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Disc-shocking and the mass function of Galactic globular clusters*

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ABSTRACT

A new homogeneous analysis of the available data confirms that the slope of the stellar mass function for Galactic globular clusters seems to depend on the position relative to the potential of the Galaxy. It is shown that the observed dependence can be accounted for by developing the idea that all globular clusters are born with identical mass functions which then evolve through rapid interactions with the Galactic disc (disc-shocking). The limits of our semi-analytical approach based on the impulse approximation are tested by N-body simulations.

Key words: stars: luminosity function, mass function – Galaxy: evolution – globular clusters: general.

1 INTRODUCTION

The advent of CCD detectors and of efficient algorithms to perform stellar photometry in crowded fields has permitted the derivation of reliable and relatively deep luminosity functions (LFs) for a fair sample of Galactic globular clusters (GCs). In turn, these LFs have led to the determination of the present-day mass function (MF) through mass–luminosity relations obtained from stellar models.

By examining the exponent x of the power law fitted to the MFs of 14 GCs taken from literature, Capaccioli, Ortolani & Piotto (1991) did not confirm the correlation with metallicity previously suggested by McClure et al. (1986). Only the most metal-rich ([Fe/H] > -1) clusters have a significantly flatter MF, but no trend is present at intermediate and low metallicities. Capaccioli et al. (1991) and Piotto (1991) found instead that the slope x depends on the position of the cluster in the Galaxy; in particular, it depends more on the height Z_g above the Galactic plane than on the galactocentric distance R_g. This result is especially surprising in that the position of a GC is an accident of the orbital phase at the epoch when the object is observed. Therefore, before attempting an explanation, we have first checked whether our finding is a mere product of the heterogeneity of the reduction procedures adopted by the various authors. After the homogenization of the MFs, preliminarily accounted for by Piotto (1991, 1992) and presented in Section 2, and with the addition of three more clusters which bring the sample to 17 GCs, the apparent dependence of the slope on R_g and Z_g still remains and calls for interpretation. Some preliminary calculations by Stiavelli et al. (1991) suggest that disc-shocking may be adequate to explain this effect. In Section 3 we present and discuss this and other possible interpretations in comparison with observations. A detailed account of our method can be found in Sections 4 and 5. Limits and possible extensions of our approach are reviewed in Section 6.

*Based on observations collected at the European Southern Observatory, La Silla, Chile.

2 EMPIRICAL CORRELATIONS

Our set of GCs is an extension of that considered by Capaccioli et al. (1991, their table 1). For M4 we have adopted the new results by Piotto, Capaccioli & Ortolani (1992), based on a larger sample of stars and a more accurate star background determination. We have also added the recent data on NGC 6171 (Ferraro & Piotto 1992), ω Cen (Richer et al. 1991; Ortolani, private communication) and NGC 5053 (Fahlman, Richer & Nemec 1991).

Basic parameters for the present sample of clusters are summarized in Table 1. The source for columns 2–5 is Chernoff & Djorgovski (1989) and that for column 6 is Webbink (1985).

Capaccioli et al. (1991) limited themselves to compilation of the MF slopes for CCD LFs published in the literature. Here the MF slopes have been recomputed in a consistent way starting from the original LFs and under the following conditions.

(i) Star counts affected by an incompleteness larger than 50 per cent are not taken into account.

(ii) Due to the lack of empirical data on metal-poor star masses, LFs are transformed into MFs using the mass–luminosity relation from VandenBerg & Bell's (1985) models. As discussed by Ferraro & Piotto (1992), the results do not

Table 1. Cluster parameters.

