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DIRECT CALCULATION OF LENGTH CONTRACTION AND CLOCK RETARDATION

D. V. Redžić

Faculty of Physics, University of Belgrade,
PO Box 44, 11000 Beograd, Serbia

E–mail: *redzic@ff.bg.ac.rs*

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SUMMARY: For simple electromagnetic models of a rod and a clock, a change of shape of a rod and of the rate of clock when they are set in uniform motion is calculated exactly, employing the correct equation of motion of a charged particle in electromagnetic field and the universal boostability assumption. Thus it is demonstrated that, for the simple system considered, the length contraction and clock retardation can be interpreted as dynamical cause-and-effect phenomena, and not as kinematical effects as usually construed in conventional presentations of Special Relativity. It is argued that the perspective relativistic change of an object (corresponding to observations from two inertial frames), while certainly being an acausal effect, has a dynamical content in the sense that it is *tantamount to* **an actual dynamical change of the object in one frame.**

Key words. relativistic processes

1. INTRODUCTION

Recently, I attempted to clarify some basic concepts and results of Einstein's Special Relativity theory (Redžić 2008), noting the paramount importance of what I called 'the universal boostability assumption' for construction of the theory. The fundamental assumption reads: 'It is possible to set a measuring rod or clock in a uniform motion or bring it back to a permanent rest without changing the rest length of the rod or the rest rate of the clock, i.e. it is possible to boost them in such a way that they remain standards of length and time in their rest frame, regardless of the constitution of these objects.' I pointed out that Einstein (1905) used a stronger assumption in his original foundation of Special Relativity, namely that the measuring capacity of a measuring rod or clock remains untouched under *arbitrary* boosts; I argued that the stronger assumption is unwarranted. Particularly, I analysed in detail the well-known relation for relativistic length contraction,

$$l_v = l'_0\sqrt{1 - v^2/c^2}\,, \tag{1}$$

relating lengths l_v and l'_0 of one and the same rod as measured in two inertial frames S and S' in standard configuration (S' is uniformly moving at speed v along the common positive x, x'-axes, and the y- and z-axis of S are parallel to the y'- and z'-axis of S'), respectively; S' is the rest frame of the rod, and S is the lab frame, with respect to which the rod is in uniform motion along its length at speed v. I recalled that Einstein (1905) stated that if the rod to be measured is at rest in S, then, 'in accordance with the principle of relativity', its length as measured in S, l_0, must be equal to l'_0,

$$l_0 = l'_0\,, \tag{2}$$

employing the same measuring rod as in the earlier measurements. Eqs. (1) and (2) imply

$$l_v = l_0\sqrt{1 - v^2/c^2}\,. \tag{3}$$

Thus, according to Einstein, a rod initially at rest in an inertial frame, after a constant velocity $\boldsymbol{v}$ is imparted to it *in an arbitrary way* so that the rod moves freely and uniformly along its length is contracted (its length is reduced), all with respect to that frame, as expressed by Eq. (3). However, I argued, noting the relevance of rest properties–preserving accelerations, that all one may infer on the basis of Special Relativity is that, in general, Eq. (1) always applies, whereas Eq. (2) and thus Eq. (3) do not necessarily apply. While length contraction and clock retardation are generally regarded, starting from Einstein, as purely kinematical results of Special Relativity, obtained directly from the Lorentz transformations, I pointed out that Eq. (3) (which involves rest length–preserving accelerations), encapsulates the actual dynamical change of the rod in the S frame due to the action of some forces on the rod in that frame. Moreover, even Eq. (1), which expresses *the relativistic perspective change* of the rod (involving measurements from two different frames S and S' and, clearly, involving no forces acting on the rod by a mere transition to another inertial frame) has a natural dynamical content.

In a recent paper (Redžić 2014a), I continued my attempts to clarify Special Relativity. To avoid possible terminological and conceptual muddle, I proposed to call the contents of Eqs. (1) and (3), the relativistic length reduction and the relativistic FitzGerald–Lorentz contraction, respectively. I noted what I consider to be fallacies in the existing literature devoted to teaching of relativity, particularly the contention that in the perspective change of an object in Special Relativity (corresponding to observations from two different inertial frames), there is no change in the object, it is only the reference frame that is changed from S to S'. (More precisely, some authors explain the differences in observations between two inertial frames as a purely kinematical effect due to the relativity of simultaneity, 'a consequence of our way of regarding things' (cf, e.g., Born 1965, Franklin 2010), while other authors (cf, e.g., Feĭnberg 1975, Miller 2010) argue that the differences are basically of a *dynamical* origin, due to a dynamical change of standards of length and time when transferring the standards between the frames S and S'.) On the one hand, the relativistic perspective change of an object is certainly an acausal phenomenon (there is no change in the object in the standard physicists' sense of the word, referring to different properties of the object with time in one frame); on the other hand, as is pointed out by Redžić (2008, 2014a), there is a dynamical content of the phenomenon which seems to be somewhat neglected in the literature.

