Discrete model for foundation-soil-foundation interaction

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Abstract

A new discrete model for the study of dynamic interaction phenomena between adjacent, rigid foundations on a homogeneous, linear elastic half-space is presented. Each dynamic degree of freedom of the foundations consists of a mass connected to a rigid support through frequency independent springs and dashpots. The interaction between the foundations is achieved by imposing spring and damping couplings developed in this work. The time lagging effects of coupled dynamic input due to wave propagation is also considered through a proposed modified vector approach.

1   Introduction

At the inception of SSI analysis, the engineering profession had very little theoretical and observational data for developing reliable methods of predicting dynamic foundation responses. It is for this reason that early researchers borrowed from the principles of structural dynamics and utilized discrete, lumped parameter models to account for the interaction between structure and soil. The introduction of the finite element method (FEM), and boundary element method (BEM), triggered the development of a number of sophisticated techniques for the solution of dynamic SSI problems. With the advancements in solution accuracy, the complexity of analyzing SSI problems has increased and, in addition, the above methods are usually computer intensive and costly. They are also not conducive to parametric studies, and due to their complexity, cannot provide an intuitive insight to the nature of the problem being solved. In recognition of these disadvantages, efforts to establish approximate methods through discrete, lumped parameter models have again been undertaken.

The interaction of nearby masses on the surface of the soil was first introduced by Whitman [1] as an important problem requiring study. As a result, Warburton *et al.* [2] performed a study of two masses with identical circular bases attached to an elastic half space. The FEM has also been used to study the dynamic interaction between foundations, e.g., Roesset and Gonzalez [3]. Whittaker and Christiano [4] developed Green's function methods to analyze the coupled effects between foundation systems, while Wong and Luco [5] used a boundary integral equation approach to calculate the dynamic response of a group of massless foundations. Several authors have used the BEM to determine the through-the-soil coupling of two square foundations , e.g., Mohammadi [6], and Huang [7]. Simple mass-spring-mass systems have been developed by Wong and Trifunac [8] and Luco and Contesse [9] to study dynamic structure-soil-structure interaction (SSSI) problems. All of the above studies indicate the importance of the presence of closely spaced foundations on individual responses.

This work addresses the interaction of adjacent, massive, square foundations perfectly bonded to a homogeneous, isotropic, linear elastic half-space. The foundations undergo vertical, horizontal, rocking and torsional motions due to the application of an external forcing function to one of the foundations. The direct interaction between each foundation and the half space is achieved using frequency independent lumped parameter mass-spring-dashpot models. The interaction between foundations is imposed by coupling springs and dampers.

2   Formulation

The geometry, coordinates and terminology for the foundation-soil-foundation system studied in this work are as shown in Figure 1. This system can be viewed as the assemblage of two independent foundation-soil (FS) systems with soil coupling. The problem is solved by first defining the FS system in terms of lumped parameter models consisting of lumped masses, springs and dampers, where each mode of vibration is considered as an independent degree-of-freedom. The second step is to define the coupling between the two foundations' degrees-of-freedom. Finally, the time lagging due to wave propagation is enforced, and responses to loading of the foundations can be computed.

2.1   Foundation-Soil Interaction Models

The relationship between the amplitude of an applied load on a rigid, massive foundation attached to an elastic medium, and the corresponding displacement of the foundation, can be determined for each foundation DOF using a simple SDOF lumped parameter model. The mass m representing the mass of the foundation plus a virtual soil mass fixed to the foundation is attached to a rigid

support by a massless spring having a stiffness k and a massless dashpot having a damping coefficient c.
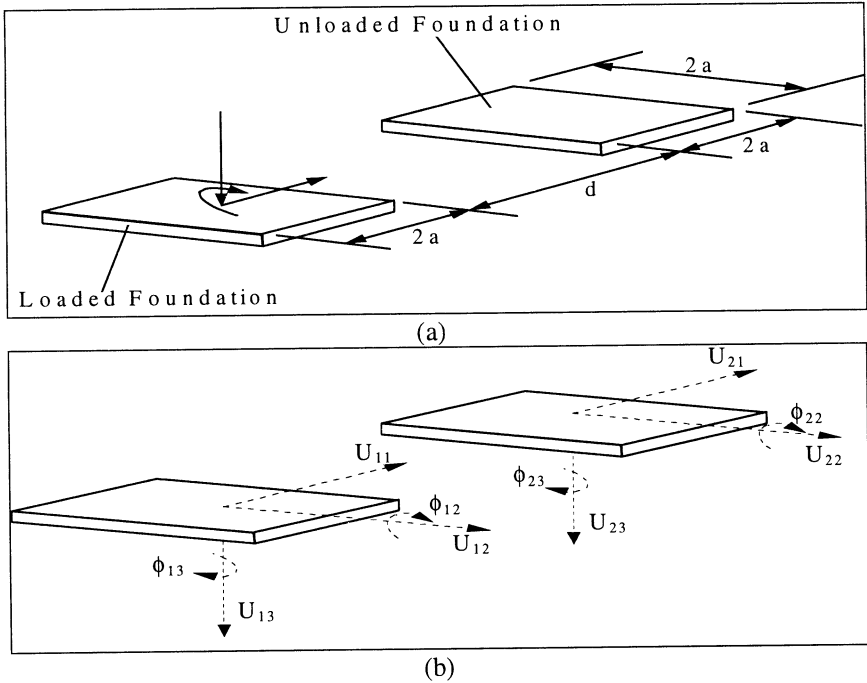

