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Christian Haas and Karol Kempa

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Coordination: Bernd Hayo • Philipps-University Marburg
School of Business and Economics • Universitätsstraße 24, D-35032 Marburg
Tel: +49-6421-2823091, Fax: +49-6421-2823088, e-mail: hayo@wiwi.uni-marburg.de

Directed Technical Change and Energy Intensity Dynamics:
Structural Change vs. Energy Efficiency*

Christian Haas[†] and Karol Kempa[†‡]

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Abstract

This paper uses a theoretical model with Directed Technical Change to analyse the observed heterogeneous energy intensity developments. Based on the empirical evidence on the underlying drivers of energy intensity developments, we decompose changes in aggregate energy intensity into structural changes in the economy (*Sector Effect*) and within-sector energy efficiency improvements (*Efficiency Effect*). We analyse how energy price growth and the relative productivity of both sectors affect the direction of research and hence the relative importance of the aforementioned two effects. The relative importance of these effects is determined by energy price growth and relative sector productivity that drive the direction of research. In economies that are relatively more advanced in sectors with low energy intensities, the *Sector Effect* dominates energy intensity dynamics given no or moderate energy price growth. In contrast, the *Efficiency Effect* dominates energy intensity developments in economies with a high relative technological level within their energy-intensive industries if moderate energy price growth is above a certain threshold. We further show that temporal energy price shocks might induce a permanent redirection of innovation activities towards sectors with low-energy intensities.

Keywords: directed technical change, energy efficiency, energy intensity, structural change.

JEL-Classification: O33, Q43, Q55.

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[†]Department of Economics, Justus Liebig University of Giessen, Chair for International Economics (VWL III), Licher Str. 66, 35394 Giessen, Germany. Corresponding author (Karol Kempa): Phone: +49 (0)641 9922112, Fax: +49 (0)641 9922119, E-mail: Karol.Kempa@wirtschaft.uni-giessen.de.

[‡]Frankfurt School of Finance and Management, FS-UNEP Collaborating Centre for Climate and Sustainable Energy Finance, Sonnemannstrasse 9-11, 60314 Frankfurt, Germany.

1. Introduction

The relationship of energy use and economic activity has been a recurring theme in the political and academic debate, particularly since the energy crisis in the 70s. A main reason is the high dependence on fossil fuel energy carriers in energy generation - 80.6% in 2014 (IEA, 2015) - and the resulting consequences for the world climate. Another aspect is the increasing energy prices due to progressing depletion of exhaustible resources and their effect on economic activity. A promising way to lower emission levels and meet climate policy targets is reducing energy intensity, i.e. decreasing the input of energy for production of a given output.

Since the energy crisis in the 1970s, numerous studies have analysed the development of energy intensities. Studies covering the period before the energy crisis, i.e. 1950 - 1970, show increasing or constant energy intensities across most of the analysed developed and emerging economies (see, e.g., Casler and Hannon (1989), Hannesson (2002), and Proops (1984)). For the period after the energy crisis, however, there is strong evidence for substantial reductions in energy intensity (increases in energy productivity) in the majority of developed economies (see, e.g., Greening, Davis, and Schipper (1998), Liddle (2012), Mulder and Groot (2012), Sun (1998), Voigt et al. (2014), and Wang (2013)).[1] In addition to analysing trends of overall energy intensities across countries, numerous studies use index- or structural decomposition analyses to disaggregate energy intensity into its driving forces (see e.g. Ang (2004) and Mendiluce, Pérez-Arriaga, and Ocaña (2010)).[2] Most studies decompose energy intensity into an *Efficiency Effect*[3] and a *Sector Effect*. The former describes energy efficiency improvements within sectors, i.e. reductions in sectoral energy intensities due to e.g. substitution of energy by other factors or energy-saving technological progress. The *Sector Effect* means structural adjustments towards sectors with low energy intensities.

Mulder and Groot (2012) decompose the development of energy intensities across 50 sectors in 18 OECD countries for the period 1970-2005. The authors find an important contribution of the *Sector Effect* for energy intensity reductions (25% in all analysed OECD economies). However, the relative importance of the Efficiency Effect seems to be stronger.[4] A recent and very comprehensive decomposition analysis was conducted by Voigt et al. (2014).

[1] Greening, Davis, and Schipper (1998) analyse ten developed economies from 1971-1991 and find energy intensity reductions between 37.5% (Norway) and 61.7% (Japan). Sun (1998) finds for a similar period (1973-1990) a reduction of energy intensity of 26.2% across OECD countries. Liddle (2012) and Wang (2013) find similar results using more recent data. In spite of continuous reductions in energy intensities, there is still a high potential for energy efficiency improvements (Velthuijsen, 1993, Worrell et al., 2009).

[2] Ang and Zhang (2000) found 124 studies applied decomposition analyses related to energy-based emissions and energy demand. Only ten years later, the number of studies almost doubled (Su and Ang, 2012).

[3] The Efficiency Effect is also referred to as Technology or Intensity Effect.

[4] Sun (1998) finds a contribution of the Efficiency Effect of 75.5% from 1973-1980 that even increased to 90% from 1980-1985 and 92.8% from 1985-1990. Greening, Davis, and Schipper (1998) also finds that energy efficiency improvements within sectors are the main drivers of energy intensity decline.

Using the World Input-Output Database (WIOD) covering 34 sectors in 40 countries from 1995-2007, Voigt et al. (2014) show a conspicuous divergence in the importance of the Sector and the Efficiency Effect for energy intensity developments across countries. In around a third of all developed economies energy intensity reductions are primarily caused by a restructuring of the economy towards sectors with low energy intensities (Sector Effect). In the reminder of all industrial countries, the Efficiency Effect is primarily responsible for the decline in energy intensity. Overall, the data-analyses on energy intensities shows the following trends:

i. while energy intensities were constant or increasing in the majority of economies until the early 1970s, they systematically decreased since the energy crisis across economies;

ii. the contribution of energy intensity reductions within industries (Efficiency Effect), e.g. through technological progress, or a structural change towards less energy-intensive economic activities (Sector Effect) to energy intensity reduction differs substantially across countries.

In contrast to the extensive data analyses on energy intensity developments, there is a lack of theoretical approaches to analyse the underlying mechanisms of the trends described above. Recent studies, as Mulder and Groot (2012) and Voigt et al. (2014), point out the exploration of the determinants of these developments including the role of technological change as directions of future research. Our paper aims to fill this gap.

Futhermore, our paper contributes to the literature on technological change and energy use in production that can be roughly attributed to two literature strands. One broad strand of literature analyses the so-called rebound effect on a macroeconomic, sector or product level. The analyses are based on substitution and income effects and differentiates (i) a direct rebound (ii) an indirect rebound and (iii) economy wide (or growth) effect.[5]

Our theoretical approach is in the spirit of another strand of literature that analyses (the direction of) technological change. Di Maria and Valente (2008) show, using a Directed Technical Change (DTC) framework (Acemoglu, 1998, 2002) that, in a model with a scarce natural resource, technological change will ultimately be complementary to energy. Further papers discuss reductions in the energy use in the context of the 'Pollution Haven' and 'Carbon Leakage' debates (see, e.g., Di Maria and Smulders, 2004, Di Maria and Werf, 2008).

In this paper, we provide a first attempt to theoretically analyse the differences in the contribution of the Sector and the Efficiency Effect to the development of energy intensities based

[5] Technological progress resulting in a decrease of the quantity of energy per unit output may induce substitution effects on the production (towards energy) and the consumption side (towards energy-intensive products) ((i) and (iii)) and may induce an increase in the overall consumption of products (ii) (Binswanger, 2001, Brookes, 2004, Greening, Greene, and Difiglio, 2000, Khazzoom, 1980, 1987, Qiu, 2014, Schipper and Grubb, 2000).

on the concept of DTC. We analyse how heterogeneous countries might react differently to identical changes of the exogenous energy price according to our model and discuss which historical developments in energy intensities outlined above are consistent with our results. We show (i) how increasing energy intensities prior to the first oil price crisis could be explained, (ii) how this energy crisis could have caused and increased energy saving and might have changed the direction of research towards sectors with low energy-intensity in certain economies, and (iii) how directed technical change might have triggered the substantial differences in the relative contribution of the Sector Effect and the Efficiency Effect in aggregate energy intensity developments.

The reminder of the paper is structured as follows. In Section 2 we introduce the model and derive the static and dynamic equilibrium. In Section 3, energy intensity is decomposed into a Sector- and Efficiency Effect and we show how both Effects are affected by innovation in both sectors and energy price growth. Section 4 contains the main results. We first show under which conditions directed technical change might induce increasing energy intensity for constant energy price. We then show that, for moderate energy price growth above a certain threshold, energy intensity declines and that the relative importance of the Sector and the Efficiency Effect largely depends on the relative technological level of the labour- and the energy-intensive sector and hence on the direction of technical change. Finally, we analyse how strong energy price growth might induce a change in the direction of technical change towards the labour-intensive sector. The final Section 5 discusses the results and concludes.

2. The Model

Our analysis is based on a marginally modified version of the model with exhaustible resources of Acemoglu et al. (2012). These authors model the energy price as function of the resource stock, since they analyse how the depletion of an exhaustible resource might induce a redirection of technical change towards a clean sector due to continuously increasing price. In contrast, we model an exogenous price for energy and endogenous energy use, as our focus is the analysis of energy intensity dynamics in alternative (historical) scenarios with different energy price growth rates.[6] Furthermore, we formulate our model in continuous time. This redefinition of the time dimension is a prerequisite for our main analysis provided in Sections 3 and 4, as it allows an extension of the model by an analytical decomposition of energy intensity into a Sector- and an Efficiency Effect.

[6]This exogenous price could be interpreted as the world market price of crude oil.

2.1. Model Framework

Consider an economy with infinitely-lived households consisting of scientists, entrepreneurs, and workers. Consumer behaviour can be described by a representative household that maximises its utility (U) through consumption, $C(t)$, of the only final product at time t with the utility function

$$U \equiv \int_0^\infty e^{-\rho t} u\left(C(t)\right) dt, \tag{1}$$

where ρ is the rate of time preference. The unique final good ($Y(t)$) is assembled from sectoral outputs of a labour-intensive sector ($Y_l(t)$) and an energy-intensive sector ($Y_e(t)$) according to

$$Y(t) = \left(Y_l(t)^{\frac{\epsilon-1}{\epsilon}} + Y_e(t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

The outputs of the labour-intensive and the energy-intensive sector are imperfect substitutes, where ϵ (with $\epsilon > 0$) is the elasticity of substitution between both goods. In the following, the two goods will be referred to as (gross) substitutes when $\epsilon > 1$ and (gross) complements in the case of $\epsilon < 1$. $\epsilon = 1$ is not considered, as in this case the production function converges to the Cobb-Douglas type and hence technical change is neutral to the input factors.