The purpose of the present note is to illustrate deliberations presented by Redžić (2008, 2014a) with simple examples, using elementary models of standards of length and time. Since measuring rods and clocks are physical devices and are subject to the laws of physics in accordance with which they are constructed, one must employ physical laws whose validity is well confirmed in an inertial frame (laboratory). The good candidates are Maxwell's equations and the Lorentz force expression for the force acting on a charge q^* in an electromagnetic field,

$$\boldsymbol{F}_L = q^*\boldsymbol{E} + q^*\boldsymbol{v}\times\boldsymbol{B}\,, \tag{4}$$

where $\boldsymbol{v}$ is the instantaneous velocity of the charge, $\boldsymbol{E}$ is the electric field and $\boldsymbol{B}$ is the magnetic flux density. This has to be combined with the equation of motion of charge q^* in electromagnetic field

$$\frac{d}{dt}\left(\frac{m^*\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right) = q^*\boldsymbol{E} + q^*\boldsymbol{v}\times\boldsymbol{B}\,, \tag{5}$$

where m^* is the charge mass and c is the speed of light *in vacuo*; the last equation fits the experimental facts if the *additional independent* assumption that m^* is time-independent is introduced (cf, e.g. Redžić et al. 2011). For simple models of a rod and clock, operating on the basis of Maxwell's equations and Eq. (5), it will be shown that when the rod and the clock are set in uniform motion with respect to the laboratory frame, they exhibit the FitzGerald–Lorentz contraction and the Larmor clock retardation in the lab, assuming rest properties–preserving accelerations. Thus, for the rod and clock under consideration, a dynamical content of the effects is clearly revealed.

Dynamical analyses of length contraction and clock retardation in the spirit of the present one, based on electromagnetic laws, have been published occasionally (Bell 1976, Jefimenko 1996, 1998, Miller 2010). Unfortunately, models proposed in those papers either cannot be solved analytically (Bell 1976), or introduce for clocks, *tacitly* (Miller 2010) or explicitly (Jefimenko 1996, 1998), a confusing assumption that the velocity of moving clock is much larger than the maximum velocity occurring in its clockwork; moreover, the desired conclusion is reached due to the additional approximation of *small* oscillations. Thus they are not very convincing. Another point is that some authors (Bell 1976, Miller 2010) attempt a constructive dynamical approach to Special Relativity, what seems to be an impossible mission, apart from the fact that Maxwell's theory cannot account for the empirical stability of solid matter. Namely, if one starts from known and conjectured *good* laws of physics in any *one* inertial frame, one can learn that if a constant velocity is imparted to a rod and a clock, the moving rod is contracted and the moving clock runs slower. However, the rod contraction and clock retardation are the necessary but not sufficient conditions for the Lorentz transformations: to construct another inertial frame and to derive the Lorentz transformations, one has to introduce, at one place or another, Einstein's two postulates of Special Relativity aided with the universal boostability assumption. (Rod contraction and clock retardation in the S frame imply that one clock–two way speed of light is c also in the S' frame but this does not suffice to 'spread time over space' in S'.) Moreover, only on the basis of Einstein's principle approach one knows that candidates for *good* physical laws in one inertial frame must be (or can be made to be) Lorentz–covariant. (Note that despite repeated statements in the literature (Møller 1972, Feĭnberg 1975, Rindler 1991), Lorentz–covariance of Maxwell's equations is not fulfilled *automatically*, as pointed out e.g. in Rosser 1964, Redžić 2014b.). Thus, it appears that the laws of physics in any *one* reference frame *cannot* 'account for all physical phenomena, including observations of moving observers,' contrary to Bell's (1976) claim.[1]

On the other hand, the standard 'kinematical' derivation of rod contraction and clock retardation (Einstein 1905) conceals the fundamental fact that rods and clocks in hand must be *relativistically valid*, that is, they have to represent physical apparatuses or devices operating in accordance with the laws of physics which are (or can be made to be) Lorentz–covariant. In the standard approach, unexpected qualities of rods and clocks in motion appear as a dry consequence of Lorentz transformations, which are achieved from logically entangled postulates, and which deal with rods and clocks *in abstracto*, regarded as primitive entities (cf Redžić (2008), p 199). However, paraphrasing Møller (1955), one would like to see—at least in a simple model—that rod contraction and clock retardation indeed follow from the structure of a physical system and the dynamical laws governing it, considered *in one frame only* (cf also Redžić 2006). Also, a dynamical content of the relativistic perspective change in an object, and the universal boostability assumption, seem to be either neglected or misrepresented in the literature. Thus the present note, which aims to complement the standard principle approach to Special Relativity by providing simple illustrations of its dynamical contents, could perhaps be of some interest.

2. DIRECT CALCULATION OF LENGTH CONTRACTION

Firstly, as a relativistically valid standard of length I discuss an elementary model of solid body proposed by Sorensen (1995).

2.1. Rod at rest

Consider four equal charges q of the same sign, at rest in the S frame (laboratory), placed at the vertices of a square $ABCD$ of a side a (A is the bottom left hand vertex, and the vertices B, C and D run counterclockwise). Employing the Coulomb law, one finds that placing a charge of opposite sign, $q_c = -q(1 + 2\sqrt{2})/4$, at the centre of the square, the resultant of the forces acting on each charge is zero. Thus, the system of the five charges is in the electrostatic equilibrium. From Earnshaw's theorem, we know that the equilibrium is unstable (cf, e.g. Tamm 1979). Clearly, some other forces are necessary to ensure the stability of the system, in addition to the electromagnetic ones.

One can verify that for five point charges q, q, q, q and q_c, the only static equilibrium shape is a square and not a rectangle or any other shape, as Sorensen (1995) pointed out. Note that equilibrium conditions fix only the shape of the equilibrium configuration and not its size (the side of the square can be arbitrary). Incidentally, the electrostatic potential energy of the static configuration is always zero, regardless of the value of a.

