# Direction Dependence of the Deceleration Parameter

Rong-Gen Cai* and Zhong-Liang Tuo†
*Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China*
(Dated: August, 2011)

**Abstract**

In this paper we study the possibly existing anisotropy in the acceleration expansion by use of the full sample of Union2 data. Using the hemisphere comparison method to search for a preferred direction, we take the deceleration parameter $q_0$ as the diagnostic to quantify the anisotropy level in the $wCDM$ model. We find that the maximum accelerating expansion direction is $(l, b) = (314^{\circ -13^\circ}_{+20^\circ}, 28^{\circ -33^\circ}_{+11^\circ})$, with the maximum anisotropy level of $\delta q_{0,max}/\bar{q}_0 = 0.79^{-0.28}_{+0.27}$, and that the anisotropy is more prominent when only low redshift data ($z \leq 0.2$) are used. We also discuss this issue in the $CPL$ parameterized model, showing a similar result.

PACS numbers: 98.80.Es, 98.80.Jk

## I. INTRODUCTION

Since the discovery of cosmic acceleration in the late of last century [1], the type Ia supernovae (SNIa) have become an important tool in determining the cosmological parameters. Given certain cosmological models, one usually implements the joint analysis with SNIa in combination with other observations, such as large scale structure [2], the cosmic microwave background (CMB) radiation [3], and so on to give constraints of cosmological parameters.

Two pillars of modern cosmology are general relativity and the cosmological principle. It is assumed that Einstein's general relativity still holds on the cosmic scales, which means that the evolution of the universe is governed by general relativity. The cosmological principle [4] says that our universe is homogeneous and isotropic on the cosmic scales. Indeed, the assumption of homogeneity and isotropy is consistent with currently accurate data coming from the cosmic microwave background (CMB) radiation, especially from the Wilkinson Microwave Anisotropy Probe (WMAP) [5], the statistics of galaxies [6], and the halo power spectrum [7], etc. And current astronomical observations are in agreement with $\Lambda CDM$ model [8].

However, the standard model is also challenged by some observations [9] (for more details see [10] and references therein). Thus it is necessary to revisit the assumption of the homogeneity and isotropy. As more and more supernovae data are released [11, 12], this study becomes possible.

From the theoretical point of view, the anisotropy may arise in some cosmological models. For example, a vector field may lead to anisotropy of the universe and gives rise to an anisotropic equation of state of dark energy [13]. Peculiar velocities are also associated with dipole-like anisotropies, triggered by the fact that they introduce a preferred spatial direction, and as a result of the drift motion, observers may find that the acceleration is maximized in one direction and minimized in the opposite [14]. In this paper, we focus on the issue related to the possible existence of anisotropy, and search for a preferred direction by using the SN Union2 (consisting of 557 SNIa) data[1]. Some previous works have payed attention to this issue using the SNIa data and found no statistically significant evidence for anisotropies [15]. Using the Union2, some authors derived the angular covariance function of the standard candle magnitude fluctuations searching for angular scales where the covariance function deviates from 0 in a statistically significant manner and no such angular scale was found [16]. Yet on the other hand, using the SNIa data in the frame work of an anisotropy Bianchi type I cosmological model and in the presence of a dark energy fluid with anisotropy equation of state, Ref. [17] found that a large level of anisotropy was allowed both in the ge-

[1] http://supernova.lbl.gov/Union/.

ometry of the universe and in the equation of state of dark energy. Ref. [18] constructed a "residual" statistic, which is sensitive to systematic shifts in the SNIa brightness, in a direction, and used this to search in different slices of redshift for a preferred direction on the sky, and found that at low redshift ($z < 0.5$) an isotropic model was barely consistent with the SNIa data at 2-3$\sigma$. In addition, Ref. [19] took use of the hemisphere comparison method to fit the $\Lambda CDM$ model to the supernovae data on several pairs of opposite hemispheres, and a statistically significant preferred axis was found. More recently, Antoniou and Perivolaropoulos [20] have applied the hemisphere comparison method to the standard $\Lambda CDM$ model and found that the hemisphere of maximum accelerating expansion was in the direction $(l, b) = (309^{\circ}{}^{-3^{\circ}}_{+23^{\circ}}, 18^{\circ}{}^{-10^{\circ}}_{+11^{\circ}})$ with Union2 data. This result is consistent with other observations, such as CMB dipole [21], CMB quadrupole [22], CMB octopole [23], large scale velocity flows [24] and large scale alignment in the QSO optical polarization data [25]. In these observations, all the preferred directions appear to be towards the North Galactic Hemisphere. They obtained the average direction of the preferred axes as $(l, b) = (278^{\circ} \pm 26^{\circ}, 45^{\circ} \pm 27^{\circ})$.

