Discrete Mathematics and Proof in the High School Introduction

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This special issue and the next one of ZDM are on discrete mathematics and proof in the high school and undergraduate curriculum. The issue contains contributions to a workshop which took place in Oberwolfach in 2003. The workshop was organized by Gila Hanna (Toronto), Kristina Reiss (Augsburg), Jürgen Richter-Gebert, (München) and Jacobus H. van Lint (Eindhoven).

In the last 25 years, discrete mathematics has rapidly changed in its methodologies, in the way in which it is viewed by mathematicians, and in particular in the range of its applications. This is partly due to an extended use of computer technology in the past decades. Moreover, discrete mathematics has proved to be an important tool for research and development for example in biology, chemistry, and computer science. Discrete mathematics has its research roots in different parts of mathematics, most prominently in group theory, geometry, number theory, algebraic combinatorics, graph theory, and cryptography. Accordingly, it has been influenced by a variety of mathematical results, methods, and representations. Their combination and integration in a profound theory is essential for research in discrete mathematics. In particular, the use of computer technology has not only influenced mathematical results but mathematical methods as well. It is necessary to discuss these methods, as they represent an important development in mathematical argumentation.

A serious discussion of the principles of proof, for example, has recently arisen as a result of work in discrete mathematics. The validity of computer-based proofs first became a significant issue among mathematicians when Appel and Haken (1978) presented their computer-based proof of the four-color problem. The critics focussed mainly on the fact that (a) the length of the proof meant that its detailed steps could not be published and (b) all computer programs are subject to error (Kleiner, 1991). The fundamental question is how reliable computer-based proofs are (Lam, 1990). Proof, however, is essential to mathematical research. Indeed, mathematics is sometimes referred to as the proving science. From this point of view, the recent results in discrete mathematics which have been obtained by computer assisted methods may be regarded as a challenge for the mathematical community. Have these new developments changed the role of proof in mathematics? Have they yielded a new understanding of the nature of proof, and what is that understanding? These are questions which concern not only mathematicians but mathematics educators as well. In schools and universities, mathematics educators as well as mathematicians have an influence on the view of mathematics which their students come to hold (Pehkonen & Törner, 1999). Accordingly, it is important that a discussion between these groups on the role of proofs is initiated.

Because discrete mathematics is regarded an important branch of mathematics, its integration into the school mathematics curriculum has often been discussed by mathematicians and mathematics educators (e.g., Kenney & Hirsch 1991; Rosenstein, Franzblau & Roberts, 1997; Anderson et al., 2001; Thies, 2001). It has been thought that discrete structures might foster a deeper understanding of mathematics as they are sometimes easier to understand than continuous structures (Weigand, 2001). Moreover, problems in discrete mathematics can play a specific role in the learning and teaching of proofs (Grenier & Payan, 1999). The underlying heuristic, conceptual, and applicational aspects of discrete mathematics can more easily be translated into elementary problems (which are not necessarily easy to solve, however). The value of discrete structures for mathematics instruction has been emphasized by the National Council of Teachers of Mathematics (NCTM) and they are featured as an important topic in the standards of the NCTM which serve as recommendations for mathematics instruction in US schools at all grade levels. As an active branch of contemporary mathematics that is widely used in business and industry, it is clear that discrete mathematics should be an integral part of the school mathematics curriculum, and in fact some topics of discrete mathematics naturally occur in other areas of the mathematics curriculum (NCTM, 2000). Combinatorics, iteration and recursion, and vertex-edge graphs, for example, are mentioned explicitly as topics to be taught in all grades from kindergarten to high school. Thus, the NCTM recommendations mention some specific elements of a discrete mathematics curriculum but there is still a substantial amount of research needed to identify in a more systematic way those topics in discrete mathematics which are most relevant for mathematics instruction. Some first steps have been taken by both, mathematicians and mathematics educators, but these attempts seem sporadic and isolated (Kenney & Hirsch 1991; Rosenstein, Franzblau & Roberts, 1997; DIMACS 2001).

Mathematics education in secondary schools aims at providing a certain level of understanding of mathematics and mathematical methods. Most of the students will not continue their studies of mathematics at college. But many students will have to apply their knowledge of mathematics in such fields as sciences, business administration, psychology, and engineering. Discrete mathematics is widely held to be a topic that has much to offer in the promotion of mathematical understanding. For this reason alone, the discussion of integrating discrete mathematics into the school curriculum should be continued and intensified. In addition, however, it is clear that in many countries there has been a dramatic decline in the interest shown by high school students in the field of mathematics. A greater emphasis on discrete mathematics and its many obvious applications has the potential to change that attitude.

Mathematicians and mathematics educators should work together, first of all, to identify those elements which are important for high school and undergraduate students' understanding of mathematics. Moreover, they should exchange their ideas concerning the teaching and learning processes with respect to discrete mathematics. As Schoenfeld (2000) has pointed out, mathematics and mathematics education differ significantly in their views on this topic. In particular, mathematicians and mathematics educators should work together in order to identify important new elements for the curriculum and to develop models for teaching and learning these topics.

The workshop focussed on discrete mathematics and its teaching and sought to address the following questions:

- What have been the most important aspects of the development of discrete mathematics in recent years?
- How can these aspects be used to make discrete mathematics more prominent in the school curriculum?
- How can discrete mathematics be taught most effectively?
- How can discrete mathematics contribute to an understanding of mathematical structures?
- How has the role of proof changed over the past few years and how should these changes be reflected in mathematics instruction?
- Can discrete mathematics make a special contribution to the learning of argumentation and proof in the mathematics classroom?

The articles included in this special issue give an overview on the results of the workshop.

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