DIRECT MEASUREMENTS OF THE INHERENT FREQUENCY STABILITY OF QUARTZ CRYSTAL RESONATORS

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Abstract

A technique is presented that allows one to measure directly the fluctuations of the natural resonant frequency of quartz crystal resonators in a passive circuit. This technique greatly aids in modeling the noise in both crystal resonators and crystal controlled oscillator circuits. Definite changes of slope in the spectral density of the frequency fluctuations, as a function of frequency offset from the natural resonant frequency of the crystals indicate that several mechanisms are contributing to the frequency instabilities in crystals. Our measurements also indicate that the electronics in the oscillators seriously degrade the frequency stability for sample times less than 100 s. The effects are especially dramatic for times less than 1 s.

Key words: Allan variance, Crystal controlled oscillator, Flicker of frequency modulation, Johnson noise, Linewidth, Quartz crystal resonators, Random walk frequency modulation, Spectral density of frequency fluctuations $S_y(f)$, White phase modulation.

Introduction

Crystal oscillators play a key role in frequency metrology. Nearly all precision frequency measurement and generation devices employ a crystal controlled oscillator in one form or another. Our measurements on the quartz resonators, which were made via a passive technique which we will describe shortly, indicate that the electronics used in the oscillators very seriously degrade the frequency stability for times shorter than 100 s. These measurement techniques should greatly aid in modeling the frequency stability of crystals, evaluating different oscillator circuits and in the selection of better ways to produce crystal resonators.

Measurement Techniques

The frequency stability of the crystal resonators has been measured in the system schematically shown in Figure 1. Two crystals which are as identical as possible are driven from the same low noise source. By careful adjustment of crystal tuning and the balancing of the relative Q's, the output from the mixer can be made first order insensitive to both amplitude and frequency modulation of the source. This helps reduce the system noise due to the source and permits one to measure small time varying frequency deviations of the crystals. This can be seen from analyzing the equivalent circuit shown in Figure 2. The amplitude of the voltage appearing at the mixer input from either crystal, labeled Vout, is proportional to the following frequency dependent terms.

$$V_{out} = K V_{in} \left(1 - i \frac{R_S}{R_S + R_L} \frac{\Delta \omega}{\gamma_x} \right) \frac{1}{\omega C_L} \quad (1)$$

Under the assumption that the resonant frequency of the crystal is ω_o which is also equal to the series resonance frequency of the external LC circuit, that the driving source frequency is equal to to ω_o plus a small frequency deviation $\Delta\omega$ and that $\Delta\omega/\gamma_x$ is very much less than 1 where $\omega_o/2\gamma_x$ is the unloaded Q of the crystal. Further we assume that γ_L is much larger than γ_x where $\omega_o/2\gamma_L$ is the unloaded Q of the external LCR circuit. The final assumption is that $\frac{1}{\omega C_1}$, the impedance due to the crystal holder, is much larger than R_S which is larger than R_L. One notes that there is a phase shift across the circuit which is proportional to the frequency difference between the source and the natural resonant frequency of the crystal. This phase shift under the above assumptions is just $\theta = \Delta\omega/\gamma_x$. Identical results are of course obtained for the other crystal. The 90° phase shifter was made from a variable length delay line. The double balance mixer is used as a phase detector and produces a dc voltage which is proportional to the instantaneous phase difference from 90° between the two signals. The sensitivity was typically three mV per degree, which translates to 170 mV per 1 Hz change in the natural resonant frequency of either quartz resonator, assuming both have a bandwidth of 2 Hz, i.e. $V = \theta(.003) = \frac{\Delta\omega}{\gamma} \frac{360}{2\pi} (.003)$, therefore $V = .170$ V for $\frac{\Delta\omega}{2\pi} = 1$ Hz and $\frac{\gamma}{2\pi} = 1$ Hz.

The signal from the mixer is then processed for either frequency or time domain data. Spectral analysis of the mixer output yields $\sqrt{S_y(f)}$, $S_y(f)$ being the spectral density of frequency fluctuations.

