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ABSTRACT

The ρ Regge amplitude for πN charge exchange scattering with helicity flip is partial-wave analyzed in the direct channel. This produces resonance circles on the Argand diagram corresponding to the prominent experimental N^* resonances.

We consider the ρ Regge amplitude for πN charge exchange scattering with helicity flip, B_{CEX}. The usual parametrization and fit to the high-energy data, $p_L = 5.8 - 18.2$ BeV/c, is:[1]

$$B_{CEX} = \beta \left(\frac{E}{E_0}\right)^{\alpha-1} \frac{1 - e^{-i\pi\alpha}}{\sin \pi\alpha \; \Gamma(\alpha)} , \tag{1}$$

where $\alpha(t) = 0.57 + 1.08\, t$, and E is the laboratory energy of the π. Choosing $E_0 = 0.7$ BeV and $\beta(t) = \text{const.} = 60.4$ mb one correctly fits the relative height of the near forward peak and the secondary peak. We now extrapolate this expression down to $p_L = 1.7$ BeV/c and check that it roughly fits $d\sigma/dt$.[2] Note in particular that the dip at $t \approx -0.6$ BeV^2 persists down to this energy and that the magnitudes of the near forward peak and of the secondary peak are correctly given by the extrapolation of the high-energy fit.

We partial-wave analyze $B(E, z)$ in the direct channel,

$$B_\ell(E) \equiv \frac{1}{2} \int_{-1}^{+1} dz\, P_\ell(z)\, B(E, z). \tag{2}$$

We integrate out to 180 deg, but we ignore the small backward peak coming from N-exchange. Furthermore if we want to use $\beta(t)$ in (1) only for those t-values which have been measured,[3] $|t| \leqslant 3.0\ \mathrm{BeV}^2$, then we should stay below $p_L \approx 2.0$ BeV/c. On the other hand we should stay above $p_L \approx 1.7$ BeV/c in order to have (1) give a rough fit to the $d\sigma/dt$ data. Actually we let p_L vary beyond these limits and consider $1.0 \leqslant p_L \lesssim 2.4$ BeV/c.

Because $\beta(t)$ is constant it is irrelevant for most of the following considerations, and we put $\beta = 1$. The complex amplitude B_ℓ, calculated from (1) and (2), is shown in Fig. 1, in (a), (b), and (c) for fixed ℓ versus p_L.

Surprisingly the B_ℓ describe circles on the Argand diagram. Such circles are usually associated with resonances. We read off the momentum p_L at which B_ℓ reaches the top of the circle, and in Fig. 2 we plot these "resonance" momenta versus ℓ and compare them to the experimental resonances (1920, 2190, 2420). In Fig. 1d we show B_ℓ for fixed p_L as a function of ℓ. Such a circle corresponds to an N* Regge pole and indicates that the N* resonances plotted in Fig. 2 actually belong to an N* Regge trajectory.

Let us understand qualitatively why we obtain circles. (1) The zeros of Im A at $\alpha = 0, -1, -2$, etc. and the double zeros of Re A at $\alpha = 0, -2, -4$, etc. are crucial. Consider for example Fig. 1d. For $p_L = 1.4$ we have $\alpha = -1.5$ at 180 deg. Therefore Im A contains two zeros in the physical region, $-1 \leqslant z_s \leqslant +1$, and the $\ell = 2$ wave becomes particularly strong. (2) The signature factor $\left(1 - e^{-i\pi\alpha}\right)$ induces the correct counterclockwise motion on the Argand diagram because the phase of $B(E, t)$ moves counterclockwise as $(-t)$ increases, and the partial-wave amplitude $B_\ell(E)$ includes larger $(-t)$ values at higher energies. (3) The arguments given in (1) and (2) are mainly relevant at intermediate energies, $p_L \leqslant 3.0$ BeV/c, since at high energies the secondary peak is so much suppressed that the presence of the nodes becomes less relevant.

