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DIRECTIONAL CONJUGATE SMOOTHING OF REFLECTION MOIRE IMAGES

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Abstract. The construction of digital images of isolines of directional derivatives of the deflection of the plate builds the ground for hybrid numerical - experimental procedures and enables to analyse the experimental results with greater precision. The procedure for the generation of digital images of isolines of directional derivatives of the deflection of the eigenmodes of bending vibrations of the circular plate is developed. A displacement based FEM formulation of this problem is used for building of digital image of isolines based on the deflection derivatives. The presented smoothing procedure enables the generation of digital images on rather coarse conventional finite element meshes. The introduction of directional smoothing (radial or angular) enables to obtain digital images of isolines with higher precision, as one of the digital images requires only radial while the other only angular smoothing. Thus the smoothing in the unnecessary direction is avoided and does not influence the image.

Keywords: conjugate approximation, directional conjugate smoothing, finite elements, digital image of isolines.

1. Indroduction

The importance of the problem of smoothing in the process of construction of digital images from the results of numerical calculations is described in [1].

Reflection moire is one of the popular methods for experimental analysis of bending vibrations of structures and can be effectively exploited in hybrid numerical – experimental techniques [2]. The most often analysed problem is that of the bending vibrations of a plate.

Reflection moiré is a typical optical engineering experimental method producing a pattern of interference fringes. Interpretation of these patterns produces the field of derivative of the deflection.

In order to better interpret the experimental results the digital images of the isolines of the directional derivatives are constructed. Here the plotting procedure for the isolines of the directional derivatives of the circular plate is developed including the directional smoothing of the represented values in order to obtain more precise images for the analysis of the structural vibrations.

The principle of the reflection moire analysis [3] is shown in Figure 1. x, y and z are the orthogonal Cartesian axes of coordinates (y axis is not shown in the figure). The plate in the status of equilibrium is in the plane of the x and y axes. Moire grating and the photographic plate are parallel to the analysed plate in the status of equilibrium and the distance between them and the analysed plate in the direction of the z axis is d. The deflection of the plate is w and for the analysed small vibrations it is assumed that $d >> w$. The subscript denotes partial derivative and N is the normal vector to the surface of the plate. u and v are the shifts of the reflected moiré grating in the directions of the x and y axes with respect to the initial moiré grating (the shift v is not shown in the figure). This is the schematic representation because in real experimental implementation a semi-silvered mirror is to be introduced in order to assure that the moiré grating and the photographic plate are not one over another.

Conventional FEM analysis techniques are based on the approximation of nodal displacements (not their derivatives which may be considered as the components of strains) via the shape functions [4, 5, 6].

Conventional FEM would require dense meshing for producing sufficiently smooth digital images of isolines of displacement derivatives. Therefore, the technique for directional smoothing of the generated isolines representing the distribution of the displacement derivatives and calculated from the displacement distribution is proposed for the analysis of a circular plate. The smoothing technique is similar to conjugate approximation used for the calculation of nodal values of stresses in [5] and enables to obtain the isolines of better quality on a rather coarse mesh by using the displacement formulation for the calculation of the eigenmodes.

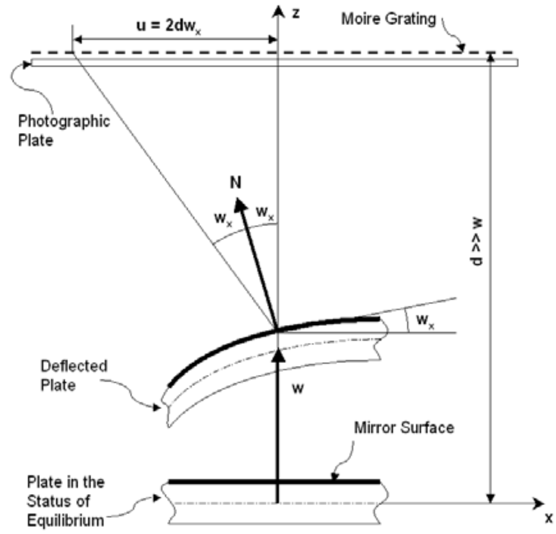

Figure 1. The schematic of vibration analysis by reflection moiré

2. Construction of the digital images of isolines using conjugate approximation with directional smoothing

First of all the eigenmodes for the bending vibrations of a circular plate are calculated by using the displacement formulation common in the finite element analysis. The plate bending element with the independent interpolation of the displacement w and the rotations about the appropriate axes θ_x and θ_y is used [4]. It is assumed that the structure performs vibrations according to the eigenmode (the frequency of excitation is about equal to the eigenfrequency of the corresponding eigenmode and the eigenmodes are not multiple).

