NBER WORKING PAPER SERIES

DIRECTED TECHNICAL CHANGE

Daron Acemoglu

Working Paper 8287
http://www.nber.org/papers/w8287

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2001

This paper is a revised version of the Review of Economic Studies lecture delivered at the 2001 The Royal Economic Society meeting in Durham. I thank Rubén Segura-Cayuela for excellent research assistance and David Autor, Olivier Blanchard, Nobu Kiyotaki, Rubén Segura-Cayuela, Jaume Ventura, and seminar participants at MIT, Institute for International Economics at Stockholm and the Royal Economics Society for comments. The views expressed herein are those of the author and not necessarily those of the National Bureau of Economic Research.

Directed Technical Change
Daron Acemoglu
NBER Working Paper No. 8287
May 2001
JEL No. E25, J31, O30, O31, O33

ABSTRACT

For many problems in macroeconomics, development economics, labor economics, and international trade, whether technical change is biased towards particular factors is of central importance. This paper develops a simple framework to analyze the forces that shape these biases. There are two major forces affecting equilibrium bias: the price effect and the market size effect. While the former encourages innovations directed at scarce factors, the latter leads to technical change favoring abundant factors. The elasticity of substitution between different factors regulates how powerful these effects are, and this has implications about how technical change and factor prices respond to changes in relative supplies. If the elasticity of substitution is sufficiently large, the long-run relative demand for a factor can slope up.

I apply this framework to discuss a range of issues including: Why technical change over the past 60 years was skill-biased, and why the skill bias may have accelerated over the past twenty-five years. Why new technologies introduced during the late eighteenth and early nineteenth centuries were unskill-biased. Why biased technical change may increase the income gap between rich and poor countries. Why international trade may induce skill-biased technical change. Why a large wage-push, as in continental Europe during the 1970s, may cause capital-biased technical change. Why technical change may be generally labor-augmenting rather than capital-augmenting.

Daron Acemoglu
Department of Economics
MIT, E52-380B
50 Memorial Drive
Cambridge, MA 02138
and NBER
daron@mit.edu

1 Introduction

There is now a large and influential literature on the determinants of the aggregate technical progress (see, among others, Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Young (1993)). This literature does not address questions related to the *direction* and *bias* of technical change. In most situations, however, technical change is not neutral: it benefits some factors of production more than others. In this paper, I develop a simple framework of directed technical change to study these biases. In this framework, profit incentives determine the amount of research and development directed at different factors and sectors.

To see the potential importance of the biases in technical change, consider a number of examples:

1. Figure 1 plots a measure of the relative supply of skills (number of college equivalent workers divided by noncollege equivalents) and a measure of the return to skills, the college premium. It shows that over the past 60 years, the U.S. relative supply of skills has increased rapidly, but there has been no tendency for the returns to college to fall in the face of this large increase in supply—on the contrary, there has been an increase in the college premium over this time period. The standard explanation for this pattern is that new technologies over the post-war period have been *skill-biased*. The figure also shows that beginning in the late 1960s, the relative supply of skills increased much more rapidly than before, and the skill premium increased very rapidly beginning in the late 1970s. The standard explanation for this increase is an acceleration in the skill bias of technical change (e.g., Autor, Katz and Krueger, 1998).

2. In contrast, technical change during the late eighteenth and early nineteenth centuries appears to have been *unskill-biased* (skill-replacing). The artisan shop was replaced by the factory and later by interchangeable parts and the assembly line (e.g., James and Skinner, 1985, Goldin and Katz, 1998). Products previously manufactured by skilled artisans started to be produced in factories by workers

with relatively few skills, and many previously complex tasks were simplified, reducing the demand for skilled workers. According to Mokyr (1990, p. 137): "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file."

3. Over the past 150 years of growth, the prices of the two key factors, capital and labor, have behaved very differently. While, both in the U.S. and in other Western economies, the wage rate has increased steadily, the rental rate of capital has been approximately constant. As is well known in growth theory, this pattern indicates that most of the new technologies are *labor-augmenting*.

4. Beginning in the late 1960s and the early 1970s, both unemployment and the share of labor in national income increased rapidly in a number of continental European countries. During the 1980s, unemployment continued to increase, but the labor share started a steep decline, and in many countries, ended up below its initial level. Blanchard (1997) interprets the first phase as the response of these economies to a wage-push by workers, and the second phase as a possible consequence of *capital-biased* technical change.

These examples document a variety of important macroeconomic issues where biased technical change plays a key role. They also pose a number of questions: why has technical change been skill-biased over the past 60 years? Why was technical change biased in favor of unskilled labor and against skilled artisans during the nineteenth century? Why has there been an acceleration in the skill bias of technical change during the past twenty-five years? Why is much of technological progress labor-augmenting rather than capital-augmenting? Why was there rapid capital-biased technical change in continental Europe following the wage-push by workers during the 1970s?

These questions require a framework where the equilibrium bias of technical change can be studied. The framework I present here builds on the existing endogenous technical change models, in particular, the expanding product variety models of Romer (1990) and Grossman and Helpman (1991).[1] It generalizes these theories to allow for technical change to be directed towards different factors: firms can invest resources to develop technologies that complement a particular factor. The relative profitabilities of the different types of technologies determine the direction of technical change.

I show that there are two competing forces determining the relative profitability of different types of innovation:

- The price effect: there will be stronger incentives to develop technologies when the goods produced by these technologies command higher prices.

- The market size effect: it is more profitable to develop technologies that have a larger market. This reiterates a point that Schmookler (1966) emphasized in his pioneering study, *Invention and Economic Growth* (p. 206): "invention is largely an economic activity which, like other economic activities, is pursued for gain;... expected gain varies with expected sales of goods embodying the invention." In contrast to Schmookler, who stressed sector-specific innovations, my interest here is with the factor bias of innovations, so the relevant market size is the supply of the factor that will be combined with the new technology.

These two effects are competing because, while the price effect implies that there will be more rapid technological improvements favoring scarce factors, the market size effect creates a force towards innovations complementing the abundant factor.[2] I will show that the elasticity of substitution between the factors determines the relative strengths of these two effects. When the elasticity of substitution is low, scarce factors command much higher prices, and the price effect is relatively more powerful.

The first major result of this framework is that irrespective of the elasticity of substitution between factors (as long as it is not equal to 1), an increase in the relative

[1] Equivalently, I could use the quality ladder model as in Aghion and Howitt (1992)—see Acemoglu (1998). The algebra turns out to be somewhat simpler with the expanding product variety model, which motivates my choice here.

[2] Another important determinant of the direction of technical change is the form of the "innovation possibilities frontier"—i.e., how the relative costs of innovation are affected as technologies change. I discuss the impact of the innovation possibilities frontier on the direction of technical change in Section 4.

abundance of a factor creates some amount of technical change biased towards that factor. The second major result is that if the elasticity of substitution is sufficiently large (in particular, greater than a certain threshold between 1 and 2), the induced bias in technology can overcome the usual substitution effect and increase the relative reward to the factor that has become more abundant. That is, directed technical change can make the long-run relative demand curve slope up. The long-run relative demand curve may be upward sloping in this set-up because of the underlying "increasing returns to scale" in the R&D process: a new machine, once invented, can be used by many workers.[3]

Figure 2 illustrates these results diagrammatically. The relatively steep downward-sloping lines show the constant technology relative demand curves. The economy starts at point A. In the absence of endogenous technical change, the increase in the supply shown in the figure moves the economy along the constant technology demand to point B. The first result of this framework implies that, as long as the elasticity of substitution between factors is not equal to 1, the increase in the supply will induce biased technical change and shift the constant technology demand curve out. The economy will therefore settle to a point like C. In other words, the (long-run) endogenous technology demand curve will be flatter than the constant technology curve. The second result implies that the induced bias in technology can be powerful enough to create a sufficiently large shift in the constant technology demand curve and take the economy to a point like D. In this case, the endogenous technology demand curve of the economy is upward sloping and the relative reward of the factor that has become more abundant increases.

After outlining the general forces shaping the direction of technical change and deriving the main results, I return to a number of applications of this framework. I discuss:

1. Why technical change over the past 60 years was skill-biased, and why skill-biased technical change may have accelerated over the past twenty-five years. Also why new technologies introduced during the late eighteenth and early nineteenth centuries were labor-biased.

2. Why biased technical change is likely to increase the income gap between rich and

[3]This is related to the nonrivalry in the use of ideas emphasized by Romer (1990).

poor countries.

3. Why international trade may induce skill-biased technical change.

4. Under what circumstances labor scarcity will spur faster technological progress as suggested by Rothbarth (1946) and Habakkuk (1962).

5. Why technical change tends to be generally labor-augmenting rather than capital-augmenting.

6. Why a large wage-push, as in continental Europe during the 1970s, may cause capital-biased technical change and affect the factor distribution of income.

This list is by no means exhaustive, and there is much research to be done to understand the implications of biased technical change and the determinants of equilibrium bias of new technologies. It is part of my aim in this paper to stress the importance of thinking about biased technical change, and to provide a set of tools that are likely to be useful for future research on these biases.

Although there is relatively little current research on biased technical change, an earlier literature was devoted to studying related issues. It was probably Hicks in *The Theory of Wages* (1932) who first discussed the determinants of equilibrium bias.[4] He wrote:

> "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive." (pp. 124-5).

Hicks' reasoning, that technical change would attempt to economize on the more expensive factor, was criticized by Salter (1960) who pointed out that there was no particular reason for saving on the more expensive factor—firms would welcome all cost reductions. Moreover, the concept of "more expensive factor" did not make much sense, since all factors were supposed to be paid their marginal product.

[4] Marx also touched on these issues. He argued that labor scarcity—the exhaustion of the reserve army of labor—would induce the capitalist to substitute machinery for labor and spur growth. See for example the discussion in Habakkuk (1962, p. 44).

These questions were revived by the "induced innovation" literature of the 1960s. An important paper by Kennedy (1964) introduced the concept of "innovation possibilities frontier" and argued that it is the form of this frontier—rather than the shape of a given neoclassical production function—that determines the factor distribution of income. Kennedy, furthermore, argued that induced innovations would push the economy to an equilibrium with a constant relative factor share (see also Samuelson, 1965, and Drandakis and Phelps, 1965). Around the same time, Habakkuk (1962) published his important treatise, *American and British Technology in the Nineteenth Century: the Search for Labor Saving Inventions*, where he argued that labor scarcity and the search for labor saving inventions were central determinants of technological progress (see also Rothbarth, 1946). The flavor of Habakkuk's argument was one of induced innovations: labor scarcity increased wages, which in turn encouraged labor-saving technical change.

This whole literature was also criticized for lack of micro-foundations. First, with specifications as in Kennedy, the production function at the firm level exhibited increasing returns to scale because, in addition to factor quantities, firms could choose "technology quantities". Second, as pointed out by Nordhaus (1973), it was not clear who undertook the R&D activities and how they were financed and priced. These shortcomings reduced the interest in this literature, and there was little research for almost 30 years, with the exception of some empirical work, such as that by Hayami and Ruttan (1970) on technical change in American and Japanese agriculture.

The analysis here, instead, starts from the explicit microfoundations laid out by the endogenous technical change models. In addition to providing an equilibrium framework for analyzing these issues, I demonstrate the presence of the market size effect, which did not feature in the earlier literature. More explicitly, the framework I present here synthesizes previous work I did in Acemoglu (1998, 1999a, b) and Acemoglu and Zilibotti (2001), as well as work by Kiley (1999) (see also Lloyd-Ellis, 1999, for a different perspective). The results in these papers show that whether technical change results from quality improvements, expanding variety of products, or expanding variety of machine types is not essential. For this reason, I choose one of the specifications and highlight the modeling choice that turns out to be more important—the form of the innovation possibilities frontier.

The rest of the paper is organized as follows. In the next section, I define some of the terms that will be used throughout the paper and clarify the distinction between factor-augmenting and factor-biased technical change. In this section, I also give a brief overview of the main results. In Section 3, I introduce the basic framework that determines the demand for innovation and I highlight the price and market size effects on the direction of technical change. Section 4 introduces the innovation possibilities frontier and shows how different forms of this frontier will affect equilibrium bias of technology. Sections 5 and 6 apply the framework developed in Sections 3 and 4 to a variety of situations where biased technical change appears to be important. Section 7 concludes.

2 Factor-Augmenting, Factor-Biased Technical Change and An Overview

This section defines some terms that will be used throughout the paper and gives a basic overview.

Consider an aggregate production function, $F(L, Z, A)$, with two inputs, L, labor, and Z, which could be capital, skilled labor or land. A is a technology index. Without loss of generality imagine that $\partial F/\partial A > 0$, so a greater level of A corresponds to "better technology" or to "technological progress". Technical change is L*-augmenting* if

$$\frac{\partial F}{\partial A} \equiv \frac{L}{A}\frac{\partial F}{\partial L},$$

which is equivalent to the production function taking the more special form $F(AL, Z)$. Z*-augmenting* technical change is defined similarly.

Technical change is L*-biased*, on the other hand, if

$$\frac{\partial \frac{\partial F/\partial L}{\partial F/\partial Z}}{\partial A} > 0,$$

that is, if technical change increases the marginal product of L more than the marginal product of Z.

To clarify the difference between these two concepts, consider the more specialized constant elasticity of substitution (CES) production function

$$y = \left[\gamma \left(A_L L\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\gamma\right)\left(A_Z Z\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where A_L and A_Z are two separate technology terms, $\gamma \in (0,1)$ is a distribution parameter which determines how important the two factors are, and $\sigma \in (0,\infty)$ is the elasticity of substitution between the two factors. When $\sigma = \infty$, the two factors are perfect substitutes, and the production function is linear. When $\sigma = 1$, the production function is Cobb-Douglas, and when $\sigma = 0$, there is no substitution between the two factors, and the production function is Leontieff. When $\sigma > 1$, I refer to the factors as gross substitutes, and when $\sigma < 1$, I refer to them as gross complements.[5]

By construction, A_L is L-augmenting and A_Z is Z-augmenting. I will also sometimes refer to A_L as labor-complementary, since in the model of Section 3, A_L will correspond to machines that are used by labor. Similarly, I refer to A_Z as Z-complementary.

