepted in the mid 1970s but then it lost momentum, due to contradictory results that led the lowest-lying candidate members of a diquark-antidiquark nonet, σ and κ, to disappear from the Particle Data Tables.

As an alternative, the possibility was considered that $a(980)$ and $f(980)$ may be $K - \bar{K}$ bound states, kept together by hadron exchange forces, the same that bind nucleons in the nuclei, color-singlet remnants of the confining color forces (hence the name $K - \bar{K}$ molecules [2] used in this connection). If they are indeed $K - \bar{K}$ molecules, scalar mesons do not need to make a complete SU(3) multiplet so that this idea would be consistent with the lack of evidence of light σ and κ. On the contrary, since the latter particles would in any case lie considerably higher than the respective thresholds, it would be very hard to consider either of them as a $\pi - \pi$ or $\pi - K$ molecule. We see that the existence or absence of the light scalars is crucial in assessing the nature of $a(980)$ and $f(980)$.

From this point of view, the recent observations of $\sigma(480)$ and $\kappa(800)$ in D nonleptonic decays at Fermilab [3] and in the $\pi\pi$ spectrum in $\phi \to \pi^0\pi^0\gamma$ at Frascati [4] have considerably reinforced the case of a full nonet with inverted spectrum, as expected for $[qq][\bar{q}\bar{q}]$ states and

fully antisymmetric diquark ($[qq]$: color $= \bar{\mathbf{3}}$, flavor $= \bar{\mathbf{3}}$, spin $= 0$). The isolated $I = 0$ state is the lightest and it likes to decay in $\pi\pi$; the heaviest particles have $I = 1, 0$ and like to decay in states containing strange quark pairs.

In the late 1970s, diquark-antidiquark mesons have been considered also from a different point of view, the so-called baryonium [5]. Baryonium resonances are called for by the extension to baryon-antibaryon scattering of the Harari-Rosner duality [6], which is well obeyed by meson-baryon amplitudes. In the latter case, the exchange of $q\bar{q}$ mesons in the t channel is dual to (indeed it implies the existence of) nonexotic baryon resonances in the s channel. Similarly, $q\bar{q}$ exchange in the t channel of baryon-antibaryon scattering would give rise to $[qq][\bar{q}\bar{q}]$ states in the s channel.

In the constituent quark model, emphasis for the binding of the diquark is on spin-spin forces, which may lead to strong attraction in the completely antisymmetric state. The baryonium picture adds a further element, the internal string structure associated with the confining, spin-independent, color forces [5]. QCD vacuum restricts to a one-dimensional string the color lines of force emerging from each quark [7]. In baryons, the strings from the three quarks join in a point to form a gauge invariant color singlet and give rise to a Y-shaped topological structure, with a quark sitting at each end of the Y. Decay of baryon resonances is produced by the breaking of one of these strings, to give a $q\bar{q}$ meson and a lighter baryon (e.g. $\Delta \to \pi + N$). In the same picture, duality implies the string structure of the diquark-antidiquark states to be that of an H, with the two quarks sitting on one side and the two antiquarks on the other side of the H. Topologically, an

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*Electronic address: luciano.maiani@roma1.infn.it
†Electronic address: fulvio.piccinini@pv.infn.it
‡Electronic address: antonio.polosa@cern.ch
§Electronic address: veronica.riquer@cern.ch

H-shaped state can be seen to arise from the fusion of two *Y*-shaped objects, the baryon-antibaryon pair. Conversely, Okubo-Zweig-Iizuka allowed decays [8] originating from the breaking of a string correspond to decays into either a lighter $qq\bar{q}\bar{q}$ state plus a $q\bar{q}$ meson or into baryon-antibaryon. For the lightest scalar mesons of each flavor these channels are forbidden by energy conservation and we expect basically narrow states. The decay into meson-meson pairs has to proceed via the tunneling of the *H*-shaped configuration into two particles with quarks and antiquarks joined by a single string: $S \rightarrow (q\bar{q}) + (q\bar{q})$. This picture is shown in [9] to give a reasonable description of the decay amplitudes of the lightest scalar mesons.

If the lightest scalar mesons are diquark-antidiquark composites, it is natural to consider analogous states with one or more heavy constituents [9,10] (see [11] for an early proposal).

The aim of the present paper is a study of diquark-antidiquark states with hidden or open charm of the form: $[cq][\bar{c}\bar{q}']$ and $[cq][\bar{s}\bar{q}']$, $q, q' = u, d$. With respect to Ref. [9], we add two new elements, the near spin independence of heavy quark forces and isospin breaking from quark masses. We find some unexpected results and predictions, summarized in the following.

For $[cq][\bar{c}\bar{q}']$ states, the approximate spin independence of heavy quark interactions [12], which is exact in the limit of infinite charm mass, implies spin one diquarks to form bound states if spin zero diquarks do so ("bad" and "good" diquarks, in Jaffe's terminology [13]). A rich spectrum is implied, with states with $J = 0$, 1, 2 and both natural and unnatural J^{PC}. We describe the mass spectrum in terms of (i) the constituent diquark mass and (ii) spin-spin interactions. We derive the strength of the latter interactions from the known meson and baryon spectrum, where possible, or from educated guesses from one-gluon exchange, otherwise.

We identify the $X(3872)$ [14] with the $J^{PC} = 1^{++}$ state with the symmetric spin distribution $[cq]_{S=1} \times [\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}$ (the charmonium assignment and its difficulties are described in Ref. [15]). Our assignment is consistent with the observed decays into charmonium plus vector mesons. It also implies a pure $S = 1$ configuration for the $c - \bar{c}$ pair, thus complying with the selection rules derived in Ref. [16]. We have one $J^{PC} = 2^{++}$ state at 3952 MeV that, within the accuracy of the model, could be identified with the $X(3940)$ seen in Belle data [17]. The scheme features two, $J^{PC} = 0^{++}$, states not yet identified. One, at 3830 MeV, could decay into $D - \bar{D}$ while the other is below the $D - \bar{D}$ threshold and should decay into $\eta_c + $ ps mesons or multihadron states. Finally, there are two $J^{PC} = 1^{+-}$ levels, predicted around 3760 MeV and 3880 MeV, also not yet seen.

It is unclear to us if bad diquarks with light flavors can bind to $[\bar{c}\bar{q}]$, let alone to a completely light-flavored anti-diquark, or if the stronger repulsion in the $S = 1$ state will suppress bound state formation. In the first case, an even richer spectrum is implied, due to the flavor symmetry of the light diquark, with many exotic states.

We extend the previous calculation to the spectrum of $[cq][\bar{s}\bar{q}']$ states, which can be computed on the basis of the same parameters. The resulting spectrum can accommodate the $X(2632)$ claimed by the SELEX Collaboration [18], as well as two other states previously discovered, namely $D_s(2317)$ (with $J^P = 0^+$) and $D_s(2457)$ ($J^P = 1^+$) [19], which could be at odds with a $c\bar{s}$ assignation [20]. The latter hypothesis has been discussed in various papers [21]. Also a molecular composition of these states has been taken in consideration, see, for example, Ref. [22]. For a review and a more extended collection of references on this topic see e.g. [23].