In each sector $j \in \{l, e\}$, labour ($L_j(t)$) and a sector specific set of machines are used for production. Each machine type i in sector j, $x_{ji}(t)$, has an individual productivity $A_{ji}(t)$. The production in the energy-intensive sector additionally requires energy $E(t)$. The production functions of both sectors are:

$$Y_l(t) = L_l(t)^{1-\alpha} \int_0^1 A_{li}(t)^{1-\alpha} x_{li}(t)^{\alpha} di, \tag{3}$$

$$Y_e(t) = E(t)^{\alpha_2} L_e(t)^{1-\alpha} \int_0^1 A_{ei}(t)^{1-\alpha_1} x_{ei}(t)^{\alpha_1} di, \tag{4}$$

with $\alpha = \alpha_1 + \alpha_2$, $\alpha \in (0, 1)$. The aggregate productivity of sector $j \in \{l, e\}$ is defined as

$$A_j(t) \equiv \int_0^1 A_{ji}(t) di. \tag{5}$$

This definition will be used for the subsequent analysis of the direction of research. Labour is assumed to be supplied inelastically. Normalising labour supply to 1, the labour market clearing condition is

$$L_l(t) + L_e(t) \leq 1. \tag{6}$$

Energy $E(t)$ is supplied at per unit costs of $c_E(t)$. With respect to the evolution of energy costs over time, we consider three different cases that are discussed in Section 4.

Machines are produced with an identical, linear production technology at identical costs of ψ units of the final product and supplied under monopolistic competition. Market clearing for the unique final good implies

$$Y(t) = C(t) + \psi\left(\int_0^1 x_{li}(t)di + \int_0^1 x_{ei}(t)di\right) + c_E(t)E(t). \tag{7}$$

Technological progress is driven by quality improvements of machines. Each machine is owned by an entrepreneur, the measure of entrepreneurs in each sector is normalised to one, respectively. At the same time, scientists (entrants) attempt to enter the market (become an entrepreneur) through innovation. Scientist direct their research at either the labour- or energy-intensive sector. With the probability $\eta_j \in (0,1)$, the innovation attempt is a success and the scientists is randomly allocated to a specific machine, increases its quality by $\gamma > 0$, receives a patent, and becomes an entrepreneur producing this machine variety. The entrepreneur that used the old version of this machine leaves the market and joins the pool of scientists. Normalising the mass of scientists to one, the market clearing condition for scientists is

$$s_l(t) + s_e(t) \leq 1, \tag{8}$$

with s_{jt} denoting the mass of scientists directing their research at sector $j \in \{l, e\}$. Due to this innovation process, together with (5), the aggregate sector productivity, $A_j(t)$, improves over time according to the following law of motion:

$$\dot{A}_{ji}(t) = s_j(t)\eta_j\gamma A_{ji}(t). \tag{9}$$

2.2. Equilibrium

In this Subsection, the model equilibrium is analysed.

Definition 1. *An equilibrium is given by prices for sector outputs ($p_j(t)$), machines ($p_{ij}(t)$) and labour ($w_j(t)$), demands for machines ($x_{ji}(t)$), sector outputs ($Y_j(t)$), labour ($L_j(t)$) and energy ($E(t)$) of sector $j = \{e, l\}$, such that at t: $p_{ij}(t), x_{ij}(t)$ maximizes profits of producers of machine i in sector j; $L_e(t), E(t)$ maximizes profits of producers in the energy intensive sector, $L_l(t)$ maximizes profits of producers in the labour intensive sector; $Y_j(t)$ maximizes profits of final good producer; $s_j(t)$ maximizes expected profits of researchers in sector j.*

As long as we analyse the equilibrium at time t with constant technologies, we drop the time index to simplify notation. Due to perfect competition on market for the final product, the profit-maximising behaviour of the final good producer result in the following relative demand for both intermediate goods:

$$\frac{p_l}{p_h} = \left(\frac{Y_l}{Y_e}\right)^{-\frac{1}{\epsilon}}. \tag{10}$$

This price ratio implies that the relative price is inversely related to the relative supply of both sectors. The magnitude of the price reaction to relative intermediate outputs is determined by the elasticity of substitution ϵ among both goods. In case of $\epsilon > 1$, the relative price response due to changes in relative outputs is smaller compared to the case of $\epsilon < 1$. Defining the final good as numeraire, the price index can be written as

$$\left(p_l^{1-\epsilon} + p_e^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 1. \tag{11}$$

The goods in the energy-intensive and the labour-intensive sector are produced competitively. Producers maximise their profits by choosing the quantities of the respective sector specific machines and labour,

$$\max_{x_{li},L_l} \left\{\Pi_{Y_l} = p_l L_l^{1-\alpha} \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di - wL_l - \int_0^1 p_{li} x_{li} di\right\}, \tag{12}$$

as well as, in the case of the e-sector, the amount of energy,

$$\max_{x_{ei},L_e,E} \left\{\Pi_{Y_e} = p_e E^{\alpha_2} L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di - wL_e - \int_0^1 p_{ei} x_{ei} di - c_E E\right\}. \tag{13}$$

Profit-maximisation yields the sectoral demands for machine i in the labour-intensive sector,

$$x_{li} = \left(\frac{\alpha p_l}{p_{li}}\right)^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{14}$$

and in the energy-intensive sector,

$$x_{ei} = \left(\frac{\alpha_1 p_e E^{\alpha_2} L_e^{1-\alpha}}{p_{ei}}\right)^{\frac{1}{1-\alpha_1}} A_{ei}. \tag{15}$$

The demands for machines increase in the price of the respective sector's output (p_j), employed labour in the sector (L_j), and the quality of the individual technology (A_{ji}). [7]

Machines are produced under monopolistic competition. The producer of each variety maximises her profit ($\pi_{ji} = \left(p_{ji} - \psi\right) x_{ji}$) given the demand for her variety. The optimisations yield the price setting rules for monopolists in both sectors, that are $p_{li} = \psi/\alpha$ for machine producers in the l-sector and $p_{hi} = \psi/\alpha_1$ for machine producers in the e-sector. Using these prices and

[7] Sectoral demands for labour and energy are provided in Appendix (A.1).

the demands for machines in both sectors, (14) and (15), the equilibrium profits of machine producers in the labour-intensive sector are

$$\pi_{li} = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{1}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_l^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{16}$$

whereas the profits in the energy-intensive sector are

$$\pi_{ei} = (1-\alpha_1)\,\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \left(\frac{1}{\psi^{\alpha_1}}\right)^{\frac{1}{1-\alpha_1}} p_e^{\frac{1}{1-\alpha_1}} E^{\frac{\alpha_2}{1-\alpha_1}} L_e^{\frac{1-\alpha}{1-\alpha_1}} A_{ei}. \tag{17}$$

Equilibrium prices, employment of labour and the resources in both sectors, as well as the production quantities in both sectors are provided in Appendix (A.1).

2.3. Technical change and research incentives

The direction of research, that is the sector a scientist chooses to direct her research effort, is determined by the expected firm value consisting of the current and the discounted future profits as entrepreneur. In case of a successful innovation, a scientists improves the quality of a machine, receives a patent, and becomes producer for this machine. Following Acemoglu et al. (2012) and Daubanes, Grimaud, and Rougé (2013), the patent is enforced for the smallest definable unit of time. This assumption simplifies the expected firm value to the profit in t.[8]

Since scientists only direct a sector and are randomly allocated to a specific machine variety, the average sectoral productivity is used as defined in (5). Taking into account the probabilities of a successful innovation, η_j, the expected firm value (i.e. expected profit) of an innovation in the l-sector, $\Pi_l(t)$, relative to an innovation in the e-sector, $\Pi_e(t)$, is:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \omega \frac{\eta_l}{\eta_e} \cdot \underbrace{\frac{p_l(t)^{\frac{1}{1-\alpha}}}{p_e(t)^{\frac{1}{1-\alpha_1}}}}_{\text{price effect}} \cdot \underbrace{\frac{L_l(t)}{E(t)^{\frac{\alpha_2}{1-\alpha_1}} L_e(t)^{\frac{1-\alpha}{1-\alpha_1}}}}_{\text{market size effect}} \cdot \underbrace{\frac{A_l(t)}{A_e(t)}}_{\substack{\text{direct}\\ \text{productivity effect}}} \tag{18}$$

using $\omega \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha_1)^{-1} \alpha_1^{-\frac{1+\alpha_1}{1-\alpha_1}} \psi^{\frac{\alpha+\alpha_1}{(1-\alpha)(1-\alpha_1)}}$. Analogously to the Directed Technical Change literature (Acemoglu, 1998, 2002), relative profitability of innovating is affected by a price- and a market size effect. The *price effect* directs innovation in the sector with the higher price. The *market size effect* makes innovations more attractive in the sector, where

[8] A detailed analysis of the direction of technical change with longer (infinite) duration, where the scientist derives monopoly profits until another scientist improves her machine variety and hence replaces her, can be found in Appendix (A.2). Although this approach is more general, this simplification does not affect our further analysis.

more factors of production, labour and energy, are employed. Since a larger market size is associated with a lower price for the output of the respective sector, both effects are opposite forces. Finally, the term $A_l(t)/A_e(t)$ captures a *direct productivity effect* as introduced by Acemoglu et al. (2012). This effect directs innovation to the sector that is technologically further advanced and hence follows the concept of "building on the shoulders of giants". In addition to these three forces, the respective probabilities of successful research, η_l and η_e, affect the relative profits.

In the following, the magnitude of these individual effects and the aggregate effect is analysed. The relative price (10) yields, together with the sectoral production quantities,(A.4) and (A.6), the relative supply in both sectors. Combining relative supply and with relative demands yields the relative employment as:

$$\frac{L_l(t)}{L_h(t)} = \left(\frac{c_E(t)^{\alpha_2} \alpha^{2\alpha}}{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}} \right)^{\epsilon - 1} \frac{A_l(t)^{-\varphi}}{A_h(t)^{-\varphi_1}} \tag{19}$$

with $\varphi_1 \equiv (1 - \alpha_1)(1 - \epsilon)$ and $\varphi \equiv (1 - \alpha)(1 - \epsilon)$. As can be seen in this expression, the market size effect favours the more advanced sector only if $\epsilon > 1 (\Leftrightarrow \varphi, \varphi_1 < 0)$. If both sectors are complements (($\epsilon < 1$ and hence $\varphi, \varphi_1 > 0$)) then the market size effect works in the direction of the technologically backward sector.

