CHAPTER 4

DIRECTIONAL SPECTRUM ESTIMATION FROM A BAYESIAN APPROACH

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ABSTRACT

A new directioal spectral estimation method using a Bayesian approach is proposed. The proposed method is examined for numerical simulation data, and the validity of the method is discussed. Some examples of the directional spectra estimated from field observation data attained at an offshore oil rig utilizing seven wave probes are also shown in this report. The major conclusions of the report are : (1) The proposed method can be applied for more than four arbitrarily mixed instrument array measurements. (2) It has a higher resolution power than other existing methods for estimating directional spectrum. (3) It is a better method for estimating directional spectra from the cross-power spectra contaminated with estimation errors. (4) It is more adaptable to reformulation of the estimation equations as the study of structures of directional spectrum progesses.

1. INTRODUCTION

Directional spectra are the fundamental properties of ocean waves expressing the energy distribution as a function of the wave frequency and direction of wave propagation. Many efforts have been made to estimate directional spectra on the bases of point measurements utilizing various wave probes, and several methods have been proposed to improve the directional resolution of the estimation.

These methods are based on a mathematical relationship between the directional spectrum and the cross-power spectra. The cross-power spectra are computed from time series records of various wave properties. In practice, wave records are often contaminated with noises. This leads to errors in the cross-power spectral estimation. Thus, the estimate of the directional spectrum is often biased by the noises and errors associated with the observed cross-power spectra.

However, no current methods take into account the existence of such errors. The directional spectrum is estimated only to satisfy the above mentioned relationship with the observed cross-power spectra, and this may be one of the causes that estimates of the directional spectrum sometimes result in poorly conditioned shape, i.e. negative values or zigzags, for instance.

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The same type of problems are seen in the field of the regression analysis problems where the number of the parameters to be estimated is large compared with the sample size. In order to overcome these difficulties, Akaike(1980) introduced a Bayesian model which better approximates the sample data, and which is compatible with an a priori condition subsistent in the phenomenon to be analyzed.

The estimation of the directional spectrum can be considered as a regression analysis problem to find the most suitable model from limited data. Therefore, the Bayesian approach should be useful to obtain the most reasonable model (directional spreading function) which best approximates the sample data (cross-power spectra) and which also conforms to the subsistent nature of the physical phenomenon, i.e., continuous and smooth variation of its value. This is accomplished by maximizing the likelihood of the model with a priori condition that the directional spreading function varies smoothly over the wave direction.

In the practical computation, another parameter which is called a hyperparameter, is introduced to consider the balance of the two requirements imposed on the model : to maximize the likelihood of the model and to maintain the smoothness of the model. In order to select the most suitable value of the hyperparameter for the given cross-power spectra, the ABIC (Akaike's Bayesian Information Criterion, 1980) is also introduced as a criterion to determine the most suitable model.

The proposed method is examined by numerical simulations, and its application to a practical directional wave analysis is also presented with the data recently recorded on an offshore oil rig at a water depth of 150 meters.

2. FUNDAMENTAL EQUATIONS RELATED TO DIRECTIONAL SPECTRUM

The relationship between the cross-power spectrum for a pair of arbitrary wave properties and the directional spectrum is introduced by Isobe et al.(1984) as the following equation :

$$\begin{aligned}\Phi_{mn}(f)=\int_0^{2\pi} H_m(f,\theta)H_n^*(f,\theta)[\cos\{k(x_{mn}\cos\theta+y_{mn}\sin\theta)\}\\ -i\sin\{k(x_{mn}\cos\theta+y_{mn}\sin\theta)\}]S(f,\theta)d\theta\end{aligned} \tag{1}$$

where f is the frequency, k is the wave number, θ is the wave propagation direction, $\Phi_{mn}(f)$ is the cross-power spectrum between the m-th and the n-th wave properties, $H_m(f,\theta)$ is the transfer function from the water surface elevation to the m-th wave property, i is the imaginary unit, $x_{mn}=x_n-x_m$, $y_{mn}=y_n-y_m$, (x_m,y_m) is the location of the wave probe for the m-th wave property, $S(f,\theta)$ is the directional spectrum and the superscript * denotes the complex conjugate.

The directional spectrum $S(f,\theta)$ is often expressed as a product of the frequency spectrum and the directional spreading function.

$$S(f,\theta)=S(f)G(\theta|f) \tag{2}$$

where, $S(f)$ is the frequency spectrum and $G(\theta|f)$ is the directional spreading function. The directional spreading function takes non-negative values and satisfies the following relationship.

