

Vilém Novák · Irina Perfilieva (Eds.)

Discovering the World with Fuzzy Logic

With 36 Figures
and 13 Tables

Physica-Verlag

A Springer-Verlag Company

Table of Contents

Preface	V
---------------	---

Part I. Fuzzy Logic Approaches to Human-Consistent Systems

Introduction to Part I	3
Toward a Logic of Perceptions Based on Fuzzy Logic	4
<i>Lotfi A. Zadeh</i>	
1 Introduction	4
2 The Basics of the Logic of Perceptions	9
3 Reasoning with Perceptions — Generalized Constraint Propagation	17
4 Concluding Remarks	25
Uncertainty-Based Information: A Critical Review	29
<i>George J. Klir</i>	
1 Uncertainty and Information	29
2 Uncertainty Formalization	30
3 Uncertainty Measurement	37
4 Uncertainty Utilization	44
5 Conclusions	50

Part II. Structure of Truth Values

Introduction to Part II	57
Triangular Norms — Basic Properties and Representation Theorems	63
<i>Erich Peter Klement, Radko Mesiar, Endre Pap</i>	
1 Triangular Norms and Conorms	63
2 Properties of t-Norms	67
3 Ordinal Sums	71
4 Representation of Continuous t-Norms	74
5 Concluding Remarks	80
Semantics for Fuzzy Logic Supporting Truth Functionality	82
<i>Jeff Paris</i>	
1 Introduction	82
2 Voting Semantics	85
3 Rerandomising Semantics	88
4 Similarity Semantics	88

5	Risk, or Dialogue, Semantics	92
6	Acceptability Semantics	93
7	Measurement-Theoretic Justifications	96
8	Approximation Semantics	96
9	Implication	99
10	Conclusion	102

States on Perfect MV-Algebras 105

Antonio Di Nola, George Georgescu, Ioana Leuştean

1	Introduction	105
2	Preliminaries on MV-Algebras	106
3	Pseudo-Subalgebras of an MV-Algebra	109
4	Completeness Properties	112
5	States on MV-Algebras	114
6	Local States on Perfect MV-Algebras	116
7	Extension Theorem	117

A Glance at Implication and T-Conditional Functions 126

Enric Trillas, Adolfo R. de Soto, Susana Cubillo

1	Introduction	126
2	Preliminaries	127
3	Implication Functions	130
4	T-Conditional Generating Functions	136
5	μ -Relative T-Conditionality: Fuzzy Logical T-States	139
6	Monotonic Fuzzy Relations, T-Logical States and Implication Functions	143

Part III. Metamathematical Aspects of Fuzzy Logic

Introduction to Part III 151

On the Metamathematics of Fuzzy Logic 155

Petr Hájek

1	Introduction	155
2	Fuzzy Propositional Logics	157
3	Predicate Calculi	167
4	Comments and Conclusions	172

Fuzzy Metalogic for Crisp Logics 175

Giangiacomo Gerla

1	Introduction	175
2	Abstract Fuzzy Deduction Systems	176
3	Fuzzy Deduction Systems in Hilbert Style	178
4	An Extension Principle for Necessities and Fuzzy Herbrand Models	180
5	Abstract Similarity Logic	182

6	Similarity Logic Associated with a Crisp Logic	184
7	Similarity-Based Prolog	186
8	Sequent Calculus and Graded Consequence Relations	187

Degrees of Truth and Degrees of Validity

Stephan Lehmke

1	Introduction	192
2	Logics of Incomplete Truth	198
3	Logics of Incomplete Knowledge	204
4	A Comparison of Logics of Incomplete Truth and Logics of Incomplete Knowledge ¹	212
5	Logics of Incomplete Truth and Incomplete Knowledge	222
6	Conclusions and Further Research	232