Using relative employment and the relative price in equilibrium, the profit of engaging in research in sector l relative to sector e can be expressed as:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon - 1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \tag{20}$$

with $\kappa \equiv \frac{(1-\alpha)\alpha}{(1-\alpha_1)} \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_2^{\alpha_2}} \right)^{\epsilon - 1} \alpha_1^{\frac{2\alpha_1}{1-\alpha} - \frac{1+\alpha_1}{1-\alpha_1} + 2\alpha_1(1-\epsilon) - \frac{\alpha_2}{1-\alpha_1} \frac{2\alpha_1}{1-\alpha}}$. This resulting relative profitability is a function of time-invariant parameters, the energy price, research efforts in both sectors as well as productivities. The following lemma can be derived from expression (20)[9]

Lemma 1. *In equilibrium, research is directed to the energy-intensive sector when* $A_e(t)^{(-\varphi_1)} \eta_e > \kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$, *to the labour-intensive sector when* $A_e(t)^{(-\varphi_1)} \eta_e < \kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$ *and to both sectors when* $A_h(t)^{(-\varphi_1)} \eta_e = \kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$.

This means that for $\epsilon > 1$, research is directed to the technically more advanced sector whereas for $\epsilon < 1$ the less advanced sector is favoured. In addition to the technological level of both sectors, the exogenous energy price affects research incentives. In general, an increasing energy price increases (decreases) the profitability for innovation in the labour-intensive

[9]Proof: See Appendix A.3.

sector for $\epsilon > 1$ ($\epsilon < 1$). Whether this effect of the energy price ultimately dominates the direct productivity effect depends on the growth rates of the energy price and the technologies. Analysing the growth rate of the relative profit yields the following lemma:

Lemma 2. *i. With moderate growth of the energy price, i.e. the growth rate remains in the band $-\eta_l\gamma(1-\alpha)/\alpha_2 \leq \gamma_c \leq \eta_e\gamma(1-\alpha_1)/\alpha_2$, the direction of technical change is determined by relative productivity that dominates the effect of energy price growth.*

ii. Strong growth of the energy price, i.e. $\gamma_c > \eta_e\gamma(1-\alpha_1)/\alpha_2$, will ultimately lead to research in the l- (e-) sector for $\epsilon > 1$ ($\epsilon < 1$).

iii. Strong negative growth of the energy price, i.e. $\gamma_c < -\eta_l\gamma(1-\alpha)/\alpha_2$, will ultimately lead to research in the e- (l-) sector only for $\epsilon > 1$ ($\epsilon < 1$).

Proof. See Appendix A.3.. □

This lemma is important for our subsequent analysis of the development of energy intensities. We will show that, although it does not affect the research direction, changes in the moderate growth rate of the energy price can substantially affect the direction and magnitude of changes in energy intensity as well as the relative strength and directions of the sector- and the efficiency effect.

3. Energy Intensity Dynamics

After characterising the model equilibrium and the determinants of the direction of technological progress, we analyse the energy intensity of the whole economy. We first show that the evolution of the energy intensity can be disaggregated in two driving forces: a *Sector Effect* and an *Efficiency Effect*. Subsequently, we analyse the directions and relative strengths of these effects for given changes in the costs of energy and changes in the aggregate productivities in both sectors. In order to simplify notation, the time index t is dropped throughout this Section.

3.1. The Sector and the Efficiency Effect

Defining the energy intensity as total energy input relative to total output, E/Y, and using the production function for the final product, the energy intensity of the whole economy can be written as

$$\frac{E}{Y} = \frac{E}{\left(Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}} = \frac{E}{Y_e}\left(\left(\frac{Y_l}{Y_e}\right)^{\frac{\epsilon-1}{\epsilon}} + 1\right)^{\frac{\epsilon}{1-\epsilon}}. \tag{21}$$

The growth rate of the energy intensity, $\gamma_{\frac{E}{Y}}$, is obtained by taking logarithms and differentiating with respect to time as

$$\gamma_{\frac{E}{Y}} \equiv \frac{d\ln\left(\frac{E}{Y}\right)}{dt} = \underbrace{\gamma_{\frac{E}{Y_e}}}_{\text{Efficiency Effect}} + \underbrace{\left(-\frac{Y_l^{\frac{\epsilon-1}{\epsilon}}}{Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}}\right)\gamma_{\frac{Y_l}{Y_e}}}_{\text{Sector Effect}}, \tag{22}$$

where $\gamma_{\frac{E}{Y_e}}$ denotes the growth rate of the energy intensity in the energy-intensive sector and $\gamma_{\frac{Y_l}{Y_e}}$ is the growth rate of the labour-intensive sector relative to the energy-intensive sector. As shown in the equation above, the development of the energy intensity can be decomposed into an *Efficiency Effect* and a *Sector Effect*. The Efficiency Effect refers to changes in the energy intensity in the *e*-sector. Since only this sector uses energy, any changes in the energy intensity translate directly into the energy intensity of the whole economy. A more productive use of energy in the energy-intensive sector is driven by improvements in the production technology in this sector. Hence, this effect could also be referred to as a technology effect. The Sector Effect is capturing the relative size of the labour-intensive sector. Since this sector does not use any energy for production, an increase of the share of the labour-intensive sector in total production leads, c.p., to a reduction of the economy wide energy intensity.

Using the previously derived equilibrium values, the strength and direction of both the Efficiency and the Sector Effect can be analysed. Using the equilibrium values for energy use and production in the *e*-sector, (A.12) and (A.13) in Appendix (A.1), we can analyse how the energy intensity in the *e*-sector is affected by changes of the energy costs as well as changes of the productivity levels in both sectors. The equilibrium energy intensity in the energy-intensive sector is:

$$\frac{E}{Y_e} = \frac{\alpha_2 \alpha^{2\alpha} c_E^{\alpha_2 - 1} A_l^{1-\alpha}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2}\right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}\right)^{-\frac{1}{1-\epsilon}}}. \tag{23}$$

Taking the logarithms and differentiating with respect to time yields the following expression for the development of the energy intensity in the energy-intensive sector, i.e. the efficiency effect:

$$\textit{Efficiency Effect} \equiv \gamma_{\frac{E}{Y_e}} = (\alpha_2 S - 1)\,\gamma_{c_E} + S\left[(1-\alpha)\gamma_{A_l} - (1-\alpha_1)\gamma_{A_e}\right], \tag{24}$$

with $S \equiv \frac{\left(\psi^{\alpha_2}(\alpha_1)^{2\alpha_1}(\alpha_2)^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}}{\left(\alpha^{2\alpha} c_E^{\alpha_2}\right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2}(\alpha_1)^{2\alpha_1}(\alpha_2)^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}} = Y_l^{\frac{\epsilon-1}{\epsilon}} / \left(Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}\right) \in (0,1)$, γ_{c_E} denoting the growth rate of the energy price, and γ_{A_l} (γ_{A_e}) denoting the growth rate of the technology level in the labour-intensive (energy-intensive) sector.

Since $S \in (0, 1)$, the growth rate of the energy price negatively affects the growth rate of the energy intensity in the energy-intensive sector. This effect is mainly driven by substituting away from the increasingly expensive production factor energy in the production process. The lower S, the higher is the magnitude of this effect, where S increases (decreases) in A_l/A_e and c_E for $\epsilon > 1$ ($\epsilon < 1$).

In a next step, we derive the growth rate of the relative size of the labour-intensive sector. Using again the equilibrium values derived initially, the relative output of the labour-intensive sector is:

$$\frac{Y_l}{Y_e} = \alpha^{2\alpha\epsilon}\alpha_1^{-\frac{2\alpha_1}{1-\alpha}(\epsilon-\epsilon\alpha}\alpha_2^{-\frac{\alpha_2\epsilon(1+\alpha)}{1-\alpha}}\psi^{\frac{\alpha_1\alpha_2\epsilon}{1-\alpha}}A_e^{-\frac{1-\alpha_1}{1-\alpha}(1-\alpha-\varphi)}A_l^{1-\alpha-\varphi}c_E^{\epsilon\alpha_2}. \tag{25}$$

Taking the logarithms and differentiating with respect to time yields the growth rate of the relative sector size as:

$$\gamma_{\frac{Y_l}{Y_e}} = \epsilon(1-\alpha)\gamma_{A_l} - \epsilon(1-\alpha_1)\gamma_{A_e} + \epsilon\alpha_2\gamma_{c_E}. \tag{26}$$

Multiplying this growth rate with the relative size of the l-sector yields the Sector Effect:

$$\textit{Sector Effect} \equiv -S \cdot \gamma_{\frac{Y_l}{Y_e}} = S \cdot \epsilon\left(-\alpha_2\gamma_{c_E} - (1-\alpha)\gamma_{A_l} + (1-\alpha_1)\gamma_{A_e}\right) \tag{27}$$

Both a positive growth rate of the energy price and research occurring in the l-sector lead to a reallocation of labour from the e- to the l-sector. This leads to an increase of the relative size of the l-sector. Hence, the Sector Effect reduces the overall energy intensity. Research in the e-sector has the opposite effect. It induces a structural change towards the energy-intensive sector at the expense of the labour-intensive sector. Hence, in this case, the Sector Effect increases the overall energy intensity in the economy. Furthermore, the magnitude of the Sector Effect depends on the relative size of the l-sector. In the case of substitutes (complements), the magnitude of the Sector Effect increases (decreases) with the relative l-sector size.

3.2. Innovation, Energy Price, and Energy Intensity Dynamics

Substituting the expressions (24) and (27) into (22) yields the the growth rate of the economy wide energy intensity as the sum of the Efficiency Effect (EE) and the Sector Effect (SE):

$$\begin{aligned}\gamma_{\frac{E}{Y}} = [&\underbrace{(\alpha_2 S - 1)}_{\text{EE}} \underbrace{-S\epsilon\alpha_2}_{\text{SE}}]\gamma_{c_E} + [\underbrace{(1-\alpha)S}_{\text{EE}} \underbrace{-S(1-\alpha)\epsilon}_{\text{SE}}]\gamma_{A_l} \\ &+[\underbrace{-(1-\alpha_1)S}_{\text{EE}} \underbrace{+S\epsilon(1-\alpha_1)}_{\text{SE}}]\gamma_{A_e}.\end{aligned} \tag{28}$$

This expression for the growth rate of the energy intensity establishes the following proposition that show how innovation in the *e*- and the *l*-sector as well as energy price growth affect the Efficiency and the Sector Effect, respectively. These results will be the basis of our discussion of the Sector- and Efficiency Effect's impact on the development of energy intensity for heterogeneous countries in different energy price scenarios.

Proposition 1. *i. Innovation in the e-sector,* $\gamma_{A_e} > 0$*, leads to a positive Sector Effect and a negative Efficiency Effect, where, in the case of* $\epsilon > 1$ $(\epsilon < 1)$*, the Sector (Efficiency) effect dominates the Efficiency (Sector) Effect, i.e. it increases (decreases) the growth rate of energy intensity.*

ii. Innovation in the l-sector, $\gamma_{A_l} > 0$*, leads to a negative Sector Effect and a positive Efficiency Effect, where, in the case of* $\epsilon > 1$ $(\epsilon < 1)$ *the Sector (Efficiency) effect dominates the Efficiency (Sector) effect, i.e. it decreases (increases) the growth rate of the energy intensity.*

iii. A positive (negative) growth rate of the energy price, $\gamma_{c_E} > 0$ $(\gamma_{c_E} < 0)$*, leads to a negative Sector Effect and a negative Efficiency Effect and hence always decreases (increases) the growth rate of the energy intensity.*

Proof. Follows from equation (28). □

The effect of technological change in the energy-intensive sector depends on the elasticity of substitution between the goods produced in both sectors (ϵ). When the sectoral outputs are substitutes, the net effect of technical progress in the energy-intensive sector is positive, i.e. it causes an increase of the energy intensity. This counter-intuitive result is due to the fact that, in spite of technological improvements in the energy-intensive sector, the increase of the share of this sector's output overcompensates the energy saving effect of technical change and hence leads to an increase of the energy intensity. The Sector Effect dominates the Efficiency Effect in this case.

