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Direct Numerical Simulation of Turbulent Wall Flows at Constant Power Input

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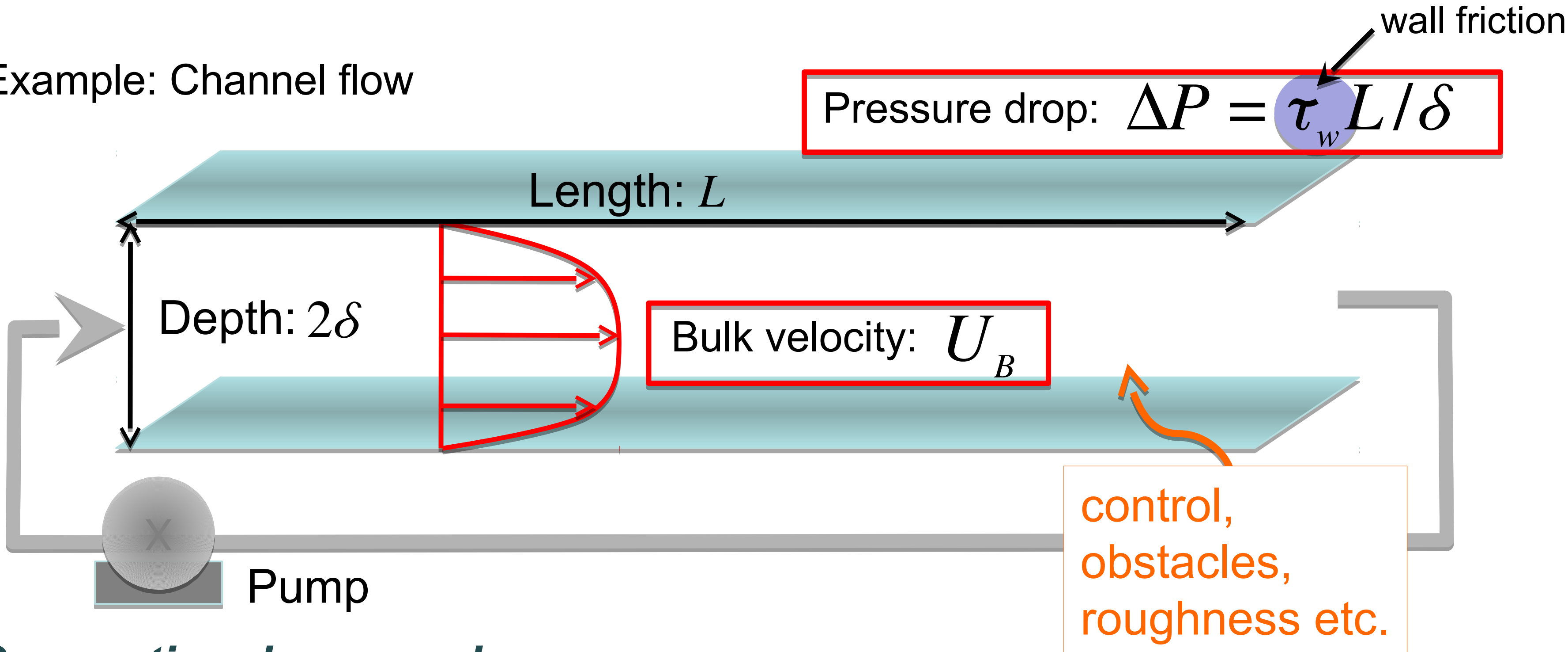
Flow Condition in Numerical Simulation

Example: Channel flow



Conventional approaches

✓ **Constant Flow Rate (CFR)**: *pressure drop (wall friction)* fluctuates in time

Successful Control **Reduction of pressure drop**

✓ **Constant Pressure Gradient (CPG)**: The flow rate fluctuates in time

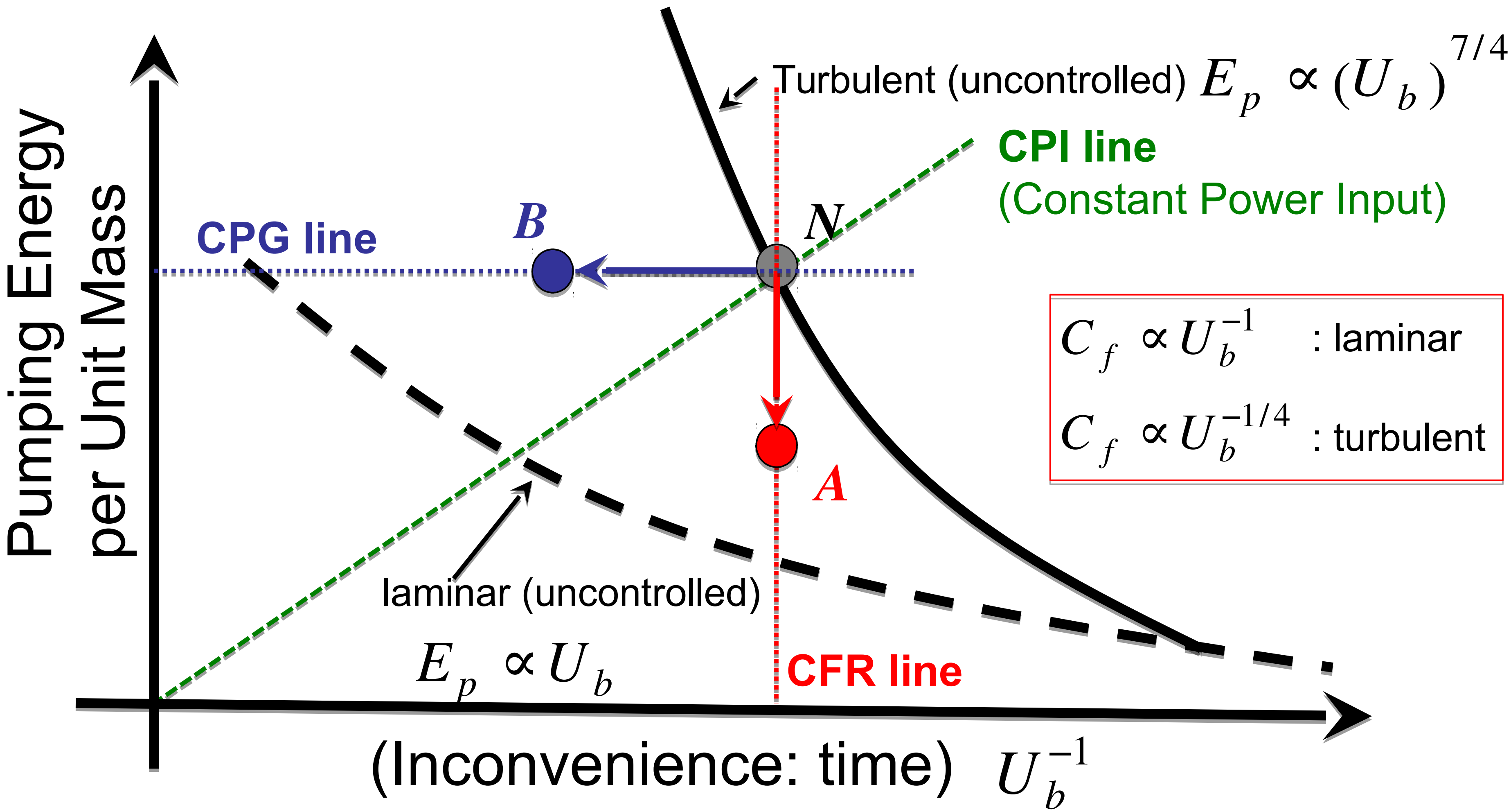
Successful Control **Increase of flow rate**

Are they the only available options ? No !

Money versus Time (Frohnapfel, Hasegawa & Quadrio, JFM 2012)

