Discounting the distant future: How much does model selection affect the certainty equivalent rate?

Ben Groom[*] Phoebe Koundouri[†] Ekaterini Panopoulou[‡]
Theologos Pantelidis[§]

March 2006

Abstract

Recent work in evaluating investments with long-term consequences has turned towards establishing a schedule of Declining Discount Rates (DDRs). Using US data we show that the employment of models that account for changes in the interest rate generating mechanism has important implications for operationalising a theory of DDRs that depends upon uncertainty. The policy implications of DDRs are then analysed in the context of climate change for the US, where the use of a state space model can increase valuations by 150%, compared to conventional constant discounting.

JEL classification: *C13, C53, Q2, Q4*
Keywords: *long-run discounting, interest rate forecasting, state-space models, regime-switching models, climate change policy.*

[*]Department of Economics, School of Oriental and African Studies.

[†]Department of Economics, University of Reading, UK and Department of Economics, University College London, UK.

[‡]Department of Banking and Financial Management, University of Piraeus, Greece and Department of Economics, National University of Ireland Maynooth. *Correspondence to:* Ekaterini Panopoulou, Department of Economics, National University of Ireland Maynooth, Co.Kildare, Republic of Ireland. E-mail: apano@may.ie. Tel: 00353 1 7083793. Fax: 00353 1 7083934.

[§]Department of Banking and Financial Management, University of Piraeus, Greece.

Acknowledgements: We are grateful to two anonymous referees, Nikitas Pittis, Christian Gollier, Cameron Hepburn, Dimitrios Malliaropulos, David Pearce, participants in the 2003 Royal Economic Society Conference, the 2004 Applied Environmental Economics Conference Royal Society, the First Hispanic Portuguese Congress of Environmental and Natural Resource Economics, the 13th Annual EAERE Conference, and seminar participants at the University College London and Reading University for helpful comments and suggestions. Panopoulou and Pantelidis thank the EU for financial support under the "PYTHAGORAS: Funding of research groups in the University of Piraeus" through the Greek Ministry of National Education and Religious Affairs.

1. Introduction

The dramatic effects of conventional exponential discounting on present values of costs and benefits that accrue in the distant future and the related issues of intergenerational equity that arise are well documented (see e.g. Portney and Weyant, 1999, Pearce *et al.,* 2003). The emergence of a long-term policy arena containing issues as diverse as climate change, nuclear build and decommission, biodiversity conservation, groundwater pollution, and the use of social Cost Benefit Analysis (CBA) to guide decision-makers in this arena has brought the discussion of long-run discounting to the fore. Discount rates that decline with the time horizon (Declining Discount Rates or DDRs) have often been touted as an appropriate resolution to what Pigou (1932) described as the 'defective telescopic faculty' of conventional discounting, while there has been much discussion about the moral and theoretical justification for such a strategy (see e.g. Dybvig *et al.,* 1996, Sozou, 1998, Weitzman, 1998, 2001, Portney and Weyant, 1999, Gollier, 2002a).

Of particular interest are the declining yet socially efficient discount rates resulting from the analyses of Weitzman (1998, 2004) and Gollier (2002a, 2002b, 2004) both of which appear to offer a theoretical path through the 'dark jungles of the second best' (Baumol, 1968) and the intergenerational equity-efficiency trade-off contained therein. In the case of Gollier (2002a) and Weitzman (1998), it is uncertainty that drives DDRs; with regard to future growth of consumption and the discount rate respectively, and in each case the presence of DDRs hinges upon the features of uncertainty underlying the primals. For instance, the nature of the decline of Weitzman's Certainty Equivalent Rate (CER) over time rests upon assumptions concerning the persistence of the interest rate. Weitzman (2004) shows how such persistence can originate in a combined neoclassical model of optimal growth under uncertainty and Bayesian statistical model. His model is able to produce persistent uncertainty in the interest rate and as a result DDRs stemming mainly from the uncertainty over future technological progress. Further contributions reveal the importance of other features of uncertainty for DDRs. Gollier (2004), for example, builds upon a burgeoning financial literature concerning the term structure of interest rates and finds that when the representative agent is prudent, a positively correlated growth process leads to a decreasing yield curve due to increased uncertainty for the distant future. Gollier also highlights the importance of second order stochastic correlation, and draws parallels with the Cox, Ingersoll and Ross model (1985) (henceforth CIR) by introducing the analogue of heteroscedasticity in his process for the interest rate.

More generally, the literature spawned by the seminal contribution of Vasicek (1977) provides additional insight into the important features of interest rates. Important aspects include: i) the specification of the variance process, i.e. the diffusion function (CIR, Chan *et al.* 1992) and ii) the relaxation of the time-homogeneity assumptions of the one-factor approaches of CIR and Chan *et al.* (1992) (e.g. Ho and Lee 1986). Ho and Lee (1986) account for the evolution of the instantaneous return and volatility over time by specifying both the drift and the diffusion process of the instantaneous stochastic rate via time-varying functions of the level of interest rates.[1]

Newell and Pizer (2003) (henceforth N&P) effectively combine some of these insights in an econometric analysis of the interest rate model described in Weitzman (1998), which results in an empirical schedule of DDRs for CBA.[2] In sum, for N&P the uncertainty surrounding the interest rate is characterised by the parameter uncertainty typically found in econometric models and the authors assume that the past behaviour of the interest rate is informative about the future. They describe the behaviour of the US long-term real interest rate with a reduced-form model, which solely takes into account the evolution of the mean of the process, but which guarantees a CER which declines at a rate dependent upon persistence. Their model is the direct analogue of the Vasicek (1977) model for the term structure of interest rates, in the sense that only the conditional mean equation is specified, while the conditional variance is held constant. With these assumptions the authors obtain a working definition of the CER based upon an econometric model and estimate the schedule of the CER via a simulation exercise.