If both sectors are complements, a too uneven input of both sectors goods leads to a decreasing productivity in the production of the final good. Hence, the complementarity of both sectoral outputs restricts the drifting apart of both sectors' output quantities. The Sector Effect is reduced and dominated by the Efficiency Effect.

The intuition of the decreasing effect of the Sector Effect on the energy intensity is as follows. Research in the l-sector increases the average productivity of this sector and hence leads to a reallocation of labour from the e- to the l-sector. The resulting restructuring of the composition of final output towards the l-sector yields a negative Sector Effect, i.e. a reduction of energy intensity in the economy. The induced decrease in labour input in the e-sector induces a substitution of labour by energy which, c.p., yields a positive Efficiency Effect, i.e. an increase of the energy intensity in the e-sector.

Substituting for S in (28) yields the growth rate of energy intensity depending on relative technologies and energy price:

$$\gamma_{\frac{E}{Y}} = \left[\frac{(\alpha_2 - 1)A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - \frac{\epsilon\alpha_2 A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{c_E}$$

$$+\left[\frac{(1-\alpha)A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - \frac{(1-\alpha)\epsilon A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_l} + \left[\frac{-(1-\alpha_1)A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} + \frac{\epsilon\,(1-\alpha_1)\,A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_e}$$

$$= \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{\varphi\,A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_l} + \left[\frac{-\varphi_1\,A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_e} \tag{29}$$

with $A \equiv \left(\frac{A_e^{1-\alpha_1}}{A_l^{1-\alpha}}\right)$, $\theta \equiv \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}}\right)^{1-\epsilon} > 0$.

4. Analysing observed Energy Intensity Developments

After having shown how innovation and energy price growth respectively affect the development of the energy intensity through the Sector and Efficiency Effect, we now turn to the analysis of energy intensity developments that can be observed in the data. We analyse both the case of substitutes, $\epsilon > 1$, as well complements, i.e. $\epsilon < 1$. As we will show throughout this Section, gross substitutes seem to be more plausible with respect to actual data. The following two assumptions will be useful for the subsequent analysis.

Assumption 1. $A_e(t)^{(-\varphi_1)}/A_l(t)^{(-\varphi)} > \kappa c_E(t)^{\alpha_2(\epsilon-1)}\eta_l/\eta_e$.

Following Lemma 1, this assumption implies that the e-sector's sufficient advancement at time t induces research in the e-sector (l-sector) only for $\epsilon > 1$ ($\epsilon < 1$).

Assumption 2. $A_e(t)^{(-\varphi_1)}/A_l(t)^{(-\varphi)} < \kappa c_E(t)^{\alpha_2(\epsilon-1)}\eta_l/\eta_e$.

Similarly, under Assumption 2 the l-sector's technological advancement results in research in the l-sector (e-sector) only for $\epsilon > 1$ ($\epsilon < 1$). For the analysis, we use natural baseline scenarios, namely research directed to one sector only for $\epsilon > 1$ and research directed to both sectors in case of $\epsilon < 1$. The intuition follows from Lemma 1. If both sectoral goods are gross substitutes and Assumption 1 holds, research is and will remain directed to the e-sector, as research increases the relative profitability of innovation in this sector through the direct productivity effect that dominates for $\epsilon > 1$. Similarly, when Assumption 2 holds, research is directed to the labour-intensive sector only and further increases the profitability of innovation in the l-sector.

In contrast, when both goods are gross complements and Assumption 1 holds, i.e. the energy-intensive sector is more advanced, research will be directed to the less advanced l-sector as the price effect dominates. Similarly, if Assumption 2 holds, research is directed to the more backward e-sector. Hence, ultimately the equilibrium must be characterized by innovation in both sectors. In this inner equilibrium, the share of scientists directing their research towards the low energy sector is $s_l = \frac{\alpha_2(\epsilon-1)\frac{\gamma_C}{\gamma}+\eta_l\varphi_1}{\varphi\eta_l+\varphi_1\eta_e}$ (Proof: See Appendix A.3).

The analysis is structured by three different scenarios of energy price growth, that can be used as a stylised way to describe different historical periods. We start with the constant energy price as our base case (4.1), which could be applied to the situation before the energy crisis. Between 1950 and 1973 the price of crude oil remained relatively constant (Hannesson, 2002). Subsequently, we analyse the energy intensity dynamics under moderate energy price growth (4.2). This is the relevant scenario for the last decades as most of the period subsequent to the high energy prices in the aftermath of the energy crises in the 1980s was characterised by increasing oil and energy prices, particularly since the late 1990s (see, e.g., Lee and Lee (2009), Narayan and Narayan (2007), and Regnier (2007)). In Subsection 4.3, we consider strong energy price growth. This scenario could be applied to periods of the oil crises and their aftermath (1974-1986) that were characterised by dramatic increases in energy costs (see, e.g., Alpanda and Peralta-Alva (2010), Linn (2008)).

4.1. Constant Energy Price

As a base case, we analyse the development of energy intensities with constant energy prices. Consider an economy where the l-sector is more advanced in period t, i.e. Assumption 1 holds. Hence, research is directed entirely to the l-sector and the technology grows with the rate $\gamma_{A_l} = \gamma\eta_l$. As $\gamma_{A_e} = \gamma_{c_E} = 0$, results of Proposition 1 (ii) can be applied directly: innovation in the l-sector induces a structural change of the economy towards the l-sector such that the Sector Effect negatively affects energy intensity. The Efficiency Effect, the increasing energy intensity in the e-sector caused by substituting energy for labour, is dominated by Sector Effect such that the energy intensity in the whole economy declines at the rate

$$\gamma_{\frac{E}{Y}} = \left[\frac{\varphi A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \right] \gamma \eta_l < 0. \tag{30}$$

In contrast, consider an economy where research is directed to the energy-intensive sector only, i.e. Assumption 2 holds. Hence, productivity in the *e*-sector grows with the rate $\gamma_{A_e} = \gamma\eta_e$. As $\gamma_{A_l} = \gamma_{c_E} = 0$, Proposition 1 can be applied directly: research directed at the energy-intensive sector induces an allocation of labour towards this sector and the energy intensity in the *e*-sector decreases, i.e. the Efficiency Effect negatively affects energy intensity. However, the restructuring of the economy away from the labour-intensive towards the energy-intensive sector, i.e. the Sector Effect, positively affects the energy intensity. As the latter effect is stronger in this scenario, the overall energy intensity increases in the economy with the rate

$$\gamma_{\frac{E}{Y}} = \left[\frac{-\varphi_1 A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \right] \gamma \eta_e > 0. \tag{31}$$

If both sectoral goods are gross complements ($\epsilon < 1$), research eventually will take place in both sectors. In this case, the results of Propositions 1 (*i* and *ii*) have to be combined. The growth rate of technology in the labour-intensive sector is $s_l\gamma\eta_l$, whereas technology in the energy-intensive sector grows with the rate $s_e\gamma\eta_e$. Hence, the growth rate of energy intensity is

$$\gamma_{\frac{E}{Y}} = \left[\frac{\varphi A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \right] s_l \gamma \eta_l + \left[\frac{-\varphi_1 A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \right] s_e \gamma \eta_e = 0.^{10} \tag{32}$$

As research is directed to both sectors, both sectoral outputs grow at the same rate. Hence, the Sector Effect is zero as the relative size of both sectors does not change. Furthermore, in equilibrium there is no reallocation of labour among both sectors and hence no substitution between labour and energy in the *e*-sector. Hence, output and energy input in the energy-intensive sector grow with the same rate yielding an Efficiency Effect of zero.

Data on energy intensities in the period prior to the energy crises reveal heterogeneous developments across countries. Hannesson (2002) analyses the energy intensities in 5 developed and 11 developing and emerging economies. Within the group of developed countries, Japan experienced a moderate and Italy a very strong increase in energy intensity. This development can also be observed among most emerging countries. Hannesson (2002, p. 219) argues that,

[10] Proof: See Appendix A.4.

particularly in Italy and Japan, the increase in energy use is mainly due to industrialisation. This development is predicted by our model when research is directed to the energy-intensive sector only. Due to research in the sector, the relative size of the energy-intensive sector increases. This restructuring of the economy towards the e-sector (industrialisation) increases overall energy intensity in the economy. When energy prices don't grow - or even fall as it could be observed in certain periods prior the energy crises - there are incentives to substitute labour by energy in the production process (Casler and Hannon, 1989).

Within the group of developed countries, two economies, France and UK, experienced a notable decline in energy intensity. This decrease in energy intensities could be an indication that these comparatively advanced economies were relatively advanced in sectors with low energy intensity. In spite of the constant energy price, research is directed to these sectors such that overall energy intensity decreases.

The model with $\epsilon < 1$ seems not applicable to most observations, as it predicts constant energy intensity. Exceptions could be the US, where energy intensities remained almost constant before the energy crisis (Casler and Hannon, 1989; Hannesson, 2002). However, most economies experienced considerably big changes in energy intensity.

4.2. Moderate Energy Price Growth

In contrast to the previous Subsection, we now assume moderate energy price growth, i.e. Lemma 2 (i) holds. Consider an economy where the l-sector is sufficiently advanced, i.e. Assumption 2 holds, such that research is directed entirely to the l-sector and the technology grows with the rate $\gamma_{A_l} = \gamma\eta_l$. Hence, results of Proposition 1 (ii and iii) can be applied. The decreasing effect of innovation in the l-sector on energy intensity is similar to the case with constant energy price. Positive energy price growth has an additional negative effect on energy intensity growth. Hence, energy intensity in the whole economy sinks with the rate:

$$\gamma_{\frac{E}{Y}} = \left[\frac{\alpha_2(1-\epsilon)\, A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{\varphi\, A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_l < 0. \tag{33}$$

Due to the additional effect of energy price growth, the rate of energy intensity reduction is larger than for constant energy price. The direction of the Efficiency Effect, however, depends on the magnitude of the energy price growth.

