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Direct CP violation in Λb→n(Λ)π+π- decays via ρ-ω mixing Physical Review D, 1998; 58(9):096013

http://link.aps.org/doi/10.1103/PhysRevD.58.096013

http://hdl.handle.net/2440/10906

Direct CP violation in $\Lambda_b \to n(\Lambda)\pi^+\pi^-$ decays via ρ-ω mixing

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We study direct CP violation in the bottom baryon decays $\Lambda_b \to f\rho^0(\omega) \to f\pi^+\pi^-$ ($f=n$ or Λ). It is found that in these decays via ρ-ω mixing the CP violation could be very large when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. With a typical value $N_c=2$ in the factorization approach, the maximum CP-violating asymmetries are more than 50% and 68% for $\Lambda_b \to n\pi^+\pi^-$ and $\Lambda_b \to \Lambda\pi^+\pi^-$, respectively. With the aid of heavy quark symmetry and phenomenological models for the hadronic wave functions of Λ_b, Λ and the neutron, we estimate the branching ratios of $\Lambda_b \to n(\Lambda)\rho^0$. [S0556-2821(98)06419-4]

PACS number(s): 11.30.Er, 12.15.Hh, 12.39.Hg, 13.20.He

I. INTRODUCTION

CP violation is still an open problem in the standard model, even though it has been known in the neutral kaon system for more than three decades [1]. The study of CP violation in other systems is important in order to understand whether the standard model provides a correct description of this phenomenon through the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Recent studies of direct CP violation in the B meson system [2] have suggested that large CP-violating asymmetries should be observed in forthcoming experiments. It is also interesting to study CP violation in the bottom baryon system in order to find the physical channels which may have large CP asymmetries, even though the branching ratios for such processes are usually smaller than those for the corresponding processes of bottom mesons. The study of CP violation in the bottom system will be helpful for understanding the origin of CP violation and may provide useful information about the possible baryon asymmetry in our universe. Actually, some data on the bottom baryon Λ_b have appeared just recently. For instance, OPAL has measured its lifetime and the production branching ratio for the inclusive semileptonic decay $\Lambda_b \to \Lambda l^- \bar{\nu} X$ [3]. Furthermore, measurements of the nonleptonic decay $\Lambda_b \to \Lambda J/\psi$ have also been reported [4]. More and more data are certainly expected in the future. It is the purpose of the present paper to study the CP violation problem in the hadronic decays $\Lambda_b \to n\pi^+\pi^-$ and $\Lambda_b \to \Lambda\pi^+\pi^-$.

The CP-violating asymmetries in the decays we are considering arise from the nonzero phase in the CKM matrix, and hence we have the so-called direct CP violation which

occurs through the interference of two amplitudes with different weak and strong phases. The weak phase difference is determined by the CKM matrix elements while the strong phase is usually difficult to control. In Refs. [5,6], the authors studied direct CP violation in B hadronic decays through the interference of tree and penguin diagrams, where ρ-ω mixing was used to obtain a large strong phase (as required for large CP violation). The data for $e^+e^- \to \pi^+\pi^-$ in the ρ-ω interference region strongly constrains the ρ-ω mixing parameters. Gardner *et al.* established not only that the CP-violating asymmetry in $B^\pm \to \rho^\pm\rho^0(\omega) \to \rho^\pm\pi^+\pi^-$ is more than 20% when the invariant mass of the $\pi^+\pi^-$ pair is near the ω mass, but that the measurement of the CP-violating asymmetry for these decays can remove the $\mathrm{mod}(\pi)$ uncertainty in $\arg[-V_{td}V_{tb}^*/(V_{ud}V_{ub}^*)]$ [6]. In the present work we generalize these discussions to the bottom baryon case. It will be shown that the CP violation in Λ_b hadronic decays could be very large.

In our discussions hadronic matrix elements for both tree and penguin diagrams are involved. These matrix elements are controlled by the effects of nonperturbative QCD which are difficult to handle. In order to extract the strong phases in our discussions we will use the factorization approximation so that one of the currents in the nonleptonic decay Hamiltonian is factorized out and generates a meson. Thus the decay amplitude of the two body nonleptonic decay becomes the product of two matrix elements, one related to the decay constant of the factorized meson and the other to the weak transition matrix element between two hadrons.

There have been many discussions concerning the plausibility of the factorization approach. Since bottom hadrons are very heavy, their hadronic decays are energetic. Hence the quark pair generated by one current in the weak Hamiltonian moves very fast away from the weak interaction point. Therefore, by the time this quark pair hadronizes into a meson it is far away from other quarks and is therefore unlikely to interact with the remaining quarks. Hence this quark pair

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is factorized out and generates a meson. This argument is based on the idea of ''color transparency'' proposed by Bjorken [7]. Dugan and Grinstein proposed a formal proof for the factorization approach by constructing a large energy, effective theory [8]. They established that when the energy of the generated meson is very large the meson can be factorized out and the deviation from the factorization amplitude is suppressed by the energy of the factorized meson.

Furthermore, we will estimate the branching ratios for the decay modes $\Lambda_b \to n(\Lambda)\rho^0$. In the factorization approach the decay rates for these processes are determined by the weak matrix elements between Λ_b and $n(\Lambda)$. With the aid of heavy quark effective theory (HQET) [9] it is shown that in the heavy quark limit, $m_b \to \infty$, there are two independent form factors. We will apply the model of Refs. [10,11] to determine these two form factors and hence predict the branching ratios for $\Lambda_b \to n(\Lambda)\rho^0$.

The remainder of this paper is organized as follows. In Sec. II we give the formalism for the CP-violating asymmetry in $\Lambda_b \to f\rho^0(\omega) \to f\pi^+\pi^-$ ($f=n$ or Λ) and calculate the strong phases in the factorization approach. Numerical results will also be shown in this section. In Sec. III we apply the result of HQET and the model of Refs. [10,11] to estimate the branching ratios for $\Lambda_b \to n(\Lambda)\rho^0$. The results from the nonrelativistic quark model [12] will also be presented for comparison. Finally, Sec. VI is reserved for a brief summary and discussion.

