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Discrete Optimization of Trusses by Simulated Annealing

At present, several methods are available for optimization of structures. The application of such methods to real structural problems, however, has not been as intense as the development of the techniques themselves. One of the main reasons is that the great majority of the methods, based on mathematical programming, consider a continuous search space. This paper presents an application of the Simulated Annealing method to the optimization of trusses considering the cross sections of the members as discrete variables. The constraints imposed to the analysis were the allowable stresses and the displacements on nodes. Some examples are presented in order to demonstrate the effectiveness of the method when compared with other methods found in literature.
***Keywords**: Optimization, trusses, heuristics, discrete, simulated annealing*

Introduction

The analysis and design of structures usually involve both highly complex procedures and a great number of variables. As a consequence, the solution has to be found iteratively while initial values are set to the variables based mainly on designer's sensitivity and experience. Also, the number of analysis steps is remarkably increased if optimum values are to be found among all possible alternatives. Describing, however, the physical behaviour of the structure through mathematical functions, extreme function values of such functions can be searched with the aim of optimization techniques.

In a general way, a minimization problem can be expressed as:

minimize $f(x_i)$ $\quad i = 1, n,$ (1)

subjected to $g_j(x_i) \le 0$ $\quad j = 1, m,$ (2)

$h_k(x_i) = 0$ $\quad k = 1, l,$ (3)

$x_i^l \le x_i \le x_i^u,$ (4)

where *f* designates the objective function and $X = (x_1, x_2, \ldots x_n)^T$ the design variables vector. The remaining functions are constraint functions corresponding to inequality constraints (*g*), equality constraints (*h*) and side constraints with lower and upper limits indicated by the superscripts *l* and *u*, respectively. These functions, which can be solved analytically or numerically, may be linear or non-linear and contain the design variables in an explicit or a non-explicit form.

The great development of structural optimization took place in the early 60's, when programming techniques were used in the minimization of structures weight. From then on, a great diversity of general techniques has been developed and adapted to structural optimization. However, the application of such approaches to real structural problems has not been verified in the same proportion. In their work, based on more than 500 examples taken from articles and books, Cohn et al. (1994) emphasize the big worry with mathematical aspects of optimization, although the small number of examples reported, mostly of purely academic interest.

The reasons usually claimed for such limited application of optimization techniques to real structural problems are related to the inherent complexity of the generated model, described by non-linear functions and generating a non-convex space of solutions (multiple points of optimum), problems for which the resolution by traditional techniques of mathematical programming has shown little efficiency. To the resolution of these kind of problem the heuristic methods have been performing an important role, since they involve solely function values in the analysis, with unimodality or even continuity in its derivatives unnecessary. On the other hand, a great number of function evaluations are needed. This apparent shortcoming, however, has been questioned by some researchers like Powell (1998), who argues that, instead of performing additional calculations to numerically determinate the gradient in mathematical programming, the effort should be used to exploit more intensely the space of solutions.

Among the main heuristic methods, it can be verified the growing application of Simulated Annealing method, which is an approach of global optimization developed in analogy upon the mechanical procedure of annealing of metals. Although only a few applications of Simulated Annealing in structural optimization have been reported, the method can be easily implemented into computers, dealing with few control parameters regarding the Genetic Algorithms.

The present work shows an application of Simulated Annealing to the optimization of trusses, considering the cross sections as discrete variables and generating an optimal solution which is feasible not only from a mathematical but also from a practical point of view. Even if due to economic and aesthetic limitations only a reduced number of distinct cross sections are considered, the number of possible combinations is high enough. In addition, if the structure has some degree of indeterminacy, the stresses might be redistributed by varying the relative stiffness of the elements as the cross sections of a single element is changed.

Nomenclature

A = cross-sectional area, m^2
E = elastic modulus, kN/m^2
F = penalized objective function, kN
f = objective function, kN
g = inequality contraint
h = equality constraint
K = Boltzmann's constant
L = length of element, m
P = penalization
p = probability function
T = temperature of the body
u = nodal dispacement, m
W = weight, kN

Greek Symbols

ΔE = energy variation
σ = stress, MPa

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ρ = specific weight, kN/m^3

Subscripts

i = relative to the number of design variables
j = relative to the number of inequality constraints
k = relative to the number of equality constraints
l = relative to lower limit
u = relative to upper limit

