Direct Multisearch for Multiobjective Optimization

Ana Luísa Custódio[1] José F. Aguilar Madeira[2]

A. Ismael F. Vaz[3] Luís Nunes Vicente[4]

[1]Universidade Nova de Lisboa [2]IDMEC-IST, ISEL

[3]Universidade do Minho [4]Universidade de Coimbra

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Outline

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1. Introduction and motivation
2. Direct MultiSearch
3. Numerical results
4. Further improvements on DMS
5. Conclusions and references

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Derivative-free multiobjective optimization

MOO problem

$$\min_{x \in \Omega} F(x) \equiv (f_1(x), f_2(x), \ldots, f_m(x))^\top$$

where

$$\Omega = \{x \in \mathbb{R}^n : \quad \ell \leq x \leq u\}$$

$f_j : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}, _{j=1,\ldots,m}$, $\ell \in (\mathbb{R} \cup \{-\infty\})^n$ and $u \in (\mathbb{R} \cup \{+\infty\})^n$

- Several objectives, often conflicting.
- Functions with unknown derivatives.
- Expensive function evaluations, possibly subject to noise.
- Impractical to compute approximations to derivatives.

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DMS algorithmic main lines

- Does not aggregate any of the objective functions.
- Generalizes ALL direct-search methods of directional type to MOO.
- Makes use of search/poll paradigm.
- Implements an optional search step (only to disseminate the search).
- Tries to capture the whole Pareto front from the polling procedure.
- Keeps a list of feasible nondominated points.
- Poll centers are chosen from the list.
- Successful iterations correspond to list changes.

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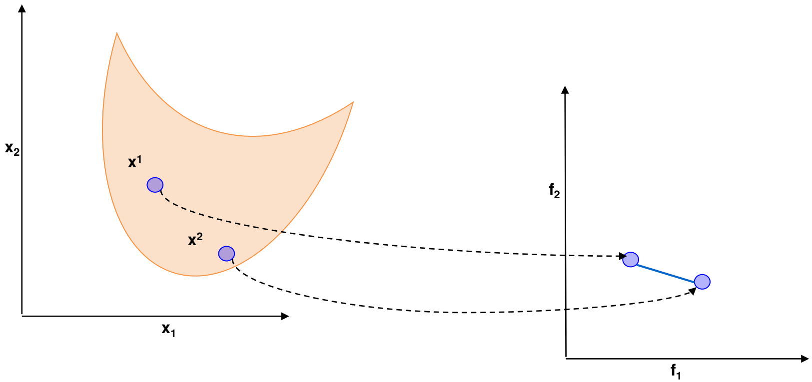

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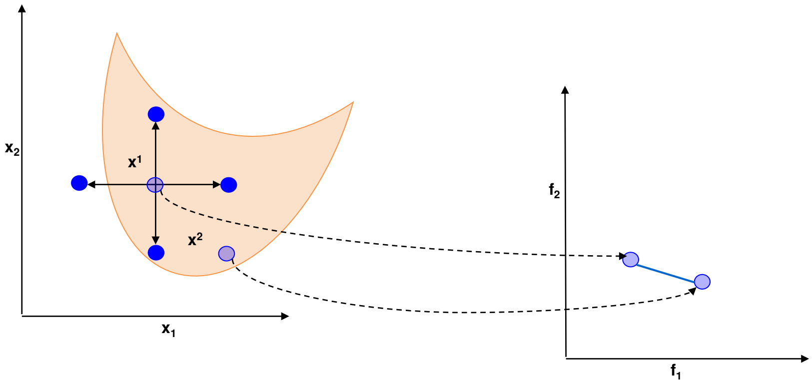

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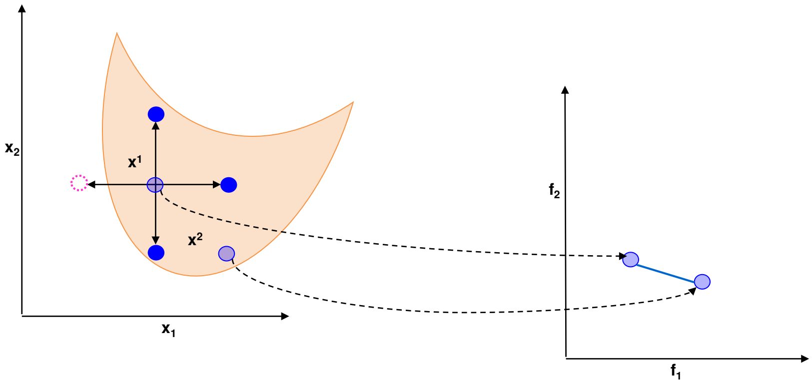

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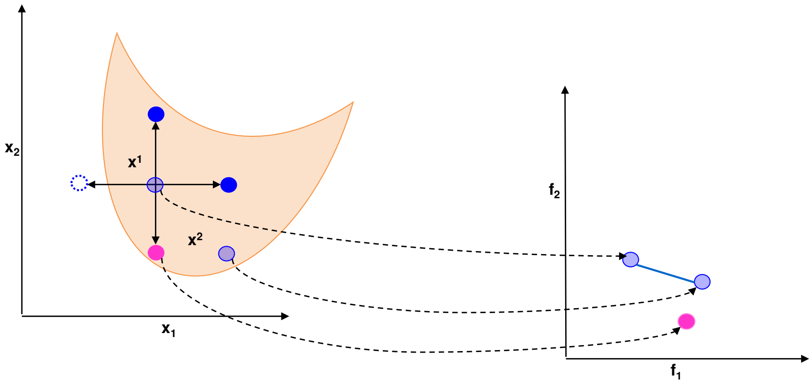

