Direct finite element analysis of flux and current distributions under specified conditions

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DIRECT FINITE ELEMENT ANALYSIS OF FLUX AND CURRENT DISTRIBUTIONS UNDER SPECIFIED CONDITIONS

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ABSTRACT

When the flux distribution of a magnetic circuit is analyzed by using the conventional finite element method, the magnetizing currents must be given. Therefore, if the flux distribution is specified, it is difficult to obtain the distributions of magnetomotive forces or configuration of magnets producing the specified field distribution by the conventional finite element method.

New methods which are called the "finite element method taking account of external power source" and the "finite element method with shape modification" have been developed. The processes of calculation in these methods are contrary to the conventional technique. These new methods have the following advantages:

(a) If there are many unknown independent magnetizing currents, these currents are directly calculated by the new method.

(b) When a flux distribution is specified, the optimum shapes of the magnets can be directly calculated.

(c) As these new methods need no repetition, computing time can be considerably reduced.

The principles and the finite element formulations of these new methods are described, and a few examples of application of these methods are shown.

These new methods make it possible to design the optimum magnetic circuits by using the finite element method.

1. INTRODUCTION

Magnetic fields of power apparatus and electronic instruments should be analyzed under the specified terminal voltages, because these are usually excited by constant voltage power sources. Sometimes the flux distributions in some parts of apparatus are specified, and the most suitable configurations and sizes of these parts have to be designed. We call such problems "inverse problems." In order to use the finite element method in the practical design of a magnetic circuit, new analyzing methods for these inverse problems should be developed.

If the terminal voltages are specified and the corresponding magnetizing currents are unknown, many iterative modifications of assumed magnetizing currents are necessary for the usual finite element analysis. Especially, if there are multiple unknown magnetizing currents as in the case of three-phase transformers, the analysis is almost impossible, because there are infinite combinations of unknown magnetizing currents to be assumed [1].

Moreover, in the usual finite element analysis, the sizes of magnetic materials are fixed. Therefore, when the sizes of magnetic materials satisfying a specified field distribution are required, many iterative modifications of assumed finite element subdivision are necessary.

Two new methods whose processes of calculation are contrary to the conventional ones have been developed, and as these new methods need no repetition, computing time can be considerably reduced. As a few examples of the application, the magnetic characteristics of a capacitor motor and the sizes of permanent magnets are computed.

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2. CLASSES OF INVERSE PROBLEMS

The inverse problems can be divided into the following three classes:

(1) Class A: the configurations and the sizes of magnetic materials are fixed, and the magnetizing currents which produce the specified terminal voltages are unknown.

This class of problem corresponds to the calculation of the characteristics of electrical machinery excited by constant voltage power sources.

The newly developed analysis method for the class A problem is called the "finite element method taking into account of external power source" [2].

(2) Class B: the magnetizations are fixed, and the sizes of magnets which produce the specified flux distribution are unknown.

The calculation of the sizes of the magnet which gives the specified flux distribution is important for a designer of magnetic circuits. Because the magnetization of a magnet is automatically determined by the quality of the magnet used and the sizes of it. Therefore, the determination of the magnetization is not so important.

The newly developed analysis method for the class B problem is called the "finite element method with shape modification".

(3) Class C: the shape of electrical machinery for which the maximum value of the electric field strength or the iron loss becomes a minimum is unknown.

Figure 1 shows the transformer windings. The configuration and size of the winding should be designed so that the maximum value of the electric field strength becomes a minimum under the condition that the total cross-sectional area of the winding is constant. Class C problems correspond to the optimum design of the shape of electrical machinery as mentioned above.

In this paper, new methods for solving the Class A and B problems are explained.

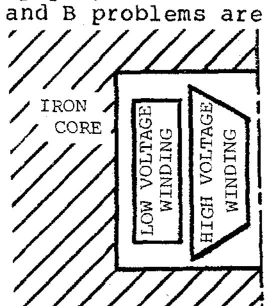

Fig. 1 Transformer winding.