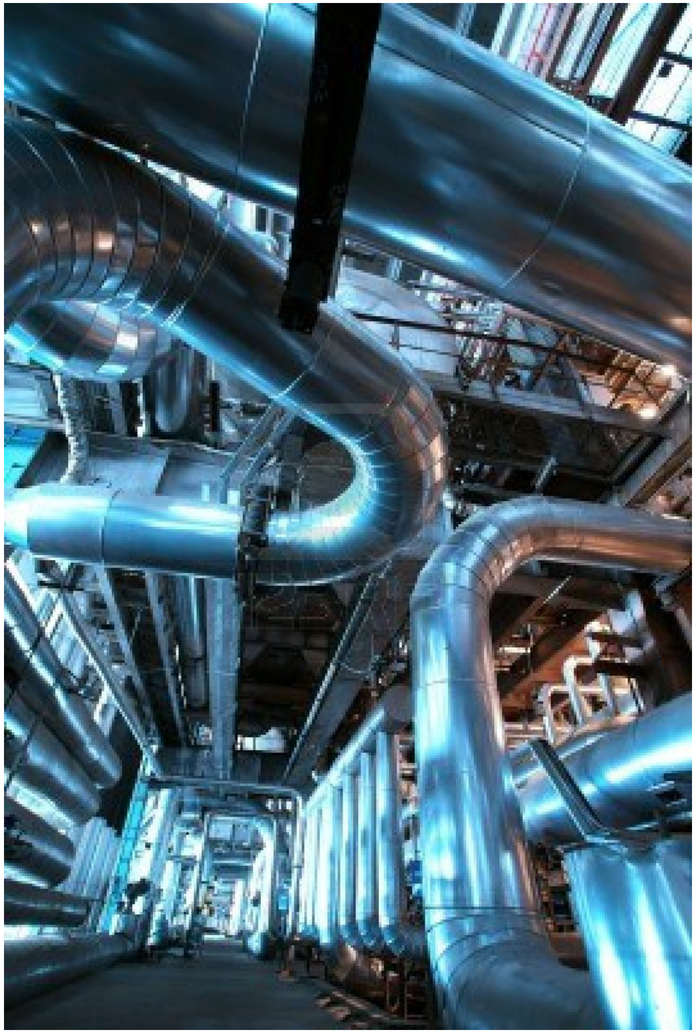
Flow control problem

compromise between *convenience* and *energy consumption*

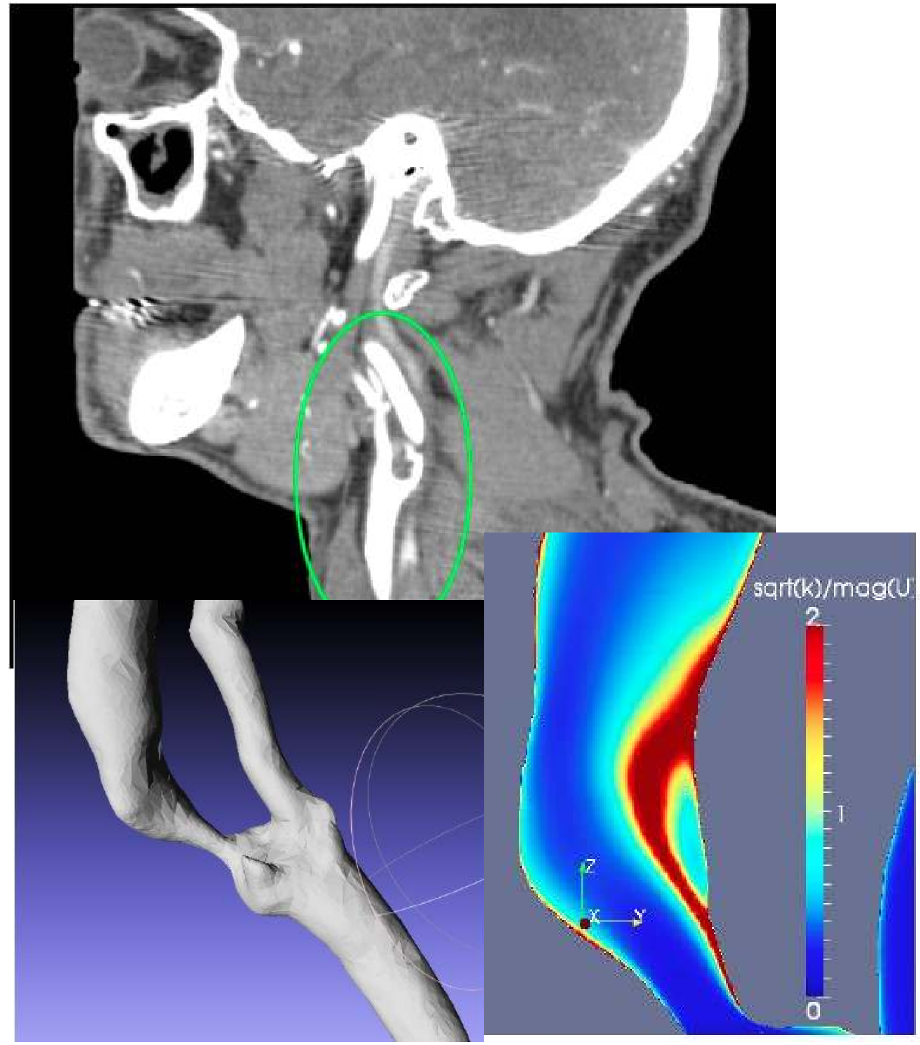


Practical Problems

Unsteady flow in piping system

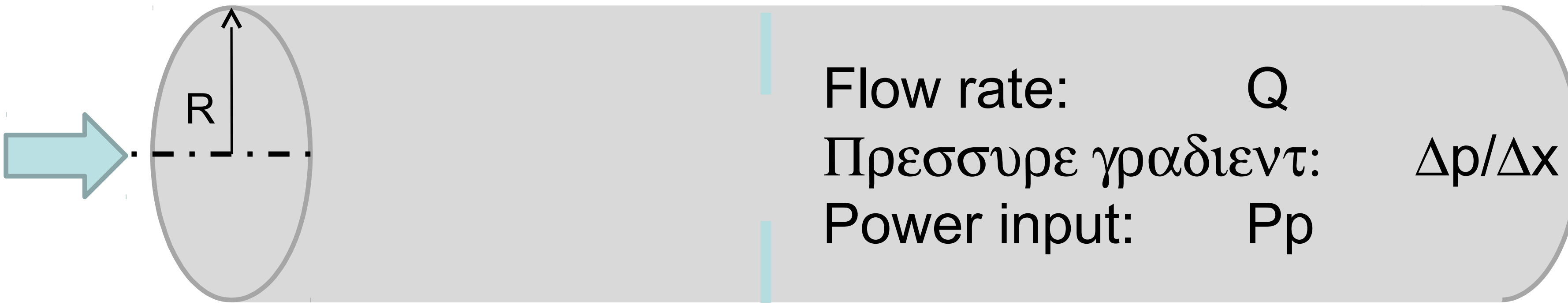


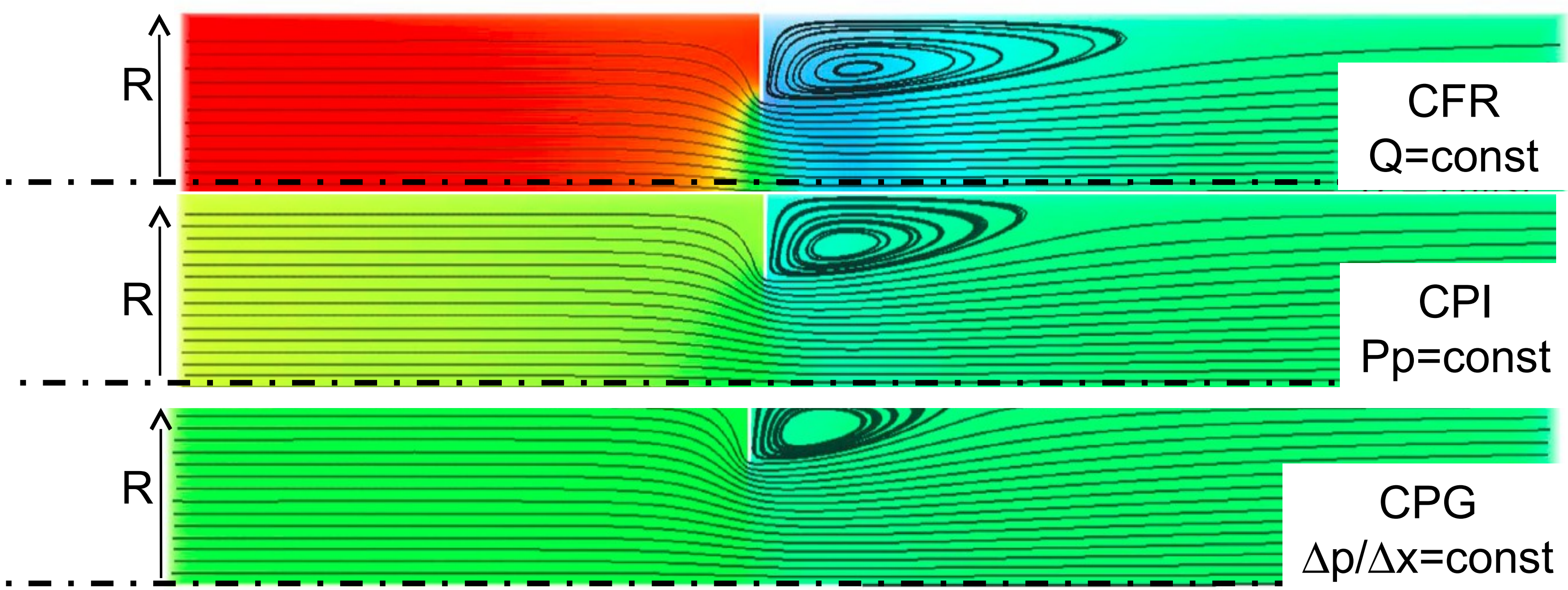
Stenosis of arteries



Most flow conditions in real systems should be neither CFR nor CPG !

laminar flow in pipe w/wo orifice





color code corresponds to pressure gradient

Comparison between Different Flow Conditions

Successful control ↑ ↓

	Ub	ΔP ($\propto \tau w$)	Pumping power ($\propto Ub\,\Delta P$)
CFR	Const.	↓	↓
CPG	↑	Const.	↑

Comparison between Different Flow Conditions

Successful control ↑ ↓

	Ub	ΔP ($\propto \tau w$)	Pumping power ($\propto Ub\,\Delta P$)
CFR	Const.	↓	↓
CPG	↑	Const.	↑
CPI	↑	↓	Const.