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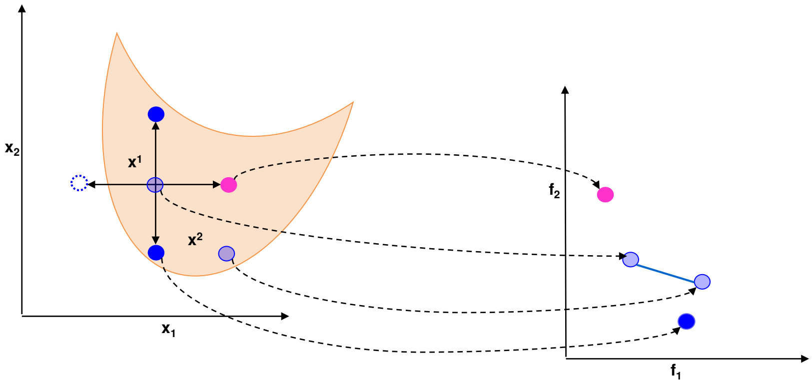

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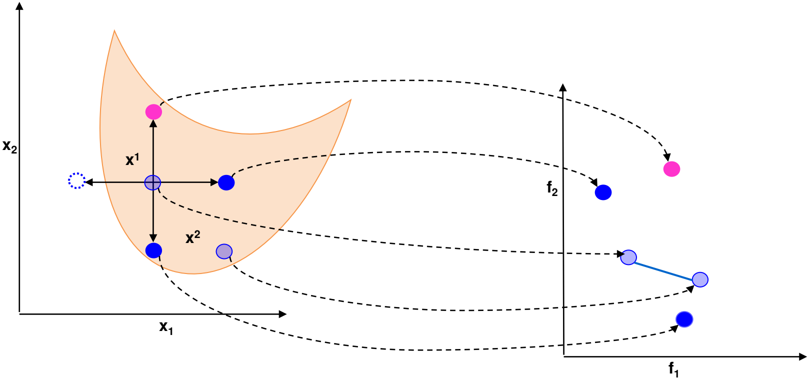

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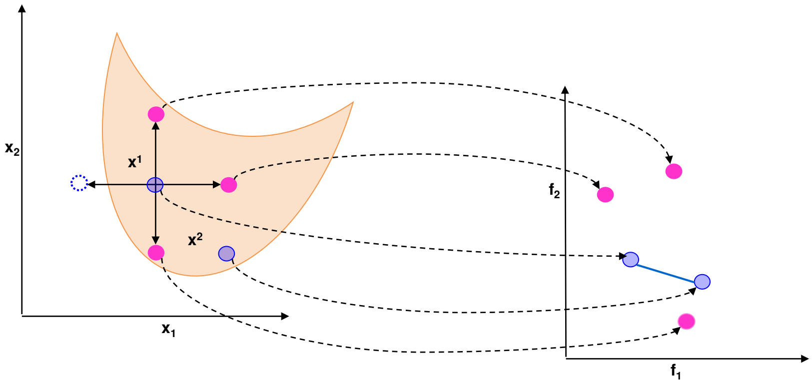

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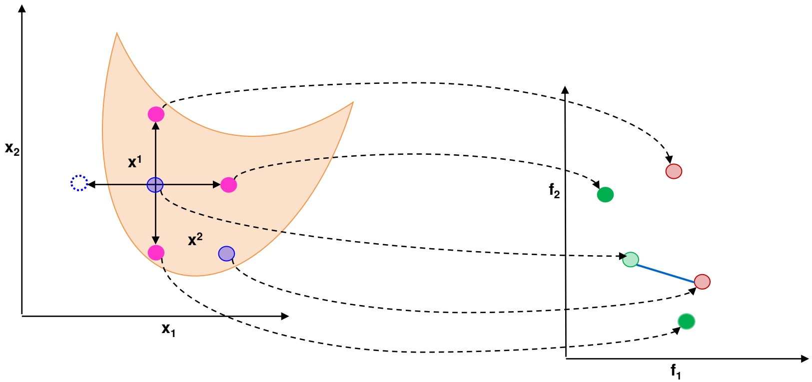

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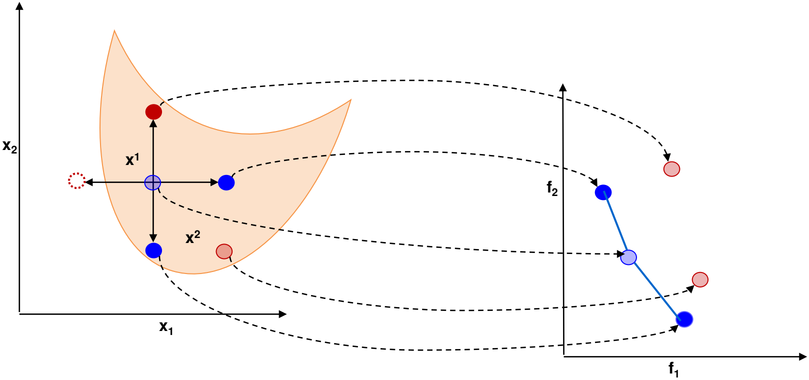

