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Dirac's Aether in Relativistic Quantum Mechanics

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The introduction by Dirac of a new aether model based on a stochastic covariant distribution of subquantum motions (corresponding to a "vacuum state" alive with fluctuations and randomness) is discussed with respect to the present experimental and theoretical discussion of nonlocality in EPR situations. It is shown (1) that one can deduce the de Broglie waves as real collective Markov processes on the top of Dirac's aether; (2) that the quantum potential associated with this aether's modification, by the presence of EPR photon pairs, yields a relativistic causal action at a distance which interprets the superluminal correlations recently established by Aspect et al.; (3) that the existence of the Einstein–de Broglie photon model (deduced from Dirac's aether) implies experimental predictions which conflict with the Copenhagen interpretation in certain specific testable interference experiments.

1. INTRODUCTION

Among all great physicists who founded quantum theory, Professor P. A. M. Dirac stands apart with Einstein and de Broglie. Indeed, once he had given (in a famous book[(1)]) the best known axiomatic presentation of the Copenhagen interpretation of this theory, he never stopped exploring new "strange" ideas, even when they were likely to destabilize an interpretation he had himself put in orbit with his crucial discoveries in electron-positron theory. In a paper written in his honor it is thus only fitting that one should discuss two of Dirac's famous "strange" ideas, i.e.,

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- his departure from a pointlike model of particles to justify the propagation of possible superluminal interactions;
- his contribution to the revival (in a new form, of course) of the old aether concept.

Since we want to concentrate essentially on the second idea, we shall only briefly recall the first idea as a possible basis for an interpretation of the experimental confirmation of nonlocal correlations in EPR experiments in photon pair emitted in the singlet state.(2)

Clearly the idea that extended particles are nonlocal in nature, i.e., that they can propagate in their interior superluminal interactions and/or information goes back to Dirac. He was the first to notice that if one treats the classical extended electron as a point charge imbedded in its own radiating electromagnetic field, the equations obtained are of the same form as those already in current use, but that in their physical interpretation the finite size of the electron reappears in a new sense: the interior of the electron being a region of space through which signals can be transmitted faster than light. Physically this can be understood as follows. If we send out a pulse from a point A and a receiving apparatus for electromagnetic waves is set up at a point B, and if we suppose that there is an extended electron on the straight line joining A to B, then the disturbed electron will be radiating appeciably at a time a/c before the pulse has reached its center, so that this emitted radiation will be detectable at B at a time $2a/c$ earlier than when the pulse, which travels from A to B with the velocity of light, arrives (here, of course, a is the electron radius). In this way a signal could be sent from A to B faster than light through the interior of an electron.

This possibility of superluminal transmission of signals, of course, is a problem of this model of extended electron in the same sense as the nonlocal correlations in an EPR experiment. As we will discuss later (see Section 3), in order to preserve the Einsteinian causality we must use the concept of relativistic action at a distance, as developed in the predictive mechanics.(3) Indeed we will be able to explain causally the nonlocal correlations by means of a nonlocal quantum potential which satisfies the compatibility conditions of the relativistic action at a distance.

This idea has engineered a long set of researches starting for example with Yukawa's bilocal particle model(4) and Bohm and Vigiier's liquid droplet model.(5) The essential point is that, independently of the internal motions which yield a classical model of spin,(6) it has generally been demonstrated by Souriau *et al.*(7) that any extended particle model yields an internal rotation of the particle's center of matter density around its center of mass with the exact frequency of de Broglie's relation $\nu_0 = m_0 c^2/h$. Of course, such extended particle models have received (until now) no direct experimental support. They open nevertheless interesting paths of research since:

- they offer the possibility to interpret the particle's newly discovered quantum numbers (T, Y, C, B, L,...) in terms of internal oscillations(8);
- they can contain (as suggested before) nonlocal hidden variables which can be utilized to support the nonlocal character of the quantum potential and lead to a causal action-at-a-distance interpretation of nonlocal correlations of EPR paradox.

Let us now come to the second idea, i.e., the reintroduction by Dirac of new possible aether models. As we shall see, this might well turn out to be one of Dirac's main contributions to the new era opened (in the author's opinion) by Aspect's confirmation of the real existence of superluminal correlations in the physical world.(2) In Dirac's own words(9):

"In the last century, the idea of an universal and all pervading aether was popular as a foundation on which to build the theory of electromagnetic phenomena. The situation was profoundly influenced in 1905 by Einstein's discovery of the principle of relativity, leading to the requirement of a four-dimensional formulation of all natural laws. It was found that the existence of an aether could not be fitted in with relativity, and since relativity was well established, the aether was abandoned.

Physical knowledge has advanced very much since 1905, notably by the arrival of quantum mechanics, and the situation has again changed. If one reexamines the question in the light of present-day knowledge, one finds that the aether is no longer ruled out by relativity, and good reasons can now be advanced for postulating an aether.

Let us consider in its simpler form the old argument for showing that the existence of an aether is incompatible with relativity. Take a region of space-time which is a 'perfect vacuum,' that is, there is no matter in it and also no fields. According to the principle of relativity, this region must be isotropic in the Lorentz sense—all directions within the light cone must be equivalent to one another. According to the aether hypothesis, at each point in the region there must be an aether, moving with some velocity, presumably less than the velocity of light. This velocity provides a preferred direction within the light-cone in space-time, which direction should show itself up in suitable experiments. Thus we get a contradiction with the relativistic requirement that all directions within the light cone are equivalent.

This argument is unassailable from the 1905 point of view, but at the present time it needs modification, because we have to apply quantum mechanics to the aether. The velocity of the aether, like other physical variables, is subject to uncertainty relations. For a particular physical state, the velocity of the aether at a certain point of space-time will not usually be a well defined quantity, but will be distributed over various possible values according to a probability law obtained by taking the square of the modulus of a wave function. We may set up a wave function which makes all values for the velocity of the aether equally probable. Such a wave function may well represent the perfect vacuum state in accordance with the principle of relativity....

Let us assume the four components v_μ of the velocity of the aether at any point of space-time commute with one another. Then we can set up a representation with the wave functions involving the v's. The four v's can be pictured as defining a point on a three-dimensional hyperboloid in a four-dimensional space, with the equation:

$$v_0^2 - v_1^2 - v_2^2 - v_3^2 = 1, \qquad v_0 > 0 \tag{1}$$

A wave function which represents a state for which all aether velocities are equally probable must be independent of the v's, so it is a constant over the hyperboloid (1). If we form the square of the modulus of this wave function and integrate over the three-dimensional surface (1) in a Lorentz-invariant manner, which means attaching equal weights to elements of the surface which can be transformed into one another by a Lorentz transformation, the result will be infinite. Thus this wave function cannot be normalized."

In other words, Dirac has bypassed all former relativistic objections to a static aether's existence by introducing a chaotic random moving subquantal aether behavior: a step subsequently revived and developed by Bohm and Vigier,[5,10] de Broglie,[11] Sudarshan *et al.*,[12] Cufaro Petroni and Vigier.[13]

To stress and clarify this essential point, we shall briefly recall a few evident results in a simplified case. One can see that Dirac's aether can be easily connected with the original "negative energy sea," which still remains the essential basis for the second quantization formalism as well as for all subsequent field theories. Indeed this negative energy sea can be considered as the first reintroduction of a material vacuum in relativistic quantum mechanics. As one knows,[14] Dirac's original vacuum is characterized (for spin-1/2 particles) by the fact that all positive energy states are not filled whereas all negative energy states are filled. In order to turn this vacuum into Dirac's aether it must be made covariant, i.e., not detectable with a Michelson and Morley experiment. As stated by Dirac,[9] we can satisfy such a condition if we consider that the four-momenta of the particles of Dirac's vacuum are uniformly distributed on the lower mass hyperboloid (see Fig. 1). Indeed, with a Lorentz transformation the equation of the hyper-

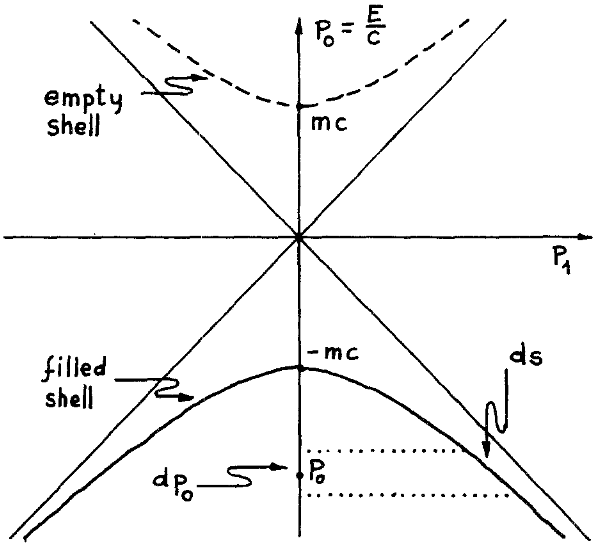

Fig. 1. Two-dimensional representation of Dirac's aether in momentum space: four-momenta are uniformly distributed on the lower filled mass shell.

boloid remains the same and, if the state distribution was uniform along this spacelike surface, i.e., if

$$dN = K\sqrt{|ds^2|} \tag{2}$$

(where dN is the number of states in a section ds of the hyperboloid and K is a constant), the new observer will see the same uniform distribution of states on his hyperboloid.

Of course, the distribution in energy is not constant in this case. We can compute this distribution starting from the obvious statement that in a section dp_0 of the p_0 axis (around a point p_0) we have a number of states $\rho(p_0)\,dp_0$ which equals the number of states in the corresponding ds on the hyperboloid (we fix here $p_1 \geqslant 0$, $p_0 \leqslant -mc < 0$), so that

$$K\,ds = dN = \rho(p_0)\,dp_0 \tag{3}$$

and hence

$$\rho(p_0) = K\frac{\sqrt{|ds^2|}}{dp_0} \tag{4}$$

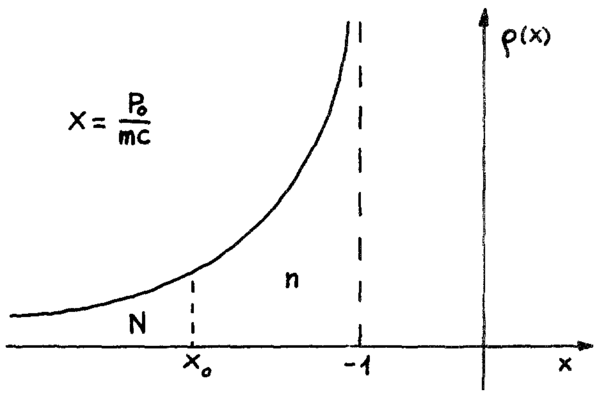

Fig. 2. Plot of the density ρ of states along the energy axis in Dirac's aether.

so that from the explicit expression of ds we have

$$\rho(p_0) = K \sqrt{\left(\frac{dp_1}{dp_0}\right)^2 - 1} = \frac{Kmc}{\sqrt{p_0^2 - m^2c^2}} \tag{5}$$

We have plotted the curve $\rho(x)$ with $x = p_0/mc$ in Fig. 2, and we can remark that our density diverges for $x \to -1$ and tends to zero for $x \to -\infty$. Nevertheless, we can prove that, if we take a fixed $x_0 \in]-\infty, -1[$, the number n of states between x_0 and -1 is always finite; on the contrary, the number N of the remaining states between $-\infty$ and x_0 always diverges. Indeed, we have

$$\begin{aligned} n &= \int_{x_0}^{-1} \rho(x)\, dx = \text{arc cosh}(x_0) \\ N &= \int_{-\infty}^{x_0} \rho(x)\, dx = +\infty \end{aligned} \tag{6}$$

This proves that in Dirac's aether distribution the weight of the almost light-like four-momenta must be predominant.