2.2. Rod in uniform motion

Assume now that the considered system has been accelerated, starting from rest until reaching a steady velocity $\boldsymbol{v}_0 = v_0\hat{\boldsymbol{x}}$, so that all five charges are uniformly moving in the plane of the initial square (the xy plane) parallel to the x axis; take that $\boldsymbol{v}_0$ is perpendicular to the sides AD and BC of the square. Assume also that the acceleration was *gentle*, in the sense that, after all transient effects have died out, the system of five uniformly moving charges is again in a stationary (time-independent) configuration. The question arises, is there such a moving configuration at all.

Now we have to take into account that at the location of each charge, in addition to the electric field, there will also be a magnetic field, since the remaining charges are in motion. The $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of a point charge q moving with constant velocity $\boldsymbol{v}_0$ were first obtained by Oliver Heaviside (1888, 1889) and the $\boldsymbol{B}$ field was rederived by J J Thomson (1889) (cf Jefimenko (1994) and references therein), long before the advent of Special Relativity. The electric field is radial but not spherically symmetrical (contrary to the electrostatic field of q), and is given by

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{q}{4\pi\epsilon_0}\frac{\boldsymbol{r}}{r^3}\frac{1 - v_0^2/c^2}{(1 - v_0^2\sin^2\theta/c^2)^{3/2}}\,, \qquad (6)$$

where $\boldsymbol{r}$ is the position vector of a field point with respect to the instantaneous (at the *same* instant t) position of q, θ is the angle between $\boldsymbol{r}$ and the velocity $\boldsymbol{v}_0$, and $c^2 \equiv 1/\epsilon_0\mu_0$. Recall that throughout the relativity paper, Einstein (1905) used the same symbol (V) for the speed of light *in vacuo* and the speed of electromagnetic waves *in vacuo* ($V \equiv 1/\sqrt{\epsilon_0\mu_0}$), linking thus Special Relativity with Maxwell's theory

[1] Miller (2010) criticizes Bell's anticipation of the equation of motion Eq. (5) as a limitation of Bell's (1976) approach. In his own constructive dynamical attempt to derive Special Relativity, Miller avoided the use of Eq. (5). Instead, he tacitly postulated that Maxwell's equations apply not only in the original rest frame of a physical system, but also in its final rest frame, cf the argument leading to Eq. (6) in Miller (2010). Thus, Miller's approach is at best a combination of a constructive dynamical, and the principle approaches.

(cf Redžić (2008), p 197). The magnetic flux density is

$$\boldsymbol{B}(\boldsymbol{r},t) = \epsilon_0\mu_0 \boldsymbol{v}_0 \times \boldsymbol{E}(\boldsymbol{r},t)\,. \qquad (7)$$

Eq. (7) and the Lorentz force expression Eq. (4) imply that the total electromagnetic force on each of the five charges of our uniformly moving system vanishes if and only if the $\boldsymbol{E}$ field vanishes at the location of each charge. The symmetry suggests that the equilibrium configuration we are looking for has a rectangular shape with q_c at the centre. Therefore, we assume that the stationary configuration is a uniformly moving rectangle $ABCD$, with sides AD and BC perpendicular to $\boldsymbol{v}_0$. Denote lengths of the sides AB (parallel to $\boldsymbol{v}_0$) and AD (transverse to $\boldsymbol{v}_0$) by b and d, respectively. Consider equilibrium conditions at the vertex A. A simple analysis reveals that the condition that the $\boldsymbol{E}$ field at A has no component in the direction perpendicular to the diagonal AC, and along AC, implies that:

$$\frac{d}{b^2} = \frac{b}{d^2(1-v_0^2/c^2)^{3/2}}\,, \qquad (8)$$

and:

$$\frac{1}{b} + \frac{1}{d(1-v_0^2/c^2)^{3/2}} = \frac{2\sqrt{2}}{\sqrt{d^2+b^2}[1-v_0^2 d^2/c^2(d^2+b^2)]^{3/2}}\,, \qquad (9)$$

respectively. Eq. (8) gives

$$b = d\sqrt{1-v_0^2/c^2}\,. \qquad (10)$$

It is easy to check that, with this value of b, Eq. (9) is satisfied identically. As can be seen, Eq. (10) is the necessary and sufficient condition for the moving rectangular configuration $ABCD$ to be the equilibrium one, i.e. the stress free state. Incidentally, Sorensen *assumed* relation (10) from the outset. Thus he did not demonstrate that 'to be in equilibrium [...] the five charges must have this rectangular shape, shortened in the dimension of the direction of motion by the Lorentz contraction as compared to the transverse direction,' contrary to his claim in Sorensen (1995). Instead, he proved only that Eq. (10) is a sufficient condition for the moving rectangle to be in equilibrium.

Note that the character of forces governing the equilibrium is such that equilibrium conditions determine the shape of the configuration and not its size (d is arbitrary), analogously to the electrostatic case. Thus, accelerating square of side a until reaching the steady velocity $\boldsymbol{v}_0$, one can arrive at a moving stationary rectangle with sides $a\sqrt{1-v_0^2/c^2}$ and a in the direction of motion and transverse to it, respectively, but also with sides $d\sqrt{1-v_0^2/c^2}$ and d, where $d \neq a$. Clearly, only in the first case acceleration was rest length–preserving. Namely, according to Special Relativity, observing the moving rectangles in equilibrium from their rest frame S', they will be squares in equilibrium of sides a and $d \neq a$, respectively, since Maxwell's equations can be made to be Lorentz–covariant, and, consequently, equation of motion Eq. (5) with $m^* =$ const can be Lorentz–covariant too (Redžić et al. 2011). Thus Eq. (1) always applies, whereas Eqs. (2) and (3) do not necessarily apply, as is pointed out by Redžić (2008).