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3. FINITE ELEMENT METHOD TAKING INTO ACCOUNT OF EXTERNAL POWER SOURCE

Outline of Method

Two-dimensional magnetic fields with some conductors and magnetic materials are analyzed by the following equation:

$$\frac{\partial}{\partial x}\left(\nu y\frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu x\frac{\partial A}{\partial y}\right) = -J_0 + J_e \qquad (1)$$

Where A, Jo and Je are the vector potential, the magnetizing current density and the eddy current density respectively. ν_x and ν_y denote the x- and y- components of reluctivity respectively.

The Rayleigh-Ritz matrix equation for (1) is denoted as follows:

$$\begin{bmatrix} H_{1,1} & \cdots & H_{1,nu} \\ \vdots & & \vdots \\ H_{nu,1} & \cdots & H_{nu,nu} \end{bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} = - \begin{bmatrix} C_{1,1} & \cdots & C_{1,k} \\ \vdots & & \vdots \\ C_{nu,1} & \cdots & C_{nu,k} \end{bmatrix} \begin{Bmatrix} I_{01} \\ \vdots \\ I_{0k} \end{Bmatrix} + \begin{Bmatrix} G_1 \\ \vdots \\ G_{nu} \end{Bmatrix} \qquad (2)$$

where [H] is the so-called coefficient matrix and [C] is the constant matrix related to the currents $\{I_0\}$ [1]. $\{G\}$ is the column matrix which is a function of the known vector potentials on the Dirichlet boundary. k is the number of unknown magnetizing currents and nu is the number of nodes whose potentials are unknown.

In the conventional finite element analysis, the vector potentials $\{A\}$ are computed by solving (2) in which the magnetizing currents $\{I_0\}$ are assumed. The terminal voltage V is calculated from the obtained vector potentials. In order to obtain the magnetizing currents satisfying the specified terminal voltage Vo, a number of repetitions are necessary until the desired results are obtained by modifying $\{I_0\}$ as shown by the thick lines in Fig.2(a).

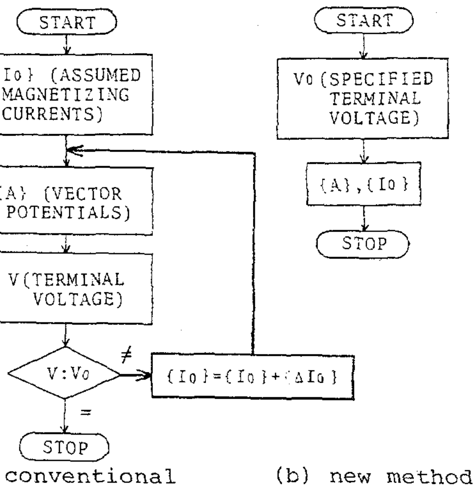
Equation (2) denotes the relation among the vector potentials $\{A\}$, the magnetizing currents $\{I_0\}$ and the co-ordinates x and y. This relation can be rewritten in the following general form:

$$f_i(x, y, \{A\}, \{I_0\}) = 0 \qquad (i = 1, \ldots\ldots, n) \qquad (3)$$

If the number n of equations in (3) is larger than nu, not only $\{A\}$ but also $\{I_0\}$ can be treated as the unknown variable. A new method which is called the "finite element method taking into account of external power source" is developed from the above-mentioned idea. The process of calculation of this new method is opposite to the conventional technique, and the magnetizing currents can be directly obtained as shown in Fig.2(b). This method has the following advantages:

(a) It is possible to obtain directly unknown magnetizing currents.

(b) As this method does not need repeated trial and error, computing time can be considerably reduced.



(a) conventional method (b) new method

Fig. 2 Processes of calculation.

Finite Element Formulation

(1) Rayleigh-Ritz matrix equation

As the term $\{I_0\}$ is unknown, $\{I_0\}$ is transposed to the left-hand side of (2). Then the following equation is obtained.

$$\left[\begin{bmatrix} H_{1,1} & \cdots & H_{1,nu} \\ \vdots & & \vdots \\ H_{nu,1} & \cdots & H_{nu,nu} \end{bmatrix} \begin{bmatrix} C_{1,1} & \cdots & C_{1,k} \\ \vdots & & \vdots \\ C_{nu,1} & \cdots & C_{nu,k} \end{bmatrix} \right] \begin{Bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} I_{01} \\ \vdots \\ I_{0k} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} G_1 \\ \vdots \\ G_{nu} \end{Bmatrix} \qquad (4)$$

As the number of the equations is nu, and the number of the unknown variables is (nu+k), (4) cannot be solved. Therefore, the following new relationships between the vector potentials, the currents and the terminal voltages are introduced.