The time domain data is taken using a voltage to frequency converter and a computing counter. The average frequency measured by the computing counter during a sample time τ is proportional to the average frequency deviation between the two crystals during the sample time τ. By taking many consecutive measurements at each sample time τ, one can compute the Allan variance $\sigma_y^2(\tau)$ in the usual way.

Results

Figure 3 shows an example of frequency domain data for a 10 MHz crystal. $S_y(f)$ is plotted versus frequency offset f. Note that the noise has a flicker of frequency character for frequencies less than 1/2 the bandwidth ($S_y(f) = h_{-1}f^{-1}$) and the random walk frequency modulation character for frequencies larger than 1/2 the bandwidth, ($S_y(f)=h_{-2}f^{-2}$).

Figure 4 shows the time domain data for this same 10 MHz crystal pair and also the frequency domain data converted to time domain [1]. The most astonishing thing here is that the frequency domain data indicates that the stability of the crystal actually improves as $\tau^{\frac{1}{2}}$ for times shorter than the inverse half bandwidth, which indicates that the frequency of the crystal cannot change rapidly compared to the inverse bandwidth.

Frequency domain measurements on a low-Q 5 MHz crystal pair are shown in Figure 5. Again, a similar result, namely that $S_y(f)$ goes as f^{-1} until f is equal to 1/2 the bandwidth thereafter $S_y(f)$ goes as f^{-2}. This indicates again that inside the bandwidth one has a flicker of frequency behavior and outside the bandwidth it goes as a random walk frequency modulation. This data indicates that the time domain stability of this crystal pair varies as $\tau^{\frac{1}{2}}$ for $\tau \le \frac{1}{\gamma_x}$

Unfortunately, it is not easy to take time domain data at a sample time of 10^{-4} seconds to verify this. Instead, let us examine the data from a high-Q crystal pair where the inverse half bandwidth is much larger. Figure 6 shows the frequency domain data from a 5 MHz crystal pair with a bandwidth of 2 Hz. The transition from f^{-1} to f^{-2} behavior should occur at 1 Hz which is the lower limit of the spectrum analyzer used. Note that $S_y(f)$ goes as f^{-2} out to at least 50 Hz so that one should expect $\sigma_y(\tau)$ to go as the square root of τ at least down to times of order $(2\pi 50)^{-1}$ or 3 ms or smaller. Figure 7 shows the time domain data for this crystal pair along with the frequency domain data transferred to time domain.

The increase in stability for times less than the inverse 1/2 bandwidth is confirmed by these measurements. As one goes to shorter and shorter times the stability becomes worse again due to noise in the isolation amplifier and the measurement system. This noise causes an uncertainty in the measurement of the position of the zero crossing. If the noise is white then the frequency fluctuations will have a white phase modulation character and $\sigma_y(\tau)$ will go as τ^{-1} down to times of order one over the bandwidth of the measurement system. For times smaller than this the phase noise is reduced by the bandwidth limiting mechanism and the stability should again improve. The line labeled "Johnson noise from amplifier" indicates the estimated contribution of our measurement system to $\sigma_y(\tau)$, in fact for nearly all circuits the electronic noise dominates over the crystal noise at very short times. For comparison the stability of these same two crystals in a high performance crystal oscillator is also indicated in Figure 7. Note the dramatic difference in stability for times less than 1 s. This vividly illustrates the power of this technique for evaluating crystal controlled oscillator circuits. We believe that it is the additive noise after the oscillator which causes the τ^{-1} behavior for times less than 1 s in the oscillators [2]