In this paper we study the dependence of this result on the dark energy model. We consider two models. One is the $wCDM$ model; the other is a dynamical dark energy model with the $CPL$ [26] parameterization. We take the present value of deceleration parameter $q_0$ as the diagnostic to quantify the anisotropy level of two opposite hemispheres.

The paper is organized as follows. In the next section we give a general introduction to the hemisphere comparison method using $q$ to quantify the anisotropy level and we apply it to $wCDM$ model fitted by the Union2 dataset. In Section III, we give the numerical results on the preferred directions from the SN data with different slices of redshift. We also give the result for the $CPL$ model. Section IV gives our conclusions.

## II. HEMISPHERE COMPARISON METHOD USING THE UNION2 DATASET

Since its first release, the SNIa data have been becoming an important tool for us to understand the evolution of the universe. And as time goes on, more accurately determined data will be released. For example, the future Joint Dark Energy Mission (JDEM) [2] is aimed to explore the properties of dark energy and to measure how cosmic expansion has changed over time. In this paper we take use of the Union2 dataset [11] using SALT2 for SNIa light-curve fitting, which contains 557 type Ia SN data and covers the redshift range $z = [0.015, 1.4]$, including samples from other surveys, such as CfA3 [27], SDSS-II Supernova Search [28] and high-z Hubble Space Telescope.

We fit the SNIa data by minimizing the $\chi^2$ value of the distance modulus. The $\chi^2_{sn}$ for SnIa is obtained by comparing theoretical distance modulus $\mu_{th}(z) = 5\log_{10}[d_L(z)] + \mu_0$, where $\mu_0 = 42.384 - 5\log_{10} h$ is a nuisance parameter, with observed $\mu_{ob}$ of supernovae:

$$\chi^2_{sn} = \sum_i^{557} \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}.$$

For a flat FRW cosmological model, one has

$$d_L(z) = (1+z)\int_0^z \frac{H_0}{H(z')}dz'. \quad (1)$$

In the case of $wCDM$ model, we have

$$H^2(z)/H_0^2 = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{3+3w}. \quad (2)$$

And the present value of the deceleration parameter can be expressed as

$$q_0 = \frac{1}{2} + \frac{3}{2}w(1-\Omega_{m0}). \quad (3)$$

In the case of the $CPL$ parametrization, the equation of state of dark energy is $w = w_0 + w_1\frac{z}{1+z}$. Accordingly one obtains

$$\begin{aligned} H(z) &= H_0^2[\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{3+3w_0+3w_1} \\ &\quad \exp -3w_1\frac{z}{1+z}], \quad (4) \\ q_0 &= \frac{1}{2} + \frac{3}{2}w_0(1-\Omega_{m0}). \quad (5) \end{aligned}$$

To eliminate the effect of $\mu_0$, we expand $\chi^2_{sn}$ with respect to $\mu_0$ [29] :

$$\chi^2_{sn} = A + 2B\mu_0 + C\mu_0^2, \quad (6)$$

[2] http://jdem.lbl.gov/.

where

$$A = \sum_i \frac{[\mu_{th}(z_i;\mu_0=0)-\mu_{ob}(z_i)]^2}{\sigma^2(z_i)},$$
$$B = \sum_i \frac{\mu_{th}(z_i;\mu_0=0)-\mu_{ob}(z_i)}{\sigma^2(z_i)},$$
$$C = \sum_i \frac{1}{\sigma^2(z_i)}.$$

Eq. (6) has a minimum as

$$\widetilde{\chi}^2_{sn} = \chi^2_{sn,min} = A - B^2/C,$$

which is independent of $\mu_0$. In fact, it is equivalent to performing a uniform marginalization over $\mu_0$, the difference between $\widetilde{\chi}^2_{sn}$ and the marginalized $\chi^2_{sn}$ is just a constant [29]. We will adopt $\widetilde{\chi}^2_{sn}$ as the goodness of fitting between theoretical model and SNIa data.