(2) Relationships between vector potentials, currents and terminal voltages

The windings in the finite element region are usually connected to the external power sources and the external loads such as resistance, inductance and capacitance. Figure 3 shows one pole pitch of the stator of an induction motor. The finite element region which is enclosed by broken line in Fig.4 corresponds to the winding shown in Fig.3 except the end coils. Vo, Ro and Lo in Fig.4 are the terminal voltage of the external power source, the resistance and the

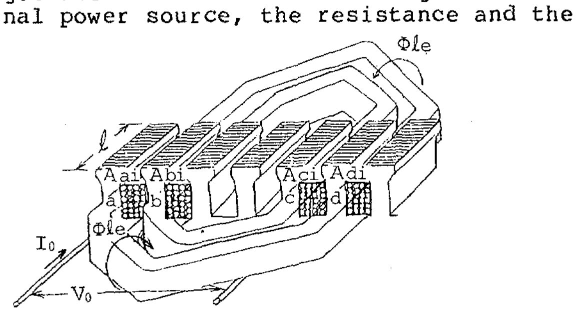


Fig. 3 Skeleton diagram.

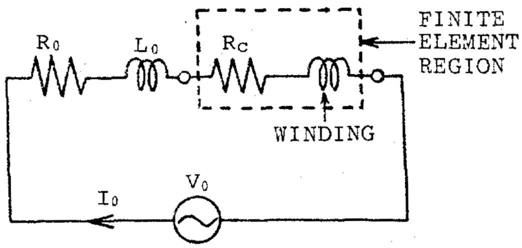


Fig. 4 Equivalent circuit.

leakage inductance of the end coils. Lo corresponds to Φ_{le} in Fig.3. Rc is the resistance of the winding in the finite element region. The following equation can be obtained from Kirchhoff's second law:

$$\int_{CFEM} \frac{\partial \mathbf{A}}{\partial t} d\mathbf{s} + (R_C + R_0) I_0 + L_0 \frac{\partial I_0}{\partial t} = V_0 \qquad (5)$$

Where CFEM is the contour along the winding in the finite element region, $\mathbf{s}$ is an unit tangent vector.

Let us calculate the first term of the left-hand side of (5) in detail. The number of conductors in a slot in Fig.3 is np, and they are connected series with the conductors in another slot. If the sectional-area of each conductor is small enough, the vector potential of each conductor is assumed to be constant at every point in the conductor. Then the following equation can be obtained:

$$\int_{CFEM} \frac{\partial \mathbf{A}}{\partial t} d\mathbf{s} = \ell \sum_{i=1}^{np} \frac{\partial}{\partial t} \{ (A_{ai} - A_{di}) + (A_{bi} - A_{ci}) \} \qquad (6)$$

Where ℓ is the thickness of the laminated core shown in Fig.3, and the subscripts a, b, c, d of vector potentials denote the slot number. For example, Aci denotes the vector potential of No.i conductor in No.c slot.

Though there is only one relationship of (5) for the case of Fig.4, two relationships similar to (5) are obtained in the case of capacitor motor shown in Fig.5, because there

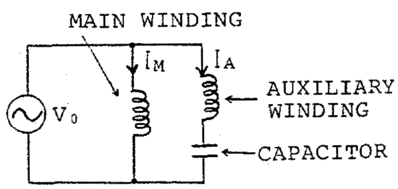


Fig. 5 Connection diagram of capacitor motor.

are two electric circuits. In general, the number k of the specified voltages $V_{01}, \ldots, V_{0k}$ is equal to that of the independent currents. Therefore, if there exist k independent currents, k relationships similar to (5) can be obtained as follows:

$$\begin{bmatrix} F_{1,1} & \cdots & F_{1,(nu+k)} \\ \vdots & & \vdots \\ F_{k,1} & \cdots & F_{k,(nu+k)} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} I_{01} \\ \vdots \\ I_{0k} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} V_{01} \\ \vdots \\ V_{0k} \end{Bmatrix} \qquad (7)$$

The elements of the matrix [F] are constants, and $\{V_0\}$ is the column matrix corresponding to the specified terminal voltages.