II. CP VIOLATION IN $\Lambda_b \to n(\Lambda)\pi^+\pi^-$ DECAYS

A. Formalism for CP violation in $\Lambda_b \to n(\Lambda)\pi^+\pi^-$

The formalism for CP violation in B meson hadronic decays [5,6] can be generalized to Λ_b in a straightforward manner. The amplitude, A, for the decay $\Lambda_b \to f\pi^+\pi^-$ is

$$A = \langle \pi^+\pi^- f|\mathcal{H}^T|\Lambda_b\rangle + \langle \pi^+\pi^- f|\mathcal{H}^P|\Lambda_b\rangle, \qquad (1)$$

where $\mathcal{H}^T$ and $\mathcal{H}^P$ are the Hamiltonians for the tree and penguin diagrams, respectively. Following Refs. [5,6] we define the relative magnitude and phases between these two diagrams as follows:

$$A = \langle \pi^+\pi^- f|\mathcal{H}^T|\Lambda_b\rangle[1 + re^{i\delta}e^{i\phi}],$$

$$\bar{A} = \langle \pi^+\pi^- \bar{f}|\mathcal{H}^T|\bar{\Lambda}_b\rangle[1 + re^{i\delta}e^{-i\phi}], \qquad (2)$$

where δ and ϕ are strong and weak phases, respectively. ϕ is caused by the phase in the CKM matrix, and if the top quark dominates penguin diagram contributions it is $\arg[V_{tb}V_{td}^*/(V_{ub}V_{ud}^*)]$ for $b \to d$ and $\arg[V_{tb}V_{ts}^*/(V_{ub}V_{us}^*)]$ for $b \to s$. The parameter r is the absolute value of the ratio of tree and penguin amplitudes:

$$r \equiv \left| \frac{\langle \pi^+\pi^- f|\mathcal{H}^P|\Lambda_b\rangle}{\langle \pi^+\pi^- f|\mathcal{H}^T|\Lambda_b\rangle} \right|. \qquad (3)$$

The CP-violating asymmetry, a, can be written as

$$a \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r\sin\delta\sin\phi}{1 + 2r\cos\delta\cos\phi + r^2}. \qquad (4)$$

It can be seen explicitly from Eq. (4) that both weak and strong phases are needed to produce CP violation. ρ-ω mixing has the dual advantages that the strong phase difference is large (passing through 90° at the ω resonance) and well known. In this scenario one has [6]

$$\langle \pi^+\pi^- f|\mathcal{H}^T|\Lambda_b\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega + \frac{g_\rho}{s_\rho}t_\rho, \qquad (5)$$

$$\langle \pi^+\pi^- f|\mathcal{H}^P|\Lambda_b\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho, \qquad (6)$$

where t_V ($V=\rho$ or ω) is the tree and p_V is the penguin amplitude for producing a vector meson, V, by $\Lambda_b \to fV$, g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective ρ-ω mixing amplitude, and s_V^{-1} is the propagator of V,

$$s_V = s - m_V^2 + im_V\Gamma_V, \qquad (7)$$

with $\sqrt{s}$ being the invariant mass of the $\pi^+\pi^-$ pair.

$\tilde{\Pi}_{\rho\omega}$ is extracted [13,14] from the data for $e^+e^- \to \pi^+\pi^-$ [15] when $\sqrt{s}$ is near the ω mass. Detailed discussions can be found in Refs. [6,13,14]. The numerical values are

$$\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \ \mathrm{MeV}^2,$$

$$\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \ \mathrm{MeV}^2.$$

We stress that the direct coupling $\omega \to \pi^+\pi^-$ is effectively absorbed into $\tilde{\Pi}_{\rho\omega}$, where it contributes some s-dependence. The limits on this s-dependence, $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}_{\rho\omega}'(m_\omega^2)$, were determined in the fit of Gardner and O'Connell, $\tilde{\Pi}_{\rho\omega}'(m_\omega^2) = 0.03 \pm 0.04$ [13]. In practice, the effect of the derivative term is negligible.

From Eqs. (1), (2), (5), (6) one has

$$re^{i\delta}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega}t_\omega + s_\omega t_\rho}. \qquad (8)$$

Defining

$$\frac{p_\omega}{t_\rho} \equiv r'e^{i(\delta_q+\phi)}, \quad \frac{t_\omega}{t_\rho} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_\rho}{p_\omega} \equiv \beta e^{i\delta_\beta}, \qquad (9)$$

where δ_α, δ_β and δ_q are strong phases, one has the following expression from Eq. (8)

$$re^{i\delta} = r'e^{i\delta_q}\frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta}s_\omega}{s_\omega + \tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha}}. \qquad (10)$$

It will be shown that in the factorization approach, for both $\Lambda_b \to n\pi^+\pi^-$ and $\Lambda_b \to \Lambda\pi^+\pi^-$, we have (see Sec. II C for details)

$$\alpha e^{i\delta_\alpha} = 1. \tag{11}$$

Letting

$$\beta e^{i\delta_\beta} = b + ci, \quad r' e^{i\delta_q} = d + ei, \tag{12}$$

and using Eq. (10), we obtain the following result when $\sqrt{s} \sim m_\omega$:

$$re^{i\delta} = \frac{C + Di}{(s - m_\omega^2 + \mathrm{Re}\,\tilde{\Pi}_{\rho\omega})^2 + (\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + m_\omega\Gamma_\omega)^2}, \tag{13}$$