DMS search & poll steps

- At each iteration considers a list of feasible nondominated points $\hookrightarrow L_k$
- Evaluate a finite set of feasible points $\hookrightarrow L_{add}$.
- Remove dominated points from $L_k \cup L_{add} \hookrightarrow L_{filtered}$.
- Select list of feasible nondominated points $\hookrightarrow L_{trial}$.
- Compare L_{trial} to L_k (success if $L_{trial} \neq L_k$, unsuccess otherwise).

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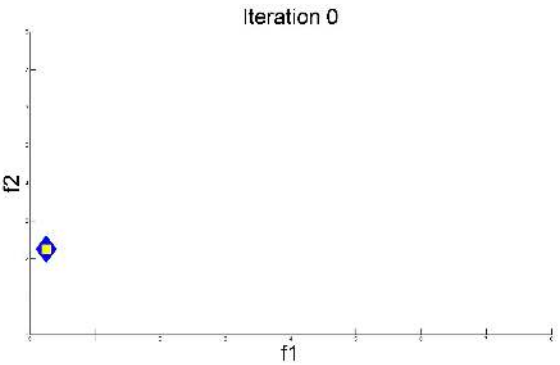

- ♦ Evaluated points since beginning.
- ■ Current iterate list.

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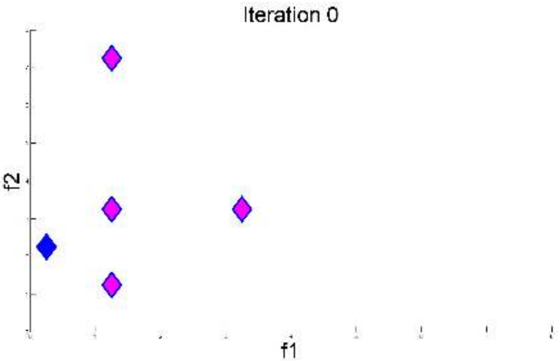

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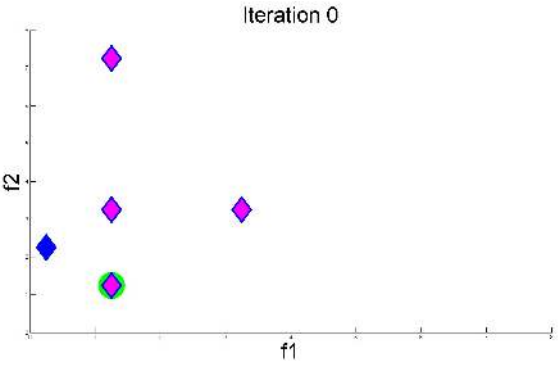

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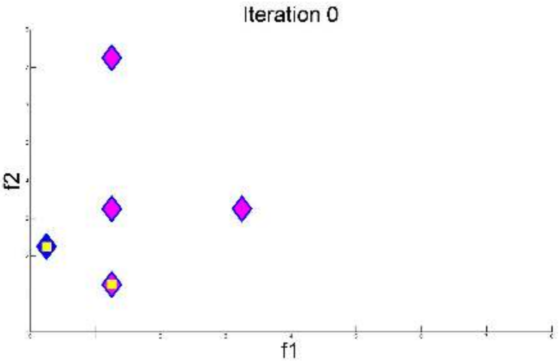

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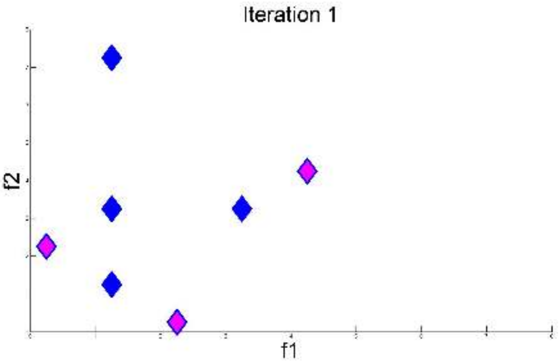

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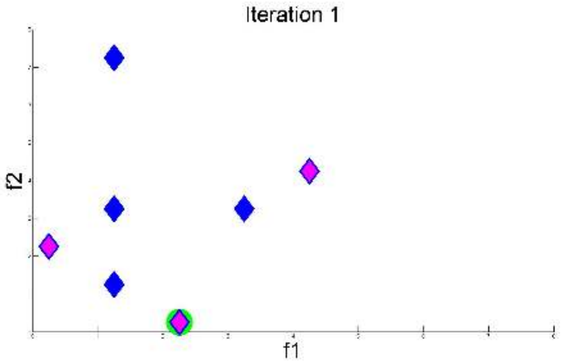

