

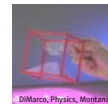
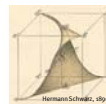
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with help from Eitan Grinspun, Mathieu Desbrun and the rest of the DDG crew  
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## A BIT OF HISTORY

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Geometry is the key!

- studied for centuries
  - Cartan, Poincaré, Lie, Hodge, de Rham, Gauss, Noether,...
- mostly differential geometry
  - differential and integral calculus



Study of invariants and symmetries

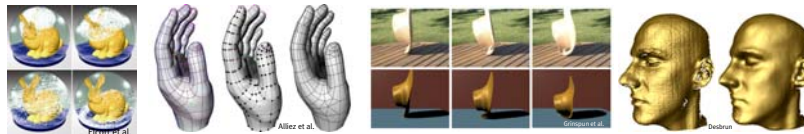
# DIFFERENTIAL GEOMETRY

Why do we care?

- geometry of surfaces
- mothertongue of physical theories

$$E = \int_S \alpha + \beta(H - H_0)^2 + \gamma K dA$$

- computation: simulation/processing



CS177 (2012) - DISCRETE DIFFERENTIAL GEOMETRY

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# GETTING STARTED

How to apply DiffGeo ideas?

- surfaces as collections of samples
  - and topology (connectivity)
- apply continuous ideas
  - BUT: setting is discrete
- what is the right way?
  - discrete vs. discretized



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# DISCRETIZED

## Build smooth manifold structure

- collection of charts
  - mutually compatible on their overlaps
- form an atlas
- realize as smooth functions
  - differentiate away...



# DISCRETIZATION OF EQS

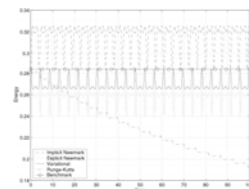
## Observation

- be careful  $M\ddot{q} + C\dot{q} + Kq = f$

$$m \frac{q_{i-1} - 2q_i + q_{i+1}}{\Delta t^2} + c \frac{q_{i+1} - q_i}{\Delta t} + kq_{i+1} = f_i$$

- **structure** may not be preserved

$$L(q, \dot{q}) = \frac{1}{2} \dot{q} M \dot{q} - V(q)$$

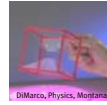
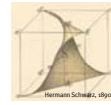


# DISCRETE GEOMETRY

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## Basic tool

- differential geometry
  - metric, curvature, etc.



## Discrete realizations

- “meshes”
  - computational geom.
  - graph theory

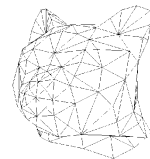


# DISCRETE DIFF. GEOMETRY

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## Building from the ground up

- discrete geometry is the given
  - meshes: triangles, tets
  - more general: cell complex
- how to do calculus?
  - preserve crucial properties



$$\int_a^b f'(x)dx = f(b) - f(a)$$

# DISCR. DIFF. GEOMETRY

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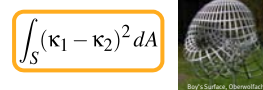
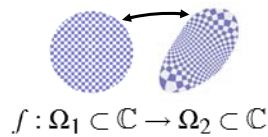
## Building from the top down

- high level theorems

- Riemann mapping

- Willmore energy

- Hamilton's prcple.



$$0 = \delta_q \int_0^T L(q(t), \dot{q}(t)) dt$$

## IS IT MUCH BETTER?

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### Magic happens

- much more robust

### Is there a recipe?

- yes, but several ones...

- discrete var. principle

### Discrete from the start