Measurements on five crystal pairs indicate that the flicker level is roughly proportional to Q^{-1}. That is, high Q crystals generally have a correspondingly better stability. However, crystals with the same Q can vary as much as a factor of 10 in stability which indicates that the fabrication techniques can make a considerable difference, for example, mounting, cut of the crystal, etc. Some measurements versus temperature were also made. The 5B-2 crystal pair demonstrated a factor of 2 higher stability at 25° C than at either -75°C or + 65°C. Another 5 MHz crystal pair with the same Q was a factor of 2 more stable at 65° than at 25° achieving a level about 2 times more stable than even the 5B-2 crystal pair illustrated in Figure 7. The improvement of oscillator stability to the level characteristic of crystal resonators would make possible a number of advances in frequency metrology. One of the most exciting ones is the multiplication from a crystal source to the infrared with orders of magnitude reduction in the linewidth over previous sources. Preliminary calculations indicate that a linewidth less than 3 Hz at .9 THz should be possible [3]. This is the frequency of the HCN laser and the first step in the well known chain used to measure the frequency of the methane stabilized lasers. Such an improvement would of course greatly improve the transfer of frequency information and stability between the infrared and rf regions of the spectrum. Other applications include improved short term stability for frequency standards, improved short term measurement stabilities for counters, etc.

Acknowledgements

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REFERENCES

[1] The translation from time domain to frequency domain can be done whenever $S_y(f)$ or $\sigma_y(\tau)$ have a simple power law dependence over a considerable extent in f or τ. See James A. Barnes, et.al. "Characterization of Frequency Stability," IEEE Trans. on I&M 20, 105 (1971).

[2] The crystal drive in the passive circuit was approximately 250μW while it was approximately 1μW in the active oscillator. As a consequence of this, the Johnson noise is proportionately a much larger fraction of the signal from the crystal in the active oscillator than in the passive circuit. Therefore, one expects the active oscillator to exhibit a much larger instability due to this effect.

The short term stability data from a crystal controlled oscillator with approximately 50μW of crystal drive given in the paper by J. Groslambert, G. Marianneau, M. Olivier and J. Uebersfeld is in very good agreement with this interpretation. Note that one has to include the contributions of Johnson noise sources in the oscillator stage which are not filtered by the crystal prior to driving the amplifier stage. Their results showed an improvement of approximately 15dB over conventional oscillators with 1μW of crystal drive.

[3] The mathematics are outlined by D. Halford in the Proceedings of the Frequency Standards and Metrology Seminar. Available from the Quantum Electronics Laboratory, Dept. of Elec. Engr., Laval University, Quebec, Canada.

PASSIVE CRYSTAL MEASUREMENT SYSTEM

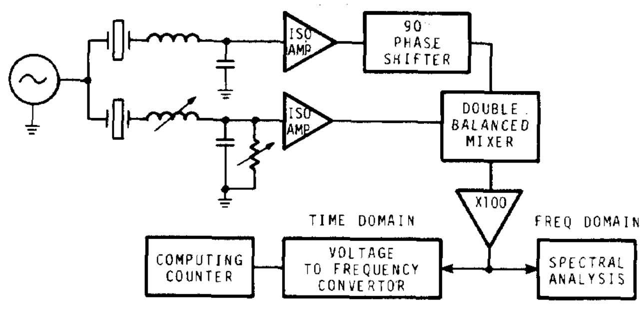
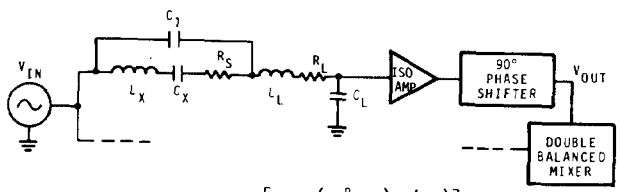


Figure 1

EQUIVALENT CIRCUIT



$$V_{OUT} = K\,V_{IN}\left[1 - i\left(\frac{R_S}{R_S + R_L}\right)\left(\frac{\Delta\omega}{\gamma_X}\right)\right]\frac{1}{\omega C_L}$$

ASSUMPTIONS

1. $\frac{1}{L_X C_X} = \omega_o^2$, $\frac{1}{L_L C_L} \doteq \omega_o^2$
2. $\omega_{SOURCE} = \omega_o + \Delta\omega$
3. $\frac{\Delta\omega}{\gamma_X} << 1$ WHERE $\frac{\omega_o}{2\gamma_X}$ = UNLOADED Q OF THE CRYSTAL
4. $\gamma_L >> \gamma_X$ $\frac{\omega_o}{2\gamma_L}$ = UNLOADED Q OF THE L_L, C_L, R_L CIRCUIT
5. $\frac{1}{\omega C_1} > R_S > R_L$