Table 1 Transfer function from small amplitude wave theory

MEASURED QUANTITY	SYMBOL	$h(k,\sigma)$	α	β
Water surface elevation	η	1	0	0
Excess pressure	p	$\rho g\dfrac{\cosh kz}{\cosh kd}$	0	0
Vertical water surface velocity	η_t	$-i\sigma$	0	0
Vertical water surface acceleration	η_{tt}	$-\sigma^2$	0	0
Surface slope (x)	η_x	ik	1	0
Surface slope (y)	η_y	ik	0	1
Water particle velocity (x)	u	$\sigma\dfrac{\cosh kz}{\sinh kd}$	1	0
Water particle velocity (y)	v	$\sigma\dfrac{\cosh kz}{\sinh kd}$	0	1
Water particle velocity (z)	w	$-i\sigma\dfrac{\sinh kz}{\sinh kd}$	0	0

k: wave number, σ: angular frequency, d: water depth, z: elevation from the bottom, ρ: fluid density, g: gravitational acceleration.

$$\int_0^{2\pi} G(\theta|f)d\theta=1 \tag{3}$$

The transfer function $H_m(f,\theta)$ in Eq.(1) is generally expressed in the form

$$H_m(f,\theta)=h_m(f)\cos^{\alpha_m}\theta\sin^{\beta_m}\theta \tag{4}$$

where the function h_m and the parameter α_m and β_m in Eq.(4) are shown in Table 1.

Equations (1) are the fundamental equations for the estimation of the directional spectrum on the basis of the simultaneous measurements of various wave properties. If the function $S(f,\theta)$ which satisfies Eq.(1) and which takes only non-negative values is obtained, the function is called a directional spectrum.

3. ESTIMATION OF DIRECTIONAL SPECTRUM FROM THE BAYESIAN APPROACH

The directional spreading function takes values greater than or equal to zero, but in this section, the function is treated as a function which always takes positive values only.

Firstly, it is assumed that the directional spreading function is expressed as a piecewise-constant function over the directional range from 0 to 2π ($K\Delta\theta=2\pi$). This assumption is commonly employed in the numerical computation of random waves.

Since $G(\theta|f)>0$, and let

$$\ln G(\theta_k|f)=x_k(f),\quad (k=1\cdots\cdots,K) \tag{5}$$

the directional spreading function is approximated by the following equation.

$$G(\theta|f)\doteqdot\sum_{k=1}^{K}\exp\{x_k(f)\}I_k(\theta) \tag{6}$$

wrere,

$$I_k(\theta)=\begin{cases}1: (k-1)\Delta\theta\leq\theta<k\Delta\theta \\ 0: \text{otherwise},\quad (k=1\cdots\cdots,K)\end{cases} \tag{7}$$

Substituting Eq.(6) into Eq.(1) and after some manipuration, the following equation is obtained considering the errors $\varepsilon_{mn}(f)$ of the cross-power spectra.

$$\phi_i(f)=\sum_{k=1}^{K}\alpha_{ik}(f)\exp\{x_k(f)\}+\varepsilon_i(f)\qquad(i=1,\cdots\cdots,2N) \tag{8}$$

where, the subscripts i=1 to N denote real parts and i=N+1 to 2N denote imaginary parts of the complex numbers ϕ_i , $\alpha_{i,k}$ and ε_i ,

$$N=M\times(M+1)/2;\quad M:\text{Number of the wave probes} \tag{9}$$

$$\alpha_{i,k}(f)=\Delta\theta\, H_m(f,\theta_k)H_n^*(f,\theta_k)[\cos\{k(x_{mn}\cos\theta_k+y_{mn}\sin\theta_k)\\ -i\sin\{k(x_{mn}\cos\theta_k+y_{mn}\sin\theta_k)\}]/\sqrt{\Phi_{mm}(f)\Phi_{nn}(f)} \tag{10}$$

$$\phi_i(f)=\Phi_{mn}(f)/\{S(f)\sqrt{\Phi_{mm}(f)\Phi_{nn}(f)}\} \tag{11}$$

$$\varepsilon_i(f)=\varepsilon_{mn}(f)/\sqrt{\Phi_{mm}(f)\Phi_{nn}(f)} \tag{12}$$

In Eq.(8), though ϕ_i , $\alpha_{i,k}$ and ε_i are the functions of the frequency f, f is omitted to simplify the expression hereafter.