(a)

(b)

Figure 1 Foundation-soil-foundation system (a) geometry and nomenclature, (b) coordinate system

Various definitions of these three parameters can be found in the literature. In this work, the definition of the virtual soil mass found in Gazetas [10] is used, while for the static stiffness and damping coefficients the expressions reported by Wolf [11] have been adopted. Following the above definitions, the equation of motion for each degree-of-freedom of a single foundation subjected to external forces is

$$m\,\ddot{y}(t) + c\,\dot{y}(t) + k\,y(t) = F(t) \tag{1}$$

where $\ddot{y}(t)$, $\dot{y}(t)$ and $y(t)$ are acceleration, velocity and displacement time histories, respectively, for the degree-of-freedom being analyzed, and $F(t)$ is the externally applied force or moment.

2.2   Foundation-Soil-Foundation Coupling Models

The coupling between the two foundations presented in Figure 1 is achieved via stiffness and damping coefficients which represent the off-diagonal terms in matrix notation. These coefficients, also frequency independent, are functions of the dimensionless ratio d/a.   For the derivation of these coupling coefficients, the time domain responses of both the loaded and unloaded

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foundations under impulsive loads are computed by assuming arbitrary distance ratio functions and, subsequently, they are compared to existing solutions obtained by any available rigorous method. The distance ratio functions are then iteratively modified until the loaded and unloaded foundation responses are in agreement with the existing data. The stiffness and damping coupling coefficients for each degree of freedom are defined as:

$$K_{c3} = \Gamma_3 \times \frac{Ga}{(1-\nu)} \qquad C_{c3} = \Psi_3 \times \frac{Ga^2}{V_S(1-\nu)}$$

$$K_{c\phi 2} = \Gamma_{\phi 2} \times \frac{Ga^3}{(1-\nu)} \qquad C_{c\phi 2} = \Psi_{\phi 2} \times \frac{Ga^4}{V_S(1-\nu)}$$

$$K_{c1} = \Gamma_1 \times \frac{Ga}{(2-\nu)} \qquad C_{c1} = \Psi_1 \times \frac{Ga^2}{V_S(2-\nu)} \qquad (2)$$

$$K_{c\phi 3} = \Gamma_3 \times Ga^3 \qquad C_{c\phi 3} = \Psi_{\phi 3} \times \frac{Ga^4}{V_S}$$

where Γ and Ψ are the distance ratio functions derived from the iterative process described above. Some of the boundary element solutions, i.e. for a mass ratio M=10, for two identical, square, massive foundations attached to an elastic half-space produced by Huang [7] are used in this work to benchmark the proposed coupling model. Thus, the stiffness and damping coupling coefficients are computed for specific distance ratios at each degree of freedom, and the following distance ratio functions were developed using appropriate regression equations:

$$\Gamma_3 = 1.614 \times 10^{-0.16257\left(d/a\right)} \qquad \Psi_3 = 8.504$$

$$\Gamma_{\phi 2} = -\left(0.04234 - 0.2396 \times \log_{10}\left(d/a\right)\right) \qquad \Psi_{\phi 2} = 7.3823 - 6.775 \times \log_{10}\left(d/a\right)$$

$$\Gamma_1 = 3.7561 \times 10^{-0.18995\left(d/a\right)} \qquad \Psi_1 = 13.2875 \qquad (3)$$

$$\Gamma_{\phi 3} = 0.05931 \qquad \Psi_{\phi 3} = 4.4429 - 2.9125 \times \log_{10}\left(d/a\right)$$

The coupling coefficients for each degree of freedom are shown graphically in Figure 2 where an extrapolation of the coupling functions beyond the distance ratios used by Huang [7] is also shown.