Whether technical change is labor-biased or Z-biased, on the other hand, depends on the elasticity of substitution. To see this, calculate the relative marginal product of the two factors:

$$\frac{MP_Z}{MP_L} = \frac{1-\gamma}{\gamma}\left(\frac{A_Z}{A_L}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Z}{L}\right)^{-\frac{1}{\sigma}}. \tag{1}$$

The relative marginal product of Z is decreasing in the relative abundance of Z, Z/L. This is the usual *substitution effect*, leading to a downward-sloping relative demand curve. The effect of A_Z on this relative marginal product depends on σ, however. If $\sigma > 1$, an increase in A_Z (relative to A_L) increases the relative marginal product of Z. When $\sigma < 1$, an increase in A_Z reduces the relative marginal product of Z. Therefore, when the two factors are gross substitutes, Z-augmenting (Z-complementary) technical change is also Z-biased. In contrast, when the two factors are gross complements, then

[5] I choose this terminology because the demand for Z increases in response to an increase in the price of L, holding its price and the quantity of L constant if and only if $\sigma > 1$, and vice versa. Mathematically,

$$\left.\frac{\partial Z}{\partial w_L}\right|_{w_Z,L} \lesseqqgtr 0 \text{ iff } \sigma \lesseqqgtr 1.$$

Z-augmenting technical change is L-biased. Naturally, when $\sigma = 1$, we are in the Cobb-Douglas case, and neither a change in A_Z nor in A_L is biased towards any of the factors.

The intuition for why, when $\sigma < 1$, Z-augmenting technical change is L-biased is simple: with gross complementarity, an increase in the productivity of Z increases the demand for the other factor, labor, by much more, effectively creating "excess demand" for labor. As a result, the marginal product of labor increases by more than the marginal product of Z.

Now to obtain an overview of the results that will follow, imagine a setup where A_L and A_Z are determined endogenously from the type and quality of machines supplied by "technology monopolists". One of the major results of the more detailed analysis below will be that the profitability of developing new Z-complementary machines relative to the profitability of labor-complementary machines will be proportional to (see equation (20))

$$\left(\frac{A_Z}{A_L}\right)^{-\frac{1}{\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{\sigma}}. \tag{2}$$

The basic premise of the approach here is that profit incentives determine what types of innovations will be developed. So when (2) is high, A_Z will increase relative to A_L. Inspection of (2) shows that when the two factors are gross substitutes ($\sigma > 1$), an increase in Z/L will increase the relative profitability of inventing Z-complementary technologies. To equilibriate innovation incentives, A_Z/A_L has to rise, reducing (2) back to its original level. Intuitively, in this case, of the two forces discussed in the introduction, the market size effect is more powerful than the price effect, so technical change is directed towards the more abundant factor. In contrast, when the two factors are gross complements ($\sigma < 1$), an increase in Z/L will lead to a fall in A_Z/A_L. However, recall that when $\sigma < 1$, a lower A_Z/A_L corresponds to Z-biased technical change. So in this case an increase in Z/L reduces the relative physical productivity of factor Z, but increases its relative *value* of marginal product, because of relative price changes. Therefore, in both cases (i.e., as long as $\sigma \neq 1$) an increase in the relative abundance of Z causes Z-biased technical change.

We will also see that if σ is sufficiently large, this induced biased technical change can be so powerful that the increase in the relative abundance of a factor may in fact increase its relative reward—i.e., the long-run relative demand curve for a factor may be upward sloping. Finally, depending on the form of the innovation possibilities frontier (i.e., the relative costs of different types of innovations) and the elasticity of substitution, the induced technical change may create a force that either increases, decreases or stabilizes the income share of the factor that has become more abundant.

3 The Demand Side

I now develop the basic framework for analyzing the determinants of the factor bias of technical change. The framework is a synthesis of the models in Acemoglu (1998, 1999a,b), Acemoglu and Zilibotti (2001) and Kiley (1999), and builds on the standard endogenous technical change models (e.g., Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992). In this section, the focus is on the demand for new technology (innovation). The next section then introduces "the innovation possibilities frontier" and discusses the supply side of innovations.

3.1 The Environment

Consider an economy that admits a representative consumer[6] with the usual constant relative risk aversion (CRRA) preferences

$$\int_0^\infty \frac{C^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{3}$$

where ρ is the rate of time preference and θ is the coefficient of relative risk aversion (or the intertemporal elasticity of substitution). I suppress the time arguments to simplify the notation, and I will do so throughout as long as this causes no confusion. The budget constraint of the consumer is:

$$C + I + R \leq Y \equiv \left[\gamma Y_L^\alpha + (1-\gamma) Y_Z^\alpha\right]^{1/\alpha} \tag{4}$$

[6]Since with CRRA utility functions, individual preferences can be aggregated into a CRRA representative consumer, whether there are different types of workers or not is not essential. See, for example, Caselli and Ventura (2000).

where I denotes investment, and R is total R&D expenditure, if any. I also impose the usual no-Ponzi game condition, requiring the lifetime budget constraint of the representative consumer to be satisfied. The production function in (4) implies that consumption, investment and R&D expenditure come out of an output aggregate produced from two other (intermediate) goods, Y_L and Y_Z, with elasticity of substitution $\varepsilon \equiv 1/(1-\alpha)$ where $-\infty < \alpha \leq 1$, and γ is a distribution parameter which determines how important the two goods are in aggregate production. Of the two intermediates, Y_L is (unskilled) labor-intensive, while Y_Z uses another factor, Z, intensively. In this section and the next, I will not be specific about what this factor is, but the reader may want to think of it as skilled labor for concreteness.[7]

The two intermediate goods have the following production functions[8]

$$Y_L = \frac{1}{1-\beta}\left(\int_0^{N_L} x_L(j)^{1-\beta}\, dj\right) L^{\beta}, \tag{5}$$

and

$$Y_Z = \frac{1}{1-\beta}\left(\int_0^{N_Z} x_Z(j)^{1-\beta}\, dj\right) Z^{\beta}, \tag{6}$$

where $\beta \in (0,1)$, and L and Z are the total supplies of the two factors, assumed to be supplied inelastically for now.[9] The labor-intensive good is therefore produced from labor and a range of labor-complementary machines. $x_L(j)$ denotes the amount of the j-th labor-complementary machine used in production. The range of machines that can be

[7]This specification also introduces a distinction between the final good, Y, and the two intermediate goods. This distinction is only useful for exposition, enabling me to use actual relative prices (rather than shadow prices) to give the intuition. None of the results in the paper depend on it.

Moreover, in this basic framework, there is no distinction between factor-specific and sector-specific technical change. The key to the results is the factor bias of technical change, not its sector bias. In the Appendix, I outline a more general model with both sector- and factor-specific technical change, and show that the same forces determine equilibrium factor bias.

[8]The firm level production functions are also assumed to be constant returns to scale, so there is no loss of generality in focusing on the aggregate production functions.

[9]A somewhat more general formulation which leads to similar results is

$$Y_L = \frac{1}{1-\beta}\left(\int_0^{N_L} x_L(j)^{v}\right)^{\frac{1-\beta}{v}} L^{\beta},$$

and similarly for Y_Z. The formulation in the text corresponds to the special case where $v = 1-\beta$. This simplifies the algebra without any loss of generality.

used with labor is denoted by N_L. The production function for the other intermediate, (6), uses Z-complementary machines and is explained similarly. Notice that given N_L and N_Z, the production functions (5) and (6) exhibit constant returns to scale. There will be aggregate increasing returns, however, when N_L and N_Z are endogenized.

I assume that machines in both sectors are supplied by "technology monopolists". In this section, I take N_L and N_Z as given, and in the next section, I analyze the innovation decisions of these monopolists (the supply of innovations) to determine N_L and N_Z. Each monopolist sets a rental price $\chi_L(j)$ or $\chi_Z(j)$ for the machine it supplies to the market. For simplicity, I assume that all machines depreciate fully after use, and that the marginal cost of production is the same for all machines and equal to ψ in terms of the final good.[10]

The important point to bear in mind is that the set of machines used in the production of the two intermediate goods are different, allowing technical change to be biased. As in models of technical change based on expanding product variety (e.g., Romer, 1990, Grossman and Helpman, 1991), the range of machines, N_L and N_Z, will determine aggregate productivity. In addition, N_Z/N_L will determine the relative productivity of factor Z.

3.2 Equilibrium

An equilibrium (given N_L and N_Z) is a set of prices for machines, $\chi_L(j)$ or $\chi_Z(j)$, that maximize the profits of technology monopolists, machine demands from the two intermediate goods sectors, $x_L(j)$ or $x_Z(j)$, that maximize intermediate good producers profits, and factor and product prices, w_L, w_Z, p_L, and p_Z, that clear markets. I now characterize this equilibrium and show that it is unique. The levels of N_L and N_Z will be determined in the next section once I introduce the innovation possibilities frontier of this economy.

[10]Slow depreciation of machines does not change the balanced growth path equilibrium, and only affects the speed of transitory dynamics. For example, if machines depreciate at some exponential rate δ, monopolists will produce the required stock of machines after the discovery of the new variety, and then will replace the machines that depreciate. The rental price will then be a markup over the opportunity cost of machines rather than over the production cost.

The product markets for the two intermediates are competitive, so market clearing implies that their relative price, p, has to satisfy

$$p \equiv \frac{p_Z}{p_L} = \frac{1-\gamma}{\gamma}\left(\frac{Y_Z}{Y_L}\right)^{-\frac{1}{\varepsilon}}, \tag{7}$$

where recall that $\varepsilon \equiv 1/(1-\alpha)$. The greater the supply of Y_Z relative to Y_L, the lower is its relative price, p. The response of the relative price to the relative supply depends on the elasticity of substitution, ε.

I choose the price of the final good as the numeraire, so

$$\gamma p_L^{1-\varepsilon} + (1-\gamma)p_Z^{1-\varepsilon} = 1. \tag{8}$$

Since product markets are competitive, firms in the labor-intensive sector solve the following maximization problem

$$\max_{L,\{x_L(j)\}} p_L Y_L - w_L L - \int_0^{N_L} \chi_L(j)\, x_L(j)\, dj, \tag{9}$$

taking the price of their product, p_L, and the rental prices of the machines, denoted by $\chi_L(j)$, as well as the range of machines, N_L, as given. The first-order conditions for the problem (9) give the following machine demands:

$$x_L(j) = \left(\frac{p_L}{\chi_L(j)}\right)^{1/\beta} L. \tag{10}$$

This equation implies that the desired amount of machine use is increasing in the price of the product, p_L, and in the firm's employment, L, and is decreasing in the price of the machine, $\chi_L(j)$. Intuitively, a greater price for the product increases the value of the marginal product of all factors, including machines, encouraging firms to rent more machines. A greater level of employment, on the other hand, implies more workers to use the machines, raising demand. Finally, because the demand curve for machines is downward sloping, a higher cost implies lower demand.

Next, the first-order condition with respect to L gives the equilibrium wage rate for unskilled workers as

$$w_L = \frac{\beta}{1-\beta} p_L \left(\int_0^{N_L} x_L(j)^{1-\beta}\, dj\right) L^{\beta-1}. \tag{11}$$

The maximization problem facing firms in the Z-intensive sector is similar to (9) and yields machine demands identical to (10):

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)}\right)^{1/\beta} Z, \tag{12}$$

and the reward to factor Z is similar to (11):

$$w_Z = \frac{\beta}{1-\beta} p_Z \left(\int_0^{N_Z} x_Z(j)^{1-\beta}\, dj\right) Z^{\beta-1}. \tag{13}$$

My interest is with the determinants of the direction of technical change. As discussed above, profit-maximizing firms will generate more innovations in response to greater profits, so the first step is to look at the profits of the technology monopolists. Recall that each monopolist faces a marginal cost of producing machines equal to ψ. Therefore, the profits of a monopolist supplying labor-intensive machine j can be written as $\pi_L(j) = (\chi_L(j) - \psi)\, x_L(j)$. Since the demand curve for machines facing the monopolist, (10), is iso-elastic, the profit-maximizing price will be a constant markup over marginal cost: $\chi_L(j) = \frac{\psi}{1-\beta}$. To simplify the algebra, I normalize the marginal cost to $\psi \equiv 1 - \beta$.[11] This implies that in equilibrium all machine prices will be given by

$$\chi_L(j) = \chi_Z(j) = 1.$$

Using this price and machine demands given by (10) and (12), profits of technology monopolists are obtained as

$$\pi_L = \beta p_L^{1/\beta} L \text{ and } \pi_Z = \beta p_Z^{1/\beta} Z. \tag{14}$$

What is relevant for the monopolists is not the instantaneous profits, but the net present discounted value of all profits. This net present discounted value can be expressed using a standard dynamic programming equation:

$$rV_L - \dot{V}_L = \pi_L \text{ and } rV_Z - \dot{V}_Z = \pi_Z, \tag{15}$$

where r is the interest rate, which is potentially time-varying. This equation relates the present discounted value of future profits, V, to the flow of profits, π. The $\dot{V}$ term takes

[11] This is without loss of any generality, since I am not interested in comparative statics with respect to β.

care of the fact that future profits may not equal current profits, for example because prices are changing.

To gain intuition, let us start with a steady state where the $\dot{V}$ terms are 0 (i.e., profits and the interest rate are constant in the future). Then,

$$V_L = \frac{\beta p_L^{1/\beta} L}{r} \text{ and } V_Z = \frac{\beta p_Z^{1/\beta} Z}{r}. \tag{16}$$

The greater V_Z is relative to V_L, the greater are the incentives to develop Z-complementary machines, N_Z, rather than labor-complementary machines, N_L. Inspection of (16) reveals two forces determining the direction of technical change:

1. The price effect: there will be a greater incentive to invent technologies producing more expensive goods, as shown by the fact that V_Z and V_L are increasing in p_Z and p_L.

2. The market size effect: a larger market for the technology leads to more innovation. Since the market for a technology is effectively the workers who use it, the market size effect encourages innovation for the more abundant factor. This can be seen from the fact that V_L and V_Z are increasing in Z and L, the total supplies of the factors combined with these technologies.

Notice that an increase in the relative factor supply, Z/L, will create both a market size effect and a price effect. The latter simply follows from the fact that an increase in Z/L will reduce the relative price $p \equiv p_Z/p_L$. Equilibrium bias in technical change— whether technical change will favor relatively scarce or abundant factors— is determined by these two opposing forces. An additional determinant of equilibrium bias is the form of the innovation possibilities frontier, which will be introduced in the next section.

This discussion emphasizes the role of price and market size effects, while factor prices do not feature in (16). The early induced innovations literature, instead, argued that innovations would be directed at "more expensive" factors (e.g., Hicks, 1932, Fellner, 1961, Habakkuk, 1962). Using equations (10), (11), (12), and (13), we can reexpress (16) as

$$V_L = \frac{(1-\beta)\, w_L L}{r N_L} \text{ and } V_Z = \frac{(1-\beta)\, w_Z Z}{r N_Z}. \tag{17}$$

This expression shows that an equivalent way of looking at the incentives to develop new technologies is to emphasize factor costs, w_L and w_Z (as well as market sizes, L and Z). As conjectured by the induced innovations literature, there will be more innovation directed at factors that are more expensive.