The empirical issues stemming from the environmental literature on DDRs, along with the development of an econometric model versatile enough to reproduce the empirical reg-

[1] See also Black *et al.* (1990), Hull and White (1990) and Black and Karasinski (1991).

[2] Weitzman (2001) conducted his own empirical analysis. However his interpretation of uncertainty is conceptually different to that of N&P. Specifically, Weitzman is concerned with current uncertainty about the future, while N&P are concerned with uncertainty in the future.

ularities typically encountered in interest rate data, are the main concern of this paper. If, like N&P, we believe that the past is informative about the future, it is important to characterise the past as accurately as possible when the estimated parameters form the basis of the empirical schedule of the CER. Given that each model differs in the assumptions made concerning the time series process, interest rate simulations and the attributes of the resulting schedule of the CER will also differ. It therefore becomes immediately obvious that model selection will have important policy implications, particularly in the long-term policy arena.

In this paper, we build upon N&P and, using the same US interest rate data for comparison purposes, show that misspecification testing generates a natural progression away from the simple N&P specification towards models which account for second-order dependence and explicitly consider changes in the time series process over time. The policy implications of interest rate uncertainty and model selection are exemplified in the evaluation of carbon damages and hence climate change policy.

The paper is organised as follows. In Section 2, we present the theory of the CER offered by Weitzman (1998), the econometric models employed to replicate the stochastic nature of US interest rates and our methodology for model selection. The results of the estimation and the simulations are presented in Section 3. Section 4 shows the implications of model selection for climate change policy and Section 5 concludes.

2. From Theory to Practice

2.1 The Certainty Equivalent Discount Factor and Rate

Discounting future consequences in period t back to the present is typically calculated using the discount factor P_t, where $P_t = \exp(-\sum_{i=1}^{t} r_i)$ and r_i is the real interest rate. When r_i is stochastic, the expected discounted value of a dollar delivered after t years is:

$$E(P_t) = E\left(\exp(-\sum_{i=1}^{t} r_i)\right) \tag{1}$$

Following Weitzman (1998) we define (1) as the *certainty equivalent discount factor*, and the corresponding *certainty-equivalent forward rate* for discounting between adjacent periods at time t as equal to the rate of change of the expected discount factor:

$$\frac{E(P_t)}{E(P_{t+1})} - 1 = \widetilde{r}_t \tag{2}$$

where $\widetilde{r}_t$ is the forward rate from period t to period $t+1$ at time t in the future, or the marginal discount rate. Weitzman (1998) and N&P show that $\widetilde{r}_t$ as defined in (2) is a declining function of time provided that there is sufficient persistence in the series over time.[3]

2.2 Parameterisation of Real Interest Rates

N&P employed a simulation method to forecast discount rates in the distant future, which was properly designed to account for uncertainty in the future path of interest rates and was mainly based on the estimation results of two econometric models, namely an autoregressive Mean-Reverting (MR) model and a Random Walk (RW) model. They estimated the following $AR(p)$ model for r_t (in logs):

$$r_t = \eta + e_t, \quad e_t = \sum_{i=1}^{p} a_i e_{t-i} + \xi_t \tag{3}$$

where $\xi_t \sim N(0, \sigma_\xi^2), \eta \sim N\left(\overline{\eta}, \sigma_\eta^2\right)$ and $\sum_{i=1}^{p} a_i < 1$ for the MR model, while $\sum_{i=1}^{p} a_i = 1$ for the RW model. The authors prove that in the case of an AR(1) model for r_t (in levels), the CER takes the following form:

$$\widetilde{r}_t = \overline{\eta} - t\sigma_\eta^2 - \sigma_\xi^2 f(\rho, t) \tag{4}$$

where $\overline{\eta}$ is the unconditional mean discount rate, ρ is the autoregressive coefficient, $f(\rho, t) = \frac{1-\rho^2-2\log(\rho)\rho^{t+1}\left(1+\rho-\rho^{t+1}\right)}{2(1-\rho)^3(1+\rho)}$ for MR and $f(\rho, t) = \frac{1}{12}(1 + 6t + 6t^2)$ for RW. It is straightforward to see that (4) is a declining function of t (see N&P for details).

This model, although simple, is successful in capturing the basic features of the underlying Data Generation Process (DGP) which lead to DDRs, namely persistence and uncertainty. However, given the abundance of models already designed to capture the

[3] Gollier (2002a) provides an arbitrage argument for Weitzman's model of the interest rate, albeit with respect to the average certainty equivalent rate, rather than the instantaneous rate.

dynamics of the interest rate data either in discrete or continuous time the question remains: Is this simple model an adequate parameterisation of reality? As early as 1985, CIR introduce second-order dependence in the stochastic process of the interest rate by letting the conditional variance vary with the level of the interest rate.[4] The simpler discretised diffusion model motivated by the CIR model is the GARCH(1,1) model, in which the conditional variance depends on its own lag as well as the lag of squared innovations. In our study, we employ the $AR(p)$ - $GARCH(l,m)$ model to account for both mean and volatility effects in the US interest rate process. Specifically our model is as follows[5]:

$$\begin{aligned} r_t &= \eta + e_t, \quad e_t = \sum_{i=1}^{p} a_i e_{t-i} + \xi_t \\ \xi_t &= h_t^{1/2} z_t, \quad h_t = c + \sum_{i=1}^{l} \gamma_i h_{t-i} + \sum_{i=1}^{m} \beta_i \xi_{t-i}^2 \end{aligned} \tag{5}$$

where h_t is the conditional volatility of ξ_t (given all available information at time $t-1$) and $z_t \sim IIDN(0,1)$. However, when fitting a GARCH model to interest rates, one often finds that the parameter estimates imply that the conditional variance process is either integrated or explosive.[6] In the case that $\sum_{i=1}^{m}\beta_i + \sum_{i=1}^{l}\gamma_i = 1$, we have an integrated GARCH process (IGARCH) for the volatility of the process resulting to an $AR(p)$ - $IGARCH(l,m)$ model.