where

$$\begin{aligned}
C = {}& (s - m_\omega^2 + \mathrm{Re}\,\tilde{\Pi}_{\rho\omega})\{d[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega} + b(s - m_\omega^2) - cm_\omega\Gamma_\omega] \\
& - e[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + bm_\omega\Gamma_\omega + c(s - m_\omega^2)]\} \\
& + (\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + m_\omega\Gamma_\omega)\{e[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega} + b(s - m_\omega^2) - cm_\omega\Gamma_\omega] \\
& + d[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + bm_\omega\Gamma_\omega + c(s - m_\omega^2)]\},
\end{aligned}$$

$$\begin{aligned}
D = {}& (s - m_\omega^2 + \mathrm{Re}\,\tilde{\Pi}_{\rho\omega})\{e[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega} + b(s - m_\omega^2) - cm_\omega\Gamma_\omega] \\
& + d[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + bm_\omega\Gamma_\omega + c(s - m_\omega^2)]\} \\
& - (\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + m_\omega\Gamma_\omega)\{d[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega} + b(s - m_\omega^2) - cm_\omega\Gamma_\omega] \\
& - e[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega} + bm_\omega\Gamma_\omega + c(s - m_\omega^2)]\}. \tag{14}
\end{aligned}$$

$\beta e^{i\delta_\beta}$ and $r' e^{i\delta_q}$ will be calculated later. Then from Eqs. (13) and (14) we obtain $r\sin\delta$, $r\cos\delta$ and r. In order to get the CP-violating asymmetry a in Eq.(4) $\sin\phi$ and $\cos\phi$ are needed. ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [16], and in the approximation that the top quark dominates the penguin diagrams, we have

$$(\sin\phi)^n = \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}},$$

$$(\cos\phi)^n = \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}, \tag{15}$$

for $\Lambda_b \to n\pi^+\pi^-$, and

$$(\sin\phi)^\Lambda = -\frac{\eta}{\sqrt{[\rho(1+\lambda^2\rho) + \lambda^2\eta^2]^2 + \eta^2}},$$

$$(\cos\phi)^\Lambda = -\frac{\rho(1+\lambda^2\rho) + \lambda^2\eta^2}{\sqrt{[\rho(1+\lambda^2\rho) + \lambda^2\eta^2]^2 + \eta^2}}, \tag{16}$$

for $\Lambda_b \to \Lambda\pi^+\pi^-$. Note that here, and in what follows, all the quantities with the superscript n (or Λ) represent those for $\Lambda_b \to n\rho^0(\omega)$ [or $\Lambda_b \to \Lambda\rho^0(\omega)$].

B. The effective Hamiltonian

With the operator product expansion, the effective Hamiltonian relevant to the processes $\Lambda_b \to f\rho^0(\omega)$ is

$$\begin{aligned}
H_{\Delta B=1} = \frac{G_F}{\sqrt{2}}\bigg[& V_{ub}V_{uq}^*(c_1 O_1^u + c_2 O_2^u) \\
& - V_{tb}V_{tq}^*\sum_{i=3}^{10} c_i O_i\bigg] + \mathrm{H.c.}, \tag{17}
\end{aligned}$$

where the Wilson coefficients, $c_i (i = 1,\dots,10)$, are calculable in perturbation theory and are scale dependent. They are defined at the scale $\mu \approx m_b$ in our case. The quark q could be d or s for our purpose. The operators O_i have the following expression

$$O_1^u = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha,$$

$$O_2^u = \bar{q}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)b,$$

$$O_3 = \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',$$

$$O_4 = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,$$

$$O_5 = \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',$$

$$O_6 = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,$$

$$O_7 = \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',$$

$$O_8 = \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,$$

$$O_9 = \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',$$

$$O_{10} = \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \tag{18}$$

where α and β are color indices, and $q' = u$, d, s quarks. In Eq. (18) O_1^u and O_2^u are the tree level operators. $O_3 - O_6$ are QCD penguin operators, which are isosinglet. $O_7 - O_{10}$ arise from electroweak penguin diagrams, and they have both isospin 0 and 1 components.

The Wilson coefficients, c_i, are known to the next-to-leading logarithmic order [17,18]. At the scale $\mu = m_b = 5$ GeV their values are

$$c_1 = -0.3125, \quad c_2 = 1.1502, \quad c_3 = 0.0174,$$

$$c_4 = -0.0373, \quad c_5 = 0.0104, \quad c_6 = -0.0459,$$

$$c_7 = -1.050 \times 10^{-5}, \quad c_8 = 3.839 \times 10^{-4},$$

$$c_9 = -0.0101, \quad c_{10} = 1.959 \times 10^{-3}. \tag{19}$$

To be consistent, the matrix elements of the operators O_i should also be renormalized to the one-loop order. This results in the effective Wilson coefficients, c_i', which satisfy the constraint

$$c_i(\mu)\langle O_i(\mu)\rangle = c_i'\langle O_i^{\text{tree}}\rangle, \tag{20}$$

where $\langle O_i(\mu)\rangle$ are the matrix elements, renormalized to one-loop order. The relations between c_i' and c_i read

$$c_1' = c_1, \quad c_2' = c_2, \quad c_3' = c_3 - P_s/3,$$

$$c_4' = c_4 + P_s, \quad c_5' = c_5 - P_s/3,$$

$$c_6' = c_6 + P_s, \quad c_7' = c_7 + P_e,$$

$$c_8' = c_8, \quad c_9' = c_9 + P_e, \quad c_{10}' = c_{10}, \tag{21}$$

where

$$P_s = (\alpha_s/8\pi)c_2[10/9 + G(m_c,\mu,q^2)],$$

$$P_e = (\alpha_{em}/9\pi)(3c_1 + c_2)[10/9 + G(m_c,\mu,q^2)],$$

with

$$G(m_c,\mu,q^2) = 4\int_0^1 dx\, x(1-x)\ln\frac{m_c^2 - x(1-x)q^2}{\mu^2},$$

where q^2 is the momentum transfer of the gluon or photon in the penguin diagrams. $G(m_c,\mu,q^2)$ has the following explicit expression [19]:

$$\text{Re}\, G = \frac{2}{3}\left(\ln\frac{m_c^2}{\mu^2} - \frac{5}{3} - 4\frac{m_c^2}{q^2} + \left(1 + 2\frac{m_c^2}{q^2}\right)\right.$$
$$\left.\times \sqrt{1 - 4\frac{m_c^2}{q^2}}\ln\frac{1 + \sqrt{1 - 4(m_c^2/q^2)}}{1 - \sqrt{1 - 4(m_c^2/q^2)}}\right),$$

$$\text{Im}\, G = -\frac{2}{3}\pi\left(1 + 2\frac{m_c^2}{q^2}\right)\sqrt{1 - 4\frac{m_c^2}{q^2}}. \tag{22}$$