Object	R_g [kpc]	$\|Z_g\|$ [kpc]	$\log(M/M_\odot)$	c	[Fe/H]	x	x_g	References
NGC 104 (47 Tuc)	8.1	3.2	6.06	2.03	−0.71	−0.1	−0.3	Hesser *et al.* (1987)
NGC 4590 (M68)	10.1	5.6	5.10	1.60	−2.09	1.7	1.2	Pryor *et al.* (1986)
NGC 5053	16.7	15.5	4.84	0.8	−2.02	2.0	1.5	Fahlman *et al.* (1991)
NGC 5139 (ωCen)	7.0	1.3	6.16	1.7	−1.60	1.8	0.7	Ortolani (priv. comm.)
NGC 5272 (M3)	12.6	10.2	5.94	1.85	−1.66	1.9	1.4	Paez *et al.* (1990)
NGC 5904 (M5)	6.6	5.5	5.91	1.40	−1.40	1.1	0.7	Pryor *et al.* (1986)
NGC 6121 (M4)	6.8	0.6	5.15	1.70	−1.28	−1.0	−1.0	Piotto *et al.* (1992)
NGC 6171 (M107)	3.9	2.4	5.13	1.60	−0.88	−1.0	−1.0	Ferraro and Piotto (1992)
NGC 6205 (M13)	8.9	4.6	5.82	1.35	−1.65	1.6	1.0	Pryor *et al.* (1986)
NGC 6218 (M12)	5.0	2.3	5.39	1.70	−1.61	−0.7	−0.8	Sato *et al.* (1989)
NGC 6254 (M10)	5.3	1.8	5.39	1.60	−1.60	0.2	0.1	Hurley *et al.* (1989)
NGC 6341 (M92)	9.8	4.4	5.62	1.70	−2.24	1.6	1.0	Stetson and Harris (1988)
NGC 6397	6.9	0.5	4.77	1.63	−2.20	0.0	−0.2	Ortolani and Piotto (1992)
NGC 6752	5.9	1.8	5.33	1.59	−1.54	0.6	0.3	Piotto and Ortolani (1992)
NGC 6838 (M71)	7.4	0.3	4.98	1.50	−0.58	−0.4	−0.4	Richer and Fahlman (1989)
NGC 7078 (M15)	10.4	4.5	5.95	2.54	−2.15	1.5	0.8	Pryor *et al.* (1986)
NGC 7099 (M30)	7.2	5.3	5.18	2.50	−2.19	1.0	0.7	Piotto *et al.* (1990)

change by more than $\Delta x = \pm 0.2$, using the more recent models by Straniero & Chieffi (1991).

(iii) MFs are modelled as power laws. The fitting range is conservatively confined to the interval $0.5 \leq m/M_\odot \leq 0.8$, even if several LFs would allow smaller masses to be reached.

(iv) Since the result of local sampling depends upon mass segregation, the *global* MF slope (x_g) applicable to the stellar population of the *entire* cluster is computed from the measured *local* MF slope x using multicomponent King–Michie models with power-law MFs (Pryor, Smith & McClure 1986).

We want to state explicitly that the lower limit in the mass range [item (iii) above] has been suggested by the fact that, for $m \lesssim 0.5$ $M_\odot$, there is no reliable mass–luminosity relation: besides the scarcity of any empirical data on metal-poor stars, the models are also uncertain (Renzini & Fusi Pecci 1988). It is also clear, however, that our restriction to a more secure range causes another source of concern for the accuracy of the MF slope, since it forces a small mass interval ($\Delta m = 0.3$ $M_\odot$). Note, in this context, that x may well depend on the mass interval considered, given the arbitrariness of the power-law representation combined with the changes induced by the mass segregation. (One consequence is that MF slopes derived over different mass intervals cannot be straightforwardly intercompared.) In the present paper the mass range has been kept strictly constant in an attempt to minimize the above effects. Even so, our very small mass interval stays as the major source of uncertainty for the value of x. Another source is the number of stars used to calculate the mass function. In any case, for most of the clusters, the uncertainty is of the order of $\Delta x = \pm 0.5$ units.

Local and *global* values of the power-law indexes are listed in columns 7 and 8 of Table 1. The *global* MF slopes have been plotted in Fig. 1 against the distance from the Galactic plane, Z_g, and the distance from the Galactic Centre R_g. The new objects added to the sample (NGC 6171, ω Cen and NGC 5053) seem to strengthen the original evidence (Piotto 1991; Capaccioli et al. 1991) of a dependence of the globular cluster mass function slopes on the position of the cluster within the Galaxy. A similar trend is present also in the *local* MF slopes x. The dependence on Z_g is the tightest one (Fig. 1, left): the clusters closest to the Galactic plane have the flattest MFs. None of the clusters with $Z_g \geq 4$ kpc has a flat MF ($x_g \geq 0.7$). A similar dependence is visible for R_g, but with a higher dispersion (Fig. 1, right). We were unable to think of any observational bias which may account for this somewhat surprising result. Note that a dependence of the MF slopes on the position does not necessarily exclude a dependence on other cluster parameters. For example, as discussed in Piotto (1991, fig. 1), on average the most metal-rich clusters have a flatter MF than do the intermediate and metal-poor clusters, suggesting that metallicity can play the role of a second parameter in determining the MF slope (Djorgovski 1991). Indeed, a more detailed analysis, involving appropriate multivariate statistical methods, shows that the MF slopes in the range $0.5 < m/M_\odot < 0.8$ are mainly determined by the position in the Galaxy, and, to a lesser extent, by the metallicity, while other parameters (like concentration, ellipticity, etc.) have little effect, at least in the present sample (Djorgovski, Piotto & Capaccioli 1993).

