BOLETÍN DE LOS OBSERVATORIOS DE TONANTZINTLA Y TACUBAYA, VOL. 4, N°30, 1968.

DIRECT DESIGN SOLUTION FOR CASSEGRAIN SHIELDS

Alejandro Cornejo and Daniel Malacara

ABSTRACT

A direct algebraic design of Cassegrain light shields is proposed, using only the assumption that the mirror surfaces are flat.

Introduction

During the design and construction of a Cassegrain or Ritchey-Chrètien telescope, the design of an adequate shielding is required in order to avoid direct sky light falling on the focal plane. A solution to this problem has been worked out by means of a graphical method, using succesive approximations[1], and later applied to electronic computers[2]. Recently, a different graphical solution has been explained, based also on succesive approximations[3]. Therefore, the possibility of a direct solution was studied. The final goal was reached using in this method only the approximation of considering the surfaces of the mirrors as plane surfaces.

Parameters of a Cassegrain system.

The main purpose of the shields in a Cassegrain system is to avoid direct sky light falling on the image plane. As the field increases the required shielding introduces two new problems, vignetting and central obscuration. The central obscuration is produced by the large size of the diameter of the shield at the secondary mirror and the vignetting is due to both shields. (See Fig. 2). Hence as it is pointed out by Young[(2)], these factors will set the conditions for the solution of the problem and will establish a compromise between the allowable field size and central obscuration.