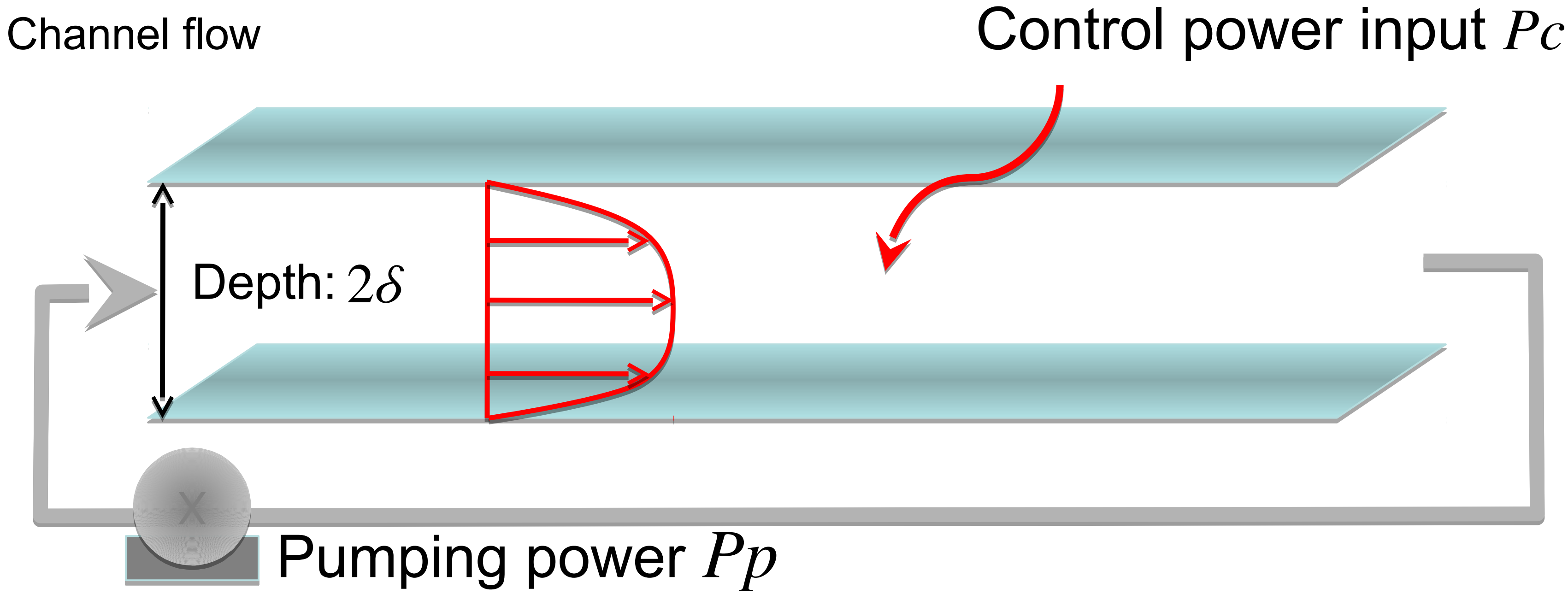
Advantage of CPI

- ✓ **Close to real operational condition (mechanical pump, heart, ……)**
- ✓ **Constant power input = constant dissipation = constant energy transfer rate**
- ✓ **Optimal ratio of total power *Ptotal* and control power input *Pc***

$$\gamma = \frac{control\ power\ input}{total\ power\ input} = \frac{P_c}{P_{total}} = \frac{P_c}{P_p + P_c}$$

Introduction to CPI concept

Problem Setting



Prescribed quantities

- ✓ **Channel half depth** δ
- ✓ **Fluid physical properties (kinetic viscosity:** ν **)**
- ✓ **Total power input:** $Ptotal = Pp + Pc = \text{const.}$

Velocity Scale based on Power Input

"The lower-limit of power consumption under CFR is achieved in the Stokes flow"
Bewley (JFM, 2009), Fukagata et al. (Physica D, 2009)

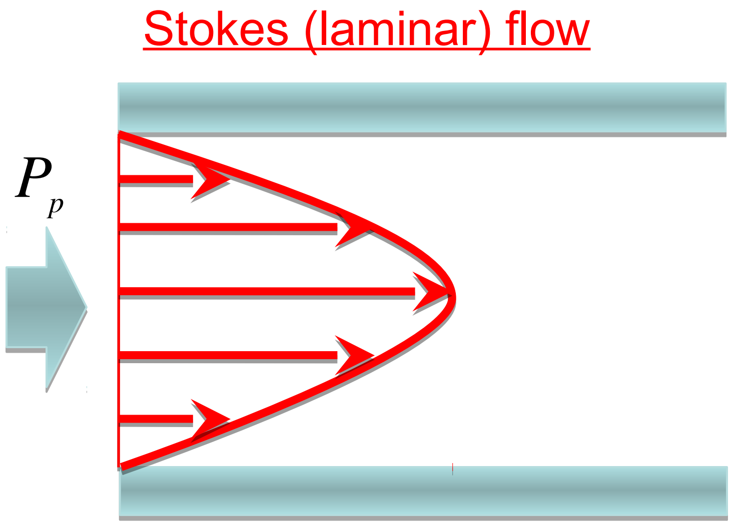
→ The flow rate becomes maximum under CPI in the Stokes flow.

✓ **Pumping power per unit wetted area**

$$P_p = \left(-\frac{dp}{dx}\right)\delta \cdot U_b$$

✓ **Bulk velocity in the Stokes flow**

$$U_b = \frac{1}{3\mu}\left(-\frac{dp}{dx}\right)\delta^2 = \sqrt{\frac{P_p \delta}{3\mu}}$$



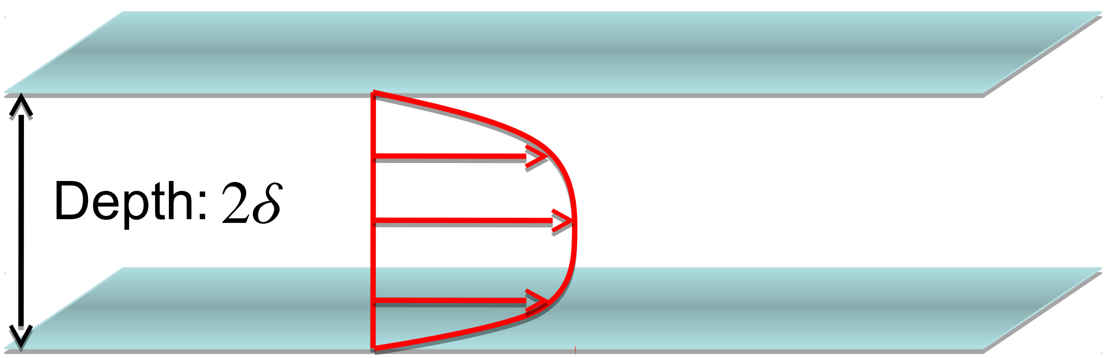
✓ **The upper-limit of the bulk mean velocity under CPI**

$$U_p = \sqrt{\frac{P_t \delta}{3\mu}}$$

→ Velocity scale based on the total power consumption

Non-dimensionalization

Channel flow



All quantities are normalized by

✓ $Up = (Ptotal\,\delta/3\mu)1/2$

✓ δ

Power-based Reynolds number

$$\mathrm{Re}_p = \frac{U_p \delta}{\nu} \cong 6500$$

$$(\mathrm{Re}_{\tau,0} = 200)$$

Navier-Stokes & Continuity Equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}_p} \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad \frac{\partial u_i}{\partial x_i} = 0$$