Both the $AR(p)$ and $AR(p)$ - $GARCH(l,m)$ models assume that the parameters driving the stochastic process are constant over the sample period, i.e. they are time-homogenous. This is likely to be an unrealistic assumption for the CER, particularly over the long-term policy horizon in hand which, following N&P, extends for 400 years. It is well known that periods of economic crises increase the volatility of interest rates.[7] Such turbulent periods are likely to induce persistence in volatility, which is often an artifact of the changes in the economic mechanism generating the interest rate (see Lamourex and Lastrapes, 1990, Gray, 1996). In this sense, regime shifts are mistaken for periods of

[4] Chan *et al.* (1992) extend the CIR model to include any power function for the diffusion function.

[5] Henceforth, r_t represents the logarithm of the real interest rate unless stated otherwise.

[6] See, for example, Engle *et al.* (1987, 1990) and Kees *et al.* (1997).

[7] Examples of such periods in the US are the period 1979-1982 that the Federal Reserve Bank switched to targeting non-borrowed reserves, the OPEC oil crisis (1973-1975), the October 1987 stock market crash and wars involving the US.

volatility clustering. These findings are corroborated by various studies (Hamilton, 1988, Gray, 1996, Naik and Lee, 1997) in the term structure literature in which the spot interest rate process experiences discrete regime shifts. These models typically posit a spot interest rate process that can shift randomly between two or more regimes. The diffusion and drift functions are kept the same but the specific parameter values are different in each regime, leading to a time-heterogeneous process. In our study we consider the following Regime-Switching (RS) model with two states:

$$r_t = \eta_k + e_t, \quad e_t = \sum_{i=1}^{p} a_i^k e_{t-i} + \xi_t \tag{6}$$

where $\xi_t \sim IIDN(0, \sigma_k^2)$, $k = 1, 2$ for the first and second regime, respectively. Each regime incorporates a different speed of mean-reversion to a different long-run mean and a different unconditional variance. At any particular point in time there is uncertainty as to which regime we are in. The probability of being in each regime at time t is specified as a Markov 1 process, i.e. it depends only on the regime at time $t-1$. We define the probability that the process remains at the first regime as P and the respective one for the second regime as Q. The matrix of the transition probabilities is assumed to be constant.[8]

While the parameterisation of an RS model allows us to define a finite number of states that the interest rate process goes through, it does not allow for cases in which both the level and the variance of the process slowly evolve over time. Such an evolution can be captured by models with time-dependent parameters.[9] Fan *et al.* (2003) compare various specifications of both time-dependent and time-independent models and propose a time-varying coefficient model which captures better the time-variation of short-term dynamics of the interest rate. This finding, along with a similar conclusion of Ait-Sahalia (1996) who finds strong non-linearity of the drift for the US interest rate, leads us to introduce a

[8] We define the following matrix of transition probabilities:

$$\begin{aligned} \text{Prob}(R_t &= 1 \mid R_{t-1} = 1) = P, \;\; \text{Prob}(R_t = 2 \mid R_{t-1} = 2) = Q \\ \text{Prob}(R_t &= 2 \mid R_{t-1} = 1) = 1 - P, \;\; \text{Prob}(R_t = 1 \mid R_{t-1} = 2) = 1 - Q \end{aligned}$$

where R_t refers to the regime at time t.

[9] See Ho and Lee (1986), Black *et al.* (1990), Hull and White (1990) and Black and Karasinski (1991) for time-dependent models in the continuous time literature.

time varying parameter model. We model the interest rate as a State Space (SS) process and specifically as an $AR(1)$ process with an $AR(p)$ coefficient as follows:

$$r_t = \eta + \alpha_t r_{t-1} + e_t, \quad \alpha_t = \sum_{i=1}^{p} \eta_i \alpha_{t-i} + u_t \tag{7}$$

where e_t and u_t are serially independent, zero-mean normal disturbances such that:

$$\begin{pmatrix} e_t \\ u_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \right). \tag{8}$$

This specification is able to capture non-linearities in the mean of the interest rate and accommodates changes in the conditional variance of the series under consideration. Tsay (1987) shows that the conditional variance of an ARCH model can be (under specific assumptions) identical to that of a Random Coefficient Autoregressive (RCA) model, which is nested in the class of AR models with AR coefficients. A simple RCA model allows for the conditional variance to evolve with previous observations, accommodating in this way the high volatility observed in periods of high interest rates. With the addition of an $AR(p)$ structure to the coefficient of our model, we are able to capture both the volatility dynamics and the observed non-linearity in the drift of the interest rate process. This time-varying coefficient model can be thought of as an infinite regime-switching model which allows for a rather elevated degree of time-heterogeneity.

The abundance of econometric models gives rise to two further questions: How is one to select among these models? and: Do the policy prescriptions of the more complex models differ sufficiently to justify their use? Our aim is to select the model that captures the dynamics of the data generating process in order to achieve an adequate description of the series under scrutiny and show that model selection has important implications for policy via the resulting schedule of discount rates. The complexity of the model and the restrictions it imposes should correspond to the level of uncertainty of the true data generating process. Otherwise, inference can be misleading and the forecasting performance of the model may be very poor. In the following section we employ misspecification tests, such as tests for stationarity, autocorrelation, heteroscedasticity or parameter instability, to provide a benchmark to our selection procedure in conjunction with both an in-sample and an out-of-sample forecasting exercise.