Coming back to the problem of the mass range for the MF fitting, we note that Richer et al. (1991) have actually published deep star counts for six GCs down to $M_I = 11$, and, at variance with our conservative approach [item (iii) above], they have tried to derive a mass function using ad hoc mass–luminosity relations (Fahlman et al. 1989). They were able to trace the mass functions down to 0.15 $M_\odot$. For all these six MFs there are indications of a steepening around 0.5–0.4 $M_\odot$, i.e. at the same mass where the mass–luminosity relation is expected to flatten (Renzini & Fusi Pecci 1988). This effect is evident in the data of M30 presented by Piotto et al. (1990); changes in the MF slopes have been also reported for M92 (Stetson & Harris 1988) and NGC 6752 (Piotto & Ortolani 1992), always at about 0.5–0.4 $M_\odot$. A close inspection of the mass functions of other clusters with less deep photometry, but reaching stars with masses smaller than 0.5 $M_\odot$ (e.g. 47 Tuc, M10, M12), shows evidence for a similar feature. In conclusion, even taking into account the arbitrariness of the power-law representation of MFs and the occurrence of mass segregation, the change of slope remains very suspiciously coincident with the limit of reliability of the mass–luminosity relation.

Keeping in mind the above cautionary remarks, it may none the less be of some interest to redraw Fig.1 for those clusters with deep enough MFs ($m \leq 0.4$ M$_{\odot}$). Fig. 2 plots five of the six clusters studied by Richer et al. (1991), together with the representative data for our small-mass-range sample (open circles), arbitrarily shifted by two units to help comparison. M5 has not been included because its MF is uncertain, as stated by the authors. Although the sample is still too small to draw any conclusion, the trends of the two data sets show no obvious disagreement.

For the six clusters observed by Richer et al. (1991), these authors suggest a possible correlation of the MF slopes with the *time of disruption*, defined as the inverse of the disruption rate calculated by Aguilar, Hut & Ostriker (1988). This correlation is not confirmed by the larger sample of 17 clusters considered in this study (Fig. 3).

3 DYNAMICAL EFFECTS

Since the data indicate that a correlation of the MF slope with Z_g exists, it is worthwhile to investigate whether it could be originated by dynamical effects rather than being genetically imprinted in the GC population. The major dynamical effects acting on a Galactic GC are bulge-shocking, disc-shocking, evaporation and dynamical friction. Their influence has been analysed by Aguilar et al. (1988), using a single-mass GC model. Dynamical effects can alter the MF slope of a GC by selectively stripping off stars at large radii. Since mass segregation concentrates the most massive stars

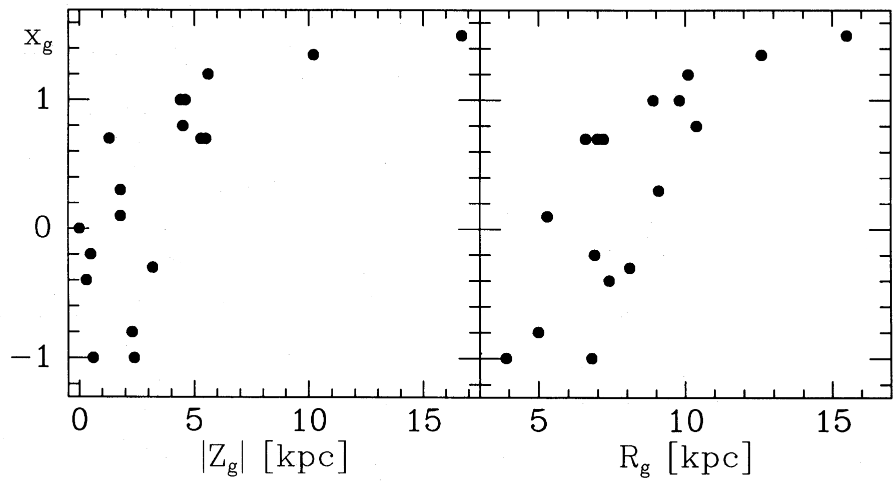

Figure 1. Global mass-function power-law index x_g for a sample of 17 globular clusters plotted against the distance from the Galactic disc $|Z_g|$ (left panel) and the distance from the Galactic Centre R_g (right), disclosing a dependence of x on the galactocentric coordinates, with the clusters closest to the disc and to the centre having the flattest mass functions. The power law has been interpolated in the mass interval $0.5 \leq m/\mathrm{M}_{\odot} \leq 0.8$. Note that the mass-function slope adopted here for ω Cen is from Ortolani (private communication); the adoption of the rather different value obtained from the mass function by Richer et al. (1991) is of no consequence for our results.

