Direct calculation of permeability and permittivity for a left-handed metamaterial

D. R. Smith,[a)] D. C. Vier, N. Kroll, and S. Schultz
Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 June 2000; accepted for publication 14 August 2000)

Recently, an electromagnetic metamaterial was fabricated and demonstrated to exhibit a ''left-handed'' (LH) propagation band at microwave frequencies. A LH metamaterial is one characterized by material constants—the permeability and permittivity—which are simultaneously negative, a situation never observed in naturally occurring materials or composites. While the presence of the propagation band was shown to be an inherent demonstration of left handedness, actual numerical values for the material constants were not obtained. In the present work, using appropriate averages to define the macroscopic fields, we extract quantitative values for the effective permeability and permittivity from finite-difference simulations using three different approaches. © *2000 American Institute of Physics.* [S0003-6951(00)03440-9]

In a paper published in 1968, Veselago introduced the concept of a left-handed (LH) medium—a medium in which both the permittivity ϵ and the permeability μ are simultaneously negative.[1] Veselago predicted that LH media would have unique and potentially useful properties, as many basic electromagnetic phenomena, such as the Doppler shift, Snell's law, and Cerenkov radiation would be ''reversed.'' The limitation in demonstrating a LH media, however, was the lack of a material example possessing a magnetic response to electromagnetic radiation sufficient to result in a negative permeability. We recall that the magnetic susceptibility can be negative (e.g., diamagnetism), but is typically very small, and therefore, insufficient to bring $\mu=1+\chi$ negative.

In 1999, Pendry *et al.* proposed an artificial system, made only of nonmagnetic conducting materials that could exhibit a large response to the magnetic field of electromagnetic radiation.[2] By building a resonance into the structure, very large positive and negative effective magnetic susceptibilities could be obtained. While the idea of creating magnetic materials from conductors is not new,[3] the structures that Pendry *et al.* suggested were particularly suited for this purpose, having a very high Q factor and nearly exclusive magnetic-field coupling. The generic frequency dependence of the effective permeability that Pendry *et al.* calculated was

$$\mu_{\text{eff}}(\omega)=1-\frac{F\omega_0^2}{\omega^2-\omega_0^2-i\omega\Gamma}, \tag{1}$$

where F, Γ, and ω_0 are constants relating to the geometry and materials composing the artificial medium. Note that, due to the resonance, a region of negative permeability occurs over a finite frequency band.

The effective magnetic medium used in the recent demonstration of an artificial LH medium[4] was a periodic array of split ring resonators (SRRs), which we also study here. All of the simulation data presented here result from finite-difference calculations of SRRs with a square cross section, utilizing a commercial electromagnetic-mode solver.[5] The standard procedure is to discretize a unit cell of the structure (Fig. 1, inset), and apply periodic boundary conditions, allowing a phase advance in one of the directions. In the case where we are looking for an effective magnetic response from the rings, we focus on electromagnetic modes polarized such that the magnetic field is parallel to the ring axes. Figure 1 shows the resulting dispersion curve, which displays a frequency band gap. The effective refractive index $n(\omega)$ is related to both the permeability and permittivity according to $n(\omega)=\sqrt{\epsilon_{\text{eff}}(\omega)\mu_{\text{eff}}(\omega)}$; thus, the band gap of Fig. 1, characterized by imaginary values of $n(\omega)$, only indicates that one or the other material parameter has fallen below zero.

A means of demonstrating the negative permeability was put forward by Smith *et al.*,[4] who combined the SRR medium with a second medium of interacting wires (Fig. 2, inset). The wire medium alone can be characterized by an effective permittivity having the same form as a dilute, collisionless plasma, or