Next, it is useful to investigate the strength of the price and market size effects in more detail. To do this, let us substitute (10) and (12) into the production functions, (5) and (6). This gives:

$$Y_L = \frac{1}{1-\beta} p_L^{(1-\beta)/\beta} N_L L \text{ and } Y_Z = \frac{1}{1-\beta} p_Z^{(1-\beta)/\beta} N_Z Z. \tag{18}$$

Substituting these into (7) and using some algebra, we obtain the relative price of the two intermediates simply as a function of relative technology, N_Z/N_L, and the relative factor supply, Z/L:

$$p = \left(\frac{1-\gamma}{\gamma}\right)^{\beta\varepsilon/\sigma} \left(\frac{N_Z Z}{N_L L}\right)^{-\beta/\sigma}, \tag{19}$$

where $\varepsilon \equiv 1/(1-\alpha)$ is the elasticity of substitution in consumption between the two intermediate goods, Y_L and Y_Z, while σ is the elasticity of substitution between the two factors, Z and L. In particular, σ is defined as

$$\sigma \equiv \frac{1-\alpha(1-\beta)}{1-\alpha}.$$

and is derived from the elasticity of substitution between the two goods, and $\sigma > 1$ if and only if $\varepsilon > 1$ or $\alpha > 0$—that is, the two factors are gross substitutes only if the two intermediate goods are gross substitutes.

Now using (16) and (19), we can write the relative profitability of creating new Z-complementary machines as (still assuming steady state):

$$\frac{V_Z}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_Z}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{\sigma}}. \tag{20}$$

This expression shows that an increase in the relative factor supply, Z/L, will increase V_Z/V_L as long as the elasticity of substitution between factors, σ, is greater than 1 and it will reduce V_Z/V_L if $\sigma < 1$. Therefore, the elasticity of substitution between the two intermediate goods regulates whether the price effect dominates the market size effect so that there are greater incentives to improve the (physical) productivity of scarce factors, or whether the market size effect dominates, creating greater incentives to improve the productivity of abundant factors. When the intermediate goods are gross substitutes, the market size effect dominates. And when they are gross complements, the price effect dominates.

Finally, to see another important role that the elasticity of substitution between the two goods plays, consider the relative factor rewards, w_Z/w_L. Using equations (10), (11), (12), and (13) and then substituting for (19), we have

$$\frac{w_Z}{w_L} = p^{1/\beta}\frac{N_Z}{N_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_Z}{N_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Z}{L}\right)^{-\frac{1}{\sigma}}. \tag{21}$$

First, note that the relative factor reward, w_Z/w_L, is decreasing in the relative factor supply, Z/L. This is simply the usual substitution effect: the more abundant factor is substituted for the less abundant one, and has a lower marginal product.

As was the case with the goods, the two factors are gross substitutes when $\sigma > 1$. We also see from equation (21) that the same combination of parameters, $(\sigma - 1)/\sigma$, which determines whether innovation for more abundant factors is more profitable also determines whether a greater N_Z/N_L increases w_Z/w_L: when $\sigma > 1$, it does, but when $\sigma < 1$, it has the opposite effect and reduces the reward of the Z-factor relative to labor. This is because N_Z/N_L is the relative physical productivity of the two factors, not their relative value of marginal product. The latter also depends on the elasticity of substitution between the two factors (recall equation (1) in Section 2). Therefore, as in Section 2, Z-biased technical change corresponds to an increase in $(N_Z/N_L)^{(\sigma-1)/\sigma}$, not simply to an increase in N_Z/N_L. Consequently, when $\sigma < 1$, a decrease in N_Z/N_L corresponds to Z-biased technical change. This feature, that the relationship between relative physical productivity and the value of marginal product is reversed when the two goods (factors) are gross complements, will play an important role in the discussion below.

4 The Supply of Innovations: The Innovation Possibilities Frontier

The previous section outlined how the production side of the economy determines the return to different types of innovation—the demand for innovation. The other side of this equation is the cost of different innovations, or using the term introduced by Kennedy (1964), the "innovation possibilities frontier". The analysis in this section will reveal that in addition to the elasticity of substitution, the form of the innovation possibilities frontier—more specifically the degree of *state-dependence*—will have an important effect on the direction of technical change. The degree of state-dependence relates to how future relative costs of innovation are affected by the current composition of R&D (current "state" of R&D). I refer to the innovation possibilities frontier as "state-dependent" when current R&D directed at factor Z reduces the relative cost of Z-complementary R&D in the future.

To start with, I follow the endogenous growth literature in limiting attention to innovation possibilities frontiers that allow steady growth in the long run. I consider the case without sustained growth later. Sustained growth requires that the innovation possibilities frontier takes one of two forms. The first, which Rivera-Batiz and Romer (1991) refer to as the lab equipment specification, involves only the final good being used in generating new innovations. With this specification, steady-state growth is generated with an intuition similar to the growth model of Rebelo (1991) whereby the key accumulation equation is linear and does not employ the scarce (non-accumulated) factors, such as labor. The second possibility is the knowledge-based R&D specification of Rivera-Batiz and Romer (1991) where spillovers from past research to current productivity are necessary to sustain growth. It is useful to distinguish between these two formulations because they naturally correspond to different degrees of *state-dependence* in R&D.

4.1 The Direction of Technical Change with the Lab Equipment Model

Consider the following production function for new machine varieties

$$\dot{N}_L = \eta_L R_L \text{ and } \dot{N}_Z = \eta_Z R_Z, \tag{22}$$

where R_L is spending on R&D for the labor-intensive good (in terms of final good), and R_Z is R&D spending for the Z-intensive good. The parameters η_L and η_Z allow the costs of these two types of innovations to differ. The innovation production functions in (22) imply that one unit of final good spent for R&D directed at labor-complementary machines will generate η_L new varieties of labor-complementary machines (and similarly, one unit of final good directed at Z-complementary machines will generate η_Z new machine types). A firm that discovers a new machine variety receives a perfectly enforced patent on this machine and becomes its sole supplier—the technology monopolist. Notice that with the specification in (22), there is no state-dependence: $\left(\partial\dot{N}_Z/\partial R_Z\right) / \left(\partial\dot{N}_L/\partial R_L\right) = \eta_Z/\eta_L$ is constant irrespective of the levels of N_L and N_Z.

I start with a balanced growth path (BGP)—or steady-state equilibrium—where the prices p_L and p_Z are constant, and N_L and N_Z grow at the same rate. This implies that the $\dot{V}$ terms in equation (15) are 0, and V_Z/V_L is constant. Moreover, this ratio has to be equal to the (inverse) ratio of η's from (22) so that technology monopolists are willing to innovate for both sectors. This immediately implies the following "technology market clearing" condition:[12]

$$\eta_L \pi_L = \eta_Z \pi_Z. \tag{23}$$

This condition requires that it is equally profitable to invest money to invent labor- and Z-complementary machines, so that along the balanced growth path N_L and N_Z can both grow. Defining $\eta \equiv \eta_Z/\eta_L$ to simplify notation and using (14) and (19), this technology market clearing condition can be solved for

$$\frac{N_Z}{N_L} = \eta^{\sigma}\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{Z}{L}\right)^{\sigma-1}, \tag{24}$$

[12] This assumes no corner solution in which R&D is only directed to one of the factors. This does not rule out the economy converging to a long-run equilibrium with only one type of innovation.

where recall that $\varepsilon \equiv 1/(1-\alpha)$ is the elasticity of substitution between the two intermediates and $\sigma \equiv (1-\alpha(1-\beta))/(1-\alpha)$ is the elasticity of substitution between the two factors. Equation (24) shows that with the direction of technical change endogenized, the relative bias of technology, N_Z/N_L, is determined simply by the relative factor supply and the elasticity of substitution between the two factors.

First, consider the case in which $\sigma > 1$, so that the two factors are gross substitutes. Then, an increase in Z/L will raise N_Z/N_L, hence the physical productivity of abundant factors tends to be higher. Moreover, since $\sigma > 1$, a higher level of N_Z/N_L will also correspond to Z-biased technical change. Therefore, technology will be endogenously biased in favor of the more abundant factor.

In the case where the two factors are gross complements, i.e., when $\sigma < 1$, equation (24) implies that N_Z/N_L is lower when Z/L is higher. Nevertheless, because $\sigma < 1$, this lower relative physical productivity translates into a higher *value* of marginal product. Therefore, even when $\sigma < 1$, technology will be endogenously biased in favor of the more abundant factor.

The exception to this result is when $\sigma = 1$, i.e., when the production function is Cobb-Douglas. In this case, the elasticity of substitution between the two intermediates is equal to 1, and technical change is never biased towards one of the factors.[13]

Next consider factor prices. Using equations (21) and (24), we obtain

$$\frac{w_Z}{w_L} = \eta^{\sigma-1}\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{Z}{L}\right)^{\sigma-2}. \tag{25}$$

Comparing this equation to (21), which specified the relative factor prices as a function of relative supplies and technology, we see that the response of relative factor rewards to changes in relative supply is always more elastic in (25): $\sigma - 2 > -1/\sigma$. This is not surprising given the LeChatelier principle, which states that demand curves become more elastic when other factors adjust. Here, the "other factors" correspond to the machine varieties N_L and N_Z.

The more surprising result here is that if σ is sufficiently large, in particular if $\sigma > 2$ (i.e., if $\alpha(1+\beta) > 1$), the relationship between relative factor supplies and relative

[13]Naturally, different types of technical change might affect γ, and therefore the marginal product of the two factors differently.

factor rewards can be upward sloping. I will discuss the relevance of this result later. Intuitively, with exogenous technology when a factor becomes more abundant, its relative reward falls: i.e., given N_Z/N_L, w_Z/w_L is unambiguously decreasing in Z/L due to the usual substitution effect (see equation (21)). Yet, because technology is endogenously biased towards more abundant factors, the overall effect of factor abundance on factor rewards is ambiguous.

Next define the relative factor shares as

$$\frac{s_Z}{s_L} \equiv \frac{w_Z Z}{w_L L} = \eta^{\sigma-1}\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{Z}{L}\right)^{\sigma-1}. \tag{26}$$

This equation states that with endogenous technology, the share of a factor in GDP will increase in the abundance of that factor as long as $\sigma > 1$.

For completeness, it is also useful to determine the long-run growth rate of this economy. To do this, note that the Euler equation from the maximization of (3) gives $g_c = \theta^{-1}(r-\rho)$, where g_c is the growth rate of consumption and recall that r is the interest rate. In BGP, this growth rate will also be equal to the growth rate of output, g. So $r = \theta g + \rho$. Next, using (16), the free-entry condition for the technology monopolists working to invent labor-complementary machines is obtained as $\eta_L V_L = 1$ (naturally, using the other sector's free entry condition gives the same result). In steady state, this condition implies

$$\frac{\eta_L \beta p_L^{1/\beta} L}{r} = 1.$$

Now using (8), (19), and (24) to substitute for p_L, we obtain[14]

$$g = \theta^{-1}\left(\beta\left[(1-\gamma)(\eta_Z Z)^{\sigma-1} + \gamma(\eta_L L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} - \rho\right).$$

Finally, it is useful to briefly look at the behavior of the economy outside the balanced growth path. It is straightforward to verify that outside the balanced growth path, there will only be one type of innovation.[15] If $\eta V_Z/V_L > 1$, there will only be firms creating new Z-complementary machines, and if $\eta V_Z/V_L < 1$, technology monopolists will only

[14] The no-Ponzi game condition requires that $(1-\theta)g < \rho$.

[15] See Proposition 1 in Acemoglu and Zilibotti (2001) for a formal proof that only one type of innovation will take place outside the balanced growth path and for a proof of global stability in a related model. The proof here is identical.

undertake R&D for labor-complementary machines. Moreover, V_Z/V_L is decreasing in N_Z/N_L (recall equation (20)). This implies that the transitory dynamics of the system are locally stable and will return to the balanced growth path. When N_Z/N_L is higher than in (24), there will only be labor-augmenting technical change until the system returns back to balanced growth, and vice versa when N_Z/N_L is too low.

4.2 The Direction of Technical Change with Knowledge-Based R&D

With the lab equipment model of the previous subsection, there is no state-dependence. I now discuss an alternative specification which allows for potential state-dependence. In the lab equipment specification there are no scarce factors that enter the accumulation equation of the economy. If there are scarce factors used for R&D, then growth cannot be maintained by increasing the amount of these factors used for R&D. So for sustained growth, these factors need to become more and more productive over time, because of spillovers from past research. This is the essence of the knowledge-based R&D specification, whereby spillovers ensure that current researchers "stand on the shoulder of giants", ensuring that the marginal productivity of research does not decline. Here for simplicity, I assume that R&D is carried out by scientists, and there is a constant supply of scientists equal to S.[16] If there were only one sector, the knowledge-based R&D specification would require that $\dot{N}/N \propto S$ (proportional to S).

With two sectors, instead, there is a variety of specifications with different degrees of state-dependence, because productivity in each sector can depend on the state of knowledge in both sectors. A flexible formulation is:

$$\dot{N}_L = \eta_L N_L^{(1+\delta)/2} N_Z^{(1-\delta)/2} S_L \text{ and } \dot{N}_Z = \eta_Z N_L^{(1-\delta)/2} N_Z^{(1+\delta)/2} S_Z, \tag{27}$$

for some $\delta \leq 1$. In this specification, δ measures the degree of state-dependence: when $\delta = 0$, there is no state-dependence—$\left(\partial \dot{N}_Z/\partial S_Z\right) / \left(\partial \dot{N}_L/\partial S_L\right) = \eta_Z/\eta_L$ irrespective of the levels of N_L and N_Z—because both N_L and N_Z creates spillovers for current

[16] The results generalize to the case where the R&D sector uses labor (or when Z is taken to be skilled labor, a combination of skilled and unskilled labor), but the analysis of the dynamics becomes substantially more complicated.

research in both sectors. In this case, the results are very similar to those in the previous subsection. In contrast, when $\delta = 1$, there is an extreme amount of state-dependence. In this case, $\left(\partial \dot{N}_Z / \partial R_Z\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_Z N_Z / \eta_L N_L$, so an increase in the stock of labor-complementary machines today makes future labor-complementary innovations much cheaper, and has no effect on the cost of Z-complementary innovations.

