

# Discrete Polarization with an Application to the Determinants of Genocides

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## Abstract

The economic literature has recognized since some time ago that inequality and polarization are two different concepts. As in the case of inequality, the measurement of polarization was initially developed in the context of a continuous dimension which defined the "closeness" of the characteristics of individuals and clusters. However, in many important dimensions, like ethnicity, there are not available measures of distance across ethnic groups. Additionally, when it comes to ethnicity individuals are mostly interested in the dichotomous perception "we versus they". In this paper we analyze the theoretical properties of a measure of polarization based on classifications (discrete polarization) instead of continuous distances across groups. We also show that, opposite to some recent empirical applications of discrete polarization, the range of suitable parameters of the original index is not correct for the measurement of polarization without distances. The second part of the paper presents an application of the index of discrete ethnic polarization to the explanation of genocides.

JEL Classification numbers: D74, D72, Z12, D63

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\*Instituto Valenciano de Investigaciones Economicas. We are grateful for comments by A. Alesina, T. Besley, P. Collier, J. Fearon, D. Ray, two anonymous referees and, specially, A. Villar. We thank the participants of seminars at the World Bank, Universitat Pompeu Fabra, Institute de la Mediterranea, Toulouse, Brown University, the European Economic Association Meetings and the Winter Meetings of the Econometric Society.

*"In the twentieth century, genocides and state mass murder have **killed more people** than have all wars".* Institute for the Study of Genocide. International Association of Genocide Scholars.

## 1 Introduction

The concept of polarization has been used frequently in political science and economics, although its precise conceptualization has turned to be evasive and difficult. For this reason many authors used it in a very imprecise, and sometimes conflicting, fashion. The contributions of Esteban and Ray (1994) and Wolfson (1994)<sup>1</sup> represent the first attempts to provide a precise definition of polarization. These recent definitions explain why the empirical measurement of different dimensions of polarization is a very recent phenomenon, opposite to the somehow related concept of inequality which has generated thousands of contributions on measurement.

The concept of polarization was initially developed in the context of a continuous dimension, in particular income, which defined the "closeness" of the characteristics of the individuals. Esteban and Ray (1994) present the properties of a precise axiomatization of a class of polarization measures based on distances in the real line<sup>2</sup>. However, in many important dimensions (like ethnicity or religion), there is no information on a continuous variable to measure distances across groups, or the "distances" have to be discretize, and there is no precise information on the latent variable used for the discretization. In this paper we introduce the theoretical properties of a class of measures of discrete polarization based on classifications instead of continuous distances. We also show that it is empirically quite different from the analogous inequality measure (fractionalization). There are many reasons that favor the use of a discrete metric to construct the index of ethnic or religious polarization<sup>3</sup>. First, there are no measures of distance across ethnic (religious) groups available and generally accepted<sup>4</sup>. The measure of the "distance" across ethnic groups is much more controversial than the identification of the list of ethnic groups. Second, the measurement of distances across

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<sup>1</sup>In fact Wolfson (1994) emphasizes the difference between polarization and inequality and develops a theoretical measure of the former that can be interpreted using the Lorenz curve.

<sup>2</sup>Empirical applications of this measure can be found in Gradin (2000) and Duclos, Esteban and Ray (2004). Anderson (2004) considers measures of polarization in terms of stochastic dominance. Keefer and Knack (2002) have argued that income based measures of polarization are similar to their corresponding Gini coefficients

<sup>3</sup>This comment also applies to many other characteristics that cannot be ordered in the real line (country of birth, immigration status, etc.).

<sup>4</sup>Fearon (2003) presents a proposal to measure "cultural fractionalization" based on the calculation of resemblance factors.

groups may generate a larger measurement error than the "belong/does not belong to" criterion. Third, if the distance across groups is measured using the strength of the sentiment of identity or political relevance then there is an important endogeneity problem. At the end we will be explaining conflict using conflict as the explanatory variable, since the sentiment of identity is high when there is conflict. Fourth, as argued by Duclos, Esteban and Ray (2004), "there are many interesting instances in which individuals are interested only in the dichotomous perception Us/They." We believe the case of ethnic (religious) groups is a important example of this situation. Finally, the "distance" across groups is most likely a function of polarization. It is reasonable to argue that in a very polarized society the distance across groups will be large, in the sense of generating a strong sentiment of identity and opposition to the other groups. If this is the case the index of discrete polarization is also capturing, in a way, distances.

Some authors<sup>5</sup> have applied the measure of Esteban and Ray (1994) to data on groups, without information on distances, assuming that the same properties of the original measure extend to the "belong-do not belong" situation. The first objective of this paper is to emphasize the fact that with discrete distances the mechanic application of the Esteban and Ray polarization index is not correct. We show that some properties of discrete polarization are different from the polarization measure in Esteban and Ray (1994)<sup>6</sup>. This is not a minor point since the empirical calculation of any the index of discrete polarization relies heavily on the choice of the parameter in the set of feasible degrees of polarization sensitivity.

The second part of the paper analyzes the effect of discrete ethnic polarization on the probability of genocides. Recently, many academic economist have turned to the study of conflict and its main determinants. In empirical applications, civil wars are the most frequently variable used to proxy for conflict. However, genocides, which is one of the most violent and bloody forms of social violence, have not receive much attention, even though they result in more deaths than civil wars. Since World War II nearly 50 genocides and political mass murder have happened and these episodes have cost the lives of at least 12 million combatants, and as many 22 million noncombatants. Those are more than all the victims of internal and international wars since 1945<sup>7</sup>. We use data on genocides because, in principle, they are more appropriate to test the relevance of (ethnic) polarization for the analysis of extreme violence.

Genocides are characterized by the extermination of members of a target group, a phenomena

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<sup>5</sup>For instance Aghion et al. (2004), Collier and Hoeffler (2004), or Alesina et al.(2003).

<sup>6</sup>From now on ER94.

<sup>7</sup>See the State Failure Task Force.

that not always happens during a civil war. Since ethnic disputes are usually very violent it is reasonable to infer that ethnicity may have an important effect on the probability of genocides. However, recent papers on the determinants of genocides, and civil wars in general, have found no evidence of the effect of ethnic fractionalization<sup>8</sup>. These findings have led some researchers to dismiss ethnicity as a potential source of extreme violence, in clear contrast with traditional theories. However, the properties of discrete polarization discussed in this paper suggest that the higher is the level of violence the more relevant is the effect of ethnic polarization. In this paper we find that ethnic heterogeneity is a statistically significant determinant of the probability of a genocide if we use an index of discrete ethnic polarization instead of ethnic fractionalization. We also find that ethnic polarization increases more the likelihood of genocides than the probability of a civil war.

A particular index of the general family of discrete polarization measures, the RQ, was used by Montalvo and Reynal-Querol (2005) in their empirical study of the causes of civil wars. The final message of that paper was strictly empirical: the RQ index is a significant explanatory variable for the incidence of civil wars. There was no theoretical justification for the use of that particular index besides a diffuse claim of analogy with the index of Esteban and Ray (1994) without distances. This new paper is basically a theoretical piece, where we present three basic contributions: first of all, we provide a precise mathematical characterization of a class of discrete polarization measures, and characterize theoretically the properties of the particular index (RQ) that we use in the AER paper. Secondly, we prove that the only polarization sensitivity compatible with reasonable properties of polarization is  $\alpha = 1$ . We consider this to be an important contribution since many authors (as we mentioned before) have wrongly understood that they can use any degree of polarization sensitivity compatible with the index of Esteban and Ray (1994),  $(0 \approx 1.6]$ , to construct indices of discrete polarization. Finally, as we argue in the theory part, discrete polarization should be more relevant the more intense is the conflict. Genocides are a perfect example since they represent the most violent form of conflict. The third original contribution of this paper is to show that the contribution of ethnic polarization to the likelihood of a conflict increases with the intensity of the conflict.

The outline of the paper is the following. Section 2 introduces the concept of discrete polarization and discusses its theoretical properties. Section 3 analyzes the empirical performance of discrete ethnic polarization in the explanation of genocides. Section 4 concludes.

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<sup>8</sup>For instance, Harff (2003) finds that none of the numerous indicators of ethnic and religious cleavages was significant. Only one variable, weakly connected with ethnicity (political elite based on an ethnic minority), was statistically significant.

## 2 A class of discrete polarization measures

Traditionally the study of the impact of ethnicity on growth or civil wars has rested on the construction of indices of fractionalization, even though most of the theories refer to "polarized societies"<sup>9</sup>. Several authors have interpreted the finding of a negative relationship between ethnic fractionalization and growth as evidence of a high probability of conflict in very heterogeneous societies. However, the empirical evidence on the direct relationship between fractionalization and civil wars is at most very weak<sup>10</sup>. We argue that the reason why ethnicity does not seem to have any impact on conflict is the use of the index of fractionalization. Reynal-Querol (2002) proposed an index of ethnic heterogeneity, RQ, that tried to capture polarization instead of fractionalization

$$RQ = 1 - \sum_{i=1}^N \left( \frac{1/2 - \pi_i}{1/2} \right)^2 \pi_i$$

The original purpose of this index was to capture how far is the distribution of the ethnic groups from the (1/2,0,0,...,0,1/2) distribution (bipolar), which represents the highest level of polarization<sup>11</sup>. There was no analysis of theoretical properties or implications. Montalvo and Reynal-Querol (2005) show that the RQ index was somehow related with the index of polarization of ER94. However, the argument based on certain analogy between the RQ index and the ER94 index is quite ad-hoc. Montalvo and Reynal-Querol (2005) do not discuss any property of the index nor its relationship with the general class of discrete polarization measures. In fact, Montalvo and Reynal-Querol (2005) is basically an empirical paper which shows that if we measure ethnic heterogeneity in terms of bipolarity then ethnicity is key to explain the probability of civil wars.

One of the objectives of our current paper is to provide a theoretical foundation to the family of discrete polarization measures given the lack of development of the theory. ER94 provide a particular conceptualization for polarization, emphasizing the difference between inequality and polarization. They argue that there are "significant problems concerning racial, religious, tribal and nationalistic conflict which clearly have more to do with the clustering of attributes than with the inequality of their distribution over the population." What do they mean by polarization? A population of individuals may be grouped according to some vector of characteristics into "clusters" such that each cluster is similar in terms of the attributes of its members, but different clusters have members with "dissimilar" attributes. Such a society is polarized even though the measurement of inequality

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<sup>9</sup>Easterly and Levine (1997).

<sup>10</sup>See for instance Collier and Hoeffler (2004).

<sup>11</sup>See also Reynal-Querol (2001).

could be low. ER94 use the following example: suppose that initially the population is uniformly distributed over the deciles of income. Suppose that we collapse the distribution in two groups of equal size in deciles 3 and 8. Polarization has increased since the "middle class" has disappeared and group identity is stronger in the second situation. However inequality, measured by the Gini index or by any other inequality measure, has decreased.<sup>12</sup>

By using three axioms, Esteban and Ray (1994) narrow down the class of allowable polarization measures to only one measure,  $P$ , with the following form

$$P = k \sum_{i=1}^N \sum_{j=1}^N \pi_i^{1+\alpha} \pi_j |y_i - y_j|$$

for some constants  $k > 0$  and  $\alpha \in (0, \alpha^*]$  where  $\alpha^* \simeq 1.6$ . When  $\alpha = 0$ <sup>13</sup> and  $k = 1$  this polarization measure is precisely the Gini coefficient. Therefore the fact that the share of each group is raised to the  $1 + \alpha$  power, which exceeds one, is what makes the polarization measure significantly different from inequality measures. The parameter  $\alpha$  can be treated as the degree of "polarization sensitivity."

In the case of ethnic diversity the identity of the groups is less controversial than the "distance" between different ethnic groups, which is much more difficult to measure than income or wealth. Then, it is reasonable to treat the "distance" across groups,  $\delta(.,.)$ , as generated by a discrete metric (1-0). If we want to measure ethnic diversity, the distance between ethnic groups may be a very difficult concept to measure. If we consider the criteria "belongs" or "does not belong" to an ethnic (religious) group, instead of the distance, then we should substitute the Euclidean metric  $\delta(y_i, y_j) = |y_i - y_j|$ , by a discrete metric

$$\begin{aligned} \delta(y_i, y_j) &= 0 && \text{if } i = j \\ &= 1 && \text{if } i \neq j \end{aligned}$$

In addition, any classification of ethnic groups implies a criterion to transform the differences of the characteristics of ethnic groups into a discrete decision rule (for instances, same family-different family). For example, following the classification of the World Christian Encyclopedia, the ethnic subgroup of the Luba, the Mongo and the Nguni belong to the Bantu ethnolinguistic group. The Akan, the Edo and the Ewe belong to the Kwa ethnolinguistic group. This implies that the "cultural distance" (defined informally by the Encyclopedia) between the subgroups of the Bantu group is

<sup>12</sup>This result does not imply that polarization and inequality have always a negative relationship.