The directions to the SNIa we use here are given in Ref. [16], and are described in the equatorial coordinates (right ascension and declination). In order to use the hemisphere comparison method, we need to convert these coordinates to usual spherical coordinates $(\theta, \phi)$, and finally to the galactic coordinates $(l, b)$ [30].

We apply the hemisphere comparison method to find the preferred axis. This method was first proposed in Ref. [19], and further developed in Ref. [20], while the difference between them is the choice of the direction which separates the data into two subsets. Since the latter method can generate more random directions compared with the number of the data, therefore we will take the method in [20]. The hemisphere comparison method involves the following steps:

1. Generate a random direction

$$\hat{n} = (\cos\phi\sqrt{1-u^2}, \sin\phi\sqrt{1-u^2}, u) \qquad (7)$$

where $\phi \in [0, 2\pi)$ and $u \in [-1, 1]$ are random numbers with uniform probability distribution.

2. Split the dataset under consideration into two subsets according to the sign of the value $\hat{n} \cdot \hat{n}_{dat}$, where $\hat{n}_{dat}$ is the unit vector describing the direction of each SNIa in the dataset. Then we divide the data in two opposite hemispheres, denoted by up and down, respectively.

3. Find the best-fitting values of $(\Omega_{m0}, w)$ ($(\Omega_{m0}, w_0)$ for $CPL$) on each hemisphere. Naturally, one can take use of the deceleration parameter $q_0$ to quantify the anisotropy level through the normalized difference

$$\frac{\Delta q_0}{\bar{q}_0} = 2\frac{q_{0,u} - q_{0,d}}{q_{0,u} + q_{0,d}} \qquad (8)$$

where the subscripts $u$ and $d$ denote the up and down hemispheres, respectively. Note that the deceleration parameter is a good diagnostic to quantify the anisotropy level, and the physically meaning is even clear. The direction with smaller $q_0$ is expanding faster than the opposite.

4. Repeat 400 times from step 1 to step 3, and find the maximum standardized difference for the Union2 data, thus one can obtain the corresponding direction of maximum anisotropy. (The repeat times is well above the number of SN data points on each hemisphere and hence it is enough in this study [20]).

## III. RESULTS

Following the steps introduced in Sec.II, one can obtain the best-fitting values of $q_0$ in each direction, and then find the maximum anisotropy direction. But in order to obtain the $1\sigma$ errors of the maximal anisotropic direction and the anisotropy level, we need to get the errors of the parameters ($(\Omega_{m0}, w)$ for $wCDM$ and $(\Omega_{m0}, w_0)$ for $CPL$), which would propagate to $q_0$ and then to $\frac{\Delta q_0}{\bar{q}_0}$. The analysis here is performed by using the Monte Carlo Markov Chain in the multidimensional parameter space to derive $1\sigma$ errors on each hemispheres in the maximum anisotropic direction. Accordingly, the $1\sigma$ deviation from the maximum anisotropy level can be expressed as $\frac{\Delta q_0}{\bar{q}_0} = \frac{\Delta q_{0,max}}{\bar{q}_{0,max}} \pm \sigma_{\delta q}$, and correspondingly the axes with $1\sigma$ error can also be obtained.

Further we explore the possible redshift dependence of the anisotropy. We implement a redshift tomography of the data and take the same procedure as before for all the following redshift slices: 0-0.2, 0-0.4, 0-0.6, 0-0.8, 0-1.0. Our results for $wCDM$ model are summarized in Table I. And we also show the result for the full sample of union2 ($0 < z \leq 1.4$) in Figure 1.