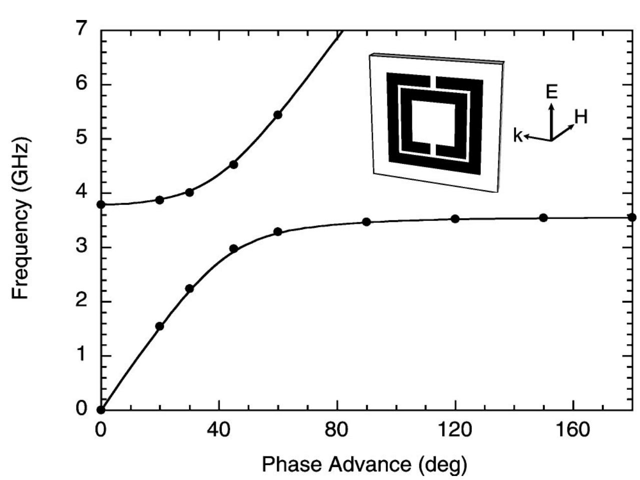

FIG. 1. Dispersion curve for an array of split ring resonators (SRRs). The strips composing the SRRs are 0.8 mm wide and 0.05 mm thick. The outer ring has an outer width of 6.6 mm, with a gap of 0.2 mm between outer and inner rings. The width of the splits is 0.3 mm. The rings are in a unit cell with dimensions $8\times8\times10$ mm (10 mm in the direction of the wire). The horizontal axis is the phase advance per unit cell, or kd, where k is the wavenumber (2π divided by the free-space wavelength).

[a)]Author to whom correspondence should be addressed; electronic mail: drs@sdss.ucsd.edu

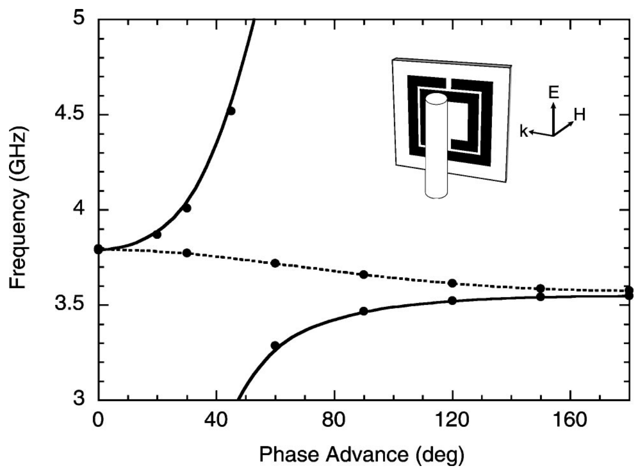

FIG. 2. Dispersion curve (dashed line) for the combined SRR and wire mediums, using the same dimensions as in Fig. 1. The solid lines correspond to the dispersion curves obtained without the wires (as in Fig. 1), shown here to help identify the LH region.

$$\epsilon_{\text{eff}}(\omega)=1-\frac{\omega_p^2}{\omega^2}, \tag{2}$$

where ω_p is an effective plasma frequency relating to the details of the wire radius and spacing. The essential feature of Eq. (2) is that below ω_p, the effective permittivity is negative. When the wire and SRR mediums are combined, the overlap of the negative permittivity of the wire medium with the frequency gap region of the SRR medium results in a propagation band (Fig. 2, dashed curve) where there was previously attenuation for the SRR medium alone. The presence of a propagation band in the combined structure thus implies that the SRR gap is due to a negative permeability, and furthermore, indicates that for this frequency region the structure is left handed.

While this is a valid demonstration of an effective LH medium, it is necessary to obtain quantitative values for the material constants, both to rule out other possible mechanisms for the propagation band, and to facilitate the design of practical structures. In designing improved microwave absorbing materials, for example, achieving the condition $\epsilon_{\text{eff}}=-\epsilon_0$, $\mu_{\text{eff}}=-\mu_0$ would be very desirable, as electromagnetic waves *at any angle of incidence* would be entirely transmitted into such a material, with nearly zero-reflected power.[1]