<sup>13</sup>Strictly speaking for  $\alpha = 0$  this is not an index of polarization.

smaller than the difference between one subgroups of the Bantu family and one of the Kwa family. In terms of a discrete metric, if we use the family classification as the base for the difference across groups, this means that the subgroup of the Bantu family are inside the ball of radius  $r$  that defines the discrete metric while the subgroups in the family Kwa are outside that ball. Therefore, any classification involves implicitly a concept and a measure of “distance” that is discretized.

The class of indices of discrete polarization,  $DP$ , can be described as

$$DP(\alpha, k) = k \sum_{i=1}^N \sum_{j \neq i} \pi_i^{1+\alpha} \pi_j$$

which depends on the values of the parameters  $\alpha$  and  $k$ .<sup>14</sup>

Embedding a discrete metric into ER’s polarization measure  $P$  alters the original formulation of the index as a polarization measure. It is known that the discrete metric and the Euclidean metric are not equivalent in  $R$ . For this reason the apparently minor change of the metric implies that the discrete polarization measure does not satisfy anymore the properties of polarization for all the range of possible values of  $\alpha$ . Therefore, for each possible  $\alpha$ , we have a different shape for the DP index. The question we want to analyze is the following: What is the admissible set of values for the coefficient  $\alpha$  if the DP measure has to satisfy the basic properties of polarization?<sup>15</sup> Our objective is, therefore, to check if the basic properties of polarization are satisfied by what we call discrete polarization. In this section we also show that for  $\alpha = 1$  and  $k=4$  the DP index has the usual properties of a polarization measure bounded between 0 and 1. This particular case is the RQ index of polarization.

The definition of polarization as a concept closely related to social tensions implies several characteristics. ER94 define the conditions imposed by polarization using the interaction between changes in the euclidean distance of groups (for instance in terms of income) and their relative size. We are

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<sup>14</sup>By analogy, what we call a discrete Gini index is a discrete polarization measure with  $\alpha = 0$  and  $k = 1$  (also called index of fractionalization or FRAC). We distinguish FRAC ( $DP(0, k)$ ) from the general discrete polarization ( $DP(\alpha > 0, k)$ ) even though the earlier is a particular case of the  $DP$  family. As we argued before the fact that  $\alpha > 0$  is the basic attraction of polarization measures versus inequality indices.

<sup>15</sup>Notice that this is somehow different from the original objective of Esteban and Ray (1994). They characterize the general form of a polarization measure in a particular family using several axioms. This implies a limit for the parameter  $\alpha$  that cannot be larger than 1.6, but also a special form for any polarization measure. Our objective in this section is to check if the basic properties of polarization are satisfied by what we call discrete polarization. However we are not claiming that the DP index is the only possible form for a polarization measure based on discrete distances.

going to redefine those conditions only in terms of groups size, since we are not using distances across groups. We work with three groups, since this is the minimum number of groups that make the measure of polarization different from the index of fractionalization. An index of discrete polarization should have basically two properties. The first property (PR1) is that if we join the two smallest groups polarization should increase.<sup>16</sup> The second property (PR2) is that any new distribution formed by shifting probability mass from one group equally to the other two groups must increase polarization.<sup>17</sup> We can show formally that the only DP measure that satisfies the two properties exposed above is the one with a value of  $\alpha = 1$ . Let's restate these two properties<sup>18</sup> formally and derive the implications with respect to the values of  $\alpha$ . Define  $\pi_1 = p, \pi_2 = q$  and  $\pi_3 = r$ .

**Property 1:**

*If there are three groups of sizes,  $p, q,$  and  $r,$  and  $p > q$  and  $q \geq r,$  then if we merge the two smallest groups into a new group,  $\tilde{q},$  the new distribution is more polarized than the original one. That is,  $POL(p, q, r) < POL(p, \tilde{q})$  where  $\tilde{q} = q + r$ <sup>19</sup>.*

We define  $POL(x)$  as the proper index of polarization when  $x = (p, q, r)$ . Property 1 states that when we join the two smallest groups the index of polarization should increase.

*Theorem 1:  $DP(\alpha, k)$  satisfies property 1 if and only if  $\alpha \geq 1.$  (see proof in Appendix I)*

As we argued before we need  $\alpha \geq 1$  to have a DP measure that satisfies property 1. We can prove that this is the case for any number of groups and not only three.

**Property 1b:** Suppose that there are two groups with size  $\pi_1$  and  $\pi_2$ . Take any one group and split it into  $m \geq 2$  groups in such a way that  $\pi_1 = \tilde{\pi}_1 \geq \tilde{\pi}_i \forall_{i=2, \dots, n+1},$  where  $\tilde{\pi}$  is the new vector of population sizes, and clearly  $\sum_{i=2}^{n+1} \tilde{\pi}_i = \pi_2$ . Then polarization under  $\tilde{\pi}$  is smaller than under  $\pi$ .

*Theorem 2: The  $DP(\alpha, k)$  measure satisfies property 1b if and only if  $\alpha \geq 1.$  (see proof in appendix I)*

Another property of polarization measures is that they attain their maximum at a bipolar symmetric distribution. We can generalize the result using the following lemma.

*Lemma 1: The  $DP(\alpha, k)$  index attains its maximum at a bipolar symmetric distribution if  $\alpha \geq 1.$  (see proof in Appendix I).*

Property 2 can be stated formally in the following way.

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<sup>16</sup>This property corresponds basically with axiom 1 and 2 in Esteban and Ray (1994).

<sup>17</sup>This is the analog for discrete distances of axiom 3 in ER94.

<sup>18</sup>Notice that, in our case, we do not use the term "axiom" since we are not interested in describing and narrowing down a general class of discrete polarization measures. We only want to check if the DP measure proposed in this paper satisfies those properties.

<sup>19</sup>This property is the analog to axioms 1 and 2 in ER94.



**Property 2:** Assume that there are three groups of sizes  $p, q, p$ . Then if we shift mass from the  $q$  group equally to the other two groups, polarization increases. That is,  $POL(p, q, p) < POL(p + x, q - 2x, p + x)$ .<sup>20</sup>

*Theorem 3:* The only  $DP(\alpha, k)$  measure that satisfy property 2 for any distribution is the one such that  $\alpha = 1$  (see proof in Appendix I).

*Corollary:* The only family of DP measures that satisfies properties 1 and 2 is the one with  $\alpha = 1, DP(1, k)$ .

If we fix  $\alpha = 1$ , and choose  $k = 4$  (which makes the range of the index  $DP(1, k)$  to lie between 0 and 1) then we obtain the RQ index

$$\begin{aligned} DP(1, 4) &= 4 \sum_{i=1}^N \sum_{j \neq i} \pi_i^2 \pi_j = 4 \sum_{i=1}^n \pi_i^2 [1 - \pi_i] = \sum_{i=1}^n \pi_i [1 - (1 + 4\pi_i^2 - 4\pi_i)] = \\ &= \sum_{i=1}^n \pi_i - 4 \sum_{i=1}^n (0.5 - \pi_i)^2 \pi_i = 1 - \sum_{i=1}^N \left( \frac{0.5 - \pi_i}{0.5} \right)^2 \pi_i = RQ \end{aligned}$$

Figure 1 shows the graph of the fractionalization index (FRAC) and the RQ index of polarization as a function of the number of groups when all of them have the same size. The index of fractionalization is defined by the expression

$$FRAC = 1 - \sum_{i=1}^N \pi_i^2 = \sum_{i=1}^N \pi_i (1 - \pi_i) \quad (1)$$

where  $\pi_i$  is the proportion of people that belong to the ethnic group  $i$  and  $N$  is the number of groups. This index has a simple interpretation as the probability that two randomly selected individuals from a given country will not belong to the same ethnic group and it increases monotonically with the number of groups. By contrast, the RQ index reaches a maximum when there are two groups.<sup>21</sup>

The simplest way to look at the implications of different choices of parameters for the discrete polarization measures is to describe the shape of different surfaces using several examples. Figures 2 to 5 show the shape of the index DP as a function of  $\alpha$  in the case of three groups.<sup>22</sup> Figure 2 corresponds to the case of  $\alpha = 0$ , which is the index of fractionalization. Figure 3 represents the index of discrete polarization when  $\alpha = 1$ , which corresponds to the RQ index. Figures 4 and 5 present

<sup>20</sup>This property corresponds to axiom 3 in ER94.

<sup>21</sup>Certainly the number of groups that maximize conflict in the context of a rent seeking contest is model specific. However the original justification for polarization measures (Esteban and Ray 1994) is to produce an index that obtains a maximum for the distribution  $(1/2, 0, 0, \dots, 0, 1/2)$ .

<sup>22</sup>The parameter  $k$ , which is just a scale factor, is fixed to 4 in all the cases.

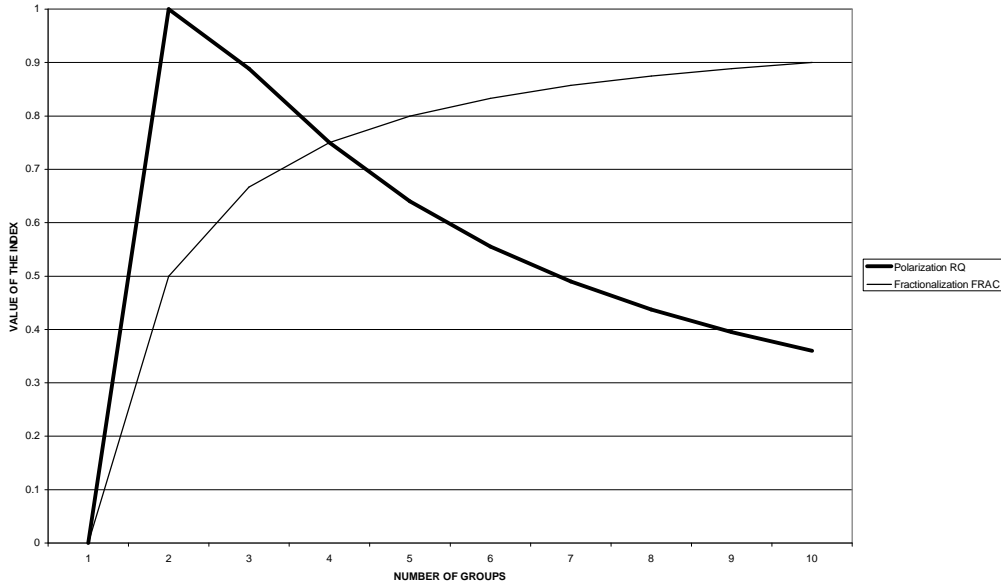


Figure 1: Fractionalization and polarization as a function of the number of groups (same size)

the surfaces generated by values of  $\alpha$  equal to 0.5 and 1.5, respectively. A first look at these figures shows that we can classify them in two groups in terms of the location of their respective maximum. In particular the surfaces presented in figures 2 and 4 reach a maximum when all the groups have the same size while in figures 3 and 5 the maximum is located at the bimodal distribution where two groups have equal size (0.5). In fact the functions with  $\alpha < 1$  can be considered as variations of the fractionalization index while the functions with  $\alpha \geq 1$  are variations of the polarization index.