The main poblem now raised by this exposition is: How does Dirac's aether interact with a positive energy particle put in it? Beyond the precise mechanism of this interaction, what about the conservation laws? We can make here some remarks: It is clear that the theory of Dirac's equation requires only that all negative energy levels must be filled with just one particle for each level. Then, if we consider the four-momentum of this particle (for example of energy $\bar{E}$), we see that we have an infinity of possibilities for the p^μ direction (at least two for the two-dimensional case,

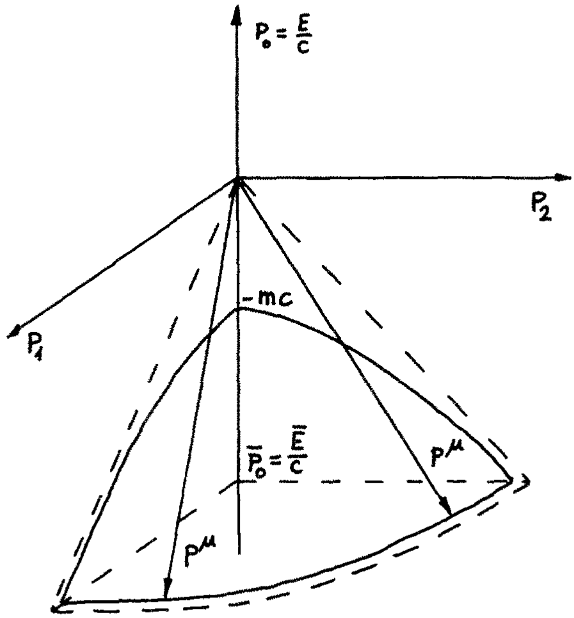

Fig. 3. Three-dimensional representation of Dirac's aether in momentum space: for each $\bar{p}_0$ value there is an infinity of equiprobable possible directions of the corresponding four-momentum p^μ.

but infinite in the other cases; see Fig. 3). So, if the level $\bar{E}$ is filled by only one particle, its p^μ is not completely determined and is uniformly distributed on the corresponding section of the hyperboloid. In this case, even if all the energy levels are filled, a vacuum particle can interact with a positive energy particle by exchanging momentum but no energy at all (with the obvious exception of the case of pair creation or annihilation). Hence we can say that in our subquantal medium a positive energy particle can travel without loss of energy (without "friction") but it can change the direction of its momentum $\mathbf{p}$ by interacting with the aether particles or produce pair creations and annihilations. As we will see in the subsequent section, these are exactly the possible interactions we need in order to describe the quantum statistics of the Klein–Gordon equation as a stochastic process.

Of course this transition from "hole theory" to Dirac's aether is too simple. In order to interpret the relativistic wave equations of quantum mechanics, Sudarshan *et al.*[12] and Vigier[14] had to complexify Dirac's aether model, i.e., introduce aether models built as superfluid states of particle–antiparticle pairs. In such model, the de Broglie waves are considered as real collective motions in which localized soliton-like energy-carrying particles[15] are surrounded by real physical "pilot waves" which interpret interference phenomena as well as nonlocal quantum correlations in configuration space. One is thus now confronted with the possible confrontation of the offsprings of Dirac's aether with present experimental possibilities.

The preceding discussion determines the plan of our paper. In the second section we shall illustrate in a simplified model how random stochastic jumps at the velocity of light yield (in a particle–antiparticle mixture) the basic relativistic second order equation of wave mechanics, i.e., the Klein–Gordon equation, which, as we will see, contains action at a distance tied to the quantum potential of Bohm[10] and de Broglie.[11] In the third section we shall discuss in terms of such a causal action at a distance the nonlocal quantum interactions which result from the experiments of Aspect.[2] In the fourth section we shall confront the conflicting predictions of the Copenhagen interpretation and of the stochastic interpretation of quantum mechanics in a particular situation in which Bohr's wave-packet-collapse concept conflicts with Maxwell's (i.e., Einstein's and de Broglie's) theory of light.

2. STOCHASTIC DERIVATION OF THE RELATIVISTIC QUANTUM EQUATIONS

According to our plan, we now utilize the concept of Dirac's chaotic aether (which assumes that the particles imbedded in it undergoes random jumps at the velocity of light) as a physical basis for the construction of the so-called stochastic interpretation of quantum mechanics. From this standpoint, the probabilistic character of quantum mechanics is not an irreducible limit of human knowledge but (following Einstein and de Broglie) appears as a natural consequence of the random character of the irregular deviations from the deterministic movement of a classical particle induced by the action of Dirac's chaotic aether. The explicit derivation of the relativistic quantum equations from such a stochastic process is, of course, very important in this type of model, because it materializes the link between the quantum and subquantum features of the microscopic world, so that all the correct predictions of the quantum mechanics can be reproduced in principle in this stochastic interpretation. This, evidently, realizes a hidden-variable theory, but (as we will later see) not a local one, as required by Bell's theorem.[16]

From the beginning of this line of thought[5,10] many demonstrations were published in the nonrelativistic[17] as well as in the relativistic[18] domain, both for spinless and spinning particles.[19] In such stochastic models the nature of the subjacent chaotic medium was not always clearly defined. In the authors' opinion, we are now left with only two lines of research in which this problem is clearly discussed, i.e., (1) the stochastic electrodynamics,[20] which consider charged particles imbedded in covariant electromagnetic vacuum; (2) the stochastic model based on Dirac's aether. In the later case one can deduce[13] from the features of this chaotic relativistic aether the fact that our particle must jump at the velocity of light, and (as we will also see later[19]) this is a fundamental characteristic in deducing the relativistic quantum equations. To show this explicitely we will give here an example of the derivation of the relativistic quantum equation for spinless particles (the Klein–Gordon equation) based on the hypothesis that the stochastic jumps are made at the velocity of light.[21]

To simplify our demonstration, we will limit ourselves to the case in which we assume that the random walks occur on a square lattice in a two-dimensional space-time (see Fig. 4) with coordinates x^0, x^1. We will describe a limit process where in each step we will suppose that our particle, starting from an arbitrary point $P_0(x^0, x^1)$, can only make jumps of fixed length, always at the velocity of light. Of course, this prescription completely determines the lattice of all possible particle positions. On such a lattice the particle can follow an infinity of possible trajectories. In our calculation we will consider first a lattice with fixed dimensions: Indeed for each jump we pose

$$\Delta x^0 = t\tau, \qquad \Delta x^1 = s\tau \qquad (t, s = \pm 1) \tag{7}$$

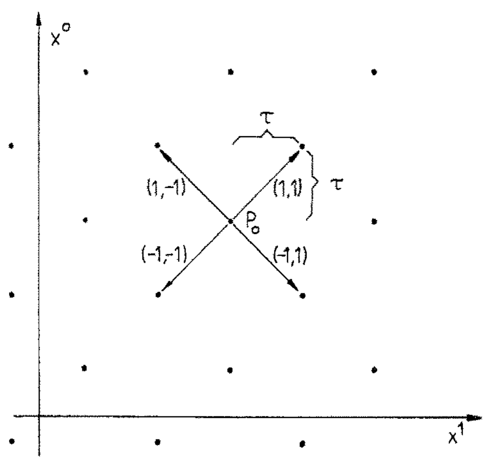

Fig. 4. Space-time lattice of dimension τ and starting point P_0. For each possible direction of the first jump we marked the corresponding value of the couple (t, s).

so that for the velocity we always have (for $\hbar = c = 1$)

$$v = \frac{\Delta x^1}{\Delta x^0} = \frac{s}{t} = \pm 1 \tag{8}$$

Here τ is the parameter which fixes the lattice dimensions: Of course, in order to recover the quantum equations, we will consider later the limit $\tau \to 0$. Moreover, it is clear from (7) and Fig. 4 that on this lattice we also consider the possibility of trajectories running backward in time: We will interpret them as trajectories of antiparticles running forward in time, following the usual Feynman interpretation.[22]

In order to describe random walks on this lattice, let we consider the following Markov process on the set of the four possible velocities of each jump: We define two sets of stochastic variables $\{\varepsilon_j\}$, $\{\eta_j\}$, with $j \in N$, in such a way that the only possible values of each ε_j and η_j are ± 1, following this prescription:

$$\varepsilon_j = \begin{cases} 1 \\ -1 \end{cases} \text{if in the } (j+1)\text{th jump the sign of velocity} \begin{cases} \text{doesn't change} \\ \text{changes} \end{cases}$$

$$\eta_j = \begin{cases} 1 \\ -1 \end{cases} \text{if in the } (j+1)\text{th jump the direction}$$

$$\text{of the time} \begin{cases} \text{doesn't change} \\ \text{changes} \end{cases}$$

with respect to the preceding jth jump. It means that the realization of the signes of ε_j, η_j determines one of the four possible directions of the $(j+1)$th jump on the ground of the direction of the jth jump, as we can see in Fig. 5.

Of course, a sequence $\{\varepsilon_j, \eta_j\}$, with $j \in N$, of values of these stochastic variables completely determines one of the infinite possible trajectories, except for the first jump, because there is no "preceding" jump for it. Thus, starting from $P_0(x^0, x^1)$, in the first jump we can get one of the four possible points $P_1(x^0 + t\tau, x^1 + s\tau)$, and after N jumps, as we can easily see by direct calculation, one of the points $P_N(x^0 + tT_N, x^1 + sD_N)$, where

$$\begin{aligned} T_N &= \tau(1 + \eta_1 + \eta_1\eta_2 + \cdots + \eta_1\eta_2 \cdots \eta_{N-1}) \\ D_N &= \tau(1 + \varepsilon_1\eta_1 + \varepsilon_1\varepsilon_2\eta_1\eta_2 + \cdots + \varepsilon_1\varepsilon_2 \cdots \varepsilon_{N-1}\eta_1\eta_2 \cdots \eta_{N-1}) \end{aligned} \tag{9}$$

We come now to the problem of the assignment of a statistical weight to each trajectory. In order to do that, we introduce for each $(j+1)$th jump a probability for each realization of the signs of the corresponding jth couple ε_j, η_j. In Table 1 we have listed these probabilities for a general ε_j, η_j couple.