As a historical aside, recall that Lorentz argued long ago that if to a system Σ' of particles in the equilibrium configuration at rest relative to the ether 'the velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ is imparted, it will *of itself* change into the system Σ' which is got from Σ' by the deformation $(\sqrt{1-v^2/c^2}, 1, 1)$ (Lorentz et al. (1952), pp 5–7, 21–23, 27–28, cf also Redžić (2014a), pp 60–1). However, Lorentz was wrong here; the change $\Sigma' \to \Sigma$ can also be effectuated by the transformation $(l\sqrt{1-v^2/c^2}, l)$, where $l \neq 1$, as the present Section 2 reveals. (From $vdl/dv = 0$, Lorentz deduced that $dl/dv = 0$, $l =$ const, and concluded: 'The value of the constant must be unity, because we already know that, for $v = 0$, $l = 1$' (Lorentz et al. (1952), p 27). But, all one can deduce from $vdl/dv = 0$ is that $dl/dv = 0$ for $v \neq 0$! Thus, l may have arbitrary (constant) value for $v \neq 0$.).

3. DIRECT CALCULATION OF CLOCK RETARDATION

The same equilibrium system of five charges, providing the standard of length in the preceding section, will be employed as an exact and yet simple model of a relativistically valid clock.

3.1. Clock at rest

Let four identical charges q be now fixed at the vertices of the square $ABCD$ of side a at rest in the lab frame S. Denote the axis perpendicular to the plane of the square which passes through its centre as the z axis; choose the origin at the centre and the x and y axes parallel to the sides AB and AD of the square, respectively. Remove the charge q_c from its central equilibrium position to the point on the positive z axis with $z = \mathcal{A}$ and release it with zero initial velocity to move under the action of the electrostatic field of the remaining four charges.

The exact equation of motion of the charge q_c in the electrostatic field is

$$m\frac{d}{dt}\left(\frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right) = q_c\boldsymbol{E}\,, \qquad (11)$$

where the mass m of q_c is assumed to be time-independent, as is pointed out in Introduction; the time parameter t in the S frame is interpreted in the standard way employing propagation of light signals *in vacuo* as *time keeper*, assuming that Einstein's clock synchronization is a valid procedure (Møller 1972). Eq. (11) and identity:

$$\boldsymbol{v}\cdot\frac{d}{dt}\left(\frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right)\equiv c^2\frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right), \tag{12}$$

imply that:

$$mc^2\frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)=q_c\boldsymbol{E}\cdot\boldsymbol{v}\,. \tag{13}$$

Specifying to our problem, $\boldsymbol{E}$ is the electrostatic field given by:

$$\boldsymbol{E}(0,0,z)=\frac{\kappa 4qz\hat{\boldsymbol{z}}}{(z^2+a^2/2)^{3/2}}, \tag{14}$$

where $\kappa = 1/4\pi\epsilon_0$, and $\boldsymbol{v} = v_z\hat{\boldsymbol{z}}$, since the motion is along the z axis. Using Eqs. (11), (13) and (14) one obtains[2]

$$\frac{dv_z}{dt}=\frac{q_c}{m}\left(1-\frac{v_z^2}{c^2}\right)^{3/2}\frac{\kappa 4qz}{(z^2+a^2/2)^{3/2}}\,. \tag{15}$$

Obviously, the charge q_c does not perform a simple harmonic motion. However, noting that $q_cq < 0$, and also that at $t = 0$, $z = \mathcal{A}$, $v_z = 0$, a simple analysis reveals that the solution of Eq. (15), satisfying these initial conditions, is a periodic function of t,

$$z_R = f(t)\,; \tag{16}$$

subscript R serves as a reminder that the square is at rest. Incidentally, the conclusion applies for *all* values of q_c satisfying $q_cq < 0$ and not only for the one employed in Section 2. Denote the period of that function by T_0; the period comprises continuous changes of position of the charge q_c from $z = \mathcal{A}$ to $z = -\mathcal{A}$ and *vice versa*. Clearly, the square and the charge may be considered as a simple model of a clock, and can be used for measuring time in terms of the number of periods T_0. Namely, as Jefimenko (1996, 1998) pointed out, 'as a physical entity, time is defined in terms of specific measurement procedures, which for the purpose of the present discussion may be described simply as "observing the rate of the clocks."'

3.2. Clock in uniform motion

Assume now that the same clock is set in uniform motion with constant velocity $\boldsymbol{v}_0 = v_0\hat{\boldsymbol{x}}$ along the positive x axis, so as to be *relativistically* valid, i.e. to serve as an *identical* standard of time also for a co-moving inertial observer. From the preceding discussion it follows that now four identical charges q have to be fixed in their rest length–preserving *equilibrium* positions, that is at the vertices of the moving rectangle $ABCD$ with sides $AB = a\sqrt{1-v_0^2/c^2}$ and $AD = a$. Remove the charge q_c co-moving with the rectangle from its central equilibrium position, to the co-moving point on the axis of the rectangle with $z = \mathcal{A}$, and release it with initial velocity $\boldsymbol{v}_0$ to move under the action of the electromagnetic field of the remaining four charges.