When ϕ_i (i=1,2,....,2N) are given and ε_i (i=1,.......,2N) are assumed to be independent from each other and have the probability of their occurrence expressed by the normal distribution having the mean 0 and the variance σ^2 , the likelihood function of x_k (k=1,.....,K) and σ^2 is given by Eq.(13).

$$L(x_1,\cdots\cdots,x_K;\sigma^2)=\frac{1}{(2\pi\sigma^2)^N}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{2N}\left\{\phi_i-\sum_{k=1}^{K}\alpha_{i,k}\exp(x_k)\right\}^2\right] \tag{13}$$

In the derivation of the equations mentioned above, the directional spreading function $G(\theta|f)$ is expressed by a piecewise constant function and the correlation between the wave energy falling on each segment of θ has not yet been taken into account considering the background of the linear wave theory. However, it is not unreal to assume that the energy distribution over the wave directions is discontinuous. Therefore, the additional condition must be added. As the simplest additional condition, the following condition is added by assuming that the second order difference of $\{x_k\}$ are close to zero for k=1,....,K. That is,

$$\sum_{k=1}^{K}\{x_k-2x_{k-1}+x_{k-2}\}^2 \qquad (x_0=x_K,\ x_{-1}=x_{K-1}) \tag{14}$$

becomes smaller as the estimate of the directional spreading function $G(\theta\mid f)$ becomes smoother.

The estimate of x_k maximizing the likelihood (Eq.(13)) and minimizing the above mentioned quantity are determined as the estimates which maximize the following quantity :

$$\ln L(x_1,\cdots\cdots,x_K\,;\sigma^2)-\frac{u^2}{2\sigma^2}\sum_{k=1}^{K}(x_k-2x_{k-1}+x_{k-2})^2 \tag{15}$$

where, u^2 is a hyperparameter.

The maximization of Eq.(15) is achieved by maximizing the following exponential function having the power expressed by Eq.(15).

$$L(x_1,\cdots\cdots,x_K\,;\sigma^2)\exp\left\{-\frac{u^2}{2\sigma^2}\sum_{k=1}^{K}(x_k-2x_{k-1}+x_{k-2})^2\right\} \tag{16}$$

When normalized, the second term can be regarded as a following distribution of $\boldsymbol{x}=(x_1,\ldots\ldots,x_K)$.

$$p(\boldsymbol{x}\mid u^2,\sigma^2)=\left(\frac{u}{\sqrt{2\pi}\sigma}\right)^K\exp\left\{-\frac{u^2}{2\sigma^2}\sum_{k=1}^{K}(x_k-2x_{k-1}+x_{k-2})^2\right\} \tag{17}$$

In Bayesian statistics, the distribution $p(y\mid x)$, which is called the posterior distribution, is proportional to the likelihood $L(y\mid x)$ and the distribution $p(y)$, which is called the prior distribution, as follows.

$$p(y\mid x)\propto L(y\mid x)p(y) \tag{18}$$

Thus, the distribution of Eq.(17) corresponds to the prior distribution in Bayesian statistics. The estimate of $\boldsymbol{x}$ obtained by maximizing Eq.(16) is regarded as the mode of the posterior distribution of $p_{\mathrm{post}}(\boldsymbol{x}\mid u^2,\sigma^2)$.

If the value of u is given , the value of $\boldsymbol{x}$ which maximizes Eq.(18) are determined by minimizing Eq.(19), regardless of the values of σ^2.

$$\sum_{i=1}^{2N}\left\{\phi_i-\sum_{k=1}^{K}\alpha_{i,k}\exp(x_k)\right\}^2+u^2\left\{\sum_{k=1}^{K}(x_k-2x_{k-1}+x_{k-2})^2\right\} \tag{19}$$

The most suitable values of the hyperparameter u^2 and the variance σ^2 are determined so that the following ABIC is minimum.

$$\mathrm{ABIC}=-2\ln\int L(\boldsymbol{x},\sigma^2)p(\boldsymbol{x}\mid u^2,\sigma^2)d\boldsymbol{x} \tag{20}$$

4. NUMERICAL COMPUTATION OF DIRECTIONAL SPECTRUM

In order to estimate the directional spectrum by means of the Bayesian approach, the minimization of Eq.(19) and the integration and the minimization of Eq.(20) must be performed. The following is the summary of the procedure of the computation presented by Ishiguro (1985) and Sakamoto (1985).

1) For a proper value of hyperparameter u and initial values $\boldsymbol{x}_0$ of $\boldsymbol{x}$, compute the estimate of $\hat{\boldsymbol{x}}$ by means of the least square method utilizing Eq.(21).