3. Empirical Results

3.1 Data

We use the US data employed by N&P for comparison purposes. Specifically, annual market interest rates for long-term government bonds for the period 1798 to 1999 are employed. Starting in 1955, nominal interest rates are converted to real ones by subtracting a ten-year moving average of the expected inflation rate of the Consumer Price Index (CPI), as measured by the Livingston Survey of professional economists. For the previous years, expected inflation is assumed to equal zero and thus nominal and real interest rates coincide. The real interest rates are then converted to their continuously compounded equivalents. Finally, the estimation is based on a three-year moving average of the real interest rates series to smooth any short-term fluctuations, since we focus on the long-term behaviour of the series.[10] Following N&P, we estimate our models based on the logarithms of the series. This logarithmic transformation precludes negative rates and makes interest rate volatility more sensitive to the level of interest rates.[11]

3.2 Results

First of all, we test the stationarity of the US real interest rates (in logs). In general, the results of a variety of unit-root tests favour the existence of a unit-root in the series, in accordance to the results of N&P.[12] However, it is well-known that unit-root tests often suffer from a lack of power to reject a false hypothesis of a unit-root for alternatives that lie in the neighbourhood of unity. Furthermore, mean shifts and non-linearities are often mistaken for unit-root behaviour (see, for example, Perron, 1990, Nelson *et al.*, 2001).[13] For completeness, we estimate both a Random Walk (RW) and a Mean-Reverting (MR)

[10] More details about the data can be found in N&P.

[11] See N&P, footnote 15, pp.60 for a detailed discussion on this issue.

[12] For brevity, the results are not reported but are available upon request.

[13] The existence of a unit root suggests that real interest rates become potentially unbounded with no economic forces at work to bring them back to some equilibrium level. However, it is interesting to note that, despite the potentially unbounded nature of the Random Walk model, high interest rate states of the world are not important in the long-run in Weitzman's model. Furthermore, this potentially unbounded nature of the interest rates could be dealt with by employing a logit transformation of the data instead of a log one. We thank an anonymous referee for the latter insight.

model. Three lags are included in both models.[14] Our estimates are virtually identical to N&P and we do not report or discuss them for brevity.

Tests for serial correlation in the residuals of the regression model suggest that mean dependence is sufficiently captured by this $AR(3)$ model. Not surprisingly though, this constant-variance model does a poor job in modelling the conditional volatility of interest rates as there is autocorrelation in the squared residuals. Specifically, the Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) in the residuals rejects the null hypothesis of homoscedasticity. In this respect, we estimate an $AR(3)-GARCH(1,1)$ model. In line with other empirical studies employing GARCH models to estimate the volatility of interest rates, we find that $\beta_1 + \gamma_1 = 1.007$ (in equation 5), implying that the unconditional variance of the process is unbounded.[15] However, the results of the Wald test indicate that the null hypothesis of β_1 and γ_1 summing up to unity cannot be rejected. This implies that the process of the conditional variance of the interest rate follows an integrated GARCH (IGARCH) process.[16] In this respect, we estimate an $AR(3) - IGARCH(1,1)$ model (see Table 1, Panel A).

[INSERT TABLE 1]

However, as discussed above, this strong persistence in the volatility of the estimated GARCH model is an indication of a regime-switching mechanism in the generating process of the interest rate.[17] In this mode, we estimate a two-regime model with each regime being an AR(2) process. The estimates of this model, reported in Table 1 (Panel B), reveal that the two regimes display significantly different characteristics.[18] The first regime is a "low-mean, high-variance" one and the second is a "high-mean, low-volatility" one, as suggested by their respective unconditional means of 3.28% and 5.55% along with the related figures for the unconditional variance. Moreover, the probabilities P and Q of the

[14]Throughout this paper, we use the Schwarz Information Criterion (SIC) to select the lag-length of the alternative models.

[15]Engle *et al.* (1990) report $\beta_1 + \gamma_1 = 1.0096$ for a portfolio of US securities and Kees *et al.* (1997) report $\beta_1 + \gamma_1 = 1.10$ for the US one-month T-bills.

[16]Neither the results for the unrestricted $AR(3) - GARCH\,(1,1)$ model nor the aforementioned misspecification tests are reported for brevity and are available from the authors upon request.

[17]Moreover, recursive and rolling estimates of the MR model reveal that its parameters are not stable over time. This set of results is available upon request from the authors.

[18]The estimates suggest that the process is (globally) second-order stationary (see Francq and Zakoian, 2001, for the exact stationarity conditions).

transition matrix which approach or even exceed 0.9 indicate that both regimes are fairly persistent. Further insights can be gained by the visual examination of Figure 1, which shows the estimated probability of the process being in the first regime. Apparently, the process spends more time in the "high-mean" regime than in the "low-mean" regime (62% compared to 38%). Consequently, the expected duration of the "low-mean" and the "high-mean" regime is 7.5 and 12 years, respectively. The nature of these regimes, along with their respective durations suggests that they may signal expectations over future booms and recessions. Since the actual pattern of the regimes in calendar time do not correspond to official dates of peaks and troughs in US business cycles, the regimes do not reflect business cycles per se.[19] Indeed, recessions or expansions could not be captured by our model since models of real long-term interest rates tend to signal real long-run economic growth.[20] Our interpretation is reinforced by the fact that the low-mean regime has high volatility: in the low-mean regime low growth is anticipated, uncertainty over the future increases and so volatility goes up.[21] In the finance literature, high volatility is commonly associated with high nominal interest rates.