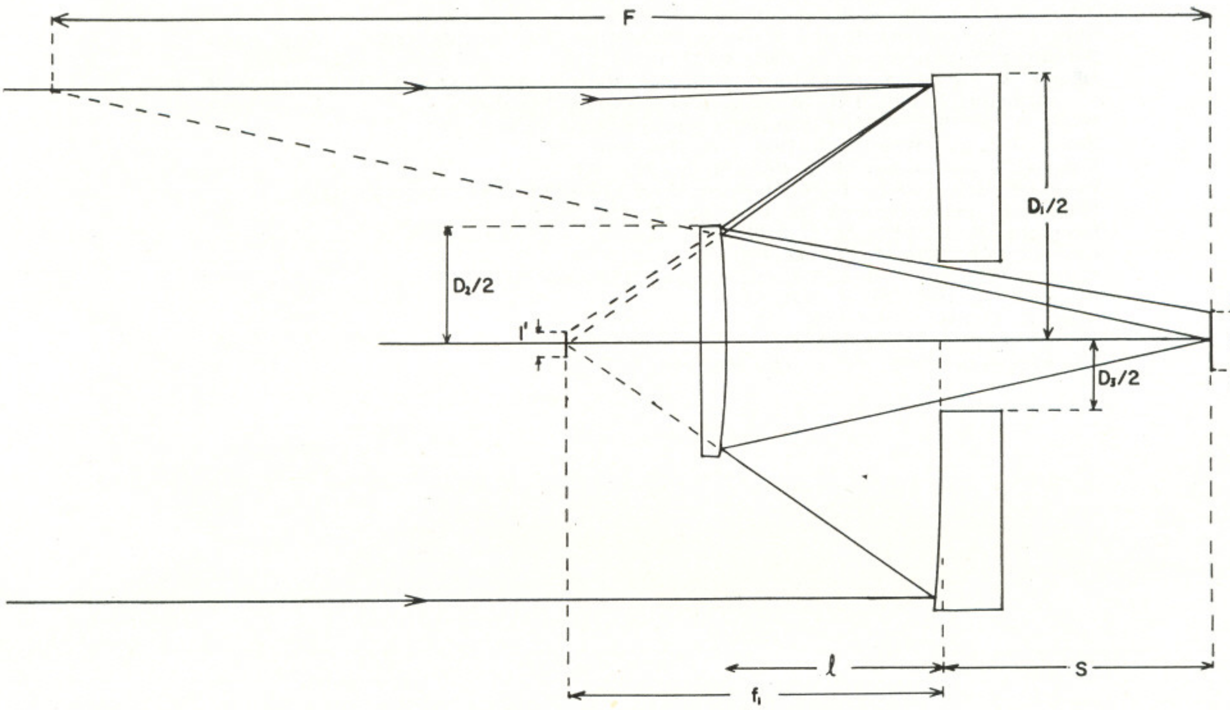

Fig. 1.—*Parameters of a Cassegrain System.*

This shielding solution was incorporated as a section, of a more general computer program for the designing of Ritchey-Chrètien and Cassegrain systems in which the equations come from a previous paper by one of us[(4)]. There, it was shown that for the design of a Cassegrain system five initial parameters must be known. The selected ones are D_1, f_1, F, S and I (or f_1/f_2 instead of f_1) as shown in Fig. 1. From them, the rest of those parameters shown also in Fig. 1 can be calculated. Of course, any other set of initial parameters could be selected. From the knowledge of all parameters the shielding