The base of the figures 2 to 5 can be interpreted as a probability triangle, where each point describes a vector of probabilities  $(\pi_1, 1 - \pi_1 - \pi_3, \pi_3)$ . We can represent the two properties of polarization in terms of movement on this triangle. Figure 6a shows movement in the probability triangle that satisfies PR1. Starting at point  $Y_0$  (0.25,0.25,0.5) movement towards point  $Y_1$  (0,0.5,0.5) should increase polarization by PR1 since we are joining the two smallest groups into one larger group. Looking at figure 2, the index of fractionalization ( $\alpha = 0$ ) and at figure 4 ( $\alpha = 0.5$ ), we realize that in those surfaces the movement from  $Y_0$  to  $Y_1$  implies a decrease of the index and, therefore they do not satisfy PR1. For  $\alpha = 1$  and  $\alpha = 1.5$  the movement from  $Y_0$  toward  $Y_1$  implies an increase in polarization as stated by PR1. Figure 6b depicts movement from  $Z_0$  (0,1,0) toward  $Z_1$ (0.5, 0, 0.5), where the mass of group 2 is shifted equally to the other two groups (1 and 3). By

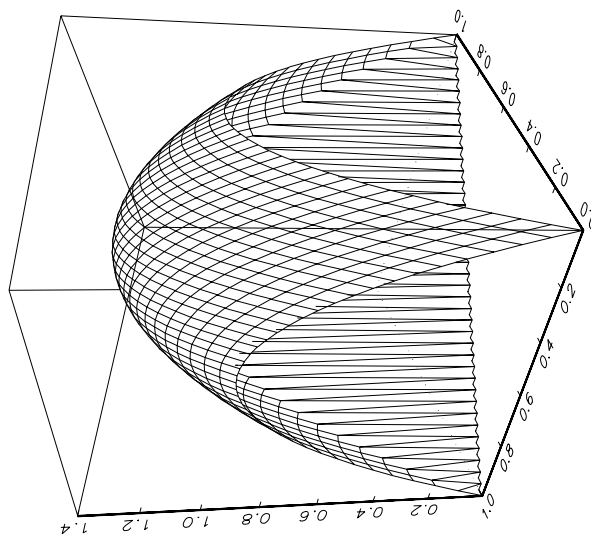


Figure 2: Index of fractionalization or discrete polarization with  $\alpha = 0$

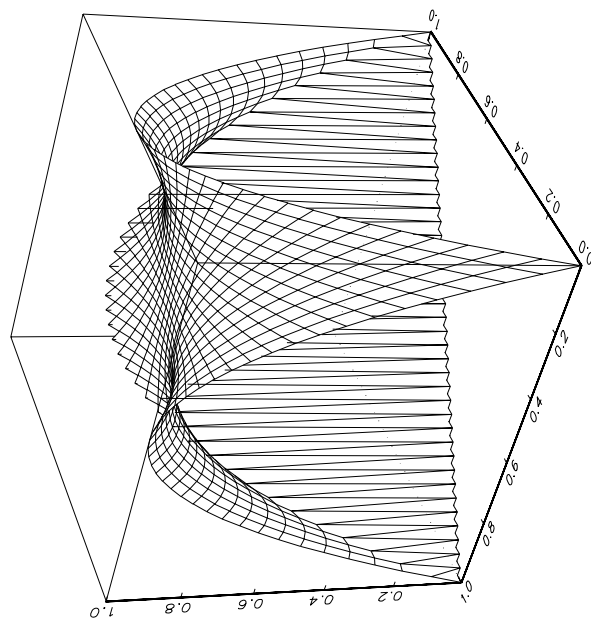


Figure 3: RQ index or discrete polarization with  $\alpha = 1$

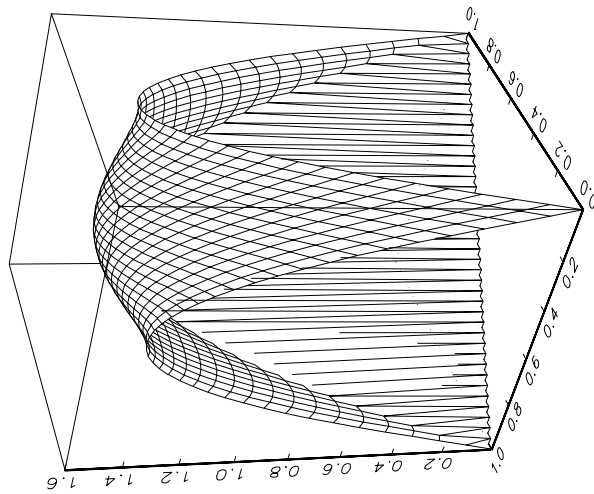


Figure 4: Discrete polarization for  $\alpha = 0.5$

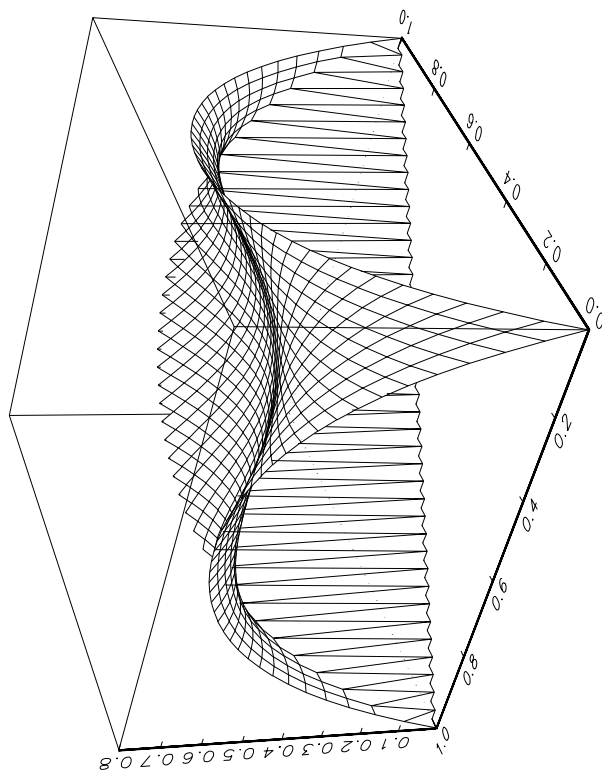


Figure 5: Discrete polarization for  $\alpha = 1.5$

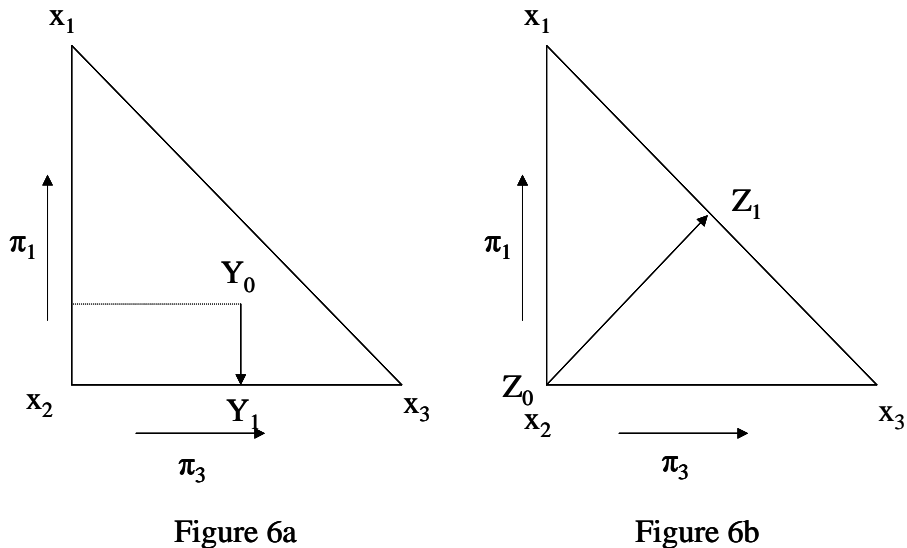


Figure 6: Properties 1 and 2.

property 2 any movement in this direction should generate an increase in polarization. Again we can easily see that the surfaces of figures 2 and 4 do not satisfy PR2 since this movement implies first an increase of the index and then a decrease. The surface in figure 3, corresponding to  $\alpha = 1$ , satisfies PR2. However, and opposite to what happened in the case of property 1, the surface in figure 5 ( $\alpha = 1.5$ ) fails to show a monotonic increase in polarization in the movement from  $Z_0$  toward  $Z_1$ .

We can see more clearly this effect in figure 7 where we depict the two dimensional plane generated by cutting the surfaces along the line from  $Z_0$  to  $Z_1$ . The X-axis represents the equal-sized transfer from group 2 to groups 1 and 3. The Y-axis represents the value of the index. For  $\alpha = 0.5$  we can see that the index increases until a transfer of  $1/3$  and then decreases. For  $\alpha = 1.5$  we see that the index increases and then decreases, hitting a local minimum at  $1/3$  and increasing again after that point. Finally for  $\alpha = 1$  we see that the index increases over the whole range with an inflection point at a transfer equal to  $1/3$ . Therefore from this informal discussion of four examples we see that only when  $\alpha = 1$ , which is equivalent to the RQ index, the discrete polarization measure satisfies both properties.

The previous results show that the range of suitable values of polarization sensitivity in a measure based on Euclidean distances cannot be translated directly to a measure based on discrete distances.

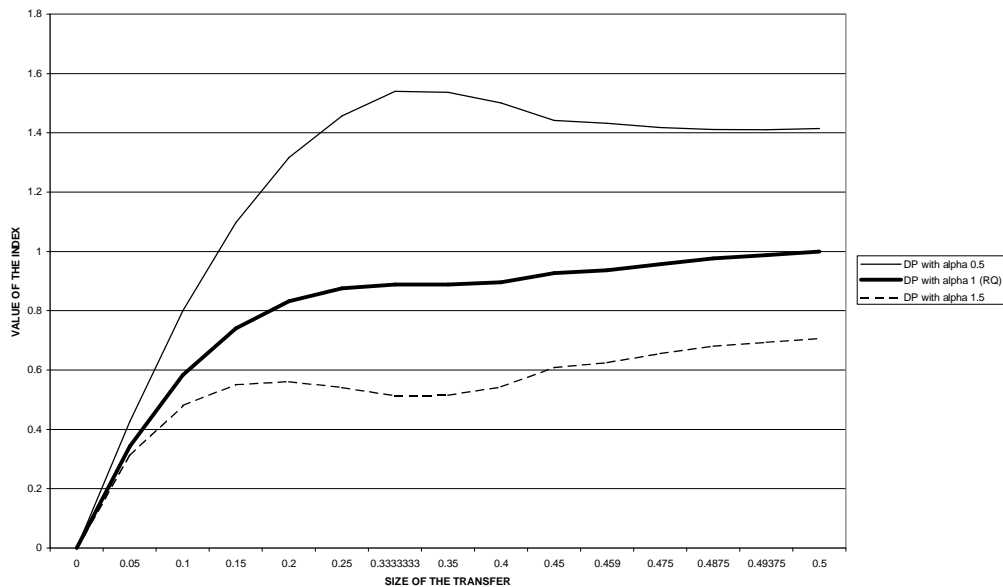


Figure 7: Value of the index of discrete polarization as a function of the parameter  $\alpha$  and the size of the transfer

However, several authors have constructed empirical measures of discrete polarization "over the range proposed by Esteban and Ray (1994)". Aghion et al (2004) construct an index of discrete polarization where the parameter  $\alpha = 4/5$ . As we show before this is actually an index of the family of fractionalization measures. Alesina et al. (2003) and Collier and Hoeffler (2004) construct three indices of polarization for three values of  $\alpha$  (0, 0.8 and 1.6)<sup>23</sup>. Obviously, the value 0 corresponds exactly to the traditional index of fractionalization while the index with  $\alpha = 0.8$  belongs to the same family. Finally, the index of discrete polarization with  $\alpha = 1.6$ , admissible for the class of  $P$  measures, is not appropriate for discrete polarization. As we have shown before the only measure of discrete polarization (with information about proportions but not distances across groups) that satisfies the above mentioned properties of polarization is the one that sets  $\alpha = 1$ , which is the RQ index.

<sup>23</sup>Alesina et al.(2003) argue that "Esteban and Ray 1994 do not point to which values was "better" to capture polarization - all values of  $\alpha$  in the specified range satisfy the properties that the class of polarization measures should satisfy. There is therefore no a priori reason to prefer one value over the other." (Page178, footnote 21)



### 3 Discrete ethnic polarization and genocides

Since World War II nearly 50 genocides and political mass murder have taken place. These episodes have killed at least 12 million of combatants and as many as 22 million noncombatants. These are more than all the victims of internal and international wars since 1945<sup>24</sup>. The human, social and economic consequences of genocides are extreme. The lost of human capital and trust among social groups as well as the economic disruption, bring countries to an economic collapse after a genocide episode. Since early international intervention is crucial to avoid genocides, the study of the main determinants of these episodes of social violence is an increasingly important issue in development economics. The crime of genocide is define in international law by the United Nations Genocide Convention (in force since January of 1951). Article II states that "in the present Convention, genocide means any of the following acts committed with intent to destroy, in whole or in part, a national, ethnical, racial or religious groups, as such: (a) Killing members of the group, (b) Causing serious bodily or mental harm to members of the group; (c) Deliberately inflicting on the group conditions of life calculated to bring about its physical destruction in whole or in part; (d) imposing measures intended to prevent births within the group; (e) Forcibly transferring children of the group to another group."