(3) Matrix equation for inverse problem

If (4) is combined with (7), the number of unknown variables becomes equal to the number of equations. Therefore, the unknown vector potentials and the magnetizing currents can be directly calculated by the following equation obtained from (4) and (7).

$$\begin{bmatrix} \begin{bmatrix} H_{1,1} & \cdots & H_{1,nu} \\ \vdots & & \vdots \\ H_{nu,1} & \cdots & H_{nu,nu} \end{bmatrix} & \begin{bmatrix} C_{1,1} & \cdots & C_{1,k} \\ \vdots & & \vdots \\ C_{nu,1} & \cdots & C_{nu,k} \end{bmatrix} \\ F_{1,1} & \cdots \quad F_{1,(nu+k)} \\ \vdots & \vdots \\ F_{k,1} & \cdots \quad F_{k,(nu+k)} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} I_{01} \\ \vdots \\ I_{0k} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} G_1 \\ \vdots \\ G_{nu} \end{Bmatrix} \\ \begin{Bmatrix} V_{01} \\ \vdots \\ V_{0k} \end{Bmatrix} \end{Bmatrix} \qquad (8)$$

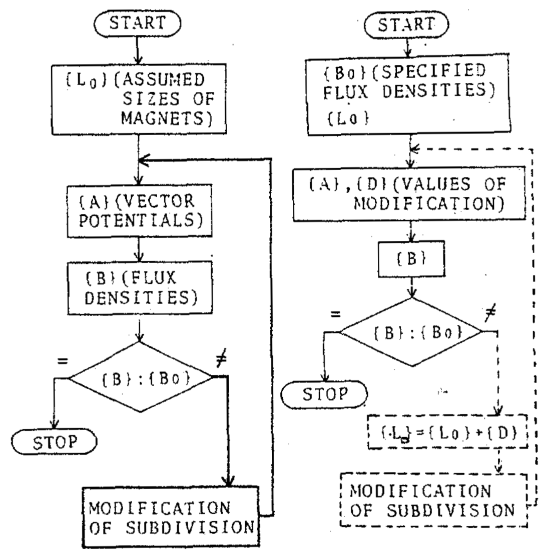
Although the matrix of (8) is nonsymmetrical, it is easily solvable by treating it as a banded matrix with edges.

4. FINITE ELEMENT METHOD WITH SHAPE MODIFICATION

Outline of Method

If the class B problem is solved by using the conventional finite element method, the process of calculation should be as Fig.6(a). The vector potentials are calculated using a temporary subdivision in which the sizes of magnets are adequately assumed. In order to obtain the sizes of magnets producing the specified flux density $\{B_0\}$, a number of repetitions are necessary until the desired results can be obtained by modifying the subdivision as shown by the thick lines in Fig.6(a).

By using the newly developed "finite element method with shape modification", the



(a) conventional method (b) new method

Fig. 6 Processes of calculation.

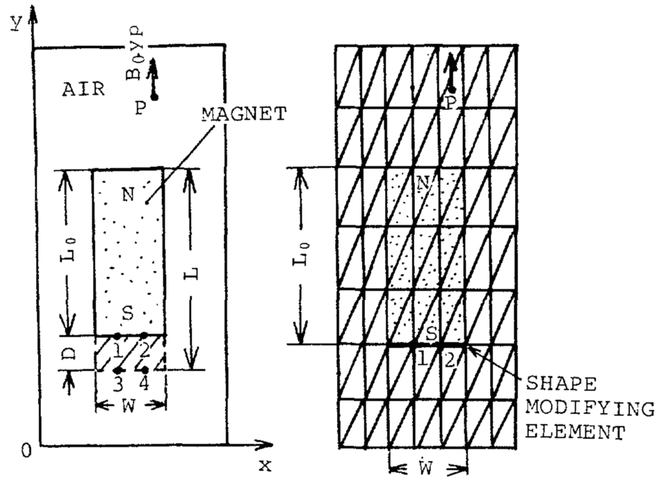
modified value {D} of sizes is directly obtained as denoted by the solid lines in Fig.6(b). The desired size {L} of the magnets is calculated by the following equation:

$$\{L\} = \{L_0\} + \{D\} \tag{9}$$

When the value {D} is very large, the result obtained without any repetition is not satisfactory because of the error due to the shape modifying element. In such a case, a few repetitions are necessary until the desired result can be obtained by modifying the subdivision by {D} as shown by the broken lines in Fig.6(b). The number of repetitions is much less than in the case of the conventional method.