The redshift tomography analysis here shows that the preferred axes are all located in a relatively small part of the North Galactic Hemisphere (around $(l, b) = (314°, 28°)$), which is consistent with what have been achieved in other papers [20] and other large scale data [24], which means that under the assumption of $wCDM$ model, the universe has a maximum acceleration direction. For

| redshift range | $l$[degree] | $b$[degree] | $\frac{\Delta q_0}{\bar{q}_0}$ |
|---|---|---|---|
| $0-0.2$ | $332^{-35}_{+3}$ | $-29^{-11}_{+42}$ | $3.103^{-0.54}_{+0.50}$ |
| $0-0.4$ | $319^{-12}_{+20}$ | $-28^{-25}_{+40}$ | $1.00^{-0.51}_{+0.47}$ |
| $0-0.6$ | $300^{-16}_{+33}$ | $-16^{-10}_{+24}$ | $0.94^{-0.39}_{+0.37}$ |
| $0-0.8$ | $309^{-31}_{+8}$ | $21^{-41}_{+28}$ | $0.91^{-0.29}_{+0.30}$ |
| $0-1.0$ | $311^{-20}_{+15}$ | $15^{-19}_{+17}$ | $0.85^{-0.28}_{+0.28}$ |
| $0-1.4$ | $314^{-13}_{+20}$ | $28^{-33}_{+11}$ | $0.79^{-0.28}_{+0.27}$ |

TABLE I: Directions of maximum anisotropy for several redshift ranges of the Union2 data fitting with the $wCDM$ model. The last column corresponds to the $1\sigma$ errors propagated from $\Omega_{m0}$ and $w$, which are obtained by MCMC method.

different redshift slices, there are slight differences in the direction of preferred axes, and the difference between the two opposite hemispheres is extremely obvious for the low-redshift slice ($z \leq 0.2$).

Here we also give the best-fitting parameters of the $wCDM$ model in Table II, where the subscripts $u$ and $d$ denote the up and down hemispheres. The results show that for the case with full SNIa data, see the last arrow in Table II, the difference between $\Lambda CDM$ model and $wCDM$ model is not obvious. In the $\Lambda CDM$ model, $\Omega^u_{m0} = 0.30$ and $\Omega^d_{m0} = 0.19$ [20].

| redshift range | $\Omega^u_{m0}$ | $\Omega^d_{m0}$ | $w^u$ | $w^d$ |
|---|---|---|---|---|
| $0-0.2$ | 0.45 | 0.10 | $-0.80$ | $-0.51$ |
| $0-0.4$ | 0.44 | 0.11 | $-1.93$ | $-0.65$ |
| $0-0.6$ | 0.41 | 0.11 | $-1.91$ | $-0.70$ |
| $0-0.8$ | 0.37 | 0.12 | $-1.68$ | $-0.67$ |
| $0-1.0$ | 0.37 | 0.18 | $-1.68$ | $-0.76$ |
| $0-1.4$ | 0.35 | 0.22 | $-1.46$ | $-0.77$ |

TABLE II: Best-fitting parameters of the $wCDM$ model for different redshift slices on the opposite hemispheres in the direction of the maximum anisotropy, where $u$ denotes the hemisphere corresponding to larger accelerations, while $d$ denotes the opposite hemisphere.

Note that $\Omega_{m0}$ is used as the diagnostic to the anisotropy level in [20]. Here we use $q_0$ as the diagnostic to the anisotropy level. We found here that $q_0$ is more sensitive to the errors of the data, since $q_0$ is relative to the second order derivative of the luminosity distance. And also because more parameters involved in the analysis will introduce more uncertainties when we try to determine the error of $q_0$, therefore the errors here are larger than those in [20]. For example, the dots colored according to the sign and magnitude of the anisotropy level will be more scattered on the unit sphere, as shown in Figure 1.

With the same procedure we also study this issue for a dynamical dark energy model with the $CPL$ parametrization. By use of all Union2 data points, we find that the direction of preferred axis is $(l, b) = (309^{\circ}{}^{-23^{\circ}}_{+30^{\circ}}, 21^{\circ}{}^{-26^{\circ}}_{+35^{\circ}})$, and the corresponding the maximum anisotropy is $\frac{\Delta q_{0,max}}{\bar{q}_{0,max}} = 0.76^{-0.46}_{+0.41}$. The result shows that it is not much different from the case of the $wCDM$ model. This shows that the best-fitting values of the preferred direction is