Total power input: $P_{total} = \frac{3}{\mathrm{Re}_p} (= const.)$

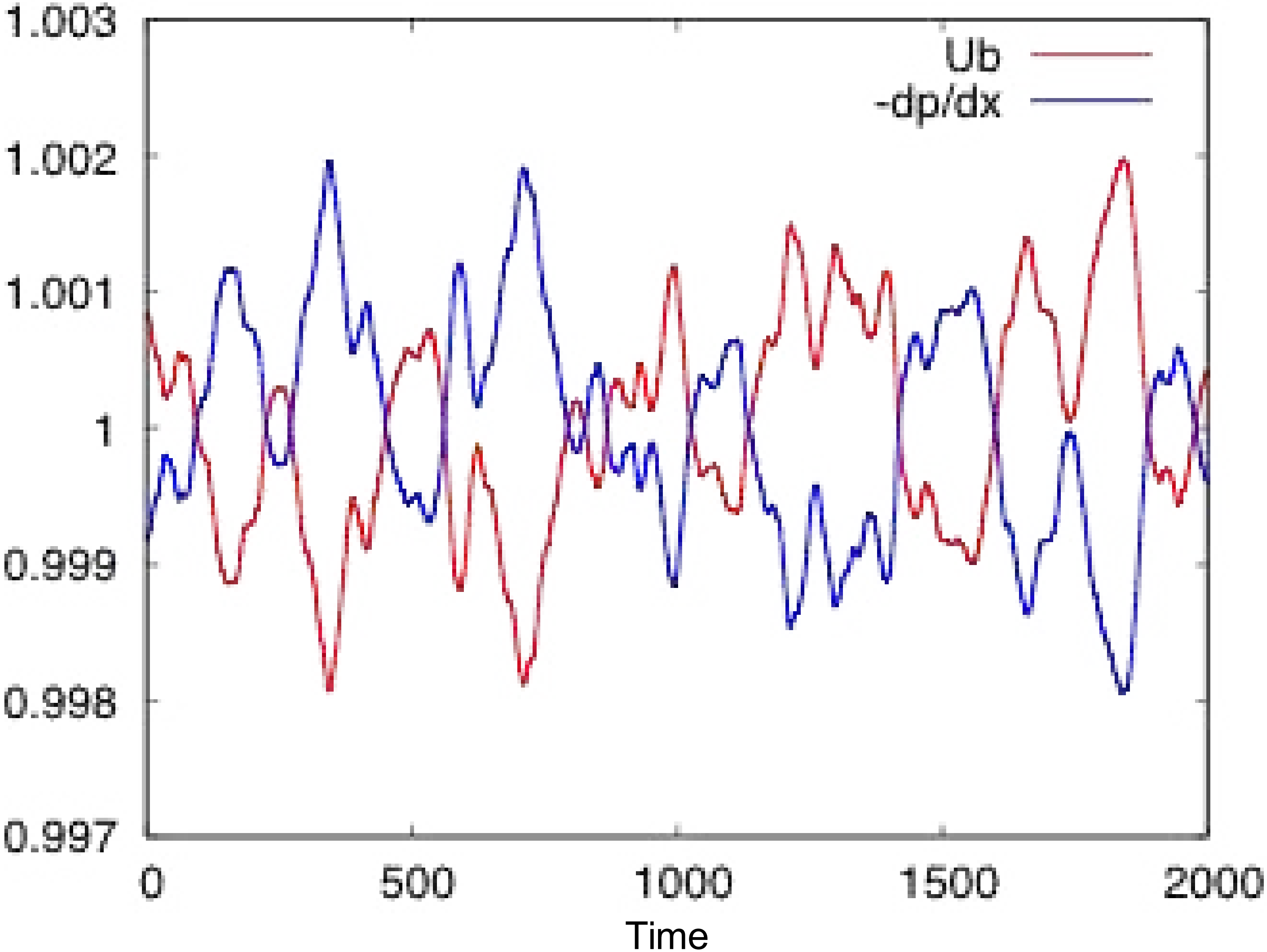
Evaluation of control performance   Gain in flow rate $U_b / U_p (\leq 1)$

Uncontrolled flow under CPI

Relationship between Different Reynolds Numbers in Uncontrolled Flow

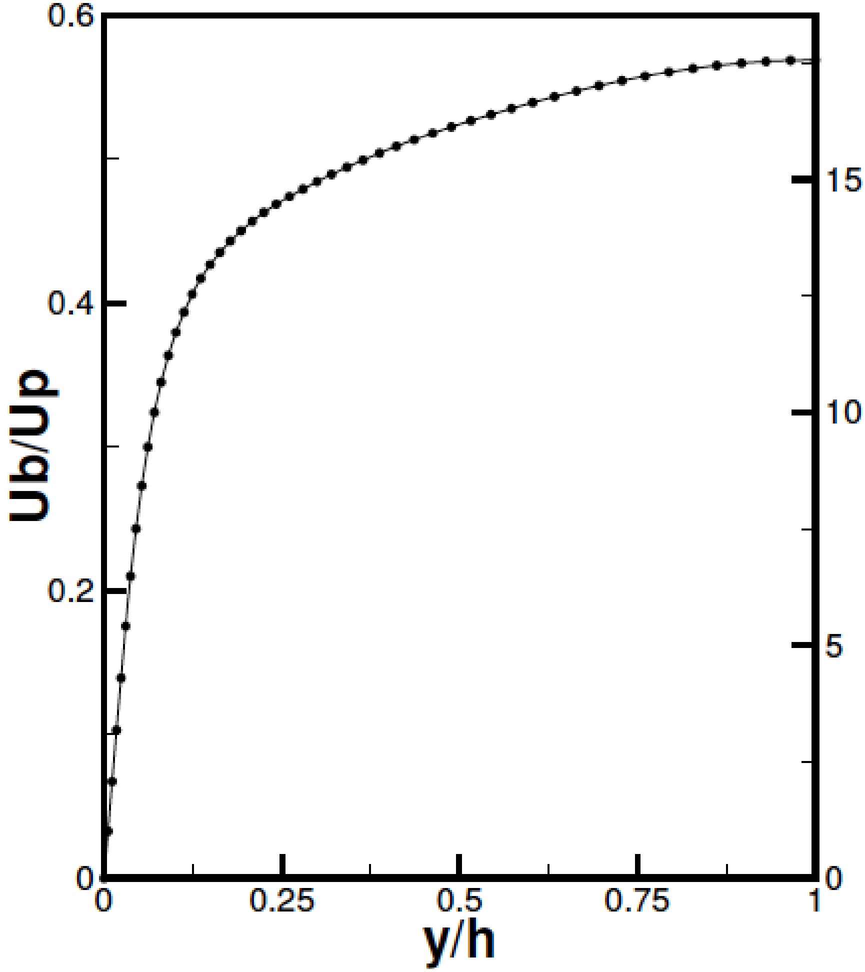
Re_τ	Re_b	Re_p	U_b/u_τ	U_p/u_τ	U_p/U_b
100	1440	2191	14.4	21.9	1.52
150	2289	4143	15.3	27.6	1.81
200	3179	6511	15.9	32.6	2.05
300	5054	12310	16.9	41.0	2.44
450	8032	23280	17.9	51.7	2.90
650	12230	41500	18.8	63.8	3.39