Are the circles in Fig. 1 really resonances? The problem is that (1) and (2) are regular at $s = m_{Res}{}^2 - i(m\Gamma)_{Res}$, they do not contain poles corresponding to resonances. Let us give a mathematical analogue. Take $\psi(z)$, the logarithmic derivative of the Γ-function. It has poles (resonances) at $z = 0, -1, -2, \cdots$. The asymptotic representation (Regge representation) is the Stirling expansion. ψ_{asy} is a good approximation to ψ for $|z| \to \infty$ as long as we exclude a wedge $|\arg z| < \pi - \epsilon$. But ψ_{asy} does not contain the poles at $z = 0, -1, -2, \cdots$, the approximation breaks down if we penetrate the wedge. Similarly the Regge asymptotic form, B_{asy}, is a good approximation to the full amplitude, B, for real s. Both amplitudes contain the resonance circles. But if we go below the physical axis the approximation breaks down, and B_{asy} does not contain the resonance poles. In a sense we are in a situation similar to the phase-shift analyst's: We must stick to real energies, and if we see a circle we never know with absolute certainty whether the extrapolation to $\mathrm{Im}\, s < 0$ would lead to a pole or not.

We now make the comparison of our circles with the established N^* resonances more quantitative, and we clarify the meaning of the parameter ℓ. The connection between B_ℓ and the partial waves $f_{\ell\pm}$ is[4]

$$\frac{1}{4\pi} B_\ell = \sum_{n=0}^{\infty} \left[\frac{1}{E+M} \left(f_{(\ell+2n)+} - f_{(\ell+2+2n)-} \right) + \frac{1}{E-M} \left(f_{(\ell+1+2n)-} - f_{(\ell+1+2n)+} \right) \right] . \tag{3}$$

For our comparison we approximate the right-hand side of this equation by the resonances,[5] we leave out the low-energy tails of high-energy resonances, corresponding to $n \neq 0$, and we neglect the term with $(E + M)^{-1}$. This reduces (3) to

$$\frac{1}{4\pi} B_\ell \approx \frac{1}{E-M} \left[f_{(\ell+1)-} - f_{(\ell+1)+} \right] . \tag{4}$$

Note that the parameter ℓ in B_ℓ mainly corresponds to an orbital angular momentum of $(\ell + 1)$. Using the established resonances we obtain from (4)

$$\frac{1}{4\pi} B_3^{CEX} \approx \frac{\sqrt{2}}{3} \frac{1}{E-M} f_{2190} \tag{5}$$

$$\frac{1}{4\pi} B_4^{CEX} \approx \frac{\sqrt{2}}{3} \frac{1}{E-M} f_{2420} \tag{6}$$

$B_2^{CEX} \approx \cdots (f_{1920} + f_{1688})$. From Fig. 1 we read off the widths $M\Gamma = (0.7,\ 0.8\ \mathrm{GeV}^2)$ for $\ell = (3, 4)$. The corresponding experimental N^* widths are $M\Gamma = (0.44,\ 0.67)$. The elasticities are determined by the height of the circles in Fig. 1, and using $\beta = 60.4$ mb and (5) (6) we obtain $M\Gamma x = (0.12, 0.09 \text{ GeV}^2)$. This agrees well with Rosenfeld's values $M\Gamma x = (0.13, 0.07)$.

Let us compare our procedure with the N/D scheme.[6] Both times one uses as an input the particles in the crossed channel ("forces"), in the N/D scheme elementary ρ exchange, in our case Regge ρ exchange. One computes the contribution of this exchange to a definite partial wave in the direct channel, $a_\ell(s)$. In our scheme this already produces the direct-channel resonances. In the N/D scheme this only gives a real Born amplitude, and the additional step of unitarization generates the resonance. In contrast the Regge ρ exchange amplitude is automatically unitary, since it roughly represents the full amplitude. For the same reason it automatically includes absorption.

We have shown in Ref. 7 how finite-energy sum rules (FESR) imply that direct-channel resonances are, on the average over the low and intermediate energy region, already contained in the Regge amplitude of the crossed channel. The present letter shows that the equivalence between t-channel Regge poles and s-channel resonances does not only hold on the average, but even locally at each intermediate energy. (At low energies the equivalence must break down, the resonances no longer overlap). The equivalence $B_{\rho\text{Regge}} \approx B_{N^*\text{Res.}}$ suggested by Fig. 1 shows that the interference model,[8] which puts $B \approx B_{\text{Regge}} + B_{\text{Res.}}$ in the intermediate energy region, involves severe double counting.