$$J(\boldsymbol{x}) \cong |\tilde{\boldsymbol{A}}\boldsymbol{x}-\tilde{\boldsymbol{B}}|^2+u^2|\boldsymbol{D}\boldsymbol{x}|^2 \tag{21}$$

where,

$$\tilde{\boldsymbol{A}}=\boldsymbol{A}\boldsymbol{E}(\boldsymbol{x}_0) \tag{22}$$

$$\tilde{\boldsymbol{B}}=\boldsymbol{B}-\boldsymbol{A}\boldsymbol{F}(\boldsymbol{x}_0)+\boldsymbol{A}\boldsymbol{E}(\boldsymbol{x}_0)\boldsymbol{x}_0 \tag{23}$$

$$\boldsymbol{A}=\begin{bmatrix} \alpha_{1,1}, & \cdots\cdots, & \alpha_{1,K} \\ \vdots & \ddots & \\ \alpha_{2N,1}, & \cdots, & \alpha_{2N,K} \end{bmatrix} \tag{24}$$

$$\boldsymbol{B}=(\phi_1, \cdots\cdots, \phi_{2N})^t \tag{25}$$

$$\boldsymbol{D}=\begin{bmatrix} 1 & 0 & 0\cdots\cdots 0 & 1 & -2 \\ -2 & 1 & 0\cdots\cdots 0 & 0 & 1 \\ 1 & -2 & 1\cdots\cdots 0 & 0 & 0 \\ \vdots & \vdots & \vdots \quad \ddots & & \vdots \\ 0 & 0 & 0\cdots\cdots 1 & -2 & 1 \end{bmatrix} \tag{26}$$

$$\boldsymbol{E}(\boldsymbol{x})=\begin{bmatrix} \exp(x_1) & 0 & \cdots\cdots & 0 \\ 0 & \exp(x_2) & \cdots\cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots\cdots & \exp(x_K) \end{bmatrix} \tag{27}$$

$$\boldsymbol{F}(x)=\{\exp(x_1), \cdots\cdots, \exp(x_K)\}^t \tag{28}$$

That is, for a certain initial value $\boldsymbol{x}_0$, the values of $\boldsymbol{x}_1$ are computed by means of the least square method utilizing Eq.(21) through Eq.(23). Substituting these $\boldsymbol{x}_1$ for the initial $\boldsymbol{x}_0$ in Eq.(22) to Eq.(23), and repeating the same process, we obtain the another set of $\boldsymbol{x}_2$. Iterating these process until the values of $\boldsymbol{x}$ converges to $\tilde{x}$, the estimate of $\hat{\boldsymbol{x}}$ is obtained for the given value of u^2.

2) Compute the ABIC by Eq.(29) for the given u and the estimate of $\hat{\boldsymbol{x}}$ obtained above.

$$\mathrm{ABIC}=2N\ln(2\pi)+2N+2N\ln(\hat{\sigma}^2)-K\ln(u^2)+\ln\{\det(\hat{\boldsymbol{A}}^t\hat{\boldsymbol{A}}+u^2\boldsymbol{D}^t\boldsymbol{D})\} \tag{29}$$

where,

$$\hat{\sigma}^2=\frac{1}{2N}\{|\hat{\boldsymbol{A}}\hat{\boldsymbol{x}}-\hat{\boldsymbol{B}}|^2+u^2|\boldsymbol{D}\hat{\boldsymbol{x}}|^2\} \tag{30}$$

and $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ are the coefficient matrix renewed in Eq.(22) and (23), which are computed for the least square estimate $\hat{\boldsymbol{x}}$.

3) Changing the value of u, repeat the process of 1) and 2).

4) From various estimates of $\hat{x}$ obtained through the process 1) through 3), choose the values $\hat{u}^2$ and $\hat{\sigma}^2$ as well as the estimate $\hat{x}$ which yield the minimum ABIC as the final estimate of x as the estimate of the directional spreading function.

In the computation of the estimate of the directional spreading function mentioned above, the initial values of x_0 are given uniformly ($x_i = \ln(1/2\pi)$). The iteration of Eq.(21) is terminated when the standard deviation of the difference between the values of x_n of n-th step and that of the previous step is smaller than or equal to 10^{-3} , i.e., $|\sigma_{\varepsilon_n}| \leqq 10^{-3}$.

In addition, the hyperparameter which minimizes the ABIC is found by the method of trial and error. The value of the hyperparameter is given by utilizing Eq.(31) with the value of m changing in a sequential manner.

$$u = ab^m \qquad (m = 1, 2, \cdots\cdots) \tag{31}$$

The derivation of the Eq.(21) through Eq.(30) and the details of the computation is given in Hashimoto et al.,(1987).

5. EXAMINATION OF THE ESTIMATION METHOD BY NUMERICAL SIMULATION

5.1 procedure of numerical simulation

Numerical simulations were performed to examine the validity of the proposed method. The procedure is the same as the one used for examining the EMLM (Isobe et al.,1984).

The directional spectrum is expressed as a product of the directional spreading function and the frequency spectrum and is computed frequency by frequency. Hence, in this section, a directional spreading function for an arbitrarily chosen frequency is examined. The practical procedure of the numerical simulation is as follows.