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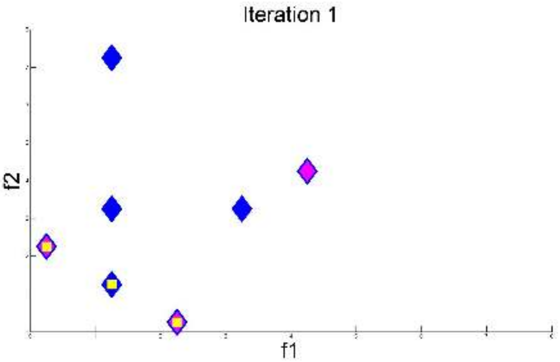

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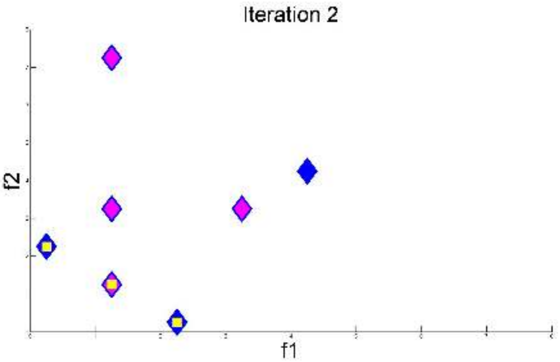

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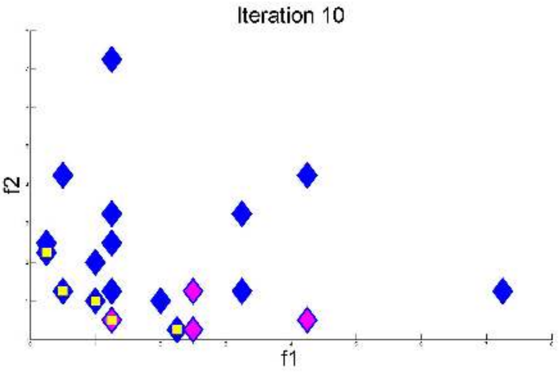

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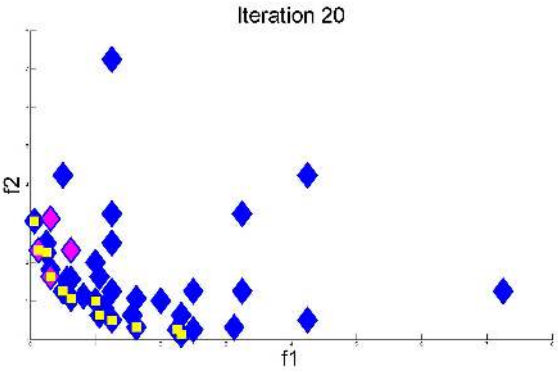

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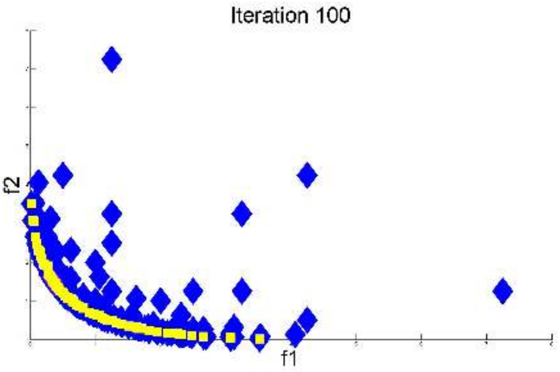

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Refining subsequences and directions

For both globalization strategies (using the mesh or the forcing function in the search step), one also has:

Theorem (existence of refining subsequences)

There is at least a convergent subsequence of iterates $\{x_k\}_{k \in K}$ corresponding to unsuccessful poll steps, such that $\alpha_k \longrightarrow 0$ in K.

Definition

Let x_* be the limit point of a convergent refining subsequence.

Refining directions for x_* are limit points of $\{d_k/\|d_k\|\}_{k \in K}$ where $d_k \in D_k$ and $x_k + \alpha_k d_k \in \Omega$.

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Pareto-Clarke critical point

Let us focus (for simplicity) on the unconstrained case, $\Omega = \mathbb{R}^n$.

Definition

x_* *is a Pareto-Clarke critical point of* F *(Lipschitz continuous near* x_**) if*

$$\forall d \in \mathbb{R}^n, \exists j = j(d), f_j^\circ(x_*; d) \geq 0.$$

Analysis of DMS

Assumption

- $\{x_k\}_{k \in K}$ refining subsequence converging to x_*.
- F Lipschitz continuous near x_*.

Theorem

If v is a refining direction for x_* then

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Convergence analysis of DMS

Theorem

If the set of refining directions for x_* is dense in $\mathbb{R}^n$, then x_* is a Pareto-Clarke critical point.

Notes

- When $m = 1$, the presented results coincide with the ones reported for direct search.
- This convergence analysis is valid for multiobjective problems with general nonlinear constraints.

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Numerical testing framework

Problems

- 100 bound constrained MOO problems (AMPL models available at http://www.mat.uc.pt/dms).
- Number of variables between 1 and 30.
- Number of objectives between 2 and 4.

Solvers

- DMS tested against 8 different MOO solvers (complete results available at http://www.mat.uc.pt/dms).
- Results reported only for
  AMOSA – simulated annealing code.
  BIMADS – based on mesh adaptive direct search algorithm.
  NSGA-II (C version) – genetic algorithm code.