Based on simple arguments for q^2 at the quark level, the value of q^2 is chosen in the range $0.3 < q^2/m_b^2 < 0.5$ [5,6]. From Eqs. (19), (21) and (22) we can obtain numerical values of c_i'. When $q^2/m_b^2 = 0.3$,

$$c_1' = -0.3125, \quad c_2' = 1.1502$$

$$c_3' = 2.433 \times 10^{-2} + 1.543 \times 10^{-3}i,$$

$$c_4' = -5.808 \times 10^{-2} - 4.628 \times 10^{-3}i,$$

$$c_5' = 1.733 \times 10^{-2} + 1.543 \times 10^{-3}i,$$

$$c_6' = -6.668 \times 10^{-2} - 4.628 \times 10^{-3}i,$$

$$c_7' = -1.435 \times 10^{-4} - 2.963 \times 10^{-5}i,$$

$$c_8' = 3.839 \times 10^{-4},$$

$$c_9' = -1.023 \times 10^{-2} - 2.963 \times 10^{-5}i,$$

$$c_{10}' = 1.959 \times 10^{-3}, \tag{23}$$

and when $q^2/m_b^2 = 0.5$,

$$c_1' = -0.3125, \quad c_2' = 1.1502$$

$$c_3' = 2.120 \times 10^{-2} + 5.174 \times 10^{-3}i,$$

$$c_4' = -4.869 \times 10^{-2} - 1.552 \times 10^{-2}i,$$

$$c_5' = 1.420 \times 10^{-2} + 5.174 \times 10^{-3}i,$$

$$c_6' = -5.729 \times 10^{-2} - 1.552 \times 10^{-2}i,$$

$$c_7' = -8.340 \times 10^{-5} - 9.938 \times 10^{-5}i,$$

$$c_8' = 3.839 \times 10^{-4},$$

$$c_9' = -1.017 \times 10^{-2} - 9.938 \times 10^{-5}i,$$

$$c_{10}' = 1.959 \times 10^{-3}, \tag{24}$$

where we have taken $\alpha_s(m_Z) = 0.112$, $\alpha_{em}(m_b) = 1/132.2$, $m_b = 5$ GeV and $m_c = 1.35$ GeV.

C. CP violation in $\Lambda_b \to n(\Lambda)\pi^+\pi^-$

In the following we will calculate the CP-violating asymmetries in $\Lambda_b \to n(\Lambda)\pi^+\pi^-$. With the Hamiltonian in Eq. (17) we are ready to evaluate the matrix elements. In the factorization approximation $\rho^0(\omega)$ is generated by one current which has the proper quantum numbers in the Hamiltonian.

First we consider $\Lambda_b \to n\rho^0(\omega)$. After factorization, the contribution to t_ρ^n from the tree level operator O_1^u is

$$\langle \rho^0 n | O_1^u | \Lambda_b \rangle = \langle \rho^0 | (\bar{u}u) | 0 \rangle \langle n | (\bar{d}b) | \Lambda_b \rangle \equiv T, \qquad (25)$$

where $(\bar{u}u)$ and $(\bar{d}b)$ denote the V-A currents. Using the Fierz transformation the contribution of O_2^u is $(1/N_c)T$. Hence we have

$$t_\rho^n = \left(c_1 + \frac{1}{N_c} c_2 \right) T. \qquad (26)$$

It should be noted that in Eq. (26) we have neglected the color-octet contribution, which is nonfactorizable and difficult to calculate. Therefore, N_c should be treated as an effective parameter and may deviate from the naive value 3. In the same way we find that $t_\omega^n = t_\rho^n$, hence, from Eq. (9),

$$(\alpha e^{i\delta_\alpha})^n = 1. \qquad (27)$$

In a similar way, we can evaluate the penguin operator contributions p_ρ^n and p_ω^n with the aid of the Fierz identities. From Eq. (9) we have

$$(\beta e^{i\delta_\beta})^n = \frac{-2\left(c_4' + \frac{1}{N_c}c_3'\right) + 3\left(c_7' + \frac{1}{N_c}c_8'\right) + \left(3 + \frac{1}{N_c}\right)c_9' + \left(1 + \frac{3}{N_c}\right)c_{10}'}{2\left(2 + \frac{1}{N_c}\right)c_3' + 2\left(1 + \frac{2}{N_c}\right)c_4' + 4\left(c_5' + \frac{1}{N_c}c_6'\right) + c_7' + \frac{1}{N_c}c_8' + (c_9' - c_{10}')\left(1 - \frac{1}{N_c}\right)}, \qquad (28)$$

$$(r' e^{i\delta_q})^n = -\frac{2\left(2 + \frac{1}{N_c}\right)c_3' + 2\left(1 + \frac{2}{N_c}\right)c_4' + 4\left(c_5' + \frac{1}{N_c}c_6'\right) + c_7' + \frac{1}{N_c}c_8' + (c_9' - c_{10}')\left(1 - \frac{1}{N_c}\right)}{2\left(c_1 + \frac{1}{N_c}c_2\right)} \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right|, \qquad (29)$$

where

$$\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \frac{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}{(1 - \lambda^2/2)(\rho^2 + \eta^2)}. \qquad (30)$$