1) The directional spreading functions to be employed in the examination are Mitsuyasu-type ones which are given by Eq.(32).

$$G(\theta) = \sum_i \alpha_i \cos^{2S_i}\left(\frac{\theta - \theta_i}{2}\right) \tag{32}$$

where α_i is the proportionality coefficient and is given so that Eq.(32) satisfies Eq.(3). For the simulation of a uni-directional sea, the directional spreading function is given by Eq.(32) with i=1 only. On the other hand, for a bi-directional sea, two different wave groups having different values of α_i , S_i, θ_i :(i=1,2) are superimposed.

2) The cross-power spectra are computed for the directional spreading function given by process 1) utilizing Eq(1).

3) On the basis of the cross-power spectra obtained above, the directional spreading function is estimated by means of the methods mentioned in the previous section. The estimate of the

directional spreading function is compared with the input directional spreading function, i.e. the one given by Eq.(32).

In addition, the directioanl spreading functions are also estimated by the EMLM for the comparison of those given by the Bayesian approach.

5.2 Examination of the estimation method

The results of the simulations are shown in Fig.1 for the three types of arrays of wave probes. The same wave condition is employed for all the arrays : wind waves characterized by $S=10$ coming in the direction $\theta_1=0°$ and a swell characterized by $S=100$ coming in the direction $\theta_2=100°$ coexist, and the ratio of the peak spectral density of the two wave groups is $a_1/a_2=0.5$. In addition, the distance between the wave probes D is given as D/L= 0.2 (L is the wave length of the component wave to be examined) for all the arrays. The solid lines denoted by TRUE in Fig.1 show the input directional spreading function, and the lines denoted by BDM shows the estimate of the directional spreading function given by the Bayesian directional spectral estimation method (which is called BDM hereafter). The ordinate of Fig.1 is normalized utilizing the peak value of the TRUE directional spreading function for respective cases as reference value.

It is observed in Fig.1 by comparing the estimates of the directional spreading functions result from Type-1 through Type-3 that as the number of wave probes increases, the directional resolution shown by the BDM and the EMLM is improved. In paticular, the BDM for Type-2 and Type-3 are almost the same as the TRUE. On the other hand, though the EMLM seems to be closer to the TRUE than the BDM for Type-1 layout, the EMLM yields an erroneous peak inbetween the two peaks exhibited by the TRUE for Type-2 layout. The EMLM is considerably improved for Type-3 in comparison with the estimate for Type-2, but the resolution is inferior to that of the BDM. For the wave prove arrays consisting more than or equal to four probes, it is seen that the BDM shows a better directional resolution than that of the EMLM.

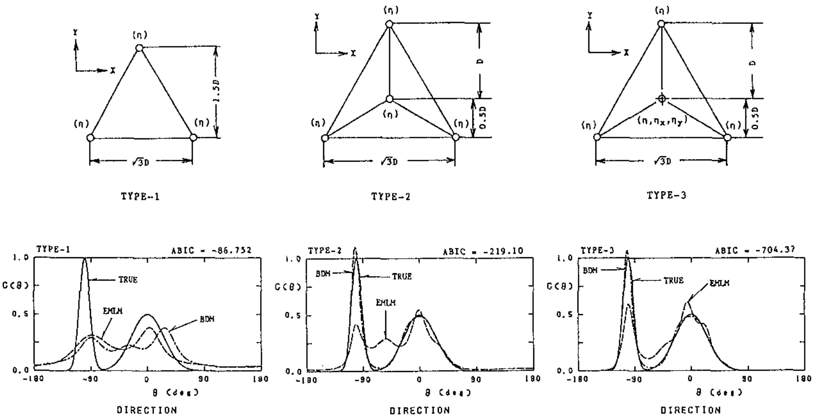

Fig.1 Directional spreading functions estimated from various types of probe arrays

For Type-1 layout, only three wave probes are utilized to measure the directional seas, the BDM does not find a suitable statistical model to explain directional seas. The reason is supposed that three independent wave properties are the minimum data to analyze the directional spectrum, and so it is impossible to estimate the errors introduced in Eq.(8).

The effect of the errors contained in the cross-power spectral estimates is illustrated in Fig.2 for six different magnitudes of errors. The probe array employed is Type-2. The values of r in the figure show the ratio of the magnitude of the errors added to the cross-power spectra and the absolute values of the cross-power spectrum. In the computation, the same magnitude of errors is equally added to all the four wave properties.

It is noted that as the magnitude of error increases, the information of the directional spectrum carried by the cross-power spectra become more biased. In fact, as seen in Fig.2, the estimates of the BDM become flatter as the magnitudes of errors increases. On the other hand, when the cross-power spectra contain larger errors, the EMLM estimates erroneous peaks and sometimes fails to yield a smooth and continuous estimates of directional spreading function. However, in these cases, the BDM detects the directional peaks properly, though it underestimates the peak values. Thus, the BDM seems to be very robust method against the errors.