Given that we have an effective medium, with only vacuum and responding currents ($\epsilon=\epsilon_0$, $\mu=\mu_0$) it is clear that we must apply appropriate averaging methods to compute *effective* material parameters. In fact, we may view the process as a discretization of Maxwell's equations, in which we replace the details of each unit cell by averaged fields and material constants. We follow Pendry *et al.*[2] in introducing the following averaging scheme for the fields, and will discuss this further in a separate publication:

$$\langle E_i\rangle=d^{-1}\int \mathbf{E}\cdot d\mathbf{x}_i, \quad \langle D_i\rangle=d^{-2}\epsilon_0\int \mathbf{E}\cdot d\mathbf{s}_i,$$
$$\langle H_i\rangle=\mu_0^{-1}d^{-1}\int \mathbf{B}\cdot d\mathbf{x}_i, \quad \langle B_i\rangle=d^{-2}\int \mathbf{B}\cdot d\mathbf{s}_i. \tag{3}$$

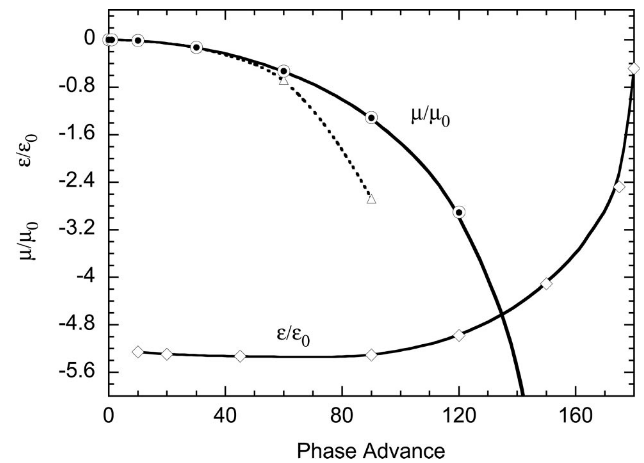

FIG. 3. Calculated permeability (open and closed circles, triangles) and permittivity (diamonds) for the left-handed propagation band of the composite SRR-wire structure. Open circles: $\langle B\rangle/\langle H\rangle$ method; closed circles: *z,n* method; triangles: currents method. The permittivity was calculated by the *z,n* method only.

d is the length of an edge of a cell, and i is one of the x, y, or z components. The averages for $\langle E\rangle$ and $\langle H\rangle$ involve line integrals along the edges of the unit cell, while the averages of $\langle B\rangle$ and $\langle D\rangle$ involve surface integrals over the faces of the cell. By following this scheme, the details of the actual structure are removed into the field averages, from which we can define the effective material parameters as

$$\mu_{\text{eff}}^{i,j}\equiv\frac{\langle B_i\rangle}{\langle H_j\rangle}, \quad \epsilon_{\text{eff}}^{i,j}\equiv\frac{\langle D_i\rangle}{\langle E_j\rangle}. \tag{4}$$

Having defined these field averages, we can now outline three computational methods to calculate the material parameters. The most straightforward of these is to use the field averages of Eq. (3) to compute the material parameters directly by the prescription shown in Eq. (4). When there is appreciable phase advance across a unit cell, however, we note that the averages are ''incorrect'' by a factor relating to the phase variation. To illustrate the problem, suppose we have a plane wave propagating through vacuum ($\epsilon=\epsilon_0$, $\mu=\mu_0$), and we discretize the space into cubes with length d. From Eqs. (3) and (4), we have, for example,

$$\langle B\rangle=\mu_0 H_0(e^{ikd}-1)/ikd=\mu_0 H_0 f(k), \tag{5}$$

where H_0 is the applied magnetic field. Clearly, the ratio $\langle B\rangle/\langle H\rangle$, where $\langle H\rangle$ is taken at the zero phase advance side of the cell, does not equal μ_0 when kd is finite. Nevertheless, we can still maintain a sensible definition of the quantities averaged over a surface, by dividing out the factor $f(k)$ appearing in Eq. (5) from all flux averaged quantities.