For $\Lambda_b \to \Lambda\rho^0(\omega)$, the evaluation is the same. Defining

$$\langle \rho^0 \Lambda | O_1^u | \Lambda_b \rangle = \langle \rho^0 | (\bar{u}u) | 0 \rangle \langle \Lambda | (\bar{s}b) | \Lambda_b \rangle \equiv \tilde{T}, \qquad (31)$$

we have

$$t_\rho^\Lambda = \left(c_1 + \frac{1}{N_c} c_2 \right) \tilde{T}. \qquad (32)$$

After evaluating the penguin diagram contributions we obtain the following results:

$$(\alpha e^{i\delta_\alpha})^\Lambda = 1, \qquad (33)$$

$$(\beta e^{i\delta_\beta})^\Lambda = \frac{3\left(c_7' + \frac{1}{N_c}c_8' + c_9' + \frac{1}{N_c}c_{10}'\right)}{4\left(c_3' + \frac{1}{N_c}c_4' + c_5' + \frac{1}{N_c}c_6'\right) + c_7' + \frac{1}{N_c}c_8' + c_9' + \frac{1}{N_c}c_{10}'}, \qquad (34)$$

$$(r' e^{i\delta_q})^\Lambda = -\frac{4\left(c_3' + \frac{1}{N_c}c_4' + c_5' + \frac{1}{N_c}c_6'\right) + c_7' + \frac{1}{N_c}c_8' + c_9' + \frac{1}{N_c}c_{10}'}{2\left(c_1 + \frac{1}{N_c}c_2\right)} \left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*} \right|, \qquad (35)$$

TABLE I. Values of β, r', δ_β and δ_q for $\Lambda_b \to n\rho^0$.

N_c	q^2/m_b^2	β	r'	δ_β	δ_q
2	0.3	0.339	1.149	-3.096	0.0769
2	0.5	0.328	1.011	-2.935	0.297
3	0.3	0.649	2.537	-3.103	0.0766
3	0.5	0.629	2.233	-2.970	0.296

where

$$\left| \frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*} \right| = \frac{\sqrt{[\rho(1+\lambda^2\rho)+\lambda^2\eta^2]^2+\eta^2}}{\lambda^2(\rho^2+\eta^2)}. \quad (36)$$

It can be seen from Eqs. (28) and (34) that β and δ_β are determined solely by the Wilson coefficients. On the other hand, r' and δ_q depend on both the Wilson coefficients and the CKM matrix elements, as shown in Eqs. (29) and (35). Substituting Eqs. (27), (28), (29), (33), (34), (35) into Eqs. (12), (13), (14) we can obtain $(r\sin\delta)^{n(\Lambda)}$ and $(r\cos\delta)^{n(\Lambda)}$. Then in combination with Eqs. (15) and (16) the CP-violating asymmetries a can be obtained.

In the numerical calculations, we have several parameters: q^2, N_c, and the CKM matrix elements in the Wolfenstein parametrization. As mentioned in Sec. II B, the value of q^2 is chosen in the range $0.3 < q^2/m_b^2 < 0.5$ [5,6].

The CKM matrix elements should be determined from experiment. λ is well measured [20] and we will use $\lambda = 0.221$ in our numerical calculations. However, due to the large experimental errors at present, ρ and η are not yet fixed. From $b \to u$ transitions $\sqrt{\rho^2+\lambda^2}=0.363\pm0.073$ [21,22]. In combination with the results from B^0-$\bar{B}^0$ mixing [23] we have $0.18 < \eta < 0.42$ [22]. In our calculations we use $\eta=0.34$ as in Refs. [5,6]. Recently, it has been pointed out [24] that from the branching ratio of $B^\pm \to \eta\pi^\pm$ a negative value for ρ is favored. Hence we will use $\rho=-0.12$, corresponding to $\eta=0.34$. These values lead to $\phi^n=126°$ and $\phi^\Lambda=-72°$ from Eqs. (15) and (16).

The value of the effective N_c should also be determined by experiments. The analysis of the data for $B \to D\pi$, $B^\pm \to \omega\pi^\pm$ and $B^\pm \to \omega K^\pm$ indicates that N_c is about 2 [25,26]. For the Λ_b decays, we do not have enough data to extract N_c at present. Finally, we use $m_b=5$ GeV, $m_c=1.35$ GeV, $\alpha_s(m_Z)=0.112$ and $\alpha_{em}(m_b)=1/132.2$ to calculate the Wilson coefficients, c_i', as discussed in Sec. II B [see Eqs. (23) and (24)]. The numerical values of β, r', δ_β and δ_q for $\Lambda_b \to n\rho^0$ and $\Lambda_b \to \Lambda\rho^0$ are listed in Tables I and II, respectively.

TABLE II. Values of β, r', δ_β and δ_q for $\Lambda_b \to \Lambda\rho^0$.

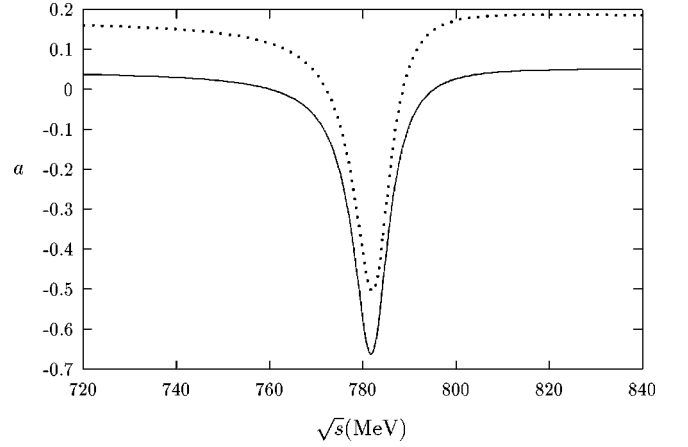| N_c	q^2/m_b^2	β	r'	δ_β	δ_q
2	0.3	0.299	9.925	-0.0611	0.0675
2	0.5	0.332	8.833	-0.235	0.257
3	0.3	3.086	3.715	-1.766×10^{-4}	6.353×10^{-3}
3	0.5	3.087	3.668	-6.071×10^{-4}	0.0216

FIG. 1. The CP-violating asymmetry for $\Lambda_b \to n\pi^+\pi^-$ with $N_c=2$. The solid (dotted) line is for $q^2/m_b^2=0.3$ (0.5).