The vibrations of the structure are registered when the structure moves sinusoidally according to the eigenmode. In this case the problem is to obtain the nodal derivatives of the transverse displacement (which may be considered as related with the components of the strains and further called as strains) of the plate of acceptable quality for the eigenmode (the eigenmode of strains), which are further used for the calculation of the images of isolines.

The derivatives of the displacement (the components of the strains) at the points of numerical integration of the finite element are calculated in the usual way:

$$\begin{Bmatrix} w_x \\ w_y \end{Bmatrix} = [B]\{\delta_0\}, \qquad (1)$$

where $\{\delta_0\}$ is the vector of nodal displacements of the eigenmode; w_x, w_y are the components of the strains; $[B]$ is the matrix relating the strains with the displacements:

$$[B] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & 0 & 0 & \vdots & \dfrac{\partial N_n}{\partial x} & 0 & 0 \\ \dfrac{\partial N_1}{\partial y} & 0 & 0 & \vdots & \dfrac{\partial N_n}{\partial y} & 0 & 0 \end{bmatrix}, \qquad (2)$$

where N_i are the shape functions of the finite element; n is the number of nodes of the finite element.

It can be noted that the displacements are continuous at inter-element boundaries, but the calculated strains due to the operation of differentiation are discontinuous. In this paper a circular plate with the fixed internal radius is analysed.

The appropriate eigenmode of strains is obtained by minimising the following errors:

$$\frac{1}{2}\iint \begin{pmatrix} ([N]\{\delta_x\} - w_x)^2 + \\ + [w_x \quad w_y][T]^T[D][T]\begin{Bmatrix} w_x \\ w_y \end{Bmatrix} \end{pmatrix} dxdy =$$

$$= \frac{1}{2}\iint \begin{pmatrix} ([N]\{\delta_x\} - w_x)^2 + \\ + \{\delta_x\}^T[B^*]^T[T]^T[D][T][B^*]\{\delta_x\} \end{pmatrix} dxdy,$$

$$\frac{1}{2}\iint \begin{pmatrix} ([N]\{\delta_y\} - w_y)^2 + \\ + \{\delta_y\}^T[B^*]^T[T]^T[D][T][B^*]\{\delta_y\} \end{pmatrix} dxdy, \qquad (3)$$

where:

$$[T] = \begin{bmatrix} \dfrac{x}{\sqrt{x^2+y^2}} & \dfrac{y}{\sqrt{x^2+y^2}} \\ \dfrac{-y}{\sqrt{x^2+y^2}} & \dfrac{x}{\sqrt{x^2+y^2}} \end{bmatrix},$$

$$[D] = \begin{bmatrix} \lambda_r & 0 \\ 0 & \lambda_\Theta \end{bmatrix}, \qquad (4)$$

where λ_r is the radial smoothing parameter; λ_Θ is the angular smoothing parameter; $\{\delta_x\}$ is the vector of nodal values of w_x; $\{\delta_y\}$ is the vector of nodal values of w_y; $[N]$ is the row of the shape functions of the finite element; $[B^*]$ is the matrix of the derivatives of the shape functions (the first row with respect to x; the second – with respect to y):

$$[N] = [N_1 \quad N_2 \quad \cdots \quad N_n],$$

$$[B^*] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_2}{\partial x} & \cdots & \dfrac{\partial N_n}{\partial x} \\ \dfrac{\partial N_1}{\partial y} & \dfrac{\partial N_2}{\partial y} & \cdots & \dfrac{\partial N_n}{\partial y} \end{bmatrix}. \qquad (5)$$

The matrix $[T]$ consists of the directional cosines transforming the derivatives to the radial and angular directions of the polar system of coordinates.

The previously mentioned concept of the eigenmode of strains is understood as the set of vectors $\{\delta_x\}$, $\{\delta_y\}$. This leads to the following systems of linear algebraic equations for the determination of each of the components of the strains:

$$\iint\left([N]^T[N]+[B*]^T[T]^T[D][T][B*]\right)dxdy\cdot\{\delta_x\}= \\ =\iint[N]^T w_x dxdy,$$

$$\iint\left([N]^T[N]+[B*]^T[T]^T[D][T][B*]\right)dxdy\cdot\{\delta_y\}= \\ =\iint[N]^T w_y dxdy. \quad (6)$$

The choice of the smoothing parameters is performed interactively from the qualitative view of the digital images of isolines. When the parameters are too small the images are insufficiently smooth because of the strains calculated from the displacement formulation. When the parameters are too big an over-smoothed image is obtained which may look acceptable but be incorrect (far from the real image of isolines). So the best value of the parameter might be considered when most of the image is of acceptable quality without the unphysical behaviour produced by the approximation.