We do not know yet whether this equivalence is a very general feature of strong interactions. But we might mention one additional example, $K^{\pm}p$. Assume for the moment that the Pomeranchon is a special case with $\alpha_P(t) \equiv 1$, and that for purposes of describing $K^{\pm}p$ we can lump together all odd signature meson trajectories (ρ, ω, Φ') into one trajectory X, and all even signature trajectories (A_2, f, f') into Y. We further assume that X and Y are exchange degenerate.[9] If we now partial wave analyze the Regge amplitudes we find that the Pomeranchon does not generate resonance circles because it is purely imaginary. The meson signature factor in K^+p factor is real, $(+1 - e^{-i\pi\alpha})_X - (-1 - e^{-i\pi\alpha})_Y = 2$, and we cannot obtain resonances. On the other hand for K^-p the phase of the Regge amplitude rotates, $(+1 - e^{-i\pi\alpha})_X + (-1 - e^{-i\pi\alpha})_Y = -2e^{-i\pi\alpha}$, and resonances can be generated.

FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. (a) - (c) The complex amplitude $B_\ell(p_L)$ for fixed ℓ versus the laboratory momentum p_L. The numbers along the circles give the values of p_L.

(d) $B_\ell(p_L)$ for fixed p_L versus ℓ. The numbers along the circle give the values of ℓ. For normalization we have put $\beta = 1$.

Fig. 2. Chew-Frautschi plot: (A) The established πN resonances, (B) The N^* resonances from our partial-wave analysis of the ρ Regge exchange. E_L = lab. energy of $\pi \approx p_L$. E_L is linear in $(M_{Res})^2$.

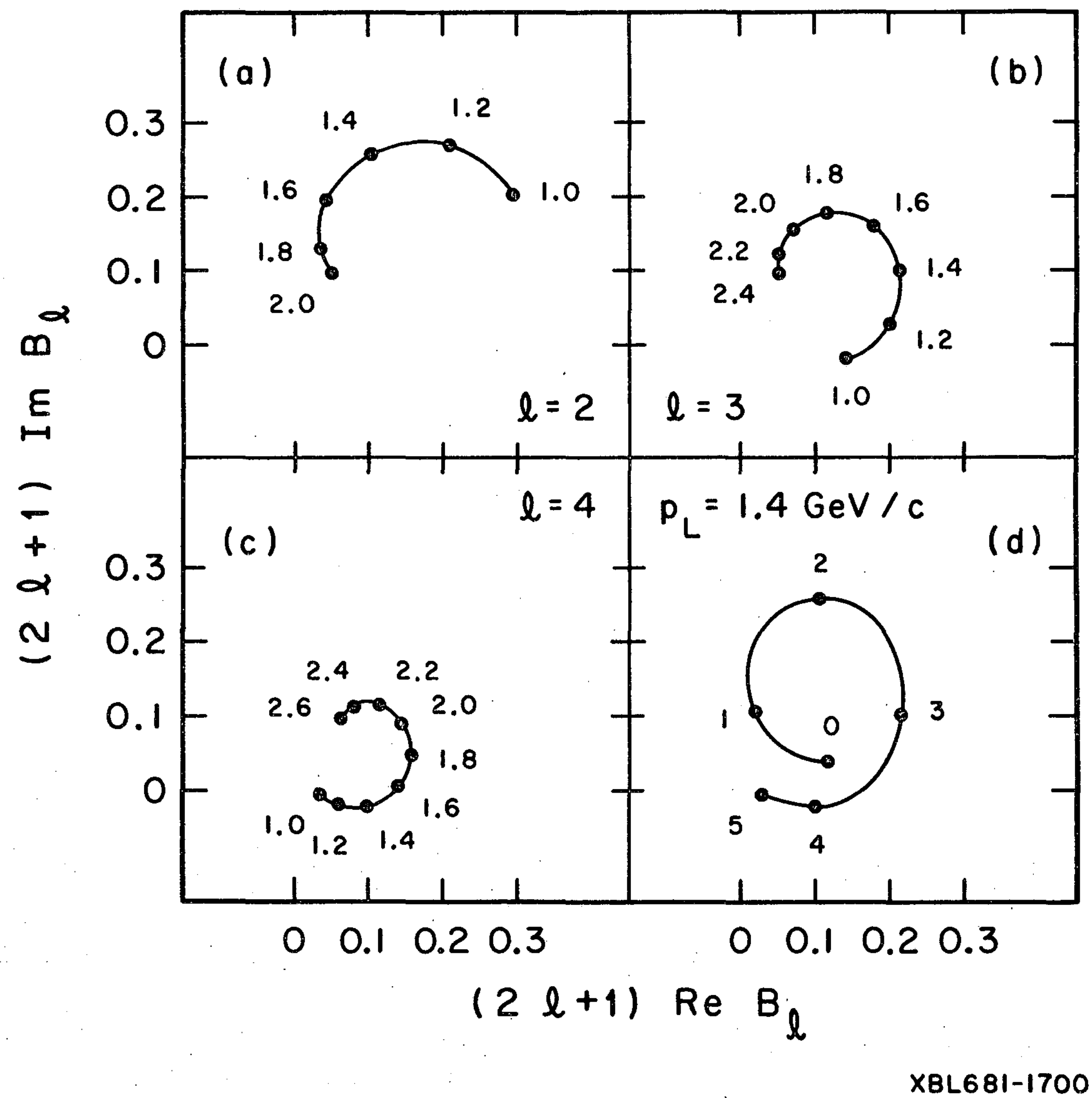
(a)
1.4
1.2
1.6
1.0
1.8
2.0
ℓ = 2
(b)
1.8
2.0
1.6
2.2
1.4
2.4
1.2
1.0
ℓ = 3
ℓ = 4
(c)
2.4
2.2
2.6
2.0
1.8
1.6
1.0
1.2
1.4
p_L = 1.4 GeV / c
(d)
2
1
0
3
5
4
0.3
0.2
0.1
0
(2 ℓ+1) Im B_ℓ
0 0.1 0.2 0.3
(2 ℓ+1) Re B_ℓ
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Fig. 1