6. FIELD DATA ANALYSIS

6.1 Facilities of directional wave measurement

The new method for the estimation of directional spectrum on the bases of the Bayesian approach mentioned above is applied to the analysis of the wave records acquired at an offshore oil rig 42 km off the Iwaki coast (Northeastern coast of the main island of Japan, see Fig.3). The Onahama Port Construction Office (OPCO), the Second Port Construction Bureau, Ministry of Transport, is conducting a multi-element measurement of ocean waves at this location. Figure 4 shows

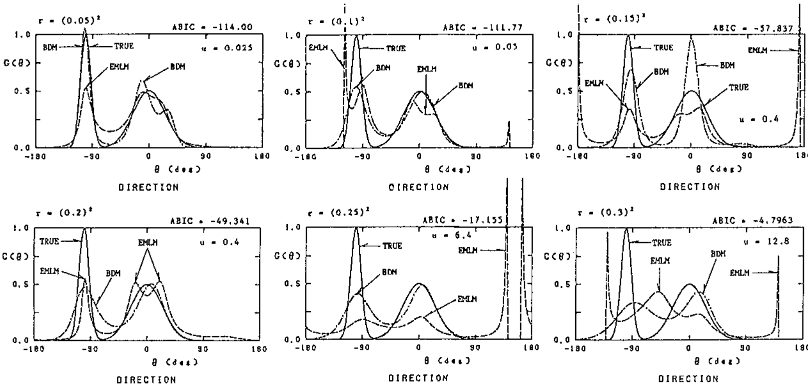

Fig.2 Effect of noise in cross-power spectra on the estimates of directional spreading functions

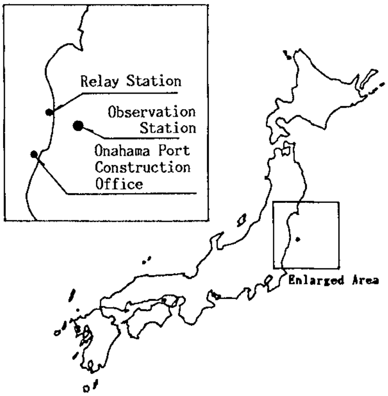

Fig.3 Location of the observation station

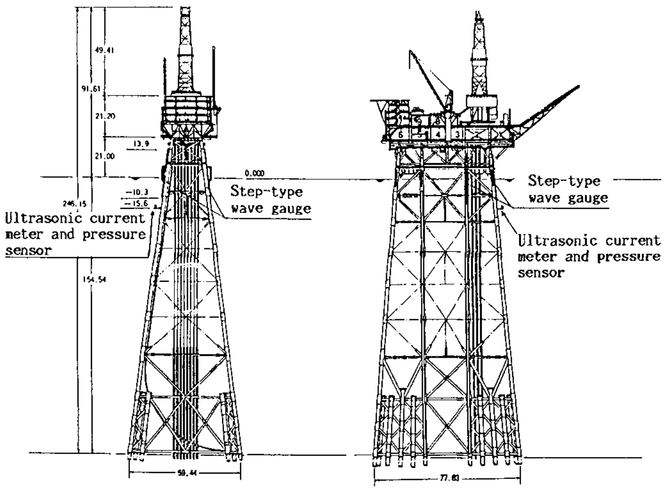

Fig.4 Offshore oil rig and the location of wave probes

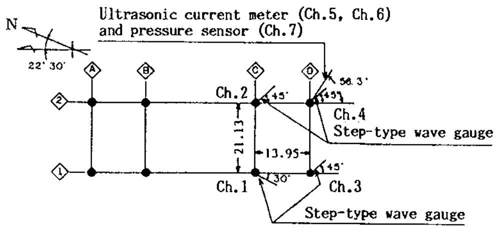

Fig.5 Wave probe array

the oil rig where four step type wave gauges and a two-axis directional current meter with a pressure sensor are installed on its legs as shown in Fig.5. The location of the rig is 37° 17' 49" N and 141° 27' 47" E, in a water depth of 155m below C.D.L.

The simultaneous measurement of 7 elements is performed for 20 minutes at a time interval of two hours. The wave records as well as the wind records are immediately transmitted by a radio telemetering system to a nearby coastal relay station for landline transmission to the OPCO.

The time series wave data are recorded on a digital magnetic tape. Data are also analyzed immediately following each observation using a mini-computer of the OPCO for real time information such as significant wave height, wave period and mean wave direction, etc. The EMLM is employed for the real time directional analysis at the office. The directional wave analysis presented herein is performed at the Port and Harbour Research Institute using the wave data recorded on the magnetic tapes.