For the independent motion of each foundation, i.e., with no influences from adjacent foundations, the mass, stiffness, and damping expressions given in Refs [10,11] are used, while the vibration coupling of adjacent foundations is obtained via Eqs (2). Thus, the static stiffness and damping of the two foundations are stored in one set of matrices, while the coupling stiffness and damping are maintained as off-diagonal terms in separate matrices. This is essential to the FSFI problem investigated in this work since, as it turns out, the presence of the nearby foundations, modeled by the coupling terms, does not influence (except for rocking motion in the very near field, d/a<0.5) the
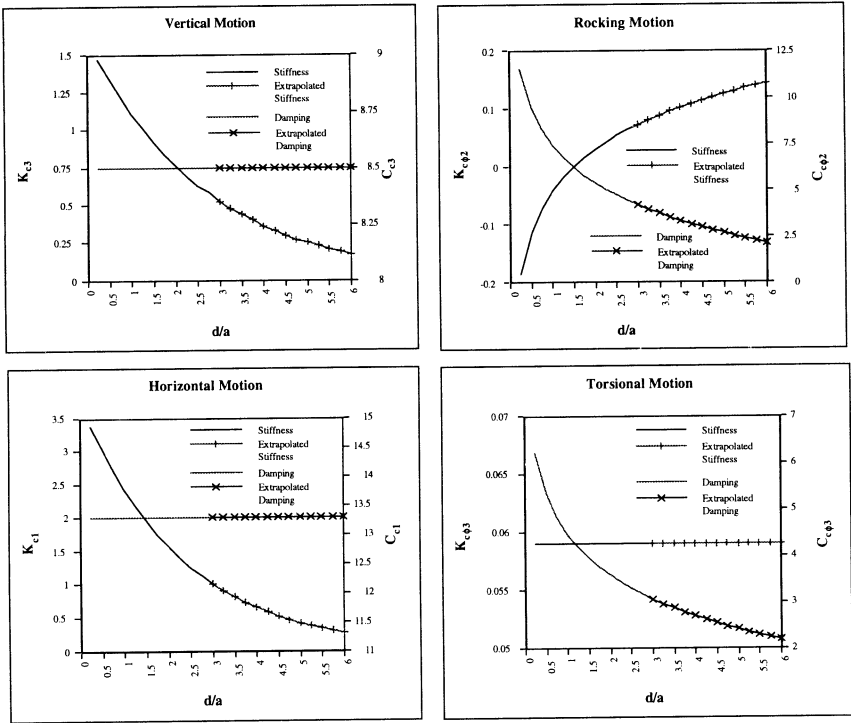
Figure 2   Coupling stiffness and damping coefficients for vertical, rocking, horizontal, and torsional motions.

static stiffness and damping of single foundations.   In addition,   if constant stiffness and damping matrices with off-diagonal terms is used, it would introduce direct coupling between degrees of freedom, and time lagging between motions of adjacent foundations would not be achieved.   However, this work evidences the fact that ignoring this time lag significantly alters the motion characteristics (frequency and amplitude) in the time domain. The time lagging between the coupled input is determined via modified displacement, velocity and acceleration vectors.

3   Numerical Results

The dynamic response of two massive, rigid, square surface foundations on a homogeneous elastic half-space is studied first.   The soil medium constants   are: mass   density $\rho=10.368$ lb-sec^2/ft^4,   shear   modulus

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$G=9.71175 \times 10^8$ lb/ft^2, and Poisson ratio $\nu=1/3$. The side of the foundation is $2a=5$' and its mass, m, is defined by the dimensionless mass ratio $M = \dfrac{m}{\rho \, r_0^3}$.