At the large momentum scales implied by the heavy quark, the strength of self-energy annihilation diagrams decreases. As a consequence, particle masses should be approximately diagonal with quark masses, *even for the up and down quarks* [10,24]. Neglecting annihilation diagrams, the neutral mass eigenstates coincide with

$$X_u = [cu][\bar{c}\bar{u}]; \qquad X_d = [cd][\bar{c}\bar{d}]. \tag{1}$$

Deviations from this ideal situation are described by a mixing angle between X_u and X_d. Considering the higher (*h*) and lower (*l*) eigenvalues, we predict

$$M(X_h) - M(X_l) = 2(m_d - m_u)/\cos(2\theta) =$$
$$= (7 \pm 2)/\cos(2\theta) \text{ MeV} \tag{2}$$

in terms of the up and down quark mass difference [25].

Isospin is broken in the mass eigenstates and, consequently, in their strong decays. In particular, we expect this to be the case for X_h and X_l, which are predicted to decay into both $J/\Psi + \rho$ and $J/\Psi + \omega$, as indeed seems to be the case [26] for $X(3872)$. A precise measurement of the branching ratios can provide a determination of $\sin\theta$ and therefore a precise prediction of the mass difference.

We analyze, in this context, the process in which the light vector meson from X decay goes into a lepton pair, with $\rho - \omega$ interference, which allows one to distinguish between the two states X_h and X_l.

Finally, we analyze the nonleptonic decay amplitudes $B \rightarrow KX$, for both B^+ and B^0, restricting for simplicity to zero mixing.

From the limit to the width of $X(3872)$ as observed by Belle [14], we infer that only one particle should dominate the final state of B^+, either X_u or X_d. The $\Delta I = 0$ rule of the weak transition implies then that B^0 decay is dominated by the other state, X_d or X_u: *a precise measurement of the X mass in B^+ and B^0 decay should reveal the mass difference given in* (2). The observation of the decays $X \rightarrow J/\Psi + e^+e^-$, mentioned in the previous paragraph, would allow an independent check of which particle is which in B^+ and B^0 decays. We derive also bounds for the production of the

TABLE I. Constituent quark masses derived from the $L = 0$ mesons (first row) or from the $L = 0$ baryons (second row).

	q	s	c
Constituent	305	490	1670
mass (MeV)	362	546	1721

charged states $X^{\pm}$ in B decays, which are close to, but not in conflict with the negative results published by *BaBar* [27].

It is hardly necessary to remark that our scheme is alternative to the $D - D^*$ molecule picture proposed for the $X(3872)$ [28]. Albeit in some case one gets similar predictions (like isospin breaking decays) the particle content and the pattern of predictions is quite different, in a way that we believe can be put to a test in the near future.

The plan of the paper is the following. We discuss in Sec. II spin-spin interactions in the constituent model and in Secs. III and IV the spectrum of the hidden and open charm states. Section V is devoted to isospin breaking; in Sec. VI we discuss the X decays. The production of X states in nonleptonic decays of $B^{+,0}$ is discussed in Sec. VII. We present our conclusions in Sec. VIII.

II. CONSTITUENT QUARKS AND SPIN-SPIN INTERACTIONS

In its simplest terms, the constituent quark model [2,29] derives hadron masses from three ingredients: quark composition, constituent quark masses and spin-spin interactions. The Hamiltonian is

$$H = \sum_i m_i + \sum_{i<j} 2\kappa_{ij}(S_i \cdot S_j) \qquad (3)$$

and the sum runs over the hadron constituents. The coefficients κ_{ij} depend on the flavor of the constituents i, j and on the particular color state of the pair.

It is not at all clear how this simple *Ansatz* can be derived from the basic QCD interaction, in particular, how the effect of the spin-independent color forces, responsible for quark confinement, can be summarized additively in the constituent masses. However, it is a fact that Eq. (3)

TABLE II. Spin-spin couplings for quark-antiquark pairs in color singlet from the hyperfine splittings of $L = 0$ mesons (first row). The values in the second row show the approximate scaling of the couplings with inverse masses (masses from meson spectrum).

	$q\bar{q}$	$s\bar{q}$	$s\bar{s}$	$c\bar{q}$	$c\bar{s}$	$c\bar{c}$
$(\kappa_{ij})_0$ (MeV)	315	195	121[a]	70	72	59
$(\kappa_{ij})_0 m_i m_j$ (GeV)^3	0.029	0.029		0.036	0.059	0.16

[a]The $s\bar{s}$ coupling which is not experimentally accessible is obtained by rescaling the $s\bar{q}$ one by the factor m_q/m_s.

describes well the spectrum of mesons and baryons, with approximately the same values of the parameters for different situations. The spin-spin interaction coefficients scale more or less as expected with constituent masses and, when compared in different color states, with the values of the color Casimir coefficients derived from one-gluon exchange (as we shall see, this is less accurate). Be as it may, we shall accept the simple Hamiltonian of Eq. (3). The rest of the section is devoted to determining the parameters from the meson and baryon masses. We summarize the (well-known) mass formulas for the $s\bar{q}$ and sq pairs (throughout the paper: $q = u, d$) and give a summary of the parameters in Tables I, II, and III.

Applied to the $L = 0$ mesons, K and K^*, Eq. (3) gives [30]

$$M = m_q + m_s + \kappa_{s\bar{q}}\left[J(J+1) - \frac{3}{2}\right]. \qquad (4)$$

Adding the analogous equations for $\pi - \rho$, $D - D^*$, $D_s - D_s^*$, we find four relations for the constituent masses and the values of the four couplings, as reported in Tables I and II. There is one consistency condition for the constituent masses, which can be written as

$$(m_c + m_q)_D + (m_s + m_q)_K - (2m_q)_\pi = 2157 \text{ MeV},$$
$$(m_c + m_s)_{D_s} = 2076 \text{ MeV}. \qquad (5)$$

This relation is representative of the inaccuracy of the model. Spin-spin interactions scale as expected, like the inverse product of the masses of participating quarks, a most remarkable feature.

Adding the $J/\Psi - \eta_c$ complex, we obtain the $c\bar{c}$ coupling, also reported in Table II, and a considerably smaller constituent charm mass:

$$(m_c)_{J/\Psi} = 1534 \text{ MeV}. \qquad (6)$$

The parameters from the J/Ψ system deviate appreciably from the rest, a not unexpected feature since the charmonium wave function is determined by the charmed quark mass and therefore is considerably different from those of the mixed flavor mesons, which are determined by the light quark masses.

Baryon masses allow us to obtain quark-quark spin interaction in a color antitriplet state. We consider the uds states, Λ (good diquark, $S = 0$), Σ and Y^* (bad di-

TABLE III. Spin-spin couplings for quark-quark pairs in color $\bar{\mathbf{3}}$ state from $L = 0$ baryons. One-gluon exchange implies $(\kappa_{ij})_{\bar{\mathbf{3}}} = 1/2(\kappa_{ij})_0$. The values in the second row show the approximate scaling of the couplings with inverse masses (masses from the baryon spectrum).

	qq	sq	cq	cs
$(\kappa_{ij})_{\bar{\mathbf{3}}}$ (MeV)	103	64	22	25
$(\kappa_{ij})_{\bar{\mathbf{3}}} m_i m_j$ (GeV)^3	0.014	0.013	0.014	0.024

quark, $S = 1$). One finds

$$M(S, J) = 2m_q + m_s + (\kappa_{qq})_{\bar{3}}\left[S(S + 1) - \frac{3}{2}\right]$$
$$+ (\kappa_{qs})_{\bar{3}}\left[J(J + 1) - S(S + 1) - \frac{3}{4}\right]. \quad (7)$$