The condition for technology market clearing in BGP now changes because relative costs of R&D depend on the current state of technology. In particular, we now have

$$\eta_L N_L^{\delta} \pi_L = \eta_Z N_Z^{\delta} \pi_Z, \tag{28}$$

where δ captures the importance of state-dependence in the technology market clearing condition. When $\delta = 0$, this condition is identical to (23) from the previous subsection. Next solving condition (28) together with equations (14) and (19), we obtain the equilibrium relative technology as

$$\frac{N_Z}{N_L} = \eta^{\frac{\sigma}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{1-\delta\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{1-\delta\sigma}}. \tag{29}$$

Now the relationship between the relative factor supplies and relative physical productivities depends on δ. This is intuitive: as long as $\delta > 0$, an increase in N_Z reduces the relative costs of Z-complementary innovations, so for technology market equilibrium to be restored, π_L needs to fall relative to π_Z. Substituting (29) into (21) gives

$$\frac{w_Z}{w_L} = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}. \tag{30}$$

Relative factor shares are then obtained as

$$\frac{s_Z}{s_L} \equiv \frac{w_Z Z}{w_L L} = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1+\delta-\delta\sigma}{1-\delta\sigma}}. \tag{31}$$

It can be verified that when $\delta = 0$, so that there is no state-dependence in R&D, all of these equations are identical to their counterparts in the previous subsection.

The growth rate of this economy is determined by the number of scientists. In BGP, both sectors grow at the same rate, so we need $\dot{N}_L / N_L = \dot{N}_Z / N_Z$, or $\eta_Z N_Z^{\delta-1} S_Z = \eta_L N_L^{\delta-1} S_L$, which gives $S_L = \eta_Z S / (\eta_Z + \eta_L)$, and $g = \eta_L \eta_Z S / (\eta_Z + \eta_L)$.

Recall that with the lab equipment specification, the economy always tends to the balanced growth path. In contrast, with the knowledge-based R&D specification, the existence of state-dependence implies that the dynamics of the system can be unstable. In particular, there will now only be labor-augmenting technical change if $\eta N_Z^\delta V_Z / N_L^\delta V_L < 1$, and only Z-augmenting technical change if $\eta N_Z^\delta V_Z / N_L^\delta V_L > 1$. However, $\eta N_Z^\delta V_Z / N_L^\delta V_L$ is not necessarily decreasing in N_Z/N_L. Inspection of (20) shows that this depends on whether $\sigma < 1/\delta$ or not. When $\sigma < 1/\delta$, $\partial \left(N_Z^\delta V_Z / N_L^\delta V_L \right) / \partial \left(N_Z/N_L \right) < 0$ and transitory dynamics will take us back to the BGP. In contrast, when $\sigma > 1/\delta$, equilibrium dynamics are unstable and will take us to a corner where only one type of R&D is undertaken. Intuitively, a greater N_Z/N_L creates the usual price and market size effects, but also affects the relative costs of future R&D. If δ is sufficiently high, this latter effect becomes more powerful and creates a destabilizing influence. For example, in the extreme state-dependence case where $\delta = 1$, the system is stable only when $\sigma < 1$, i.e., when the two factors are gross complements.

In what follows, I restrict my attention to cases where the stability condition is satisfied, so we have

$$\sigma < 1/\delta \qquad \text{or} \qquad \delta\alpha\beta < (1-\delta)(1-\alpha). \tag{32}$$

What is the relationship between relative supplies and factor prices? This depends on whether

$$\sigma > 2 - \delta \qquad \text{or} \qquad \alpha\beta > (1-\delta)(1-\alpha). \tag{33}$$

If (33) is satisfied, an increase in the relative abundance of a factor raises its relative marginal product. When $\delta = 0$, this condition is equivalent to $\sigma > 2$, which was obtained in the previous subsection. It is clear that (32) and (33) cannot be simultaneously satisfied when $\delta = 1$. In that case, stability of the system requires that the relative demand curves slope down. However, for all values of δ less than 1, both conditions can be satisfied simultaneously, so the general conclusions reached in the previous subsection continue to apply as long as $\delta < 1$. In fact, some degree of state-dependence makes it more likely that long-run demand curves slope up.

The case of extreme state-dependence, $\delta = 1$, on the other hand, leads to a number of special results. First, when $\delta = 1$, we need $\sigma < 1$—i.e., the two factors to be gross complements—for stability (recall that we need $\sigma < 1/\delta$ for stability). Intuitively, extreme state-dependence implies that when the market size effect is more powerful than the price effect, eventually all innovations will be for one of the sectors. Second, to have innovations for both sectors, we need equation (31) to be satisfied, which implies:

$$\frac{s_Z}{s_L} \equiv \frac{w_Z Z}{w_L L} = \eta^{-1}. \tag{34}$$

Technical change is now acting to equate relative factor shares (and this will be a stable outcome as long as $\sigma < 1$). This formulation of the innovation possibilities frontier therefore leads to the result that Kennedy (1964) conjectured. Recall that Kennedy suggested that induced innovations will push the economy towards constant factor shares (see also Samuelson, 1965, and Drandakis and Phelps, 1965). Here we find that this result obtains when the innovation possibilities frontier takes the form given in (27) with an extreme amount of state-dependence, $\delta = 1$. In the next section, I discuss a number of applications, and I start with the case of limited state-dependence, i.e., $\delta < 1$. I return to applications of the case with $\delta = 1$ later.

4.3 The Scale Effect and the Market Size Effect

The market size effect, the feature that there will be more R&D directed towards the more abundant factor, is crucial for the results so far. I derived this effect using a standard endogenous growth model exhibiting steady long-run growth in the absence of population growth. As is well known, such models also exhibit a "scale effect", in the sense that as population increases, the growth rate of the economy also increases. Jones (1995) convincingly shows that there is little support for such a scale effect, and instead proposes a model where steady growth in income per capita is driven by population growth. One might wonder whether once we remove the scale effect, the market size effect is still present, especially since both effects are related to the presence of aggregate increasing returns to scale due to R&D. In this subsection, I show that the market size effect is independent of the scale effect, so evidence against the scale effect does not constitute evidence against the market size effect. To show this I analyze a model related to that of Jones (1995) without such scale effects.[17]

Suppose that we are in the case with knowledge-based R&D model, but only with limited spillovers from past research. In particular, suppose that equation (27) is modified to

$$\dot{N}_L = \eta_L N_L^{\lambda} S_L \text{ and } \dot{N}_L = \eta_Z N_Z^{\lambda} S_Z, \tag{35}$$

where $\lambda \in (0, 1]$. In the case where $\lambda = 1$, we have the knowledge-based R&D formulation with no state-dependence.[18] When $\lambda < 1$, the extent of spillovers from past research are limited, and this economy will not have steady growth in the absence of population growth. However, as shown by Jones (1995), there will be steady growth when population grows steadily. In particular, suppose that population grows at the exponential rate n. Then it is straightforward to see that output in this economy will grow at the rate

$$g = \frac{n}{1-\lambda},$$

so, there will be steady income per capita growth at the rate $\lambda n/(1-\lambda)$. If $\lambda = 1$, there is no balanced growth, and output would reach infinity in finite time.

The important point for the focus here concerns the market size effect on the direction of technical change. To investigate this issue, note that the technology market clearing condition implied by (35) is

$$\eta_L N_L^{\lambda} \pi_L = \eta_Z N_Z^{\lambda} \pi_Z,$$

which is parallel to (28). Exactly the same analysis as there implies that equilibrium relative technology is

$$\frac{N_Z}{N_L} = \eta^{\frac{\sigma}{1-\lambda\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{1-\lambda\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{1-\lambda\sigma}}, \tag{36}$$

and long-run relative factor price is

$$\frac{w_Z}{w_L} = \eta^{\frac{\sigma-1}{1-\lambda\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\lambda)\varepsilon}{1-\lambda\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-2+\lambda}{1-\lambda\sigma}}. \tag{37}$$

[17] Other alternatives include Young (1998) and Howitt (1999). I choose the formulation similar to that of Jones (1995) for simplicity.

[18] It is also straightforward to introduce additional state-dependence into this formulation, but I choose not do so to simplify the discussion.

Therefore, we obtain exactly the same results as before. Specifically, there will always be technical change biased towards the factor that has become more abundant, and if $\sigma > 2 - \lambda$, the long-run relative demand curve for factors will be slope up.

4.4 Discussion

The analysis so far has highlighted two important determinants of the direction of technical change. The first is the degree of substitution between the two factors, which is derived from the degree of substitution between the two goods that these factors produce. When the two factors are more substitutable, the market size effect is stronger, and endogenous technical change is more likely to favor the more abundant factor. The second determinant of the direction of technical change is the the degree of state-dependence in the innovation possibilities frontier.[19]

A natural question to ask at this point is whether existing empirical evidence can help us determine whether there is substantial state-dependence in the R&D process. To the best of my knowledge, there has been no direct investigation of this issue. The data on patent citations analyzed by, among others, Jaffe, Trajtenberg and Henderson (1993), Trajtenberg, Henderson and Jaffe (1992) and Caballero and Jaffe (1993) may be useful in this regard. These papers study the extent of subsequent citations of patents. A citation of a previous patent is interpreted as evidence that a current invention is exploiting information generated by the previous invention. This corresponds to the presence of N_L and N_Z terms in equation (27) in the model here. We can therefore use

[19] The analysis assumed that all technical change results from purposeful R&D. Plausibly, there may also be some "technology drift" that is not directed. For example, some new technologies may be invented as a result of non-profit maximizing incentives. Perhaps more important, big inventions, what Mokyr (1991) refers to as macro-innovations or what Breshnahan and Trajtenberg (1994) call general-purpose technologies, may reflect the natural progress of science, and respond little to profit incentives. The direction of technical change would then determine how these macro-innovations are developed for commercial use. It is straightforward to generalize the above results to this case. For example, suppose the innovation possibilities frontier is

$$\dot{N}_L = \xi_L N_L + \eta_L R_L \text{ and } \dot{N}_Z = \xi_Z N_Z + \eta_Z R_Z,$$

where the ξ_L and ξ_Z terms capture the technology drift, that is, improvements in technology that are unrelated to R&D efforts directed at different types of innovation. The presence of the ξ's terms does not change the marginal return to different types of innovation. Therefore, for an equilibrium in which there is both types of R&D, we still need the innovation equilibrium condition, (23), to hold.

patent citations data to investigate whether there is state-dependence at the industry level. Industry level state-dependence corresponds to patents being cited in the same industry in which they originated. Unfortunately, it is currently impossible to investigate state-dependence at the factor level. This is because, although we have information about the industry for which the patent was developed, we do not know which factor the innovation was directed at.

Results reported in Table 1 in Trajtenberg, Henderson and Jaffe (1992) suggest that there is some amount of industry state-dependence, but this is limited. For example, patents are likely to be cited in the same three-digit industry from which they originated, but on average, they are more often cited in a different three-digit industry. In the future, it may be possible to develop a classification of different innovations into capital, labor and skill-complementary groups to investigate state-dependence at the factor level. This seems a fruitful area for future empirical research.

5 Applications (With Limited State-Dependence)

In this section, I discuss some of the applications of directed technical change. I will emphasize both why models with endogenously biased technical change are useful in thinking about a range of problems and how the results depend on the elasticity of substitution. I start with a range of applications where a formulation with limited state-dependence, i.e., $\delta < 1$, appears to be more appropriate.

5.1 Endogenous Skill-Biased Technical Change

Figure 1 plots a measure of the relative supply of skills (the number of college equivalent workers divided by noncollege equivalents) and a measure of the return to skills (the college premium). It shows that over the past 60 years, the U.S. relative supply of skills has increased rapidly, and in the meantime, the college premium has also increased. The figure shows that beginning in late 1960s, the relative supply of skills increased much more rapidly than before. In response, the skill premium fell during the 1970s, and then increased sharply during the 1980s and the early 1990s.

Existing explanations for these facts emphasize exogenous skill-biased technical change. Technical change is assumed to naturally favor skilled workers, perhaps because new tasks are more complex and generate a greater demand for skills. This skill bias explains the secular behavior of the relative supply and returns to skills. Moreover, the conventional wisdom is that there has been an acceleration in the skill bias of technology precisely around the same time as the relative supply of skills started increasing much more rapidly. This acceleration explains the rapid increase in the skill premium and wage inequality during the 1980s.

A model based on directed technical change suggests an alternative explanation. Suppose that the second factor in the model above, Z, is skilled labor, which I denote by H. Also assume that the innovation possibilities frontier is given by (27), so that we are in the knowledge-based R&D model. This is without any loss of generality since for the focus here, the lab equipment specification corresponds to the case with $\delta = 0$ in terms of (27). Then, equation (30) from Section 4 implies that the skill premium in the long run is given by

$$\text{skill premium}=\frac{w_H}{w_L} = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}. \tag{38}$$

The framework developed here implies that the increase in the supply of skills creates a tendency for new technologies to be skill-biased. This offers a possible explanation for the secular skill-biased technical change of the twentieth century. Equally interesting, if $\sigma > 2 - \delta$, then the long-run relationship between the relative supply of skills and the skill premium is positive: greater skill abundance leads to a greater skill premium. Intuitively, with σ large enough, the market size effect is so powerful that it not only dominates the price effect on innovations, but also dominates the usual substitution effect between skilled and unskilled workers at a given technology (as captured by equation (21) above). As a result, an increase in the relative supply of skills makes technology sufficiently skill-biased to raise the skill premium. In this case, the framework predicts sufficiently biased technical change over the past 60 years to actually increase the skill premium, consistent with a pattern depicted in Figure 1.

The same model, with $\sigma > 2 - \delta$, also offers an explanation for the behavior of the college premium during the 1970s and the 1980s shown in Figure 1. Suppose that the economy is hit by a large increase in H/L. If this increase is not anticipated sufficiently in advance, technology will not have adjusted by the time the supply of skills increases. The initial response of the skill premium will be given by equation (21) which takes technology as given. Therefore, the skill premium will at first fall, but then as technology starts adjusting, it will rise rapidly. Figure 3 draws this case diagramatically.

This framework also provides a possible explanation for why technical change during the late eighteenth and early nineteenth centuries may have been biased towards unskilled labor. The emergence of the most "skill-replacing" technologies of the past two hundred years, the factory system, coincided with a large change in relative supplies. This time, there was a large migration of unskilled workers from villages and Ireland to English cities and a large increase in population (see, for example, Habakkuk, 1962, Bairoch, 1988, or Williamson, 1990). This increase created profit opportunities for firms in introducing technologies that could be used with unskilled workers. In fact, contemporary historians considered the incentive to replace skilled artisans by unskilled laborers as a major objective of technological improvements of the period. Ure, a historian in the first half of the nineteenth century, describes these incentives as follows:

> "It is, in fact, the constant aim and tendency of every improvement in machinery to supersede human labor altogether, or to diminish its costs, by substituting the industry of women and children for that of men; of that of ordinary labourers, for trained artisans." (quoted in Habakkuk, 1962, p. 154).

The framework developed here is consistent with the notion that the incentives for skill-replacing technologies were shaped by the large increase in the supply of unskilled workers.

Therefore, with a sufficiently large elasticity of substitution, this model provides an attractive explanation for the skill-replacing technical change of the nineteenth century, the secular skill bias of technology throughout twentieth century, and the recent acceleration in this skill bias and inequality dynamics.