The exact equation of motion of the charge q_c in the field, obtained from Eq. (5) assuming that m is constant, reads:

$$m\frac{d}{dt}\left(\frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right)=q_c\boldsymbol{E}+q_c\boldsymbol{v}\times\boldsymbol{B}\,; \tag{17}$$

obviously, Eq. (13) applies in this case too. Using Eq. (6), after a somewhat cumbersome but in every step simple calculation, one finds that the electric field on the co-moving axis (which is perpendicular to the plane of the moving rectangle $ABCD$ and passes through its centre) is:

$$\boldsymbol{E}=\frac{\kappa 4qz\hat{\boldsymbol{z}}}{(z^2+a^2/2)^{3/2}}\frac{1}{\sqrt{1-v_0^2/c^2}}=E_z\hat{\boldsymbol{z}}\,. \tag{18}$$

For the magnetic flux density at the same point on the co-moving axis, using Eq. (7), one finds:

$$\boldsymbol{B}=-\frac{v_0}{c^2}E_z\hat{\boldsymbol{y}}=-\frac{v_0}{c^2}\frac{\kappa 4qz\hat{\boldsymbol{y}}}{(z^2+a^2/2)^{3/2}}\frac{1}{\sqrt{1-v_0^2/c^2}}\,. \tag{19}$$

Using Eqs. (17), (13), (18) and (19), and taking into account initial conditions (at the time $t = 0$, the charge q_c is at the point $z = \mathcal{A}$ on the co-moving axis, and components of its velocity are $v_x = v_0$, $v_y = v_z = 0$), a simple analysis reveals that the charge q_c will move forever along the co-moving axis, i.e. so that $v_x = v_0$, $v_y = 0$. Skipping details, we give the final equation of motion of the charge along the co-moving axis:

$$\frac{dv_z}{dt}=\frac{q_c}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2}\frac{\kappa 4qz}{(z^2+a^2/2)^{3/2}}\frac{1}{\sqrt{1-v_0^2/c^2}}\,. \tag{20}$$

Now, since $v^2 = v_0^2 + v_z^2$ one has:

$$\left(1-\frac{v^2}{c^2}\right)^{3/2}=\left(1-\frac{v_0^2}{c^2}\right)^{3/2}\left[1-\frac{v_z^2}{c^2(1-v_0^2/c^2)}\right]^{3/2}, \tag{21}$$

and introducing:

$$v_z^*=\frac{v_z}{\sqrt{1-v_0^2/c^2}}=\frac{dz}{dt^*}\,, \tag{22}$$

[2]The force $q_c\boldsymbol{E}$ is always parallel to the instantaneous velocity $\boldsymbol{v}$ of the charge q_c so that one can derive Eq. (15) using the concept of 'longitudinal' mass, taking into account that the Lorentz force expression is a *pure* force (cf Rindler 1991, Redžić et al. 2011). I preferred not to employ here the potentially misleading concepts of 'transverse' and 'longitudinal' mass, while they appear occasionally in the literature (Rindler 1991, Jefimenko 1996).

where:

$$t^* \equiv t\sqrt{1 - v_0^2/c^2}\,, \tag{23}$$

Eq. (20) can be recast into:

$$\frac{dv_z^*}{dt^*} = \frac{q_c}{m}\left(1 - \frac{v_z^{*2}}{c^2}\right)^{3/2}\frac{\kappa 4qz}{(z^2 + a^2/2)^{3/2}}\,. \tag{24}$$

Eq. (24) for the clock in motion has exactly the same form as Eq. (15) for the *same* clock at rest, the only difference being that variable t in the latter is replaced by $t^* \equiv t\sqrt{1 - v_0^2/c^2}$ in the former. Since the solution of Eq. (15) is a periodic function (16) with period T_0, it is clear that the solution of Eq. (24) satisfying identical initial conditions ($z = \mathcal{A}$ and $v_z = v_z^* = 0$, at $t = t^* = 0$) is the same function of t^*:

$$z_M = f(t^*) = f(t\sqrt{1 - v_0^2/c^2})\,, \tag{25}$$

where subscript M serves as a reminder that the clock is in uniform motion; period of the clock in motion is obviously:

$$T_M = \frac{T_0}{\sqrt{1 - v_0^2/c^2}}\,. \tag{26}$$

Thus, the above model of a clock, however fragile, provides a simple and yet exact illustration of the Larmor clock retardation.[3] For this clock, the retardation appears to be a dynamical, cause–and–effect phenomenon, as was the case with length contraction discussed in Section 2.[4] A more detailed analysis of clock retardation is given in next Subsection.

Rewrite Eq. (26) in the form:

$$T_v = \frac{T_0}{\sqrt{1 - v^2/c^2}}\,, \tag{27}$$

where now v is the speed of the rectangle in uniform motion, so as to be analogous to Eq. (3). Since the rest rate–preserving acceleration is assumed in the above derivation of Eq. (27), one has that

$$T_0 = T_0'\,, \tag{28}$$

where T_0' is the period of the moving clock as observed by an inertial co-moving observer. Eqs. (27) and (28) imply:

$$T_v = \frac{T_0'}{\sqrt{1 - v^2/c^2}}\,, \tag{29}$$

which is analogous to Eq. (1). As can be seen, *mutatis mutandis*, remarks analogous to those for the uniformly moving rod, presented in the last two paragraphs of Section 2, apply to the case of the uniformly moving clock. Particularly, Eq. (29) always applies, whereas Eqs. (28) and (27) need not apply.