We can write similar equations for $P - \Delta^+$, involving only $(\kappa_{qq})_{\bar{3}}$, and for Λ_c, Σ_c, Σ_c^*, involving $(\kappa_{qq})_{\bar{3}}$ and $(\kappa_{qc})_{\bar{3}}$. We find three determinations of $(\kappa_{qq})_{\bar{3}}$, the values of $(\kappa_{qs})_{\bar{3}}$ and $(\kappa_{qc})_{\bar{3}}$, as well as a new determination of the three constituent masses. The new information is given in Tables I and III. The consistency conditions read

$$(\kappa_{qq})_{\bar{3}}(P - \Delta) = 97 \text{ MeV},$$
$$(\kappa_{qq})_{\bar{3}}(\Lambda - \Sigma - \Sigma^*) = 103 \text{ MeV}, \quad (8)$$
$$(\kappa_{qq})_{\bar{3}}(\Lambda_c - \Sigma_c - \Sigma_c^*) = 107 \text{ MeV}.$$

For completeness, we consider also the three Ξ_c states, involving $(\kappa_{qs})_{\bar{3}}$ again and $(\kappa_{sc})_{\bar{3}}$. From the masses, we find

$$(\kappa_{qs})_{\bar{3}}(\Xi_c) = 78 \text{ MeV}, \qquad (\kappa_{sc})_{\bar{3}}(\Xi_c) = 25 \text{ MeV}. \quad (9)$$

The overall agreement is quite satisfactory, in particular, for the spin-spin couplings. The decreasing strength with increasing mass is evident, until we go to $cs(\bar{s})$ or $c\bar{c}$ states which may have considerable distortions in their wave functions.

For our purposes, however, we need to consider further couplings, which refer to the quark-antiquark interactions to which we have not yet experimental access. Inside our states, these pairs are in a superposition of color singlet and color octet. Omitting spinor and space-time variables, we write

$$[cq][\bar{c}\,\bar{q}] = \epsilon^{abc}\epsilon_{ab'c'}(c_b q_c)(\bar{c}^{b'}\bar{q}^{c'})$$
$$= (c_b q_c)(\bar{c}^b\bar{q}^c) - (c_b q_c)(\bar{c}^c\bar{q}^b). \quad (10)$$

Color indices in the last term of Eq. (10) can be rearranged with the use of the familiar color Fierz identities:

$$\sum_a \lambda_{ij}^a \lambda_{kl}^a = 2\left(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk}\right),$$

to put into evidence the state of color of the $c\bar{c}$ pair:

$$[cq][\bar{c}\,\bar{q}] = \frac{2}{3}(c_b q_c)(\bar{c}^b\bar{q}^c) - \frac{1}{2}(\bar{c}\lambda^A c)(\bar{q}\lambda^A q). \quad (11)$$

It is not difficult from Eq. (11) to see that the probability to find a particular $q\bar{q}$ pair in color octet is twice the probability of the color singlet, so that (the same holds for the other flavors as well)

$$\kappa_{c\bar{c}}([cq][\bar{c}\,\bar{q}]) = \frac{1}{3}(\kappa_{c\bar{c}})_0 + \frac{2}{3}(\kappa_{c\bar{c}})_8. \quad (12)$$

Of course, we do not know $(\kappa_{c\bar{c}})_8$. We resort to the rule derived from one-gluon exchange:

$$(\kappa_{c\bar{c}})_X = \text{const}[C^{(2)}(X) - C^{(2)}(3) - C^{(2)}(\bar{3})], \quad (13)$$

where $C^{(2)}(X)$ is the value of the quadratic Casimir operator in the representation **X**: $C^{(2)}(X) = 0, 3, 4/3, 4/3$ for $X = \mathbf{0}, \mathbf{8}, \mathbf{3}, \bar{\mathbf{3}}$. Equations (12) and (13) give, in conclusion,

$$\kappa_{c\bar{c}} = \kappa_{c\bar{c}}([cq][\bar{c}\,\bar{q}]) = \frac{1}{4}(\kappa_{c\bar{c}})_0. \quad (14)$$

We apply the previous results to determine the constituent mass of light diquarks, considering explicitly the case of the $a_0(980)$:

$$a_0(980) = [sq]_{S=0}[\bar{s}\,\bar{q}]_{S=0}. \quad (15)$$

We write the Hamiltonian according to

$$H = 2m_{[cs]} + 2(\kappa_{sq})_{\bar{3}}[(S_s \cdot S_q) + (S_{\bar{s}} \cdot S_{\bar{q}'})]$$
$$+ 2\kappa_{q\bar{q}}(S_q \cdot S_{\bar{q}'}) + 2\kappa_{s\bar{q}}[(S_s \cdot S_{\bar{q}'})$$
$$+ (S_{\bar{s}} \cdot S_q)] + 2\kappa_{s\bar{s}}(S_s \cdot S_{\bar{s}}). \quad (16)$$

The state given in Eq. (15) is not an eigenstate of this Hamiltonian, which is diagonalized only within the states with different diquark spin composition, see Sec. III below. However, the latter could as well not exist, so we content ourselves with the mean value:

$$\langle a_0|H|a_0 \rangle = 984 \text{ MeV} = 2m_{[sq]} - 3(\kappa_{sq})_{\bar{3}} \quad (17)$$

and, using the value in Table III, we find

$$m_{[sq]} = 590 \text{ MeV}. \quad (18)$$

For $\sigma(480)$, with $(\kappa_{qq})_{\bar{3}}$, we find

$$m_{[ud]} = 395 \text{ MeV}. \quad (19)$$

Light diquarks constituent are not at all much heavier than constituent quarks.

III. THE SPECTRUM OF $[cq][\bar{c}\bar{q}']$ STATES

States can be conveniently classified in terms of the diquark and antidiquark spin, S_{cq}, $S_{\bar{c}\bar{q}'}$, total angular momentum, J, parity, P, and charge conjugation, C. We have the following states.

(i) Two states with $J^{PC} = 0^{++}$:

$$|0^{++}\rangle = |0_{cq}, 0_{\bar{c}\bar{q}'}; J = 0\rangle;$$
$$|0^{++\prime}\rangle = |1_{cq}, 1_{\bar{c}\bar{q}'}; J = 0\rangle. \quad (20)$$

(ii) Three states with $J = 1$ and positive parity:

$$|A\rangle = |0_{cq}, 1_{\bar{c}\bar{q}'}; J = 1\rangle;$$
$$|B\rangle = |1_{cq}, 0_{\bar{c}\bar{q}'}; J = 1\rangle; \quad (21)$$
$$|C\rangle = |1_{cq}, 1_{\bar{c}\bar{q}'}; J = 1\rangle.$$

Under charge conjugation, $|A\rangle$ and $|B\rangle$ interchange while $|C\rangle$ is odd. Thus the 1^+ complex contains one C-even and two C-odd states:

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle);$$

$$|1^{+-}\rangle = \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle); \qquad (22)$$

$$|1^{+-\prime}\rangle = |C\rangle.$$

One can analyze these states in terms of the states with definite values for the spin of $c\bar{c}$ and $q\bar{q}'$. The state with both spins equal to zero cannot appear, because $J = 1$; among the others, the only one with $C = +$ is that with both spins equal to 1. Thus, the state $|1^{++}\rangle$ in Eq. (22) has a definite value of the $c\bar{c}$ spin, $S_{c\bar{c}} = 1$.