Is it empirically plausible to think that $\sigma > 2 - \delta$—or to think that there is a high degree of substitution? In the case of skilled and unskilled workers, a high degree of substitution seems reasonable. Although the elasticity of substitution is generally difficult to estimate, there are a number of estimates using aggregate data that give a range of plausible values. The difficulty arises from the fact that the elasticity in question refers to the elasticity of substitution in the aggregate and holding technology constant. For this reason, the most reliable estimates appear to be those using time-series variation, since technology may be thought to respond relatively little to high frequency variations in relative supply. The great majority of these estimates are above $\sigma = 1$ (see, for example, Freeman, 1986). An interesting study by Angrist (1995) uses a "natural experiment" arising from the large increase in university enrollments for Palestinian labor and estimates an elasticity of substitution between workers with 16 years of schooling and those with less than 12 of schooling of approximately $\sigma = 2$.

Whether the elasticity of substitution is large enough to make the long-run relative demand curve to slope up will also depend on the degree of state-dependence, δ. To start with, take the case with no state-dependence, $\delta = 0$ (or the lab equipment specification). This implies that with $\delta = 0$, there will be an upward-sloping long-run relative demand curve for skills, when σ is greater than 2. This is a relatively large elasticity, but not outside the plausible range of values. When $\delta > 0$, so that there is some amount of state-dependence, smaller values of the elasticity of substitution are sufficient to generate an upward sloping long-run demand curve.[20]

Finally, to see why I chose to focus of the case of limited state-dependence, recall from equation (34) that when $\delta = 1$, in the long-run we must have $s_H/s_L = \eta^{-1}$. Therefore, the relative shares of skilled and unskilled labor in GDP should be constant (cfr. equation (34)). This is clearly not consistent with the pattern depicted in Figure 1, where both the number of skilled workers and their relative renumeration have been increasing over time.

[20]However, now, we have to check whether the system is stable. For example, if $\sigma = 1.4$, (32) implies that we need $\delta < 0.71$ for stability.

5.2 Directed Technical Change and Cross-Country Income Differences

Many less developed countries (LDCs) use technologies developed in the U.S. and other OECD economies (the North). A number of economists, including Atkinson and Stiglitz (1969), David (1975), Stewart (1978), and Basu and Weil (1998), have pointed out that imported technologies may not be "appropriate" to LDCs' needs. Directed technical change increases these concerns: it implies that technologies will be designed to make optimal use of the conditions and factor supplies in the North. Therefore, they will be highly inappropriate to the LDCs' needs (Acemoglu and Zilibotti, 2001). Because it is still often profitable for the LDCs to use these technologies rather than develop their own, the extent of directed technical change will determine how inappropriate technologies in use in the LDCs' are to their needs. Via this channel, directed technical change will influence the income gap between the North and the LDCs. I now use the above framework to discuss this issue.

Suppose that the model outlined above applies to a country I refer to as "the North" (either the U.S. or all the OECD countries as a whole). Also to simplify the algebra, I now focus on the case with no state-dependence $\delta = 0$, though all the results generalize to the case of limited state-dependence immediately. Suppose also that there are a set of LDCs in this world economy who will use the technologies developed in the North. Take Z to be skilled labor, though for the results in this subsection, it could also stand for physical capital. For simplicity, I take all of these countries to be identical, with L' unskilled workers and H' skilled workers, and normalize the mass of these countries to 1, so that L' and H' are also the total supplies of unskilled and skilled workers in the LDCs. A key characteristic of the LDCs is that they are less abundant in skilled workers than the North, so

$$\frac{H'}{L'} < \frac{H}{L}.$$

I assume that because of lack of intellectual property rights, all LDCs can copy new machine varieties invented in the North without paying royalties to Northern technology monopolists. This assumption implies that the relevant markets for the technology monopolists will be given by the factor supplies in the North. I also assume that the cost of producing machines in the LDCs could be higher, $\kappa^{-\beta/(1-\beta)}$ rather than $\psi \equiv 1-\beta$ as in the North. This cost differential may result from the fact that firms in the LDCs do not have access to the same knowledge base as the technology monopolists in the North. Suppose also that there is free entry to copying Northern machines, so that all machines sell at marginal cost in the LDCs (but LDC firms cannot reexport machines to the North). Finally, for now there is no international trade between the North and the LDCs.

An analysis similar to above immediately gives intermediate output in the LDCs parallel to those in the North (recall equation (18)):

$$Y'_L = \frac{1}{1-\beta}\left(p'_L\right)^{(1-\beta)/\beta}\kappa N_L L' \text{ and } Y'_H = \frac{1}{1-\beta}\left(p'_H\right)^{(1-\beta)/\beta}\kappa N_H H',$$

where p''s denote prices in the LDCs, which differ from those in the North because factor proportions are different and there is no international trade. The parameter κ features in these equations since machine costs are different in the South. It is natural to think of κ as less than 1, so that machine prices are higher and fewer machines are used in the South than in the North. Notice also that the technology terms, N_L and N_H, are the same as in the North, since these technologies are copied from the North. Using this equation and (4) and (18), the ratio of aggregate income in the South to that in the North can be written as:

$$\frac{Y'}{Y} = \frac{\kappa\left[\gamma\left(\left(p'_L\right)^{(1-\beta)/\beta}N_L L'\right)^{\alpha} + (1-\gamma)\left(\left(p'_H\right)^{(1-\beta)/\beta}N_H H'\right)^{\alpha}\right]^{1/\alpha}}{\left[\gamma\left(p_L^{(1-\beta)/\beta}N_L L\right)^{\alpha} + (1-\gamma)\left(p_H^{(1-\beta)/\beta}N_H H\right)^{\alpha}\right]^{1/\alpha}}. \tag{39}$$

In the Appendix, I show that

$$\frac{\partial Y'/Y}{\partial N_H/N_L} \propto \frac{1-\beta}{\sigma}\left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}}\left(\left(\frac{H'}{L'}\right)^{\frac{\sigma-1}{\sigma}} - \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}\right). \tag{40}$$

Since $H'/L' < H/L$, this expression implies that when $\sigma > 1$, i.e., when the two factors are gross substitutes, an increase in N_H/N_L increases the income gap between the LDCs and the North (i.e. reduces Y'/Y). In contrast, when $\sigma < 1$, so that the two factors are gross complements, an increase in N_H/N_L narrows the income gap. The intuition for both results is straightforward. An increase in N_H/N_L increases the productivity of skilled workers relative to the productivity of the unskilled. Therefore, everything else equal, a society with more skilled workers benefits more from this type of technical change. However, this change also affects the relative scarcity, and therefore the relative price, of the two goods. In particular, the skill-intensive good becomes more abundant and its relative price falls. When $\sigma > 1$, this effect is weak, and the North still gains more in proportional terms, and the ratio of LDC income to income in the North falls. However, when $\sigma < 1$, the price effect is strong, and as a result, in proportional terms, the skill-abundant North benefits less than the LDCs.

Next, recall that when $\sigma > 1$, an increase in N_H/N_L corresponds to skill-biased technical change, while when $\sigma < 1$, it is a decrease in N_H/N_L that corresponds to skill-biased technical change. Therefore, irrespective of the value of σ, skill-biased technical change increases the income gap between the LDCs and the North. This extends the results in Acemoglu and Zilibotti (2001) to a slightly more general model, and also more importantly, to the case where the two factors are gross complements, i.e., $\sigma < 1$.

This result is important since most economists believe that the past twenty-five years have witnessed rapid skill-biased technical change (as discussed in subsection 5.1). It suggests that an unforeseen consequence of this skill-biased technical change is to create a force widening the income gap between LDCs and rich economies (see Berman, 2000, for an empirical investigation of this point).

Now to highlight the role of directed technical change more sharply, contrast two situations:

1. New technologies are developed for the Northern market.

2. New technologies are developed for the world market.

The first situation applies when there are no intellectual property rights in the LDCs for Northern companies, and I view it as a good approximation to reality. I compare it to the second case to highlight how directed technical change will create a force towards a larger income gap between the LDCs and the North.

When new technologies are developed for the Northern market, equation (24) applies and $N_H/N_L \propto (H/L)^{\frac{\sigma-1}{\sigma}}$. In contrast, when new technologies can also be sold in the LDC markets, we have $N_H/N_L \propto \left(H^W/L^W\right)^{\frac{\sigma-1}{\sigma}}$ where $H^W/L^W = (H + \kappa H')/(L + \kappa L')$ is the world (effective) ratio of skilled to unskilled workers. The parameter κ features in this equation because it parameterizes the relative productivity/demand for machines of LDC workers. By the assumption that $H'/L' < H/L$, we have $H^W/L^W < H/L$.

Suppose $\sigma > 1$, then in the world without intellectual property rights, technologies developed in the North will feature too high a level of N_H/N_L for the LDCs' needs. In contrast, when $\sigma < 1$, then technologies developed in the North will feature too low a level of N_H/N_L for the LDCs. However, in the latter case, a lower N_H/N_L corresponds to more skill-biased technology, since skilled and unskilled labor are gross complements. Therefore, in both cases technologies developed in the North will be "too skill-biased" for the LDCs. So directed technical change creates a force towards a larger income gap between the rich and the poor. This result is intuitive: there are more skilled workers in the North, and directed technical change induces technology monopolists in the North to develop technologies appropriate for skilled workers (i.e., when $\sigma > 1$, higher N_H/N_L, and when $\sigma < 1$, lower N_H/N_L). These skill-biased technologies are less useful for LDCs, so LDCs benefit less than the North, and the income gap is larger than it would have been in the absence of directed technical change.

5.3 The Effect of International Trade on Technical Change

The literature on trade and growth shows how patterns of international trade affect the rate of technical change (e.g., Grossman and Helpman, 1991). Similarly, when the direction of technical change is endogenous, changes in the amount of international trade may affect the type of technologies that are developed. This may be relevant for current economic concerns, for example because, as claimed by Adrian Wood, trade opening may have affected wage inequality through its effect on "defensive" innovations.[21]

To address these issues, consider the same setup as in the previous subsection with a

[21] This point is also suggested in Acemoglu (1998) and analyzed in detail in Acemoglu (1999a). Xu (2001) extends the set up of Acemoglu (1999a), which I use here, to have both goods employ both factors.

set of LDCs who can copy innovations from the North, again maintaining the assumption of no state-dependence (i.e., $\delta = 0$ or the lab equipment specification). Assume that at first there is no international trade, so the equilibrium characterized in the previous subsection applies. In particular, the relative price of skill-intensive goods in the North is given by a similar equation to (19):

$$p = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\beta\varepsilon}{\sigma}} \left(\frac{N_H H}{N_L L}\right)^{-\frac{\beta}{\sigma}}, \tag{41}$$

and the skill premium is given by an equation equivalent to (21). When skill bias of technology is endogenized (with no state-dependence), N_H/N_L is given by (24).

Next, suppose that there is full opening to international trade, with both intermediate goods traded costlessly. Assume that the structure of intellectual property rights is unchanged as a result of this trade opening.[22] International trade will generate a single world relative price of skill-intensive goods, p^W. To determine this price, note that the total supply of skill-intensive goods will be $\left(p_H^W\right)^{(1-\beta)/\beta} N_H^W \left(H + \kappa H'\right) / (1-\beta)$, and the total supply of labor-intensive goods will be $\left(p_L^W\right)^{(1-\beta)/\beta} N_L^W \left(L + \kappa L'\right) / (1-\beta)$. Using these expressions and equation (7), the world relative price is obtained as

$$p^W = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\beta\varepsilon}{\sigma}} \left(\frac{N_H^W \left(H + \kappa H'\right)}{N_L^W \left(L + \kappa L'\right)}\right)^{-\frac{\beta}{\sigma}} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\beta\varepsilon}{\sigma}} \left(\lambda^{-1} \frac{N_H^W H}{N_L^W L}\right)^{-\frac{\beta}{\sigma}}, \tag{42}$$

where the last equality defines $\lambda \equiv (H/L) / \left(\left(H + \kappa H'\right) / \left(L + \kappa L'\right)\right) > 1$. The fact that $\lambda > 1$ follows because $H'/L' < H/L$. I also use the notation N_H^W and N_L^W to emphasize that world technologies may change from their pre-trade levels in the North as a result of international trade.

Since skills are scarcer in the world economy than in the North alone, trade opening will increase the relative price of skill-intensive goods in the North, i.e. $p^W > p$. This is a straightforward application of standard trade theory. What is different here is that this change in product prices will also affect the direction of technical change. In particular, recall that the two forces shaping the direction of technical change are the market size effect and the price effect. Because trade does not affect the structure of intellectual

[22] See Acemoglu (1999a) for a discussion of the results when the enforcement of property rights changes.

property rights, the market sizes for different types of technologies do not change. But product prices are affected by trade, so the price effect will be operational. Since the price effect encourages innovations for the scarce factor, international trade, which makes skills more scarce in the North, will induce more innovations directed at skilled workers.

To see this more formally, once again consider the case of no state-dependence in R&D (i.e., the lab equipment specification, or (27) with $\delta = 0$). For the technology market to clear, we need condition (23) to be satisfied. Combining the relative price of skill-intensive goods, now given by (42), with technology market clearing condition (23), we obtain

$$\frac{N_H^W}{N_L^W} = \eta^\sigma \lambda \left(\frac{1-\gamma}{\gamma}\right)^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}. \tag{43}$$

Comparing this equation to (24), we see immediately that, because $\lambda > 1$, trade increases the physical productivity of skilled workers more than that of unskilled workers. As usual, this increase in N_H/N_L may not correspond to skill-biased technical change if $\sigma < 1$.

To study the effect of this induced change in technology on factor prices, first note that (42) implies that without a change in technology trade opening would increase the skill premium in the North to

$$\frac{w_H}{w_L} = \left(p^W\right)^{1/\beta} \frac{N_H}{N_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \lambda^{\frac{1}{\sigma}} \left(\frac{N_H}{N_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \tag{44}$$

Comparing this equation to (21) from before, we see that trade opening necessarily increases the skill premium, which is a simple application of standard trade theory.

When technology responds to trade opening, the skill premium in the North, instead, changes to

$$\frac{w_H}{w_L} = \left(p^W\right)^{1/\beta} \frac{N_H^W}{N_L^W} = \eta^{\sigma-1} \lambda \left(\frac{1-\gamma}{\gamma}\right)^\varepsilon \left(\frac{H}{L}\right)^{\sigma-2}. \tag{45}$$

This equation differs from (44) because the skill bias of technology is now given by (43) rather than being held constant at the pre–trade level.