[INSERT FIGURE 1]

However, a regime-switching model that accommodates abrupt changes in the behavior of interest rates may not be able to capture the relatively gradual evolution of economic fundamentals. Such a change in the evolution of the economy and interest rates as well might be captured better with a state space model. We specifically model the interest rate process as an $AR(1)$ process with an $AR(1)$ coefficient. This parametrization allows both the degree of mean reversion and the variance of the process to change over time. The parameter estimates for this model, presented in Table 1 (Panel C), show that the

[19] As defined by National Bureau of Economic Research.

[20] Business cycles are generally modelled via nominal short term interest rates and denote mainly changes in the monetary policy stance.

[21] One mechanism for this is as follows: expected future shifts in output growth (shifts in the IS curve) affect future short rates and hence the current long-term rate. The capital asset pricing model of Lucas (1978) also predicts a positive relation between the level of real interest rates and expected future consumption growth (over the holding period of the bond) and consequently future output growth (see also De Lint and Stolin, 2003). The "low-mean, high-volatility" regime can then be explained as follows: since real long-term interest rates signal expectations over output growth in the long-run, their low level signals an expected slowdown and overall increased uncertainty.

autoregressive coefficient process is strongly persistent.[22] The evolution of the estimated coefficient, $\widehat{\alpha}_t$ over time, illustrated in Figure 2, suggests that at the end of our sample, interest rates are quickly mean reverting ($\widehat{\alpha}_t$ equals 0.47). Finally, the constant in our model suggests a minimum for the real interest rate, rather than a mean value, which is estimated at 1.67%.

[INSERT FIGURE 2]

3.3 Certainty-equivalent Discount Rates and Discount Factors

We follow N&P and simulate 100,000 possible future discount rate paths for each estimated model starting in 2000 and extending 400 years into the future. The simulations, which are described briefly in the Appendix, are based on the estimates presented in Table 1.[23] The initial value of the real interest rate is set at 4%, which as N&P argue reflects the best comparison with a constant rate. Table 2 reports the simulated expected discount factors, as defined in equation (1), for the various models.

[INSERT TABLE 2]

As expected, the models produce considerably different discount factors and the differences between them are evident even from the first 60 years. For example, for a 60-year horizon the SS model produces substantially higher valuations than the rest of the models (the difference is over 100 % in some cases). Overall, the higher valuations come from either the SS or the RW model. The present value of $1 delivered after 100 years is $0.05 and $0.08 with respect to RW and SS, while the corresponding figure for the remaining models is about $0.02. The higher value at the end of the forecasting horizon is retained by the RW model followed by the AR-IGARCH and the SS models.

Naturally, the differences among the models with respect to the discount factor projections are reflected in the projected schedule of the CERs. Figure 3 displays the CERs as calculated by employing equation (2). All the models accommodate declining interest rates mainly stemming from the persistence and uncertainty built in them. They differ, however, at the path they follow and the terminal values they attain. For example, SS and RW produce the lower rates for the first 100 years, reaching a CER of around 2%

[22] Our estimates satisfy the stability condition derived by Weiss (1985) .

[23] The reader is referred to N&P for the estimates of the RW and the MR models (Table 1, page 63).

(half the initial value). During the same period, the MR and the AR-IGARCH models follow similar paths yielding a reduction of just 50 basis points. In the case of RS, the CER increases slightly due to some overshooting during the first 40 years. Except for this overshooting, the RS model regains its quick declining path for the rest of the period reaching a rate of 0.7% after 400 years. The highest terminal rate is produced by the SS model, which projects a rate of 1.6%, followed by MR at 1.4%.

[INSERT FIGURE 3]

3.4 Model Selection

So far, typical misspecification testing has shown that a constant coefficient model may not be able to fully capture the dynamics of the US interest rates over the period examined.[24] Along this line of reasoning, we suggested two time-varying coefficient models (RS and SS), one accommodating abrupt changes and the other allowing for a gradual change over time in the generating mechanism of the interest rates. Given the different schedules of discount rates that result, model selection is likely to be very important for the outcome of CBA.

We now perform both an in-sample and an out-of-sample forecast exercise to select between the two models. For comparison purposes, all the estimated models are included in the analysis. Our criterion by which we judge the forecasting performance of the estimated models is the commonly-used Mean Square Forecast Error (MSFE).[25] Our results for both the experiments are presented in Table 3.[26]

Specifically, the in-sample exercise is performed for the second half of our sample (1900-1999) and as a result 100 forecasts are generated. In this respect, the forecast value of the interest rate (in logs) is estimated for all the aforementioned models and then the mean of the squared deviations from the actual values is calculated. The out-of-sample exercise is quite different. More in detail, we employ the annual forward rates implied by the

[24]By typical misspecification testing, we refer to tests for serial correlation (e.g. the Breusch-Godfrey LM test), for heteroskedasticity (e.g. the ARCH LM test), for the validity of parameter restrictions (the Wald test) and for the stability of parameters (the recursive and rolling estimation of models).

[25]We have employed a number of kernels in the calculation of the MSFE and yet this did not affect the ranking of the models. The results are not presented for brevity.