Lemma 3. *With moderate energy price growth and research directed to the l-sector, the Efficiency Effect is negative when*

$$\gamma_{c_E} > \frac{(1-\alpha)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_l.$$

Proof. Follows from equation (29) with $\gamma_{A_e} = 0$:

$$\textit{Efficiency Effect} = \frac{(\alpha_2 - 1)A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma_{c_E} + \frac{(1-\alpha)A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \overset{(>)}{<} 0$$
$$\Leftrightarrow \gamma_{c_E} \overset{(<)}{>} \frac{(1-\alpha)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_l$$

□

In contrast, when Assumption 1 holds, i.e. the economy is more advanced in the energy-intensive sector, research is directed to e-sector and the growth rate of energy intensity is:

$$\gamma_{\frac{E}{Y}} = \left[\frac{\alpha_2(1-\epsilon)\, A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[-\varphi_1 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e. \tag{34}$$

As shown in Proposition 1 (*i* and *iii*), a positive growth rate of the energy price reduces energy intensity (negative Efficiency and Sector Effect) whereas research in the energy intensive sector increases energy intensity (negative Efficiency and positive Sector Effect). Whether the overall effect is negative depends on the growth rate of the energy price relative to the growth rate of the technology in the e-sector.

Proposition 2. *i. With moderate energy price growth and research directed to the e-sector, energy intensity declines when*

$$\gamma_{c_E} > \frac{\varphi_1 A^{1-\epsilon}}{(\alpha_2(1-\epsilon) - 1)\, A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_e.$$

ii. The higher the relative technology, A, the lower is the threshold Λ_e.

Proof. i. Follows from equation (29) with $\gamma_{A_l} = 0$:

$$\gamma_{\frac{E}{Y}} = \left[\frac{\alpha_2(1-\epsilon)\, A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{-\varphi_1\, A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e \overset{(>)}{<} 0$$
$$\Leftrightarrow \frac{\gamma_{c_E}}{\gamma\eta_e} \overset{(<)}{>} \frac{\varphi_1 A^{1-\epsilon}}{(\alpha_2(1-\epsilon) - 1)\, A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} > 0$$

ii.

$$\frac{\partial \Lambda_e}{\partial A} = \frac{(1-\epsilon)\varphi_1 A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(\alpha_2(1-\epsilon) - 1)\, A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}\right]^2} \overset{(>)}{<} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$$

□

With $\epsilon < 1$ and moderate energy price growth, research is ultimately taking place in both sectors. Hence, the equilibrium growth rate of energy intensity is:

$$\gamma_{\frac{E}{Y}} = \left[\frac{\alpha_2 (1-\epsilon) A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2 (1-\epsilon)}} - 1 \right] \gamma_{c_E} + \left[\frac{\varphi A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2 (1-\epsilon)}} \right] s_l \gamma \eta_l + \left[\frac{-\varphi_1 A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2 (1-\epsilon)}} \right] s_e \gamma \eta_e = -\gamma_{c_E}.\text{[11]} \tag{35}$$

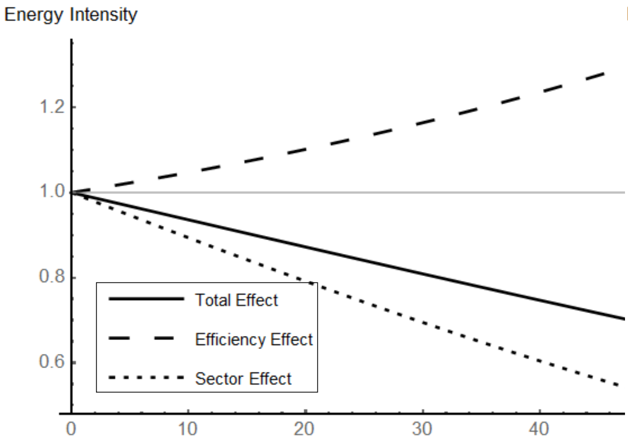
As research is directed to both sectors, the relative sector size does not change and hence the Sector Effect is equal to zero. The Efficiency Effect, however, is $-\gamma_{c_E} < 0$.

We illustrate the results for an economy, where research is directed to the relatively more advanced labour-intensive sector ((33) and Lemma 3) in Figure 1. Figure 1(a) depicts the energy intensity development for very low energy price growth, i.e. $\gamma_{c_E} < \Lambda_l$, and Figure 1(b) shows the development for $\gamma_{c_E} > \Lambda_l$.[12] In both cases, the overall energy intensity in the economy, the sum of the Efficiency and the Sector Effect (Total Effect), decreases over time. A comparison of both Figures shows that the reduction is stronger for larger energy price growth. Furthermore, in both cases the decline in energy intensity is mainly driven by the Sector Effect. As research is directed to the *l*-sector, labour is reallocated from the *e*- to the *l*-sector resulting in an increasing relative sector-size of the labour-intensive sector (negative Sector Effect). Through this structural adjustment towards the *l*-sector, the share of the energy-intensive sector's output in overall output decreases which drives the reduction of energy intensity in the economy. A major difference between both cases is the direction of the Efficiency Effect. For very low energy price growth, Figure (a) shows a positive Efficiency Effect, whereas this effect is negative for all other moderate energy price growth rates. The stronger the growth of the energy price, the higher is the incentive to substitute energy by other factors in the *e*-sector. An energy price increase larger than Λ_l induces a substitution of energy by other factors of production within the energy-intensive sector. Hence, input of energy grows slower than output in this sector, and hence energy intensity in the *e*-sector decreases (negative Efficiency Effect). Very low energy price growth rates are not sufficient to induce this substitutions. Energy input grows more than proportionally and the energy intensity in the *e*-sector increases (positive Efficiency Effect).

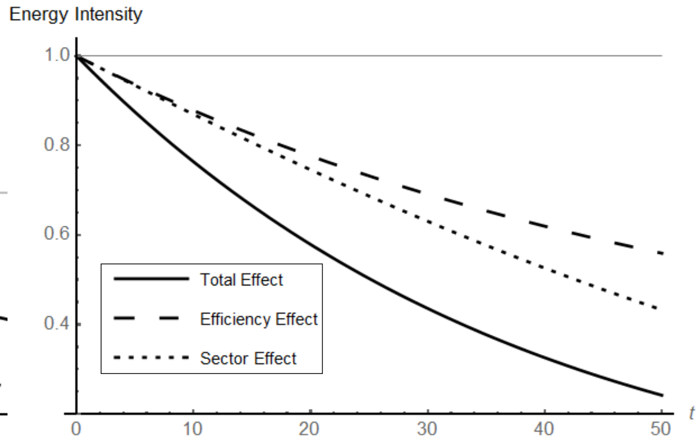
Our results on energy intensity dynamics when research is directed to the energy-intensive sector (Expression (34) and Proposition 2) are depicted in Figure 2. Figure 2(a) illustrates the

[11]Proof: See Appendix A.4.

[12]The figures throughout this paper are based on numerical simulations based on the following parameters. We assume in the case of substitutes (complements) $\epsilon = 2$ ($\epsilon = 0.8$), $\alpha_1 = \alpha_2 = \frac{1}{6}$ and therefore $\alpha = 0.3$, so that the national income spent on machines is approximately equal to the share of capital and the labour income is approximately equal to the share of labour. Furthermore we assume $\eta_e = \eta_l = 0.1$ and $\gamma = 0.2$ so that the long run growth rate of GDP is approximately 2%. We interpret the difference between to points in time $(t = z) - (t = (z-1)) = 1$ as one year.



(a)



(b)

Figure 1: Efficiency, Sector and Total Effect with moderate energy price growth (a) below threshold Λ_l (here: $\gamma_{c_E} = 0.1\%$) and (b) above threshold Λ_l (here: $\gamma_{c_E} = 2\%$) and research directed to the labour-intensive sector.

energy intensity development for very low energy price growth, i.e. $\gamma_{c_E} < \Lambda_e$, and Figure 2(b) depicts the development for $\gamma_{c_E} > \Lambda_e$. For both energy price growth rates, the Efficiency Effect is negative. The reason is that, even at low energy price growth rates, the effect of technological progress directed to the e-sector overcompensates the growing input of energy. Hence, output in the energy intensive sector grows faster due to technological improvements. Furthermore, the Sector Effect is positive in both cases. As research is directed to the e-sector, labour is reallocated towards this sector and hence the relative sector size of the labour-intensive sector declines over time, whereas the sector that uses energy for production gains in relative importance. This positive Sector Effect increases energy intensity in the economy. This positive Sector Effects dominates the energy efficiency improvements in the e-sector for growth rates below Λ_e (Figure (a)). If the energy price grows at a larger rate, the Efficiency Effect dominates this positive Sector Effect. In addition to the energy efficiency improvements due to technological change in the energy-intensive sector, the growing energy price further incentivises a substitution from energy by other factors of production. This additional substitution induced by the energy price yields a decrease of the energy intensity over time (Figure (b)).

Finally, the results for complements (Expression 35) are depicted in Figure 3. Energy intensity declines, while the magnitude is increasing with the growth rate of the energy price. Furthermore, the drop in energy intensity is solely driven by the Efficiency Effect, as for moderate energy price growth rates, research is always directed to both sectors. The growth rate of energy price only affects the allocation of researchers to both sectors: the higher energy price growth, the higher the share of scientists directing their research to the energy-intensive sec-

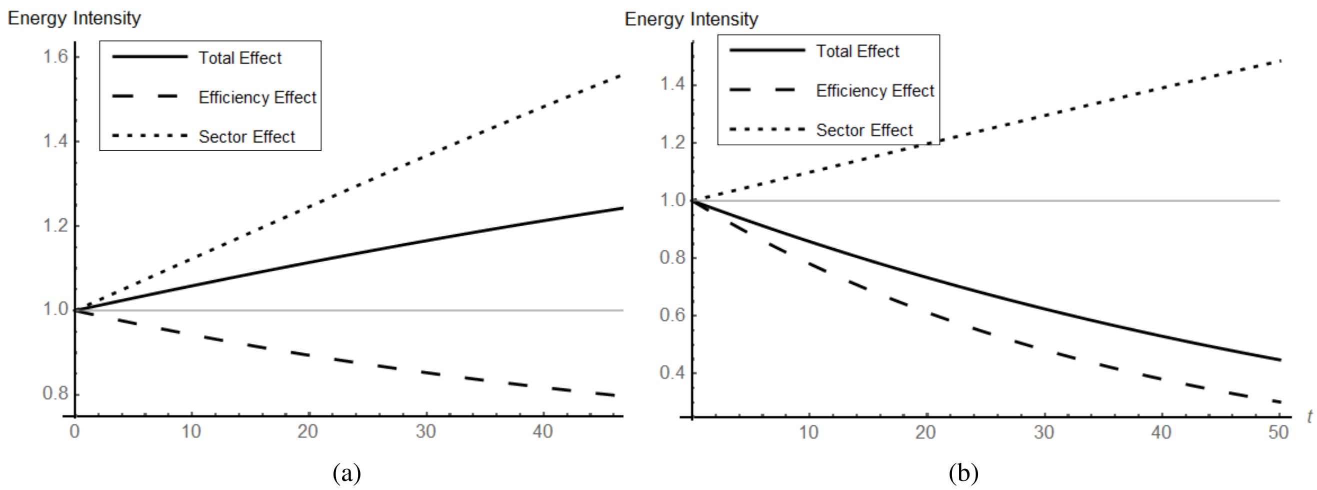


Figure 2: Efficiency, Sector and Total Effect with moderate energy price growth (a) below threshold Λ_e (here: $\gamma_{c_E} = 0.1\%$) and (b) above threshold Λ_e (here: $\gamma_{c_E} = 2\%$) and research directed to the energy-intensive sector.

tor. This reallocation compensates the increasing growth rate of the energy price such that the relative sector size remains constant (Sector Effect is zero). Increasing costs of energy induce substitution of energy by machines in the energy-intensive sector which leads to a reduction of energy intensity in the e-sector. Given the constant relative sector size, this directly translates to a energy intensity reduction in the whole economy.