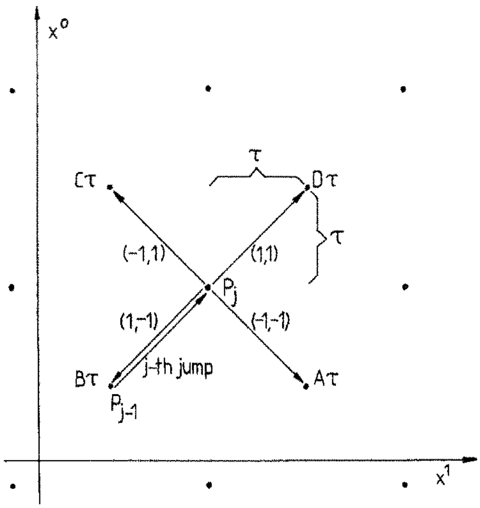

Fig. 5. An example of the four possible successions of two jumps. For each possible $(j+1)$th jump we marked the value of the couple (ε_j, η_j) and the corresponding probability.

Moreover, we suppose that A, B, C, D are constant over all the space-time.

Among these four constants we can also posit the usual relation

$$(A + B + C + D)\tau = 1 \tag{10}$$

In order to derive the Klein–Gordon equation, we consider a function $f(x^0, x^1)$ defined over all the space-time and, generally speaking, with complex values, and then we define the following set of functions:

$$F_N^{t,s}(x^0, x^1) = \langle f(P_N)\rangle = \langle f(x^0 + tT_N, x^1 + sD_N)\rangle \tag{11}$$

Table I. Probabilities for the Four Possible Successions of Two Jumps

ε_j	η_j	Probability
−1	−1	$A\tau$
1	−1	$B\tau$
−1	1	$C\tau$
1	1	$D\tau$

Here $\langle\cdot\rangle$ indicates an average for all the possible points P_N attained, following trajectories constituted by N jumps, starting from $P_0(x^0, x^1)$ with a first jump made in the direction fixed by (t, s).

In fact, it is clear that the terminal point P_N is not uniquely determined by the initial point P_0 and the number of jumps N, because the possibility to choose different trajectories of N jumps. Of course, on the average, the statistical weight of each P_N is calculated from the probabilities associated with the trajectories which lead to P_N, as stated in the previous section. We remark finally that, because the arbitrariness of the starting point P_0, the function $F_N^{t,s}$ is defined over all space-time.

We can start to make this average from the first jump so that, remembering (9) and (11):

$$\begin{aligned} F_N^{t,s}(x^0, x^1) &= \langle f[x^0 + t\tau + t\tau\eta_1(1 + \eta_2 + \cdots + \eta_2 \cdots \eta_{N-1}), \\ &\qquad x^1 + s\tau + s\tau\varepsilon_1\eta_1(1 + \varepsilon_2\eta_2 + \cdots + \varepsilon_2 \cdots \varepsilon_{N-1}\eta_2 \cdots \eta_{N-1})]\rangle \\ &= \langle f(x^0 + t\tau + t\eta_1 T_{N-1}, x^1 + s\tau + s\varepsilon_1\eta_1 D_{N-1})\rangle \\ &= D\tau F_{N-1}^{t,s}(x^0 + t\tau, x^1 + s\tau) + A\tau F_{N-1}^{-t,s}(x^0 + t\tau, x^1 + s\tau) \\ &\quad + B\tau F_{N-1}^{-t,-s}(x^0 + t\tau, x^1 + s\tau) + C\tau E_{N-1}^{t,-s}(x^0 + t\tau, x^1 + s\tau) \end{aligned} \tag{12}$$

and using (10), that is $D\tau = 1 - (A + B + C)\tau$, we get

$$\begin{aligned} F_N^{t,s}(x^0, x^1) &= F_{N-1}^{t,s}(x^0 + t\tau, x^1 + s\tau) \\ &\quad + A\tau[F_{N-1}^{-t,s}(x^0 + t\tau, x^1 + s\tau) - F_{N-1}^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\ &\quad + B\tau[F_{N-1}^{-t,-s}(x^0 + t\tau, x^1 + s\tau) - F_{N-1}^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\ &\quad + C\tau[F_{N-1}^{t,-s}(x^0 + t\tau, x^1 + s\tau) - F_{N-1}^{t,s}(x^0 + t\tau, x^1 + s\tau)] \end{aligned} \tag{13}$$

We pass now to the limit $N \to \infty$ (and τ fixed): If we indicate with $F^{t,s}$ the functions for $N \to \infty$ we have, from (13),

$$\begin{aligned} F^{t,s}(x^0, x^1) &= F^{t,s}(x^0 + t\tau, x^1 + s\tau) \\ &\quad + A\tau[F^{-t,s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\ &\quad + B\tau[F^{-t,-s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\ &\quad + C\tau[F^{t,-s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x^1 + s\tau)] \end{aligned} \tag{14}$$

and then

$$
\begin{aligned}
&-[F^{t,s}(x^0 + t\tau, x^1) - F^{t,s}(x^0, x^1)]/t\tau \\
&\quad = (s/t)[F^{t,s}(x^0, x^1 + s\tau) - F^{t,s}(x^0, x^1)]/s\tau \\
&\qquad + [F^{t,s}(^0 + t\tau, x^1 - s\tau) - F^{t,s}(x^0, x^1 + s\tau)]/t\tau \\
&\qquad - [F^{t,s}(x^0 + t\tau, x^1) - F^{t,s}(x^0, x^1)]/t\tau \\
&\qquad + (A/t)[F^{-t,s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\
&\qquad + (B/t)[F^{-t,-s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x^1 + s\tau)] \\
&\qquad + (C/t)[F^{t,-s}(x^0 + t\tau, x^1 + s\tau) - F^{t,s}(x^0 + t\tau, x_1 + s\tau)]
\end{aligned} \tag{15}
$$

In the limit $\tau \to 0$, when our lattice tends to recover all of space-time, we get the following set of four partial differential equations (one for each possible value of the couple t, s of the first jump):

$$
\begin{aligned}
-\frac{\partial F^{t,s}}{\partial x^0} = \frac{s}{t}\frac{\partial F^{t,s}}{\partial x^1} + \frac{A}{t}(F^{-t,s} - F^{t,s}) + \frac{B}{t}(F^{-t,-s} - F^{t,s}) \\
+ \frac{C}{t}(F^{t,-s} - F^{t,s})
\end{aligned} \tag{16}
$$

where we neglected the arguments (x^0, x^1) of the functions.

If we define now the following four linear combinations of the four functions $F^{t,s}$:

$$
\begin{cases}
\phi = F^{1,1} + F^{-1,-1} + F^{1,-1} + F^{-1,1} \\
\psi = F^{1,1} + F^{-1,-1} - F^{1,-1} - F^{-1,1} \\
\chi = -F^{1,1} + F^{-1,-1} - F^{1,-1} + F^{-1,1} \\
\omega = -F^{1,1} + F^{-1,-1} + F^{1,-1} - F^{-1,1}
\end{cases} \tag{17}
$$

we can build a new equivalent set of equations by combining Eqs. (16):

$$
\begin{cases}
\dfrac{\partial\phi}{\partial x^0} + \dfrac{\partial\psi}{\partial x^1} = -2(A + B)\chi \\[2ex]
\dfrac{\partial\psi}{\partial x^0} + \dfrac{\partial\phi}{\partial x^1} = -2(C + B)\omega \\[2ex]
\dfrac{\partial\chi}{\partial x^0} + \dfrac{\partial\omega}{\partial x^1} = 0 \\[2ex]
\dfrac{\partial\omega}{\partial x^0} + \dfrac{\partial\chi}{\partial x^1} = -2(A + C)\psi
\end{cases} \tag{18}
$$

By derivation and successive linear combination of equations (18), we have:

$$\begin{cases} \Box\phi = -2(A+B)\dfrac{\partial\chi}{\partial x^0} + 2(C+B)\dfrac{\partial\omega}{\partial x^1} = -2(A+2B+C)\dfrac{\partial\chi}{\partial x^0} \\ \Box\psi = -2(C+B)\dfrac{\partial\omega}{\partial x^0} + 2(A+B)\dfrac{\partial\chi}{\partial x^1} = -2(A+2B+C)\dfrac{\partial\omega}{\partial x^0} \\ \qquad -4(A+B)(A+C)\psi \\ \Box\chi = 2(A+C)\dfrac{\partial\psi}{\partial x^1} = -2(A+C)\dfrac{\partial\phi}{\partial x^0} - 4(A+C)(A+B)\chi \\ \Box\omega = -2(A+C)\dfrac{\partial\psi}{\partial x^0} = 2(A+C)\dfrac{\partial\phi}{\partial x^1} + 4(A+C)(C+B)\omega \end{cases} \tag{19}$$

(where $\Box$ is a two-dimensional d'Alembert operator); and if we pose

$$-B = \frac{A+C}{2}, \qquad 2(A^2 - C^2) = m^2 \tag{20}$$

we finally get

$$\begin{cases} (\Box + m^2)\psi = 0 \\ \Box\phi = 0 \\ (\Box + m^2)\chi = -2(A+C)\dfrac{\partial\phi}{\partial x^0} \\ (\Box + m^2)\omega = 2(A+C)\dfrac{\partial\phi}{\partial x^1} \end{cases} \tag{21}$$

We now make the following remarks:

(a) We can interpret the first equation of (21) as a Klein–Gordon equation. The function ψ, which satisfies this Klein–Gordon equation, is the average of a function f over all the possible final points reached following all the possible trajectories of infinite jumps. In this average, as we can see from (17), we consider also the first jump by supposing that the four possibilities for the signes of t, s are equiprobables.

(b) The functions χ, ϕ, ω which satisfies the remaining equations in (21) have no direct physical interpretation and seem to us to constitute only a formal tool in the deduction of the equation for the complete average ψ. However, we see that in (21) the equation for ψ is not coupled at all with the other equations for χ, ϕ, ω, so that the solution of the Klein–

Gordon equation is absolutely independent of the solutions of the rest of the system.