[26]The MSFE is defined as the average squared deviations between actual and predicted values of a model. The values in Table 3 are interpreted in a similar way to the Schwarz-Bayes information criteria in the sense that the model with the lowest average forecast error (MSFE) is chosen.

term structure of the inflation-indexed US government bonds for a 30-year horizon and calculate the respective deviations from our simulated forward rates.[27,28] As previously we calculate the average squared deviations and rank the models inversely with respect to their MSFEs. Interestingly, both experiments (in-sample and out-of-sample) rank the SS model first followed by the RW and MR models. More importantly, the RS model performs poorly in terms of forecasting both in-sample and out-of-sample.[29]

[INSERT TABLE 3]

The forecasting accuracy of the SS model combined with its ability to capture the dynamics in the generating mechanism of the US interest rate does not leave much room for questioning its use in the present context. In the following section we illustrate the policy implications of each model in an evaluation of the damages from carbon emissions.

4. Policy Implications of Model Selection

The foregoing has established the importance of model selection in determining a schedule of DDRs for use in CBA. In this section we highlight the policy implications of DDRs and the impact of model misspecification by considering the same case study as N&P, that is, climate change and the value of carbon mitigation.[30] Specifically, we establish the present value of the removal of 1 ton of carbon from the atmosphere, and hence the present value of the benefits of the avoidance of climate change damages for each of the specified models. The respective figures are reported in Table 4.

[INSERT TABLE 4]

Interestingly, the higher valuations are attained by the SS model followed by the RW model. In detail, the values produced by SS and RW are 150% and 80% higher than the constant discounting rate values, respectively. On the other hand, the RS model

[27] Evaluating the out-of-sample forecasting performance of the models under consideration for the long run is impossible due to limitation of data, as forward rates exist for a maximum period of 30 years.

[28] Some disparity between our forecast rates, which are smoothed rates, and the rates on inflation-indexed bonds, is possible due to the transformations of the original dataset (i.e. taking a 3-year moving average of the long-term real interest rate). However, we did not account for it, since this exercise is just a benchmark for the sub-period of the first 30 years.

[29] Elsewhere, the estimated posterior odds ratio has been used in model selection in this context. With equal probabilities for RW and MR models as priors, the revised odds were 60:40 in favour of the former (see N&P). Through this lens we need not accept any particular model with certainty, although the candidates still need to be selected. We are grateful to an anonymous referee for this insight.

[30] See N&P for the assumptions concerning the modelling of carbon emissions damages.

provides roughly equivalent values of carbon to those generated by conventional constant discounting (there is only a 9% difference). Based on the MR's forecasts, the present value of the removal of 1 ton of carbon emissions from the atmosphere increases by only 12% compared to the constant rate discounting approach.

It is important to recognise the role played by the time profile of benefits in this calculation. With its lower CER in the long term it is easy to envisage a profile of benefits for which the less preferred RS model would yield a higher evaluation. Nevertheless, given these different valuations, it is crucial from a policy perspective to make a clear judgment as to which model is most appropriate.

In short, in the US context, the selection of econometric models on the basis of forecasting performance and the preferred schedule of discount rates makes climate change prevention a more desirable investment.

5. Conclusions

In response to the need to appraise projects over very long time horizons, a number of theoretical discussions have arisen concerning the appropriateness of discount rates that fall with the time horizon considered (Declining Discount Rates, DDRs). In the theoretical studies of Weitzman's (Weitzman 1998), it is persistence and uncertainty that drive the DDRs, features that are captured by Newell and Pizer (2003) (N&P) via an econometric forecasting approach.

In this paper, we extend N&P's approach on the influence of discount rate uncertainty on the expected present value of benefits in the far future. Taking their approach, we also assume that the past is informative about the future and therefore characterizing the past as accurately as possible can assist us in forecasting the future and determining the path of CERs. Given that econometric models contain different assumptions concerning the probability distribution of the object of interest, model selection is crucial in operationalising a theory of DDRs that depends upon uncertainty. After specifying a variety of models, we compared the restrictions they impose on the interest rate mechanism and evaluated their forecasting performance both in-sample and out-of-sample as a means of selecting among them. Using US interest rate data, we have shown that a state space model that allows for changes over time in the data generating process is more appropriate for the case at hand. The importance of model selection is easily seen in the starkly different paths for the certainty equivalent rate that each model implies. This conclusion is made more concrete in our case study which shows that utilisation of the preferred state space model to estimate the discount factors results in an increase of 150% in the present value of carbon emissions reduction compared to a constant rate discounting approach. Clearly this could have important implications for climate change policy.

References

Ait-Sahalia Y. 1996. Testing continuous-time models of the spot interest rate. *Review of Financial Studies* **9**: 385-426.

Baumol WJ. 1968. On the social rate of discount. *American Economic Review* **57**: 788-802.

Black F, Derman E, Toy W. 1990. A one-factor model of interest rates and its application to Treasury bond options. *Financial Analysts Journal* **46**: 33-39.

Black F, Karasinski P. 1991. Bond and option pricing when short rates are lognormal. *Financial Analysts Journal* **47**: 52-59.

Chan KC, Karolyi GA, Longstaff FA, Sanders AB. 1992. An empirical comparison of alternative models of the short-term interest rate. *Journal of Finance* **47**: 1209–1227.

Cox JC, Ingersoll JE, Ross SA. 1985. A theory of the term structure of interest rates. *Econometrica* **53**: 385-407.

De Lint C, Stolin D. 2003. The predictive power of the yield curve: A theoretical assessment. *Journal of Monetary Economics* **50:** 1603-1622.

Dybvig PH, Ingersoll JE, Ross SA. 1996. Long forward and zero-coupon rates can never fall. *Journal of Business* **69**: 1-24.

Engle RF, Lilien DM, Robins RP. 1987. Estimating time varying risk premia in the term structure: The ARCH-M model. *Econometrica* **55**: 391-407.