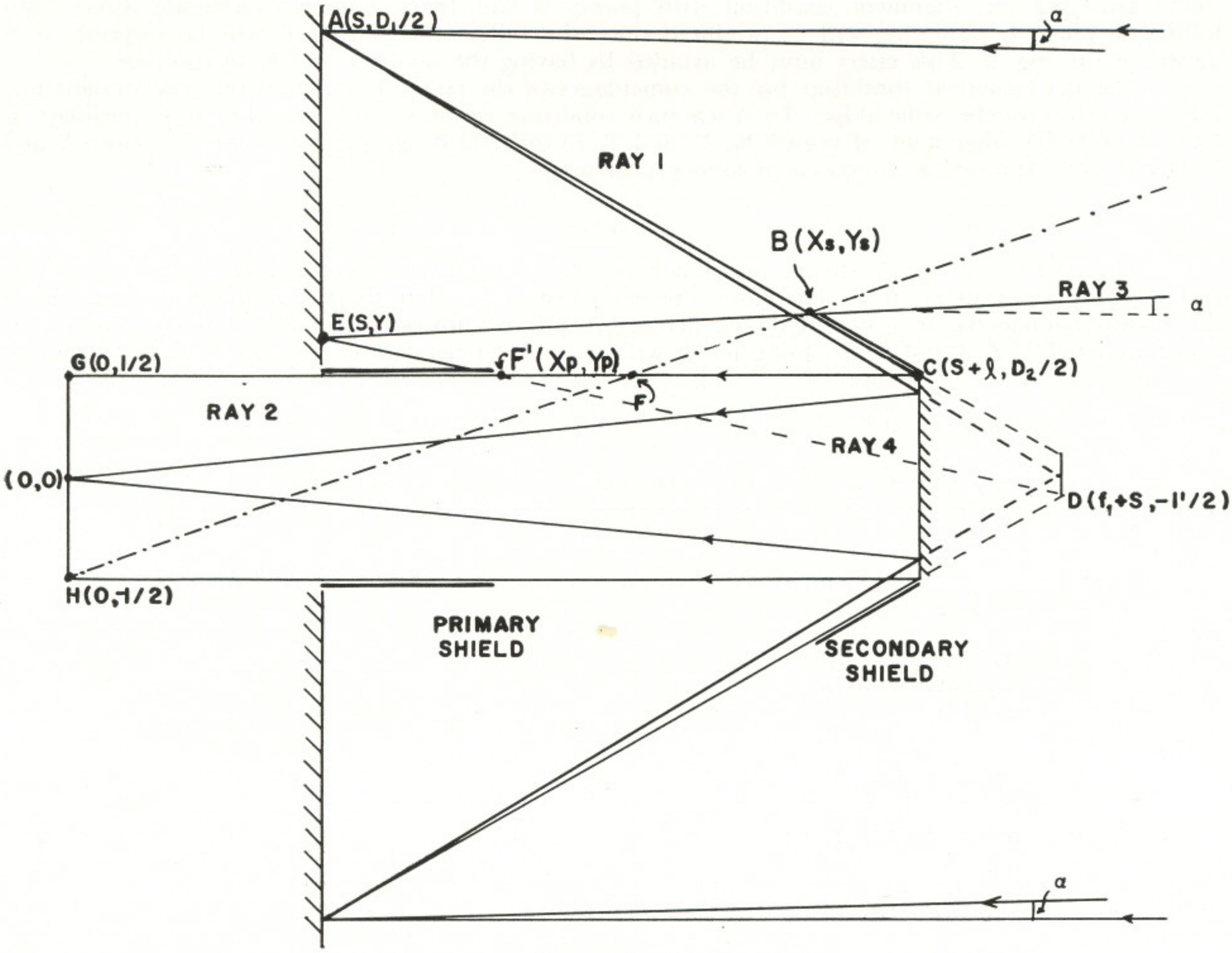

Fig. 2.—*Parameters for the Shielding Design.*

design can be started as shown in Fig. 2; where the known values are used as the coordinates of several convenient points.

Shield design

As it was previously mentioned, the shield design must avoid stray light on the image plane I, and vignetting effects. Once these problems have been solved, the decision for the value to be allowed for the central obscuration is left to the criteria of the designer, which can be modified by changing the field.

In Fig. 2, the total angular field 2 α produces the image-size fields I' and I at the primary and Cassegrain focus, respectively. The middle point of the image plane I will be taken as the origin of the coordinate system. As a first approximation, the surfaces of the mirrors will be considered as plane surfaces.

In order to solve at least partially the vigneting problem we will consider an extreme ray, at the edge of the primary mirror. The ray forms an angle α with respect to the horizontal incident ray. The ray at the angle α is reflected at the primary and secondary mirrors as ray 1 and ray 2 respectively of Fig. 2. This gives us the path at an extreme ray in the optical system. Therefore, the lower limits for the diameters of the shields must be along the ray 1 (secondary shield) and ray 2 (primary shield); otherwise such an extreme ray could be stopped and vignetting is produced.

Looking at the problem of direct light falling on the image plane I, we easily conclude that the end points of the shields mut be along ray 1 and ray 2. Such points also have to be in alignment with point H of the image plane I, in order to avoid direct light falling on it; this means that B, F' and H of Fig. 2 must be aligned. Unfortunately, the problem is still unsolved because if we consider a ray (at an angle α) passing throught the point B, we have the following vignetting effect: the ray incident on the primary mirror at point E (See Fig. 2) is reflected toward point D of the image plane I', intersecting the ray 2 at point F'. If point F has been taken as the end of the primary shield satisfying the alignment condition with points B and H in order to eliminate direct light falling on plane I, vignetting will be produced since the reflected ray from E will be stopped, as is shown in the Fig. 2. This effect must be avoided by having the points F and F' to coincide.

The mathematical condition for the coincidence of the points F and F' is that the straight line HF' and F'B have the same slope. To reach such condition we must find the value of Y (ordinate of E) that gives the alignment of points H, F' and B. In order to do so, the coordinates of points B and F' have to be expressed as functions of known parameters.

Algebra

The coordinates of all the points shown in Fig. 2 are known except those of the points B,F' and the ordinate Y of point E. Therefore, the main task is to relate their coordinates as functions of the known parameters. In order to do so, mathematically we proceed as follows: The equation of the rays numbered 1, 2, 3 and 4 are going to be written in the general form $y = mx + b$. It is possible to write:

Ray 1 ($\overline{AC}$):

$$y = -\left(\frac{D_1 - D_2}{2\,l}\right) x + \frac{D_1}{2} + \frac{D_1 - D_2}{2\,l} \cdot s \tag{1}$$

Ray 2 ($\overline{CG}$)

$$y = \frac{D_2 - I}{2\,(s+l)}\, x + \frac{I}{2} \tag{2}$$

Ray 3 ($\overline{BE}$)

$$y = \alpha\, x - (\alpha\, s - Y) \tag{3}$$

Ray 4 ($\overline{DE}$)

$$y = -\frac{I'/2 + Y}{f_1}\, x + Y\left(1 + \frac{s}{f_1}\right) + \frac{I' \cdot s}{2\,f_1} \tag{4}$$