Simulated Annealing Method

The optimization approaches normally employed are based on descending strategies. In these, from an initial solution, a new function value is generated and compared to the initial one. If a reduction in the function value is verified, the new value is adopted as the current solution and the procedure is repeated until any better value is achieved. The final result obtained, depending on the characteristics of the functions involved, might be the best solution in the vicinity of the initial solution, but not necessarily the best in the whole search space. A strategy usually used to improve the solution consists in analyzing the problem from several initial solutions. Following an alternative strategy, the Simulated Annealing method tries to avoid convergence to a local minimum by accepting also, according to a specific criterion, solutions that increase the value of the function. The method is recognized as a procedure for optimization problems of anticipated difficult solution, and is developed in analogy upon the process of annealing of a solid, when a state of minimum energy is searched. The denomination annealing is given to the process of heating of a solid to its point of fusion followed by a slow cooling. In this process, slow cooling is essential to maintain a thermal equilibrium in which the atoms are able to reorganize themselves in a structure with minimum energy. If the solid is cooled abruptly, the atoms will form an irregular and weak structure, with high energy as a consequence of the internal effort spent. In computational terms, the annealing can be seen as a stochastic procedure for the determination of the atomic organization with minimum energy. At high temperatures, the atoms move freely being able to achieve, with high probability, positions that increase the energy of the system. When temperature is reduced, the atoms move gradually to form a regular structure, reducing the probability of energy increase.

According to Metropolis et al. (1953), the probability of a change in the energy of the system is given by

$$p(\Delta E) = \exp\left(\frac{-\Delta E.K}{T}\right) , \qquad (5)$$

where T is the body temperature and K, the Boltzmann constant.

The simulation of annealing as an optimization technique was originally presented by Kirkpatrick et al. (1983), with the objective function corresponding to the energy of the solid. Similarly to the annealing in thermodynamics, the process initiates with a high value of T, from which a new solution is generated. This new solution is automatically accepted if it generates a decrease in the function value. Otherwise, if the new value is greater than the previous one, the acceptance is given according to a probabilistic criterion, being the acceptance function:

$$p = \exp\left(\frac{-\Delta f}{T}\right) . \qquad (6)$$

The new solution is accepted if p is larger than a randomly generated number between zero and one. Once T is high, the majority of the solutions are accepted, being T reduced gradually at each trial series in the vicinity of the current solution.

Examples

The effectiveness of Simulated Annealing is illustrated in this work by two classical examples, taken from Rajeev and Krishnamoorthy (1992).

In these problems, the objective functions are set to minimize the weight of the structure, W, composed by n elements:

$$f(x) = W = \sum_{i=1}^{n} \rho A_i L_i , \qquad (7)$$

where A_i and L_i are, respectively, the cross-sectional area and the length of the *ith* member. In addition, both problems are subjected to the following constraints, expressed in the normalized form as:

$$\frac{\sigma_i}{\sigma_a} - 1 \leq 0 \quad \text{and} \quad \frac{u_i}{u_a} - 1 \leq 0 , \qquad (8)$$

where σ_i is the stress in member i, σ_a is the allowable stress for all members, u_i is the displacement of each node (vertical and horizontal) and u_a the allowable displacement for all nodes.

Aiming at the computational implementation, the constraints were considered by using a dynamic penalty technique, known as annealing penalty (Michalewicz and Schoennauer, 1996). Similarly to the optimization technique, a penalty factor has an initial value relatively low, which is gradually increased as the temperature reduces. The penalized function *F(x)* can be written as:

$$F(x) = f(x)+P(x). \qquad (9)$$

Being

$$P(x) = \sum \left(\frac{1}{2T}\right) g(x)^2 , \qquad (10)$$

where *P(X)* is the function that represents the assembly of the penalized constraints. In this way, even if the problem starts from unfeasible solutions, small violations of constraints are initially allowed.

Example 1: 10-bar truss

The geometry of the 10-bar truss structure employed in the first example is shown in Fig.1.

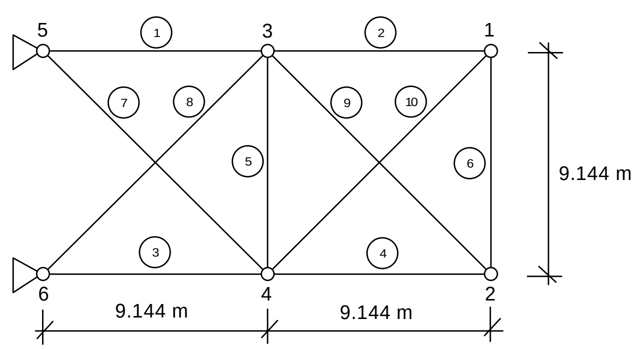

Figure 1. Example 1: 10-bar plane truss geometry.