(iii) One state with $J^{PC} = 2^{++}$:

$$|2^{++}\rangle = |1_{cq}, 1_{\bar{c}\bar{q}'}; J = 2\rangle. \qquad (23)$$

The 2^{++} state has also $S_{c\bar{c}} = 1$.

Next, we consider the Hamiltonian, which is the same as in Eq. (16) with $s \rightarrow c$:

$$\begin{aligned}
H = {} & 2m_{[cq]} + 2(\kappa_{cq})_{\bar{3}}[(S_c \cdot S_q) + (S_{\bar{c}} \cdot S_{\bar{q}'})] \\
& + 2\kappa_{q\bar{q}}(S_q \cdot S_{\bar{q}'}) + 2\kappa_{c\bar{q}}[(S_c \cdot S_{\bar{q}'}) + (S_{\bar{c}} \cdot S_q)] \\
& + 2\kappa_{c\bar{c}}(S_c \cdot S_{\bar{c}}). \qquad (24)
\end{aligned}$$

The Hamiltonian is diagonal on the 1^{++} and 2^{++} states, with eigenvalues:

$$M(1^{++}) = 2m_{[cq]} - (\kappa_{cq})_{\bar{3}} + \frac{1}{2}\kappa_{q\bar{q}} - \kappa_{c\bar{q}} + \frac{1}{2}\kappa_{c\bar{c}}, \quad (25)$$

$$M(2^{++}) = 2m_{[cq]} + (\kappa_{cq})_{\bar{3}} + \frac{1}{2}\kappa_{q\bar{q}} + \kappa_{c\bar{q}} + \frac{1}{2}\kappa_{c\bar{c}}. \quad (26)$$

A tedious but straightforward calculation (see the Appendix) leads to two, 2×2, matrices for the other states.

$$M(0^{++}) = \begin{pmatrix} -3(\kappa_{cq})_{\bar{3}} & \frac{\sqrt{3}}{2}(\kappa_{q\bar{q}} + \kappa_{c\bar{c}} - 2\kappa_{c\bar{q}}) \\ \frac{\sqrt{3}}{2}(\kappa_{q\bar{q}} + \kappa_{c\bar{c}} - 2\kappa_{c\bar{q}}) & (\kappa_{cq})_{\bar{3}} - (\kappa_{c\bar{c}} + \kappa_{q\bar{q}} - 2\kappa_{c\bar{q}}) \end{pmatrix},$$

$$M(1^{+-}) = \begin{pmatrix} -(\kappa_{cq})_{\bar{3}} + \kappa_{c\bar{q}} - \frac{(\kappa_{c\bar{c}} + \kappa_{q\bar{q}})}{2} & \kappa_{q\bar{q}} - \kappa_{c\bar{c}} \\ \kappa_{q\bar{q}} - \kappa_{c\bar{c}} & (\kappa_{cq})_{\bar{3}} - \kappa_{c\bar{q}} - \frac{(\kappa_{q\bar{q}} + \kappa_{c\bar{c}})}{2} \end{pmatrix}.$$

The state 1^{++} is an almost perfect candidate to explain the properties of $X(3872)$:

(i) it is expected to be narrow, like all diquark-antidiquark systems below the baryon-antibaryon threshold;

(ii) the unnatural spin parity forbids the decay in $D - \bar{D}$, which is not observed;

(iii) it can decay in the observed channels $J/\Psi +$ light vector meson, with conservation of the spin of the heavy flavor system;

(iv) it decays into both ρ and ω, due to isospin breaking in its wave function (Sec. IV).

How narrow is narrow we shall consider in Sec. VI. For the moment, we identify the 1^{++} with the $X(3872)$ and proceed to compute the spectrum via the couplings of Table III, directly, and those of Table II, scaled according to Eqs. (11)–(13). By Eq. (25), the diquark constituent mass is fixed to be

$$m_{[cq]} = 1933 \text{ MeV.} \qquad (27)$$

We report in Fig. 1 the full spectrum computed numerically. The energy levels have an error which is difficult to quantify at the moment, maybe in the order of $10-20$ MeV. A few observations are in order.

*(i)* From Eqs. (25) and (26) we read

$$M(2^{++}) = M(1^{++}) + 2[(\kappa_{cq})_{\bar{3}} + \kappa_{c\bar{q}}] = 3952 \text{ MeV.} \qquad (28)$$

This places the 2^{++} close to the recently observed [17] resonance at 3940 MeV. Note that the coupling κ_{cq} is well determined and the other, $\kappa_{c\bar{q}}$, could easily be smaller than we estimate with the color factor. The identification of the 2^{++} with the $X(3940)$ is quite attractive. The 2^{++} can decay in $J/\Psi +$ light vector meson respecting the conservation of the heavy flavor spin and also in $D - \bar{D}$. The decay $X(3940) \rightarrow J/\Psi + \omega$ is seen by Belle, the $D - \bar{D}$ decay should be searched for, but it could be somewhat suppressed by the decay in D wave.

(ii) Of the two 0^{++} states, one is below the $D - \bar{D}$ threshold. It can decay in $\eta_c\pi$ or $\eta_c\eta$ or multihadron states. The other should be seen to decay in $D - \bar{D}$. There are no candidates, at present, for the 0^{++} states. The same holds for the two 1^{+-} states. Allowed decays of the latter are $J/\Psi + \pi(\eta)$, $\eta_c + \rho(\omega)$.

IV. THE $[cq][\bar{s}\bar{q}']$ STATES

We extend the calculation of the previous section to the states $[cq][\bar{s}\bar{q}']$, leaving aside the issue whether they can
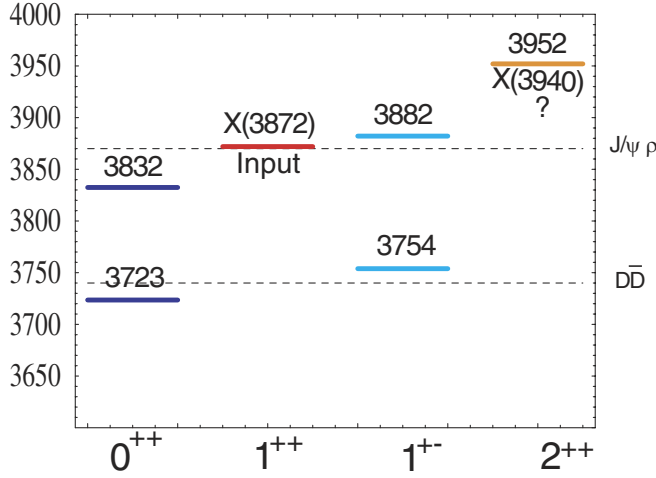
FIG. 1 (color online).   The full spectrum of the X particles.