In Figs. 1 and 2 we plot the numerical values of the CP-violating asymmetries, a, for $\Lambda_b \to n\pi^+\pi^-$ and $\Lambda_b \to \Lambda\pi^+\pi^-$, respectively, for $N_c=2$. It can be seen that there is a very large CP violation when the invariant mass of the $\pi^+\pi^-$ is near the ω mass. For $\Lambda_b \to n\pi^+\pi^-$ the maximum CP-violating asymmetry is $a_{max}^n=-66\%$ ($q^2/m_b^2=0.3$) and $a_{max}^n=-50\%$ ($q^2/m_b^2=0.5$), while for $\Lambda_b \to \Lambda\pi^+\pi^-$, $a_{max}^\Lambda=68\%$ ($q^2/m_b^2=0.3$) and $a_{max}^\Lambda=76\%$ ($q^2/m_b^2=0.5$). It would be very interesting to actually measure such large CP-violating asymmetries.

Although N_c is around 2 for B decays, it might be different in the Λ_b case. We also calculated the numerical values when $N_c=3$. It is found that, in this case, we still have large CP violation for $\Lambda_b \to n\pi^+\pi^-$, with $a_{max}^n=-52\%$ ($q^2/m_b^2=0.3$) and $a_{max}^n=-40\%$ ($q^2/m_b^2=0.5$). However, for $\Lambda_b \to \Lambda\pi^+\pi^-$, a_{max}^Λ is much smaller, only about 6%.

III. BRANCHING RATIOS FOR $\Lambda_b \to n(\Lambda)\rho^0$

In this section we estimate the branching ratios for $\Lambda_b \to f\rho^0$. In the factorization approach, ρ^0 is factorized out and hence the decay amplitude is determined by the weak tran-
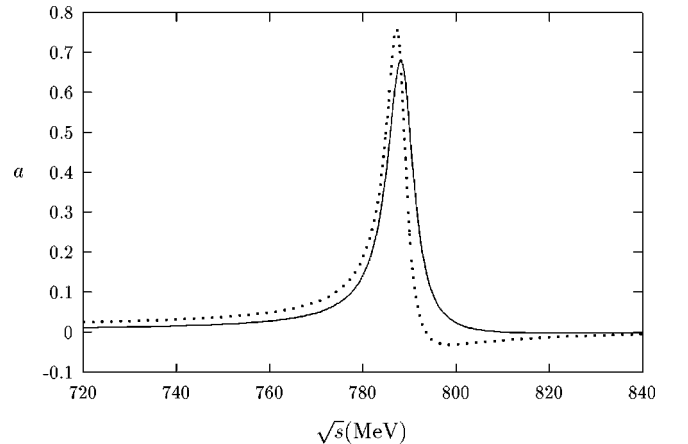

FIG. 2. The CP-violating asymmetry for $\Lambda_b \to \Lambda\pi^+\pi^-$ with $N_c=2$. The solid (dotted) line is for $q^2/m_b^2=0.3$ (0.5).

sition matrix elements $\Lambda_b \to f$. In the heavy quark limit, $m_b \to \infty$, it is shown in the HQET that there are two form factors for $\Lambda_b \to f$ [27]:

$$\langle f(p_f)|\bar{q}\gamma_\mu(1-\gamma_5)b|\Lambda_b(v)\rangle$$

$$= \bar{u}_f(p_f)[F_1(v \cdot p_f) + \not{v}F_2(v \cdot p_f)]$$

$$\times \gamma_\mu(1-\gamma_5)u_{\Lambda_b}(v), \tag{37}$$

where $q = d$ or s; u_f and u_{Λ_b} are the Dirac spinors of f and Λ_b, respectively; p_f is the momentum of the final baryon, f, and v is the velocity of Λ_b. In order to calculate F_1 and F_2 we need two constraints.

In Ref. [28] the author proposed two dynamical assumptions with respect to the meson structure and decays: (i) in the rest frame of a hadron the distribution of the off-shell momentum components of the constituents is strongly peaked at zero with a width of the order of the confinement scale; (ii) during the weak transition the spectator retains its momentum and spin. These two assumptions led to the result that the matrix element of the heavy to light meson transition is dominated by the configuration where the active quarks' momenta are almost equal to those of their corresponding mesons. This argument is corrected by terms of order $1/m_b$ and Λ_{QCD}/E_f, and hence is a good approximation in heavy hadron decays. Some relations among the form factors in the heavy to light meson transitions are found in this approximation. In Ref. [10] the above approach is generalized to the baryon case and a relation between F_1 and F_2 is found.