Shape Modifying Element

When the sizes of magnets are unknown variables, (3) becomes a non-linear equation. If the equation is a linear function of the co-ordinates x and y, (3) can be solved easily. From this point of view, a new element which is called a "shape modifying element" is conceived. The new element is explained by an example shown in Fig.7(a). For simplicity, the width W and



(a) model (b) finite element subdivision

Fig. 7 Shape modifying element

the magnetization of the magnet are given. Lo and L of the magnet are the estimated and the modified lengths . Therefore, length D is unknown. As the number of the specified flux densities $\{B_0\}$ should be equal to the number of the unknown variables {D}, only the y- component Boyp of the flux density at a point P is specified for this example, because there is only one magnet in Fig. 7.

If the length D is small, the flux distribution in the hatched part is almost uniform in the y- direction. Then the following equations can be assumed among the vector potentials A_1 , ..., A_4 at the nodes 1, ..., 4 in Fig. 7(a) [3].

$$A_1 = A_3 \ , \ A_2 = A_4 \tag{10}$$

As the aim of this analysis is to obtain the size of the magnet producing the specified flux density, the accuracy of the flux distribution near the hatched area is not so important.

Figure 7(b) is the subdivision corresponding to Fig.7(a). The newly developed so-called "shape modifying element" is denoted by the thick line in Fig.7(b). It corresponds to the hatched area in Fig.7(a). If the shape modifying element has the same energy as the hatched area in Fig. 7(a), the nodes 3 and 4 may be superposed on the nodes 1 and 2 as shown in Fig 7(b).

The energy $\chi^{(e)}$ of the shape modifying element e is calculated by the following equation.

$$\chi^{(e)} = -\iint_{S^{(e)}} \left(\int_0^A J_m \, dA \right) dx\,dy \tag{11}$$

Where $S^{(e)}$ is the area of the element and J_m is the equivalent magnetic current density [4,5].

As the shape modifying element has no area, it is easy to set it on an arbitrary position of the subdivision without any re-subdivision.

Finite Element Formulation

(1) Rayleigh-Ritz matrix equation

From (10), (11) is rewritten as follows:

$$\chi^{(e)} = -D\int \left(\int_0^A J_m \, dA \right) dx \tag{12}$$

For simplicity, let us assume that the magnetization has only the y- component My. Then the following equation can be obtained from (12).

$$\frac{\partial \chi^{(e)}}{\partial A_i} = -\nu_0 D M_y \tag{13}$$

Where Ai is the vector potential of a node i.

If D in (13) is treated as an unknown variable, the matrix equation like (4) is obtained as follows:

$$\left[\begin{bmatrix} H_{1,1} & \cdots & H_{1,nu} \\ \vdots & & \vdots \\ H_{nu,1} & \cdots & H_{nu,nu} \end{bmatrix} \begin{bmatrix} C^*_{1,1} & \cdots & C^*_{1,r} \\ \vdots & & \vdots \\ C^*_{nu,1} & \cdots & C^*_{nu,r} \end{bmatrix} \right] \left\{ \begin{matrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} D_1 \\ \vdots \\ D_r \end{Bmatrix} \end{matrix} \right\} = \begin{Bmatrix} G_1 \\ \vdots \\ G_{nu} \end{Bmatrix} \tag{14}$$

Where [C*] is the coefficient matrix of {D}. Though there is only one magnet in Fig. 7, usually there are many magnets. The lengths D_1, ..., D_r of their shape modifying elements are denoted by a column matrix {D}. Where r is the number of the magnets.

In order to solve (14), the following new relationships between the vector potentials and the flux densities are introduced in the same way as in chapter 3.

(2) Relationships between vector potentials and flux densities

Figure 8 shows an first-order triangular element e. The x- and y- components Boxp and Boyp of the flux density at a point P in the element are specified.