Harff (2003) proposes an operational definition of genocide/politicide based on the previous legal definition, but avoiding several of its limitations, in order to construct a dataset for empirical analysis. Harff (2003) defines genocide and politicicide as events that involve the promotion, execution, and/or implied consent of sustained policies by governing elites or their agents- or in the case of civil war, either of the contending authorities- that result in the deaths of a substantial portion of a communal group or politicized non-communal group. In genocides, the victimized groups are defined primarily in terms of their communal (ethnolinguistic, religious) characteristics. In politicides, by contrast, groups are defined primarily in terms of their political opposition to the regime and dominant groups. Genocides and politicides are different to state repression and terror. In cases of state terror authorities arrest, persecute or execute a few members in ways designed to terrorize the majority of the group into passivity or acquiescence.

In the case of genocide, authorities physically exterminate enough (not necessarily all) members of a target group so that it can no longer pose any conceivable threat to their rule or interests. Because genocide involves frequently the confrontation of ethnic, religious or nationalistic groups, we think it is a specially important case for the study of the relationship between conflict and

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<sup>24</sup>See the State Failure Task Force.

alternative measures of ethnic diversity.

The literature that analyses the determinants of genocides is scarce. Recently, Harff (2003) constructed a dataset on genocides and politicides, as the principal investigator of the State Failure Task Force (genocide/politicide project) and tested a structural model of the antecedents of genocide and politicide. Harff (2003) identifies six causal factors and test, in particular, the hypothesis that "the greater the ethnic and religious diversity, the greater the likelihood that communal identity will lead to mobilization and, if conflict is protracted, prompt elite decisions to eliminate the group basis of actual or potential challenges". However, she finds no empirical evidence to support this hypothesis. The variables used to capture potential conflict were measures of diversity (ethnic fractionalization). For this reason, an in line with most of the literature on the determinants of civil wars, Harff (2003) concludes that the effect of ethnic diversity on genocides is not statistically relevant. Fearon and Laitin (2003) and Collier and Hoeffler (2004) find that ethnic fractionalization has no effect on the likelihood of the onset of a civil war. Easterly et al (2006) analyze the determinant of mass killing which, they clarify, should not be confused with genocides. They find that mass killing is related with the square of ethnic fractionalization. This suggests that a situation close to two large groups would be the most dangerous one even in the case of mass killing.

Theoretical models suggest that the risk of conflict is high when society is divided into two large groups of similar size. Moreover, Horowitz (1985) argues that the relationship between ethnic diversity and violence is not monotonic: there is less violence in highly homogeneous and highly heterogeneous societies, and more conflicts in societies where a large ethnic minority faces an ethnic majority. If this is so then an index of polarization should capture better the likelihood of genocides than an index of fractionalization.

Caselli and Coleman (2006) have recently proposed a theory of ethnic conflict were they argue that coalitions formed along ethnic lines compete for the economy's resources. Ethnicity enforces coalition membership. They claim, at least in the initial working paper version of the paper, that ethnic dominance could be an important factor in conflicts. The empirical results reported by Collier (2001) seems to indicate that a good operational definition of dominance implies a group that represents between 45% and 90% of the population. However, Collier and Hoeffler (2004) find that dominance, defined as mentioned before, has only a weak positive effect on the onset of civil wars. Ethnic dominance, or the existence of a large ethnic group, is related with ethnic polarization although it does not capture some subtle aspects. Dominance implies the existence of a large group. A high degree of polarization captures the idea of a large majority versus a large minority. Therefore,

dominance is, in general, a necessary condition for a high degree of polarization but it is not sufficient.

### 3.1 Data and basic specification

In this section we present the estimation of a logit model for the incidence of genocides as a function of ethnic and religious heterogeneity. The sample includes 138 countries during 1960-99. We divide the sample into five-year periods. The endogenous variable is the incidence of a genocide. The source is the State Failure Project dataset<sup>25</sup>. We analyze whether genocides, and the different intense-type of civil wars have the same ethnic roots, in terms of polarization versus fractionalization.

The explanatory variables follow the basic specifications of the civil wars literature<sup>26</sup>. Many of the determinants of civil wars were thought as causes of conflict and, therefore, may also be important in the explanation of genocides. The recent empirical literature emphasizes, in general, the role of economic and geographical determinants of conflict. Fearon and Laitin (2003) argue that income per capita is a proxy for "state's overall financial, administrative, police and military capabilities." Once a government is weak rebels can expect a higher probability of success. In addition, Collier and Hoeffler (2004) point out that a low level of income per capita reduces the opportunity cost of engaging into conflict.

The size of the population is another usual suspect in the explanation of conflict. Collier and Hoeffler (2004) consider that the size of the population is an additional proxy for the benefits of a rebellion since it measures potential labor income taxation. Fearon and Laitin (2003) indicate that a large population implies difficulties in controlling what goes on at the local level and increases the number of potential rebels that can be recruited by the insurgents. Similar arguments apply to genocides, which usually are perpetrated by rebel groups that have been recruited based on ethnic identity.

Mountains are another dimension of opportunity since this terrain could provide a safe haven for rebels. Long distances from the center of the state's power also favors the incidence of conflict, specially if there is a natural frontier between them, like a sea or other countries. Collier and Hoeffler (2004) point out that the existence of natural resources provide an opportunity for rebellion since these resources can be used to finance the war and increases the payoff if victory is achieved.

We are going to emphasize the role of ethnic divisions. Our hypothesis is that ethnic polarization will play a more important role in the explanation of genocides than in civil wars. We are going

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<sup>25</sup>See Appendix II for a precise definition.

<sup>26</sup>See, for instance, Montalvo and Reynal-Querol (2005), Fearon and Laitin (2003) and Collier and Hoeffler (2004).

to use also the concept of ethnic dominance as characterized by Collier (2001). There are basically three sources of ethnolinguistic groups across countries: the World Christian Encyclopedia (WCE), the Encyclopedia Britannica (EB) and the Atlas Narodov Mira (ANM) (1964). The most accurate description of ethnic diversity is the one in the WCE. We follow Vanhanen (1999) in taking into account only the most important ethnic divisions. Vanhanen (1999) uses a measure of genetic distance to separate different degrees of ethnic cleavage. The proxy for genetic distance is "the period of time that two or more compared groups have been separated from each other, in the sense that intergroup marriage has been very rare. The longer the period of endogamous separation, the more groups have had time to differentiate." This criterion is reasonable since we are using discrete distances and, therefore, we have to determine the identity of the relevant groups. Another source of data on ethnic diversity is the Encyclopedia Britannica (EB)<sup>27</sup> which uses the concept of geographical race. A third source of data on ethnolinguistic diversity is provided by the Atlas Narodov Mira (ANM) (1964), the result of a large project of the Department of Geodesy and Cartography of the State Geological Committee of the old USSR.

Therefore the explanatory variables for the core specification of the incidence of genocide include the log of real GDP per capita in the initial year (LGDPC), the log of the population at the beginning of the period (LPOP), primary exports (PRMEXP), mountains (MOUNTAINS), noncontiguous states (NONCONT), and the level of democracy (DEMOCRACY)<sup>28</sup>. Using this core specification we check the empirical performance of indices of ethnic fractionalization (ETHFRAC), polarization (ETHPOL) and dominance (ETHDOM). Table 1 presents the basic statistics for these variables, separating the sample by geographical regions and the aggregated results. Table 1 also includes the traditional ELF indicator of ethnolinguistic fractionalization used by Mauro (1995). Ethnic polarization and ethnic dominance are the highest in Latin America. However, it is interesting to notice that ethnic dominance has a much larger range than ethnic polarization. It is also interesting to point out that while the average degree of polarization of Sub-Saharan Africa is higher than the overall average, the opposite happens in the case of ethnic dominance. The highest degree of ethnic fractionalization (either using the ETHFRAC variable or ELF) corresponds to Sub-Saharan Africa. This result is common to all the literature on ethnic fractionalization starting with Easterly and Levine (1997).

Table 2 present the correlations across the alternative indices of ethnic heterogeneity. It also

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<sup>27</sup>This is the basic source of data on ethnic heterogeneity of Alesina et al. (2003).

<sup>28</sup>Appendix III describes the source of each of these variables.

includes the correlations among the variables conditional on the degree of ethnic fractionalization (above and below the median, the percentile 25 and the percentile 75). For the whole sample (panel A) the correlation between dominance and any measure of fractionalization is negative. Both measure of fractionalization (ELF and ETHFRAC) have a very high degree of correlation (0.86). Finally, the index of polarization has a positive correlation with both, the indices of fractionalization and the index of dominance.

Panels B-D show the correlations conditional on different levels of ethnic fractionalization. Several interesting facts are embedded in the results of those panels. First, the correlation between polarization and fractionalization is negative for countries over the median of the degree of fractionalization, and it is even more negative the higher is the percentile that defines the high level group. These results are not a surprise given the properties of discrete polarization that we discuss in the theoretical part of the paper. The correlation between polarization and dominance as a function of the degree (high or low) of fractionalization, is not monotonic: it is small for the countries over the percentile 25, but the highest degree corresponds to countries over the median and not to the sample of countries over the percentile 75.

### 3.2 Ethnic heterogeneity and the incidence of genocides

Table 3 reports the results of the estimation of the basic specification obtained using ethnic fractionalization and ethnic polarization measures<sup>29</sup>. Column 1 shows that ethnic fractionalization has no effect on genocides, confirming results from previous research. This indicates that highly fragmented societies have no higher risk of suffering a genocide than homogeneous societies. However, this does not mean that ethnicity does not matter for explaining genocides. If we substitute the index of ethnic fractionalization by the index of ethnic polarization we find a positive and statistically significant effect on the incidence of genocide, which is robust to the inclusion of the other typical controls on the core specification. Column 2 shows this result. Moreover, when including both measures together (column 3), we find that ethnic fractionalization has no effect while ethnic polarization has a positive and significant effect on the incidence of genocides/politicides. The results are mostly unchanged if we include regional dummies (columns 4-7). Therefore, ethnic heterogeneity, measured as ethnic polarization, is important for the explanation of the likelihood of genocides. Moving from social homogeneity (one group or polarization equal to 0) to the highest degree of polarization (two groups of equal size or the index equal to 1) increases the probability of a genocide in 7 percentage

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<sup>29</sup>The tables show the test statistics calculated using the corrected (clustered) standard deviation of the estimators.

points. If the polarization index increases one standard deviation (0.24) from the average (0.51) the probability of genocide increases in 1.7 percentage points.

The State Failure Project (SFP) includes together genocides and politicides. In many situations genocides and politicides are part of the same process, mainly because ethnic division are reflected in political parties. In order to separate the cases that are purely politicides, meaning that the main divisions are not ethnic, religious, racial or nationalistic, we use the information in Harff (2003). Among all the genocides/politicides in our sample, the SFP lists two cases as politicides: Chile and El Salvador. We test the robustness of our results excluding these two countries. Columns 1 to 3 of Table 4 corroborate the findings of Table 3. Once the two pure politicides are excluded the coefficient on ethnic polarization increases. The results are qualitative identical if we add as regressors the regional dummies.

The majority of the genocides and politicides coded by SFP took place during the course of an ethnic civil war, as defined by SFP. Very few episodes are considered genocides/politicides with no civil war. We test the sensitivity of the results to the exclusion of the genocides/politicides that were not considered a civil war. Table 5 shows that ethnic fractionalization has no significant effect on genocides associated to civil wars. However, if instead of fractionalization we include ethnic polarization, this variable has a positive and significant effect on the incidence of civil war (column 2). If we include both measures (column 3), we find that only polarization has a statistically significant effect on genocides. The results are unchanged if we include in the regression the regional dummies.

We also analyze whether the results presented previously are robust to the exclusion of some geographical regions. Table 6 considers the estimation of the basic specification eliminating, in sequential steps, Sub-Saharan African countries, Latin American countries and countries of Asia. The results indicate that ethnic polarization has a positive and significant effect on genocide, even in the presence of ethnic fractionalization, and even when the genocide is part of an ongoing civil war. The statistical significance of ethnic polarization is robust to all these different subsamples.

Table 7 includes in the basic regression the ethnic dominance variable. Column 1 shows that ethnic dominance is not statistically significant to explain the incidence of genocides at the usual level of significance. Column 2 shows that when dominance and polarization are included together then none of them is statistically significant. However, there are several variables that are not significant and some authors do not consider in the basic specification of the incidence of civil wars: the proportion of mountains and noncontiguous areas and the size of primary exports. Since

these variables are never statistically significant we run the regressions without them. In those specifications ethnic polarization is statistically significant (at least in some specification) while ethnic dominance is not significant.