(c) The previous derivation of (21) from (18) shows that each solution $(\phi, \chi, \psi, \omega)$ of (18) is a solution of (21), but it is possible to show that not all the solutions of (21) are solutions of (18). Indeed, for example, we can verify by direct calculation that

$$\begin{aligned} &\psi = \exp[ip \cdot x] \;\; (\text{with } p^2 = m^2) \\ &\chi = \phi = \omega = 0 \end{aligned} \tag{22}$$

is a solution of (21), but it is not a solution of (18). Therefore, it is important to analyze the following question: We proved the statement "the function ψ defined as a stochastic average in (17) and satisfying the system (18) always is a solution of a Klein–Gordon equation." What about the inverse statement "all the solutions of a Klein–Gordon equation are interpretable as stochastic averages satisfying a system like (18)?" We will show here that this inverse statement holds in the following sense: If ψ is an arbitrary solution of the Klein–Gordon equation always, we can determine the functions χ, ϕ, ω in such a way that $(\phi, \chi, \psi, \omega)$ is a solution of (18). In fact, if ψ is an arbitrary solution of the Klein–Gordon equation in (21), we can choose ϕ as an arbitrary solution of $\Box\phi = 0$, and then we determine χ and ω as follows:

$$\begin{aligned} \chi &= \frac{1}{C - A}\left[\frac{\partial\phi}{\partial x^0} + \frac{\partial\psi}{\partial x^1}\right] \\ \omega &= \frac{1}{A - C}\left[\frac{\partial\psi}{\partial x^0} + \frac{\partial\phi}{\partial x^1}\right] \end{aligned} \tag{23}$$

It is only a question of calculation to show now that our $(\phi, \chi, \psi, \omega)$ is a solution of (18) (with $-B = (A + C)/2$) and of (21).

(d) We are confronted here with an old problem characteristic of relativistic quantum mechanics,[23] namely the existence of negative probabilities. Indeed, we can immediately see from (20) that A, B, C, D cannot be simultaneously positive if we want to get the system (21). If, for example, we choose A, $C < 0$, we have

$$B = -\frac{A + C}{2} > 0 \qquad \text{and} \qquad D = \frac{1}{\tau} - (A + B + C) = \frac{1}{\tau} - \frac{A + C}{2} > 0$$

so that the probabilities of the inversion of the sign of the velocity (A, C) have an opposite sign with respect to the probabilities (B, D) of the noninversion of the sign of the velocity (see Fig. 5). This choice of the signs is also coherent with the definition (17) of ψ as an average, where $F^{1,1}$ and $F^{-1,-1}$ are considered with the same probability but with opposite sign with respect to $F^{1,-1}$ and $F^{-1,1}$. Of course, we have no final answer to the question "what is a negative probability?": We can only quote a proposition for his interpretation[24] in which the negative sign of the probability distribution is interpreted as reflecting the opposite "charge" values of antiparticles in a particle-antiparticle distribution. We further remark that this is a problem which arises every time we are dealing with particles and antiparticles, and hence that it would be very strange not to meet it here where the possibility of trajectories running backward in time on our lattice are interpreted with the presence of pair creation and annihilation.[22] On the other hand, it is clear that if we had not assumed the possibility of the trajectories running backward in time (i.e., the antiparticle behavior) all our statistics would be different since it is possible to show[25] that one obtains in this case a classical diffusion equation that one can not reduce to the quantum Klein–Gordon equation.

3. DETERMINISTIC NONLOCAL INTERPRETATION OF THE ASPECT-RAPISARDA EXPERIMENTS ON THE EPR PARADOX

In this section we are going to utilize the hydrodynamical-stochastic interpretation of the quantum mechanics, physically based on the real existence of a chaotic Dirac's aether, as a starting point for a deterministic interpretation of the recent results of the Aspect–Rapisarda experiments on correlated photon pairs. As is well-known, the paradoxical features of the quantum mechanical description of correlated systems first discussed by Einstein, Podolsky, and Rosen[26] are now experimentally tested, in the form established by Bohm[27] for discrete variables. Recent discussions have convinced physicists that an essential property of the EPR paradox lies in the nonlocal character of quantum correlations, which seem to be in striking contradiction with Einstein's relativistic causal description of nature—and imply causal anomalies.[28] Indeed, in the EPR paper the hypothesis of the noninteraction between two correlated system at a great distance is essential in order to achieve the demonstration of the incompleteness of the quantum mechanics,[29] and Bell's theorem[16] states that there is a measurable difference between the predictions of quantum mechanics and any local hidden-variable theory for correlated particles.

To illustrate this, we briefly recall a typical experimental set-up to check Bell's inequalities. Let us consider a pair of photons (1 and 2) issuing from a cascade source S in a single state of polarization, so that they move in opposite directions parallel to the same x axis. These photons are successively detected through two linear polarizers (L and N) with polarization directions A and B perpendicular to the x axis (see Fig. 6). We know[1] that a photon impinging upon a linear polarizer either passes or is stopped, thus answering yes or no (1 or 0) to the question: "Is your linear polarization found parallel or perpendicular to the direction $A(B)$ of the polarizer $L(N)$?" We can thus compute the probability of the four possible answers to the composite question: "Does the photon 1 pass the polarizer L and the photon 2 the polarizer N?" For this calculation we need only the initial and final states $|i\rangle$ and $|f\rangle$ of our composite system, so that, denoting by $(1, 1)$, $(1, 0)$, $(0, 1)$, and $(0, 0)$ the probabilities of the four possible answers, we compute the probabilities as $|\langle i\,|f\rangle|^2$.

Of course, if our initial state is

$$|i\rangle = (1/\sqrt{2})(|y_1\rangle\,|y_2\rangle + |z_1\rangle\,|z_2\rangle) \tag{24}$$

in terms of state vectors polarized along two orthogonal axes y and z in an $x = \text{const.}$ plane, the final state is, for example, for the case $(1, 1)$,

$$|f\rangle = (\cos A\,|y_1\rangle + \sin A\,|z_1\rangle)(\cos B\,|y_2\rangle + \sin B\,|z_2\rangle) \tag{25}$$

so that we have

$$(1, 1) = |\langle i\,|f\rangle|^2 = (1/2)(\cos A\,\cos B + \sin\,A\,\sin\,B)^2 = (1/2)\cos^2\alpha \tag{26}$$

if $\alpha = A - B$. In an analogous way we get immediately

$$(0, 0) = (1/2)\cos^2\alpha \quad (1, 0) = (0, 1) = (1/2)\sin^2\alpha \tag{27}$$

The crux of the new situation now lies in Bell's proof[16] that this quantum mechanical predictions on the two photon coincidences cannot result from

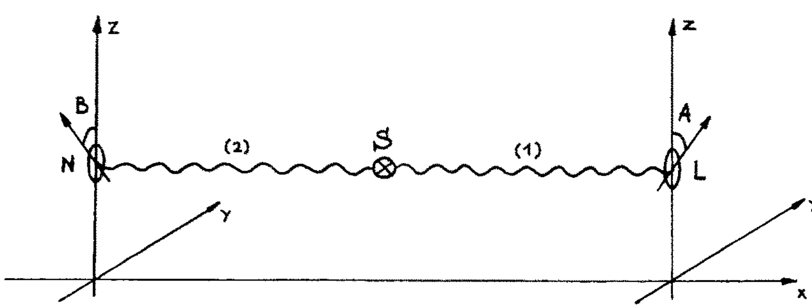

Fig. 6. Schematic view of an EPR–Bohm experiment.

the correlation functions obtained in a local hidden-variable theory. The same result was attained, with another example, in a recent paper by Feynman.[23]

As one knows, despite the almost general confirmation of such quantum predictions in EPR experiments[30] (i.e., of the violation of Bell's inequality) a supplementary device with four photon coincidences was needed to definitely prove the nonlocal character of this quantum correlation.[31] This set-up essentially rests on the use of calcite crystals acting as random switches on the photon paths which orient them, with a 1/2-probability, in the ordinary (O) or extraordinary (E) rays. The photon thus pick at random four possible paths and are subsequantly detected through two pairs of linear polarizers L, L', N, N' (see Fig. 7). The recent result of this experiment,[2] obtained by Aspect's group, confirms the quantum mechanical prediction with great precision for separation of 12 m between the polarizers $L(L')$ and $N(N')$. If the forthcoming Rapisarda experiment also confirms this result, we will be faced with the problem of the interpretation of nonlocal correlations in the microscopic domain.

Two remarks can be made at this stage of the discussion:

(1) There is no possibility left by these experiments, to construct a local hidden-variable theory for quantum mechanics,[16] but it is still possible in principle to build a nonlocal one coherent with a characteristic feature of the hydrodynamical-stochastic interpretation—since the quantum potential for correlated systems is always nonlocal.[32] The problem, as we will now see, is how to construct a coherent causal covariant nonlocal theory.

(2) There is no possibility (as claimed by the authors in another paper[33]) for a observer in L to use this EPR experiment to send macroscopic superluminal signals to the observer in N, because they are always dealing with coincidence experiments. However, we can deduce[33] from

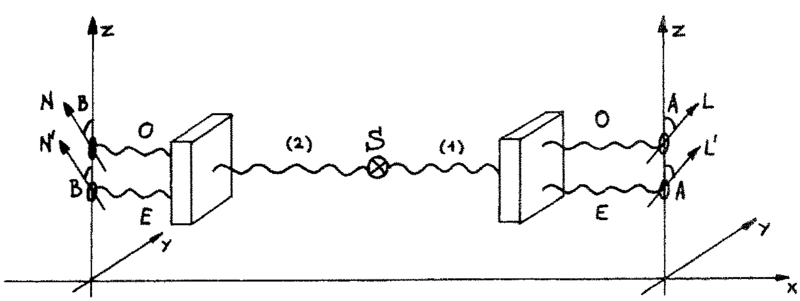

Fig. 7. Schematic view of the Aspect–Rapisarda experiment.

an "*a posteriori*" analysis of the experimental results the existence of past superluminal "exchange of information" between the two photons, in the sense that we can explain by a closed causal chain the existence of correlations and any other variation of that induced by operations on the polarizers only by a sort of nonlocal link between spatially separated events.

In this section we will extend the analysis of the nonlocal character of the quantum potential to the case of spinning particles, in order to show that, for correlated systems, also the spins (and polarizations) are nonlocally connected.

We start with a non-zero mass photon model ($m_\gamma \neq 0$). This is justified: (1) by the well-known fact[34] that the zero-mass limit of a nonzero mass spin-1 Proca particle cannot be physically distinguished from a Maxwell wave, since the so-called transverse waves just correspond to $J_3 = \pm 1$ (i.e., to opposite circular polarizations), while the longitudinal solutions $J_3 = 0$ (pratically decoupled from transverse waves when $m_\gamma \to 0$) describes the Coulomb field when $m_\gamma \to 0$; (2) by the theoretical result that (with $m_\gamma \neq 0$) one has found a classical counterpart (i.e., the Weyssenhoff particle) to the photon field,[35] so that one can determine a classical counterpart of spin for isolated "classical" photons which is distributed[36] in the hydrodynamical representation of the Proca wave equation.