The assumed data are: modulus of elasticity E = 10^4 ksi (6.89 x 10^4 MPa), weight density of the material ρ = 0.10 lb/in^3 (2,770 kg/m^3) and vertical downward loads of 100 kips (445.374 kN) at joints 2 and 4. The allowable stresses are limited to ± 25 ksi (175.25 MPa) and displacement to 2 in (50.8 mm). The 42 available sections assumed for the design variables, given in the list S, were taken from the American Institute of Steel Construction Manual (Rajeev and Krishnamoorthy, 1992). S = { 1.62, 1.80 , 1.99 , 2.13 , 2.38 , 2.62 , 2.63 , 2.88 , 2.93 , 3.09 , 3.13 , 3.38 , 3.47 , 3.55 , 3.63 , 3.84 , 3.87 , 3.88 , 4.18 , 4.22 , 4.49 , 4.59 , 4.80 , 4.97 , 5.12 , 5.74 , 7.22 , 7.97 , 11.5 , 13.5 , 13.9 , 14.2 , 15.5 , 16.0 , 16.9 , 18.8 , 19.9 , 22.0 , 22.9 , 26.5 , 30.0 , 33.5 } (in^2). Since each bar can take any of the available sections, the number of combinations is 10^{42} (aprox. 1.7x10^{16}).

Due to its heuristic nature, the starting point is less important in Simulated Annealing than in methods based on mathematical programming. The results obtained in the present work along with data for comparison reported by several authors are given in Table 1. The methods used to obtain each result are given immediately below Table 1. By comparing the results, it can be readily observed a significant reduction in the final weight of the structure obtained in the present work relatively to the results obtained by the other heuristic methods (Genetic Algorithms). Also, the results obtained by applying Simulated Annealing are the best among those others found in literature.

Example 2: 25-bar truss

Fig. 2 shows the geometry of the Example 2, a 25-bar space truss. The members are divided into eight groups, according to Table 2. The assumed data are: E = 10^4 ksi (6.89 x 10^4 MPa) and ρ = 0.10 lb/in^3 (2,770 kg/m^3), with the applied loads listed in Table 3.

Again, the objective function of the problem is set to minimize the weight of the structure *W*. The stress is constrained to ± 40 ksi (257.6 MPa) and only the displacements at joints 1 and 2 are restricted, both to less than ± 0.35 in (8.89 mm) in the x and y directions.

In this example, the available discrete values of the variables (in^2) are S = { 0.1 , 0.2 , 0.3 ,..., 2.6 , 2.8 , 3.0 , 3.2 , 3.4 }, a set of 30 values (Rajeev and Krishnamoorthy, 1992).

Table 4 presents the results obtained for the example, comparing the values obtained in present work with others found in literature. Again, the best result is given by Simulated Annealing.

Table 1. 10-bar plane truss results summary.

Method	W (lb)	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1	5491.71	33.50	1.62	22.90	15.50	1.62	1.62	7.97	22.00	22.00	1.62
2	5613.84	33.50	1.62	22.00	15.50	1.62	1.62	14.20	19.90	19.90	2.62
3	5491.71	33.50	1.62	22.90	15.50	1.62	1.62	7.97	22.00	22.00	1.62
4	5586.59	30.00	1.62	22.90	13.50	1.62	1.62	13.90	22.00	22.00	1.62
5	5490.74	33.50	1.62	22.90	14.20	1.62	1.62	7.97	22.90	22.00	1.62

1-Improved Penalty Function Method (Cai and Thiereu, 1993)
2-Genetic Algorithms (Rajeev and Krishnamoorthy, 1992)
3-Difference Quotient Method (Thong and Liu, 2001)
4-Genetic Algorithms (Coello, 1994)
5-Simulated Annealing (present work)

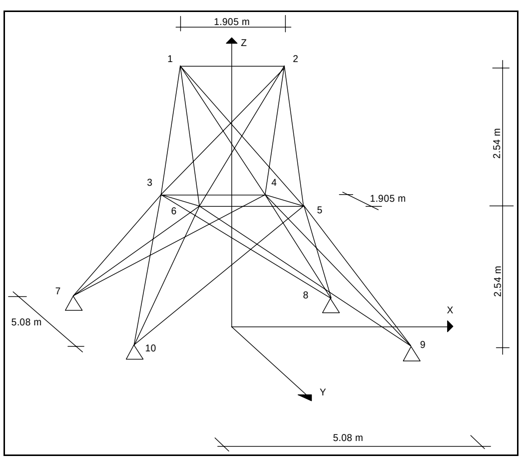

Figure 2. Example 2: 25-bar space truss geometry.

Table 2. Group membership.

Group Number	Members
1	1-2
2	1-4, 2-3, 1-5, 2-6
3	2-5, 2-4, 1-3, 1-6
4	3-6, 4-5
5	3-4, 5-6
6	3-10, 6-7, 4-9, 5-8
7	3-8, 4-7, 6-9, 5-10
8	3-7, 4-8, 5-9, 6-10

Table 3. Loading conditions.