FIG. 2 (color online).   The predicted spectrum of the particles with open charm.

bind or not. The appropriate Hamiltonian is

$$H = m_{[cq]} + m_{[sq]} + 2(\kappa_{cq})_{\bar{3}}(S_c \cdot S_q) + +2(\kappa_{sq})_{\bar{3}}(S_{\bar{s}} \cdot S_{\bar{q}'})$$
$$+ +2\kappa_{q\bar{q}}(S_q \cdot S_{\bar{q}'}) + 2\kappa_{c\bar{q}}(S_c \cdot S_{\bar{q}'})$$
$$+ 2\kappa_{s\bar{q}}(S_{\bar{s}} \cdot S_q) + 2\kappa_{c\bar{s}}(S_c \cdot S_{\bar{s}}). \tag{29}$$

The angular momentum composition of the multiplet is, of course, the same as the previous one except that this set of states is not invariant under C conjugation and the $J^P = 1^+$ states form an irreducible complex. The energy levels are given by the following formulas:

$$M(2^+) = m_{[cq]} + m_{[sq]} + \frac{1}{2}[(\kappa_{cq})_{\bar{3}} + +(\kappa_{sq})_{\bar{3}}]$$
$$+ \frac{1}{2}\kappa_{q\bar{q}} + \frac{1}{2}(\kappa_{c\bar{q}} + \kappa_{s\bar{q}}) + \frac{1}{2}\kappa_{c\bar{s}}, \tag{30}$$

$$M(1^+)_{11} = [-3(\kappa_{cq})_{\bar{3}} + (\kappa_{sq})_{\bar{3}}]/2, \qquad M(1^+)_{12} = (\kappa_{q\bar{q}} - \kappa_{c\bar{q}} - \kappa_{s\bar{q}} + \kappa_{c\bar{s}})/2,$$

$$M(1^+)_{13} = (\kappa_{q\bar{q}} - \kappa_{c\bar{q}} + \kappa_{q\bar{s}} - \kappa_{c\bar{s}})/\sqrt{2}, \qquad M(1^+)_{22} = [(\kappa_{cq})_{\bar{3}} - 3(\kappa_{sq})_{\bar{3}}]/2, \tag{31}$$

$$M(1^+)_{23} = (-\kappa_{q\bar{q}} - \kappa_{c\bar{q}} + \kappa_{q\bar{s}} + \kappa_{c\bar{s}})/\sqrt{2}, \qquad M(1^+)_{33} = [(\kappa_{cq})_{\bar{3}} + (\kappa_{sq})_{\bar{3}} - \kappa_{q\bar{q}} - \kappa_{q\bar{s}} - \kappa_{c\bar{q}} - \kappa_{c\bar{s}}]/2,$$

$$M(0^+)_{11} = -\frac{3}{2}[(\kappa_{cq})_{\bar{3}} + (\kappa_{sq})_{\bar{3}}], \qquad M(0^+)_{12} = \frac{\sqrt{3}}{2}(\kappa_{q\bar{q}} - \kappa_{c\bar{q}} - \kappa_{s\bar{q}} + \kappa_{c\bar{s}}),$$

$$M(0^+)_{22} = \frac{1}{2}[(\kappa_{cq})_{\bar{3}} + (\kappa_{sq})_{\bar{3}}] - (\kappa_{q\bar{q}} + \kappa_{c\bar{q}} + \kappa_{s\bar{q}} + \kappa_{c\bar{s}}). \tag{32}$$

The spectrum is reported in Fig. 2.

The particle claimed by the SELEX Collaboration [18] fits quite naturally in it as the 2^+ member of the multiplet. Note the change of attribution, with respect to the 0^+ assignment suggested previously [10], which makes the $X(2632)$ compatible with the diquark constituent masses found in Eqs. (18) and (27), without changing the results presented there. In addition, we associate tentatively the lowest 0^+ and one of the lowest-lying 1^+ with $D_s(2317)$ and $D_s(2457)$ [19], respectively. This is compatible with the observed decays:

$$D_s(2317) \to D_s \pi^0; \qquad D_s(2457) \to D_s \gamma \pi^0; \quad (D_s)^* \pi^0. \tag{33}$$

A four quark interpretation of the D_s particles has been advanced in Ref. [20], while the $c\bar{s}$ interpretation is pur-
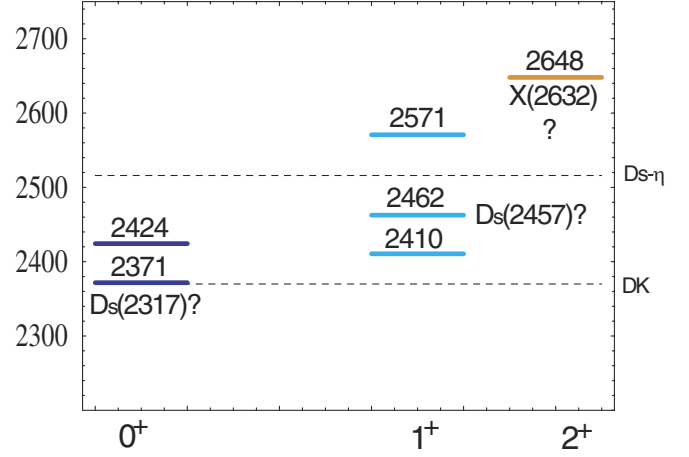
sued in [21]. We find quite suggestive that by assigning the $X(3872)$ to its natural, 1^{++}, level and using reasonable values of the spin-spin couplings we are able to fit the other four particles, which could be at odds with the conventional quark-antiquark interpretation.

V. ISOSPIN BREAKING

We consider in this section the finer structure of the $X(3872)$. In particular, we consider the neutral states with the composition given in Eq. (1). Physical states could be expected to fall in isospin multiplets with $I = 1, 0$:

$$f_{c\bar{c}} = (X_u + X_d)/\sqrt{2}; \qquad a_{c\bar{c}} = (X_u - X_d)/\sqrt{2}. \tag{34}$$

The two states in Eq. (1) are mixed by self-energy diagrams whereby a light quark pair transforms into another one by annihilation into intermediate gluons. In the

basis Eq. (1) annihilation diagrams contribute equally to all entries of the mass matrix. The contribution of quark masses, on the other hand, is diagonal in the basis Eq. (1). The resulting 2×2 matrix is

$$\begin{pmatrix} 2m_u + \delta & \delta \\ \delta & 2m_d + \delta \end{pmatrix},$$

δ being the contribution from annihilation graphs. The matrix with all equal entries δ, admits the states in (34) as eigenvectors, with split masses.

At the mass scale determined by the $c\bar{c}$ pair we expect [10] annihilation diagrams to be small, as indicated by the very small J/Ψ width. Thus, mass eigenvectors should align to the quark mass basis. For the strange quark, this happens already at the mass scale of the vector mesons, ϕ and ω. The other case is provided by the scalars $a(980)$ and $f(980)$, which are quite degenerate in mass. The upper bound [30] $|\Delta M| < 10$ MeV indicates that annihilation contributions are, at best, at the level of the normal isospin breaking mass differences, suggesting a sizable deviation from the isospin basis. At the $X(3872)$ scale, we expect the $u - d$ quark mass difference to dominate and the mass eigenstates to coincide with the states in (1) to a rather good extent.