For this example, the external load is defined as a rectangular impulse with a magnitude of 100 lb or lb.ft, depending on the loaded degree-of-freedom, and a duration of 0.00001821 sec. The results of this work indicate that choosing a time step equal to 1/20 of the duration of the square impulse is sufficiently small to produce accurate results. Further reduction in the time step was found to only improve the accuracy of the results by less than 0.5%. The accuracy of the solution is established by comparison with the results reported by Huang [7], which have been benchmarked using several other sources.
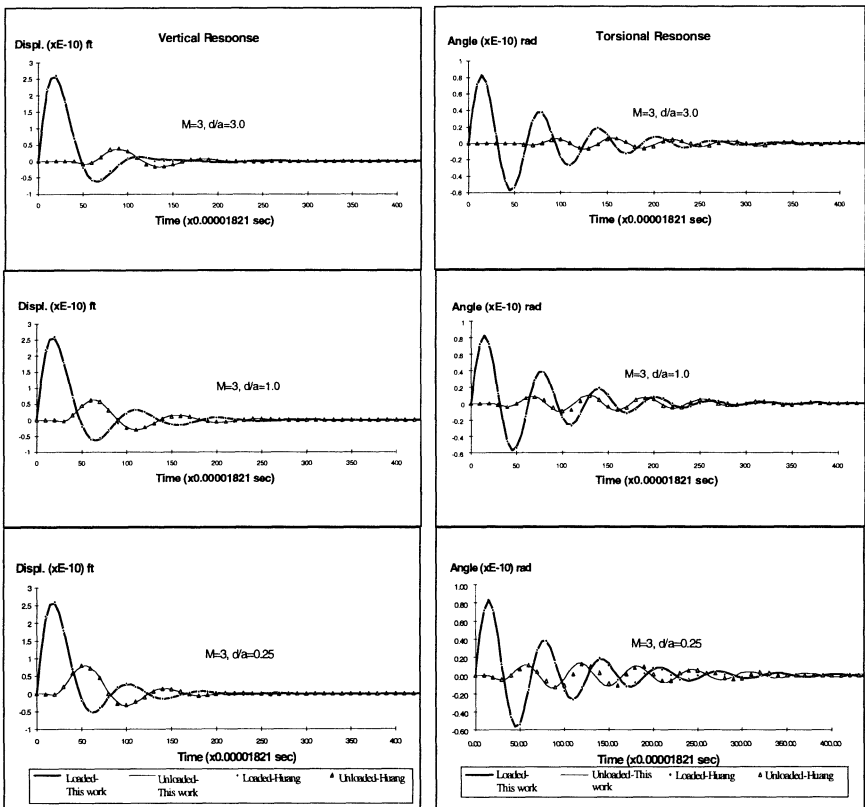

Figure 3 Vertical and torsional response versus time
due to a rectangular impulse load.

Due to limited space, only the results for vertical and torsional motions, for mass ratio M=3 and several values of the distance d/a, are shown in Figure 3. The results indicate that the model captures the influence of the coupling on both the loaded and unloaded foundations, even in the near-field zone (d/a<0.5).

As previously discussed, minimal data, is available for confirming the universal applicability of the methodology proposed in this work. In order to provide a level of confidence in the methodology, a finite element solution of a foundation-soil-foundation interaction problem involving three foundations is solved with the computer program SASSI [12] for comparison. The three foundations are coupled in groups of two foundations at a time with the coupling coefficients derived above. The geometry and mechanical constants of the system are shown in Figure 5 along with the results obtained using the proposed model and those of the FEM analysis. The close agreement of the two sets of results is apparent.
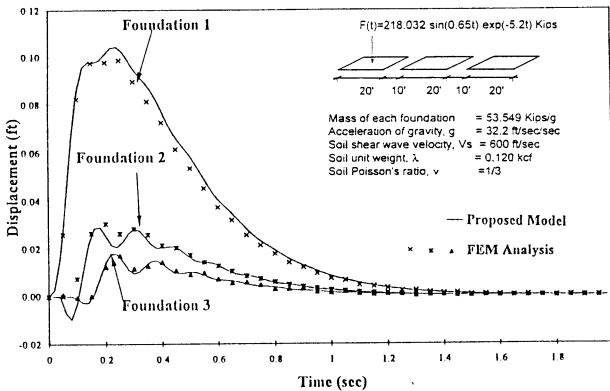

Figure 4 Dynamic response versus time of a set of three foundations due to an external impulsive load.

The interested reader can find a more complete set of parametric studies on the use of the proposed model in Mulliken [13].

Conclusions

A combination of classical foundation-soil interaction models and coupling models are used to compute the foundation-soil-foundation interaction of two adjacent, rigid foundations subjected to externally applied loads. This approach takes advantage of available data for the foundation-soil dynamic interaction phenomenon and includes familiar terminology and methodologies for incorporation of the dynamic through-the-soil coupling of adjacent foundations. Through a number of comparison studies performed in this work the proposed methodology is proven accurate and efficient. The

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advantages of this approach are: i) the lumped parameter model can be easily inc orporated in most general purpose structural dynamics computer programs used by most practicing structural engineers, ii) the simple nature of the model easily enables parametric studies and physical insight to FSFI problems, and iii) the inclusion of the time lagging between coupled motions eliminates over-prediction of the coupling influences and provides more accurate time domain responses.

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