Direct optimal growth analysis for timesteppers

Spencer J. Sherwin
Department of Aeronautics, Imperial College London, SW7 2AZ, UK
Hugh M. Blackburn
Department of Mechanical Engineering, Monash University, Vic 3800, Australia
Dwight Barkley
Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

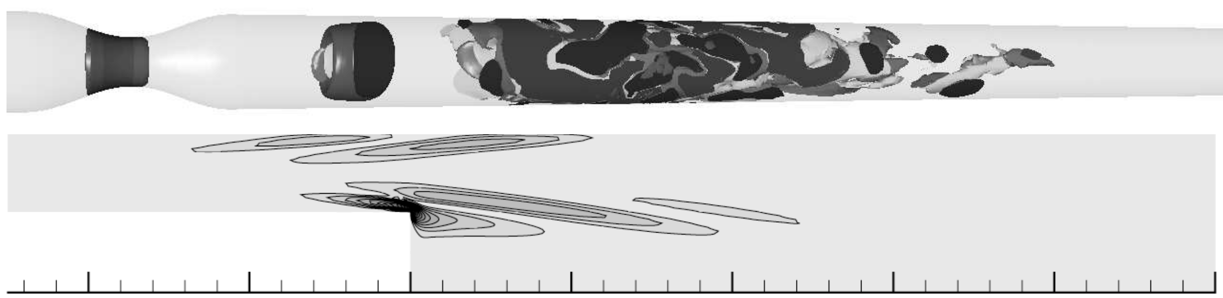

Figure 1: (a) Transition in a pulsatile constricted (stenotic) tube due to a period doubling vortex tilting mechanism identified by linear stability analysis, (b) Logarithm of energy of the maximal growth initial condition in two-dimensional flow past a backward facing step.

In complex geometry flows the dimension of the dynamic and geometric parametric spaces make the study of transitional flows for all parameters difficult if not impossible to undertake via direct numerical simulation. However, when the problem has one direction of geometric homogeneity, as the examples shown in figure 1, an alternative approach is to adopt linear stability analysis. In this technique a non-linear two-dimensional basic state is computed using a non-linear Navier-Stokes solver. The linearised Naviers-Stokes equations are then temporally integrated around this basic state and the dominant eigenvalues of this system evaluated iteratively using an Arnoldi iteration. When absolute/global instabilities are present the onset of linear instabilities can be directly obtained as shown in figure 1(a). However many problems of engineering interest do not exhibit this type of instability and so alternative techniques such as transient growth (or optimal growth) analysis can be consider, an example of which is shown in figure 1(b). The need for low numerical diffusion and dispersion motivate the use of accurate spatial discretisations such as the spectral/*hp* element method. In this talk we will outline these analysis techniques and demonstrate their application to geometrically non-trivial problems such a flow in a constricted pipe and the backward facing step.

References

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