Both in the usual quantum mechanical theory[37] and in the stochastic interpretation of quantum mechanics[38] a system of two correlated photons ($m_\gamma \neq 0$) can be represented by a second rank tensor $A_{\mu\nu}$. As one knows, this compound state of two spin-1 particles can be split [as a consequence of the group representation relation $D(1) \otimes D(1) = D(2) \oplus D(1) \oplus D(0)$] into a symmetric part $A_{\underline{\mu\nu}}$, a skew part $A_{\breve{\mu\nu}}$, and a trace $A_{\mu\mu}$, representing respectively the $J = 2$, $J = 1$, and $J = 0$ compound states. Since the aforementioned experiments utilize 0-1-0 singlet states cascades, we shall limit ourselves to the $A_{\mu\mu}$, $D(0)$, $J = 0$ singlet case.

Denoting by 1 and 2 the two photons (with coordinates x_1^μ and x_2^μ), we represent our compound state by a scalar field

$$\Phi(x_1, x_2) = A_{1\mu}(x_1) A_2^\mu(x_2) = \exp[R(x_1, x_2) + iS(x_1, x_2)]$$
$$\text{where} \quad \hbar = c = 1$$

As one knows,[3,39] such a scalar field satisfies the system of relations

$$\begin{cases} (\Box_1 + \Box_2 - 2m_\gamma^2)\Phi = 0 \\ (\Box_1 - \Box_2)\Phi = 0 \end{cases} \tag{28}$$

or, equivalently,

$$\begin{cases}(\Box_1 - m_\gamma^2)\Phi = 0 \\ (\Box_2 - m_\gamma^2)\Phi = 0\end{cases} \tag{29}$$

along with the transverse gauge conditions $\partial_{1\mu} A_1^\mu = \partial_{2\mu} A_2^\mu = 0$, the second relation (28) representing the causality constraint in the so-called predictive mechanics with action at a distance.[3] In this case, the Lagrangian of our pair will be

$$\mathscr{L} = m_\gamma^2 \Phi^* \Phi + \partial_{1\mu} \Phi^* \partial_1^\mu \Phi + \partial_{2\mu} \Phi^* \partial_2^\mu \Phi \tag{30}$$

A classical relativistic hydrodynamical analysis[36,40] then allows one to build the energy-momentum tensor for each single photon (from now on, because of the $1 \leftrightarrow 2$ symmetry, we will calculate only the quantities relative to the photon 1), i.e.,

$$\begin{aligned} t_{1\mu\nu} &= \frac{\partial \mathscr{L}}{\partial(\partial_1^\nu A_1^\lambda)} \partial_{1\mu} A_1^\lambda + c.c. - \mathscr{L}\, \delta_{\mu\nu} \\ &= \partial_{1\mu} \Phi^* \partial_{1\nu} \Phi + \partial_{1\mu} \Phi \partial_{1\nu} \Phi^* - \mathscr{L} \delta_{\mu\nu} \end{aligned} \tag{31}$$

From Belinfante's tensor,[40]

$$\begin{aligned} f_{1\mu\nu\lambda} &= \frac{\partial \mathscr{L}}{\partial(\partial_1^\lambda A_1^\rho)} \mathscr{Z}_{\mu\nu}^{\rho\sigma} A_{1\sigma} + c.c. \\ &= \tfrac{1}{2}(\partial_{1\lambda} \Phi^*)(A_{2\mu} A_{1\nu} - A_{1\mu} A_{2\nu}) + c.c. \end{aligned} \tag{32}$$

where $\mathscr{Z}_{\mu\nu}^{\rho\sigma} = \frac{1}{2}(\delta_{\rho\mu}\, \delta_{\sigma\nu} - \delta_{\rho\nu}\, \delta_{\sigma\mu})$, the spin density tensor becomes (if u_i^μ are the unitary four-velocity of the photons)

$$\tfrac{1}{2} S_{1\mu\nu} = -u_1^\lambda f_{1\mu\nu\lambda} = (A_{1\mu} A_{2\nu} - A_{1\nu} A_{2\mu})\, u_1^\lambda\, \partial_{1\lambda} \Phi^* + c.c. \tag{33}$$

and the spin vector can be written

$$S_{1\mu} = \frac{i}{2} \varepsilon_{\nu\alpha\beta\mu} u_1^\nu S_1^{\alpha\beta} \tag{34}$$

Moreover, denoting now by a dot the derivative along a current line, we can show that, because of the $t_{1\mu\nu}$ symmetry, we have[40]

$$\dot{S}_{1\mu\nu} = \partial_{1\lambda}(u_1^\lambda S_{1\mu\nu}) = t_{1\mu\nu} - t_{1\nu\mu} = 0 \tag{35}$$

From this results that:[41]

(1) $S_{1\mu}$ has a constant length in the sense that $\dot{S}_1^2 = 0$; indeed we see that[40] $S_1^2 = S_{1\mu} S_1^\mu = \frac{1}{2} S_{1\alpha\beta} S_1^{\alpha\beta}$ because of the properties of $\varepsilon_{\alpha\mu\nu\beta}$, the antisymmetry of $S_{1\mu\nu}$, $u_1^\mu u_{1\mu} = -1$, and $u_{1\mu} A_1^\mu = u_{1\mu} A_2^\mu = 0$. Hence $\dot{S}_1^2 = 0$, because we showed that $\dot{S}_{1\mu\nu} = 0$.

(2) The derivative of S_1 is $\dot{S}_1 = (i/2) \varepsilon_{\nu\alpha\beta\mu} S_1^{\alpha\beta} (u_1^\lambda \partial_{1\lambda} u_1^\nu)$, so that it depends in a nonlocal way from the $A_2(x_2)$ contained in $S_1^{\mu\nu}$.

(3) The elements of the photon pairs interact permanently not by exchanging tachyons but through action at a distance, which reflects the disturbance of Dirac's covariant stochastic aether.[13,14] In present experiments the photons are "holding hands" over 12 meters an any disturbance of one is carried superluminally to the other by a phase-like disturbance of the stochastic quantum potential—which includes a quantum torque.

Despite the presence of an action at a distance, it is possible to show[41] that this system is relativistically deterministic, in the sense that we shall now show:

- The system of two $J = 1$ particles can be solved in the forward (or backward) time direction in the sense of the Cauchy problem.
- The paths of the two particles are time-like.
- The formalism is invariant under the Poincaré group $P = T \otimes \mathscr{L}\uparrow$.

Indeed, writing $P_i^\mu = \partial S / \partial q_{i\mu}$ $(i = 1, 2)$, we can split internal from external variables by writing $P^\mu = p_1^\mu + p_2^\mu$; $y^\mu = (1/2)(p_1^\mu - p_2^\mu)$; $Q^\mu = (1/2)(q_1^\mu + q_2^\mu)$; $z^\mu = q_1^\mu - q_2^\mu$; q_i^μ, and p_i^μ representing pairs of canonical variables. Splitting (29) into real and imaginary parts, we obtain, for the real parts,

$$\begin{cases} (1/2)\, \partial_{1\mu} S \partial_1^\mu S + U_1 = (1/2)\, m_\gamma^2 \\ (1/2)\, \partial_{2\mu} S \partial_2^\mu S + U_2 = (1/2)\, m_\gamma^2 \end{cases} \tag{36}$$

where we have $U_i = -(1/2)(\Box_i R + \partial_i^\mu R \partial_{i\mu} R)$. This separation can be performed in the rest frame of the center of mass of the two photons, where we consider the case of an eigenstate of P_μ, i.e., $\Phi = \varphi(z^\mu) \exp[ik_\mu(x_1^\mu + x_2^\mu)/2]$, where k^μ is a constant timelike vector. In that case we have $(\partial_1^\mu + \partial_2^\mu) R = 0$ and $(\partial_1^\mu + \partial_2^\mu) S = k^\mu$, so that, subtracting Eq. (36), we get $k^\mu (\partial R / \partial z^\mu) = 0$, and hence R only depends on $z_\perp^\mu = z^\mu - (z_\nu k^\nu) k^\mu / k^2$. In order to satisfy the condition $\{y \cdot P, U\} = 0$ for the existence of causal timelike world lines,[3,39] we must now make the substitution $z_\perp^\mu \to \tilde{z}^\mu = z^\mu + (z_\nu P^\nu) P^\mu / P^2$, so that $(\partial_1^\mu + \partial_2^\mu) R = 0$ and $U_1 = U_2 = U(\tilde{z}_\mu)$. In that case the relations (36) represent a pair of causally bound photons connected by a causal action at a distance. Moreover:

(1) The causality condition $P \cdot y = 0$ implies that the Poisson bracket of the two photon Hamiltonians $\{H_1, H_2\}$ is zero, i.e., that their corresponding proper times τ_1 and τ_2 are independent.

(2) $q_i^\mu = x_i^\mu$ in the rest frame of the center of mass Σ_0 $(k_j = 0)$.

(3) Subtracting Eq. (36) with $U_1 = U_2$, we get[13] $P \cdot y = 0$, so that $\dot{P}_\mu = 0$, where the dot denotes the operation $(1/2)(\partial/\partial\tau_1 + \partial/\partial\tau_2)$. This yields $P \cdot \dot{y} = 0$, which shows that no energy can be exchanged between the photons in Σ_0, so that no causal anomaly results from this particular type of action at a distance.

(4) We have $\dot{p}_i \cdot P = 0$, so that the paths of both photons remain timelike.

(5) The formalism shows that[42] our causal covariant action at a distance is instantaneous only in Σ_0, and its velocity η can thus be calculated in any other frame Σ by the $\Sigma_0 \to \Sigma$ corresponding Lorentz transformation. In the particular case of the Aspect–Rapisarda experiment, this immediately yields $\eta = 7{,}57c$ in the laboratory frame.

This analysis implies that the hydrodynamical-stochastic interpretation of the quantum mechanics based on the physical existence of a chaotic Dirac's aether can provide all the essential elements needed to build a nonlocal hidden-variable theory, since the nonlocal quantum potentials and quantum torques satisfy the compatibility conditions[3] required by the predictive mechanics in order to have a relativistically deterministic theory. This also implies that both EPR paradox and the experimentally confirmed violation of Bell's inequality can be completely interpreted in a model that does not imply mysterious retrodictions[43] or *a priori* limitations of our comprehension[44]—since quantum mechanics appears as a statistical manifestation of a subquantum classical, relativistic and deterministic world in which there is also place for actions at a distance whose physical basis is the nonlocal quantum potential or, in other words, the physical existence of de Broglie's waves on Dirac's aether.