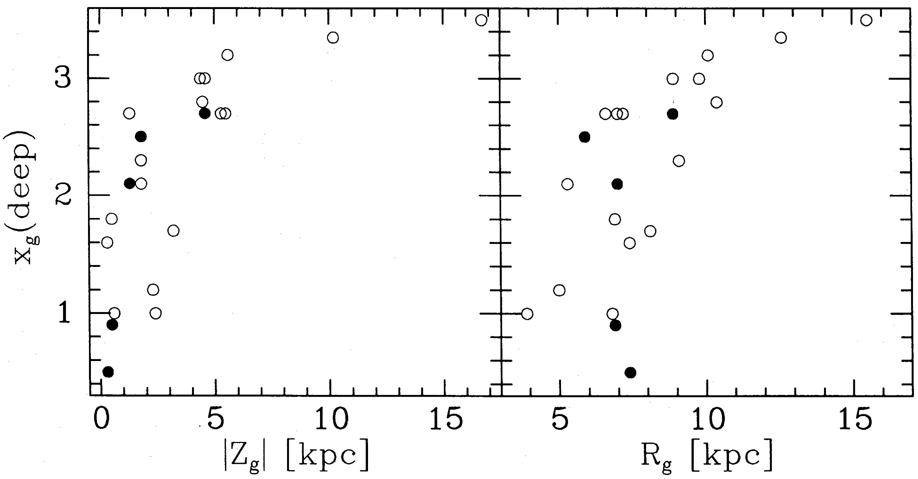

Figure 2. As Fig. 1, but for MF slopes computed for masses $m \leq 0.4$ M$_{\odot}$ (full dots). The trend is similar to that in Fig. 1 (whose points are reproduced here as open circles after shifting by $\Delta x = 2$ units), but the slopes are systematically steeper.

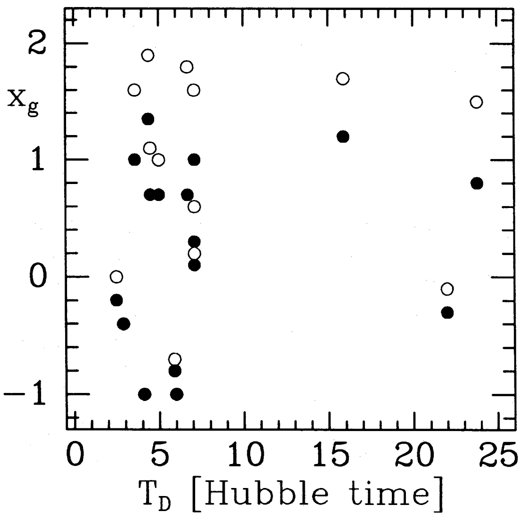

Figure 3. Global (full dots) and local (open circles) MF slopes against the disruption time (in Hubble time units), as computed by Aguilar et al. (1988).

into the centre and pushes the least massive to the outer regions, low-mass stars will be preferentially lost, thus flattening the MF.

Dynamical friction, evaporation and bulge-shocking can affect the observed distribution, but only indirectly. Evaporation is likely to influence the observed MF slope but, since it is an 'internal' mechanism, it does not directly produce a correlation with position. Actually, the effect on the MF slope is far from being well understood, since it depends strongly on the orbital distribution. Stars on orbits confined at large radii are very unlikely to be unbound compared to those on radial orbits which are able to traverse the dense cluster core (Binney & Tremaine 1987). A correlation with position may in any case be produced at a 'higher order', since the position of a GC influences its evolutionary state and hence the details of evaporation. A correlation with galactocentric distance could be induced by evaporation, since metallicity, which is itself correlated with the galactocentric distance, could change the initial mass function and consequently the dynamical evolution. However, similar effects would not produce a correlation with the height over the Galactic disc as is observed, and, in any case, our data show that the correlation between the MF slope and the metallicity is weak (Piotto 1991; Djorgovski et al. 1993).

Another indirect contribution to the observed correlation may actually come from dynamical friction. Clusters orbiting the Galaxy at large radii ($>R_\odot$, where $R_\odot$ is the Sun's distance from the Galactic Centre) on orbits close to the plane of the disc are qualitatively expected to suffer only a modest evolution. They would thus be characterized by a steep MF slope (close to the original one) and small height over the Galactic disc, Z_g. Dynamical friction in the field of disc stars should, however, affect their orbits which would spiral down toward the bulge (Aguilar et al. 1988). This may be the reason why such clusters are not observed (see e.g. Fig. 1).

We are thus left with bulge- and disc-shocking. While the first is the dominant disruption mechanism for GCs with small perigalactic radii and at early epochs, it does not appear to affect clusters strongly at $R_g > 4$ kpc (Aguilar et al. 1988). If the primordial cluster population were characterized by many GCs in nearly radial orbits, then a strong evolution of the MF slope should be induced by bulge-shocking. Such an effect would tend to smear out any correlation with the observed position of the cluster (statistically close to the apocentre), which would be essentially unrelated to the strength of bulge-shocking (dependent on the pericentric distance). However, clusters on nearly radial orbits are quickly destroyed, so that a correlation of the MF slope with the position within the Galaxy can be restored. Indeed, the current distribution of GC orbits is biased toward circular orbits (see e.g. Innanen, Harris & Webbink 1983), so that there is a strong correlation between the observed galactocentric radius of a cluster, R_g, and the pericentric distance or the distance where the disc is crossed.