All solvers tested with default values.

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DMS numerical options

- No search step.
- List initialization: sample along the line ℓ–u.
- List selection: all current feasible nondominated points.
- List ordering: new points added at the end of the list, poll center moved to the end of the list.
- Positive basis: $[I \ \ -I]$.
- Step size parameter: $\alpha_0 = 1$, halved at unsuccessful iterations.
- Stopping criteria: minimum step size of 10^{-3} or a maximum of 20000 function evaluations.

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Performance metrics — Purity

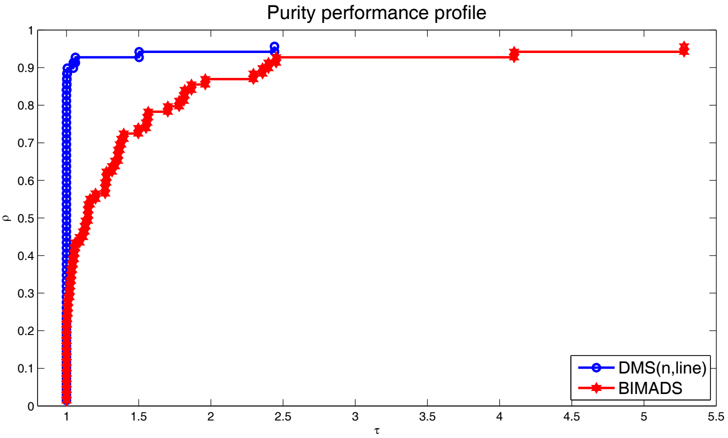
$F_{p,s}$ (approximated Pareto front computed by solver s for problem p).

F_p (approximated Pareto front computed for problem p, using results for all solvers).

Purity value for solver s on problem p:

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}.$$

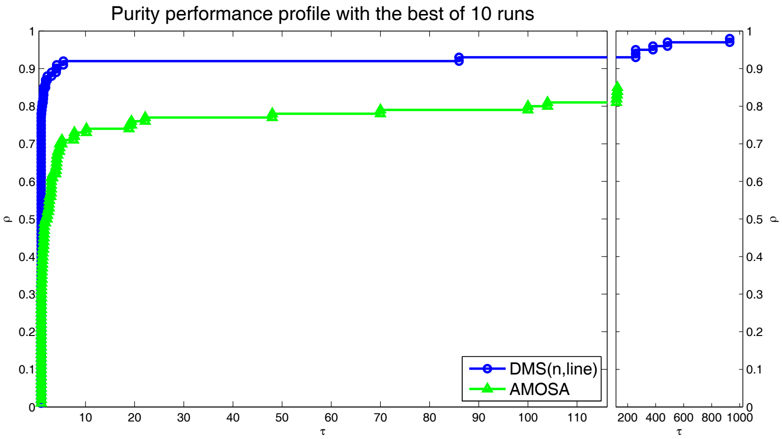
Comparing DMS to other solvers (Purity)



Purity Metric (percentage of points generated in the reference Pareto front)

$$t_{p,s} = \frac{|F_{p,s}|}{|F_{p,s} \cap F_p|}$$

Comparing DMS to other solvers (Purity)



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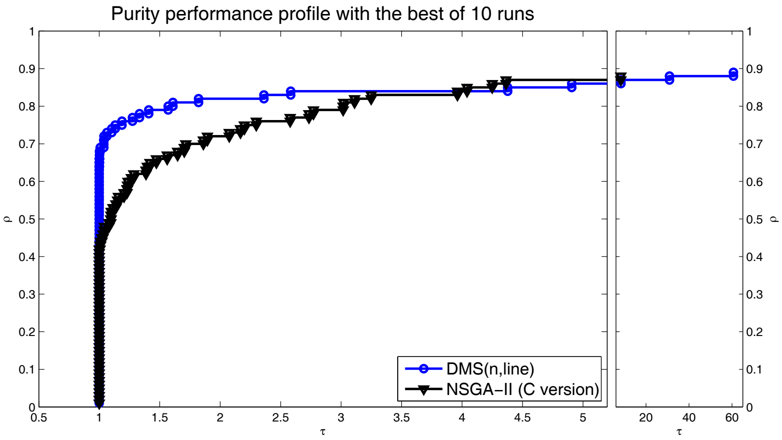


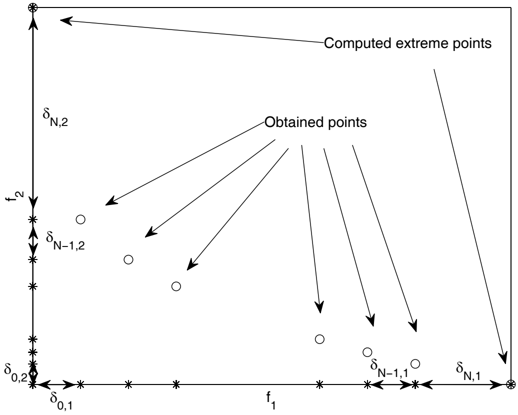
Purity Metric (percentage of points generated in the reference Pareto front)