Another relation between F_1 and F_2 comes from the overlap integral of the hadronic wave functions of Λ_b and f. In the heavy quark limit Λ_b is regarded as a bound state of a heavy quark b and a light scalar diquark $[ud]$ [10,11,29]. On the other hand, the light baryon f has various quark-diquark configurations [30] and only the $q[ud]$ component contributes to the transition $\Lambda_b \to f$. This leads to a suppression factor, C_s, which is the Clebsch-Gordan coefficient of $q[ud]$. $C_s = 1/\sqrt{2}$ for n and $C_s = 1/\sqrt{3}$ for Λ, respectively [30]. In the quark-diquark picture, the hadronic wave function has the following form:

$$\psi_i(x_1, \vec{k}_\perp) = N_i x_1 x_2^3 \exp[-b^2(\vec{k}_\perp^2 + m_i^2(x_1 - x_{0i})^2)], \tag{38}$$

where $i = \Lambda_b$, n or Λ; x_1, x_2 ($x_2 = 1 - x_1$) are the longitudinal momentum fractions of the active quark and the diquark, respectively; $\vec{k}_\perp$ is the transverse momentum; N_i is the normalization constant; the parameter b is related to the average transverse momentum, $b = 1.77$ GeV and $b = 1.18$ GeV, corresponding to $\langle k_\perp^2 \rangle^{1/2} = 400$ MeV and $\langle k_\perp^2 \rangle^{1/2} = 600$ MeV respectively; and x_{0i} ($x_{0i} = 1 - \epsilon/m_i$, ϵ is the mass of the diquark) is the peak position of the wave function. By working in the appropriate infinite momentum frame and evaluating the good current components [10,11], another relation between F_1 and F_2 is given in terms of the overlap integral of the hadronic wave functions of Λ_b and f. Therefore, F_1 and F_2 are obtained as the following,

$$F_1 = \frac{2E_f + m_f + m_q}{2(E_f + m_q)}C_s I(\omega),$$

$$F_2 = \frac{m_q - m_f}{2(E_f + m_q)}C_s I(\omega), \tag{39}$$

where $I(\omega)$ is the overlap integral of the hadronic wave functions of Λ_b and f,

$$I(\omega) = \left(\frac{2}{\omega+1}\right)^{7/4} y^{-9/2}[A_f K_6(\sqrt{2}b\epsilon)]^{-1/2}$$

$$\times \exp\left(-2b^2\epsilon^2\frac{\omega-1}{\omega+1}\right)\int_{-2b\epsilon/\sqrt{\omega+1}}^{y-2b\epsilon/\sqrt{\omega+1}} dz$$

$$\times \exp(-z^2)\left(y - \frac{2b\epsilon}{\sqrt{\omega+1}} - z\right)\left(z + \frac{2b\epsilon}{\sqrt{\omega+1}}\right)^6, \tag{40}$$

and $y = bm_f\sqrt{\omega+1}$, with ω being the velocity transfer $\omega = v \cdot p_f/m_f$ and A_f and K_6 defined as

$$A_f = \int_0^1 dx\, x^6(1-x)^2 \exp[-2b^2 m_f^2(x-\epsilon/m_f)^2],$$

$$K_6(\sqrt{2}b\epsilon) = \int_{-\sqrt{2}b\epsilon}^{\infty} dx \exp(-x^2)(x+\sqrt{2}b\epsilon)^6. \tag{41}$$

It should be noted that in Eqs. (40) and (41) we have taken the limit $m_b \to \infty$.

It can be shown that $\omega = 3.03$ for $\Lambda_b \to n\rho^0$ and $\omega = 2.58$ for $\Lambda_b \to \Lambda\rho^0$. Taking $\epsilon = 600$ MeV, from Eq. (40), we find that $I^n = 0.0258(0.0509)$ for $b = 1.77$ GeV^{-1} ($b = 1.18$ GeV^{-1}), and $I^\Lambda = 0.0389(0.0781)$ for $b = 1.77$ GeV^{-1} ($b = 1.18$ GeV^{-1}). Substituting these numbers into Eq. (39) we obtain the following results,

$$F_1^n = -0.0199(-0.0393), \quad \text{for } b = 1.77(1.18) \text{ GeV}^{-1},$$

$$F_2^n = 0.00168(0.00332), \quad \text{for } b = 1.77(1.18) \text{ GeV}^{-1}, \tag{42}$$

and

$$F_1^\Lambda = 0.0245(0.0492), \quad \text{for } b = 1.77(1.18) \text{ GeV}^{-1},$$

$$F_2^\Lambda = -0.00205(-0.00411), \quad \text{for } b = 1.77(1.18) \text{ GeV}^{-1}, \tag{43}$$

where we have taken $m_d = 0.35$ GeV and $m_s = 0.50$ GeV.

To estimate the branching ratios for $\Lambda_b \to n(\Lambda)\rho^0$ we only take the O_1^u and O_2^u terms in the Hamiltonian (17), since they give dominant contributions. In the factorization approach, the amplitude for $\Lambda_b \to n(\Lambda)\rho^0$ has the following form:

$$A(\Lambda_b \to f + \rho^0) = \frac{G_F}{\sqrt{2}} V_{ub} V_{uq}^* a_1 \langle \rho^0 | \bar{u} \gamma_\mu (1 - \gamma_5) u | 0 \rangle$$

$$\times \langle f | \bar{q} \gamma^\mu (1 - \gamma_5) b | \Lambda_b \rangle, \qquad (44)$$

where

$$a_1 = c_1 + \frac{1}{N_c} c_2. \qquad (45)$$

In Eq. (44) ρ^0 has been factorized out and the matrix element $\langle \rho^0 | \bar{u} \gamma_\mu (1 - \gamma_5) u | 0 \rangle$ is related to the decay constant f_ρ. From Eq. (44) the branching ratios for $\Lambda_b \to n(\Lambda) \rho^0$ can be obtained directly [10,31]. Taking $f_\rho = 216$ MeV, $V_{ub} = 0.004$, $V_{us} = 0.22$, $V_{ud} = 0.975$ and $a_1 = 0.28$ (corresponding to $N_c \sim 2$) we obtain

$$B(\Lambda_b \to n\rho^0) = \begin{cases} 1.61 \times 10^{-8} & \text{for } b = 1.18 \text{ GeV}^{-1}, \\ 4.14 \times 10^{-9} & \text{for } b = 1.77 \text{ GeV}^{-1}, \end{cases}$$
$$(46)$$

and

$$B(\Lambda_b \to \Lambda\rho^0) = \begin{cases} 1.23 \times 10^{-9} & \text{for } b = 1.18 \text{ GeV}^{-1}, \\ 3.06 \times 10^{-10} & \text{for } b = 1.77 \text{ GeV}^{-1}. \end{cases}$$
$$(47)$$