Note that the above dynamical derivation of Eq. (26) applies to the simple clock considered. On the other hand, in the framework of relativistic *kinematics*, Møller (1972) argued: 'In view of the fact that an arbitrary physical system can be used as a clock, we see that any physical system which is moving relative to a system of inertia must have a slower course of development than the same system at rest'. Here one has to take into account that, according to Special Relativity, any physical system must conform to some Lorentz–covariant *dynamical* laws, however complex the system is. Since the exact form of the laws is generally unknown, 'an all-inclusive dynamic (causal) interpretation of time dilation is hardly possible,' as Jefimenko (1996) pointed out. Fortunately, the principle approach to Special Relativity predicts Eq. (26) *indirectly*, via the Lorentz transformations, without the need to enter into details of the phenomenon that serves as a clock. Namely, one need not know the exact laws governing the operation of a clock; it suffices to know that the laws have to be Lorentz–covariant. However, one must admit that any clock retardation hides a complex dynamical process and also involves the universal boostability assumption. Finally, note that the above simple clock model illustrating a dynamical content of Eq. (26) represents an *ideal* clock. Namely, a real clock necessarily involves damping, which is in our case due to the radiation reaction force. Construction of Special Relativity requires of course ideal standard clocks so our simple model may perhaps be to the point.

3.3. Clock retardation in details

Eq. (15) can obviously be recast into:

$$\frac{dv_z}{dz}v_z = -\frac{|q_c q|\kappa 4}{m}\left(1 - \frac{v_z^2}{c^2}\right)^{3/2}\frac{z}{(z^2 + a^2/2)^{3/2}}\,. \tag{30}$$

[3]Note that when the clock (the square + the charge q_c) moves *along* the axis of the square, so that $\boldsymbol{v}_0 = v_0\hat{\boldsymbol{z}}$ ('longitudinal' clock), in the same way as Jefimenko's (1996) clock # 1, employed also in Jefimenko (1998), or Miller's (2010) clock, an *exact one frame* derivation of Eq. (26) appears rather challenging. A 'longitudinal' clock, involving a non-uniform clock retardation, will be discussed elsewhere.

[4]What is the origin of labelling rod contraction and clock retardation as kinematical effects (i.e., that they can be dealt with without involving actions, forces, masses)? This appears to be relics of Newton's absolute space and time concepts, where it is tacitly assumed that 'a moving rigid body at the epoch t may in geometrical respects be perfectly represented by the *same* body *at rest* in a definite position' (Einstein 1905), and analogously for a moving clock. While 'kinematical' is fitting in the context of the Galilean transformation, it masquerades the dynamical contents of the Lorentz transformation.

Separating variables and integrating, setting $v_z = 0$ when $z = \pm\mathcal{A}$, and solving for v_z yields:

$$v_z = \frac{dz}{dt} = \mp c\{...\}^{1/2}\,, \tag{31}$$

where:

$$\{...\} \equiv \{1 - [1 + (|q_c q|\kappa 4/mc^2)(1/\sqrt{z^2 + a^2/2} - 1/\sqrt{\mathcal{A}^2 + a^2/2})]^{-2}\}\,, \tag{32}$$

and $-$ and $+$ sign corresponds to the motion of q_c in the direction of decreasing z and increasing z, respectively. Eq. (31) implies that, for the oscillator at rest, passage of q_c from z to $z+dz$ lasts time interval:

$$dt = \mp(1/c)dz\{...\}^{-1/2}\,. \tag{33}$$

Thus, the period T_0 of the oscillator at rest is given by:

$$T_0 = \frac{2}{c}\int_{-\mathcal{A}}^{\mathcal{A}}\{...\}^{-1/2}\mathrm{d}z\,. \tag{34}$$

Incidentally, in the case of *small* oscillations, i.e. when $\mathcal{A} \ll a$, from Eq. (34) one obtains that:

$$T_0 \approx \frac{2}{c}\int_{-\mathcal{A}}^{\mathcal{A}}\{1 - [1 + (\mathcal{K}/2mc^2)(\mathcal{A}^2 - z^2)]^{-2}\}^{-1/2}\mathrm{d}z\,, \tag{35}$$

where $\mathcal{K} \equiv |q_c q|\kappa 4/(a^2/2)^{3/2}$. As can be seen, this result coincides with the corresponding Møller's result for the period of his '"relativistic" oscillator', Eq. (60) in Møller (1955), as it should. Finally, note that when $\mathcal{K}\mathcal{A}^2 \ll mc^2$, from Eq. (35) one obtains the familiar expression for the period of a simple harmonic oscillator, $T_0 = 2\pi\sqrt{m/\mathcal{K}}$.

Analogously, using Eqs. (22)-(24), for the oscillator in uniform motion one finds that passage of q_c from z to $z + dz$ lasts time interval:

$$dt_M = \frac{1}{\sqrt{1 - v_0^2/c^2}}(\mp)(1/c)dz\{...\}^{-1/2}\,, \tag{36}$$

which is $1/\sqrt{1 - v_0^2/c^2}$ times longer than the corresponding time interval Eq. (33) for the oscillator at rest. The period of the oscillator in motion is obviously given by:

$$T_M = \frac{1}{\sqrt{1 - v_0^2/c^2}}\frac{2}{c}\int_{-\mathcal{A}}^{\mathcal{A}}\{...\}^{-1/2}\mathrm{d}z\,, \tag{37}$$

which is Eq. (26). For our specific 'transverse' clock, Eq. (36) embodies clock retardation: *any* segment of 'life' of a 'transverse' clock that is moving with the velocity v_0 relative to the S frame lasts longer by the factor $1/\sqrt{1 - v_0^2/c^2}$ than the *same* segment of 'life' of the *same* clock at rest in S.