$$t_{p,s} = \frac{|F_{p,s}|}{|F_{p,s} \cap F_p|}$$

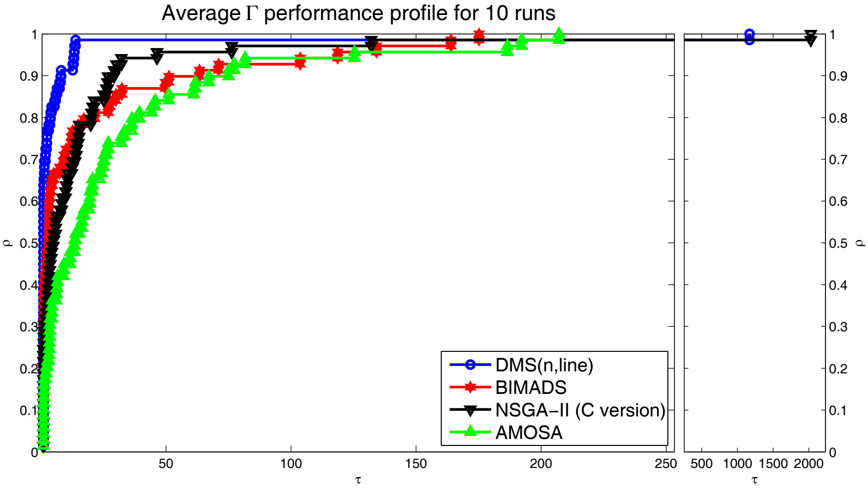
Performance metrics — Spread

Gamma Metric (largest gap in the Pareto front)

$$\Gamma_{p,s} = \max_{j \in \{1,\ldots,m\}} \left(\max_{i \in \{0,\ldots,N\}} \{\delta_{i,j}\} \right)$$



Comparing DMS to other solvers (Spread)



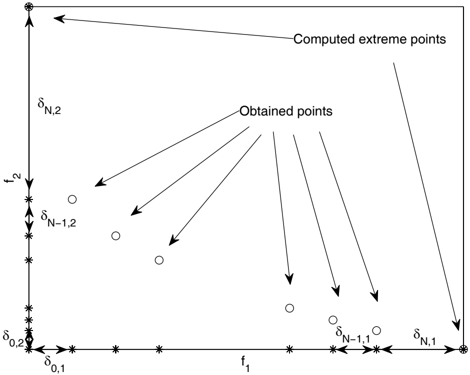
Gamma Metric (largest gap in the Pareto front)

Performance metrics — Spread

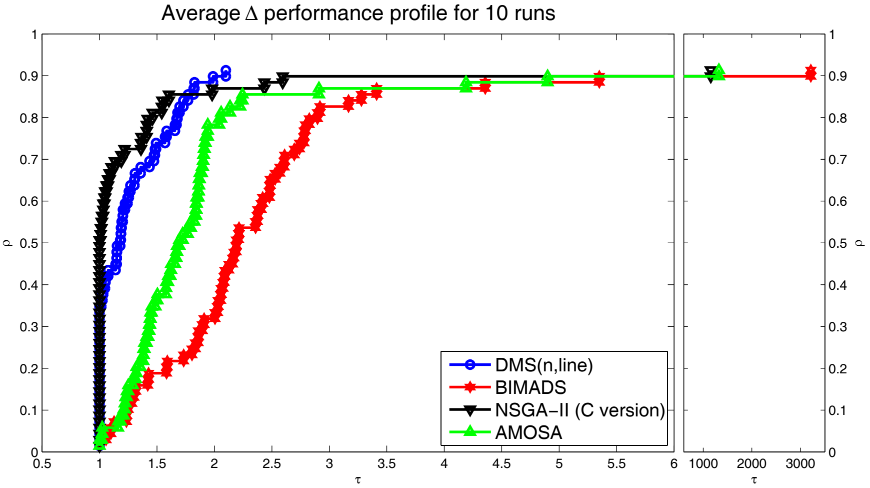
Delta Metric (uniformity of gaps in the Pareto front)

$$\Delta_{p,s} = \max_{j\in\{1,\ldots,m\}} \left(\frac{\delta_{0,j} + \delta_{N,j} + \sum_{i=1}^{N-1} |\delta_{i,j} - \bar{\delta}_j|}{\delta_{0,j} + \delta_{N,j} + (N-1)\bar{\delta}_j} \right)$$

where $\bar{\delta}_j$, for $j = 1, \ldots, m$, is the $\delta_{i,j}$'s average.