The importance of causal action at a distance is now evident. Despite the fact that we are dealing with a nonlocal theory, we claim that there is no possibility left for causal anomalies.[39] Indeed, a perfectly deterministic nonlocal theory is not at all a theory in which we can send superluminal signals in contradiction to relativity and causality,[28] since the existence of such "signals" requires the existence of a "free will," i.e., of somebody who "decides" at a given time to send something to somebody else. If, as claimed in our model, absolutely everything (bodies, men, "free will," etc.) are completely determined, all events are fixed somewhere in space-time, so that we cannot properly speak of "signals." In this scheme, the world is thus describable by a causal ensemble of particle in mutual interaction, the causality implied in it being absolute. The measuring processes themselves and the observers satisfy the same causal laws and are real physical processes with antecedents in time. The measuring process (observer plus apparatus plus observed object) can now be considered as a set of particles which belong to an overall causal process, so that the intervention of a measurement contains no extranatural "free will" or "observer consciousness." Quantum measuring devices now act as spectral analysers[10] which split into subpackets the real de Broglie's waves associated with particles: The particle entering in one of them according to its random causal motion.[45] In brief, there is no "free will" signal production and thus no possible causal paradoxes: Nothing exists beyond the motion and interactions of material particles in a random stochastic aether.

4. HOW DOES A PHOTON INTERFERE WITH ITSELF?

We conclude this paper with a brief discussion of the present theoretical and experimental status of Professor Dirac's initial views on the nature of quantum mechanics illustrated in the first pages of his famous book on quantum mechanics.[1] As every physicist knows, this book contain the deepest and clearest exposition ever made of the basic concepts underlying the Copenhagen interpretation of quantum mechanics. It is thus very important that the physical *gedanken* experiment discussed by him (based on the theory of light in single photon eases) are now about to become testable directly (a natural consequence of technical progress in the field of detection of single photons) and that explicit, realisable (in the author's opinion) experiments are now proposed and discussed in the literature in order to test the validity of the said concepts.[46]

Dirac starts his discussion of the principles of quantum mechanics by a discussion of the principle of superposition of states which he analyzed in the case of isolated photons both for polarization and interference. Since the present experiments are really built to test the validity and signifiance of his analysis for interference, we shall quote him at some length:[1]

"We shall discuss the description which quantum mechanics provides of the interference of photons. Let us take a definite experiment demonstrating interference. Suppose we have a beam of light which is passed through some kind of interferometer, so that it gets split up into two components and the two components are subsequently made to interfere. We may, as in the preceding section, take an incident beam consisting of only a single photon and inquire what will happen to it as it goes through the apparatus. This will present to us the difficulty of the conflict between the wave and corpuscular theories of light in an acute form.

Corresponding to the description that we have in the case of the polarization, we must now describe the photon as going partly into each of the two components into which the incident beam is split. The photon is then, as we may say, in a translational state given by the superposition of the two translational states associated with the two components. We are thus led to a generalization of the term 'translational state' applied to a photon. For a photon to be in definite translational state it need to be associated with one single beam of light, but may be associated with two or more beams of light which are the components into which one original beam has been split. Translational states are thus superposable in a similar way to wave functions.

Let us consider now what happens when we determine the energy in one of the components. The result of such a determination must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to be entirely in one of the beams. This sudden change is due to the disturbance in the translational state of the photon which the observation necessarily makes. It is impossible to predict in which of the two beams the photon will be found. Only the probability of either result can be calculated from the previous distribution of the photon over the two beams.

One could carry out the energy measurement without destroying the component beam by, for example, reflecting the beam from a movable mirror and observing the recoil. Our description of the photon allows us to infer that, *after* such an energy measurement, it would not be possible to bring about any interference effects between the two components. So long as the photon is partly in one beam and partly in the other, interference can occur when the two beam are superposed, but this possibility disappears whe the photon is forced entirely into one of the beams by an observation. The other beam then no longer enters into the description of the photon, so that it counts as being entirely in one beam in the ordinary way for any experiment that may subsequantly be performed on it.

On these lines quantum mechanics is able to effect a reconciliation of the wave and corpuscular properties of light."

This justifies Dirac's famous sentence: "The new theory, which connect the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs."

To summarize, this analysis evidently rests (1) on the idea that individual photons interfere only with themselves; (2) on the assumption that one cannot tell through which branch of the interference device the photon goes (i.e., through which slit it passes in the Young hole experiment), since any such detection in one branch would collapse the probability wave of the other branch thus annihilating the interference pattern—even when built by photons coming one by one in independent wave packets; (3) on the description of photons as either waves *or* particles—never the two simultaneously.

As one knows, the only alternative interpretation for the photon case rests on Einstein's[47] and de Broglie's[48] suggestion that individual photons are waves *and* particles, i.e., that there are real Maxwell waves (practically devoid of energy and momentum) which carry (pilot) localized nondispersive concentrations of energy-momentum which correspond to individual photons. In an interference device, for example, the real wave goes through both branches (both slits in the double slit experiment) the photon going throught one slit only. An individual photon is thus influenced by the wave of both slits in the interference observation region—so that it is distributed according to Maxwell's wave superposition principle. This yields in this case the quantum mechanical prediction—since Maxwell's wave are then equivalent to the probabilistic ψ field of quantum mechanics.

Clearly the only experimental way to distinguish between these two interpretation would be:

(a) to discover a means for detecting through which branch (slit) the photon goes without destroying the subsequent interference region;
(b) to utilize such a means to construct a specific precise experimental set-up in which the two preceding interpretations yield conflicting testable predictions.

Let us first discuss point (a). Curiously, the discovery of a possible mean to follow a photon path without destroying its interference properties rests on a typical consequence of wave mechanics itself, i.e., the possibility of duplicating photons by using a 3-photon resonance mechanism initially suggested by Bassini, Cagnac, *et al.*[49] and developed by Gozzini[50]—since the use of such a photon duplicator on one of the interference branches would tell us (by absorbing one of them) by which path it has gone, while the remaining one could still be used for interference detection. The principle of Gozzini's duplicator is simple. Before it, all known laser amplifiers were difficult to use due to parasitic light, specially when one wants to act with highly directional light. Moreover the "copies" of an indident photon are generally emitted in a sample of excited cells, i.e., are not in phase with the exciting photon, even when inserted in a coherent wave packet. The Pisa duplicator rests on the idea that one can stimulate with three photons the transition from the level $3^2S_{1/2}$ to the level $3^2P_{1/2}$ of the sodium (separated by an energy E_{10}) by irradiating sodium vapor with two lasers of frequency ν_1 and incident photons ν_2 such that $2\nu_1 - \nu_2 = E_{10}/h$, according to the

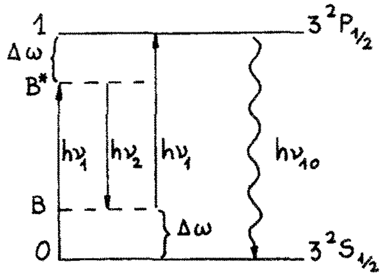

Fig. 8. Scheme of level transitions in Gozzini's duplicator.

scheme of Fig. 8, where the incident absorbed photons are represented by $\uparrow$ and the emitted photons by $\downarrow$. If this process satisfies the relations $\nu_1 = \nu_0 - \Delta\nu$ and $\nu_2 = \nu_0 - 2\Delta\nu$, ν_0 being the resonance frequency, it is then possible to induce a transition through two intermediate virtual states B and B^*: The absorption of two photons of frequency ν_1 combined with an incoming stimulating photon ν_2 induces the production of two photons $h\nu_2$ of equal phases (since theoretically built in the laser-like process $B^* \to B$) and one luminescence photon $h\nu_0$. This duplication process presents the great interest of eliminating any Doppler contribution, since it does not depend on the sodium atom's momentum if the three photons satisfy the geometry of Figs. 9 and 10, where the relation $\hbar\mathbf{k}_1 + \hbar\mathbf{k}_1 - \hbar\mathbf{k}_2 = 0$ implies total momentum conservation—so that all excited atoms enter resonance independently of their velocity. Since one can operate with the set-up of Fig. 11, one can localize the process at the point P, eliminate the fluorescence $h\nu_0$ with a Fabry–Perot device and, by pulsing the incoming $h\nu_2$ packets, individualize the time of copy creation in the duplicator.

Of course, following an argument of Selleri[51] (who has played a pioneer role in this type of proposals[46]), the use of such a duplicator as path detector implies a simple preliminary test to check that the duplication process is really associated with a passage of a photon $h\nu_2$ through the duplicator. It goes as follows: let us consider (see Fig. 12) the arrival of

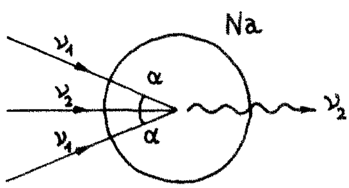

Fig. 9. Geometry of the duplicator's interactions.

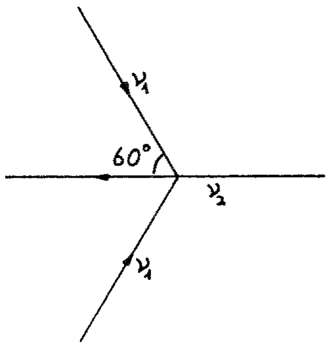

Fig. 10. Memomentum conservation in Gozzini's duplicator.

photons $h\nu_2$ one by one on a semitransparent mirror M of transmission coefficient 1/2. As one knows,[52] two photo multipliers $PM1$ and $PM2$ then necessarily detect anticoincidences, since the photon enters the reflected *or* the transmitted beam. If one then introduces Gozzini's duplicator on one of the beams (say the transmitted), two possibilities arise, i.e.:

(a) Coincidences appear, which would imply that the duplicator D is excited only by an empty wave.

(b) Anticoincidences persist, which show that D is only excited when hit by a photon $h\nu_2$.

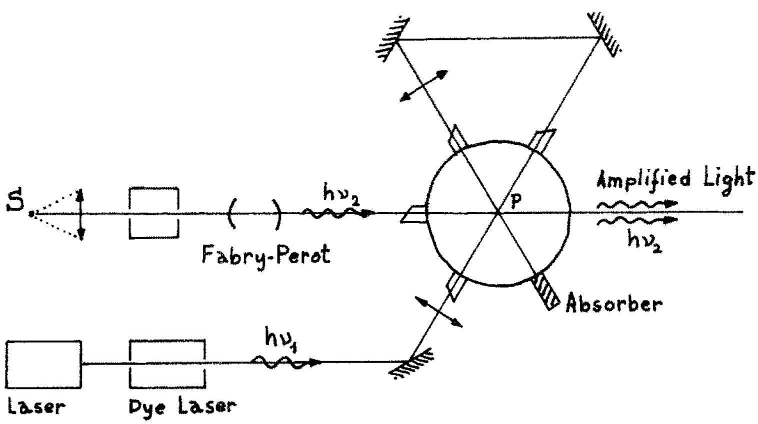

Fig. 11. Schematic representation of Gozzini's photon duplicator D. Two photons $h\nu_1$ emitted in the laser criss-cross at P with a sodium molecule and a photon $h\nu_2$ emitted at S.