In the reflection moiré images [3]:

$$u=2dw_x, \\ v=2dw_y, \quad (7)$$

where d is the distance between the reference grating and the plate (for the analysed small vibrations of the plate $d>>w$ and no distinction is made between the values of the distance between the reference grating and the plate for the deflected plate and for the plate in the status of equilibrium). So the reflection moiré images are directly related with the images of isolines of directional derivatives of the deflection of the plate.

3. Numerical results

The circular plate with the fixed internal radius is analysed. The reflected finite element meshes for the tenth eigenmode are shown in Figure 2. The reflected mesh for the plate in the status of equilibrium is grey and the reflected mesh for the plate deflected according to the eigenmode is black.

Isolines of the derivative of the displacement of the plate in the radial direction for the tenth eigenmode with radial smoothing are shown in Figure 3. Isolines of the derivative of the displacement of the plate in the angular direction for the tenth eigenmode with radial smoothing are shown in Figure 4.

Isolines of the derivative of the displacement of the plate in the radial direction for the tenth eigenmode with angular smoothing are shown in Figure 5. Isolines of the derivative of the displacement of the plate in the angular direction for the tenth eigenmode with angular smoothing are shown in Figure 6.

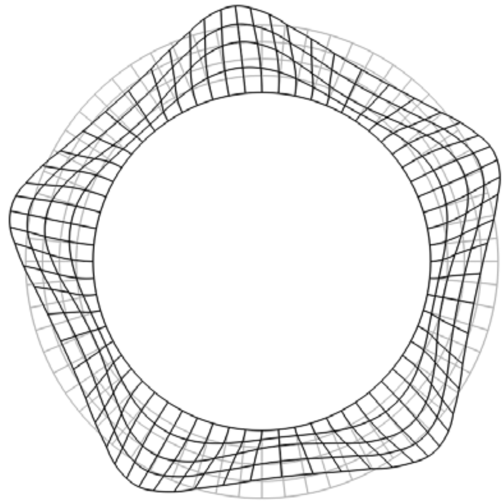

Figure 2. The reflected mesh for the tenth eigenmode of the circular plate (the reflected mesh in the status of equilibrium is grey and the reflected mesh deflected according to the eigenmode is black)

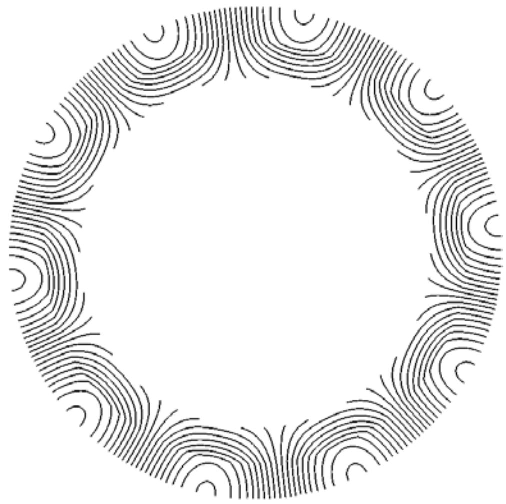

Figure 3. Isolines of the derivative of the displacement of the circular plate in the radial direction for the tenth eigenmode with radial smoothing

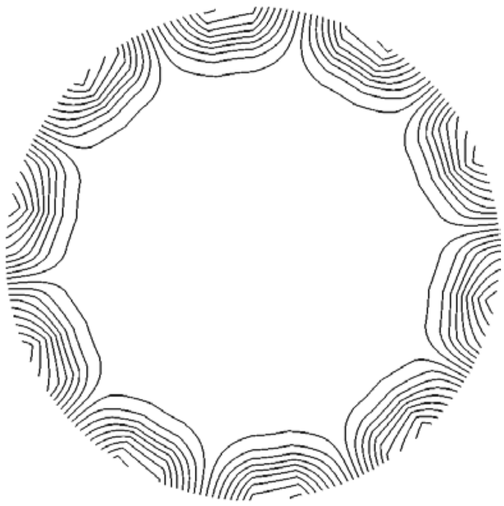

Figure 4. Isolines of the derivative of the displacement of the circular plate in the angular direction for the tenth eigenmode with radial smoothing

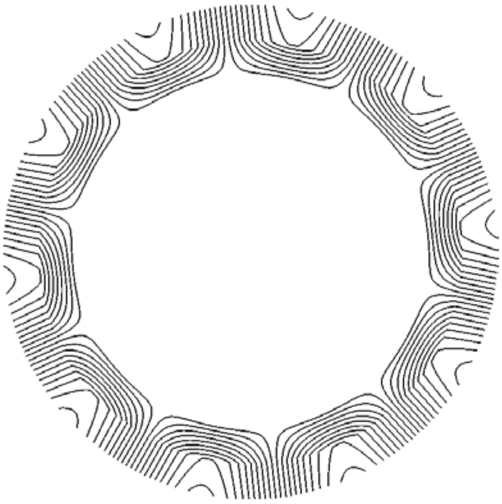

Figure 5. Isolines of the derivative of the displacement of the circular plate in the radial direction for the tenth eigenmode with angular smoothing