Comparing the post-trade skill premium, (45), to the skill premium before trade opening (e.g. equation (25) or equation (38) from subsection 5.1), we see that irrespective of whether $\sigma > 1$ or not, trade opening increases the skill premium. However, it is more informative to compare the magnitude of the increase in the skill premium with induced technical change to the increase that would have occurred without technical change, i.e., compare equation (45) to equation (44). This comparison shows that the induced technical change will increase the skill premium by more than predicted by the standard trade theory only when $\sigma > 1$. The intuition is simple: expression (43) shows that trade will induce technical change to increase the relative physical productivity of skilled workers. An increase in relative physical productivity translates into skill-biased technical change only when $\sigma > 1$. Therefore, we obtain the result that, as conjectured by Adrian Wood, trade opening could induce skill-biased technical change and increase wage inequality more than predicted by standard trade theory. Yet, this conclusion obtains only when skilled and unskilled workers are gross substitutes (which appears to be the empirically relevant case).

5.4 The Habakkuk Hypothesis

Finally, I use the above framework to discuss the Habakkuk hypothesis. According to this hypothesis, the more rapid technical change or technology adoption in the U.S. economy during the ninteenth century relative to the British economy resulted from the relative labor scarcity in the U.S.. Habakkuk (1962) argued that this labor scarcity encouraged firms to develop and adopt labor-saving technologies. Despite the prominence of this hypothesis in the economic history literature, there has been no widely accepted formalization of the argument, and we are consequently unaware of under what circumstances Habakkuk's conclusion would apply (see, David, 1975). In addition, standard economic theory suggests that higher wages may reduce investment, and via this channel, discourage innovation. Moreover, in contrast to the basic premise of the Habakkuk hypothesis, the endogenous growth literature emphasizes the presence of a scale effect which suggests that a larger workforce should encourage more innovation. Could the Habakkuk hypothesis be valid despite these countervailing effects?

To discuss this case, suppose that Z now stands for land. Moreover, assume that $\eta_Z = 0$, which implies that there are no land-complementary innovations. The only source of technical change is labor-complementary innovations. The rest of the set-up is the same as before, and to economize on space, I will only study the lab equipment specification. The question is under what circumstances greater labor scarcity (smaller L/Z) will lead to a higher level of N_L—i.e., to more innovations. From (16), the free-entry condition for technology monopolists, $\eta_L V_L = 1$, implies[23]

$$\frac{\eta_L \beta p_L^{1/\beta} L}{r} = 1.$$

Now using (8) and (19), this condition gives:

$$\eta_L \beta \left[\Lambda \left(\frac{N_L}{Z} \right)^{\frac{1-\sigma}{\sigma}} L^{-\frac{(1-\sigma)^2}{\sigma}} + \gamma L^{\sigma-1} \right]^{\frac{1}{\sigma-1}} = r, \tag{46}$$

where Λ is a suitably defined constant. Inspection of equation (46) immediately shows that $\partial N_L / \partial Z > 0$. Therefore, a greater abundance of land (for a given level of employment) always encourages the creation of more labor-complementary technologies. So comparing the U.S. to Britain, as Habakkuk did, leads to the conclusion that there should be faster technical change in the more land-abundant U.S..

On the other hand, the sign of $\partial N_L / \partial L$—i.e., the effect of labor-scarcity for a given supply of land—is ambiguous. If $\sigma > 1$, so that labor and land are gross substitutes, it can be verified that $\partial N_L / \partial L > 0$, hence in this case, greater scarcity of labor (for a given supply of land) discourages the development of new technologies. In contrast, if σ is sufficiently smaller than 1, i.e., if labor and land are sufficiently complementary, we can have $\partial N_L / \partial L < 0$.

This is intuitive. A greater scarcity of labor creates two forces: the price of the labor-intensive good is higher, but also the market size for labor-complementary technology is smaller. Which force dominates depends on the strength of the market size effect, which is again a function of the degree of substitutability. If $\sigma > 1$, the market size effect is powerful and we obtain the opposite of the result conjectured by Habakkuk—it is not labor scarcity, but labor abundance that spurs innovation. However, with sufficient complementarity between labor and land, the model gives the result conjectured by

[23] Because there is only innovation in one of the sectors, there may no longer be growth in the long run. In particular, if the two goods are gross complements, i.e., $\sigma < 1$, there will not be long-run growth. The interest rate is always given by $r = \theta g + \rho$, which implies $r = \rho$, when $g = 0$.

Habakkuk—greater labor scarcity leads to faster innovation. Therefore, in this framework the Habakkuk hypothesis requires labor and land to be highly complementary. Since the focus of Habakkuk's analysis was technical change in agriculture and low-tech manufacturing during the nineteenth century, the assumption of a high degree of complementarity between labor and other inputs may be realistic. Moreover, the finding that a low elasticity of substitution between labor and other factors is necessary for Habakkuk's hypothesis is intuitive: Habakkuk emphasized high wages as the inducement to technical change. Labor scarcity translates into higher wages when the elasticity of substitution is low.

6 Labor-Augmenting Technical Change and State-Dependence

6.1 Why Is Long-Run Technical Change Labor-Augmenting?

I have so far discussed a number of applications of directed technical change with limited state-dependence. Next consider another regularity discussed in the introduction: the share of labor and capital have been approximately constant in U.S. GDP, while the capital-labor ratio has been increasing steadily. This suggests that technical change has been predominantly labor-augmenting (unless the elasticity of substitution between capital and labor happens to be exactly equal to 1). Can a model of directed technical change be useful in thinking about why aggregate technical change appears to be labor-augmenting?

To provide an answer to this question, suppose that Z corresponds to physical capital.[24] Assume also that $Z = K$ is growing over time due to capital accumulation. Let us start with the flexible innovation possibilities frontier given by (27), where δ parameterizates the degree of state-dependence. Then (31) implies that

$$\frac{s_K}{s_L} = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\delta)\varepsilon}{(1-\delta\sigma)}} \left(\frac{K}{L}\right)^{\frac{\sigma-1+\delta-\delta\sigma}{1-\delta\sigma}} . \tag{47}$$

[24] One has to be careful with this interpretation since the model already features "machines" which can also be thought as capital.

Since there is capital accumulation, i.e., K/L is increasing, equation (47) implies that unless $\sigma = 1$ (the elasticity of substitution exactly equal to 1), or $\delta = 1$ (extreme state-dependence), factor shares will not be constant: with the capital-labor ratio growing, the share of capital will be contracting or expanding, and there will exist no balanced growth path. This implies that neither the lab equipment specification nor the knowledge-based R&D specification with limited state-dependence are consistent with a constant capital share or with purely labor-augmenting technical change.

Therefore, if we want to construct a model where all technical change is (endogenously) labor-augmenting, we need to adopt the innovation possibilities frontier (27) with $\delta = 1$—i.e., adopt an extreme amount of state-dependence. In Acemoglu (1999b), I show in more detail that when the innovation possibilities frontier takes the form in (27) with $\delta = 1$ and the elasticity of substitution is less than 1, there exists a unique equilibrium path tending to a BGP with only labor-augmenting technical change. This result can be interpreted as either a positive or negative one: on the positive side, it shows that it is possible to construct a model where equilibrium long-run technical change is labor-augmenting, even though capital-augmenting technical change is also allowed. Moreover, the economy converges to this equilibrium, and on the transition path, there will typically be capital-augmenting technical change, which implies that technical change is not always labor-augmenting, but only in the very long run. On the negative side, it shows that this result only obtains when there is an extreme amount of state-dependence in R&D, i.e. $\delta = 1$, and when the elasticity of substitution between labor and capital is less than 1. This leads to two natural questions.

First, is an elasticity of substitution less than 1 between capital and labor, i.e., $\sigma < 1$, reasonable? We know even less about the elasticity of substitution between capital and labor than about the elasticity of substitution between skilled and unskilled labor. Nevertheless, most estimates suggest that an elasticity less than 1 is reasonable. As was the case for the elasticity of substitution between skilled and unskilled workers, the relevant elasticity is for the aggregate, so the more reliable estimates exploit time-series variation. Using time-series data, Coen (1969), Eisner and Nadiri (1968), and

Lucas (1969), for example, all estimate elasticities significantly less than 1.[25]

Second, how do we reconcile this result, which requires extreme state-dependence, $\delta = 1$, with the findings in the previous subsection, which relied on limited state-dependence, i.e. $\delta < 1$? One possibility is to accept defeat, and use models of directed technical change only to discuss issues related to labor, skill and land-complementary technical change where, as discussed in the previous section, a model with limited state-dependence leads to realistic predictions. Another possibility is to develop a hybrid framework where the degree of state-dependence varies across innovations directed at different factors. I give the basic idea here with a simple unified framework.

Suppose that the final good now takes the form:

$$Y = \left[\gamma Y_K^{\alpha} + (1-\gamma) Y_L^{\alpha}\right]^{1/\alpha},$$

defined as a CES aggregator of a capital and a labor-intensive good. The capital intensive good has a production function similar to before: $Y_K = \int_0^{M_K} x_{mk}^{1-\beta}(j)\, dj K^{\beta}$, where x_{mk}'s denote the amounts of capital-complementary machines and M_K denotes the varieties of capital-complementary machines.

The labor intensive good is produced using both skilled and unskilled labor, and features three different types of technologies:

$$Y_L = \left(\int_0^{M_L} x_{ml}^{1-\beta}(j)\, dj\right)\left(\gamma_L\left(\int_0^{N_L} x_{nl}^{1-\beta}(j)\, dj L^{\beta}\right)^{\varsigma} + (1-\gamma_L)\left(\int_0^{N_H} x_{nh}^{1-\beta}(j)\, dj H^{\beta}\right)^{\varsigma}\right)^{\beta/\varsigma},$$

where M_L denotes the type of machines that complement both skilled and unskilled labor, and N_L and N_H are the range of machines that are used separately by unskilled and skilled workers respectively. The x's again denote machine quantities. Therefore, overall there are 4 different types of technical change: capital augmenting, labor-augmenting, and then separately augmenting skilled and unskilled labor. Now imagine the following innovation possibilities frontier:

$$\frac{\dot{M}_K}{M_K} = \eta_{MK} S_{MK},\ \frac{\dot{M}_L}{M_L} = \eta_{ML} S_{ML},\ \dot{N}_H = \eta_{NH} M_L S_{NH} \text{ and } \dot{N}_L = \eta_{NL} M_L S_{NL},$$

[25] Berndt (1976) claims that the use of higher quality data leads to higher estimates of the elasticity of substitution. Nevertheless, Berndt (1976) does not control for a time trend in his estimation, biasing his results towards one.

where again S denotes scientists, and the different subscripts specify in which sectors the scientists are working. This specification implies that the technology to generate capital and labor-augmenting innovations—the M's—exhibits extreme state-dependence with $\delta = 1$. However, the innovation possibilities frontier to produce new technologies for unskilled labor or skilled labor features limited state-dependence.[26] In particular, new technologies for both skilled and unskilled, N_H and N_L, build on some general-purpose "labor-augmenting" body of knowledge, captured by M_L (the variety of labor-augmenting machines).

This formulation provides a unified framework for analyzing endogenously biased technical change both between skilled and unskilled labor, and between labor and capital. The bias of technical change toward skills is determined by the elasticity of substitution (the strength of the market size effect) and the results obtained in Section 5 continue to apply. In contrast, the innovation possibilities frontier exhibits extreme path dependence between capital and labor, so the bias of technical change between labor and capital is shaped by the requirement that the relative shares of these two factors remain stable (cfr. equation (34)).

The problem with this specification is that it does not have micro-foundations. Why should there be extreme state-dependence in R&D directed at labor vs. capital, but only limited state-dependence in R&D for skilled vs. unskilled labor? An investigation of more primitive assumptions that might lead to such a result is a useful area for future study.

6.2 Wage-Push and Technical Change

Finally, I use the above framework to investigate whether a "wage-push shock" can cause capital-biased technical change. As discussed in the introduction, Blanchard (1997) documents that both unemployment and the labor share in a number of continental European economies rose sharply starting in the late 1960s. Both Blanchard and Caballero and Hammour (1998) interpret this as the response of these economies to a wage-push;

[26]Here, for simplicity, I take a formulation with no path-dependence for innovations directed at skilled and unskilled workers, though this can be generalized easily.

the militancy and/or the bargaining power of workers increased because of the changes in labor market regulations taking place over this time period, or because of the ideological effects of 1968. This wage-push translated into higher wages and lower employment. During the 1980s, we see a different pattern: unemployment in these countries continues to increase, but in most countries, the labor share falls sharply and ends below its starting level. Blanchard conjectures that this may be due to "biased technical change", while Caballero and Hammour, similarly, argue that a change in the degree of capital-labor substitution is responsible for the trends in the 1980s, and that this was a response to the wage-push of the 1970s. A similar situation occurs in Portugal following the revolution, with the labor share increasing to over 90 percent of output, but then sharply falling back to a much lower level. A model of endogenously biased technical change gives us an opportunity to study these issues.

Consider the above framework with Z interpreted as capital, K, and as before, focus on the case with elasticity of substitution less than 1, i.e., $\sigma < 1$. In addition, in order to study wage-push, let me introduce a quasi-labor supply function:

$$L = m\left(s_L\right),$$

where recall that s_L is the labor share, and I assume $m' > 0$. This function relates the supply of labor to the labor share rather than to the level of the wage, since, to be consistent with long-run facts, labor supply must remain roughly constant when the wage rate increases steadily. Also it is worth noting that this formulation captures both the labor supply decisions of workers and bargaining between firms and workers. For example, with a bargaining setup, when unemployment is high (employment is low), workers will have a weaker bargaining position, and only receive a smaller share of output, i.e. a lower s_L. To simplify the algebra, I further specialize this supply function, and assume that

$$L = \left(\mu \frac{s_L}{1 - s_L}\right)^{1/\tau} = \left(\mu \frac{s_L}{s_K}\right)^{1/\tau}. \tag{48}$$

Here $\tau > 0$, and μ is a shift parameter. A decrease in μ will correspond to a "wage-push" shock, since it will increase the labor share at a given level of employment. A high level of τ corresponds to a more inelastic quasi-supply curve. This formulation is convenient since an expression for s_L/s_K, for given bias of technology, can be immediately obtained from (21) as

$$\frac{s_L}{s_K} = \left(\frac{1-\gamma}{\gamma}\right)^{-\frac{\varepsilon}{\sigma}} \left(\frac{N_K}{N_L}\frac{K}{L}\right)^{\frac{1-\sigma}{\sigma}}. \tag{49}$$

Equation (49) states that when labor and K are gross complements, i.e., $\sigma < 1$, as we have assumed, an increase in the relative productivity of the other factors, N_K will raise the labor share. With $\sigma > 1$, we would obtain the opposite results.