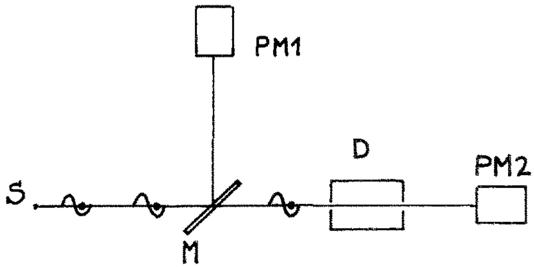

Fig. 12. Representation of Selleri's set-up to test the relation of Gozzini's duplicator D with de Broglie's waves. The semitransparent mirror M (with transmission coefficient 1/2) acting on photons coming one by one lies on the transmitted path. The existence (nonexistence) of anticorrelations between the photomultipliers $PM1$ and $PM2$ ensures the triggering (nontriggering) of D by the passage of a real photon on the transmitted path.

If case (a) is true (an unlikely possibility in the author's opinion), this would imply a direct argument for the real existence of the Einstein–de Broglie waves. If case (b) is verified (in conformity with the usual laser theory) then the appearence of two $h\nu_2$ photons is correlated with the impact of one $h\nu_2$ on the duplicator, and no photon exists in the reflected wave.

With the duplicator in hand we can now discuss possible set-ups which satisfy B. Various proposals have been made to that effect. The latest, by Garuccio, Rapisarda, and Vigier,[46] rests on the assumption (which can also be checked by experiment[46]) that the two outgoing photons $h\nu_2$ have the same frequency and *the same phase* as the incoming photon $h\nu_2$. This assumption, theoretically justified by the similarity of the $B^* \to P$ decay with the usual laser mechanism, is of course not established experimentally and it is quite possible that it would turn out that the two outgoing photons present a random phase fluctuation with respect to the incoming wave packet, so that the use of the duplicator apparently prevents the use of the outgoing $h\nu_2$ photons in interference devices. It is thus important (still in our opinion) that Andrade e Silva, Selleri, and Vigier[53] have been able to construct a proposal which modifies the Garuccio, Rapisarda, and Vigier proposal in such a way that one can compare the antagonistic predictions of the Copenhagen and the causal stochastic interpretations of quantum mechanics, i.e., show, independently of the phases of the duplicator's photons, that in the limit of one photon only the Bohr–Dirac model of purely probabilistic waves yields a different prediction from the Einstein–de Broglie real Maxwell wave model—so that their merit can be assessed by experiment.

We shall rediscuss this later proposal here not only to satisfy B, but also to show how the Einstein–de Broglie model, which rests on Dirac's

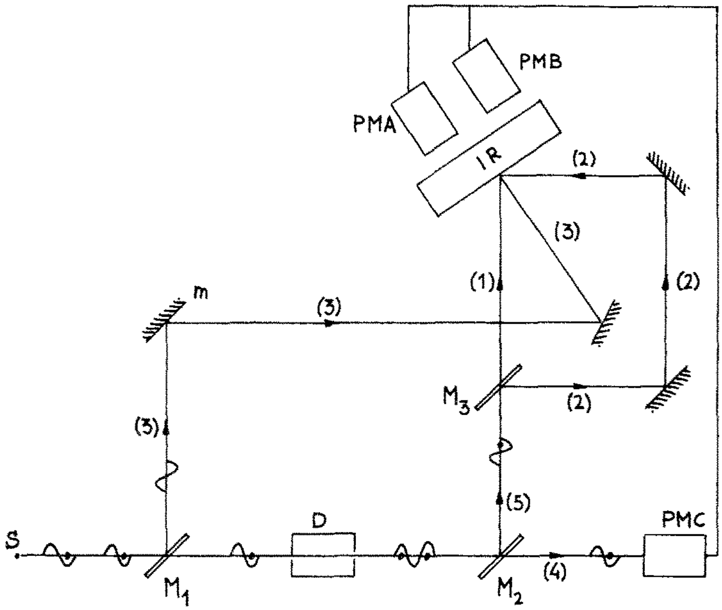

Fig. 13. Representation of the Andrade e Silva, Selleri, and Vigier set-up. M_1, M_2, and M_3 are semitransparent mirrors with transmission coefficient 1/2. *PMA* and *PMB* are photo multipliers connected with half-wave receivers coinciding with the maximum and the minimum of the interference fringes of the interference pattern of paths (1) and (2). They are put in coincidence with *PMC* so that one is sure that two photons have effectively emerged from the duplicator *D*.

aether, yields a new interpretation of Dirac's famous statement that each photon interferes only with itself. As in the Garuccio, Rapisarda, and Vigier proposal,[46] one starts (see Fig. 13) from a set of successive packets (which are issued from an incoherent source) which impinge on a semitransparent mirror M_1. If the experiment has confirmed assumption (b) (i.e., if in this experiment no coincidences have been observed), the appearance of correlated photons in *PMC* and in *PMA* and *PMB* implies that the duplicator *D* has been excited by a transmitted photon from M_1—and that no energetic photon is propagating on the M_1 reflected path (3). Following Dirac, this implies that no probability wave exists along the reflected path (3). No use can further be made of path (3). On the contrary, following Einstein and de Broglie (and also Maxwell's concept of unquantized real existing light waves), a real energy empty pilot wave propagates along (3) which can be reflected by two mirrors into an interference region *IR*, on which also converge the two beams (1) and (2) generated by the further splitting (by a semitransparent mirror M_3) of one of the two photon beams generated by a semitransparent mirror M_2 which splits (in 1/3 of the cases[46] selected by the *PMA*, *PMB*, and *PMC* coincidence) the two photons issuing from D into the respective paths (4) and (5). Following Dirac, this experiment clearly predicts interferences (independent of the phases produced in D) which can be detected (following Pfleegor and Mandel's device[54]) on a pile of half wave detectors connected with the photo multipliers *PMA* and *PMB*. Following Einstein, de Broglie, and Maxwell, the device predicts something else, since we can write for the overall intensity I observed in *IR*

$$I = I_1 + I_2 + I_3 + 2\sqrt{I_1 I_2} \cos \delta_{12} \tag{37}$$

where I_i is the intensity of the ith beam and δ_{ij} is the relative phase shift of the ith and jth beam. In the preceding relation we have suppressed terms in $\cos \delta_{13}$ and $\cos \delta_{23}$, since their phase shifts assume different values in different events. As stressed by Andrade e Silva, Selleri, and Vigier,[53] this implies that the term I_3 is always present (unless one suppresses m) and has an observable effect on the fringe visibility parameter

$$v = 2\sqrt{I_1 I_2}/(I_1 + I_2 + I_3) \tag{38}$$

which can be measured by dividing the coefficient of $\cos \delta_{12}$ by the nonoscillating term in the interference region. Of course, one could further check the existence of an empty pilot wave on (3) by a stroboscoping device on path (3). In other words, we are now

(1) in a position to check the existence of the Einstein–de Broglie–Maxwell wave;

(2) in a situation where the concept of wave packet collapse (really induced by D in our case) yields for the Copenhagen School predictions which contradict the one-photon limit of Maxwell's theory of light.

The answers to the experiment will be interesting to observe. If it confirms Maxwell, then one can only conclude that the presence of real pilot waves which accompany real photons justify Dirac's statement that photons interfere only with themselves. This statement, however, does not preclude the possibility, used by de Broglie and Andrade e Silva,[55] that for coherent beams waves which belong to different photons can interfere, as shown in their interpretation of the Pfleegor and Mandel experiment.

5. CONCLUSIONS

We conclude this paper with some remarks. As a consequence of the results of Aspect's experience (i.e., as a consequence of the violation of Bell's inequality) we are now confronted with the following question: How can we interpret the correlations between spacelike separated events? We think that two different attitudes are possible:

1. Assume that the violation of Bell's inequality proves the existence of correlations between spatially separated events and *hence* the existence of interactions (or signals) exchanged between such events. In this case the problem is to know if the correct interpretation of this new fact lies (a) in a signal exchange made via Feynman's zig-zag (with all the consequences of the possibility to really travel backward in time), as claimed by Wigner[56] and Costa de Beauregard,[43] or (b) in a completely deterministic theory based on the relativistic action at a distance of a quantum potential interpreted in the frame of Dirac's aether, as explained in Section 3.
2. Assume a no-problem attitude in the sense that, from a standpoint based only on the directly observed facts, the violation of Bell's inequality cannot directly prove the existence of a signal exchange between two spatially separated events: the necessity to choose a causal chain on a nonlocal correlation is no reason to assume the existence of nonlocal interactions. This no-problem attitude, which reflects Bohr's attitude toward the EPR paradox,[44] draws its justification from the fact, already remarked, that we cannot use an EPR mechanism to send any superluminal macroscopic signal.

It is an open question which of these two attitudes is the correct one. This makes clear that the Aspect–Rapisarda experiment, despite the importance of finally completely testing the existence of the quantum nonlocal correlations, is not a crucial epistemological experiment in the sense that it does not completely impose a choice between the various standpoints. For this reason, we think that this experiment (comparable in its importance to Michelson's experiment) only opens a new era of theoretical and experimental research. The future choice really depends on the results of the proposed experiments on the direct testing of the existence of the Broglie's waves on Dirac's aether, since only these new forthcoming results can shed new light on the old question of the real nature of the ψ field in wave mechanics.