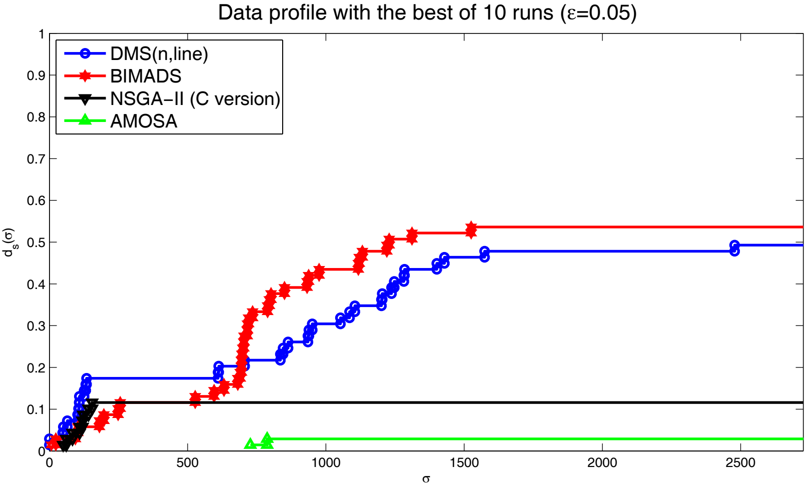


Comparing DMS to other solvers (Spread)



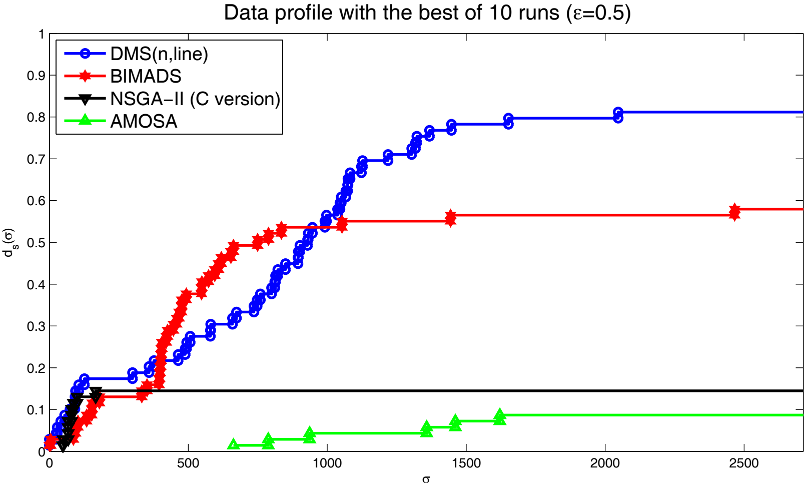
Delta Metric (uniformity of gaps in the Pareto front)

Comparing DMS to other solvers



maximum function evaluations = 5000

Comparing DMS to other solvers



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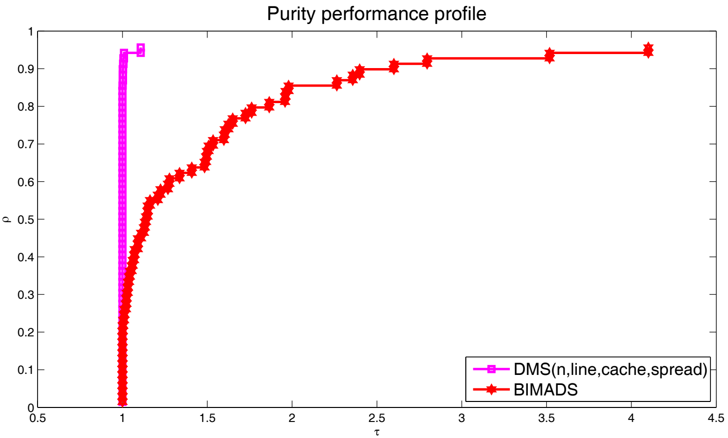
Outline

Comparing DMS to other solvers (Purity)



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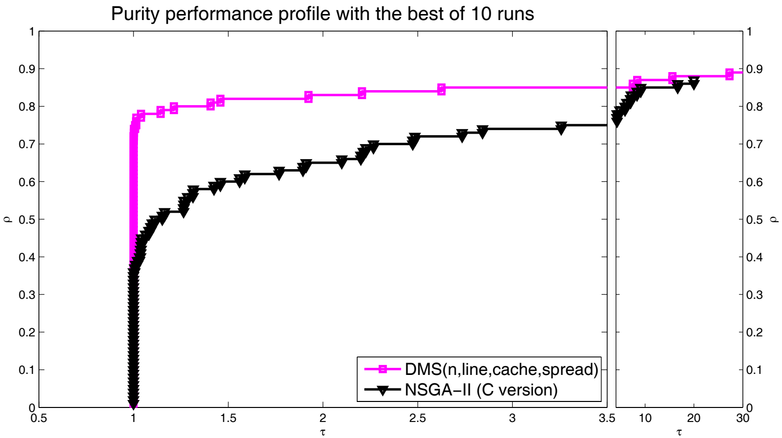

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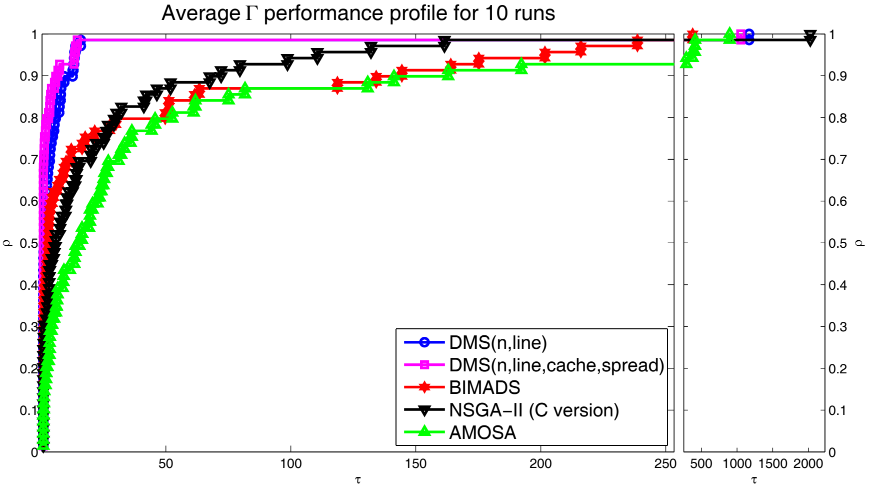