### **3.3 Ethnic heterogeneity and highly-death, intermediate and minor civil wars**

Genocide and politicide are an extreme form of civil conflicts. Given our previous discussion, it seems reasonable to expect that the effect of ethnic polarization on the probability of a conflict is reduced the less intense is the conflict. In order to test this hypothesis, we need a classification of civil wars depending on its intensity. There is no doubt that genocide and politicide are the most violent conflicts. Civil wars can be classified depending of their intensity in terms of the number of deaths. The dataset on civil wars of Uppsala/PRIO (Peace Research Institute of Oslo) is the most widely used data on civil wars. Uppsala/PRIO defines an armed conflict as a contested incompatibility that concerns government and/or territory where the use of armed force between two parties, of which at least one is the government of a state, results in at least 25 battle related deaths. We consider only civil war conflict (type 3 and 4 from Uppsala/PRIO classification), excluding interstate war. Uppsala/PRIO distinguish three types of conflicts depending on the number of deaths:

- Minor armed conflict: at least 25 battle-related deaths per year and fewer than 1000 battle-related deaths during the course of the conflict.
- Intermediate Armed Conflict: At least 25 battle-related deaths per year and an accumulated total of at least 1000 deaths, but fewer than 1000 per year.
- War: At least 1000 battle-related deaths per year.

These classification, allow us to distinguish three types of civil wars, from more to less intense, and to compare the effect of ethnic polarization on genocides, high intensity civil wars, intermediate and minor civil wars. In column 1 and 2 of table 8 we analyze the effect ethnic polarization on the incidence of civil wars that involve more than 1000 deaths a year. These results indicate that ethnic polarization has a positive and significant effect on the incidence of highly intense civil wars, even in the presence of ethnic fractionalization, which has no significant effect<sup>30</sup>. Moving from an homogenous country (polarization=0) to a totally polarized county (polarization=1) the probability of an extreme civil war increases 17 percentage points (based on column 1). In Column 3 and 4, we

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<sup>30</sup>The regression of column 2 is a replication of Montalvo and Reynal-Querol (2005). We include it here as a matter of comparison with genocides and low intensity civil war.

estimate the determinants of intermediate civil war. The results indicate that polarization is still an important determinant of this type of conflict: an increase of the index of polarization from 0 to 1 increases the probability of a medium intensity war by 14 percentage point. Finally in column 5 and 6, the dependent variable is a dummy that has value 1 if the country had a minor conflict during the period. In these cases ethnic polarization does not have a significant role in explaining the incidence of conflict.

As a final check of the robustness of the results, table 9 contains the estimation of the specification excluding the variables that were not significant in any of the previous tables. Therefore, we exclude primary exports, mountains and noncontiguous territories. Following the previous strategy columns 1, 2 and 3 present the estimation for both genocides and politicides, only genocides and genocides that happened during a civil war, respectively. In all cases we confirm that ethnic polarization is a statistically significant determinant of genocides. Finally, this result is robust to running the logit regressions in a cross section (column 4 to 6), where the dependent variable takes value 1 if a country has suffered a genocide during the whole period (1960-1999) and zero otherwise. The values of GDP per capita, population and democracy are measured at the beginning of the period (1960). The results show that ethnic polarization has a significant and positive effect in explaining the incidence of genocides even if we consider only the cross section for the whole period.

## 4 Conclusions

The economic literature has recognized since some time ago that inequality and polarization are two different concepts. As in the case of inequality, the measurement of polarization was initially developed in the context of a continuous dimension, in particular income, which defined the "closeness" of the characteristics of individuals and clusters. Esteban and Ray (1994) present the properties of a precise axiomatization of a class of polarization measures based on distances in the real line. However, in many important dimensions (like ethnicity or religion), there is no information on a continuous variable to measure distances across groups. In addition, if there was such a proxy for "ethnic distances", that measure would be much more controversial than the identification of the list of ethnic groups. Finally, in many instances, and ethnicity is one of them, individuals are only interested in the dichotomous perception "we versus they". For these reasons we analyze in this paper the theoretical properties of a measure of polarization based on classifications instead of continuous distances across groups. We show that the range of parameter values suitable for this



measure of discrete polarization is different from the ones in the original polarization measure of Esteban and Ray (1994). This is important since some recent papers have constructed measures of polarization using data on groups, without information on distances, assuming that the range of suitable parameters of the original index can be directly applied to discrete polarization.

The second part of the paper presents an application of the index of discrete ethnic polarization to the explanation of genocides. Most of the recent papers on the determinants of civil wars and genocides fail to find any significant effect for ethnic heterogeneity, measured as fractionalization. However, Horowitz (1985) argues that the relationship between ethnic diversity and violence is not monotonic: there is less violence in highly homogeneous and highly heterogeneous societies, and more conflicts in societies where a large ethnic minority faces an ethnic majority. If this is so then an index of polarization should capture better the likelihood of genocides than an index of fractionalization. We argue that the use of ethnic fractionalization instead of an index of polarization is the main reason for the failure to find a significant effect of ethnic heterogeneity on the probability of genocides. The empirical results support this interpretation.

## APPENDIX I.

### PROOFS

#### Proof of Theorem 1:

*Proof of sufficiency:*

The general discrete polarization index can be written as

$$\begin{aligned}
 DP(\alpha, k) &= k \sum_{i=1}^n \sum_{j \neq i} \pi_i^{1+\alpha} \pi_j = k \sum_{i=1}^n \pi_i^{1+\alpha} (1 - \pi_i) = \sum_{i=1}^n k[\pi_i(\pi_i^\alpha - \pi_i^{1+\alpha})] = \\
 &= \sum_{i=1}^n \pi_i(k\pi_i^\alpha - k\pi_i^{1+\alpha}) = \sum_{i=1}^n \pi_i(1 - 1 + k\pi_i^\alpha - k\pi_i^{1+\alpha}) = \sum_{i=1}^n \pi_i(1 - \frac{k}{k} + k\pi_i^\alpha - k\pi_i^{1+\alpha}) = \\
 &= \sum_{i=1}^n \pi_i - \sum_{i=1}^n \pi_i k(\frac{1}{k} - \pi_i^\alpha + \pi_i^{1+\alpha}) = 1 - \sum_{i=1}^n \pi_i k(\frac{1}{k} - \pi_i^\alpha + \pi_i^{1+\alpha}) \quad (1)
 \end{aligned}$$

For the three point distribution  $(p, q, r)$  the discrete polarization measure is

$$DP(\alpha, k)^{(p,q,r)} = 1 - pk(\frac{1}{k} - p^\alpha + p^{1+\alpha}) - qk(\frac{1}{k} - q^\alpha + q^{1+\alpha}) - rk(\frac{1}{k} - r^\alpha + r^{1+\alpha})$$

For the alternative distribution  $(p, \tilde{q})$  the DP index is

$$DP(\alpha, k)^{(p,\tilde{q})} = 1 - pk(\frac{1}{k} - p^\alpha + p^{1+\alpha}) - \tilde{q}k(\frac{1}{k} - \tilde{q}^\alpha + \tilde{q}^{1+\alpha})$$

where  $q + r = \tilde{q}$

Therefore

$$\begin{aligned}
 DP(\alpha, k)^{(p,\tilde{q})} - DP(\alpha, k)^{(p,q,r)} &= qk(\frac{1}{k} - q^\alpha + q^{1+\alpha}) + rk(\frac{1}{k} - r^\alpha + r^{1+\alpha}) - \tilde{q}k(\frac{1}{k} - \tilde{q}^\alpha + \tilde{q}^{1+\alpha}) = \\
 &= qk(\frac{1}{k} - q^\alpha + q^{1+\alpha}) + rk(\frac{1}{k} - r^\alpha + r^{1+\alpha}) - (q+r)k(\frac{1}{k} - \tilde{q}^\alpha + \tilde{q}^{1+\alpha}) = \\
 &= qk[(\frac{1}{k} - q^\alpha + q^{1+\alpha}) - (\frac{1}{k} - \tilde{q}^\alpha + \tilde{q}^{1+\alpha})] + \\
 &+ rk[(\frac{1}{k} - r^\alpha + r^{1+\alpha}) - (\frac{1}{k} - \tilde{q}^\alpha + \tilde{q}^{1+\alpha})] =
 \end{aligned}$$

Let's define  $h(\pi) = (\frac{1}{k} - \pi^\alpha + \pi^{1+\alpha})$ . The first derivative of this function is

$$h'(\pi) = -\alpha\pi^{\alpha-1} + (1+\alpha)\pi^\alpha$$

Notice that  $h'(\pi^*) = 0$  when  $\pi^* = \frac{\alpha}{1+\alpha}$ . Evaluating at the first derivative we obtain that  $h(\pi)$  is a strictly increasing for all  $\pi > \pi^*$  and a strictly decreasing function for all  $\pi < \pi^*$ .

We can write the difference in DP when we merge two small groups in function of  $h(\cdot)$  as

$$DP(\alpha, k)^{(p,\tilde{q})} - DP(\alpha, k)^{(p,q,r)} = qk(h(q) - h(\tilde{q})) + rk(h(r) - h(\tilde{q}))$$

We want to show that if  $\alpha \geq 1$  then  $h(q) > h(\tilde{q})$  and  $h(r) > h(\tilde{q})$  for all  $q, r < \frac{1}{2}$  and, therefore,  $DP(\alpha, k)^{(p,\tilde{q})} - DP(\alpha, k)^{(p,q,r)}$  is positive for any distribution of  $p, q$  and  $r$ .

In principle we should analyze two possible cases: when the merge results in a group that is smaller than the original largest group ( $\tilde{q} \leq p$ ) and when the merge of the smallest groups is large than the originally largest group ( $\tilde{q} > p$ ).

**CASE 1:**  $q + r = \tilde{q} \leq p$ .

In this case  $q + r = \tilde{q} \leq \frac{1}{2}$ . and  $r \leq q < \frac{1}{2}$

Since  $\tilde{q}$  is smaller than  $p$ , then  $\tilde{q} \leq \frac{1}{2}$ , Therefore we need that  $h(\pi_i) > h(\tilde{q})$  for all  $\pi_i \leq \tilde{q} \leq \frac{1}{2}$ .

Therefore if  $h(q, r) > h(\tilde{q})$  for all  $q, r \leq \frac{1}{2}$ , then  $h(\pi)$  has to be a decreasing function for all  $\pi \leq \frac{1}{2}$ . This is only possible if  $\pi^*(\alpha) \geq 1/2$ . But since  $\pi^* = \frac{\alpha}{1+\alpha} \geq \frac{1}{2}$ , the latter is satisfied if and only if  $\alpha \geq 1$ .

Therefore for  $h$  being strictly decreasing for all  $q, r \leq 1/2$ , implies that the DP index has to satisfy property 1 if  $\alpha \geq 1$ .

**CASE 2:**  $q + r = \tilde{q} > p$

In this case the minimum value for  $p$  is,  $p = \frac{1}{3} + \varepsilon$ , and the maximum value for  $\tilde{q} = \frac{2}{3} - \varepsilon$ . Notice that now  $q$  and  $r$  can not be any value between  $(0, \frac{2}{3})$ , otherwise would violate the assumption that  $q, r < p$ . Therefore, the maximum value for  $q$  and  $r$  is,  $q = \frac{1}{3}, r = \frac{1}{3} - \varepsilon$ . This is problematic because we don't need that  $h$  be decreasing for  $\pi \leq \frac{2}{3}$ .

Now for each value of  $\tilde{q}$ , which means a value for  $p$ , there is a possible maximum value for  $q$ , which in the limit is  $p$ . Therefore what we need to show is that  $h(\max q) > h(p) \geq h(\tilde{q})$ ,

We have to show therefore, that  $h(\max q) > h(p) \geq h(\tilde{q})$  in all the range of  $\tilde{q} \in [\frac{1}{2}, \frac{2}{3}]$ . This means that we have to analyze the range of possibilities when  $\frac{1}{3} < q < \frac{1}{2}$  when  $\frac{1}{2} \leq \tilde{q} < \frac{2}{3}$

Notice that as  $\tilde{q}$  decrease, then  $p$  increases, and then the range of possible  $q$  also increases, and therefore in the limit the maximum  $q = p$ , increases. Therefore,

If the following inequality  $h(\frac{1}{3} + \varepsilon) \geq h(\frac{2}{3} - \varepsilon)$  is satisfied for all  $\varepsilon$ , means that when  $\tilde{q} > p$ , then property 1 is satisfied.