Part IV. Formal Systems of Fuzzy Logic

Introduction to Part IV

Fuzzy Propositional Logic

Esko Turunen

1	Introduction and Preliminaries	243
2	Semantics of Fuzzy Propositional Logic	245
3	Axiom System for Fuzzy Propositional Logic	254
4	Completeness of Fuzzy Propositional Logic	262

Some Consequences of Herbrand and McNaughton Theorems

in Fuzzy Logic

Vilém Novák, Irina Perfilieva

1	Introduction	271
2	Preliminaries	272
3	Some Properties of Fuzzy Theories	274
4	Herbrand Theorem in Fuzzy Logic	277
5	McNaughton Theorem in Fuzzy Logic	281
6	Fuzzy Relations Represented by Formulas	288
7	Consistency of Fuzzy Theories	290
8	Conclusion	295

Part V. Fuzzy Quantifiers

Introduction to Part V

Many	302
<i>Petr Hájek</i>	
1 Introduction	302
2 Fuzzy Quantifiers	302
3 Many	304
On T-Quantifiers and S-Quantifiers	310
<i>Radko Mesiar, Helmut Thiele</i>	
1 Introduction	310
2 Cardinal Quantifiers	311
3 T_* and S_* Quantifiers	314
4 T^* and S^* Quantifiers	318

Part VI. Reasoning in Impreciseness

Introduction to Part VI	329
Reasoning on Imprecisely Defined Functions	331
<i>Daniele Mundici</i>	
1 Prologue	331
2 The Classical ($y \mapsto x$) Problem	332
3 The General ($y \mapsto x$) Problem	336
4 Descriptions with Constraints	349
5 Epilogue: Revisiting the Prologue	355
6 Appendix: MV-Algebras, Partitions, Logic	357
Similarity-Based Reasoning	367
<i>Francesc Esteva, Pere Garcia, Lluís Godó</i>	
1 Introduction	367
2 Similarity and Fuzzy Sets	369
3 Upper Approximation Mappings	370
4 Graded Entailment Relations Induced by Upper Approximation Mappings	372
5 A Simple Application to Interpolative Reasoning	380
6 Multi-modal Systems Based on Approximation Relations	382
7 Conclusions	393

Part VII. Relational Systems in Fuzzy Logic

Introduction to Part VII	399
---------------------------------------	-----

Generalized Solvability Behaviour for Systems of Fuzzy Equations	401
<i>Siegfried Gottwald</i>	
1 Introduction	401
2 Solvability of Fuzzy Relational Equations	403
3 Solvability of Fuzzy Arithmetical Equations	406
4 Solvability of Systems of Fuzzy Equations	408
5 Solvability Degrees and Approximate Solutions	412
6 Towards more Difficult Equations	425
7 Concluding Remarks	428
Fuzzy Points, Fuzzy Relations and Fuzzy Functions	431
<i>Frank Klawonn</i>	
1 Introduction	431
2 GL-Monoids and Fuzzy Equality	432
3 Fuzzy Points	435
4 Fuzzy Functions	442
5 Conclusions	452
<hr/>	
Part VIII. Extensions and Departures from Fuzzy Logic	
<hr/>	
Introduction to Part VIII	457
Fuzzy Galois Connections and Fuzzy Concept Lattices: From Binary Relations to Conceptual Structures	462
<i>Radim Bělohlávek</i>	
1 Imperative: Concepts and Conceptual Structures	462
2 Preliminaries	463
3 Fuzzy Galois Connections	466
4 Fuzzy Concept Lattices	469
5 Selected Topics	475
6 Retrospective and Perspective	483
7 Illustrative Examples	484
A Unified Compilation Style Labelled Deductive System for Modal, Substructural and Fuzzy Logics	495
<i>Krysia Broda, Alessandra Russo, Dov Gabbay</i>	
1 Introduction	495
2 The CLDS Approach	497
3 The E_{CLDS} System	508
4 The L_{CLDS} System	520
5 The F_{CLDS} System	535
6 Conclusions	542
Author's Index	549

Index 551