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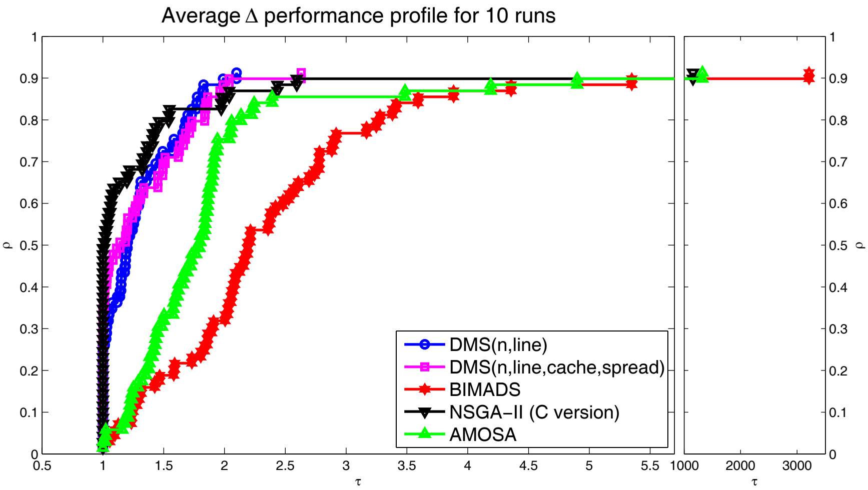
Comparing DMS to other solvers (Spread)



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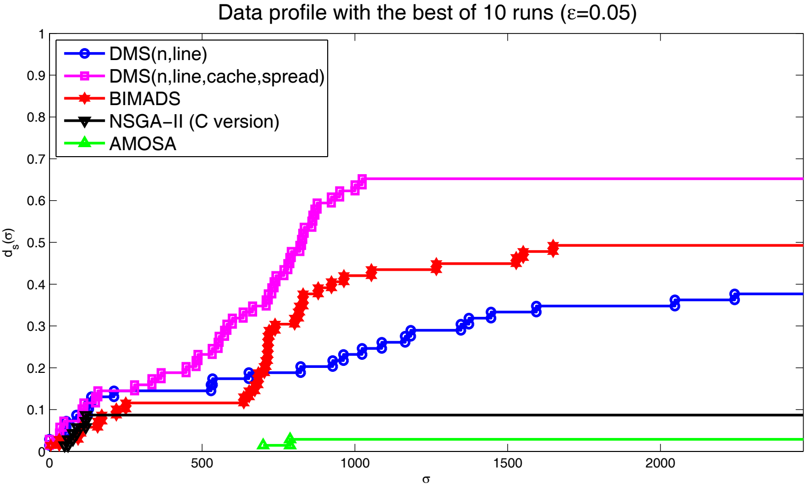
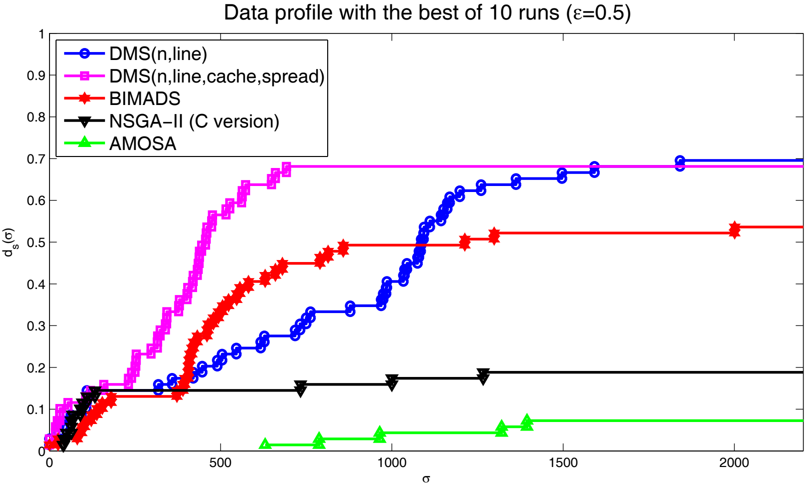
Comparing DMS to other solvers



maximum function evaluations = 5000

Comparing DMS to other solvers



maximum function evaluations = 5000

Outline

Conclusions and references

- Development and analysis of a novel approach (Direct MultiSearch) for MOO, generalizing ALL direct-search methods.

- Direct MultiSearch (DMS) exhibits highly competitive numerical results for MOO.

DMS (Matlab implementation) and problems (coded in AMPL) freely available at: `http://www.mat.uc.pt/dms`.

A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente, Direct multisearch for multiobjective optimization, to appear, SIAM Journal on Optimization.

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