Figure 2

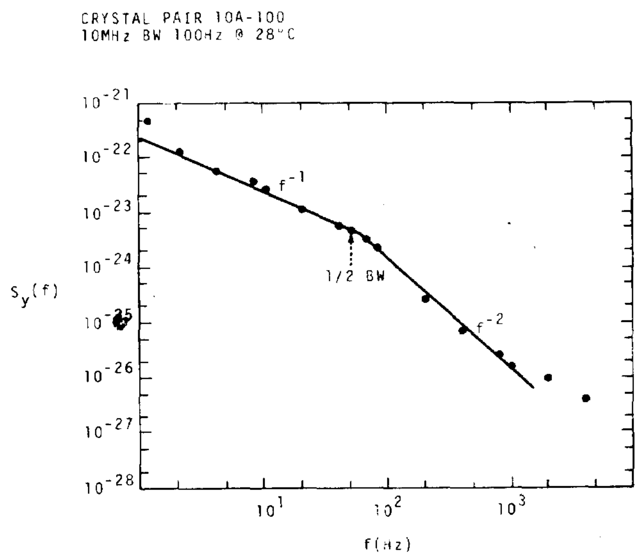


Figure 3

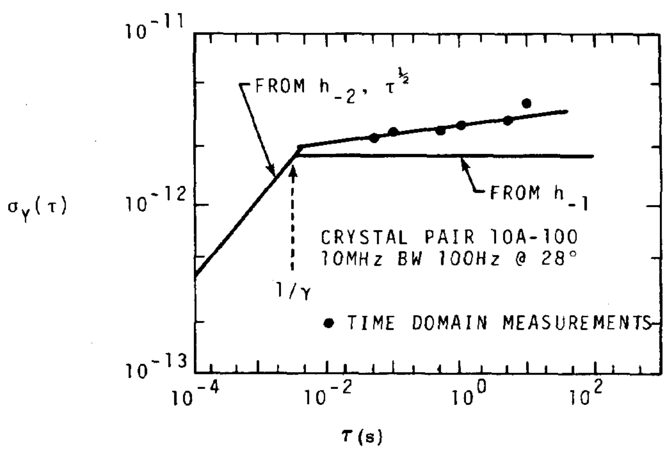


Figure 4