Time Trace of *Ub* & *dp/dx*

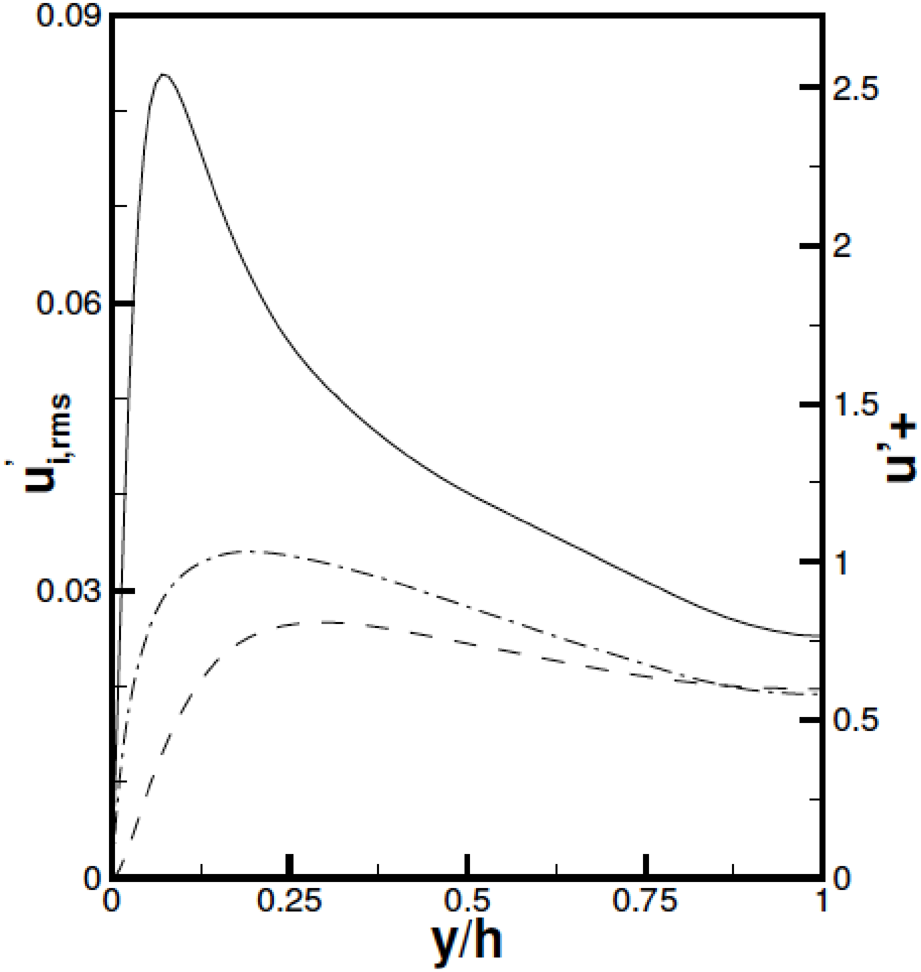


Fundamental Flow Statistics

Mean Velocity



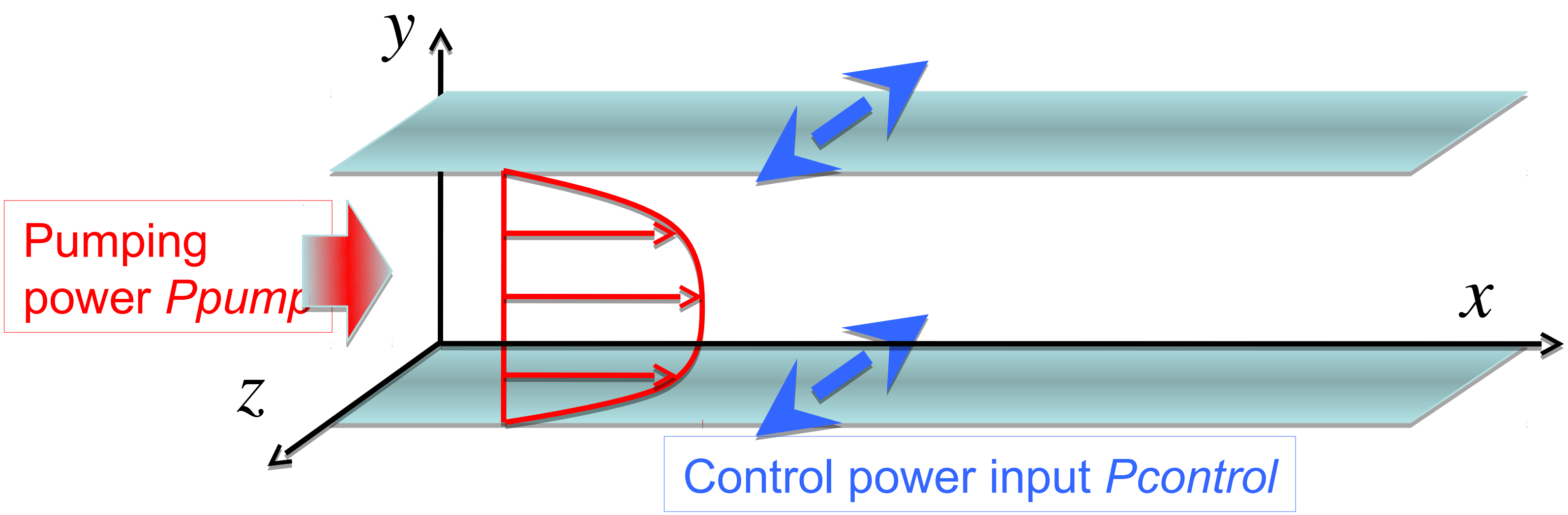
Velocity fluctuation



Results in CFR, CPG & CPI converge to the identical flow state in uncontrolled flow if Reb, Reτ, Rep are adjusted properly.

Controlled flow under CPI

(Spanwise wall oscillation)



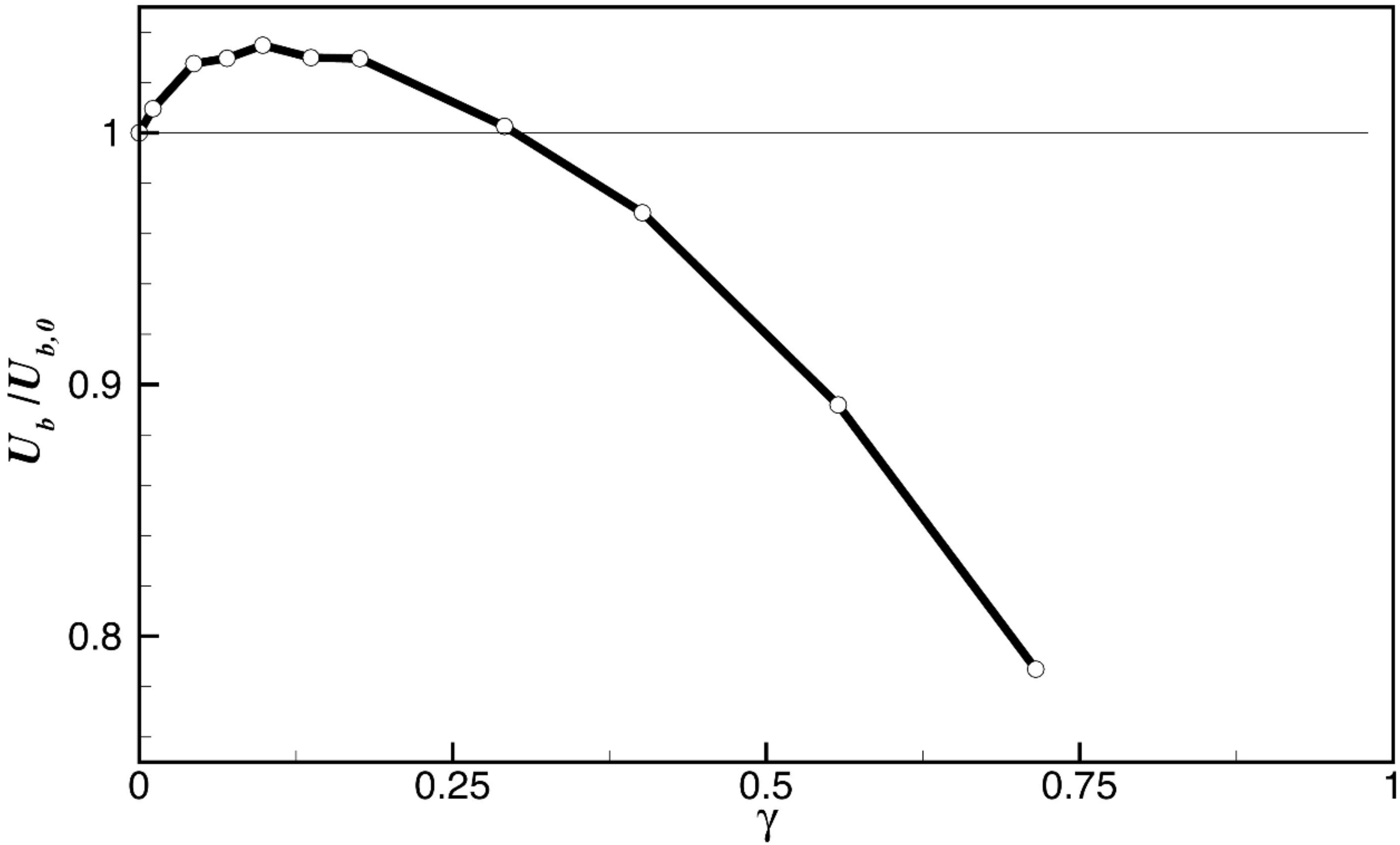
Rep = 6500

(Reτ, 0 = 200)

T+ = 125

$$P_{total} = P_{pump} + P_{control} = const.$$

Optimal Power Input

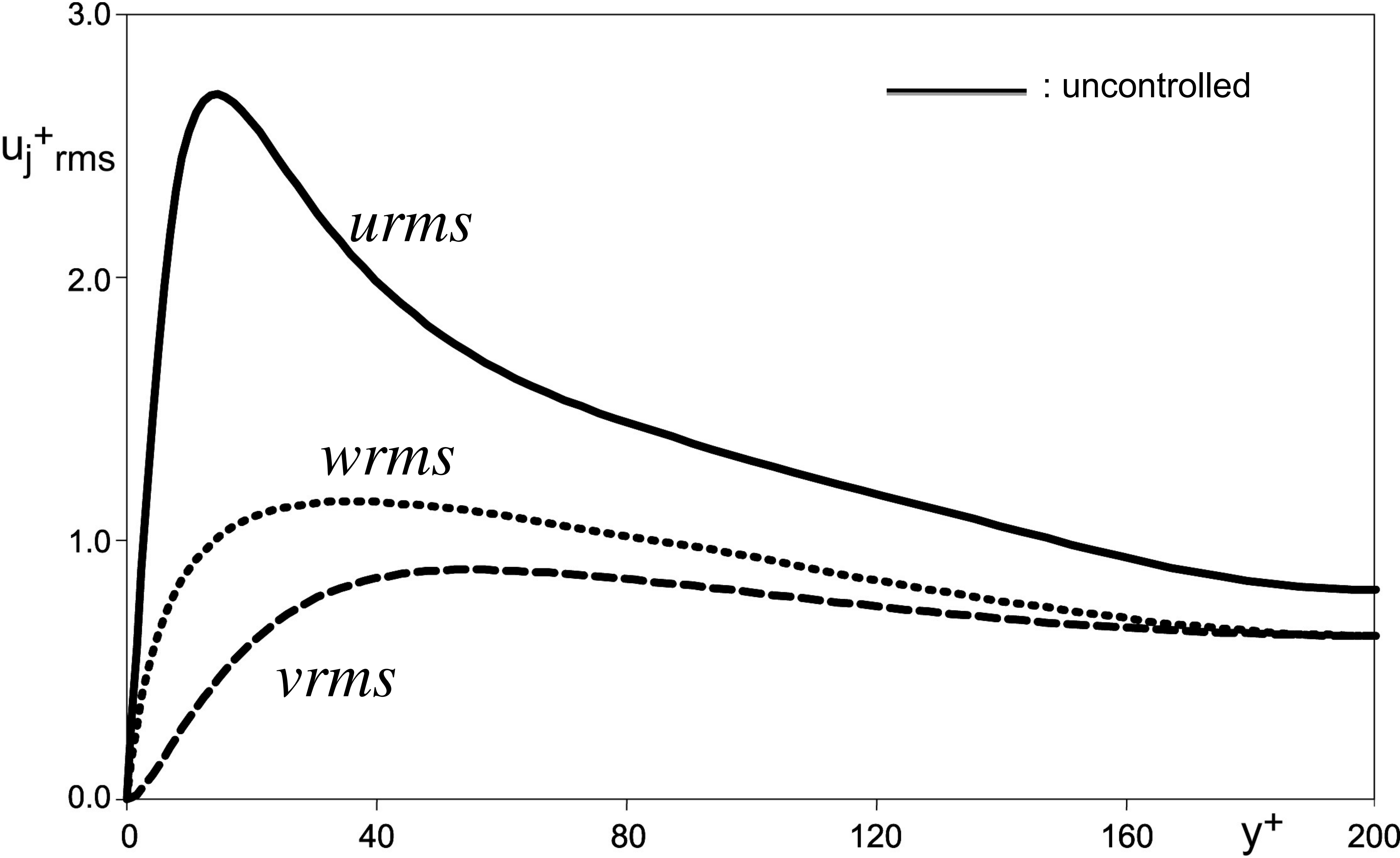


$\gamma = \frac{P_c}{P_{total}} \sim 0.1$ leads to the maximum bulk mean velocity