Node Number	Fx (N)	Fy (N)	Fz (N)
1	4,453.74	-44,537.40	-44,537.40
2	0.0	-44,537.40	-44,537.40
3	2,226.87	0.0	0.0
6	2,672.24	0.0	0.0

Table 4. 25-bar space truss results summary.

Method	W (lb)	A1	A2	A3	A4	A5	A6	A7	A8
1	487.41	0.1	0.1	3.4	0.1	2.0	1.0	0.7	3.4
2	546.01	0.1	1.8	2.3	0.2	0.1	0.8	1.8	3.0
3	562.93	0.1	1.8	2.6	0.1	0.1	0.8	2.1	2.6
4	485.05	0.1	0.5	3.4	0.1	1.9	1.0	0.4	3.4
5	539.78	1.5	0.7	3.4	0.7	0.4	0.7	1.5	3.2
6	484.33	0.1	0.4	3.4	0.1	2.2	1.0	0.4	3.4

1-Improved Penalty Function Method (Cai and Thiereu, 1993)
2-Genetic Algorithms (Rajeev and Krishnamoorthy, 1992)
3-Brach and Bound (Zhu, 1986)
4-Difference Quotient Method (Thong and Liu, 2001)
5-Genetic Algorithms (Coello, 1994)
6-Simulated Annealing (present work)

Regarding the number of function evaluations needed by Simulated Annealing, it must be emphasized that this number is very high when compared to those required by mathematical programming. Therefore, this approach is specially indicated to problems where usual techniques are not efficient. Nevertheless, it was observed that the Simulated Annealing method rapidly converged to the vicinity of the optimum solution. Fig. 3 presents the results obtained for the Example 2, where one can observe that around 14.7 percent of the total function evaluations (39,201), the error was lower than 5 percent. Hence, a less rigorous definition of the stop criterion can drive to a significant reduction in the number of calculations.

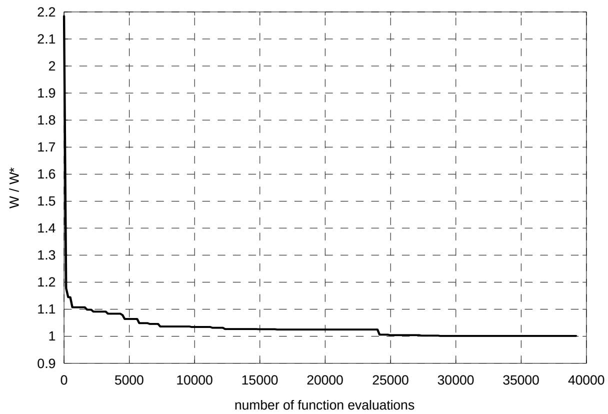

Figure 3. Convergence of 25-bar space truss.

Figure 4 presents the obtained relationship between the average number of function evaluations and the number of design variables. To construct the plot, several initial configurations (member grouping) were considered, starting from a unique group (all bars with the same section) and evolving up to 25 groups (each bar allowed to assume a different section). The results suggest that the relative efficiency of Simulated Annealing increases with the dimension of the problem, since the number of possible combinations grows exponentially.

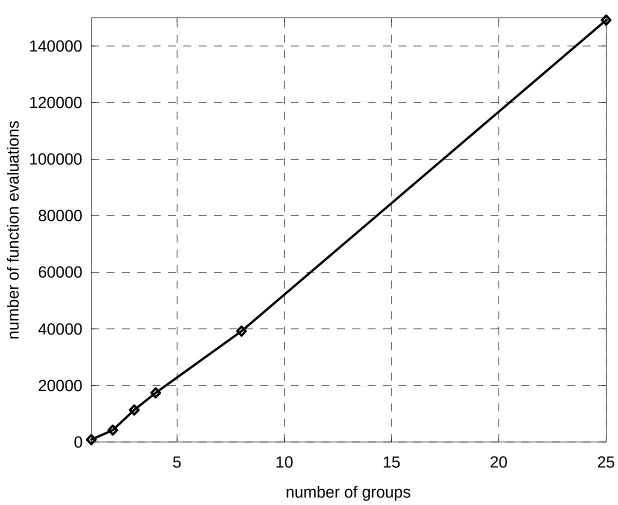

Figure 4. Relationship between function evaluations and number of variables.

Concluding Remarks

This work presented an application of the Simulated Annealing method to the determination of the minimum weight in structures analyzed by the truss model. Since the determination of the elements cross section dimensions is based mainly on the designer's former experience, optimization techniques can be a valuable tool in the phases of analysis and dimensioning, allowing the identification of the more stressed elements, as well as those that eventually could be deleted from the structure.

The heuristic methods allow an easy and efficient treatment of discrete variables, since they do not require a continuum search space. Among these methods, Simulated Annealing was found to be very adequate, especially due to the few control parameters involved.

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