Engle RF, Ng VK, Rothchild M. 1990. Asset pricing with a factor ARCH covariance structure: Empirical estimates for Treasury bills. *Journal of Econometrics* **45**: 213-237.

Fan J, Jiang J, Zhang C, Zhou Z. 2003. Time-dependent diffusion models for term structure dynamics and the stock price volatility. *Statistica Sinica* **13**: 965-992.

Francq C, Zakoian JM. 2001. Stationarity of multivariate Markov-switching ARMA models. *Journal of Econometrics* **10**: 339–364.

Gollier C. 2002a. Time horizon and the discount rate. *Journal of Economic Theory* **107**: 463-473.

Gollier C. 2002b. Discounting an uncertain future. *Journal of Public Economics* **85**: 149-166.

Gollier C. 2004. The consumption-based determinants of the term structure of discount rates. *Mimeo, University of Toulouse.*

Gray FS. 1996. Modelling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics* **42**: 27-62.

Hamilton JD. 1988. Rational expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control* **12**: 385-423.

Ho TSY, Lee SB. 1986. Term structure movements and pricing interest rate contingent claims. *Journal of Finance* **41**: 1011-1029.

Hull J, White A. 1990. Pricing interest rate derivative securities. *Review of Financial Studies* **3**: 573-592.

Kees GK, Nissen FGJA, Schotman PC, Wolff CCP. 1997. The dynamics of short-term interest rate volatility reconsidered. *European Finance Review* **1**: 105–130.

Lamourex G, Lastrapes W. 1990. Persistence in variance, structural change, and the GARCH model. *Journal of Business and Economic Statistics* **23**: 225–234.

Lucas RE. 1978. Asset prices in an exchange economy. *Econometrica* **46**: 1259-1282.

Naik V, Lee MH. 1997. Yield curve dynamics with discrete shifts in economic regimes. Faculty of Commerce, *University of British Columbia, Working Paper.*

Nelson CR, Piger J, Zivot E. 2001. Markov regime switching and unit-root tests. *Journal of Business and Economic Statistics* **19**: 404-415.

Newell R, Pizer W. 2003. Discounting the benefits of climate change mitigation: How much do uncertain rates increase valuations? *Journal of Environmental Economics and Management* **46 (1)**: 52-71.

Pearce D, Groom B, Hepburn C, Koundouri P. 2003. Valuing the future: Recent advances in social discounting. *World Economics* **4**: 121-141.

Perron P. 1990. Testing for a unit root in a time series with a changing mean. *Journal of Business and Economic Statistics* **8**: 153-162.

Pigou A. 1932. *The Economics of Welfare*, 4th edition, Mac Millan: London.

Portney P, Weyant J, (eds). 1999. *Discounting and Intergenerational Equity.* Washington DC: Resources for the Future.

Sozou PD. 1998. On hyperbolic discounting and uncertain hazard rates. *Proceedings of the Royal Society of London Series B-Biological Sciences* **265**: 2015-2020.

Tsay R. 1987. Conditional heteroskedasticity time series models. *Journal of the American Statistical Association* ***82:*** 590-604.

Vasicek O. 1977. An equilibrium characterisation of the term structure. *Journal of Financial Economics* **5**: 177-188.

Weiss A. 1985. The stability of the AR(1) process with an AR(1) coefficient. *Journal of Time Series Analysis* **6**: 181-186.

Weitzman M. 1998. Why the far distant future should be discounted at its lowest possible rate. *Journal of Environmental Economics and Management* **36**: 201-208.

Weitzman M. 2001. Gamma discounting. *American Economic Review* **91**: 261-271.

Weitzman M. 2004. Discounting a distant future whose technology is unknown. *Mimeo, Harvard University.*

Appendix: Simulation Methodology

Mean Reverting Model: We employ a multivariate normal distribution to draw random values for the coefficients of (3) taking into account the estimated variance-covariance matrix of the coefficients. Another draw from a normal distribution is employed for the estimated variance. Given this set of random parameters, we generate a future path of the interest rate. We repeat the same procedure to generate 100.000 random paths of the interest rates.

Random Walk Model: As previous.

AR(3)-IGARCH (1,1): The simulation methodology is similar to the MR model. However, in this case we use the multivariate normal distribution to obtain random draws for both the conditional mean and conditional variance parameters.

Regime Switching: The RS model offers the most computationally intensive simulation and is conducted as follows. First, we generate random values for the probabilities P and Q from a $Beta(k, j)$ distribution. The values of the parameters k and j of the Beta distribution are properly chosen in order to correspond to a Beta distribution with mean and standard deviation equal to the ones estimated. Specifically, in the case of P we set k and j equal to 28.8 and 4.42, respectively. The corresponding values for Q are 55.17 and 5, respectively. Using the random values of P and Q, we calculate the probability of being in each regime for each of the future 400 years, namely P_t and Q_t. A univariate normal distribution is used to get random draws for σ_1^2 and σ_2^2 separately according to the estimates presented in Table 1 (Panel B). Similarly to our previous simulations, the random values for the coefficient estimates, n_1, n_2, $a_{1,}^1$ $a_{2,}^1$ a_1^2 and a_2^2 are drawn from a multivariate normal distribution. Then, we simulate the future interest rate path 100.000 times on the grounds of the probabilities P_t and Q_t and the random draws of the coefficients.

State Space: The simulation design for the SS model is straightforward as we randomly draw the coefficient values from univariate normal distributions according to the estimated values (Table 1, Panel C). We then simulate the future path of interest rates in a similar way to the other models.