The following relationships exist between the vector potentials and the flux densities [5].

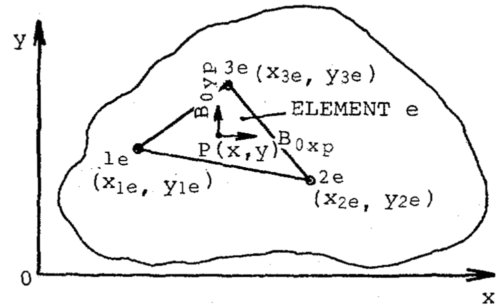


Fig. 8 An element whose flux density is specified.

$$\left.\begin{aligned} B_{0xp} &= \sum_{j=1}^{3} d_{je} A_{je}/2\Delta \\ B_{0yp} &= -\sum_{j=1}^{3} c_{je} A_{je}/2\Delta \end{aligned}\right\} \qquad (15)$$

Where c_{je}, d_{je} and Δ are constants. Then the following matrix equation corresponding to (7) is obtained.

$$\begin{bmatrix} F^*_{1,1} & \cdots & F^*_{1,(nu+r)} \\ \vdots & & \vdots \\ F^*_{r,1} & \cdots & F^*_{r,(nu+r)} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} D_1 \\ \vdots \\ D_r \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} B_{01} \\ \vdots \\ B_{0r} \end{Bmatrix} \qquad (16)$$

Where $[F^*]$ is the coefficient matrix of $\{A\}$ and $\{D\}$. B_{0i} is the specified i'th x- or y-component of flux density, and, for example, this corresponds to Boyp in Fig.7.

(3) Matrix equation for inverse problem

The unknown vector potential $\{A\}$ and the modified value $\{D\}$ of magnets can be directly calculated by the following equation obtained from (14) and (16).

$$\begin{bmatrix} \begin{pmatrix} H_{1,1} & \cdots & H_{1,nu} \\ \vdots & & \vdots \\ H_{nu,1} & \cdots & H_{nu,nu} \end{pmatrix} & \begin{pmatrix} C^*_{1,1} & \cdots & C^*_{1,r} \\ \vdots & & \vdots \\ C^*_{nu,1} & \cdots & C^*_{nu,r} \end{pmatrix} \\ \begin{pmatrix} F^*_{1,1} & \cdots & & F^*_{1,(nu+r)} \\ \vdots & & & \vdots \\ F^*_{r,1} & \cdots & & F^*_{r,(nu+r)} \end{pmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_{nu} \end{Bmatrix} \\ \begin{Bmatrix} D_1 \\ \vdots \\ D_r \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} G_1 \\ \vdots \\ G_{nu} \end{Bmatrix} \\ \begin{Bmatrix} B_{01} \\ \vdots \\ B_{0r} \end{Bmatrix} \end{Bmatrix} \qquad (17)$$

If the calculated value D is negative, the magnet should be shortened.

The case of linear equation and only one specified component of flux density is dealt in Fig.7. The case of non-linear equation and many specified components of flux density can also be analyzed.

In this paper, though only the value $\{D\}$ is unknown, the magnetization $\{M\}$ of magnets can also be unknown variables. But, in the latter case, the Rayleigh-Ritz matrix equation becomes a non-linear simultaneous equation. The details of this problem will be reported in another paper.

5. SOME EXAMPLES OF APPLICATION

Analysis of Capacitor Motor

As an example of the application of the "finite element method taking into account of external power source", flux distributions of a capacitor motor are analyzed. In this case, there are two relationships like (5) between the vector potentials, the currents and the terminal voltages. One is obtained for the main winding and the other is for the auxiliary winding.

Figure 9 shows the structure and dimensions of an analyzed capacitor

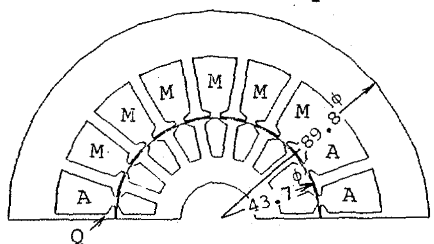


Fig. 9 Capacitor motor.

motor. This motor has 2-poles and the rated voltage and the input are 100(V) and 170(W) respectively. The core is made of 0.5mm thick non-oriented silicon steel (Grade : AISI, M-43). The rotor is die-cast. A capacitor whose capacitance is 11(μF) is connected to the auxiliary winding.