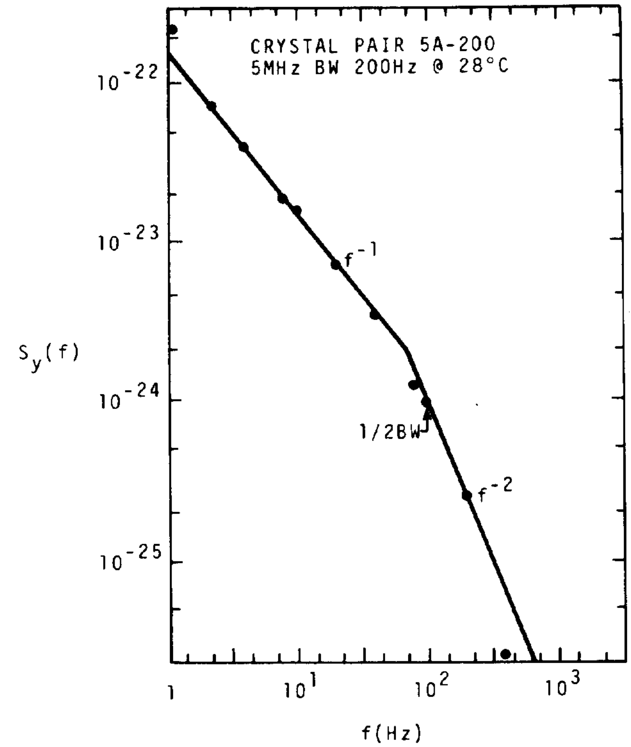


Figure 5

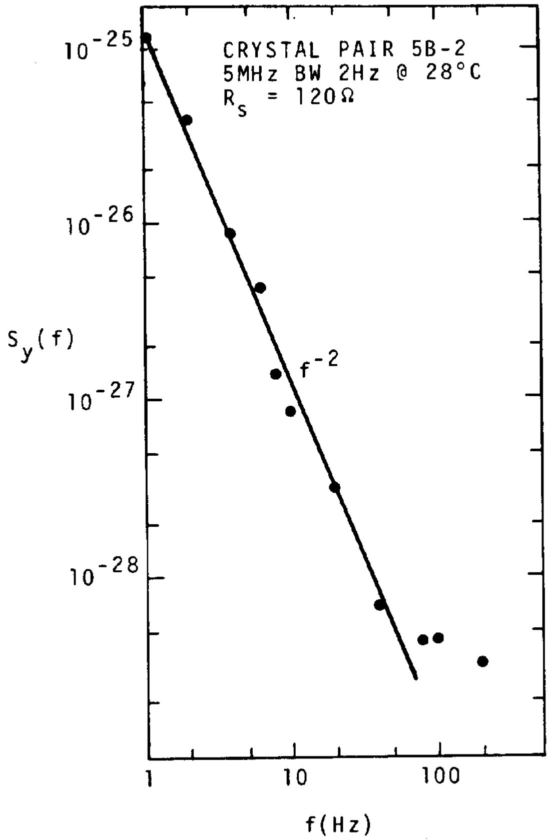


Figure 6

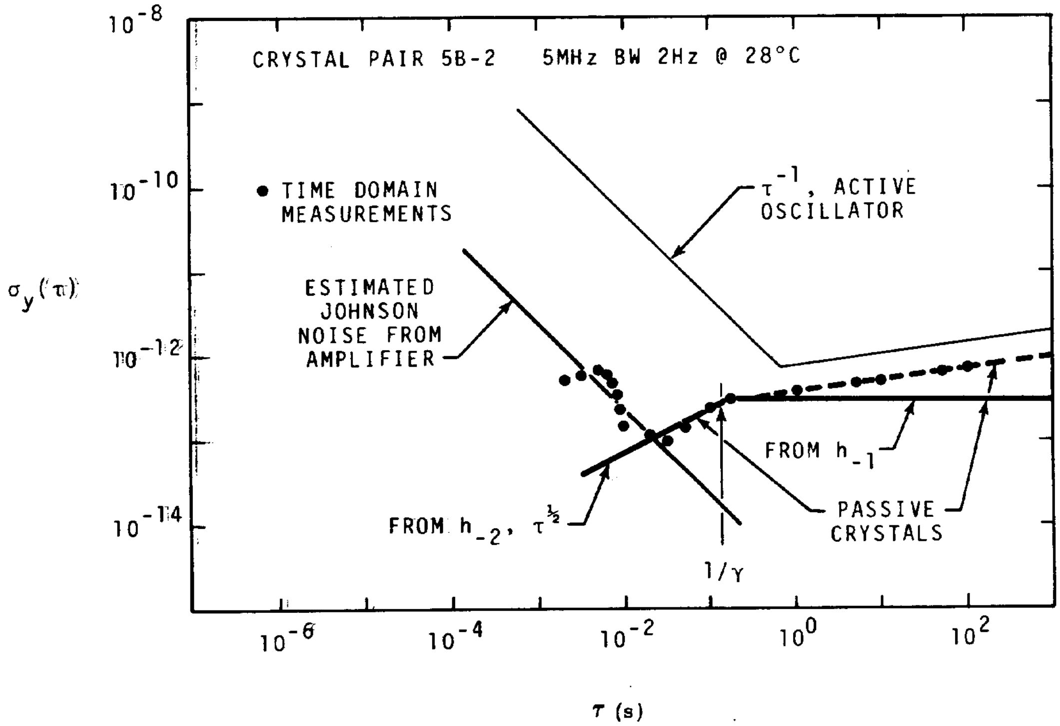


Figure 7