In contrast, as outlined by Stiavelli et al. (1991), one can account for the correlations in Fig. 1 by considering the effects produced on the stellar distribution of a GC by iterated passages through the Galactic disc. Heating due to disc-shocking indeed acts preferentially on stars located in the outer regions of the cluster (see e.g. Binney & Tremaine 1987) and thus, for the argument given above, it leads to an MF flattening.

Two effects conspire to produce a correlation with Z_g:

(i) at any R_g, the effectiveness of disc-shocking is intrinsically stronger for clusters crossing the disc at low speed, i.e. moving in orbits with lower inclination (which corresponds to statistically smaller $|Z_g|$);

(ii) on the other hand, regardless of the orbital type, a GC spends most of its time close to the apocentre in the projected plane (R, Z), i.e. there is a high probability that a GC is observed at its maximum $|Z_g|$.

Before starting the discussion of our model, it is worth noting that stellar evolution and mass loss are also very important in the early phases of GC evolution (Applegate 1986), i.e. in the first few Gyr, when probably the Galactic disc was not yet formed. Thus these processes are mostly decoupled from the disc-shocking effects that we are investigating in this paper. Our adopted model of a 'standard' GC (cf. Section 4) includes implicitly all these effects, since it does not contain massive stars. Stellar evolution affects GC MFs and dynamics in a position-dependent way (Chernoff & Weinberg 1990). However, it will probably not correlate with the height above the Galactic disc but, rather, with the galactocentric distance. Since we know observationally that the MF depends more on Z_g than on the galactocentric distance R_g, we are forced to conclude that the combined distance-dependent effect on the MF of the early-phase mass loss must be small. By assuming a 'standard' GC model initially independent of the position, we essentially single out disc-shocking from all other possible effects and investigate its effectiveness in shaping the MF. The processes discussed above, alone or combined, can be important, but their effects on the MF slope are beyond the purposes of this paper and will be the subject of a future, more detailed analysis.

4 EFFECTS OF DISC-SHOCKING

In order to evaluate the effects of disc-shocking we have adopted a semi-analytical approach based on the impulse approximation (Ostriker, Spitzer & Chevalier 1972). We also assumed that all clusters have the same initial mass function, given by the Salpeter law, $\xi(m)\propto m^{-(1+x_{\rm in})}$, with $x_{\rm in}=1.35$, and the same concentration parameter c (King 1962). The model of our 'standard' globular cluster is as follows: (i) the mass function is sampled in six mass bins as in Pryor et al. (1986), (ii) the initial (mass) concentration parameter is the average value $c=1.7$ (cf. Webbink 1985), (iii) the tidal radius depends on the galactocentric distance $R_{\rm g}$ according to the relation $r_{\rm t}=100(R_{\rm g}/{\rm R}_\odot)^\alpha$ pc, with ${\rm R}_\odot=8.5$ kpc and α in the physically interesting range from 0.67 (logarithmic potential) to 1 (Keplerian potential), (iv) the initial central velocity dispersion is $\sigma_0=6.5$ km s^{-1}.

We proceeded as follows: first we computed the change in MF slope caused by a single disc crossing in our 'standard' globular cluster model. As a second step we estimated the effect of several disc crossings. After several tests, we found a simple analytical formula relating with sufficient accuracy the effect of a single crossing to that of many. Finally, by a Monte Carlo simulation, we computed the global effect on a population of 'standard' globular clusters.

The effect of a single crossing on each stellar mass bin is obtained by computing the fraction of stars which are unbound after the stellar velocities v_z are modified according to the impulse approximation formula $\Delta v_z=K_0 z$, where (Ostriker et al. 1972)

$$K_0=2\,\frac{|g_Z(R)|}{V_Z}; \tag{1}$$

z and v_z are the Z-components of the position and velocity of a cluster star with respect to the cluster centre, R is the galactocentric radius at which the disc crossing occurs, and V_Z is the Z-component of the cluster orbital velocity. The gravitational acceleration of the Galaxy is $g_Z(R)=g_0\exp(-R/R_0)$; we adopt a disc scalelength of $R_0=3.5$ kpc, and $g_0=7.65\times10^{-8}$ cm s^{-2}, corresponding to a surface mass density at the solar radius $\Sigma_\odot=75$ M$_\odot$ pc^{-2} (Binney & Tremaine 1987). When the GC crosses the disc, the impulsive change in velocity Δv_z is computed for each phase-space element. The potential energy remains constant, so that elements previously bound may be rendered unbound. Since we 'enforce' the existence of a tidal radius or, equivalently, a cut-off energy, we define as unbound all those phase-space elements less bound than the cut-off energy. The new MF slope is derived from the remaining (still bound) mass in each bin, thus yielding the dependence of Δx on K_0. The change in mass for each bin, which is described by a separate King component, $f_{\rm K}$, is