So we look which families of DP measures satisfy this inequality:

$$\begin{aligned} h(\frac{1}{3} + \varepsilon) &\geq h(\frac{2}{3} - \varepsilon) \\ 1 - (\frac{1}{3} + \varepsilon)^\alpha + (\frac{1}{3} + \varepsilon)^{1+\alpha} &\geq 1 - (\frac{2}{3} - \varepsilon)^\alpha + (\frac{2}{3} - \varepsilon)^{1+\alpha} \\ -(\frac{1}{3} + \varepsilon)^\alpha + (\frac{1}{3} + \varepsilon)^{1+\alpha} &\geq -(\frac{2}{3} - \varepsilon)^\alpha + (\frac{2}{3} - \varepsilon)^{1+\alpha} \\ (\frac{1}{3} + \varepsilon)^\alpha [\frac{1}{3} + \varepsilon - 1] &\geq (\frac{2}{3} - \varepsilon)^\alpha [\frac{2}{3} - \varepsilon - 1] \\ (\frac{1}{3} + \varepsilon)^\alpha [-\frac{2}{3} + \varepsilon] &\geq (\frac{2}{3} - \varepsilon)^\alpha [-\frac{1}{3} - \varepsilon] \\ \left[ \frac{\frac{1}{3} + \varepsilon}{\frac{2}{3} - \varepsilon} \right]^\alpha &\geq \left[ \frac{\frac{1}{3} + \varepsilon}{\frac{2}{3} - \varepsilon} \right] \end{aligned}$$

Therefore in order this inequality be satisfied for all values of  $\varepsilon$  we need that  $\alpha \geq 1$ . It would also be true for  $r$ , given that  $r \leq q \leq \frac{1}{2}$ , and we have shown that  $h$  is decreasing function of  $\pi \leq \frac{1}{2}$ .

Therefore,  $DP(\alpha, k)^{(p\tilde{q})} \geq DP(\alpha, k)^{(p,q,r)}$  if  $\alpha \geq 1$

*Proof of necessity:*

By contradiction. We can show that if  $\alpha < 1$ , then there always exist a distribution of  $p, q, r$  such that the polarization after merging the two smallest groups is smaller than the original, that is to say  $DP(\alpha, k)^{(p\tilde{q})} < DP(\alpha, k)^{(p,q,r)}$ .

Consider the case such that  $q = r$ . Therefore  $\tilde{q} = 2q$ .

Let's now compute,

$$\begin{aligned} DP(\alpha, k)^{(p\tilde{q})} - DP(\alpha, k)^{(p,q,r)} &= \\ 2q[k(\frac{1}{k} - q^\alpha + q^{1+\alpha})] - k2q[(\frac{1}{k} - (2q)^\alpha + (2q)^{1+\alpha})] &= \\ 2kq[\frac{1}{k} - q^\alpha + q^{1+\alpha} - \frac{1}{k} + (2q)^\alpha - (2q)^{1+\alpha}] &= \\ 2kq[-q^\alpha + q^{1+\alpha} + (2q)^\alpha - (2q)^{1+\alpha}] &= \end{aligned}$$

We want to show that for  $\alpha < 1$ , there always exist a set of  $q \in [q^*, \frac{1}{3})$ , such that  $[-q^\alpha + q^{1+\alpha} + (2q)^\alpha - (2q)^{1+\alpha}] < 0$

$$q^\alpha(q-1) + (2q)^\alpha(1-2q) < 0$$

$$(2q)^\alpha(1-2q) < -q^\alpha(q-1)$$

$$(2q)^\alpha(1-2q) < q^\alpha(1-q)$$

$$(\frac{2q}{q})^\alpha < \frac{(1-q)}{(1-2q)}$$

$$2^\alpha < \frac{(1-q)}{(1-2q)}$$

Notice that if  $q = r$  then  $q < \frac{1}{3}$ . If  $q = \frac{1}{3}$ , then  $\frac{(1-q)}{(1-2q)}$  evaluated at  $\frac{1}{3}$  is 2.

Moreover, for  $\alpha < 1$ ,  $2^\alpha < 2$ . Therefore, there always exist a set of  $q' \in [q^*, \frac{1}{3})$ , such that  $2^\alpha < \frac{(1-q')}{(1-2q')} < 2$ .

Therefore, for any  $\alpha < 1$ , there exist a set of  $q' \in [q^*, \frac{1}{3})$ , such that  $DP(\alpha, k)^{(p\tilde{q})} < DP(\alpha, k)^{(p,q,r)}$  ■

## Proof of Theorem 2:

*Proof of sufficiency:*

The general discrete polarization index can be written as

$$DP(\alpha, k)^{(N=n)} = 1 - \sum_{i=1}^n \pi_i k(\frac{1}{k} - \pi_i^\alpha + \pi_i^{1+\alpha}) \quad (1)$$

For the two point distribution ( $N=2$ ) the discrete polarization measure is

$$\begin{aligned} DP(\alpha, k)^{(N=2)} &= 1 - \sum_{i=1}^2 \pi_i k(\frac{1}{k} - \pi_i^\alpha + \pi_i^{1+\alpha}) = \\ 1 - \pi_1 k(\frac{1}{k} - \pi_1^\alpha + \pi_1^{1+\alpha}) - \pi_2 k(\frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha}) & \end{aligned}$$

For the alternative  $N$  point distribution  $N = 1 + n$  the DP index is

$$\begin{aligned} DP(\alpha, k)^{(N=n+1)} &= 1 - \sum_{i=1}^{n+1} \tilde{\pi}_i k(\frac{1}{k} - \tilde{\pi}_i^\alpha + \tilde{\pi}_i^{1+\alpha}) = \\ 1 - \tilde{\pi}_1 k(\frac{1}{k} - \tilde{\pi}_1^\alpha + \tilde{\pi}_1^{1+\alpha}) - \sum_{i=2}^{n+1} \tilde{\pi}_i k(\frac{1}{k} - \tilde{\pi}_i^\alpha + \tilde{\pi}_i^{1+\alpha}) & \quad \text{where } \tilde{\pi}_1 = \pi_1 \text{ and } \sum_{i=2}^{n+1} \tilde{\pi}_i = \pi_2 \end{aligned}$$

Therefore

$$\begin{aligned}
DP(\alpha, k)^{(N=2)} - DP(\alpha, k)^{(N=n+1)} &= \sum_{i=2}^{n+1} \tilde{\pi}_i k \left( \frac{1}{k} - \tilde{\pi}_i^\alpha + \tilde{\pi}_i^{1+\alpha} \right) - \pi_2 k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) = \\
&\sum_{i=2}^{n+1} \tilde{\pi}_i k \left( \frac{1}{k} - \tilde{\pi}_i^\alpha + \tilde{\pi}_i^{1+\alpha} \right) - \sum_{i=2}^{n+1} \tilde{\pi}_i k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) = \\
&\sum_{i=2}^{n+1} \tilde{\pi}_i \left[ k \left( \frac{1}{k} - \tilde{\pi}_i^\alpha + \tilde{\pi}_i^{1+\alpha} \right) - k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) \right] = \\
&\tilde{\pi}_1 \left[ k \left( \frac{1}{k} - \tilde{\pi}_1^\alpha + \tilde{\pi}_1^{1+\alpha} \right) - k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) \right] + \\
&\tilde{\pi}_2 \left[ k \left( \frac{1}{k} - \tilde{\pi}_2^\alpha + \tilde{\pi}_2^{1+\alpha} \right) - k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) \right] + \dots + \\
&\tilde{\pi}_{n+1} \left[ k \left( \frac{1}{k} - \tilde{\pi}_{n+1}^\alpha + \tilde{\pi}_{n+1}^{1+\alpha} \right) - k \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) \right]
\end{aligned}$$

Let's define  $h(\pi) = \left( \frac{1}{k} - \pi^\alpha + \pi^{1+\alpha} \right)$ . The first derivative of this function is

$$h'(\pi) = -\alpha \pi^{\alpha-1} + (1+\alpha)\pi^\alpha$$

Notice that  $h'(\pi^*) = 0$  when  $\pi^* = \frac{\alpha}{1+\alpha}$ . Evaluating at the first derivative we obtain that  $h(\pi)$  is a strictly increasing for all the  $\pi > \pi^*$ , and a strictly decreasing function for all the  $\pi < \pi^*$ .

We can write the difference in DP in function of  $h(\cdot)$  as

$$DP(\alpha, k)^{N=2} - DP(\alpha, k)^{N=n+1} = \sum_{i=2}^{n+1} \tilde{\pi}_i [h(\tilde{\pi}_i) - h(\pi_2)]$$

We want to show that if  $\alpha \geq 1$  then  $h(\tilde{\pi}_i) > h(\pi_2)$  for all  $\tilde{\pi}_i < \frac{1}{2}$  and, therefore,  $DP(\alpha, k)^{N=2} - DP(\alpha, k)^{N=n+1}$  is positive for any distribution..

In principle we should analyze two possible cases: when we split the small group ( $\pi_2 \leq \pi_1$ ) and when we split the largest group ( $\pi_2 > \pi_1$ ).

**CASE 1: If  $\pi_2 \leq \pi_1$ .**

In this case  $\pi_2 \leq \frac{1}{2}$ . and  $\tilde{\pi}_i < \frac{1}{2}$

Since  $\pi_2$  is smaller than  $\pi_2$ , then  $\pi_2 \leq \frac{1}{2}$ , Therefore we need that  $h(\tilde{\pi}_i) > h(\pi_2)$  for all  $\tilde{\pi}_i < \pi_2 \leq \frac{1}{2}$ .

Therefore if  $h(\tilde{\pi}_i) > h(\pi_2)$  for all  $\tilde{\pi}_i \leq \frac{1}{2}$ , then  $h(\pi)$  has to be a decreasing function for all  $\pi \leq \frac{1}{2}$ . This is only possible if  $\pi^*(\alpha) \geq 1/2$ . But since  $\pi^* = \frac{\alpha}{1+\alpha} \geq \frac{1}{2}$ , the latter is satisfied if and only if  $\alpha \geq 1$ .

Therefore for  $h$  being strictly decreasing for all  $\tilde{\pi}_i \leq 1/2$ , implies that the DP index has to satisfy property 1 if  $\alpha \geq 1$ .

**CASE 2:  $\pi_2 > \pi_1$**

In that case the maximum value that  $\tilde{\pi}_i$  can take in the limit would be  $\pi_1$ , that is  $\max \tilde{\pi}_i = \pi_1 - \varepsilon$ .

The value for  $\pi_2 = (1 - \pi_1)$ . Notice that now  $\tilde{\pi}_i$  can not be any value between  $(0, 1 - \pi_1)$ , otherwise would violate the assumption that  $\tilde{\pi}_i < \pi_1$ . Therefore, the maximum value for  $\tilde{\pi}_i$  is  $\max \tilde{\pi}_i = \pi_1 - \varepsilon$ . This is problematic because we don't need that  $h$  be decreasing for  $\pi \leq \pi_2$ .

Now for each value of  $\pi_2$ , which means a value for  $\pi_1$ , there is a possible maximum value for  $\tilde{\pi}_i$ , which in the limit is  $\pi_1$ . Therefore what we need to show is that  $h(\max \tilde{\pi}_i) > h(\pi_1) \geq h(\pi_2)$ ,

We have to show therefore, that  $h(\max \pi_1) > h(\pi_1) \geq h(\pi_2)$  in all the range of  $\pi_2 \in [\frac{1}{2}, 1 - \pi_1]$ . This means that we have to analyze the range of possibilities when  $\pi_1 < \hat{\pi}_i < \frac{1}{2}$  when  $\frac{1}{2} \leq \pi_2 < 1 - \pi_1$

Notice that as  $\pi_2$  decreases, then  $\pi_1$  increases, and then the range of possible  $\tilde{\pi}_i$  also increases, and therefore in the  $\max \tilde{\pi}_i$  (that in the limit  $= \pi_1$ ) also increases. Therefore,

If the following inequality  $h(\pi_1 + \varepsilon) \geq h(1 - \pi_1 - \varepsilon)$ . is satisfied for all  $\varepsilon$ , means that when  $\pi_2 > \pi_1$ , then property 1 is satisfied.