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REFERENCES

1. P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford, London, 1958).
2. A. Aspect, P. Grangier, and G. Roger, *Phys. Rev. Lett.* **47**, 460 (1981); *Phys. Rev. Lett.* **49**, 91 (1982).
3. Ph. Droz-Vincent, *Ann. Inst. H. Poincaré*, **27**, 407 (1977); *Phys. Rev.* **D19**, 702 (1979); *Ann. Inst. H. Poincaré*, **33**, 377 (1980).
4. H. Yukawa, *Proceding International Conference of Elementary Particles* (Kyoto, 1965).
5. D. Bohm and J. P. Vigier, *Phys. Rev.* **96**, 208 (1954); *Phys. Rev.* **109**, 1882 (1958).
6. C. Fenech, M. Moles, and J. P. Vigier, *Lett. N. Cim.* **24**, 56 (1979).
7. F. Halbwachs, J. M. Souriau, and J. P. Vigier, *J. Phys. Rad.* **22**, 26 (1981).
8. Ph. Guéret, M. Moles, P. Merat, and J. P. Vigier, *Lett. Math. Phys.* **3**, 47 (1979); N. Cufaro Petroni, Z. Maric, Dj. Zivanovic, and J. P. Vigier, *J. Phys. A* **14**, 501 (1981); N. Cufaro Petroni, Z. Maric, Dj. Zivanovic, and J. P. Vigier, *Lett. N. Cim.* **29**, 565 (1980).
9. P. A. M. Dirac, *Nature* **168**, 906 (1951).
10. D. Bohm, *Phys. Rev.* **85**, 166, 180 (1952); *Phys. Rev.* **89**, 458 (1953).
11. L. de Broglie, *La thermodynamique de la particule isolée* (Gauthier-Villars, Paris, 1964).
12. K. P. Sinha, C. Sivaram, and E. C. G. Sudarshan, *Found. Phys.* **6**, 65 (1976); *Found. Phys.* **6**, 717 (1976); *Found. Phys.* **8**, 823 (1978).
13. J. P. Vigier, *Astron. Nachr.* **303**, 55 (1982); N. Cufaro Petroni and J. P. Vigier, *Causal Action at a Distance and a new Possible Deduction of Quantum Mechanics from General Relativity: The Many-Body Problem,*" *Preprint* (Institut H. Poincaré, Paris, 1982).
14. J. P. Vigier, *Lett. N. Cim.* **29**, 467 (1980).
15. Ph. Guéret and J. P. Vigier, *Nonlinear Klein–Gordon Equation Carrying a Non-dispersive Soliton-like Singularity,*" Preprint (Institut H. Poincaré, Paris, 1982); Ph. Guéret and J. P. Vigier, *Soliton Model of Einsteinian Nadelstrahlung in Real Physical Maxwell Waves,*" Preprint (Institut H. Poincaré, Paris, 1982); Ph. Guéret and J. P. Vigier, *De Broglie's Wave-Particle Duality in the Stochastic Interpretation of Quantum Mechanics: A Testable Physical Assumption*, Preprint (Institut H. Poincaré, Paris, 1982).
16. J. S. Bell, *Physics* **1**, 195 (1965); *Rev. Mod. Phys.* **38**, 447 (1966); J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).
17. E. Nelson, *Phys. Rev.* **150**, 1079 (1966); L. de la Peña Auerbach, *J. Math. Phys.* **10**, 1620 (1969); L. de la Peña Auerbach and A. M. Cetto, *Found. Phys.* **5**, 355 (1975).
18. W. Lehr and J. Park, *J. Math. Phys.* **18**, 1235 (1977); J. P. Vigier, *Lett. N. Cim.* **24**, 258, 265 (1979); F. Guerra and P. Ruggiero, *Lett. N. Cim.* **23**, 529 (1979); N. Cufaro Petroni and J. P. Vigier, *Int. J. Th. Phys.* **18**, 807 (1979); Kh. Namsrai, *Found. Phys.* **10**, 353, 731 (1980).
19. D. Bohm, R. Schiller, and J. Tiomno, *Suppl. N. Cim.* **1**, 48, 67 (1955); L. de la Peña Auerbach, *J. Math. Phys.* **12**, 453 (1971); N. Cufaro Petroni and J. P. Vigier, *Phys. Lett.* **A73**, 289 (1979); *Phys. Lett.* **A81**, 12 (1981); "Stochastic Interpretation of Relativistic Quantum Equations," in *Old and New Questions in Physics, Cosmology, Philosophy, and Theoretical Biology: Essays in Honor of Wolfgang Yourgrau*, Alwyn van der Merwe, ed. (Plenum, New York, 1983).
20. T. H. Boyer, *Phys. Rev.* **D11**, 790, 809 (1975); L. de la Peña Auerbach and A. M. Cetto,

Found. Phys. **8**, 191 (1978); *Int. J. Quant. Chem.* **12**, Suppl. 1, 23 (1978); P. Claverie and S. Diner, *Int. J. Quant. Chem.* **12**, suppl. 1, 41 (1978); L. de la Peña Auerbach and A. Jàuregui, *Found. Phys.* **12**, 441 (1982).
21. N. Cufaro Petroni and J. P. Vigier, *Random Motions at the Velocity of Light and Relativistic Quantum Mechanics*," Preprint (Insituto di Fisica, Bari, 1982).
22. R. P. Feynman, *Phys. Rev.* **76**, 749, 769 (1949).
23. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964); E. P. Wigner, *Phys. Rev.* **40**, 749 (1932); J. E. Moyal, *Proc. Camb. Phil. Soc.* **45**. 99 (1949); P. A. M. Dirac, *Proc. Roy. Soc. A* **180**, 1 (1942); W. Pauli, *Rev. Mod. Phys.* **15**, 175 (1943); R. P. Feynman, *Rev. Mod. Phys.* **20**, 367 (1948); *Int. J. Theor. Phys.* **21**, 467 (1982).
24. J. P. Vigier and Ya. P. Terletskij, *Sov. Phys. J.E.T.P.* **13**, 356 (1961).
25. A. Avez, *Interprétation Probabiliste d'équations aux derivées Partielles Hyperboliques Normales*," Preprint (Journées Relativistes, Paris, 1976).
26. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
27. D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, N.J., 1951); D. Bohm and Y. Aharonov, *Phys. Rev.* **108**, 1070 (1957).
28. C. Møller, *The Theory of Relativity* (Oxford, London, 1972).
29. V. Augelli, A. Garuccio, and F. Selleri, *Ann. Fond. L. de Broglie* **1**, 154 (1976).
30. S. J. Freedman and F. Clauser, *Phys. Rev. Lett.* **28**, 938 (1972); R. A. Holt and F. M. Pipkin, Harvard, Preprint (1974), unpublished; G. Faraci, S. Gutkowsky, S. Notarrigo, and A. R. Pennisi, *Lett. N. Cim.* **9**, 667 (1974); L. Kasday, J. Ullman, and C. S. Wu, *N. Cim.* **B25**, 663 (1975); J. F. Clauser, *Phys. Rev. Lett.* **36**, 1223 (1976); E. S. Fry and R. C. Thompson, *Phys. Rev. Lett.* **37**, 465 (1976); A. R. Wilson, J. Lowe, and D. K. Butt, *J. Phys.* **G2**, 613 (1976); M. Bruno, M. D'Agostino, and C. Maroni, *N. Cim.* **B40**, 42 (1977).
31. A. Aspect, *Phys. Lett.* **A67**, 117 (1975); *Progr. Sci. Cult.* **1**, 439 (1976); *Phys. Rev.* **D14**, 1944 (1978); F. Falciglia, G. Iaci, and V. A. Rapisarda, *Lett. N. Cim.* **26**, 327 (1979); L. Pappalardo and V. A. Rapisarda, *Lett. N. Cim.* **29**, 221 (1980); A. Garuccio and V. A. Rapisarda, *N. Cim.* **A65**, 269 (1981).
32. D. Bohm and B. Hiley, *Found. Phys.* **5**, 93 (1975); C. Philippidis and D. Bohm, *The Aharonov–Bohm Effect and the Quantum Potential*, preprint (Birbeck College, London, 1982).
33. N. Cufaro Petroni and J. P. Vigier, *Lett. N. Cim.* **25**, 151 (1979).
34. L. de Broglie, *La Mechanique Ondulatoire du Photon* (Gauthier-Villars, Paris, 1940); L. Bass and E. Schrödinger, *Proc. Roy. Soc. Lond.* A232, 1 (1955); S. Deser, *Ann. Inst. H. Poincaré* **16**, 79 (1972); M. Moles and J. P. Vigier, *C. R. Acad. Sci. Paris* **B276**, 697 (1973).
35. F. Halbwachs, F. Piperno, and J. P. Vigier, *Lett. N. Cim.* **33**, 311 (1982).
36. A. Garuccio and J. P. Vigier, *Lett. N. Cim.* **30**, 57 (1981).
37. L. D. Landau and E. M. Lifshitz, *Teoria Quantistica Relativistica* (Editori Riuniti, Roma, 1978).
38. N. Cufaro Petroni and J. P. Vigier, *Lett. N. Cim.* **26**, 149 (1979); *Phys. Lett.* **A88**, 272 (1982); Kh. Namsrai, *J. Phys.* **A14**, 1307 (1981); *Sov. J. Part. Nucl.* **12**, 449 (1981).
39. N. Cufaro Petroni, Ph. Droz-Vincent, and J. P. Vigier, *Lett. N. Cim.* **31**, 415 (1981).
40. F. Halbwachs, *Théorie Relativiste des Fluides à Spin* (Gauthier-Villars, Paris, 1960).
41. N. Cufaro Petroni and J. P. Vigier, *Causal Action at a Distance Interpretation of the Aspect–Rapisarda Experiment*, Preprint (Institut H. Poincaré, Paris, 1982).
42. A. Garuccio, V. A. Rapisarda, and J. P. Vigier, *Lett. N. Cim.* **32**, 451 (1981).
43. O. Costa de Beauregard, *N. Cim.* **B42**, 41 (1977); *Ann. Fond. L. de Broglie* **2**, 231

(1977); *Phys. Lett.* **A67**, 171 (1978); *N. Cim.* **B51**, 267 (1979); *N. Cim. Lett.* **17**, 551 (1980).
44. N. Bohr, *Phys. Rev.* **48**, 696 (1935).
45. M. Cini, M. De Maria, G. Mattioli, and F. Nicolo, *Found. Phys.* **9**, 479 (1979).
46. F. Selleri, *Lett. N. Cim.* **1**, 908 (1969); F. Selleri and J. P. Vigier, in *Old and New Questions in Physics, Cosmology, Philosophy, and Theoretical Biology: Essays in Honor of Wolfgang Yourgrau, Alwyn van der Merwe*, ed. (Plenum, New York, 1983); A. Garuccio and J. P. Vigier, *Found. Phys.* **10**, 797 (1980); A. Garuccio, K. Popper, and J. P. Vigier, *Phys. Lett.* **A86**, 397 (1981); A. Garuccio, V. A. Rapisarda, and J. P. Vigier, *Phys. Lett.* **A90**, 17 (1982); J. and M. Andrade e Silva, *C. R. Acad. Sci. Paris*, **290**, 501 (1980).
47. A. Einstein, *Ann. der Phys.* **17**, 132 (1905); *Ann. der Phys.* **18**, 639 (1905); *Zeit. Phys.* **18**, 121 (1917).
48. L. de Broglie, *Ann. Phys.* **3**, 22 (1925).
49. B. Cagnac, G. Grynberg, and F. Biraben, *Jour. de Phys.* **34**, 845 (1973); G. Grynberg, F. Biraben, M. Bassini, and B. Cagnac, *Phys. Rev. Lett.* **37**, 283 (1976).
50. A. Gozzini, in *Proceedings of the Symposium on Wave-Particle Dualism* (Reidel, Dordrecht, 1983).
51. F. Selleri, *Ann. Fond. L. de Broglie*, **7**, 45 (1982).
52. L. Mandel and K. Dajenais, *Phys. Rev. Lett.* **18**, 2217 (1978).
53. J. Andrade e Silva, F. Selleri, and J. P. Vigier, *Some Possible Experiments on Quantum Waves*, preprint (Institut H. Poincaré, Paris, 1982).
54. R. L. Pfleegor and L. Mandel, *Phys. Rev.* **159**, 1084 (1967).
55. L. de Broglie and J. Andrade e Silva, *Phys. Rev.* **172**, 1284 (1968).
56. E. P. Wigner, *Symmetries and Reflections* (MIT press, Mass., 1971).

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