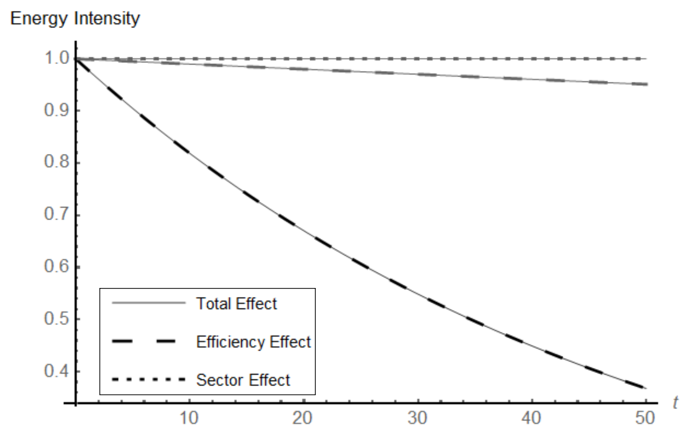


Figure 3: Efficiency, Sector and Total Effect with moderate energy price growth (here: $\gamma_{c_E} = 2\%$ (dark line), $\gamma_{c_E} = 0.1\%$) and research directed to both sectors (inner equilibrium s^{**}).

For $\epsilon > 1$ our results are in line with the data on energy intensities after the energy crises. Decomposition analyses show that the decline of energy intensity has been driven by the Sector Effect in around 25%-30% of OECD economies between 1995 and 2007 (Mulder and

Groot, 2012, Voigt et al., 2014). We offer an explanation why the Sector Effect is driving energy intensity reduction in certain economies, whereas the Efficiency Effect is the main driver in others. For the former economies, our model indicates that the productivity of sectors with low energy intensities must have been relatively higher than the productivity of energy intensive sectors. Hence, we argue that the dominance of the Sector Effect for the reduction of energy intensity is due to research being directed to the technically more advanced and hence more profitable labour-intensive sectors. Our result for research being directed to the labour-intensive sector (Figure 1) shows similar developments to the decomposed developments of energy intensities that can be seen for, e.g., Australia, Finland, Italy, Japan, Netherlands, UK, and USA (Voigt et al., 2014, Figures 7, 8, & 9). In all these economies, the Sector Effect drives energy intensity reduction. The role of the Sector Effect is particularly dominant in the UK (87%) and Japan (127%) (Mulder and Groot, 2012). In most of these economies the Efficiency Effect is negative as well, which is in line with our results for moderate energy price growth rates that are not very small. Exceptions are Italy and Japan that show a moderately positive Efficiency Effect. Although we assume a exogenous energy price that is identical for all countries, the reality is of course more complex as energy prices largely depend on, e.g., national taxes and policies. Energy price data shows that Japan experienced an approximately constant energy price between 1990 and 2005, while the energy price in Italy only grew very moderately, at least between 1990 and 2000 (Lee and Lee, 2009).

In the reminder of the 65%-70% OCED economies, energy efficiency improvements, particularly in industries with high energy-intensities, where the main drivers of energy intensity reductions. Similarly, we argue that in these countries the energy-intensive sectors were relatively more advanced to the sectors with low energy intensities. Hence, the higher relative productivity and thus profitability of energy-intensive industries directed research towards these sectors. Innovations in these industries, together with energy price growth inducing factor substitution, result in energy intensity reduction (Efficiency Effect).[13] Our model results describe the observed developments in, e.g., Austria, Canada, Germany, Ireland, Malta, and Spain (See Figure 2 and Voigt et al. (2014, Figures 8 & 9)). In all these economies, energy intensities decline due to the Efficiency Effect, while the Sector Effect is positive, indicating a restructuring towards energy-intensive industries.

Finally, the case of complements does not provide too satisfactory results. Energy intensity reductions are solely driven by the Efficiency Effect. However, decomposition analysis show

[13]Wang (2013) conducts a further decomposition of the Efficiency Effect in underlying driving forces, namely technological progress, substitution between energy and capital and labour, and changes in the structure of output. Using data from 1980-2010, the study shows that, in Europe, the main contributor to energy intensity reductions is technological progress followed by increases in the capital-energy ratio, whereas in North America, the order is reversed. Steinbuks and Neuhoff (2014) analyse various industries and show that labour is a substitute for energy.

an important role of sectoral adjustments as a driver of energy intensity reductions as they, e.g., attribute for 25% of energy intensity reduction in OECD countries (Mulder and Groot, 2012, p.1915).

4.3. Strong Energy Price Growth

In this Scenario, the energy price is assumed to grow at a high rate, i.e. $\gamma_{c_E} > (1-\alpha_1)\,\eta_e\gamma/\alpha_2$ (Lemma 2), such that it ultimately determines the direction if innovation.[14] In an economy, where the labour-intensive sector is sufficiently advanced, i.e. Assumption 2 holds, research is directed to the l-sector. Hence, as in Subsection 4.2, the technological level in the l-sector grows with the rate $\gamma_{A_l} = \gamma\eta_l$ and the equation for the growth rate of energy intensity is similar to the expression for moderate price growth (33). As the growth rate of energy intensity was negative for moderate price growth and a positive growth rate of the energy price negatively affects energy intensity, energy intensity growth is also always negative for strong energy price growth. The magnitude of energy intensity decline is even larger than for moderate price growth. The intuition for this result is straightforward. The stronger the energy price grows, the stronger is the substitution away from energy in the shrinking energy-intensive sector (Efficiency Effect).

Consider an economy that is more advanced in the energy-intensive sector, i.e. Assumption 1 holds, and hence research is initially directed to the e-sector. As outlined in Lemma 2, a sufficiently strong energy price growth will ultimately direct research to the l-sector. In spite of research being completely targeted at the e-sector, the increasing price for energy is decreasing the relative profitability (20) of innovation in the energy-intensive sector until it falls below unity and the direction of research changes in favour of the l-sector. The timing of this switch of research depends, next to the actual magnitude of energy price growth, on the relative productivity of the e-sector. The more advanced the energy-intensive sector, the longer it will take, c.p., until energy price growth will redirect innovation. Before this switch, technology in the e-sector grows with the rate $\gamma_{A_e} = \gamma\eta_e$ and energy intensity growth can be described with the same expression as in the case of moderate energy price growth (34). In contrast to the case of innovation in the e-sector and moderate price growth, energy intensity is always decreasing in this case with the rate:

$$\gamma_{\frac{E}{Y}} < -\frac{(1-\alpha_1)}{\alpha_2}\eta_e\gamma. \tag{36}$$

[14] We abstract from the case of strong decline of the energy price, i.e. $\gamma_c < -(1-\alpha)\,\eta_l\gamma/\alpha_2$, as it seems implausible with respect to actual energy price developments. All effects, however, are the exact opposite of the effects of a high increase in energy price analysed in this Subsection.

The main driver of the decline in energy intensity, next to the increased Efficiency Effect due to faster increasing energy costs, is the Sector Effect. As long as energy price grows moderately, the Sector Effect is always positive. For strong energy price growth, the Sector Effect becomes negative, i.e. the energy-intensive sector becomes relatively smaller.[15] The reason for this development is as follows. The rapidly growing costs of energy cannot be compensated by innovation. Energy input declines over time and hence the output in the energy-intensive sector shrinks. This means that strong energy price growth fosters a restructuring of the economy towards the *l*-sector even when innovation is still directed to the *e*-sector. As the relative decline in output of the energy-intensive sector is stronger than the increase of its relative price for $\epsilon > 1$, the profitability of innovation in this sector decreases. This process continues until research switches to the *l*-sector, i.e. Assumption 2 applies.

In contrast to a constant or moderately increasing energy price, research is not directed to both sectors in case of strong energy price growth. In the former cases, the magnitude of energy price growth only affects the allocation of scientists to both sectors. As can be seen from relative profitability of research (20), a higher energy price favours the energy-intensive sector for $\epsilon < 1$. Hence, the higher the energy price growth, the larger the share of scientists directing their research towards the *e*-sector (A.15), which is the typical result of DTC models. The increasing energy price induces a decline in energy input. As both sectoral outputs are complements, more research is directed to the energy-intensive sector to compensate for the energy price increase such that both sectors continue to grow at the same rate. As soon as energy price growth is strong, i.e. $\gamma_{c_E} > (1-\alpha_1)\,\eta_e\gamma/\alpha_2$, all research is directed to the *e*-sector. However, as discussed above for substitutes, research cannot overcompensate the reduction in energy input. In contrast to substitutes, the price effect is stronger than the market size effect for $\epsilon < 1$. Hence, relative price increase of output in the energy-intensive sector is stronger then the decline in relative output. Hence, growing energy prices increase the profitability of research in the energy-intensive sector. The development of energy intensity for this case is described be the same expression as in the case of substitutes and research directed to the *e*-sector (34) and energy intensity is decreasing (35).

This scenario is relevant for periods of high energy price growth / energy price shocks as e.g. during the period of energy crises. The results for both complements and substitutes show that, for high energy price grows, the reduction of energy intensity is higher than for moderate energy price growth. This result is supported by the findings of Sun (1998), who analyses the period 1973-1990 and shows that the reduction in energy intensity was particularly strong in the periods 1973-1980 (14.25%) and 1980-1985 (12.52%). An interesting implication of high energy price growth is that it can redirect technological progress from the energy-intensive

[15]Proof: *Sector Effect* $= \left[-\frac{\epsilon\alpha_2 A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{c_E} + \left[\frac{\epsilon(1-\alpha_1)A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e < 0 \Leftrightarrow \gamma_{c_E} > \frac{(1-\alpha_1)}{\alpha_2}\eta_e\gamma.$

to the labour-intensive sector, even when it is only temporary. Consider an economy, where research is directed to the e-sector prior the energy price shock. When an energy price shock occurs, energy price grows with a high rate, the relative profitability of innovation in the energy-intensive sector declines over time as can be seen in expression (20). As soon as relative profitability falls below unity, innovation in the labour-intensive sector becomes more profitable and hence scientist redirect their research activities towards this sector. Whether this redirection of research occurs depends on the length of the high energy price growth period and the relative technological advancement of the e-sector.