6.2 Field data analysis

The time series wave records analyzed here were obtained during the passage of Typhoon No.17 from September 29 to 30 in 1986. At 12:00 on September 30, the maximum significant wave height ($H1/3$ = 6.20m) and period ($T1/3$ = 12.5s) was recorded. Figure 6 is the weather maps of these two days.

Figure 7 shows the time variation of the directional spectrum estimated on the basis of the 7-element wave records by the BDM for every four hours from 16:00 on Sept. 29 to 20:00 on Sept. 30. Up to 4:00 on Sept. 30, bi-directional seas are observed : swell comes from the south and the wind generated waves come from the east-southeast at the same time. After 8:00 on the 30th, the directional spectra are uni-directional. During the passage of Typhoon No. 17, the significant wave height shows two maxima. One was observed at 2:00 on the 30th, when the spectral density of the swell reaches its maximum. The other peak significant wave height was observed at 12:00 on the 30th and it was the highest during these two days. It should be noted that the directional spreading of the directional spectra observed at 12:00 and later are constricted at the peak frequency, i.e., the concentration of the spectral density is highest at the peak frequency and becomes lower as the frequency deviates from the peak frequency. This is the same characteristics shown in the directional spreading function proposed by Mitsuyasu et al.(1975).

Examples of the directional spectra for various types of probe arrays estimated by the BDM and the EMLM are compared in Fig.8. The left figures are the estimates given by the BDM and the right figures are those by the EMLM. The major difference between the estimates given by the two different methods is that those given by the BDM show higher spectral peak than those given by the EMLM. Though the BDM yields almost the same shape of the directional spectral estimates for various types of probe arrays, the EMLM yields the different shape of the estimates depending on the arrangement and the number of wave sensors.

On the basis of these results, we conclude that the estimates of the directional spectra show considerably different shapes depending

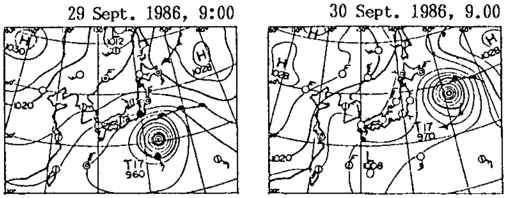

Fig.6 Weather maps of Typhoon No.17

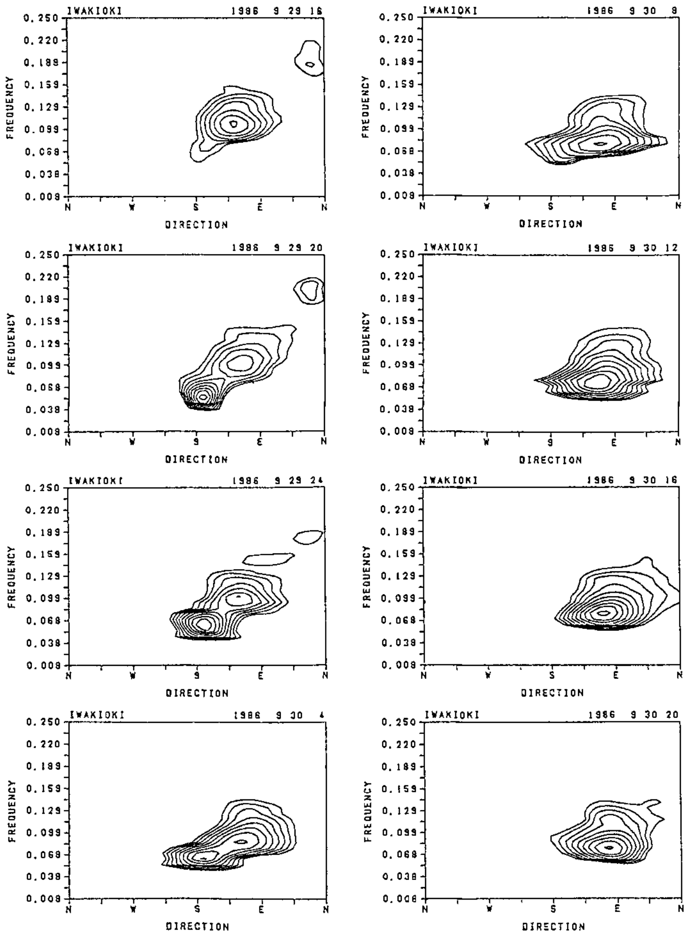

Fig.7 Time Variation of the directional spectrum
(estimated by the BDM from 7-element measurement)