A numerical estimate of the mass difference is obtained as follows. The $u - d$ quark mass difference is determined by the pseudoscalar meson spectrum [25], after separating its contribution from the background of second order electromagnetic (e.m.) corrections due to one-photon exchange. The so-called Dashen's theorem [31] states that one-photon exchange does not contribute to the isospin breaking, U-spin singlet combination of kaon and pion mass differences, which is therefore given by the quark mass difference:

$$(K^+ - K^0) - (\pi^+ - \pi^0) = C(m_u - m_d) =$$
$$= -5.3 \cdot 10^{-3} \ (\text{GeV})^2, \quad (35)$$

where particle symbols stand for squared masses. Combining with the equations for the pion and kaon masses in terms of quark masses, and assuming $m_s = 150$ MeV from the baryon mass differences, one finds

$$(m_u - m_d) = 3.3 \ \text{MeV}. \quad (36)$$

Before translating Eq. (36) in hadron mass differences, one must control the one-photon exchange contributions. We can divide the e.m. corrections in (i) corrections on the same constituent quark line, and (ii) photon crossing from one to another quark line. In the constituent quark model, the first correction goes into a renormalization of the constituent mass while the second one adds to the spin-spin interaction. A control case is that of the charmed mesons:

$$M(D^+) - M(D^0) = 4.78 \pm 0.1 \ \text{MeV},$$
$$M(D^{*+}) - M(D^{*0}) = 3.3 \pm 0.7 \ \text{MeV}. \quad (37)$$

Note that the result in the first line is *larger* than the mass difference in (36): the spin-spin interaction in total spin zero is repulsive (attractive) for nonvanishing (vanishing) total charge. The correction to the constituent masses is obtained by averaging over the two spin multiplets, see Eq. (4):

$$(m_d - m_u)_{\text{const}} = \frac{3(D^{*+} - D^{*0}) + (D^+ - D^0)}{4}$$
$$= 3.7 \pm 0.7 \ \text{MeV}. \quad (38)$$

The agreement with Eq. (36) is better than for individual mass differences. Unfortunately, at the moment, we do not have enough masses to determine and subtract the spin-spin e.m. interaction for the $[cq][\bar{c}\bar{q}']$ multiplet. We take the example as suggestive that neglecting photon exchange may introduce an error of, perhaps, $\simeq 30\%$.

Non-negligible gluon annihilation diagrams mix X_u and X_d and increase the mass difference. Writing

$$X_{\text{low}} = \cos\theta X_u + \sin\theta X_d,$$
$$X_{\text{high}} = -\sin\theta X_u + \cos\theta X_d, \quad (39)$$

we get the result already stated in the Introduction:

$$M(X_h) - M(X_l) = 2(m_d - m_u)/\cos(2\theta)$$
$$= (7 \pm 2)/\cos(2\theta) \ \text{MeV}.$$

The mixing angle can be determined from ΔM as well as from the ratio of the decay rates in $J/\Psi + \omega$ and $J/\Psi + \rho$, as we shall see in the next section. It goes without saying that the same considerations can be applied to all states in Figs. 1 and 2.

In conclusion, we predict close to maximal isospin breaking in the wave function and correspondingly in the hadronic decays of $X(3872)$.

Isospin violation in the wave function is also predicted by the DD^* molecule scheme [28], with the $X(3872)$ being essentially $(D^0\bar{D}^{*0}) + (\bar{D}^0 D^{*0})$. However, in our scheme we have two states rather than one, separated by the mass difference (2), and quite a richer phenomenology.

VI. THE $X(3872)$ DECAY WIDTH

A pair of color-singlet mesons cannot be obtained by cutting the strings that join quarks and antiquarks in the H shaped $[qq][\bar{q}\bar{q}]$ states. The baryonium picture suggests that the two-meson decays of the latter go via intermediate baryon-antibaryon states of high mass. This implies basically narrow widths.

The $X(3872)$ is expected to be particularly narrow for several additional reasons.

(i) Unnatural spin parity forbids decays into $D\bar{D}$;
(ii) the channel DD^* is below threshold;

(iii) decay in η_c + mesons is forbidden by heavy flavor spin conservations [16].

Of the charmonium channels, the only available ones are $J/\Psi + 2\pi$ and $J/\Psi + 3\pi$, dominated by ρ^0 and ω, respectively. Each mass eigenstate decays simultaneously in the two channels, due to isospin breaking in the wave function.

We describe the decay by a single switch amplitude, associated with the process:

$$[cu]_{\bar{3}}[\bar{c}\,\bar{u}]_{3} \rightarrow (c\bar{c})_0 (u\bar{u})_0, \tag{40}$$

where subscripts indicate color configurations.

We further write the invariant three-meson coupling for X_u according to

$$L_{X_u \Psi V} = g_V \epsilon^{\mu\nu\rho\sigma} P_\mu X_\nu \psi_\rho V_\sigma = g_V M_X (\mathbf{X} \wedge \psi) \cdot \mathbf{V}, \tag{41}$$

where P_μ and M_X are the decaying particle momentum and mass. To estimate the value of g_V, we compare with the similar couplings for the light scalar mesons, determined by one dimensionful constant, $A \simeq 2.6$ GeV. An admittedly bold guess, to obtain the order of magnitude, is

$$g_V M_X = \frac{A}{\sqrt{2}}. \tag{42}$$

By dominating the $2\pi(3\pi)$ decay with $\rho(\omega)$ exchange in the narrow-width approximation, we find

$$\frac{d\Gamma(X_l \rightarrow \psi + f)}{ds} = \frac{2x_{l,V}|A|^2 B_{(V \rightarrow f)}}{8\pi M_X^2} \frac{M_V \Gamma_V}{\pi}$$
$$\times \frac{p(s)}{(s - M_V^2)^2 + (M_V \Gamma_V)^2} \tag{43}$$

with $f = \pi^+ \pi^- (\pi^+ \pi^- \pi^0)$ for $V = \rho(\omega)$, s the invariant mass squared of the pions, and p the decay momentum:

$$p(s) = \frac{\sqrt{\lambda(M_X, M_\psi, M_V)}}{2M_X};$$
$$\lambda = (M_X)^4 + (M_\psi)^4 + (M_V)^4 - 2(M_X M_\psi)^2 \tag{44}$$
$$- 2(M_X M_V)^2 - 2(M_\psi M_V)^2.$$

The coefficient $x_{l,V}$ is

$$x_{l,V} = \frac{(\cos\theta \pm \sin\theta)^2}{2} \tag{45}$$

for $V = \omega(\rho)$. Similar equations hold for the higher mass state, X_h, with the appropriate substitutions.

By numerical integration, we then find

$$\langle p \rangle_\rho = \left(\frac{M_\rho \Gamma_\rho}{\pi}\right) \int_{(2m_\pi)^2}^{\infty} ds \frac{p(s)}{(s - M_\rho^2)^2 + (M_\rho \Gamma_\rho)^2}$$
$$= 126 \text{ MeV}; \tag{46}$$
$$\langle p \rangle_\omega = 22 \text{ MeV},$$

and

$$\Gamma(X_l \rightarrow J/\psi + \pi^+ \pi^-) = \frac{2x_{l,\rho}|A|^2}{8\pi M_X^2}\langle p \rangle_\rho$$
$$= 2x_{l,\rho} \cdot 2.3 \text{ MeV}; \tag{47}$$
$$\Gamma(X_l \rightarrow J/\psi + \pi^+ \pi^- \pi^0) = \frac{2x_{l,\omega}|A|^2}{8\pi M_X^2}\langle p \rangle_\omega =$$
$$= 2x_{l,\omega} \cdot 0.4 \text{ MeV}.$$