$$\Delta M_i=\int_{E\le E_{\rm c}} f_{\rm K}\,{\rm d}^3v-\int_{E'\le E_{\rm c}} f_{\rm K}\,{\rm d}^3v, \tag{2}$$

where $E=m_iv^2/2+m_i\Phi$ is the single-particle energy before disc-shocking and $E'=m_i\{v_x^2+v_y^2+[v_z+\Delta v_z(z)]^2\}/2+m_i\Phi$ is the particle energy after disc-shocking.

The MF slope of the models is computed over a mass range roughly equivalent to that derived from observations. Since after several disc crossings the MF is no longer a power law, the derived values of the slope are expected to depend upon the choice of the mass range over which they are determined. However, by carrying out some ad hoc experiments, we convinced ourselves that the correlation does not depend critically on the adopted mass range, as long as we consider unevolved stars in the upper main sequence.

The cumulative change of the MF slope is computed as

$$1+x_{\rm g}=(1+x_{\rm in})\left(1-\frac{\Delta x}{1+x_{\rm in}}\right)^n, \tag{3}$$

where Δx refers to a single crossing, and n is the total number of crossings given by the age of the disc (12 Gyr) divided by the orbital semiperiod of the cluster. Note that results do not change significantly with a linear approximation $x_{\rm g}=x_{\rm in}-n\Delta x$.

The above scheme for a single object is then applied to evaluate the statistical effects on a random population of 'standard' models. We have run Monte Carlo simulations by generating populations of GCs, all in circular orbit, with a spatial density given by $\nu_{\rm GC}\sim R_{\rm g}^\beta$, with $\beta=-3.5$ (Frenk & White 1980).

Our results are shown in Fig. 4, where $Z_{\rm g}$ is 'sampled' at a random phase. It is apparent that there is a loose correlation of $x_{\rm g}$ with $R_{\rm g}$. The only clear feature is the lack of GCs with small $x_{\rm g}$ at high $Z_{\rm g}$, which proves that disc-shocking is indeed effective in modifying the initial mass function of Galactic globular clusters. However, at variance with observations (Fig. 1, right panel), our simulations give quite a few GCs having high $x_{\rm g}$ in the low-$Z_{\rm g}$ region. Also, we find that the instantaneous spatial distribution of GCs in our Monte Carlo experiment includes objects at large galactocentric distances and low $Z_{\rm g}$, in contrast with the observed sample (Fig. 1). This discrepancy is possibly due both to the higher dynamical friction experienced by these clusters compared to high-$Z_{\rm g}$ GCs (see Section 1) and to an observational bias. It seems reasonable to suppose that clusters at large galactocentric distance and low height over the disc are likely to be excluded from observing lists because of the heavy disc-star contamination, which affects the reliability of LFs. On these grounds and in order to visualize the effect, we repeated our Monte Carlo simulations, rejecting all objects with $R_{\rm g}>{\rm R}_\odot$ and Galactic latitude $|b|<45°$. With such a selection, which reproduces the observational selection, both the distribution of clusters in the (R, Z) plane and the correlations with x are in better agreement with the observations (Fig. 5).

5 TESTING THE METHOD

We shall comment on the simplifications and assumptions of our calculations.

To check that our simple scaling of the effect of a single crossing to many crossings is not too unrealistic, we have explicitly computed, for a few values of the parameter K_0, the effect of multiple crossings, by assuming, as an extreme case, that after each crossing the cluster quickly rearranges its mass distribution to a King model with the same concentration but the new mass function. These tests indicate that our assumptions do not bear too heavily on our results.

We have assumed that the concentration of a cluster does not change throughout its life. This is certainly not correct, since clusters evolve and tend to become more and more concentrated. By experimenting with clusters of different concentration, we find that this evolution does not significantly affect our results. They depend mainly on the amount of mass segregated at large radii, rather than on the concentration itself which mainly gives a measure of the cluster core properties.