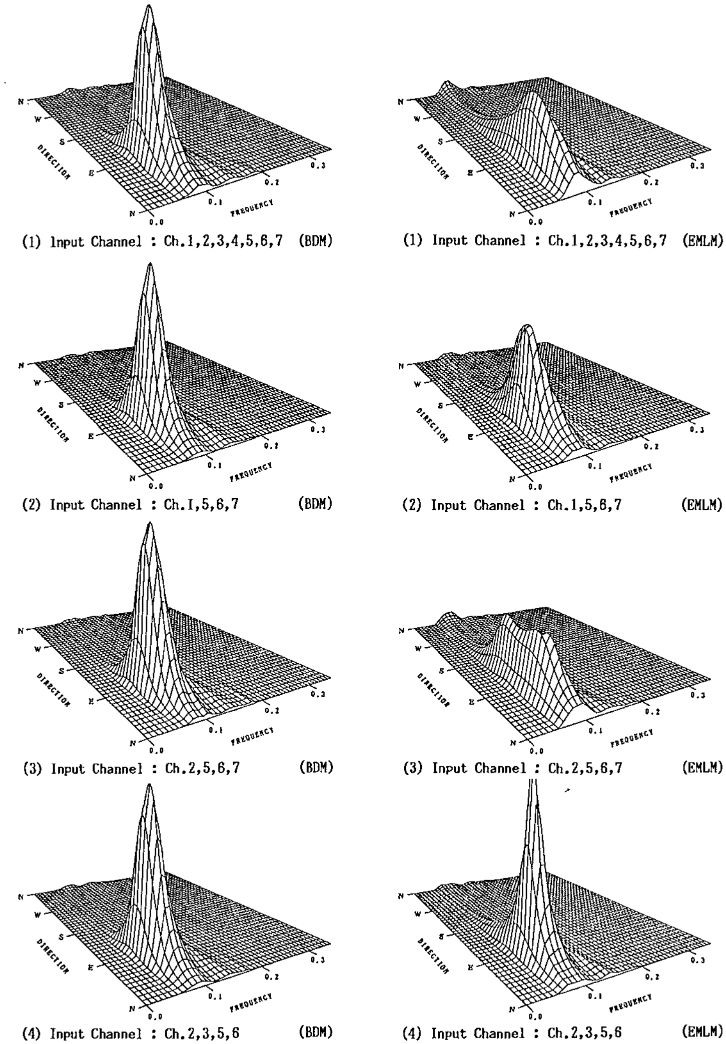

Fig.8 Examples of the directional spectra for various types of probe array estimated by the BDM and the EMLM

on the method for the directional spectral estimation. Therefore, many wave properties must be measured, and the estimation method utilized must be examined for a detailed analysis of the directional spectrum.

6. CONCLUSIONS

The following major conclusions sum up the study:

1) The proposed Bayesian Directional Spectral Estimation Method (BDM) can be applied for the directional wave analysis on the basis of an arbitrary wave probes. However, the method needs wave records consisting of at least four elements of wave properties. When only three elements are measured, the Extended Maximum Likelihood Method (EMLM) and the Maximum Entropy Principle Method (MEP : Kobune and Hashimoto ; 1986) are recommended.

2) When four or more wave probes are employed in the observation, the BDM is the preferred analytical approach. The directional resolution exhibited by the BDM in this circumstance is greater than those shown by the EMLM or other exsisting methods.

3) The BDM is fairly sound against the noises contained in the estimates of the cross-power spectra. As the rate of the noise over the cross-power spectra increases, the estimate given by the BDM becomes flatter than the true directional spectrum. This BDM tendency emerges as the method tends to rely on the a priori condition (smooth and continuous), whenever the given information (cross-power spectra) is not reliable enough. When the cross-power spectra contain large errors, the EMLM fails to yield reasonable estimates, while the BDM can detect the directional peak where the true directional spectrum shows its peak density.

4) Estimates of the directional spectrum vary widely depending on the method employed for the directional wave analysis, number of elements of wave properties to be analyzed and the layout of the probes. Though the directions where the directional spectrum shows its peak density can be detected from the directional wave analysis on the basis of three or four element measurement, it is necessary to measure directional waves with many probes to detail the shape of the directional spectrum. For field observation, especially in deep sea, simultaneous measurements of many wave properties are very difficult for technical and financial reasons. However, the BDM is a very powerful method for the directional wave analysis in laboratories.

5) In the present paper, as an a priori condition, the simplest condition is introduced to characterize the inherent nature of the directional spectrum. This is necessary, as the BDM relies heavily on the a priori condition when the given information is insufficient, to delineate the directional spectrum. However, when research reveals more detail in the structure of ocean directional waves, the method can be improved by adopting the newly attained knowledge as the a priori condition. Thus, the BDM is more adaptable to reformulation of the estimation equations as the study of structures of directional wave spectrum progresses.

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