We anticipate small widths, comparable to the resolution of Belle and *BaBar*. However, given the mass difference in (2), one would expect to observe either two peaks or one unresolved structure, broader than the stated experimental resolution, 4.5 MeV. Taking Belle data at face value, we conclude that only one of the two neutral states is produced appreciably in B^+ decay (this will be discussed in the next section). Assuming this to be the case, we can get some information on the mixing angle from the observed ratio of 3π to 2π decay rates. We get from Eq. (47)

$$\left(\frac{\Gamma(3\pi)}{\Gamma(2\pi)}\right)_{X_l} = \frac{(\cos\theta + \sin\theta)^2}{(\cos\theta - \sin\theta)^2} \cdot \frac{\langle p_\omega \rangle}{\langle p_\rho \rangle},$$
$$\left(\frac{\Gamma(3\pi)}{\Gamma(2\pi)}\right)_{X_h} = \frac{(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)^2} \cdot \frac{\langle p_\omega \rangle}{\langle p_\rho \rangle}. \tag{48}$$

Belle attributes all events with $\pi^+ \pi^- \pi^0$ mass above 750 MeV to ω decay and divides by the total number of observed 2π events. They find

$$\left(\frac{\Gamma(3\pi)}{\Gamma(2\pi)}\right)_{\text{Belle}} = 0.8 \pm 0.3_{\text{stat}} \pm 0.1_{\text{syst}}. \tag{49}$$

The central value is compatible with Eq. (48) for

$$\theta \simeq \pm 20° \tag{50}$$

for X_l or X_h, respectively. Assuming that there are no other significant decay modes, the corresponding widths and branching fractions for the particle seen in B^+ decay are

$$\Gamma = 1.6 \text{ MeV (3.7 MeV)}, \qquad \mathcal{B}(2\pi) = 0.61 \ (0.95), \tag{51}$$

where we have listed in parentheses the properties of the particle not seen in B^+ decay. The mass difference of the two states is

$$M(X_h) - M(X_l) = (8 \pm 3) \text{ MeV}. \tag{52}$$

We give also the corresponding predictions for the charged state X^+, which decays via ρ exchange only,

$$\Gamma(X^+ \rightarrow J/\psi + \pi^+ \pi^0) = \frac{2|A|^2}{8\pi M_X^2}\langle p \rangle_\rho = 4.6 \text{ MeV}; \tag{53}$$
$$\mathcal{B}(X^+ \rightarrow J/\psi + \pi^+ \pi^0) \simeq 1.$$

The value of the mixing angle in (50) is perhaps on the high side but still compatible with the general picture. More precise data are clearly needed.

We close this section by considering the leptonic decays:

$$X(3872) \rightarrow J/\Psi + e^+ e^-. \tag{54}$$

The lepton pair originates from the coherent superposition of ρ and ω produced in the decay of the X. Thus, the branching ratio can distinguish between X_l and X_h, supplementing the measurement of the mass. For simplicity we give the result for the case of vanishing mixing. A simple calculation gives

$$\frac{d\Gamma(X \rightarrow \psi + e^+ e^-)}{ds} = \frac{|A|^2 \mathcal{B}(\rho \rightarrow e^+ e^-)}{8\pi M_X^2} \frac{M_\rho \Gamma_\rho}{\pi} \cdot p(s)$$

$$\cdot \left| \frac{1}{(s - M_\rho^2) + i(M_\rho \Gamma_\rho)} \right.$$

$$\left. \pm \frac{1/3}{(s - M_\omega^2) + i(M_\omega \Gamma_\omega)} \right|^2.$$

We have assumed the quark-model ratio for the leptonic amplitudes of ρ and ω and used the narrow-width approximation. The sign $\pm$ applies to X_u and X_d, respectively. Combining with Eq. (47), with $\theta = 0$, we find

$$\mathcal{B}(X_u \rightarrow J/\Psi + e^+ e^-) = 0.8 \cdot 10^{-4},$$
$$\mathcal{B}(X_d \rightarrow J/\Psi + e^+ e^-) = 0.3 \cdot 10^{-4}. \tag{55}$$

VII. PRODUCTION OF $[cq][\bar{c}\bar{q}']$ STATES IN B NONLEPTONIC DECAYS

B^+ and B^0 decays produce superpositions of the two neutral states, Eq. (1) as well as the charged states:

$$X^+ = [cu][\bar{d}\,\bar{c}]; \qquad X^- = [cd][\bar{u}\,\bar{c}]. \tag{56}$$

For simplicity, we shall restrict to vanishing mixing.

We consider first the B^+ decay amplitudes for the allowed decay:

$$B^+ = (\bar{b}u) \rightarrow \bar{c} + c + \bar{s} + u + (\bar{u} + u \text{ or } \bar{d} + d). \tag{57}$$

One additional pair is included in the final state, created from the vacuum by the strong interaction.

If we want a K in the final state, the $\bar{s}$ must combine either with the spectator quark, u, to give a K^+ (amplitude A_1) or with one quark from the additional pair, to give either K^+ or K^0 (amplitude A_2). Thus we have two independent amplitudes.

B^+:

$$A(K^+ X_u) = A_1 + A_2; \qquad A(K^+ X_d) = A_1;$$

$$A(K_S X^+) = \frac{A_2}{\sqrt{2}}. \tag{58}$$

For B^0 decays we have simply to exchange u with d and K^+ with K^0, to get

B^0:

$$A(K_S X_d) = \frac{A_1 + A_2}{\sqrt{2}}; \qquad A(K_S X_u) = \frac{A_1}{\sqrt{2}};$$

$$A(K^+ X^-) = A_2. \tag{59}$$

We note, in passing, that these relations follow also from the $\Delta I = 0$ rule obeyed by the weak transition.

We noted already that the mass difference given in Eq. (2) is larger than the apparent width of the $X(3872)$ peak seen by Belle [14]. Thus, only one of the two neutral states is produced appreciably in B^+ decay. Orientatively, we shall assume that

$$\Gamma(K^+ X_{(u \text{ or } d)}) > 4\Gamma(K^+ X_{(d \text{ or } u)}). \tag{60}$$

Equation (60) implies some bound to the production of the charged states, $X^\pm$, not observed thus far. With three amplitudes and two parameters, Eq. (58) gives rise to the triangle inequality:

$$|A(K^+ X_u)| + |A(K^+ X_d)| > \sqrt{2}|A(K_S X^+)|$$
$$> ||A(K^+ X_u)| - |A(K^+ X_d)||. \tag{61}$$

We are interested in the lower bound to the rate of X^+, which, due to Eq. (60) or Eq. (61), is

$$|A(K_S X^+)| > \frac{1}{2\sqrt{2}}|A(K^+ X_q)| \tag{62}$$

with $q = u$ or d, according to which is the dominant decay product.

Equation (60) has two solutions:
$B^+ \rightarrow K^+ X_u$ *dominant*:

$$A_1 \simeq A_2 \tag{63}$$

or

$B^+ \rightarrow K^+ X_d$ *dominant*:

$$A_1 \simeq -\frac{1}{2}A_2. \tag{64}$$

We consider now B^0 decays, Eq. (59). It is immediate to see that if X_u dominates B^+ decays, X_d dominates B^0 decays and vice versa. Thus we are led to predict that *the X particle in B^+ and B^0 decays are different*, with a mass difference given by Eq. (2) or (52). In addition, from the corresponding triangle inequality, we find

$$\Gamma(K^+ X^-) > \frac{1}{2}\Gamma(K_S X_q) \tag{65}$$

with $q = u$ or d, whichever particle dominates B^0 decays. Relations (62) and (65) remain unchanged if one considers

an equal mixture of B^0 and $\bar{B}^0$ decays and adds K^+ and K^- events.