If the coordinates of the point B, in Eqs. 1 and 3, and those of point F' in equations 2 and 4, are introduced, we obtain:

$$y_s = \frac{D_2 - D_1}{2l}\, x_s + \frac{D_1}{2}\left(1 + \frac{s}{l}\right) - \frac{D_2}{2} \cdot \frac{s}{l} \tag{5}$$

$$y_s = \alpha\, x_s - \alpha s + Y \tag{6}$$

$$y_p = \frac{D_2 - I}{2\,(s+l)}\, x_p + \frac{I}{2} \tag{7}$$

$$y_p = -\left(\frac{I'}{2} + Y\right)\left(\frac{x_p}{f_1}\right) + \left(1 + \frac{s}{f_1}\right) Y + \frac{I'}{2f_1}\,(s) \tag{8}$$

Solving for x_s, y_s, x_p, y_p in function of the known parameters we have:

$$x_s = \frac{(-\,\alpha\, s + Y)\mathrm{l} - \dfrac{D_1}{2\,(l + s)} + \dfrac{D_2}{2s}}{\dfrac{D_2 - D_1}{2} - \alpha l} \tag{9}$$

$$y_s = \alpha\, x_s - \alpha\, s + Y \tag{10}$$

$$x_p = (s + l) \left\{ \frac{Y\,(f_1 + s) + \frac{I'}{2} \cdot s - \frac{I}{2} \cdot f_1}{\frac{D_2 - I}{2}\, f_1 + \left(\frac{I'}{2} + Y\right) \cdot (s + l)} \right\} \tag{11}$$

$$y_p = \frac{D_2 - I}{2\,(s+l)}\, x_p + \frac{I}{2} \tag{12}$$

where the only unknown parameter is the ordinate Y of the point E on the mirror and which we will find.

Now, the mathematical expression for the alignment of H, B and F'; which insures no stray light at the image plane I and unvignetted fieldwill be:

$$\frac{y_s - y_p}{x_s - x_p} - \frac{y_s + I/2}{x_s} = D\,I\,F = O \tag{13}$$

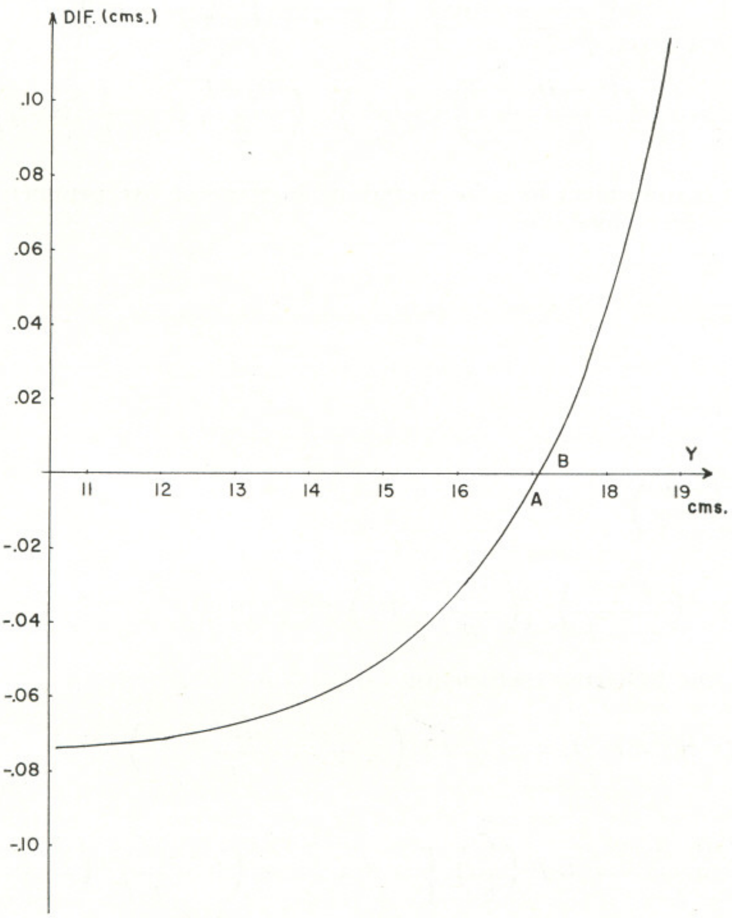

Fig. 3.—*Curve for the Solution of the Shield Design.*

Therefore, if the values of the coordinates x_s, y_s; x_p, y_p are included into Eq. 13, a quadratic equation for Y can be obtained. Fig. 3 shows the type of curve obtained by the method of succesive approximations. After some algebraic work the following equation results:

$$A_2\, Y^2 + A_1\, Y + A_o = o \tag{14}$$

where the coefficient A_2, A_1 and A_o given by:

$$A_2 = (f_1 + s) \cdot \left[\left(\alpha - \frac{D_2 - I}{2\,(s+l)} \right) \cdot l + \frac{D_2 - D_1}{2} - \alpha l \right] - I\, l \tag{15}$$

$$A_1 = \left(\alpha - \frac{D_2 - I}{2\,(s+l)} \right) \cdot \left(\frac{I's - If_1}{2}\, l + (f_1 + s) \left(\frac{D_2 - D_1}{2}\, s - \frac{D_1}{2}\, l - \alpha l s \right) \right)$$

$$+ \left(\frac{D_2 - D_1}{2} - \alpha l \right) \cdot \left((f_1 + s) \left(\frac{I}{2} - \alpha s \right) + \frac{I's - If_1}{2} \right)$$

$$- \left(\frac{I \cdot l}{s + l} \right) \cdot \left(\frac{D_2 - I}{2}\, f_1 + \frac{I'}{2}\, (s + l) \right) - I \left(\frac{D_2 - D_1}{2}\, s - \frac{D_1}{2}\, l - \alpha l s \right) \tag{16}$$

$$A_o = \left(\alpha - \frac{D_2 - I}{2\,(s+l)} \right) \cdot \left(\frac{Is - If_1}{2} \right) \cdot \left(\frac{D_2 - D_1}{2}\, s - \frac{D_1}{2}\, l - \alpha l s \right)$$

$$+ \left(\frac{D_2 - D_1}{2} - \alpha l \right) \cdot \left(\frac{I's - If_1}{2} \right) \cdot \left(\frac{I}{2} - \alpha s \right)$$

$$- \frac{I}{s+l} \left(\frac{D_2 - D_1}{2}\, s - \frac{D_1}{2}\, l - \alpha l s \right) \cdot \left(\frac{D_2 - I}{2}\, f_1 + \frac{I'}{2}\, (s+l) \right) \tag{17}$$

As in Ref. 4, it is convenient to write everything in terms of five parameters: D_1, F, f_1, s and I by using the following four equations:

$$I' = \frac{f_1}{\mathrm{F}}\, I \tag{18}$$

$$\alpha = \frac{I}{2F} \tag{19}$$

$$l = f_1 \left(\frac{F - s}{f_1 + F} \right) \tag{20}$$

$$D_2 = D_1 + \left(\frac{F - s}{f_1 + F} \right) \cdot \left(\frac{IF_1}{F} - D_1 \right)$$

obtaining in this way the following coefficients:

$$A_2 = - D_1 (f_1 + F)(f_1 + s) + I f_1 \left(f_1 + s + \frac{f_1\, s}{F} - F \right) \tag{22}$$

$$A_1 = \frac{D_1^2\, (f_1 + s)^2}{2} + I^2 \left(\frac{f_1^2}{2} \right) \left[-3 + \frac{s}{f} + \left(1 + \frac{s}{F} \right) \left(\frac{f_1 s + F^2}{F\,(f_1 + s)} \right) \right]$$

$$- D_1 I f_1 \cdot \left(\frac{f_1\, s}{F} + \frac{s^2}{2F} + S - \frac{F}{2} + f_1 \right) \tag{23}$$

$$A_o = \left(\frac{D_1^2\, I\, (f_1 + s) f_1}{4} \right) \cdot \left(\frac{s}{F} + 1 \right) + \frac{D_1\, I^2\, f_1^2}{2} \cdot \left(\frac{3}{2} - \frac{s}{F} - \frac{s^2}{2F^2} \right) \tag{24}$$