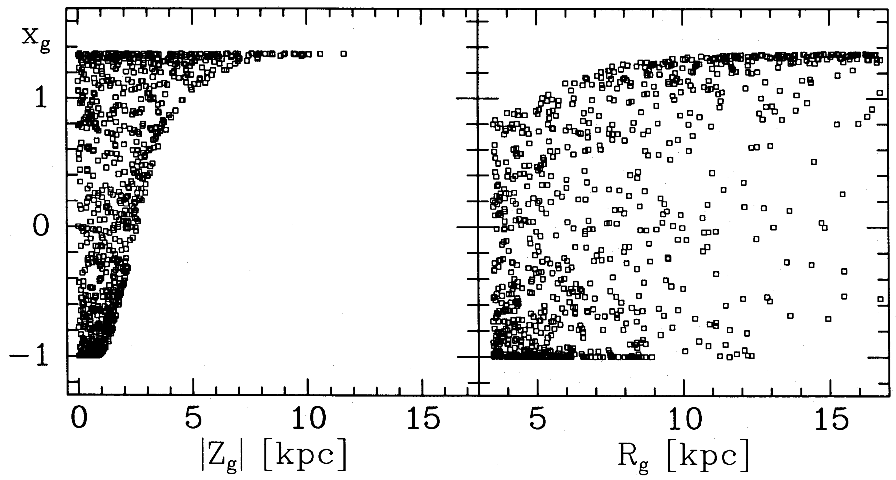

Figure 4. Results of our Monte Carlo experiments: the final mass-function slope x_g is plotted against the galactocentric radius R_g and the height over the Galactic disc Z_g for 10^3 GCs. Note how clusters with a flatter slope avoid the area with largest Z_g.

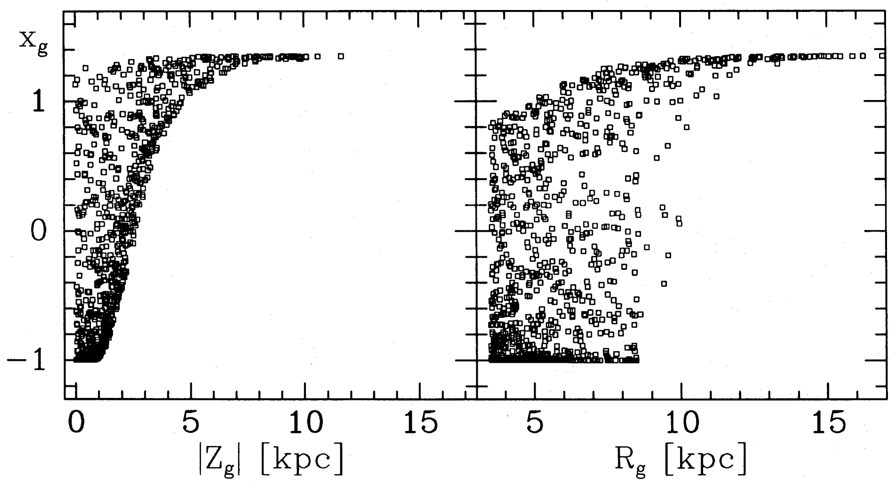

Figure 5. Same data as Fig. 1, adding a selection in Galactic latitude: clusters with $R_g > R_\odot$ and $|b| < 45°$ are rejected.

We have also explored several values for all the other parameters, and tried to assess the effects of other factors, such as the eccentricity of the orbits, the mass bins, the density distribution of the GC system and the value of the tidal radius for our 'standard' GC model. We find that our results are robust. The properties of the Galactic disc, e.g. its surface density, can influence the effectiveness of disc-shocking. Agreement with observations can be found for a disc surface density, at the solar radius, in the range 75 ± 25 $M_\odot$ pc^{-2}. Such a value is changed if one considers a disc age different from 12 Gyr.

6 *N*-BODY SIMULATIONS

In order to have an independent check of the effect of a small number of crossings, we ran a set of *N*-body simulations. The simulations, accounted in Table 2, were performed with the 3D code described by Bertin & Stiavelli (1989) and based on a spherical expansion of density and potential, by using $N = 10^4$ particles and a 'continuous' mass spectrum, i.e. by assigning to each particle a different mass so as to reproduce the desired mass function. Since the adopted code is essentially collisionless, the collisional evolution of a globular cluster could not be simulated. We checked that a cluster model would remain stationary over the time-scales of interest when no disc-shocking is present. We observed a modest evaporation. We also ran a simulation using four times as many particles to test that our results were independent of *N*. Each simulation required typically 10 hours per 10^4 steps on a VAX 8600, for a total of about 300 cpu hours. The simulations were confined to relatively short intervals, where the effect of collisionality can be neglected. Despite some random fluctuations of the intensity of the effect, the results of the numerical experiments, plotted in Fig. 6, confirm the validity of our analytical approach, namely that the change in mass-function slope determined with our semi-analytical approach agrees with that measured in the *N*-body experiments. For very small values of K_0, the change in slope

Table 2. *N*-body simulations.