So we look which families of  $DP(\alpha, k)$  measures satisfies this inequality:

$$\begin{aligned} h(\pi_1 + \varepsilon) &\geq h(1 - \pi_1 - \varepsilon) \\ 1 - (\pi_1 + \varepsilon)^\alpha + (\pi_1 + \varepsilon)^{1+\alpha} &\geq 1 - (1 - \pi_1 - \varepsilon)^\alpha + (1 - \pi_1 - \varepsilon)^{1+\alpha} \\ -(\pi_1 + \varepsilon)^\alpha + (\pi_1 + \varepsilon)^{1+\alpha} &\geq -(1 - \pi_1 - \varepsilon)^\alpha + (1 - \pi_1 - \varepsilon)^{1+\alpha} \\ (\pi_1 + \varepsilon)^\alpha [\pi_1 + \varepsilon - 1] &\geq (1 - \pi_1 - \varepsilon)^\alpha [1 - \pi_1 - \varepsilon - 1] \\ (\pi_1 + \varepsilon)^\alpha [\pi_1 + \varepsilon - 1] &\geq (1 - \pi_1 - \varepsilon)^\alpha [-\pi_1 - \varepsilon] \end{aligned}$$

$$\left[ \frac{\pi_1 + \varepsilon}{1 - \pi_1 - \varepsilon} \right]^\alpha \geq \left[ \frac{\pi_1 + \varepsilon}{1 - \pi_1 - \varepsilon} \right]$$

Therefore in order this inequality be satisfied for all values of  $\varepsilon$  we need that  $\alpha \geq 1$ . Moreover it would also be true for all  $\tilde{\pi}_i \leq \max \tilde{\pi}_i$  given that we have shown that  $h$  is a decreasing function of  $\pi$ .

Therefore,  $DP(\alpha, k)^{N=2} \geq DP(\alpha, k)^{N=n+1}$  if  $\alpha \geq 1$

*Proof of necessity:*

By contradiction. We can show that if  $\alpha < 1$ , then there always exist a distribution of  $\pi$  such that the polarization before splitting one group is smaller than the new distribution, that is to say  $DP(\alpha, k)^{(N=2)} < DP(\alpha, k)^{(N=N+1)}$ .

Consider the case such that the distribution among two groups is composed by  $\pi_1$  and  $\pi_2$ . Then the distribution of  $N + 1$  groups is composed by  $\pi_1$  and  $\tilde{\pi}_2 = \tilde{\pi}_3 = \tilde{\pi}_4 = \dots = \tilde{\pi}_{N+1} = \pi$ , such that

$$\sum_{i=2}^{N+1} \tilde{\pi}_i = N\pi = \pi_2.$$

Let's now compute,

$$\begin{aligned} DP(\alpha, k)^{(2)} - DP(\alpha, k)^{(N+1)} &= \\ N\pi \left[ k \left( \frac{1}{k} - \pi^\alpha + \pi^{1+\alpha} \right) \right] - k\pi_2 \left( \frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha} \right) &= \end{aligned}$$

$$N\pi[k(\frac{1}{k} - \pi^\alpha + \pi^{1+\alpha})] - k(N\pi)(\frac{1}{k} - (N\pi)^\alpha + (N\pi)^{1+\alpha}) =$$

$$N\pi k[\frac{1}{k} - \pi^\alpha + \pi^{1+\alpha} - \frac{1}{k} + (N\pi)^\alpha - (N\pi)^{1+\alpha}] =$$

$$N\pi k[-\pi^\alpha + \pi^{1+\alpha} + (N\pi)^\alpha - (N\pi)^{1+\alpha}] =$$

we want to show that for  $\alpha < 1$ , there always exist a set of  $\pi \in [\pi^{**}, \frac{1}{N})$ , such that  $N\pi k[-\pi^\alpha + \pi^{1+\alpha} + (N\pi)^\alpha - (N\pi)^{1+\alpha}] < 0$

$$\text{that } -\pi^\alpha + \pi^{1+\alpha} + (N\pi)^\alpha - (N\pi)^{1+\alpha} < 0$$

$$\pi^\alpha(\pi - 1) + (N\pi)^\alpha(1 - N\pi) < 0$$

$$(N\pi)^\alpha(1 - N\pi) < -\pi^\alpha(\pi - 1)$$

$$(N\pi)^\alpha(1 - N\pi) < \pi^\alpha(1 - \pi)$$

$$(\frac{N\pi}{\pi})^\alpha < \frac{(1-\pi)}{(1-2\pi)}$$

$$N^\alpha < \frac{(1-\pi)}{(1-N\pi)}$$

Notice that if  $\tilde{\pi}_2 = \tilde{\pi}_3 = \tilde{\pi}_4 = \dots = \tilde{\pi}_{N+1} = \pi$ , then  $\pi < \frac{1}{N+1}$ . If  $\pi = \frac{1}{N+1}$ , then  $\frac{(1-\pi)}{(1-N\pi)}$  evaluated at  $\frac{1}{N+1}$  is  $N$ .

Moreover, for  $\alpha < 1$ ,  $N^\alpha < N$ . Therefore, there always exist a set of  $\pi' \in [\pi^{**}, \frac{1}{N+1})$ , such that  $N^\alpha < \frac{(1-\pi')}{(1-N\pi')} < N$ .

Therefore, for any  $\alpha < 1$ , there always exist a set of  $\pi' \in [\pi^{**}, \frac{1}{N+1})$ , such that  $DP(\alpha, k)^{(N=2)} < DP(\alpha, k)^{(N=N+1)}$  ■

### Proof of Lemma 1:

*Step 1:* Suppose there are  $N$  groups of any size. Take the biggest one and separate it from the others. Then merge all the other groups into one group. By property 1b the DP measure increases if and only if  $\alpha \geq 1$ . That is, in the new distribution the index is larger than in the original one if and only if  $\alpha \geq 1$ . This means that, given any distribution of  $N$  groups, we can always find another distribution on two groups where the DP index is larger if and only if  $\alpha \geq 1$ . This does not mean that the new distribution is more polarize as explain above, but that the index is larger.

*Step 2:* Suppose now that we only have two groups of  $\pi$  and  $(1 - \pi)$  sizes. The polarization index  $DP = k \sum_{i=1}^2 \pi_i^{1+\alpha}(1 - \pi_i) = k[\pi_1^{1+\alpha}(1 - \pi_1) + (1 - \pi_1)^{1+\alpha}\pi_1]$

It is easy to verify that for any  $\alpha$  this expression is maximized at  $\pi_1 = \pi_2 = 0.5$ , which means that this is a global maximum if  $\alpha \geq 1$ . ■

### Proof of Theorem 3:

Any three points discrete distribution can be written in the form  $(x, 1-2x, x)$  such that  $x \in [0, \frac{1}{2}]$ . Our purpose is to show under what conditions  $DP$  is an increasing function of  $x$ , the shifted mass

from the q group to any other group,

$$DP(x, 1 - 2x, x) < DP(\tilde{x}, 1 - 2\tilde{x}, \tilde{x}) \text{ for all } x < \tilde{x}.$$

Therefore the comparison of  $DP(p, q, p)$  and  $DP(p+x, q-2x, p+x)$ . would be the same as the comparison of

$$DP(x', 1 - 2x', x') \text{ and } DP(\tilde{x}, 1 - 2\tilde{x}, \tilde{x}) \text{ where } x' = p \text{ and } \tilde{x} = p + x$$

We can compute DP in this case as

$$DP(\alpha, k) = k[(2x^{1+\alpha}(1-x) + (1-2x)^{1+\alpha}2x)] = k[2x^{1+\alpha} - 2x^{2+\alpha} + (1-2x)^{1+\alpha}2x]$$

The first derivative of DP is

$$\begin{aligned} \frac{\partial DP}{\partial x}(\alpha, k) &= k[2(1+\alpha)x^\alpha - 2(2+\alpha)x^{1+\alpha} + (1+\alpha)(1-2x)^\alpha(-2)2x + (1-2x)^{1+\alpha}2] = \\ &2k\{x^\alpha[(1+\alpha) - (2+\alpha)x] + (1-2x)^\alpha[-2(1+\alpha)x + (1-2x)]\} = \\ &2k\{x^\alpha[1+\alpha-2x-x\alpha] + (1-2x)^\alpha[(1-2x) - 2x - 2x\alpha]\} \end{aligned}$$

Therefore  $\frac{\partial DP}{\partial x}$  evaluated at  $\alpha = 1$  is always positive given that

$$\frac{\partial DP(1, k)}{\partial x} = 2k[1 - 3x]^2 > 0 \forall x. \text{ Therefore if } \alpha = 1 \text{ then } \frac{\partial DP(1, k)}{\partial x} > 0 \text{ for any distribution.}$$

In addition the partial derivative,  $\frac{\partial DP(\alpha, k)}{\partial x}$ , evaluated at  $x = \frac{1}{3}$  is always equal to 0

$$\begin{aligned} &2k\{(\frac{1}{3})^\alpha[1+\alpha-2\frac{1}{3}-\frac{1}{3}\alpha] + (1-2\frac{1}{3})^\alpha[(1-2\frac{1}{3}) - 2\frac{1}{3} - 2\frac{1}{3}\alpha]\} = \\ &= k\{(\frac{1}{3})^\alpha[\frac{1}{3} + \frac{2}{3}\alpha] + (\frac{1}{3})^\alpha[-\frac{1}{3} - \frac{2}{3}\alpha]\} = 0 \text{ for all values of } \alpha \end{aligned}$$

The second derivative is

$$\begin{aligned} \frac{\partial^2 DP(\alpha, k)}{\partial x \partial x} &= 2k\{\alpha x^{\alpha-1}[(1+\alpha) - 2x - x\alpha] + x^\alpha[-2 - \alpha] + \\ &+ \alpha(1-2x)^{\alpha-1}(-2)[1 - 4x - 2x\alpha] + (1-2x)^\alpha[-4 - 2\alpha]\} = \\ &2k\{\alpha x^{\alpha-1}[1+\alpha-2x-x\alpha] - x^\alpha[2+\alpha] - \\ &- 2\alpha(1-2x)^{\alpha-1}[1-4x-2x\alpha] - (1-2x)^\alpha[4+2\alpha]\} \end{aligned}$$

Evaluating the second derivative at  $x = 1/3$  we obtain

$$\frac{\partial^2 DP(\alpha, k)}{\partial x \partial x} = (\frac{1}{3})^\alpha[3(2\alpha^2 - 2)]$$

This means that for  $\alpha = 1$ , then  $\frac{\partial^2 DP(1, k)}{\partial x \partial x} = 0$ , which implies that  $x = \frac{1}{3}$  is an inflection point.

However if  $\alpha < 1$ , then  $\frac{\partial^2 DP(\alpha, k)}{\partial x \partial x} < 0$ , which means that  $x = \frac{1}{3}$  is a maximum and if  $\alpha > 1$ , then

$\frac{\partial^2 DP(\alpha, k)}{\partial x \partial x} > 0$ , which means that  $x = \frac{1}{3}$  is a minimum. Therefore if  $x = \frac{1}{3}$  is a maximum, this

means that for any ball around  $x=1/3$  then  $DP(\alpha < 1, k)^{x=\frac{1}{3}+\epsilon} < DP(\alpha < 1, k)^{x=\frac{1}{3}}$  which violates

property 2. On the other side for  $\alpha > 1$   $x = \frac{1}{3}$  is a minimum which implies that  $DP(\alpha > 1, k)^{x=\frac{1}{3}-\epsilon}$

$> DP(\alpha > 1, k)^{x=\frac{1}{3}}$  which also violates property 2. Therefore the only DP measure that satisfy

property 3 for any distribution has a parameter  $\alpha = 1$ . ■



## **APPENDIX II.**

### **OPERATIONAL CRITERIA TO DEFINE GENOCIDES.**

(1) Authorities' complicity in mass murder must be established. Any persistent, coherent pattern of action by the state and its agents, or by a dominant social group, that brings about the destruction of a people's existence, in whole or in part, within the effective territorial control of a ruling authority is prima facie evidence of that state, or other, authority's responsibility. In situations of civil war (i.e., contested territorial control) either of the contending authorities may be deemed responsible for carrying out, or allowing, such actions.

(2) The physical destruction of a people requires time to accomplish: it implies a persistent, coherent pattern of action. Thus, only sustained episodes that last six months or more are included in the final dataset. This six month requirement is to a degree arbitrary. At the other end of the time spectrum are episodic attacks on a group that recur periodically, such as Iraqi government attacks on Kurds from 1960 to 1975. Annual codings are especially important for these kinds of episodes to permit tracking of peaks and lulls.