Combining this scenario with the previously analysed energy price growth regimes provides a framework for analysing long-run developments of energy intensities since the middle of the 20th century. Suitable cases for such an analysis seem to be, e.g., Italy and Japan. Both countries experienced energy intensity increases between 1950 and 1970 that were largely driven by industrialisation (Hannesson, 2002, p. 219). This observation implies that research was directed to energy-intensive industrial sector, where the resulting increase in energy intensity is mainly due the energy price growth rates close to zero. After the period of very high energy price growth 1974-1986, both economies saw energy intensity reductions that were largely driven by the Sector Effect (Voigt et al., 2014, p. 55). This implies that, after the energy price shocks, research was directed to sectors with low energy intensities that resulted in the observed energy intensity reductions. These two adverse developments indicate that the direction of research seems to have switched during the energy price crisis, namely from the e-sector to the l-sector. Hence, our analysis does not only provide insights on the dominance of the Sector and Efficiency Effect on energy intensity dynamics within different energy price growth regimes, but also offers a framework to study these dynamics throughout a long time period with with different energy price growth rates.

5. Conclusion

In this paper, we used a Directed Technical Change framework to analyse the adverse developments of energy intensities across countries. We used a DTC model set-up with energy as input factor in one of two sectors (Acemoglu et al., 2012). We have decomposed energy intensity in this model into a Sector- and an Efficiency Effect in order to investigate its dynamics due to the direction of research and energy price growth. This allowed for a detailed analysis of the impacts of energy prices and innovation on the development of energy intensity and, in particular, the relative importance of structural adjustments between sectors and energy efficiency within sectors that can be observed across countries.

Our main contribution to the literature is a first attempt to theoretically analyse observed energy intensity developments, using on a dynamic model with endogenous technical change.

So far, studies analysing the trends in energy intensities and the interaction of the driving forces, as the Sector- and Efficiency Effect, have been empirical. With increasing availability of data and sophisticated methodologies, these studies, particularly those using decomposition methods, have shown extensive and fruitful insights into the underlying drivers of energy intensity trends that substantially differ across countries.

Our paper offers a valuable supplement to the literature by theoretically exploring the determinants of the heterogeneous trends across countries. We offer an explanation why structural adjustments drive energy intensity reductions in certain countries whereas they are dominated by within-sector efficiency improvements in other. We show how energy price growth and the relative productivity of industries with low and high energy intensities affect the direction of research and hence the relative importance of the aforementioned two effects: (i) in economies that are relatively more advanced in industries with low-energy intensities and both industries' outputs are gross substitutes, research being directed to these industries results in the Sector Effect dominating energy intensity dynamics give no or moderate energy price growth; (ii) in economies that have high technological level within their energy-intensive industries compared to their other sectors, the Efficiency Effect dominates energy intensity developments, if moderate energy price growth is above a certain threshold; (iii) during periods of high energy price growth / energy price shocks, the Efficiency Effect always dominates the Sector Effect due the induced substitution of energy by other factors of production in the energy-intensive sectors. Furthermore, our paper offers a mechanism how temporal energy price shocks might induce a redirection of innovation activities towards sectors with low-energy intensities in an economy that remains even after the shock.

Our paper is a first step in exploring the determinants of heterogeneous energy efficiency trends based on a dynamic model with endogenous technical change. An area of future work might be an empirical investigation of our underlying mechanism to test whether our predicted role of technological levels for the relative importance of the Sector- and the Efficiency Effect is supported by the data. Furthermore, future research could use the rather new disaggregated sectoral data, as WIOD, to estimate the elasticity of substitution between sectors with high and those with low energy intensities. As our approach is a first step to theoretically analyse energy intensity determinants, extensions or alternative theoretical modelling strategies seem a fruitful direction of further research. As the data indicates structural adjustments in production between countries, it would be valuable to develop a multi-country model that could be used to analyse between-country structural adjustments caused by international trade. Particularly theoretical research appears to have a potential for important additional insights, as the empirical literature has taught us a great deal about energy intensity developments and its decomposition, whereas the underlying determinants are still largely unexplored.

A. Appendix A

A.1. Prices and Quantities in Equilibrium

Profit maximisation in the energy-intensive and labour-intensive sectors yields the following first-order conditions:

$$L_l = \left(\frac{w}{(1-\alpha)\, p_l \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di} \right)^{-\frac{1}{\alpha}}, \tag{A.1}$$

$$L_e = \left(\frac{w}{(1-\alpha)\, p_e E^{\alpha_2} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di} \right)^{-\frac{1}{\alpha}}, \text{ and} \tag{A.2}$$

$$E = \left(\frac{c_E}{p_e \alpha_2 L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di} \right)^{\frac{1}{\alpha_2 - 1}}. \tag{A.3}$$

Plugging the equilibrium quantity of machines (14) into (3) yields the production of labour-intensive output:

$$Y_l = L_l A_l \left(\frac{\alpha^2 p_l}{\psi} \right)^{\frac{\alpha}{1-\alpha}}. \tag{A.4}$$

Plugging (15) into (A.3) yields the equilibrium quantity of energy:

$$E = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E} \right)^{\frac{1-\alpha_1}{1-\alpha}} p_e^{\frac{1}{1-\alpha}} L_e \tag{A.5}$$

Combining (A.5) and (15) with (4) yields the production of the energy-intensive good as:

$$Y_e = \left(\frac{(\alpha_1)^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E} \right)^{\frac{\alpha_2}{1-\alpha}} p_e^{\frac{\alpha}{1-\alpha}} L_e A_e. \tag{A.6}$$

Equilibrium on the labour market implies an identical wage in both sectors. Equating (A.1) and (A.2), together with (A.5), (15) and (14), yields the relative price of both sectoral goods:

$$\frac{p_l}{p_e} = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_e^{1-\alpha_1}}{c_E^{\alpha_2} \alpha^{2\alpha} A_l^{1-\alpha}}. \tag{A.7}$$

This price ratio, together with the price index (11), leads to the equilibrium prices in both sectors:

$$p_l = \frac{\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}A_e^{1-\alpha_1}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1}{1-\epsilon}}}, \tag{A.8}$$

$$p_e = \frac{\alpha^{2\alpha}c_E^{\alpha_2}A_l^{1-\alpha}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1}{1-\epsilon}}}. \tag{A.9}$$

Combining the prices with input demands yields the equilibrium employment of labour in both sectors

$$L_l = \frac{\left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)}, \tag{A.10}$$

$$L_e = \frac{\left(c_E^{\alpha_2}\alpha^{2\alpha}\right)^{1-\epsilon}A_l^{\varphi}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)} \tag{A.11}$$

as well as equilibrium energy use in the energy-intensive sector

$$E = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}}\alpha_2^{\frac{1-\alpha_1}{1-\alpha}}\alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon+1\right)}c_E^{\alpha_2-1-\epsilon\alpha_2}A_l^{1+\varphi}A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{1+\varphi}{\varphi}}}. \tag{A.12}$$

Plugging these optimal inputs into (A.4) and (A.6) yields the the equilibrium outputs in the labour- and energy-intensive sector as

$$Y_l = \frac{\alpha^{\frac{2\alpha}{1-\alpha}}\psi^{\frac{\alpha_1(\epsilon\alpha_2-1)}{1-\alpha}}\alpha_1^{\frac{2\alpha_1(1-\epsilon+\epsilon\alpha)}{1-\alpha}}\alpha_2^{\frac{\alpha_2(1-\epsilon-\epsilon\alpha)}{1-\alpha}}A_e^{\frac{1-\alpha_1}{1-\alpha}(\alpha+\varphi)}A_l}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}, \tag{A.13}$$

$$Y_e = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}}\alpha_2^{\frac{\alpha_2}{1-\alpha}}\alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon\right)}c_E^{-\epsilon\alpha_2}A_l^{\alpha+\varphi}A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha}c_E^{\alpha_2}\right)^{1-\epsilon}A_l^{\varphi} + \left(\psi^{\alpha_2}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\epsilon}A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}. \tag{A.14}$$

A.2. Direction of technical change with patents of infinite duration

Scientists choose to direct their research at the sector with higher expected firm value (discounted flow of future profits as entrepreneur):

$$E\left[V_{ji}(t=z)\right] = \int_z^{\infty} E\left[\pi_{ji}(t)\right] exp\left(-\int_z^t \left(1 - E\left[s_j(t)\right]\eta_j\right) dt\right) dt \quad \text{with} j \in \{e, l\}.$$

The expected relative value of firm i in sector j at time $t = z$ comprise current (at time z) and discounted future ($t > z$) expected profits ($E\left[\pi_{ji}(t)\right]$). The expected discount rate ($1 - E[s_j(t)]\eta_j$) depends on the expected research effort in sector j at each time t ($E[s_j(t)]$) and the probability of successful research (η_j). Expected relative firm value at $t = z$ is defined as

$$V(t=z) \equiv \frac{E\left[V_{li}(t=z)\right]}{E\left[V_{ei}(t=z)\right]}.$$

Substitutes (i.e. $\epsilon > 1$):

Since equilibrium research allocation depends crucially on the expected discount rate, the subsequent discussion of research equilibria is structured along three discount rate cases (for special cases see 1. & 3., general case 2.):

1. For $(1 - E[s_j(t)]\eta_j) \to 0$, $V(t=z) \to \frac{\Pi_{li}(t=z)}{\Pi_{ei}(t=z)}$, i.e. relative firm value reduces to current relative firm profits. Results of Appendix A.3 can be applied.

2. For $0 < (1 - E[s_j(t)]\eta_j) < 1$ and since $\frac{\partial E[V_{ji}(t=z)]}{\partial \Pi_{ji}(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > \frac{\partial E[\Pi_{sector \neq j,i}(t)]}{\partial A_j(t)}$, $\lim_{A_j(t)\to 0} E[\Pi_{ji}(t)] = 0$, $\lim_{A_j(t)\to\infty} E[\Pi_{ji}(t)] = \infty$ for each set of parameters there exists a unique relative technology $(A_l(t=z)/A_e(t=z))^*$ such that $\left.\frac{V_{li}(t=z)|_{s(t)=1}}{V_{ei}(t=z)|_{s(t)=0}}\right|_{\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*} = 1$. With $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, research will take place in the l-sector (e-sector) only. With $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ there exists a unique equilibrium ($s^{**} \in (0,1)$) with research directed to both sectors.

   a) With moderate energy price growth, i.e. $-\frac{\eta_l\gamma(1-\alpha)}{\alpha_2} < \gamma_c(t) < \frac{\eta_h\gamma(1-\alpha_1)}{\alpha_2}$, the expected relative profit (and therefore the expected relative firm value ($V(t)$)) increases (decreases) if research is directed to sector l (e) only (Proof: see Appendix A.3.1). Therefore a research equilibrium $s^* \in \{0,1\}$ at time z is always a research equilibrium in $t > z$. An inner equilibrium in $t = z$, $s^*(t=z) = s^{**}$, is an inner equilibrium if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$. With $s^*(t=z) \overset{(<)}{>} s^{**}$ research will take place in sector l (e) for all $t > z$ (follows from Appendix A.3.1).

   b) With strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ research will occur in sector l (e) at $t = z$ and all $t > z$ (follows from Lemma 2 and Appendix A.3.1). If $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and with strong positive (negative) energy price growth, research will at $t = z$ take place in the e-sector (l-sector) only. Since strong positive (negative) energy price growth increases (decreases) $V(t)$, there exists a time $\tau > z$ where $V(\tau) = 1$ and $V(t > \tau) \overset{(<)}{>} 1$, leading to research equilibrium in sector l (e) for all $t > \tau$ (follows from Lemma 2). There are multiple equilibria with $s^*(t = z) \in [0, 1]$ if $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and a unique equilibrium with all research in sector l (e) for all $t > z$ in the case of strong positive (negative) energy price growth.