Table 1: Estimation Results

Panel A: AR(3)-IGARCH(1,1) model			
Coefficient	Estimate	Std. Error	t-stat.
n	1.330	0.104	12.811
a_1	1.951	0.085	23.033
a_2	-1.322	0.156	-8.472
a_3	0.355	0.080	4.441
c	0.000	0.000	3.236
β_1	0.442	0.092	4.805
Panel B: Regime Switching model			
Coefficient	Estimate	Std. Error	t-stat.
n_1	1.189	0.128	9.327
a_1^1	1.589	0.078	20.36
a_2^1	-0.660	0.086	-7.630
n_2	1.714	0.238	7.206
a_1^2	1.787	0.050	35.55
a_2^2	-0.800	0.049	-16.395
σ_1^2	0.004	0.001	5.651
σ_2^2	0.000	0.000	6.070
P	0.867	0.058	14.934
Q	0.917	0.035	25.976
Panel C: State Space model			
Coefficient	Estimate	Std. Error	t-stat.
n	0.510	0.082	6.185
n_1	0.990	0.002	494.9
$ln(\sigma_e^2)$	-9.158	1.324	-6.917
$ln(\sigma_u^2)$	-6.730	0.144	-46.63

Table 2. Certainty Equivalent Discount Factors

Model Year	4% Constant	Mean Reverting	Random Walk	AR IGARCH	Regime Switching	State Space
1	0.96154	0.96154	0.96154	0.96154	0.96154	0.96154
20	0.45639	0.45906	0.46177	0.45876	0.45390	0.56424
40	0.20829	0.21661	0.22917	0.21250	0.19576	0.33136
60	0.09506	0.10471	0.12480	0.10062	0.08458	0.20296
80	0.04338	0.05150	0.07777	0.04894	0.03700	0.12889
100	0.01980	0.02567	0.05082	0.02455	0.01647	0.08408
150	0.00279	0.00476	0.02333	0.00529	0.00238	0.03132
200	0.00039	0.00095	0.01830	0.00178	0.00041	0.01255
250	0.00006	0.00022	0.01119	0.00104	0.00010	0.00526
300	0.00001	0.00006	0.00890	0.00086	0.00003	0.00227
350	0.00000	0.00002	0.00715	0.00080	0.00002	0.00100
400	0.00000	0.00001	0.00669	0.00078	0.00001	0.00044

Table 3. Mean Square Forecast Errors

Model	Mean Reverting	Random Walk	AR IGARCH	Regime Switching	State Space
Out-of-Sample	2.058	2.171	2.102	2.323	1.832
In-Sample	0.056	0.050	0.061	0.101	0.001

Table 4. Value of Carbon Damages (in $1989 values)

Model	Carbon Values ($/tc)	Relative to Constant Rate	Relative to Mean Reverting	Relative to Random Walk
Regime-Switching	5.22	-9.0%	-18.8%	-49.4%
Constant (4.0%)	5.74	—	-10.7%	-44.4%
AR-IGARCH	6.37	11.0%	-0.9%	-38.3%
Mean Reverting	6.43	12.0%	—	-37.7%
Random Walk	10.32	79.8%	60.5%	—
State Space	14.44	151.6%	124.6%	39.9%

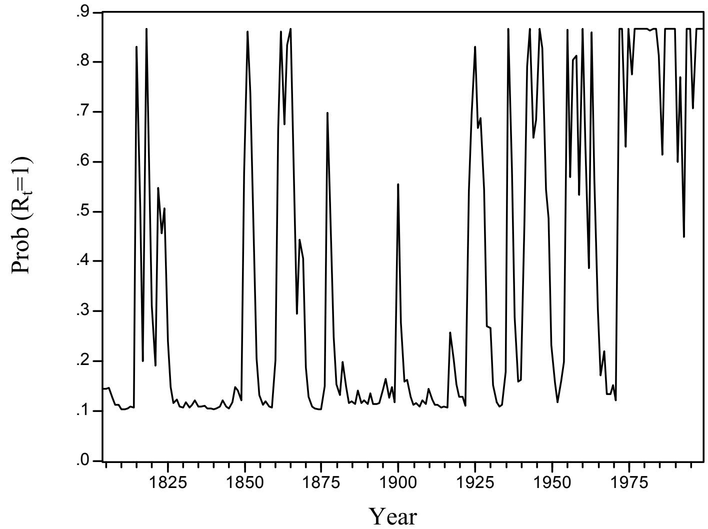

Figure 1: Smoothed Probability of Regime 1

Figure 2: Evolution of the Autoregressive Coefficient in the State Space Model

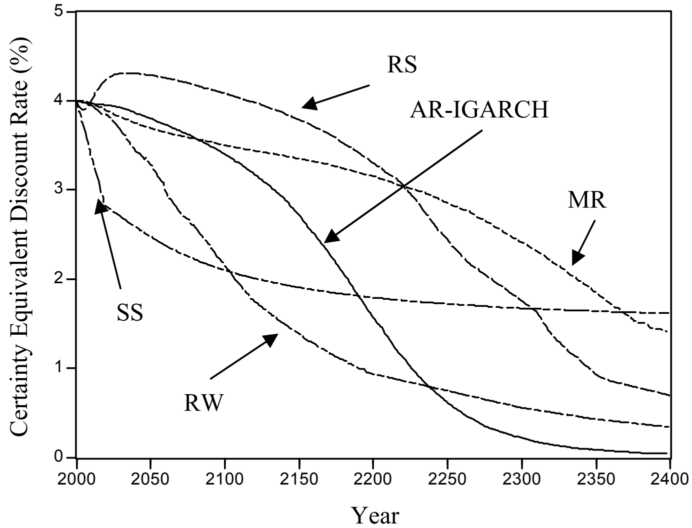

Figure 3: Forecasts of Certainty Equivalent Discount Rates