What will happen in the long run? There are two margins of adjustment: the capital-labor ratio and technology. Even without any adjustment in technology, the labor share may return back to its initial level if the capital-labor ratio returns to its initial position. This will be the case, for example, if the long-run return to capital is given (a perfectly elastic supply of capital or constant relative risk aversion preferences for the representative consumer). However, Blanchard (1997) shows that changes in capital-labor ratio do not account for the behavior of the labor share. In fact, in most countries, the capital-labor ratio rose following the wage-push shock. This raises the possibility that the major adjustment was due to changes in technology. To focus on this, I ignore capital accumulation and normalize $K = 1$ (see Acemoglu, 1999b, for the analysis of the case where both technology and capital adjust).

Now combining (48) and (49), we obtain

$$L^{SR} = \mu^{\frac{\sigma}{\tau\sigma+1-\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{-\frac{\varepsilon}{\tau\sigma+1-\sigma}} \left(\frac{N_K}{N_L}\right)^{\frac{1-\sigma}{\tau\sigma+1-\sigma}}. \tag{50}$$

Since $\sigma < 1$, (50) defines a positive relationship between μ and L. I use the superscript "SR" to emphasize that this is the short-run response of employment, for given bias of technology. (50) implies that a wage-push shock, i.e. a decrease in μ, will reduce employment. Equation (49) implies that as long as $\alpha < 0$, this shock will also increase the labor share in GDP. Therefore, for a given bias in technology, as argued by Blanchard and Caballero and Hammour, a wage-push will reduce employment and increase the labor share.

A wage-push shock will also affect the equilibrium bias of technical change. As in the previous subsection, consider the case with extreme state-dependence, $\delta = 1$.

The simplest way to proceed is to combine (48) with equation (34) from Section 4.2, which gives the long-run relative labor share as $(s_L/s_K)^{LR} = \eta^{-1}$. In other words, the technology-market clearing condition can only be satisfied if the labor share in GDP returns back to its initial level. Using this, we also calculate the long-run employment level as

$$L^{LR} = \eta^{-\frac{1}{\tau}} \mu^{\frac{1}{\tau}}, \tag{51}$$

where the superscript "LR" shows that these expressions refer to the long-run equilibrium. Next, using equation (29), we obtain the relative technology level consistent with long-run equilibrium as

$$\frac{N_K}{N_L} = \eta^{\frac{\sigma\tau-(1-\sigma)}{\tau(1-\sigma)}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \mu^{\frac{1}{\tau}} \tag{52}$$

Comparison of (50) and (51) immediately implies that the elasticity of employment with respect to μ is greater in the long run $(\sigma/(\tau\sigma + 1 - \sigma) < 1/\tau$.[27] This again can be thought of as an application of the LeChatelier principle. Intuitively, with $\sigma < 1$, despite the increase in the cost of labor resulting from the wage-push, firms cannot substantially reduce their labor demand in the short run because of the low elasticity of substitution. However, this change in factor prices increases the value of, and induces, technical change that would allow firms to be less dependent on labor. Once this technical change takes place, firms gradually reduce their labor demand.[28] Inspection of equation (52) also implies that a decline in μ—i.e., an adverse labor supply shock—will reduce N_K/N_L. This corresponds to capital-biased technical change since capital and labor are gross complements.[29]

Figure 4 draws this case diagramatically, assuming that technology is given in the short run when the wage-push first occurs, and then traces the adjustment of the economy to the shock. It shows diagramatically how, as in the case of European economies,

[27]When $\sigma > 1$, equation (51) no longer gives the long-run labor demand, since the economy would go to an equilibrium with only one type of innovation.

[28]This idea is related to that proposed by Caballero and Hammour (1998). They suggest that in order to avoid being appropriated by labor, during the 1980s European firms chose more capital intensive production techniques, reducing their labor demand.

[29]However, notice that in this model, the labor share simply returns back to its original level. So to explain why the labor share may have fallen below its original level in many European countries, we may also need some of the adverse labor supply shocks to have reversed themselves (i.e., μ to have fallen back towards its initial level).

the wage-push will first increase the labor share, and then gradually reduce it by creating capital-biased technical change. Throughout the process, employment falls.

7 Concluding Remarks

For many problems in macroeconomics, development economics, labor economics, and international trade, whether technical change is biased towards particular factors is of central importance. This paper synthesized some recent research on the determinants of biased technical change. The presumption is that the same economic forces—profit incentives—that affect the amount of technical change will also shape the direction of technical change, and therefore determine the equilibrium bias of technology. I argued that this perspective helps us understand a number of otherwise puzzling features. For example:

1. Why technical change over the past 60 years may have been skill-biased, and why the skill bias may have accelerated over the past twenty-five years. And also why new technologies introduced during the late eighteenth and early nineteenth centuries may have been unskill-biased.

2. Why biased technical change may increase the income gap between rich and poor countries.

3. Why international trade may induce skill-biased technical change.

4. Why labor scarcity may spur faster technological progress.

5. Why technical change may be generally labor-augmenting rather than capital-augmenting.

6. Why a large wage-push, as in continental Europe during the 1970s, may cause capital-biased technical change.

I also highlighted the various modeling choices involved in thinking about the direction of technical change. I demonstrated that there are two robust forces affecting equilibrium bias: the price effect and the market size effect. The elasticity of substitution between different factors regulates how powerful these effects are, and this has implications for how technical change and factor prices respond to changes in relative supplies.

Another important determinant of equilibrium bias of technology is the form of the innovation possibilities frontier—how relative costs of different types of innovation change with the current state of technology. The choice here is between a specification that emphasizes state-dependence, whereby past innovations complementing a factor make current innovations directed at that factor cheaper, and a specification without state-dependence. It appears that in thinking about skill-biased technical change a specification with only limited state-dependence leads to more plausible results, while in the case of capital- and labor-augmenting technical change, a specification with extreme state-dependence appears more appropriate. Although it is possible to combine these features to have a unified framework that can be applied both to analyzing the degree of skill bias and capital bias of technology, this framework is currently highly ad hoc and without micro-foundations. Future work towards such a unified model would be very useful.

Whether technical change exhibits this type of state-dependence, and whether this state-dependence affects different factors differentially, is ultimately an empirical question. Existing evidence does not enable us to reach firm conclusions. Nevertheless, data on patent citations seem to provide a useful starting place, and empirical work that can inform modeling choices in this field is another area for fruitful future research.

8 Appendix

8.1 Derivation of equation (40)

Substituting for the price levels using equations (8) and (19) and simplifying, we can express (39) as

$$\frac{Y'}{Y} = \frac{\delta \left(\frac{\gamma+(1-\gamma)\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon(1-\sigma)}{\sigma}} \left(\frac{N_H H'}{N_L L'}\right)^{\frac{\sigma-1}{\sigma}}}{\gamma+(1-\gamma)\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon(1-\sigma)}{\sigma}} \left(\frac{N_H H}{N_L L}\right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}}}{\times \left(\frac{\gamma+(1-\gamma)\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon(1-\sigma)}{\sigma}} \left(\frac{N_H H'}{N_L L'}\right)^{\frac{\sigma-1}{\sigma}}}{\gamma+(1-\gamma)\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon(1-\sigma)}{\sigma}} \left(\frac{N_H H}{N_L L}\right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{e-1}{\varepsilon}} \times \left(\frac{L'}{L}\right)^{\frac{e-1}{\varepsilon}}}$$

Differentiation of this expression immediately shows that both fractions are increasing in $\frac{N_H}{N_L}$ if

$$\frac{1-\beta}{\sigma}\left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}}\left(\left(\frac{H'}{L'}\right)^{\frac{\sigma-1}{\sigma}} - \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}\right) > 0$$

and decreasing if

$$\frac{1-\beta}{\sigma}\left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}}\left(\left(\frac{H'}{L'}\right)^{\frac{\sigma-1}{\sigma}} - \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}\right) < 0.$$

This establishes the claim in the text.

8.2 Sector-Specific and Factor-Specific Technical Change

In the text I discussed a model in which each factor was used only in one sector, hence there was no sector-specific technical change (different from factor-specific technical change). It is possible to generalize this framework to allow for both types of technical change.

Suppose that the final good is defined over two intermediates, denoted 1 and 2:

$$Y = \left[\gamma Y_1^{\alpha} + (1-\gamma)\, Y_2^{\alpha}\right]^{1/\alpha}.$$

Although one of the sectors is more intensive in skilled labor and the other sector in unskilled labor, they both use both factors. In particular, assume that

$$Y_1 = \left(\int_0^{M_1} x_1^{1-\beta}(j)\,dj\right)\left(\gamma_1\left(\int_0^{N_L} x_{1l}^{1-\beta}(j)\,dj L_1^{\beta}\right)^{\varsigma} + (1-\gamma_1)\left(\int_0^{N_H} x_{1h}^{1-\beta}(j)\,dj H_1^{\beta}\right)^{\varsigma}\right)^{\beta/\varsigma}$$

$$Y_2 = \left(\int_0^{M_2} x_2^{1-\beta}(j)\,dj\right)\left(\gamma_2\left(\int_0^{N_L} x_{2l}^{1-\beta}(j)\,dj L_2^{\beta}\right)^{\varsigma} + (1-\gamma_2)\left(\int_0^{N_H} x_{2h}^{1-\beta}(j)\,dj H_2^{\beta}\right)^{\varsigma}\right)^{\beta/\varsigma}$$

Here L_1 and L_2 are the amounts of unskilled labor used in the two sectors (and $L_1+L_2 = L$), and H_1 and H_2 are the amounts of skilled labor in the two sectors (with $H_1+H_2 = H$). M_1 and M_2 correspond to sector-specific technical change (and x_1 and x_2 to sector-specific machines): they benefit all the factors used in the corresponding sector. Finally, N_L and N_H correspond to factor-specific technical change, because these machines can be used with the factors irrespective of which sectors these factors are being employed in.

In this more general framework, it is no longer possible to obtain closed-form solutions. Nevertheless, a similar analysis to before reveals a number of important results. Denote the prices of the two sectors by p_1 and p_2, and consider the profit maximization of firms in the two sectors, i.e. to maximize p_1Y_1 and p_2Y_2, taking factor, product and machine prices as given. Differentiation gives relatively simple expressions for machine demands. In particular, total expenditure on sector-specific machines is given by

$$\chi_1(j)\,x_1(j) = (1-\beta)\frac{p_1Y_1}{M_1} \text{ and } \chi_2(j)\,x_2(j) = (1-\beta)\frac{p_2Y_2}{M_2}, \tag{53}$$

where $\chi_1(j)$ and $\chi_2(j)$ again denote machine prices. Some algebra also gives that

$$\chi_l(j)\,x_{1l}(j) = \frac{1-\beta}{\beta}\frac{w_L L_1}{N_L} \text{ and } \chi_l(j)\,x_{2l}(j) = \frac{1-\beta}{\beta}\frac{w_L L_2}{N_L},$$

where again $\chi_l(j)$ is the price of unskilled labor-complementary machine j. Similar expressions apply for the demand for skilled labor-complementary machines. Summing these two expressions, we have total expenditure on unskilled labor-complementary machines as

$$\chi_l(j)\,x_l(j) = \frac{1-\beta}{\beta}\frac{w_L L}{N_L}, \tag{54}$$

and similarly for skilled labor-complementary machines

$$\chi_h(j)\, x_l(j) = \frac{1-\beta}{\beta}\frac{w_H H}{N_H}. \tag{55}$$

Although it is cumbersome to characterize the full equilibrium of this model, these expressions give some useful intuition. First recall that profits will be proportional to total expenditures (since prices are given by a constant markup over marginal cost). The expressions in (53) then show that it will be more profitable to develop technologies for sectors that generate greater revenues. This result confirms the conjecture by Schmookler (1966) that sectors facing greater demand will generate more innovation.

Comparison of expressions (54) and (55) with (17) in Section 3 shows that the incentives to carry out factor-biased technical change are the same in this more general model as in the simpler framework where each factor was only used in one sector. This highlights that the key dimension of the results above was factor-specific technical change not sector-specific technical change. The incentives to develop factor-complementary technologies is identical in this more general framework to those in the simpler framework of Section 3.

9 References

.

Acemoglu, Daron, "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality" *Quarterly Journal of Economics*, CXIII (1998), 1055-1090.

Acemoglu, Daron, "Patterns of Skill Premia" NBER Working Paper no. 7018 (1999a).

Acemoglu, Daron, "Labor- And Capital-Augmenting Technical Change" NBER Working Paper no. 7544 (1999b).

Acemoglu, Daron and Fabrizio Zilibotti, "Productivity Differences" (2001), forthcoming *Quarterly Journal of Economics*.

Aghion, Philippe and Peter Howitt, "A Model of Growth Through Creative Destruction" *Econometrica*, LX (1992), 323-351.

Aghion, Philippe and Peter Howitt, *Endogenous Growth Theory*, Cambridge, MA, MIT Press, 1998.

Ahmad, Syed, "On The Theory of Induced Invention," *Economic Journal* LXXVI, (1966), 344-357.

Angrist, Joshua, "The Economic Returns to Schooling in the West Bank and Gaza Strip" *American Economic Review,* 85,1065-1087.

Atkinson, Anthony B. and Joseph E. Stiglitz "A New View of Technological Change," *Economic Journal* (1969) 79, 573-78.

Autor, David, Alan Krueger and Lawrence Katz, "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics*, CXIII (1998), 1169-1214.

Bairoch, Paul (1988), *Cities and Economic Development: From the Dawn of History to the Present*, University of Chicago Press, Chicago, translated by Christopher Braider.

Basu, Susanto and David N. Weil "Appropriate Technology and Growth," *Quarterly Journal of Economics*, CXIII, 1025-1054.

Berman, Eli, "Does the Factor Bias of Technical Change Stifle International Convergence?" Boston University mimeo (2000).

Berndt, Ernst, "Reconciling Alternative Estimates of the Elasticity of Substitution," *Review of Economics and Statistics* 58, No. 1.,(1976) pp. 59-68.

Blanchard, Olivier, "The Medium Run" *Brookings Papers on Economic Activity*, (1997), 89-158.

Caballero, Ricardo J. and Mohammad Hammour; "Jobless Growth: Appropriability, Factor Substitution and Unemployment" *Carnegie-Rochester Conference Proceedings* (1998) vol. 48, pp. 51-94.

Caballero, Ricardo J. and Adam B. Jaffe, "How High are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth" *NBER Macroeconomics Annual 1993,* Cambridge, MA, MIT Press, 1993, 15-74.

Caselli, Francesco and Jaume Ventura "A Representative Consumer Theory of Distribution" forthcoming *American Economic Review* (2000).

Coen, R. M.; "Tax Policy and Investment Behavior: A Comment," *American Economic Review*, (1969) pp.370-379.

David, Paul, *Technical Choice, Innovation and Economic Growth: Essays on American and British Experience in the Nineteenth Century*, London, Cambridge University Press, 1975.

Drandakis, E. and Edmund Phelps, "A Model of Induced Invention, Growth and Distribution" *Economic Journal*, 76 (1965), 823-840.