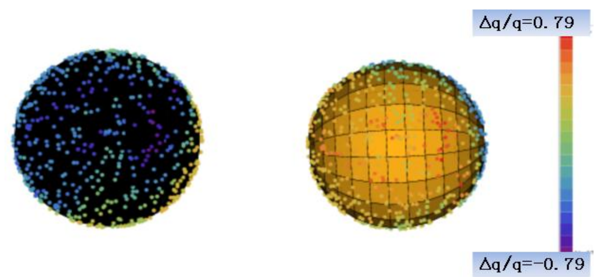


FIG. 1: The dots on the unit sphere colored according to the sign and magnitude of the anisotropy level are the directions of the random axes. The hemisphere shown on the left panel is the one corresponding to larger accelerations with the preferred axis $(l, b) = (314^{\circ}, 28^{\circ})$ [$q_0 = -0.92$ in this direction], while the right corresponds to the one with smaller accelerations and the preferred axis $(l, b) = (134^{\circ}, -28^{\circ})$ and $q_0 = -0.40$.

## IV. CONCLUSIONS

From some astronomical observations and some theoretical models of the universe, there seemingly exists some evidence for a cosmological preferred axis [10]. If such a cosmological preferred axis indeed exists, one has to seriously consider an anisotropic cosmological model as a realistic model, instead of the FRW universe model.

In this paper we investigated the existence of anisotropy of the universe by employing the hemisphere comparison method and the Union2 SNIa dataset and found this preferred direction. We used the present value of the deceleration parameter $q_0$ to quantify the anisotropy level of the two hemispheres. For the $wCDM$ model, the preferred direction is $(l,b) = (314^{\circ}{}^{-13^{\circ}}_{+20^{\circ}}, 28^{\circ}{}^{-33^{\circ}}_{+11^{\circ}})$, and $\frac{\Delta q_0}{\bar{q}_0} = 0.79^{-0.28}_{+0.27}$. While in the case of CPL model, the direction of preferred axis is $(l,b) = (309^{\circ}{}^{-23^{\circ}}_{+30^{\circ}}, 21^{\circ}{}^{-26^{\circ}}_{+35^{\circ}})$, and correspondingly the maximum anisotropy level is $\frac{\Delta q_{0,max}}{\bar{q}_{0,max}} = 0.76^{-0.46}_{+0.41}$. Comparing with the result given in [20], where the $\Lambda CDM$ model is employed, our results are basically in agreement. This means that the best-fitting preferred direction is not much sensitive to the dark energy models.

Finally let us notice that it can be seen from Table I that the result is weakly dependent on redshift if the redshift tomography analysis is employed.

We would like to thank Prof. Z. K. Guo for useful discussions and valuable comments. This work was supported in part by the National Natural Science Foundation of China (No. 10821504, No. 10975168, No.11035008 and No.11075098), by the Ministry of Science and Technology of China under Grant No. 2010CB833004 and by a grant from the Chinese Academy of Sciences.

* Electronic address: cairg@itp.ac.cn
† Electronic address: tuozhl@itp.ac.cn

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999) [arXiv:astro-ph/9812133].

[2] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633**, 560 (2005) [arXiv:astro-ph/0501171]; M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723].

[3] D. N. Spergel *et al.* [ WMAP Collaboration ], Astrophys. J. Suppl. **170**, 377 (2007). [astro-ph/0603449].

[4] S. Weinberg, *Cosmology* (Oxford University Press, New York, New York, 2008).

[5] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].

[6] S. Trujillo-Gomez, A. Klypin, J. Primack and A. J. Romanowsky, arXiv:1005.1289 [astro-ph.CO].

[7] B. A. Reid *et al.*, Mon. Not. Roy. Astron. Soc. **404**, 60 (2010) [arXiv:0907.1659 [astro-ph.CO]].

[8] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **77**, 023504 (2008) [arXiv:0710.1092 [astro-ph]].

[9] L. Perivolaropoulos, [arXiv:0811.4684 [astro-ph]]; R. -J. Yang, S. N. Zhang, Mon. Not. Roy. Astron. Soc. **407**, 1835-1841 (2010). [arXiv:0905.2683 [astro-ph.CO]]. P. Naselsky, W. Zhao, J. Kim, S. Chen, [arXiv:1108.4376 [astro-ph.CO]].

[10] L. Perivolaropoulos, arXiv:1104.0539 [astro-ph.CO].