To conclude, we give explicitly the lower bounds to the production of $X^{\pm}$ in B^+ and B^0 decays:

$$R^+ = \frac{\mathcal{B}(B^+ \to K_S X^+) \cdot \mathcal{B}(X^+ \to J/\Psi + \pi^+ \pi^0)}{\mathcal{B}(B^+ \to K^+ X_{l/h}) \cdot \mathcal{B}(X_{l/h} \to J/\Psi + \pi^+ \pi^-)}$$

$$> 0.2,$$

$$R^0 = \frac{\mathcal{B}(B^0 \to K^+ X^-) \cdot \mathcal{B}(X^- \to J/\Psi + \pi^- \pi^0)}{\mathcal{B}(B^0 \to K_S X_{h/l}) \cdot \mathcal{B}(X_{h/l} \to J/\Psi + \pi^+ \pi^-)}$$

$$> 0.53$$

to be compared with the upper limit given by *BaBar* [27]:

$$R^+ < 0.8$$

with large errors.

VIII. CONCLUSIONS

The diquark-antidiquark structure explains well the properties of the $X(3872)$: spin-parity, narrow-width, simultaneous decay into channels with different isospin. Taking the $X(3872)$ as input, we have derived a spectrum which is able to explain spin-parity and decay properties of few other particles that could be at odds with a $q\bar{q}$ picture: $X(3940)$, the previously discovered $D_s(2317)$ and $D_s(2457)$ and the $X(2632)$ claimed by SELEX.

Isospin breaking in the wave function and in strong decays of these states is a distinctive consequence of the asymptotic freedom of QCD, much in the same way as narrow widths for heavy quarkonia. Also, all these states have to be doublets, unlike the case of the $D - D^*$ molecules, with typical mass splittings given by twice the down-up quark mass difference. The two different states of $X(3872)$ should appear in B^+ and B^0 decays, respectively.

The crucial test of the scheme, of course, will be the observation of the charged or doubly charged partners of the X particles and, more generally, the observation of heavy states with really exotic quantum numbers. We have derived rather strict bounds for the production of $X^+(3872)$ in B decays, close to the present limits so that a meaningful test may be expected in the near future. The existence of exotic states at low energy is also a pressing issue.

The indications derived from the properties of $a/f(980)$ and $X(3872)$ seem to us very compelling, so as to warrant a thorough experimental and theoretical investigation.

ACKNOWLEDGMENTS

APPENDIX

We give here a simple derivation of the matrix elements of the spin-spin Hamiltonian over the $qq - \bar{q}\bar{q}$ states. We consider states which can be written schematically as

$$|S_{[cu]}, S_{[\bar{c}\bar{u}]}; J\rangle = |\Gamma, \Gamma'; J\rangle = (c^a \Gamma_{ab} u^b)(\bar{c}^c \Gamma'_{cd} \bar{u}^d). \quad \text{(A1)}$$

$S_{[cu]}$ and $S_{[\bar{c}\bar{u}]}$ are the total diquark and antidiquark spin and J the total angular momentum. Individual spins are represented by 2×2 matrices, Γ^α, with

$$\Gamma^0 = \frac{\sigma_2}{\sqrt{2}}; \qquad \Gamma^i = \frac{1}{\sqrt{2}} \sigma_2 \sigma^i \quad \text{(A2)}$$

for spin 0 and 1, respectively. The matrices Γ are normalized so that

$$\mathrm{Tr}[(\Gamma^\alpha)^\dagger \Gamma^\beta] = \delta^{\alpha\beta}. \quad \text{(A3)}$$

Spin operators are defined according to

$$\mathbf{S}_u |\Gamma\rangle = \left| \Gamma \frac{1}{2} \sigma \right\rangle; \qquad \mathbf{S}_c |\Gamma\rangle = \left| \frac{1}{2} \sigma^T \Gamma \right\rangle. \quad \text{(A4)}$$

Since

$$\sigma^T \sigma_2 = -\sigma_2 \sigma \quad \text{(A5)}$$

we recover the expected formulas for the total spin operator:

$$(\mathbf{S}_u + \mathbf{S}_c)|\Gamma^0\rangle = 0, \qquad [(S_u)^i + (S_c)^i]|\Gamma^j\rangle = i\epsilon^{ijk}|\Gamma^k\rangle. \quad \text{(A6)}$$

We find, also

$$\langle 0|\mathbf{S}_u|1\rangle = -\langle 0|\mathbf{S}_c|1\rangle = \frac{1}{2};$$

$$\langle 1|\mathbf{S}_u|1\rangle = +\langle 1|\mathbf{S}_c|1\rangle = \frac{1}{2}\langle 1|(\mathbf{S}_u + \mathbf{S}_c)|1\rangle. \quad \text{(A7)}$$

We can now compute the matrix elements of products of spin operators. We have two cases.

(1) Same diquark, e.g. $S_u \cdot S_c$. This operator is just a combination of Casimir operators:

$$2(\mathbf{S_u} \cdot \mathbf{S_c}) = (\mathbf{S_{cu}})^2 - (\mathbf{S_c})^2 - (\mathbf{S_u})^2 \quad \text{(A8)}$$

and is diagonal in the basis.

(2) Different diquarks, e.g. $S_u \cdot S_{\bar{u}}$. We consider as an example the $J = 0$ states, represented by (summation of

repeated indices understood)

$$|0, 0; 0\rangle = \frac{1}{2}(\sigma_2) \otimes (\sigma_2); |1, 1; 0\rangle = \frac{1}{2\sqrt{3}}(\sigma_2 \sigma^i) \otimes (\sigma_2 \sigma^i).$$
(A9)

Using the basic definitions, we have

$$2(\mathbf{S}_u \cdot \mathbf{S_u})|0, 0; 0\rangle = \frac{1}{4}(\sigma_2 \sigma^i) \otimes (\sigma_2 \sigma^i) = \frac{\sqrt{3}}{2}|1, 1; 0\rangle;$$

$$2(\mathbf{S}_u \cdot \mathbf{S_u})|1, 1; 0\rangle = \frac{1}{4\sqrt{3}}(\sigma_2 \sigma^i \sigma^j) \otimes (\sigma_2 \sigma^i \sigma^j)$$
$$= \frac{\sqrt{3}}{2}|0, 0; 0\rangle - |1, 1; 0\rangle.$$
(A10)

In conclusion, on the states $|0, 0; 0\rangle$ and $|1, 1; 0\rangle$ we obtain the matrices

$$2(\mathbf{S}_u \cdot \mathbf{S}_c) = \begin{pmatrix} -3/2 & 0 \\ 0 & 1/2 \end{pmatrix},$$

$$2(\mathbf{S}_u \cdot \mathbf{S}_{\bar{u}}) = \begin{pmatrix} 0 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1 \end{pmatrix}.$$
(A11)

Using the relations, Eqs. (A7), we derive from (A11) the representatives of the other spin-spin operators, to obtain, e.g., the mass matrix given in Sec. III.

We conclude by giving the tensor basis for the $J = 1$ states:

$$|A\rangle = |0, 1; 1\rangle = \frac{1}{2}(\sigma_2) \otimes (\sigma_2 \sigma^i);$$

$$|B\rangle = |1, 0; 1\rangle = \frac{1}{2}(\sigma_2 \sigma^i) \otimes (\sigma_2);$$

$$|C\rangle = |1, 1; 1\rangle = \frac{1}{2\sqrt{2}}\epsilon^{ijk}(\sigma_2 \sigma^j) \otimes (\sigma_2 \sigma^k).$$

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