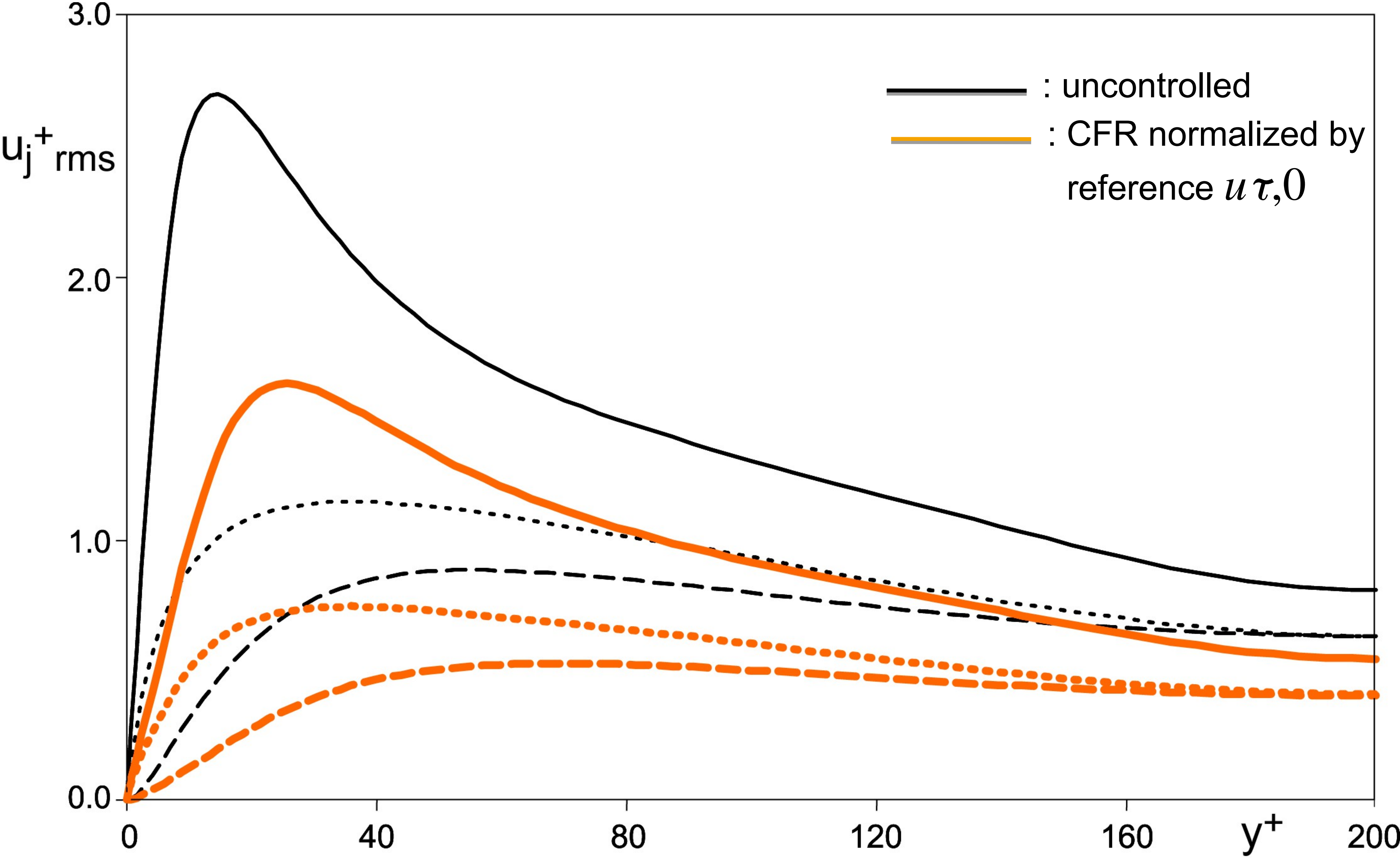
Conclusions

- **Constant power input (CPI) condition is proposed as a flow condition alternative to conventional CFR and CPG**
  - ✓ **close to real operational condition**
  - ✓ **power input (= energy transfer rate = dissipation) is kept constant**
  - ✓ **optimal ratio of total power input and control power input**
- **CPI condition is first implemented in DNS of wall turbulence**
  - ✓ **Power-based velocity scale: Up**
  - ✓ **dimensionless total power input: 3/Rep**
- **CPI simulation successfully run for the uncontrolled and controlled flows.**
  - ✓ **Uncontrolled flow under CPI is essentially same as those under CFR and CPG.**
  - ✓ **In the controlled flow, the maximum Ub is obtained when γ is around 10％.**