Alternatively, we can utilize the calculated surface current density in the SRR (and wire) to determine the material response. In this second method, we make use of the definitions relating the magnetization and polarization to the magnetic flux and electric field, respectively, or $\mathbf{M}=\chi_M\mathbf{H}$ and $\mathbf{P}=\chi_E\mathbf{E}$. The average magnetization and polarization can be found by integration of the current densities as follows:

$$\mathbf{M}=\frac{1}{2V}\int \mathbf{r}\times\mathbf{j}dV, \quad \mathbf{P}=\frac{1}{i\omega V}\int \mathbf{j}dV, \tag{6}$$

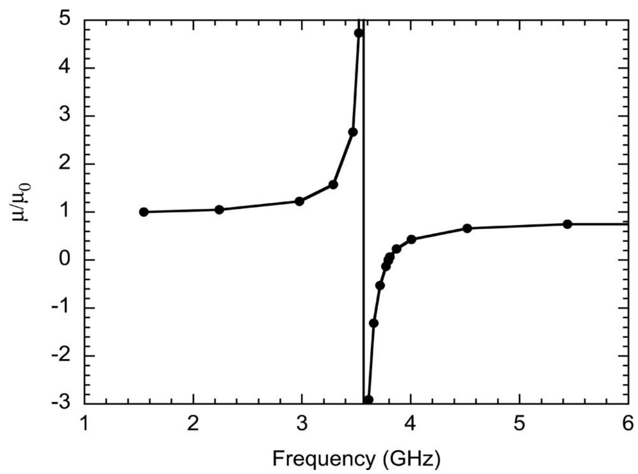

FIG. 4. Calculated permeability as a function of frequency for the SRR medium.

where it is assumed that the currents are confined within each cell.

A third method for determining ϵ_{eff} and μ_{eff} involves the calculation of the wave impedance z from the averaged field quantities. This latter method is robust, and is similar to that introduced by Contopanagos *et al.* for calculations on photonic band-gap materials, where the free-space wavelength is typically on the order of, or even smaller than, the unit-cell dimensions.[6] An average Poynting vector can be defined from the averaged fields as

$$\mathbf{S}=\frac{1}{2}\text{Re}(\langle\mathbf{E}\rangle\times\langle\mathbf{H}\rangle*)=\frac{1}{2}\text{Re}\left(\frac{\langle E\rangle^2}{z}\hat{\mathbf{k}}\right), \tag{7}$$

where $z=\sqrt{\mu_{\text{eff}}/\epsilon_{\text{eff}}}$. By combining the determination of z with the dispersion relation, which provides $n=\sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}}$ (Figs. 1 and 2), we can extract values for ϵ_{eff} and μ_{eff}.

We applied all three methods to compute the material parameters for the SRR medium alone, and for the composite structure consisting of the SRR medium and the wire medium. Figure 3 shows data for the LH composite structure.[4] The calculations confirm that over the propagation band shown as the dashed curve in Fig. 2, both the permittivity and permeability are less than zero. Both methods requiring only field averages are in agreement, while the method using currents shows a rapid divergence from the other curves, increasing with increasing phase advance. Nevertheless, the same qualitative behavior is indicated by all three methods.

We conclude by presenting in Fig. 4 the composite permeability for the SRR medium, which displays the resonant form predicted by Eq. (1). The curve was pieced together from the permeability calculation of the SRR medium alone, and the permeability calculation of the SRR with the wires. The smoothness of the transition at $\mu_{\text{eff}}=0$, where the data from the SRRs alone join the data from the SRRs and wires, is indicative that any interactions between the wires and the SRRs are negligible.

Having now demonstrated the capability to characterize the material constants of artificial media, we are in a position to efficiently design and optimize LH structures for specific applications.

This work was supported by DARPA through a grant from ONR (Contract No. N00014-00-1-0632), and the DOE (Contract Nos. DE-FG-03-093ER40793 and DE-FG-03-93ER40759).

[1] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[2] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[3] I. S. Schelkunoff and H. T. Friis, in *Antennas: Theory and Practice*, edited by S. Sokolnikoff (Wiley, New York, 1952), p. 584.
[4] D. R. Smith, W. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[5] T. Weiland, AIP Conf. Proc. **297**, 291 (1994).
[6] H. F. Contopanagos, C. A. Kyriazidou, W. M. Merrill, and N. G. Alexopoulos, J. Opt. Soc. Am. A **16**, 1682 (1999).