3. For $(1-E[s_j(t)]\eta_j) \to 1$ and moderate energy price growth, $V(t = z) \to 1$ and there exist two equilibria with all research directed to the e- or the l-sector and multiple equilibria with research directed to both sectors (i.e. $s \in (0, 1)$). With strong positive (negative) energy price growth there exists a unique equilibrium with all research directed to sector l (e), as $\frac{d\Pi_e i(t)}{dt} \to 0$ ($\frac{d\Pi_l i(t)}{dt} \to 0$) and therefore $V(t = z) \overset{(0)}{\to} \infty$.

For (plausible) discount rates smaller 1, i.e. $0 \leq (1 - E[s_j(t)]\eta_j) < 1$, from 1. and 2. it follows that alternative patent terms do not induce qualitative differences in the research equilibrium at $t = z$. Research takes place in the relatively more advanced sector. Only the value of relative technology thresholds may differ, due to model design.
Research equilibria at $t > z$ are influenced only in so far, as if the direction of research changes over time, i.e. in the case of strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, the change occurs at an earlier point in time.

Complements (i.e. $\epsilon < 1$):
Results from Appendix A.3 can be applied.

A.3. Equilibrium allocation of researchers

With strong positive (negative) energy price growth, i.e. $\eta_e\gamma(1 - \alpha_1)/\alpha_2 < \gamma_{c_E}$ ($< -\eta_l\gamma(1 - \alpha)/\alpha_2$), the direction of the change of relative profit is independent of research.

Proof.

$$\frac{d\left(\frac{\Pi_l(t)}{\Pi_e(t)}\right)}{dt} = \alpha_2(\epsilon - 1)c_E(t)^{\alpha_2(\epsilon-1)}\gamma_{c_E} + \varphi_1 A_e(t)^{\varphi_1} s_e\eta_e\gamma - \varphi A_l(t)^{\varphi} s_l\eta_l\gamma \overset{(<)}{>} 0 \ \forall \ s_e, s_l \in [0, 1].$$

□

From that it follows that for moderate energy price growth, i.e. $-\eta_l\gamma\left(1-\alpha\right)/\alpha_2 \leq \gamma_{c_E} \leq \eta_e\gamma\left(1-\alpha_1\right)/\alpha_2$, the direction of the change of relative profit is not independent of research.

A.3.1. Moderate energy price growth

In the case of substitutes ($\epsilon > 1$):

1. From equation (20) and with $s(t) \equiv s_l(t)$ it follows:

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \quad \text{if} \quad s(t) \gtreqless s^{**} = \frac{\alpha_2(\epsilon-1)\frac{\gamma_c}{\gamma} + \eta_l\varphi_1}{\varphi\eta_l + \varphi_1\eta_e}. \tag{A.15}$$

Proof.

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \Leftrightarrow 0 \lesseqgtr \frac{d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt}{\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}} = \frac{\alpha_2(\epsilon-1)}{c_E(t)}\frac{dc_E(t)}{dt} - \frac{\varphi}{A_l(t)}\frac{dA_l(t)}{dt} + \frac{\varphi_1}{A_e(t)}\frac{dA_e(t)}{dt}.$$

Using equation (7) and (8) yields:

$$\begin{aligned} 0 &\lesseqgtr \alpha_2(\epsilon-1)\gamma_c - \varphi s_l(t)\gamma\eta_l + \varphi_1 s_e(t)\gamma\eta_e \\ \Leftrightarrow s^{**} = s(t) &\gtreqless \frac{\alpha_2(\epsilon-1)\frac{\gamma_c}{\gamma} + \eta_l\varphi_1}{\varphi\eta_l + \varphi_1\eta_e}. \end{aligned}$$

□

2. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t=z) = \frac{A_l(t=z)^{(1-\alpha)}}{A_e(t=z)^{(1-\alpha_1}} \overset{(<)}{>} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}}.$$

Proof. Using equation (20) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa\frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e}\frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}} \overset{(>)}{<} \frac{A_l(t)^{(1-\alpha)}}{A_e(t)^{(1-\alpha_1}} \equiv A(t).$$

□

If $s^*(t = z) \in \{0, 1\}$ is an equilibrium in $t = z$ than it is also an equilibrium in all $t > z$ (follows from Lemma 2 and A.15).

3. At time $t = z$ there exist multiple equilibria $s \in [0, 1]$ if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} .$$

Proof.

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} = \frac{A_l(t)^{(1-\alpha)}}{A_e(t)^{(1-\alpha_1}} \equiv A(t).$$

□

If $s^*(t = z) \in (0, 1)$ is an equilibrium in $t = z$ than $s^*(t = z) \in (0, 1)$ is also an equilibrium in $t > z$ if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$. If $s^*(t) \overset{(<)}{>} s^{**}$ there will be only research in sector l (e) in all $t > z$ (follows from Lemma 2 and A.15).

In the case of complements ($\epsilon < 1$):
(all results follow from the analysis of the case of substitutes):

1. From equation (15) follows:

$$d \frac{\Pi_{li(t)}}{\Pi_{ei(t)}} / dt \gtreqless 0 \quad \text{if} \quad s(t = z) \lesseqgtr s^{**}.$$

2. At time $t = z$ there exists a unique equilibrium research allocation s^* with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} .$$

Since $s^*(t = z) \in \{0, 1\}$ decreases $\left| 1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \right|$, there exists a time $\tau > z$ where $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ ($\Leftrightarrow A(t = \tau) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}}$).

3. At time $t = z$ there exist multiple equilibria $s^* \in [0, 1]$, if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} .$$

If $s^*(t = z) \in (0, 1)$ is an equilibrium in $t = z$ than $s^*(t = z) \in (0, 1)$ is also an equilibrium in all $t > z$ if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$.
Since $s^* \neq s^{**}$ would result in a(n unrealistic) permanently alternating direction of research, we assume $s^* = s^{**}$ (i.e. the dynamically stable equilibrium) in the case of an

inner equilibrium. This is also the technical result for longer patent terms (see Appendix A.2.2).

A.3.2. Strong energy price growth

In the case of substitutes ($\epsilon > 1$):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) = \frac{A_l(t = z)^{(1-\alpha)}}{A_e(t = z)^{(1-\alpha_1}} \overset{(<)}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}}.$$

Proof. Using equation (15) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}} \overset{(>)}{<} \frac{A_l(t)^{(1-\alpha)}}{A_e(t)^{(1-\alpha_1}} \equiv A(t).$$

□

If $A(t = z) \overset{(<)}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 1$ (0) is an equilibrium in $t = z$ and in all $t > z$ (follows from Lemma 2). If $A(t = z) \overset{(>}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 0$ (1) is an equilibrium in $t = z$ and since $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with all research directed to sector l (e) for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s \in [0, 1]$ if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}}.$$

Proof.

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}} = \frac{A_l(t)^{(1-\alpha)}}{A_e(t)^{(1-\alpha_1}} \equiv A(t).$$

□

With strong positive (negative) energy price growth, $s^*(t) = 1$ (= 0) is the unique equilibrium in all $t > z$ (follows from Lemma 2).

In the case of complements ($\epsilon < 1$):
(all results follow from the analysis of the case of substitutes):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} . \tag{A.16}$$

   If $A(t = z) \overset{(<)}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 0$ (1) is an equilibrium in $t = z$ and in all $t > z$, follows from (A.16). If $A(t = z) \overset{(>}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 1$ (0) is a unique equilibrium in $t = z$ and since $\left| 1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with all research directed to sector l (e) for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s^* \in [0, 1]$, if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{1-\epsilon}} .$$

   With strong positive (negative) energy price growth, $s^*(t) = 0$ ($= 1$) is the unique equilibrium in all $t > z$ (follows from (A.16)).

A.4. Sector and Efficiency Effect

Complements
The Sector Effect can be derived from equation (29) using equilibrium allocation of researchers:

$$\begin{aligned}
\textit{Sector Effect} =& -\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \epsilon \alpha_2 \gamma_c - \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} (1-\alpha) \epsilon \gamma \eta_l \frac{\alpha_2(\epsilon - 1)\frac{\gamma_c}{\gamma} + \eta_e \varphi_1}{\eta_e \varphi_1 + \eta_l \varphi} \\
&+ \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \epsilon (1-\alpha_1) \gamma \eta_e \frac{-\alpha_2(\epsilon - 1)\frac{\gamma_c}{\gamma} + \eta_l \varphi}{\eta_e \varphi_1 + \eta_l \varphi} \\
=& -\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \epsilon \alpha_2 \gamma_c + \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \alpha_2 \epsilon \gamma_c \frac{(1-\alpha)\eta_l + (1-\alpha_1)\eta_e}{\eta_l(1-\alpha) + \eta_e(1-\alpha_1)} \\
&+ \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} \epsilon \gamma \frac{(1-\alpha)\eta_l \eta_e (1-\alpha_1) - (1-\alpha_1)\eta_e \eta_l (1-\alpha)}{\eta_l(1-\alpha) + \eta_e(1-\alpha_1)} \\
=& 0
\end{aligned}$$

The Efficiency Effect can be derived from equation (29) using equilibrium allocation of researchers:

$$
\begin{aligned}
\textit{Efficiency Effect} =& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_c + (1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \frac{\alpha_2(\epsilon-1)\frac{\gamma_c}{\gamma} + \eta_e\varphi_1}{\eta_e\varphi_1 + \eta_l\varphi} \\
&- (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \frac{-\alpha_2(\epsilon-1)\frac{\gamma_c}{\gamma} + \eta_l\varphi}{\eta_e\varphi_1 + \eta_l\varphi} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_c + \frac{(-1)(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_l\alpha_2\gamma_c}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
&- \frac{(1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_e\alpha_2\gamma_c}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
&+ \frac{(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha_1) - (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha)}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_c - \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\alpha_2\gamma_c \frac{(1-\alpha)\eta_l - (1-\alpha_1)\eta_e}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
=& -\gamma_c
\end{aligned}
$$

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