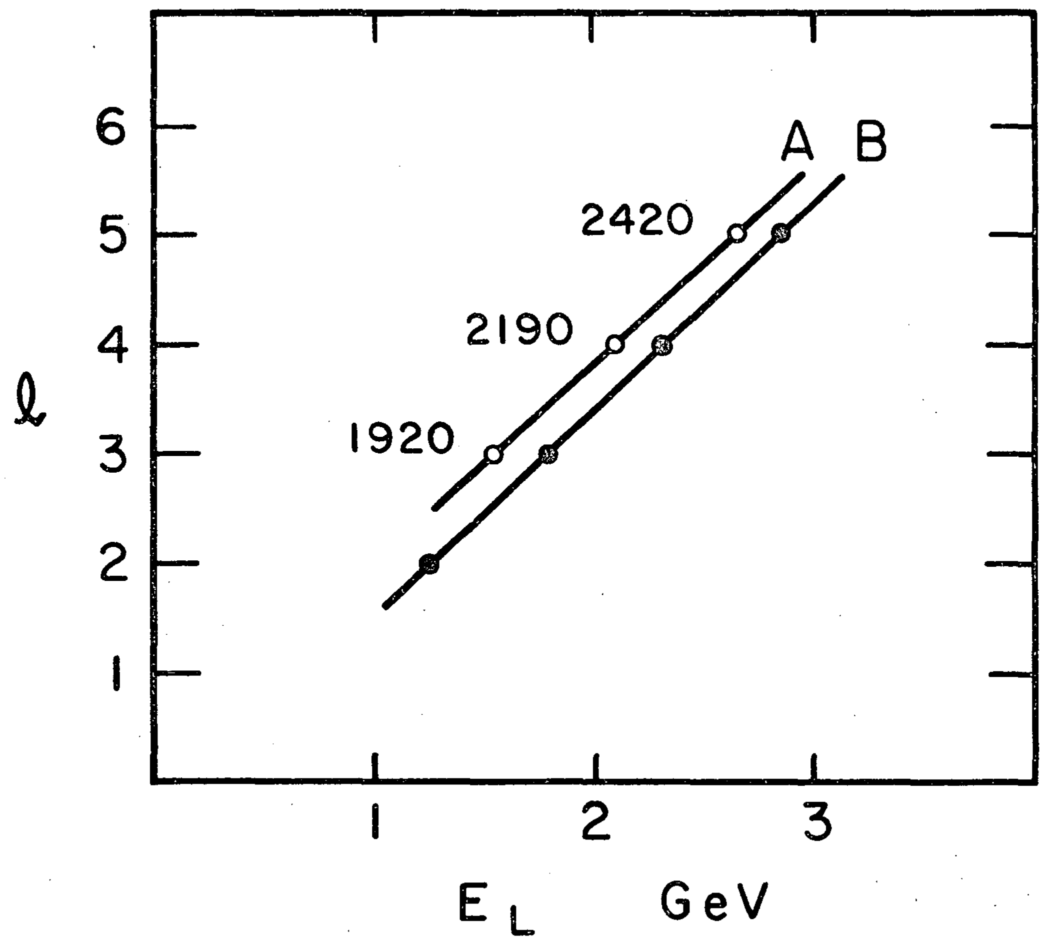

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Fig. 2