Turbulent Intensity in Spanwise Wall Oscillation Control



Turbulent Intensity in Spanwise Wall Oscillation Control



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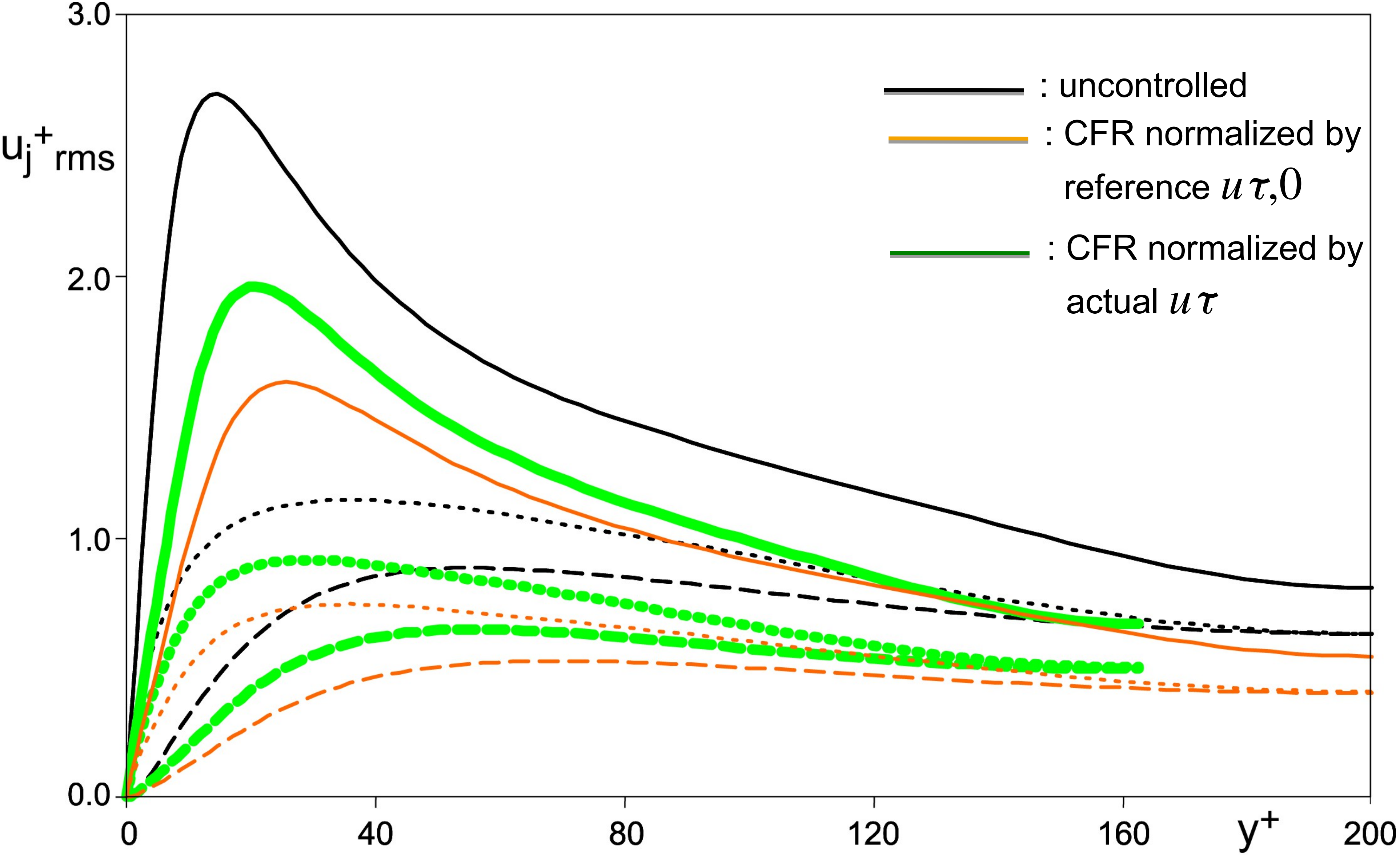
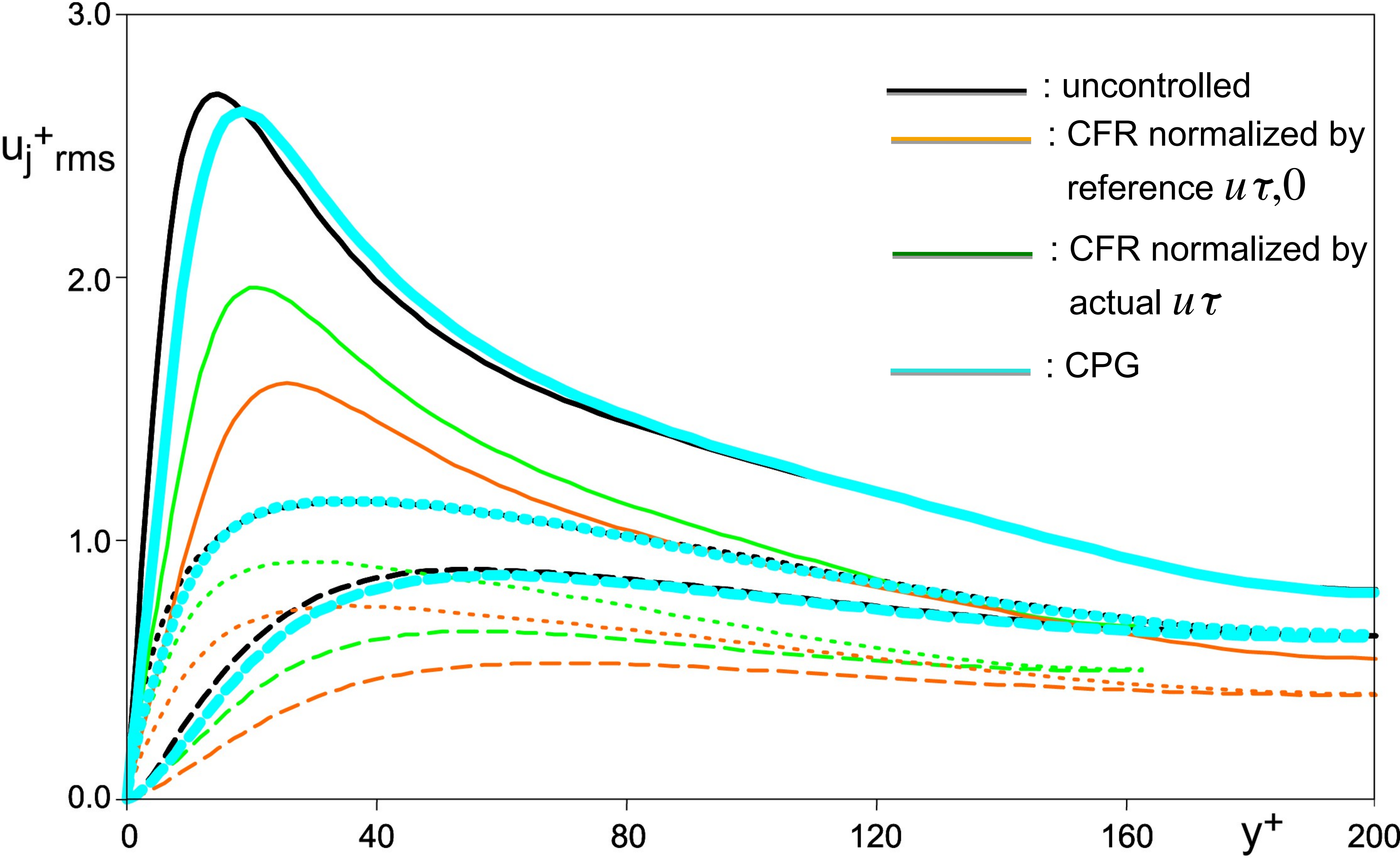


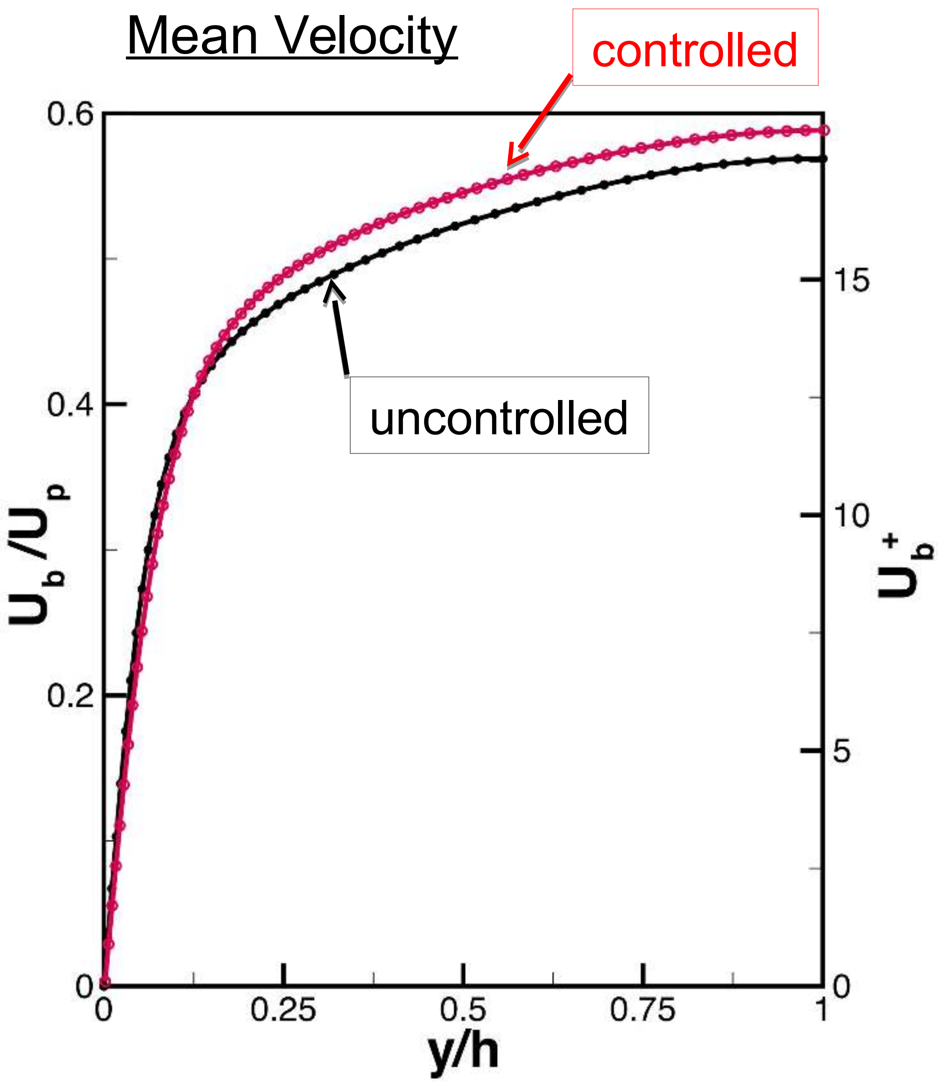
Turbulent Intensity in Spanwise Wall Oscillation Control



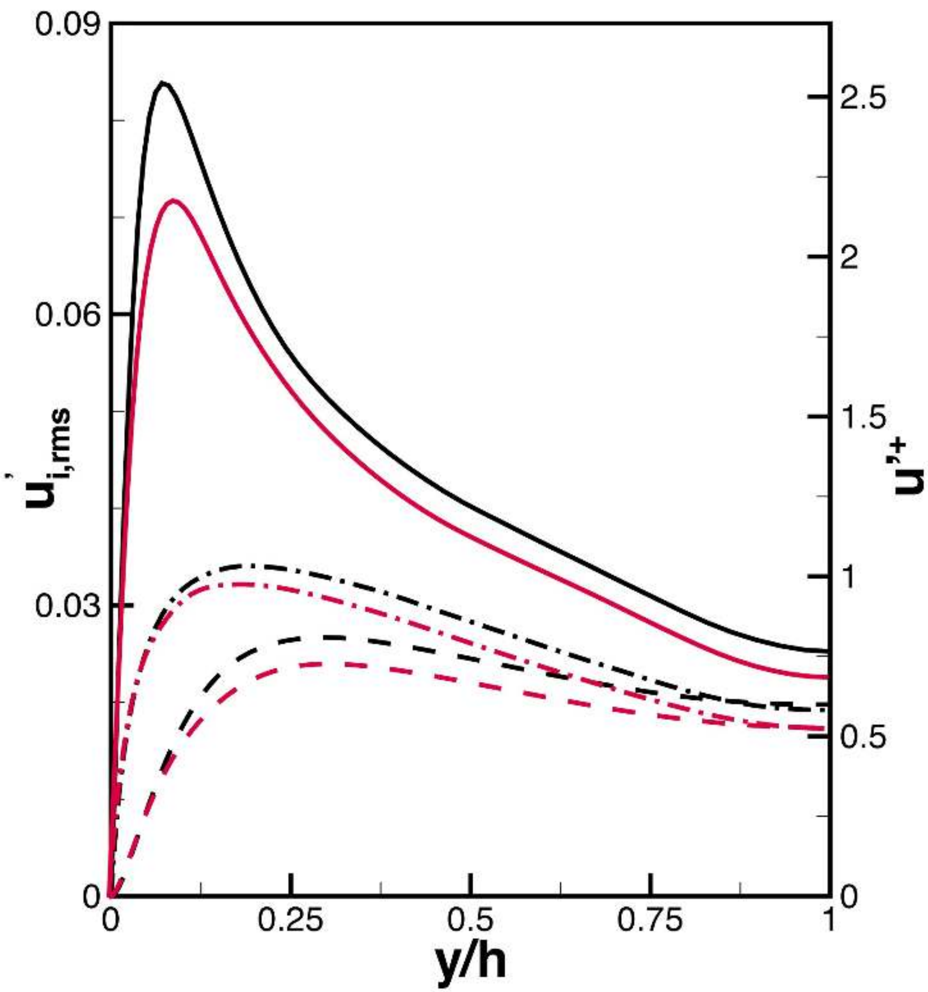
Interpretation can be changed depending on flow conditions and normalization !