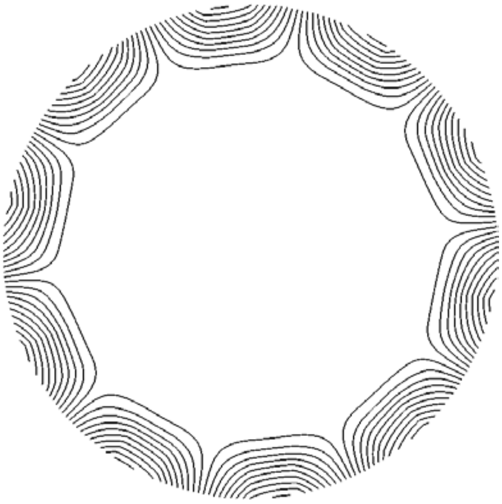

Figure 6. Isolines of the derivative of the displacement of the circular plate in the angular direction for the tenth eigenmode with angular smoothing

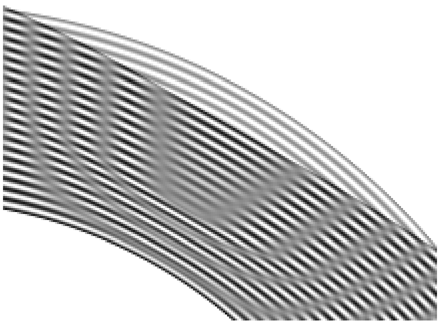

Figure 7. Zoomed reflection moiré image for the radial grating for the tenth eigenmode without smoothing

The importance of smoothing is illustrated in Figures 7-10. In Figure 7 zoomed reflection moiré image for the radial grating for the tenth eigenmode without smoothing is shown, while in Figure 8 the same image with smoothing is presented. In Figure 9 zoomed reflection moiré image for the angular grating for the tenth eigenmode without smoothing is shown, while in Figure 10 the same image with smoothing is presented. The effect of smoothing in improving the quality of images is evident.

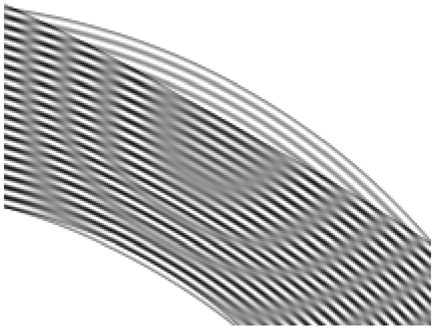

Figure 8. Zoomed reflection moiré image for the radial grating for the tenth eigenmode with smoothing

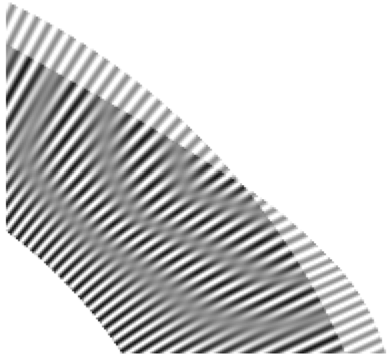

Figure 9. Zoomed reflection moiré image for the angular grating for the tenth eigenmode without smoothing

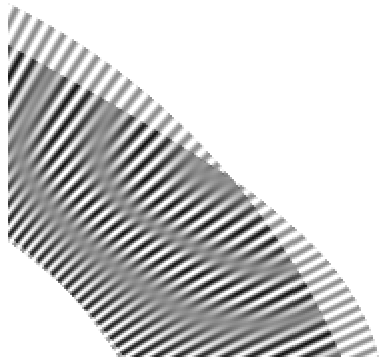

Figure 10. Zoomed reflection moiré image for the angular grating for the tenth eigenmode with smoothing

4. Conclusions

The construction of digital images of isolines of directional derivatives of the deflection of the plate builds the ground for hybrid numerical – experimental procedures and enables to analyse the experimental results with higher precision. The procedure for the generation of digital images of isolines of directional derivatives of the deflection of the eigenmodes of bending vibrations of the circular plate is developed. A displacement based FEM formulation is used for construction of digital images of isolines based on the deflection derivatives. The presented smoothing procedure enables the generation of images on rather coarse conventional finite element meshes. The proposed directional smoothing procedure enables to avoid smoothing in the unnecessary direction and thus to reduce the errors on the resulting image.

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