Figure 10 shows the flux distribution at slip S=0.05 (corresponding to the rated load). Figure 11 shows the flux density waveform at the stator teeth Q shown in Fig.9. Points denoted by o represent the calculated results and the solid line denotes the measured one.

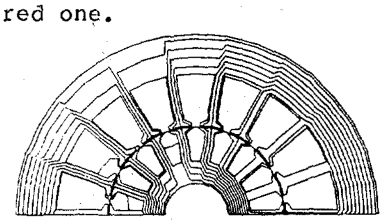


Fig.10 Flux distribution.

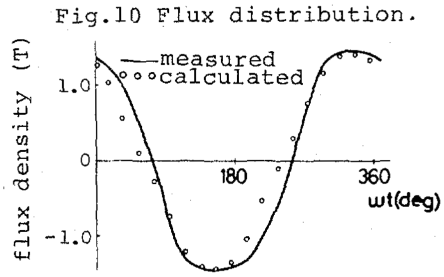


Fig. 11 Flux density waveform.

Determination of Magnet Shape

Figure 12 shows a magnetic circuit with two magnets. The magnets are anisotropic and both of the magnetizations are 1.0 (Wb/m^2). The estimated lengths Lo's are both 0.2(m). The lengths L_1 and L_2 of the magnets 1 and 2 which produce the y-directional flux densities B_{0y1} and B_{0y2} at

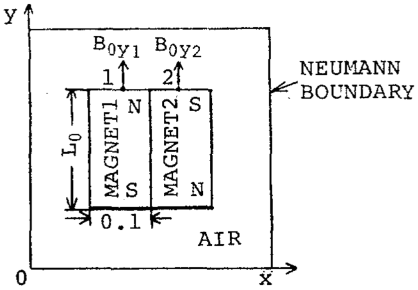
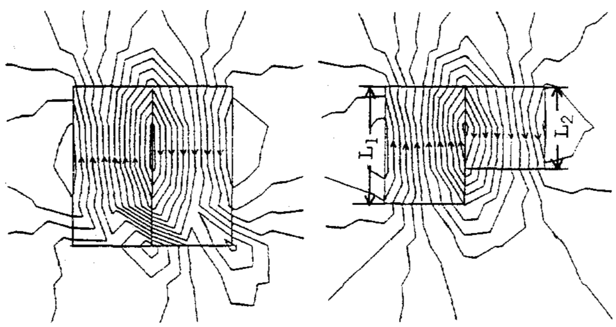


Fig. 12 Magnet model.

the points 1 and 2 on the surfaces of the magnets are calculated by setting the shape modifying elements at the edges of the magnets shown by the thick lines in Fig.12.

When B_{0y1} and B_{0y2} are specified to be 0.5 and 0.46(T), the calculated lengths L1 and L2 of the magnets are 0.147 and 0.103(m) respectively. Figure 13(a) shows the



(a) approximate calculation (b) exact calculation

Fig. 13 Flux distributions.

obtained flux distribution using the new method. The true flux distribution for the magnets the lengths of which are 0.147 and 0.103(m) is shown in Fig.13(b). The flux distribution around the shape modifying element in Fig.13(a) is different from that around the corresponding part in Fig.13(b). However, both flux distributions near the surfaces, the flux densities on which are specified, are almost the same .

6. Conclusions

The development of our new methods of analysis has made it easy to design practically the magnetic circuits using the finite element method.

We have already applied these methods to the analysis of the commutation of a universal motor [6], the characteristics of synchronous machines under load, and the optimum design of the magnetic circuit of the magnet roll in a copying machine [7].

The finite element method taking into account of external power source needs further investigation about (a) estimation of the impedance outside the finite element region, (b) improvement of the accuracy of this method.

The following problems should be investigated in order to establish the finite element method with shape modification:
(a) non-linear analysis, (b) modification of the width of magnet, (c) determination of the magnetization when the size of magnet is unknown, (d) accuracy of this method.

It is hoped that the range of application is enlarged, and the new method for class C problem will be developed.

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