Recall that in the standard kinematical approach, clock retardation is deduced either for a *point clock* or for a non-point 'transverse' clock that involves processes with $x' = \mathrm{const}$ (such as in our clock model), without, however, disclosing its dynamical contents. Namely, from the standard Lorentz transformation:

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - vx/c^2)\,,$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$, it follows that $dt = dt'/\sqrt{1 - v^2/c^2}$ when $x' = \mathrm{const}$, whereas dy' and dz' need not vanish. Note that, for 'longitudinal' clocks ($x' \neq \mathrm{const}$), clock retardation is non-uniform.

For the sake of comparison, we quote here a related passage from French's (1968) excellent book *Special Relativity*, that illustrates the subtlety of the problem we are discussing:

'... the time dilation is an expression of the definition of simultaneity, on the one hand, and of a particular type of measurement, on the other. To describe it by simply saying "Moving clocks run slow" may be convenient, but is also somewhat glib and can be misleading. For one thing, this statement suggests, quite contrary to relativistic ideas, that there is something absolute about motion. And, equally unfortunately, it suggests that some essential change occurs in the operation of the clock itself, that the physical basis of its operation has somehow been modified, whereas it is a central feature of relativity theory that just the opposite is true—that the operation of the clock as described in its own frame of reference is completely unaffected. We must recognize that whenever we speak of an object as moving, that statement has meaning only with respect to some given frame of reference (usually our own). As long as this is borne in mind, it is legitimate to speak of moving clocks or moving meter sticks. But beware!'

It seems, however, that some of the above French's statements can be misleading. Namely, our clock model reveals that character of forces governing the operation of the clock when it is in motion with respect to an inertial frame is different from character of forces governing its operation when the *same* clock is at rest with respect to the same (or any other) inertial frame!

4. WHERE DO THE PERSPECTIVE CHANGES COME FROM?

To avoid confusion, begin with a few terminological comments.

By 'the perspective relativistic change' of an object I mean that, according to Special Relativity, one and the same object (in the sense consisting of the same 'atoms') has distinct properties (say, the length of a rod or the period of a clock), depending on whether the properties are being measured in the rest frame of the object or in the laboratory frame (with respect to which the object is in a uniform translatory motion). Note that here we deal with two different states of motion of an object with respect to two inertial frames ('observers'), and two respective 'configurational states' of the object. By 'the actual physical change' of an object in the standard physicists' sense of the phrase, I mean that properties of the object become distinctly different under the action of certain forces (both external and internal), all with respect to one and the same inertial frame.

Now comes what is perhaps the key question of Special Relativity. When a single physical object is observed by two different inertial observers (or when a single observer changes frames), where do the differences between their observations come from? Particularly, when an object at rest in S' and thus in uniform motion with respect to the lab frame S is observed from the two frames, why the results of observations differ? Note that the perspective change itself has nothing to do with *previous* history of the object either in S' or in S, the history may be unknown to us; moreover, the object need not be free nor connected. Note also that there seems to be an overall consensus that we did nothing to the object by merely observing it from two different frames (or by accelerating an observer to another frame). Thus, there is no cause of the perspective relativistic change, it is an acausal effect.

Miller (2010) argued that the differences among observations of different inertial observers 'are due to the differences in their respective measuring instruments [...] these perspective effects ultimately have a dynamical origin because the properties of measuring instruments are determined by the forces that keep them in equilibrium in their respective frames.' The author explained, following Feĭnberg (1975), that 'when the measuring rods and clocks are moved between inertial observers, they suffer dynamical changes. When the observers use their dynamically altered rods and clocks to make measurements, it is not surprising that their results differ and that they differ by the same factors that are involved in the dynamical changes.'

While Feĭnberg and Miller advocate a force interpretation of the so-called kinematical effects of Special Relativity, a common thread in their discussions is that 'there are no dynamical effects in the physical object being observed'; the differences in measuring instruments used by different inertial observers are all that matters. Now, it is certainly true that nothing at all has happened to the object being observed by a mere transition to another inertial frame. ('The body received no impact, pull or boosts, but is viewed from a system moving relative to it; [...] there has been no actual change in the body itself.') However, I think that Feĭnberg and Miller's interpretation falsifies the spirit of Special Relativity. While the perspective relativistic change is an acausal effect, I will argue that there is a dynamical explanation of the effect in the sense that the perspective change is *tantamount to* an actual physical change. This seems to be the gist of Special Relativity.

Firstly, each inertial observer possesses his or her own set of measuring instruments which are identical to one another. A measuring rod at rest in the lab frame S is in all respects identical to a measuring rod of the same construction at rest in the 'moving' frame S' under identical physical conditions; the rods *embody* the same length in their respective rest frames. That the rods can indeed be *of the same construction* is secured by the universal boostability assumption, as was illustrated in Section 2.[5] Therefore, it is somewhat perplexing to account for the differences between the observations of the S– and S'–observer in terms of the differences in their respective measuring instruments, as Feĭnberg (1975) and Miller (2010) do. A natural explanation appears to be at hand.

As was noted above, in the perspective relativistic change we deal with two different states of motion of an object with respect to two inertial observers, and two respective 'configurational states' of the object. Specifying to the simple system discussed in Sections 2 and 3, taking into account that the theory employed (Maxwell's equations plus the Lorentz force Eq. (5)) is made to be Lorentz–covariant, it follows that different observations of the system considered in the lab frame S and in the rest frame S' are due to different states of motion of the system in the two frames, and thus to its different respective configurations. The differences in configurations are due to a different electromagnetic field produced by the moving field–producing charges, and hence to a different force acting on the moving field–experiencing charges.