Fundamental Flow Statistics

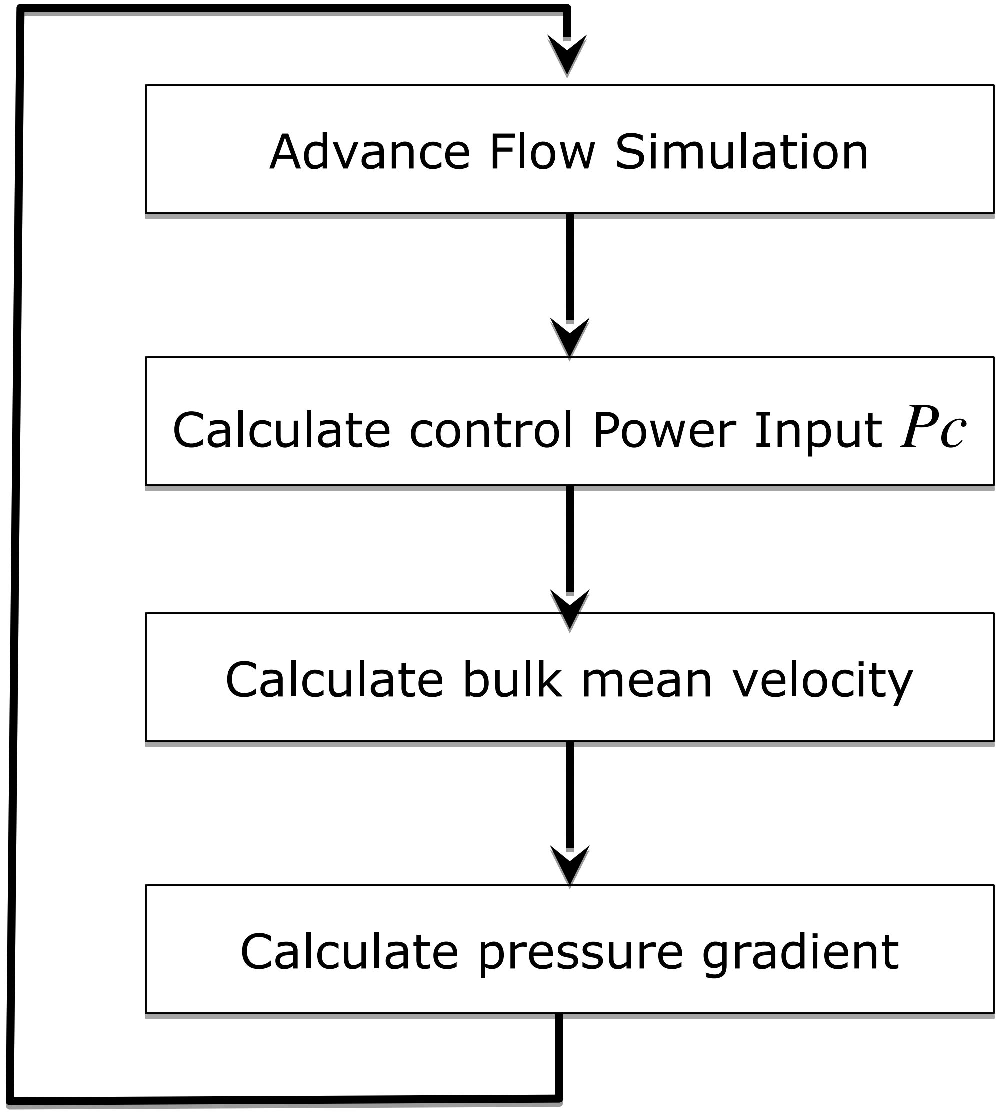
Mean Velocity



Velocity fluctuation



Numerical Implementation

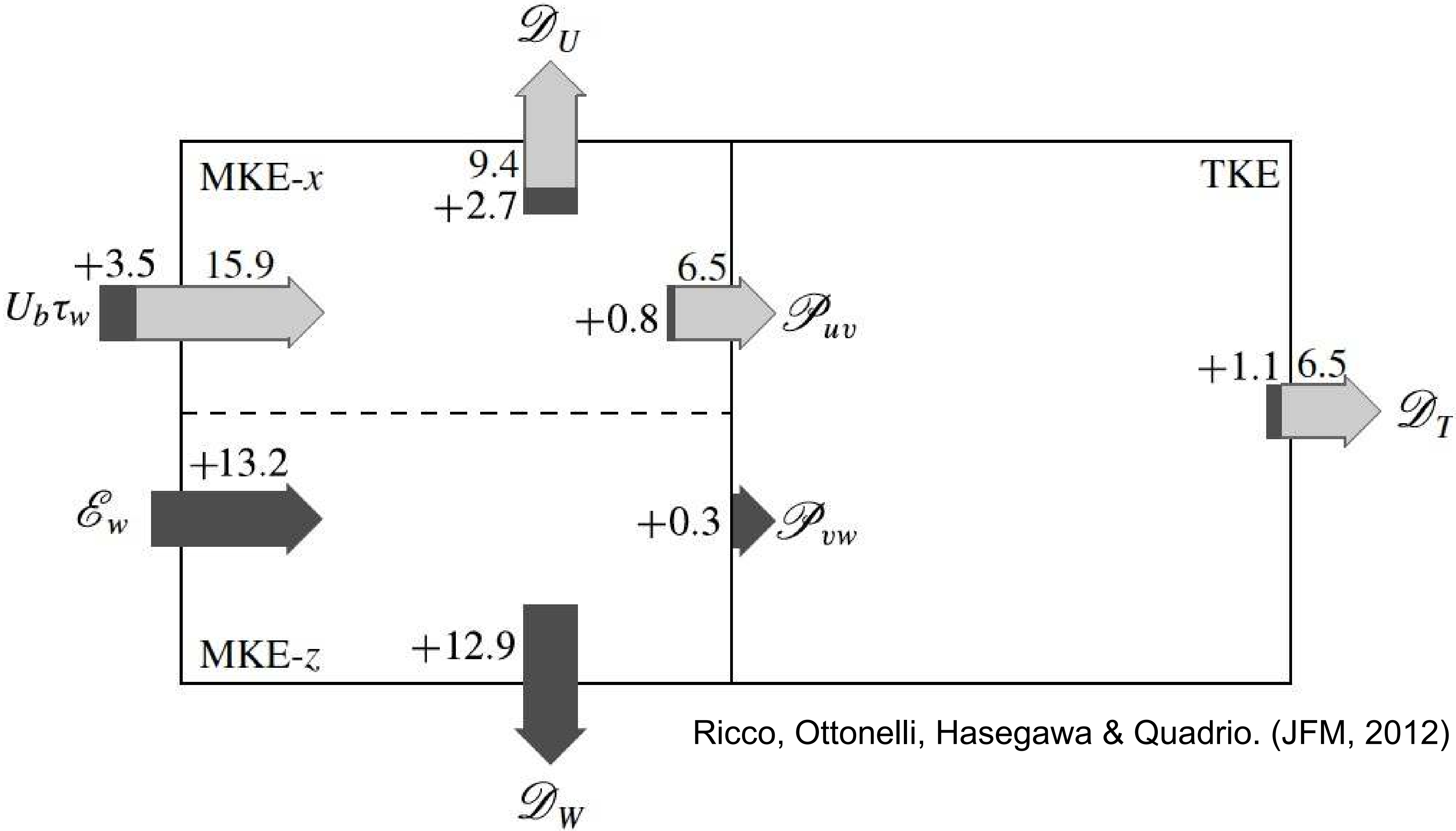


Time step: $n \rightarrow n+1$

$$P_{pump}^{n+1} = P_{total} - P_c^n$$

$$\left(-\frac{dp}{dx}\right)^{n+1} = \frac{P_{pump}^{n+1}}{U_b}$$

Energy Box



Ricco, Ottonelli, Hasegawa & Quadrio. (JFM, 2012)