[11] R. Amanullah *et al.*, Astrophys. J. **716**, 712 (2010) [arXiv:1004.1711 [astro-ph.CO]].

[12] N. Suzuki *et al.*, arXiv:1105.3470 [astro-ph.CO].

[13] C. Armendariz-Picon, JCAP **0407**, 007 (2004) [arXiv:astro-ph/0405267];
T. Koivisto and D. F. Mota, Astrophys. J. **679**, 1 (2008) [arXiv:0707.0279 [astro-ph]];
T. Koivisto and D. F. Mota, JCAP **0808**, 021 (2008) [arXiv:0805.4229 [astro-ph]];
G. Esposito-Farese, C. Pitrou and J. P. Uzan, Phys. Rev. D **81**, 063519 (2010) [arXiv:0912.0481 [gr-qc]].

[14] C. G. Tsagas, arXiv:1107.4045 [astro-ph.CO]; C. G. Tsagas, Mon. Not. Roy. Astron. Soc. **405**, 503 (2010) [arXiv:0902.3232 [astro-ph.CO]].

[15] K. Tomita, Prog. Theor. Phys. **106**, 929 (2001) [arXiv:astro-ph/0104141];
S. Gupta, T. D. Saini and T. Laskar, Mon. Not. Roy. Astron. Soc. **388**, 242 (2008) [arXiv:astro-ph/0701683];
S. Gupta and T. D. Saini, arXiv:1005.2868 [astro-ph.CO].

[16] M. Blomqvist, J. Enander and E. Mortsell, JCAP **1010**, 018 (2010) [arXiv:1006.4638 [astro-ph.CO]].

[17] L. Campanelli, P. Cea, G. L. Fogli and A. Marrone, Phys. Rev. D **83**, 103503 (2011) [arXiv:1012.5596 [astro-ph.CO]].

[18] J. Colin, R. Mohayaee, S. Sarkar and A. Shafieloo, Mon. Not. Roy. Astron. Soc. **414**, 264 (2011) [arXiv:1011.6292 [astro-ph.CO]].

[19] D. J. Schwarz and B. Weinhorst, Astron. Astrophys. **474**, 717 (2007) [arXiv:0706.0165 [astro-ph]].

[20] I. Antoniou and L. Perivolaropoulos, JCAP **1012**, 012 (2010) [arXiv:1007.4347 [astro-ph.CO]].

[21] C. H. Lineweaver, L. Tenorio, G. F. Smoot, P. Keegstra, A. J. Banday and P. Lubin, Astrophys. J. **470**, 38 (1996) [arXiv:astro-ph/9601151].

[22] M. Frommert and T. A. Ensslin, arXiv:0908.0453 [astro-ph.CO].

[23] P. Bielewicz, K. M. Gorski and A. J. Banday, Mon. Not. Roy. Astron. Soc. **355**, 1283 (2004) [arXiv:astro-ph/0405007].

[24] R. Watkins, H. A. Feldman and M. J. Hudson, Mon. Not. Roy. Astron. Soc. **392**, 743 (2009) [arXiv:0809.4041 [astro-ph]].

[25] D. Hutsemekers, R. Cabanac, H. Lamy and D. Sluse, Astron. Astrophys. **441**, 915 (2005) [arXiv:astro-ph/0507274];

D. Hutsemekers, A. Payez, R. Cabanac, H. Lamy, D. Sluse, B. Borguet and J. R. Cudell, arXiv:0809.3088 [astro-ph];
D. Hutsemekers and H. Lamy, arXiv:astro-ph/0012182.

[26] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D **10**, 213 (2001) [arXiv:gr-qc/0009008];
E. V. Linder, Phys. Rev. Lett. **90**, 091301 (2003) [arXiv:astro-ph/0208512].

[27] M. Hicken *et al.*, Astrophys. J. **700**, 1097 (2009) [arXiv:0901.4804 [astro-ph.CO]].

[28] J. A. Holtzman *et al.*, Astron. J. **136**, 2306 (2008).

[29] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **72**, 123519 (2005) [arXiv:astro-ph/0511040].

[30] P. Duffett-Smith, 'Practical Astronomy with your Calculator' Cambridge University Press (1989).