Run Id.	t_g (yrs)	N_{step}	N shocks	K_0
No Tide	1.25×10^8	10^4	0	0.0
Only Tide	1.25×10^8	10^4	0	0.0
SHK1	1.25×10^8	10^4	1	1.0
Test 40K	1.25×10^8	10^4	1	1.0
SHK2	1.25×10^8	10^4	1	0.44
SHK3	1.25×10^8	10^4	1	0.25
SHK4	1.25×10^8	10^4	1	0.13
SHK5	1.25×10^8	10^4	1	0.064
SHK6	1.25×10^8	10^4	1	0.20
SHK7	1.25×10^8	10^4	2	0.13
SHK8	1.25×10^8	10^4	2	0.20
SHK9	1.25×10^8	10^4	3	0.20
LSHK1	2.5×10^8	2×10^4	2	0.13
LSHK2	2.5×10^8	2×10^4	2	0.44
SHK10	2.5×10^8	2×10^4	7	0.025
SHK11	2.5×10^8	2×10^4	2	0.025
SHK12	2.5×10^8	2×10^4	7	0.064
SHK13	2.5×10^8	2×10^4	3	0.032
SHK14	2.5×10^8	2×10^4	3	0.10
SHK15	2.5×10^8	2×10^4	3	0.51
SHK16	2.5×10^8	2×10^4	3	0.38

becomes very difficult to determine with *N*-body simulations, since only very few stars are lost. In addition, in the limit of small K_0, we observe that disc-shocking induces large-scale oscillations of the GC, thus complicating any estimate of cluster heating and mass loss. Neglecting oscillations and graininess, our analytical calculations are expected to yield the most accurate results in this regime.

7 CONCLUSIONS

The dependence of the slope of the GC global mass function on the height over the Galactic disc is confirmed by a new homogeneous analysis of all the available data: on average, the slopes for 17 GCs, as measured in the interval $0.5 \leq m/\mathrm{M}_\odot \leq 0.8$, are $\Delta x_g \simeq 1.5$ units steeper at all $Z_g \geq 4$ kpc. There is also a dependence of x_g on the distance from the Galactic Centre. Again, the MF is steeper at larger distances: $x_g \gtrsim 1.0$ for $R_g \gtrsim 9$ kpc. Both effects seem real (i.e. not produced by some observational bias), but their statistical significance cannot be firmly assessed until more clusters are observed at $R_g \geq 10$ kpc and $Z_g \geq 4$ kpc, and a more stringent multivariate analysis is performed.

We have shown that the correlation of the MF slope with the position within the Galaxy can be explained by adopting the idea that all globular clusters are born with identical mass functions which then evolve due to disc-shocking, as originally suggested by Stiavelli et al. (1991). The effect of disc-shocking has been studied using both a semi-analytical approach and *N*-body simulations. We find that the resulting evolution does not seem to depend significantly on the specific parametrization chosen for the globular cluster structure or on the Galactic disc properties. If confirmed by more extended data sets, this result could be used to set constraints on the gravitational field of our Galaxy (Stiavelli, Piotto & Capaccioli 1992) and would help in the interpretation of the history of the globular cluster system in the Galaxy.

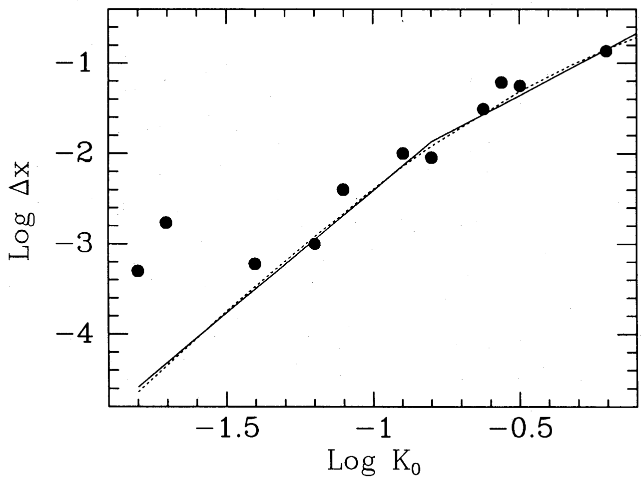

Figure 6. MF slopes from the *N*-body simulations (full dots) plotted against K_0 (equation 1). The solid line represents the fitting formula adopted in the Monte Carlo simulations; the dotted line shows the analytical values for the disc-shocking effects, derived as described in Section 3.

These further developments would probably require the use of a more complex code than the one used here. All dynamical effects should be included, together with a realistic Galactic potential and a set of initial conditions for the GC system. However, even in the absence of such a code, we are already able to set an upper limit of about 100 $\mathrm{M}_\odot$ pc^{-2} for the disc surface density in the solar neighbourhood, unless the disc itself is much younger than 12 Gyr.

If the extension of the trend to very low masses is confirmed, this could be accounted for, in this model, by assuming that all GCs have an IMF characterized by a slope $x_g = 1.35$ for intermediate masses, and $x_g = 3.5$ for low masses.

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