Once Eq. 14 is solved, the positive solution of Y is substituted into the equations for x_s, y_s, x_p and y_p. With the values of these coordinates known, the dimensions of the shields can be obtained.

The lengths L_p and L_s of the primary and secondary shield respectively will be taken using the vertex of each mirror as a reference and will be equal to:

$$L_p = x_p - s \tag{25}$$

and

$$L_s = l + s - x_p \tag{26}$$

The diameters D_p and D_s of the primary and secondary shields respectively are given by:

$$D_p = 2\,x_p \tag{27}$$

$$D_s = 2\,x_s \tag{28}$$

where x_p and x_s, from Eqs. 9 and 11 are given by:

$$x_s = f_1 + s - \frac{2\,f_1}{D_1}\,Y \tag{29}$$

TABLE I

	Dimension of the Shields in cm.				Central Obscuration	
	Primary		Secondary			
Field Size	Length	Diameter	Length	Diameter	Without Shield	With Shield
20.0	91.04	21.31	49.14	36.24	7.84 %	20.25 %
15.0	94.41	18.91	38.15	32.40	7.29 %	16.40 %
8.0	97.87	15.66	21.81	26.61	6.25 %	11.40 %
4.0	98.68	13.70	11.58	22.94	5.90 %	8.88 %
2.0	98.50	12.85	6.0	20.95	5.71 %	6.86 %
1.0	98.18	12.35	3.09	19.89	5.62 %	6.20 %
0.5	97.95	12.10	1.56	19.35	5.57 %	5.86 %
0.1	97.72	11.89	0.32	18.91	5.22 %	5.57 %
0.0	97.66	11.84	0.0	18.80	5.29 %	5.29 %

and

$$x_p = F(f_1 + s) \cdot \left(\frac{2YF \ (f_1+s) + f_1 I \ (s-F)}{2YF^2 (f_1+s) + f_1 D_1 F \ (f_1+s) + f_1 I \,(fF-Fs-f_1 s-F^2)} \right) \quad (30)$$

Final analysis

An analysis of Eq. 13 and graph 3 show how the difference in the slopes of HF', and F'B change as function of the ordinate Y; where the exact solution of Y will be the crossing point of the curve with the x axis, which is the solution of Eq. 14.

From Fig. 2 we can see how as the usable field increases, the slopes of the lines HF' and F'B become bigger, implying that the diameters of the shield increase, and with them the value of the central obscuration (due mainly to the diameter of the secondary shield).

As an example of our program, in Table I the results for the dimensions of the shields for a Ritchey-Chrètien telescope are shown where field I has been given several values, and the values of the parameters of the telescope are: $D_1 = 80$; $F = 1200$; $f_1 = 264.0$ $f_2 = 79.53$; $s = 80$; $1 = 201.97$ (all units are centimeters). In the last two columns of Table I the values for the central obscuration are shown for the cases where shields are included, and where no shields are used. Central obscurations in the last case is due only to the size of the secondary mirror.

Acknowledgements

The authors want to thank Mr. Arquimedez Morales for his help in checking the Algebra and making the drawings.

REFERENCES

1. F. Sauer, Sterne u. Weltraum **4**, 141 (1966).
2. A. T. Young, Appl. Opt. **6**. 1063 (1967).
3. R. Prescott, Appl. Opt. **7**, 479 (1968).
4. D. Malacara, Boletín de los Observatorios de Tonantzintla y Tacubaya, # 27, 127 (1965).