Eisner, R., and M. I. Nadiri, "Investment Behavior and the Neoclassical Theory," *Review of Economics and Statistics*, (1968) pp. 369-382.

Freeman, Richard "Demand For Education" Chapter 6 in Orley Ashenfelter and Richard Layard (editors) *Handbook of Labor Economics*, North Holland, Vol I (1986), 357-386.

Goldin, Claudia and Lawrence F. Katz, "The Origins of Technology-Skill Complementarity," *Quarterly Journal of Economics* 113 (1998), 693-732.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy,* Cambridge, MA, MIT Press, 1991.

Habakkuk, H. J., *American and British Technology in the Nineteenth Century: Search for Labor Saving Inventions*, Cambridge University Press, 1962.

Hayami, Yujiro and Vernon Ruttan, "Factor Prices and Technical Change in Agricultural Development: the U.S. and Japan, 1880-1960" *Journal of Political Economy*, LXXII (1970), 1115-1141.

Hicks, John *The Theory of Wages*, Macmillan, London, 1932.

Howitt, Peter "Steady Endogenous Growth with Population and R&D Inputs Growing" *Journal of Political Economy*, 107 (1999), 715-730.

Jaffe, Adam B., Manuel Trajtenberg, and Rebecca Henderson, "Geographic Localizaion of Knowledge Spillovers as Evidenced by Patent Citations" *Quarterly Journal of Economics*, CVIII (1993), 577-598.

James, John A., and Jonathan S. Skinner, (1985) "The Resolution of the Labor-Scarcity Paradox," *Journal of Economic History*, Volume 45, pp.513 - 540.

Jones, Charles I., "R & D-Based Models of Economic Growth," *Journal of Political Economy* 103, (1995), 759-784.

Jones, Larry E. and Rodolfo E. Manuelli, "A Convex Model of Equilibrium Growth: Theory and Policy Implications" *Journal of Political Economy*, 98 (1990), 1008-1038.

Jorgenson, Dale W., Frank M. Gollop, and Barbara M. Fraumeni, *Productivity and U. S. Economic Growth*, Cambridge, MA, Harvard University Press, 1987.

Kennedy, Charles, "Induced Bias in Innovation and the Theory of Distribution" *Economic Journal,* LXXIV (1964), 541-547.

Kiley, Michael, "The Supply of Skilled Labor and Skill-Biased Technological Progress" 1999, *Economc Journal*.

Lucas, Robert E., "Labor-Capital Substitution in U.S. Manufacturing," in *The Taxation of Income from Capital*, edited by Arnold Harberger and Martin J. Bailey, Washington, DC: The Brookings Institution, (1969) pp. 223-274.

Lucas, Robert, "On the mechanics of economic development." *Journal of Monetary Economics*, 22 (1988), 3-42.

Mokyr, Joel, *The Levers of Riches: Technological Creativity and Economic Progress* Oxford University Press, New York, 1990.

Nordhaus, William; "Some Skeptical Thoughts on the Theory of Induced Innovation" *Quarterly Journal of Economics*, LXXXVII, (1973), 208-219.

Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth" *Journal of Political Economy* 99 (1991), 500-521.

Rivera-Batiz, Luis A. and Paul M. Romer, "Economic Integration and Endogenous Growth" *Quarterly Journal of Economics*, 106 (1991), 531-555.

Romer, Paul M., "Increasing Returns and Long-Run Growth" *Journal of Political Economy* 94 (1986), 1002-1037.

Romer, Paul M., "Endogenous Technological Change" *Journal of Political Economy*, IIC (1990), S71-S102.

Rothbarth, E. "Causes of the Superior Efficiency of the USA Industry As Compared with British Industry" *Economics Journal* (1946).

Salter W. E.G., *Productivity and Technical Change*, 2nd edition, Cambridge University Press, (1966) Cambridge, United Kingdom.

Samuelson, Paul, "A Theory of Induced Innovations Along Kennedy-Weisacker Lines" *Review of Economics and Statistics*, XLVII (1965), 444-464.

Schmookler, Jacob, *Invention and Economic Growth*, Cambridge, Harvard University Press, 1966.

Segerstrom, P. S., T. Anant, and E. Dinopoulos, "A Schumpeterian Model of the Product Life Cycle" *American Economic Review* 80 (1990), 1077-1092.

Shapiro, Carl and Joseph Stiglitz; "Unemployment as Worker Discipline Device" *American Economic Review,* 73 (1984), 433-444.

Stewart, Frances; *Technology and Underdevelopment*, The Macmillan Press, Ltd., London, England Stewart, Frances (1977). *Technology and Underdevelopment*, The Macmillan Press, Ltd., London, England.

Manuel Trajtenberg, Rebecca Henderson, andJaffe, Adam B. "Ivory Tower Vs. Corporate Lab: an Empirical Study of Basic Research and Appropriatability" NBER working paper 4146,1992.

Williamson, Jeffrey, *Coping With City Growth During the British Industrial Revolution*, Cambridge, Cambridge University Press, 1990.

Wood, Adrian, *North-South Trade, Employment and Inequality: Changing Fortunes in a Skill Driven World*, Oxford, Clarendon Press, 1994.

Xu Bin, "Endogenous Technology Bias, International Trade and Relative Wages", University of Florida mimeo, 2001.

Young, Alwyn "Invention and Bounded Learning by Doing" *Journal of Political Economy* 101 (1993), 443-472.

Young, Alwyn "Growth without Scale Effect" *Journal of Political Economy*, 106 (1998), 41-63.

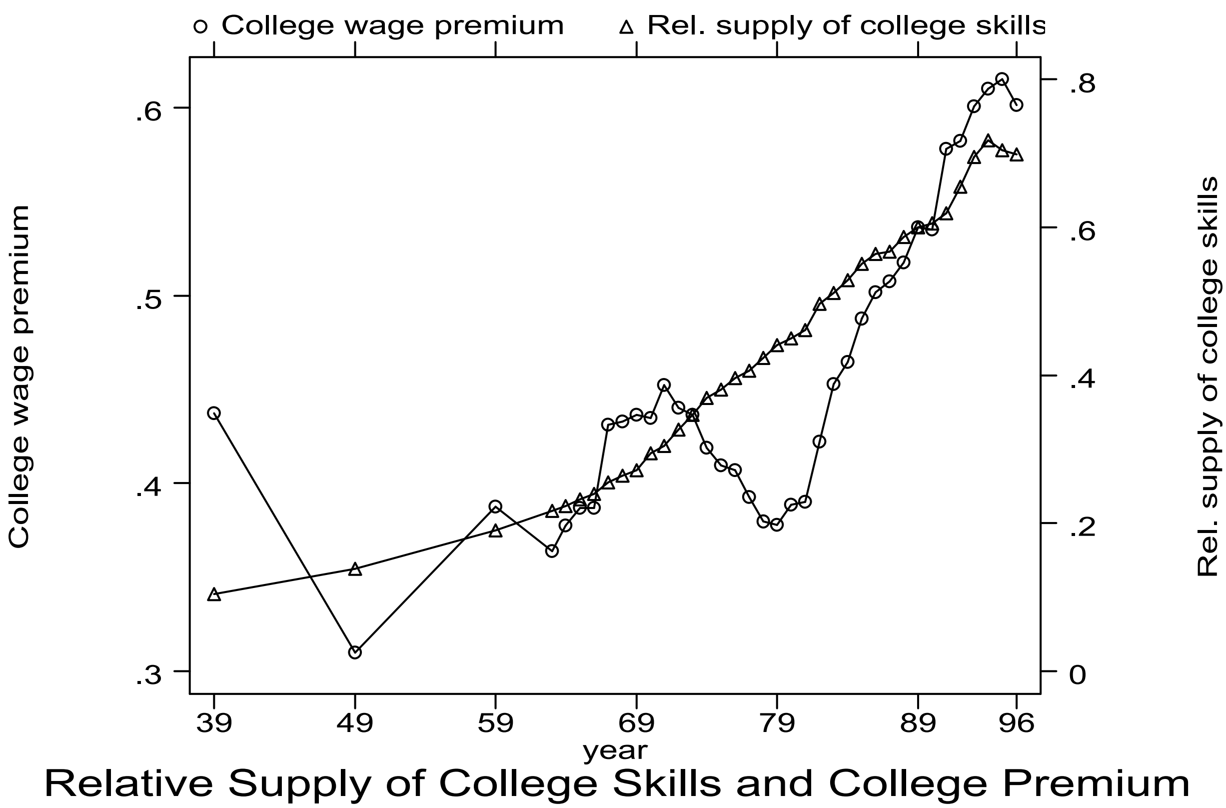

Figure 1: The behavior of the (log) college premium and relative supply of college skills (weeks worked by college equivalents divided by weeks worked of noncollege equivalents) between 1939 and 1996. Data from March CPSs and 1940, 1950 and 1960 censuses.

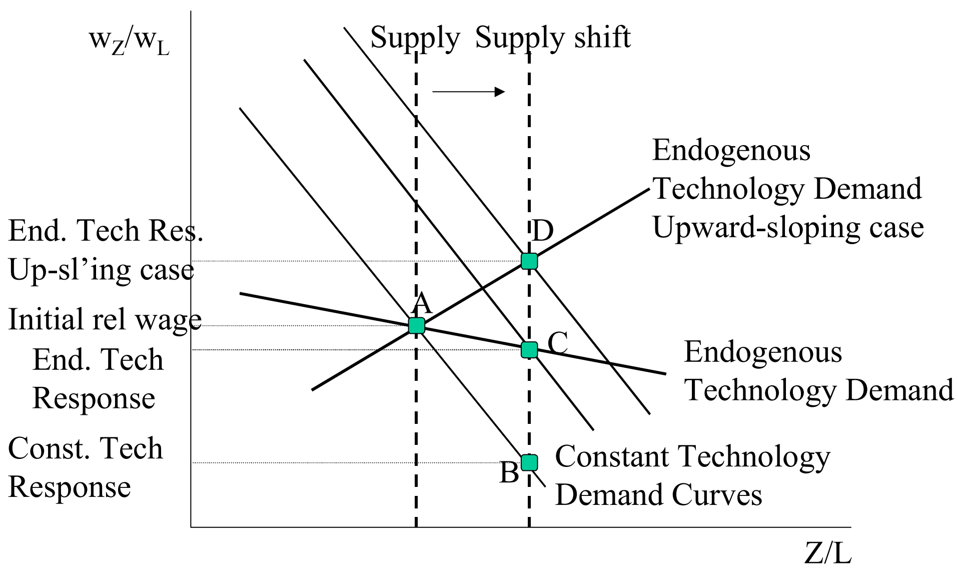

Figure 2: Constant technology and endogenous technology relative demand curves. Constant technology: A → B. Endogenous technology: A → C. Endogenous technology with powerful market size effect: A → D.

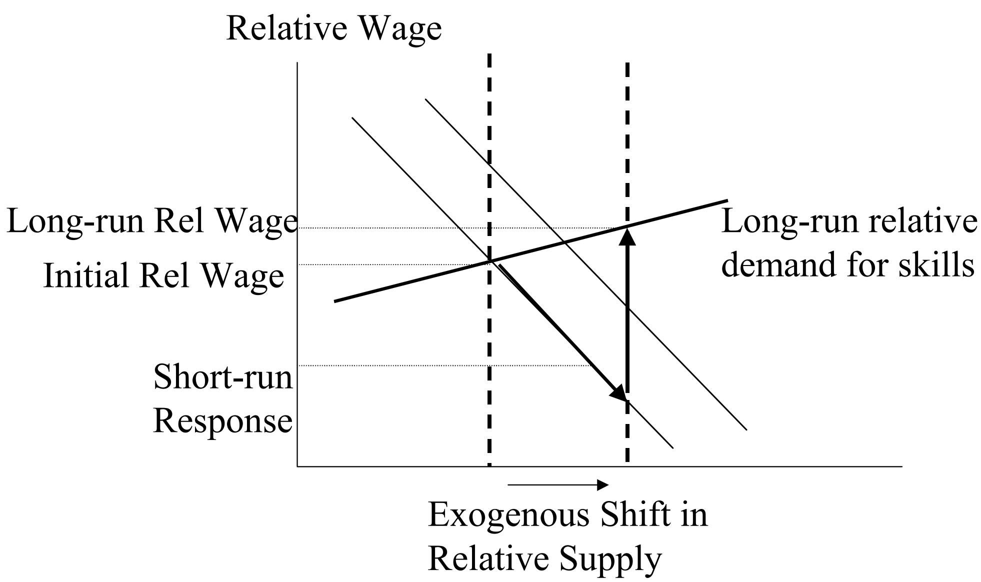

Figure 3: Short-run and long-run responses of the skill premium to an increase in the relative supply of skills when condition (33) is satisfied.

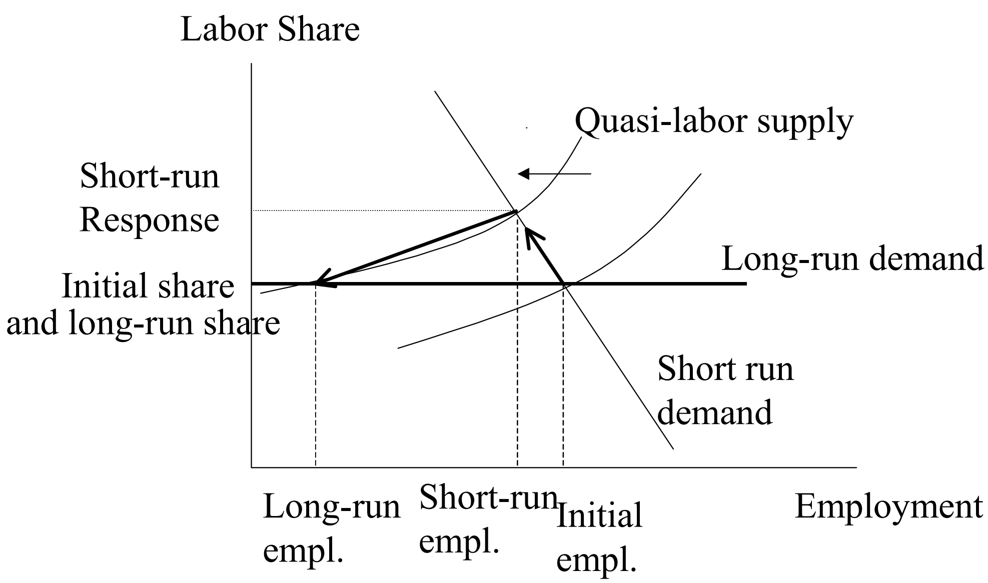

Figure 4: Short-run and long-run responses of the labor share to a shift in the quasi-labor supply when capital and labor are gross complements.