On the other hand, if we start from the object at rest in the lab frame S, which is in the same configurational state as it was the object's rest state in the 'moving' frame S', and accelerate it until reaching the steady velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ in a persistent state (thus being at rest with respect to the 'moving' frame) *and* if the acceleration was rest properties–preserving, we reach the same configurational state of the moving object as measured in the lab, as was found earlier as a result of the perspective change. Now we deal with two configurational states of the same object, which are identical to the ones discussed above in the context of the perspective change, corresponding to two different states of motion but now with respect to one inertial frame. In the 'one frame scenario', one has two *stationary* configurational states with distinct properties of the object due to different character of forces providing equilibrium in the two states of motion; this actual physical change has a clear dynamical origin. Since the 'two frames situation' is perfectly equivalent to the corresponding 'one frame situation' under the assumptions stated, one must admit that not only the actual physical change

[5] According to conventional presentations of Special Relativity, measuring instruments need not be transferred between frames; instead, they can be constructed in each frame 'from scratch', following the same recipe. But it appears that the universal boostability assumption, or its equivalent, must be introduced at some step of the procedure. At first sight, the universal boostability assumption has basically the same contents as Born's (1965) 'principle of the physical identity of the units of measure' (cf also Redžić (2008), footnote 12). However, Born seems to imply that measuring capacity of a rod or clock remains untouched under *arbitrary* boosts, which is incorrect, cf Section 2. In the same way, Feĭnberg's (1997) '*universality with respect to the acceleration regime* [of the rod contraction and the clock retardation]' does not generally hold.

but also the perspective relativistic change of an object has a dynamical content. Clearly, in the system discussed in Section 2, the change belongs to *statics* (equilibrium of forces and momenta).

Basically, all that matters is the state of uniform motion of a physical system with respect to *one* (arbitrary but fixed) inertial observer, under the proviso that the (persistent) rest configuration of the system is conserved. It is irrelevant whether two different states of motion of the system are observed from two different inertial frames, respectively, or from one frame only, if in the latter case, the two states of motion are related by a rest properties–preserving acceleration. This supremacy of any one inertial observer appears to be the gist of the principle of special relativity.[6]

One last point is worth making. In the simple system considered, changes of velocity–dependent internal forces result in a definite change of the system structure, that is changes of the internal forces induce a persistent structure change. This obviously cannot be achieved by changing the inertial frames of the observers. While the perspective relativistic change itself has nothing to do with previous history of the object in the inertial frames, clearly, the system structure in any one frame is determined by its previous dynamic history in that frame.

5. SUMMARY

The calculations of rod contraction and clock retardation presented in this paper provide a dynamical cause–and–effect type interpretation of those so-called kinematical effects of Special Relativity. A dynamical content of the effects is clearly revealed at least in the case of the simple electromagnetic model employed, in terms of various character of forces governing the equilibrium of the rod or the operation of the clock in the state of motion and in the state of rest of the system under consideration. By means of the same model, the importance of the universal boostability assumption is illustrated. A dynamical content of the perspective relativistic change is also discussed. It is argued that when a connected physical object in a persistent state is observed by observers in different inertial frames, the differences among their observations can be construed as due to changes in internal forces which determine the structure of the object with a change of its velocity, provided that the velocity change is performed *in a rest properties-preserving way* under a combined action of external and internal forces with respect to one of the inertial frames. The different inertial observers have a dynamical explanation of the differences among their observations in terms of an equivalent dynamical change in the object with respect to one inertial frame.

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[6]Incidentally, Feĭnberg (1975) pointed out that, instead of transferring a physical object from its initial rest frame S to its final rest frame S' through a rest properties–preserving acceleration, the same final state of the object in motion relative to S can be reached in a different way. Namely, instead of accelerating the object, one can accelerate a reference frame–copy of the S' frame, initially at rest relative to S', until the copy frame eventually becomes the S frame (cf Einstein (1905) and also a somewhat obscure attempt by Swann 1960); of course, the object is now assumed to be always at rest in S'. Feĭnberg asked why does the action on the measuring system of rods and clocks cause a contraction of the measured rod. After an explanation that I found obscure, he noted that 'one may naturally still wonder why a symmetric result is obtained when there is such an enormous asymmetry in the transition to the final state of motion with the same relative velocity.' It seems, however, that in his explanation Feĭnberg failed to take into account properly that the perspective change is an acausal phenomenon, and also that measuring capacity of a rod or clock remains untouched under rest properties–preserving boosts.

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ДИРЕКТНО ИЗРАЧУНАВАЊЕ КОНТРАКЦИЈЕ ДУЖИНЕ И УСПОРАВАЊА ЧАСОВНИКА

D. V. Redžić

Faculty of Physics, University of Belgrade, PO Box 44, 11000 Beograd, Serbia

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Оригинални научни рад

За једноставне електромагнетне моделе штапа и часовника, промена облика штапа и промена хода часовника када се они доведу у стање равномерног кретања израчунате су егзактно, употребљавајући коректну једначину кретања наелектрисане честице у електромагнетном пољу и претпоставку универзалне бустабилности. На тај начин је доказано да, за једноставни разматрани систем, контракција дужине и успоравање часовника могу бити интерпретирани као динамички феномени узрочно–последичног типа, а не као кинематички ефекти, како се они уобичајено тумаче у конвенционалним излагањима Специјалне релативности. Аргументисано је да перспективна релативистичка промена неког објекта (која одговара мерењима из два инерцијална система), мада несумњиво представља акаузални ефект, има динамички садржај у смислу да је *еквивалентна* једној стварној динамичкој промени тог објекта у једном инерцијалном систему.