In Ref. [12] the $\Lambda_b \to n(\Lambda)$ transition matrix elements are calculated in the nonrelativistic quark model. The form factors f_i and g_i, which are defined by ($q = p_{\Lambda_b} - p_f$)

$$\langle f(p_f) | \bar{q} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p_{\Lambda_b}) \rangle$$

$$= \bar{u}_f \{ f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu$$

$$- [g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q_\mu] \gamma_5 \} u_{\Lambda_b}, \qquad (48)$$

are found to be $f_1(0) = 0.045$, $f_2(0) = -0.024/m_{\Lambda_b}$, $f_3(0) = -0.011/m_{\Lambda_b}$, $g_1(0) = 0.095$, $g_2(0) = -0.022/m_{\Lambda_b}$, $g_3(0) = -0.051/m_{\Lambda_b}$ for $\Lambda_b \to n$, and $f_1(0) = 0.062$, $f_2(0) = -0.025/m_{\Lambda_b}$, $f_3(0) = -0.008/m_{\Lambda_b}$, $g_1(0) = 0.108$, $g_2(0) = -0.014/m_{\Lambda_b}$, $g_3(0) = -0.043/m_{\Lambda_b}$ for $\Lambda_b \to \Lambda$. Pole dominance behavior for the q^2 dependence of the form factors is assumed:

$$f_i(q^2) = \frac{f_i(0)}{\left(1 - \dfrac{q^2}{m_V^2}\right)^2}, \quad g_i(q^2) = \frac{g_i(0)}{\left(1 - \dfrac{q^2}{m_A^2}\right)^2}, \quad (49)$$

where for $b \to d$, $m_V = 5.32$ GeV, $m_A = 5.71$ GeV, and for $b \to s$, $m_V = 5.42$ GeV, $m_A = 5.86$ GeV. Substituting Eqs. (48) and (49) into Eq. (44) we find that

$$B(\Lambda_b \to n\rho^0) = 6.33 \times 10^{-8}, \quad B(\Lambda_b \to \Lambda\rho^0) = 4.44 \times 10^{-9}.$$
$$(50)$$

These results are bigger than those in Eqs. (46) and (47). Combining the predictions in these two models we expect that $B(\Lambda_b \to n\rho^0)$ is around 10^{-8} and $B(\Lambda_b \to \Lambda\rho^0)$ is about 10^{-9}. For comparison, in B decays, the branching ratio for $B^- \to \pi^- \rho^0$ is of the order 10^{-6} [32], and for $B^- \to \rho^- \rho^0$ the branching ratio is about 10^{-5} [32]. Hence $B(\Lambda_b \to n\rho^0)$ is two to three orders smaller than those for the corresponding meson decays.

IV. SUMMARY AND DISCUSSIONS

In this work we studied direct CP violation in Λ_b hadronic decays $\Lambda_b \to f\rho^0(\omega) \to f\pi^+ \pi^-$ ($f = n$ or Λ). It was found that, as a result of the inclusion of ρ-ω mixing, the CP-violating asymmetries in these two processes could be very large when the invariant mass of the $\pi^+ \pi^-$ pair is in the vicinity of the ω resonance. For $N_c = 2$, the maximum CP-violating asymmetries were more than 50% and 68% for $\Lambda_b \to n\pi^+ \pi^-$ and $\Lambda_b \to \Lambda\pi^+ \pi^-$, respectively, for reasonable values of q^2/m_b^2. Furthermore, we estimated the branching ratios for $\Lambda_b \to n(\Lambda)\rho^0$ decays by using HQET and phenomenological models for the hadronic wave functions. The results from the nonrelativistic quark model were also presented for comparison. It was shown that the branching ratios are about 10^{-8} and 10^{-9} for $\Lambda_b \to n\rho^0$ and $\Lambda_b \to \Lambda\rho^0$, respectively, which are two or three orders smaller than those for the corresponding B decays. Since there will be more data on the heavy baryon, Λ_b, from different experimental groups in the future, it will be very interesting to look for such large CP-violating asymmetries in the experiments in order to get a deeper understanding of the mechanism for CP violation. On the other hand, the smaller branching ratios for the Λ_b hadronic decays will make the measurements more difficult. Furthermore, the study of CP violation in Λ_b decays may provide insight into the baryon asymmetry phenomena required for baryogenesis.

There are some uncertainties in our calculations. While discussing the CP violation in these two channels, we have to evaluate hadronic matrix elements where nonperturbative QCD effects are involved. We have worked in the factorization approximation, which is expected to be quite reliable because the b quark decays are very energetic. However, in this approach, the color-octet term is ignored. Hence N_c has to be treated as an effective parameter which should be determined by experiment. Although there are enough data to fix N_c in B decays as $N_c \sim 2$, the best value of N_c for Λ_b decays is not certain. We gave the plots of the CP-violating asymmetries for $N_c = 2$ and discussed the situation for $N_c = 3$. If $N_c = 3$, the CP violation for $\Lambda_b \to \Lambda\pi^+ \pi^-$ is not

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large anymore. However, for both $N_c = 2$ and 3 there is large CP violation for $\Lambda_b \to n\pi^+\pi^-$. Our numerical results also depend on q^2/m_b^2, but the behavior is mainly determined by N_c. The ρ-ω mixing parameter, $\widetilde{\Pi}_{\rho\omega}$, also has some experimental uncertainty, but this has little influence on our results.

While estimating the branching ratios for $\Lambda_b \to n(\Lambda)\rho^0$ we worked in the heavy quark limit. Since m_b is much larger

than the QCD scale, Λ_{QCD}, the $1/m_b$ corrections should be small.

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