(3) The victims to be counted are unarmed civilians, not combatants. It rarely is possible to distinguish precisely between the two categories in the source materials. Certain kinds of tactics nonetheless are indicative of authorities' systematic targeting of noncombatants: massacres, unrestrained bombing and shelling of civilian inhabited areas, declaration of free fire zones, starvation by prolonged interdiction of food supplies, forced expulsion ("ethnic cleansing") accompanied by extreme privation and killings, etc.

(4) In principle, numbers provided in "body counts" do not enter the definition of what constitutes an episode. A "few hundred" killed constitutes as much a genocide or politicide as the deaths of thousands if the victim group is small in number to begin with.

Note: Definitions and operational guidelines are adapted from Barbara Harff and T. R. Gurr, "Victims of the State: Genocides, Politicides, and Group Repression from 1945 to 1995," pp. 33-58 in Albert J. Jongman (ed.), *Contemporary Genocides: Causes, Cases, Consequences* (Leiden: University of Leiden, PIOOM Interdisciplinary Research Program on Root Causes of Human Rights Violations, 1996).

### **APPENDIX III.**

#### **DEFINITION OF THE VARIABLES AND SOURCES OF INFORMATION.**

**GENOCIDE:** The data set we use comes from Gurr and Harff (1995) and Harff (2003), and it is available by in the The State Failure task force project. This is the only academically recognized dataset on genocides and politicide

**CW:** A dummy hat takes value 1 if there is a civil war during the period and zero otherwise. The data comes from Uppsala/PRIO.

**LGDP:** Log of real GDP per capita of the initial period (1985 international prices) from the Penn World Tables 5.6.

**LNPOP:** Log of the population al the beginning of the period from the Penn World Tables.5.6.

**PRIMEXP:** Proportion of primary commodity exports of GDP. Primary commodity exports. Source: Collier and Hoeffler (2001).

**MOUNTAINS:** Percent Mountainous Terrain: This variable is based on work by geographer A.J Gerard for the World Bank's "Economics of Civil war, Crime, and Violence" project.

**NONCONT:** Noncontiguous state: Countries with territory holding at least 10,000 people and separated from the land area containing the capital city either by land or by 100 kilometers of water were coded as "noncontiguous." Source: Fearon and Laitin (2003)

**DEMOCRACY:** Democracy score: general openness of the political institutions (0=low; 10=high). Source: Polity III data set. (<http://weber.ucsd.edu/~kgledits/Polity.html>). We transform the score in a dummy variable that takes value 1 if the score is higher or equal to 4. This variable is very correlated with the variable Freedom of the Freedom House.

**ETHFRAC:** index of ethnolinguistic fractionalization calculated using the data of the World Christian Encyclopedia.

**ETHPOL:** index of ethnolinguistic polarization calculated using the data of the World Christian Encyclopedia.

**ETHDOM:** ethnic dominance. It takes value 1 if an ethnic group represents between 45% and 90% of the total population following the suggestion of Collier (2001) and Collier and Hoeffler (2004). It is calculated using the data of the World Christian Encyclopedia

## References

- [1] Aghion, P., Alesina, A. and F. Trebbi (2004), "Endogenous political institutions," *Quarterly Journal of Economics*, 119 (2), 565-612.
- [2] Alesina, A., Devleeschauwer, A., Easterly, W., Kurlat, S. and R. Wacziarg (2003) "Fractionalization." *Journal of Economic Growth*, 8, 155-194.
- [3] Atlas Narodov Mira (Atlas of the People of the World). Moscow: Glavnoe Upravlenie Geodezii i Kartografii, 1964. Bruck, S.I., and V.S. Apenchenko (eds.).
- [4] Anderson, G. (2004), "Toward an empirical analysis of polarization," *Journal of Econometrics*, 122, 1-26.
- [5] Barret, D. ed. (1982) *World Christian Encyclopedia*. Oxford University Press.
- [6] Caselli and Coleman (2006) "On the theory of ethnic Conflict." NBER Working Paper 12125.
- [7] Collier, P. (2001) "Implications of Ethnic Diversity." *Economic Policy* (April).
- [8] Collier, P., and A. Hoeffler (2004) "Greed and Grievances in Civil Wars." *Oxford Economic Papers*, 56, 563-595.
- [9] Doyle, M. W., and N. Sambanis (2000) "International Peacebuilding: A Theoretical and Quantitative Analysis." *American Political Science Review* 94:4 (December).
- [10] Duclos, J., Esteban, J. and D. Ray (2004), "Polarization: concept, measurement and estimation," *Econometrica*, 74, 1737-1772.
- [11] Easterly, W., and Levine (1997) "Africa's growth tragedy: Policies and Ethnic divisions." *Quarterly Journal of Economics*.
- [12] Easterly, W., Gatti, R. and S. Kurlat (2006), "Development, democracy and mass killings," *Journal of Economic Growth*, 11 (2), 129-156.
- [13] Encyclopedia Britannica.
- [14] Esteban, J., and Ray (1994) "On the measurement of polarization." *Econometrica* 62(4): 819-851.

- [15] Fearon, J. (2003), "Ethnic and cultural diversity by country," *Journal of Economic Growth*, 8, 195-222.
- [16] Fearon J. and Laitin, D. (2003). "Ethnicity, Insurgency, and Civil War", *American Political Science Review* 97 (February )
- [17] Gradin (2002), "Polarization and inequality in Spain: 1973-91," *Journal of Income Inequality*, 11 (1-2), 34-52.
- [18] Gradin (2000), "Polarization by subpopulations in Spain: 1973-91," *Review of Income and Wealth*, 46 (4), 457-474.
- [19] Harff,B. (2003), "No Lessons Learned from Holocaust? Assessing Risks of Genocide and Political Mass Murder since 1955", *American Political Science Review*, 97(1 ),57-73.
- [20] Horowitz, D. (1985). *Ethnic Groups in Conflict*. Berkeley: University of California Press.
- [21] Keefer, P. and S. Knack (2002) "Polarization, politics and property rights: links between inequality and growth." *Public Choice* 11: 127-154.
- [22] Licklider, R. (1993) *Stopping the Killing: How Civil Wars End*. NY: NYU Press.
- [23] \_\_\_\_\_(1995) "The consequences of negotiated settlements in Civil Wars, 1945-1993." *American Political Science Review* **89**(3): 681-690.
- [24] Mauro, P. (1995)"Corruption and Growth." *Quarterly Journal of Economics*, CX, 681-712.
- [25] Montalvo, J.G. and Reynal-Querol, M. (2005) "Ethnic Polarization, Potential Conflict, and Civil Wars", *American Economic Review*, 95(3), 796-816.
- [26] Reynal-Querol, M. (2002) "Ethnicity, Political Systems, and Civil Wars." *Journal of Conflict Resolution* 46(1, February): 29-54.
- [27] \_\_\_\_\_ (2001), "Religious Conflict and Growth", chapter 1 of PhD thesis in the LSE, London.
- [28] Singer, J.D. and M. Small (1982) *Resort to arms: International and civil war, 1816-1980*. Beverly Hills, CA: Stage.
- [29] Singer, J.D. and M. Small (1994) "Correlates of War Project: International and Civil War Data, 1816-1992." (ICPSR 9905). Ann Arbor, Mich.

- [30] Taylor, C., and M.C. Hudson (1972) *The World Handbook of Political and Social Indicators*, 2nd ed.(New Haven, CT: Yale University Press.
- [31] The Statesman's Yearbook.
- [32] Vanhaven, T. (1999). "Domestic Ethnic Conflict and Ethnic Nepotism: A Comparative Analysis" *Journal Of Peace Research*, vol.36, no.1, pp. 55-73.
- [33] Wolfson, M.C. (1994), "When inequalities diverge," *American Economic Review*, 84 (2), (Papers and Proceedings), 353-58.
- [34] World Factbook (several years).

TABLE1: Summary Statistics

	All sample		Sub-Saharan Africa		Latin America		Asia		OECD	
	mean	Std.	mean	Std.	mean	Std.	mean	Std.	mean	Std
LGDP	7.72	1.05	6.76	0.62	7.936	0.60	7.82	0.72	9.07	0.51
LPOP	15.39	1.94	15.15	1.42	14.69	1.96	14.95	2.45	16.09	1.83
PRIMEXP	0.16	0.18	0.17	0.14	0.20	0.25	0.18	0.16	0.08	0.06
MOUNT	15.19	20.04	12.73	21.72	18.19	17.89	10.31	14.30	17.05	20.67
NONCONT	0.15	0.35	0.02	0.15	0.03	0.18	0.43	0.49	0.45	0.49
DEMOC	0.45	0.49	0.20	0.40	0.58	0.49	0.50	0.50	0.957	0.20
ETHPOL	0.51	0.24	0.53	0.19	0.64	0.20	0.45	0.27	0.35	0.24
ETHFRAC	0.44	0.27	0.63	0.26	0.44	0.19	0.35	0.27	0.23	0.20
ETHDOM	0.52	0.49	0.39	0.48	0.75	0.42	0.43	0.49	0.41	0.49
ELF	0.41	0.29	0.65	0.24	0.26	0.21	0.46	0.28	0.21	0.20

TABLE 2: Table of correlation across ethnic indicators

Panel A	All sample						
	ETHPOL	ETHFRAC	ETHDOM				
ETHFRAC	0.57						
ETHDOM	0.53	-0.10					
ELF	0.45	0.86	-0.165				
Panel B	Over median of ETHFRAC			Below median of ETHFRAC			
	ETHPOL	ETHFRAC	ETHDOM		ETHPOL	ETHFRAC	ETHDOM
ETHFRAC	-0.79			ETHFRAC	0.93		
ETHDOM	0.61	-0.80		ETHDOM	0.85	0.81	
ELF	-0.55	0.66	-0.60	ELF	0.67	0.65	0.587
Panel C	Over percentile 25 ETHFRAC			Below percentil25 ETHFRAC			
	ETHPOL	ETHFRAC	ETHDOM		ETHPOL	ETHFRAC	ETHDOM
ETHFRAC	0.07			ETHFRAC	0.59		
ETHDOM	0.22	-0.77		ETHDOM	0.69	-0.14	
ELF	-0.01	0.80	-0.68	ELF	0.82	0.54	0.539
Panel D	Over percentile 75 ETHFRAC			Below percentile 75 ETHFRAC			
	ETHPOL	ETHFRAC	ETHDOM		ETHPOL	ETHFRAC	ETHDOM
ETHFRAC	-0.98			ETHFRAC	0.92		
ETHDOM	0.43	-0.53		ETHDOM	0.67	0.53	
ELF	-0.54	0.57	-0.31	ELF	0.67	0.71	0.368

TABLE 3  
Logit regressions for the incidence of genocides (including politicides)

	(1)	(2)	(3)	(4)	(5)	(6)
	GEN& POLIT	GEN& POLIT	GEN& POLIT	GEN& POLIT	GEN& POLIT	GEN& POLIT
Constant	-7.13 (-2.72)	-8.17 (-2.94)	-7.76 (-2.73)	-7.86 (-2.58)	-9.21 (-3.00)	-9.88 (-3.25)
LGDPC	-0.41 (-1.28)	-0.45 (-1.47)	-0.57 (-1.83)	-0.43 (-1.52)	-0.40 (-1.51)	-0.48 (-1.71)
LPOP	0.42 (4.31)	0.43 (4.14)	0.47 (4.85)	0.46 (3.17)	0.46 (3.58)	0.55 (3.91)
PRIMEXP	2.11 (1.32)	1.93 (1.25)	2.23 (1.44)	2.19 (1.23)	1.86 (1.09)	2.27 (1.31)
MOUNTAINS	0.01 (1.42)	0.01 (0.78)	0.01 (0.59)	0.01 (1.41)	0.01 (0.82)	0.008 (0.63)
NONCONT	0.73 (0.92)	0.91 (1.21)	0.99 (1.32)	0.66 (0.65)	0.85 (0.89)	1.018 (1.09)
DEMOCRACY	-1.07 (-1.71)	-1.09 (-1.79)	-1.08 (-1.79)	-1.07 (-1.72)	-1.10 (-1.78)	-1.10 (-1.79)
ETHFRAC	0.28 (0.43)		-0.96 (-0.99)	0.12 (0.16)		-1.37 (-1.22)
ETHPOL		2.32 (2.17)	2.75 (2.35)		2.25 (1.98)	2.90 (2.25)
Regional dummies	No	No	No	Yes	Yes	Yes
PseudoR2	0.137	0.15	0.16	0.14	0.16	0.164
N	846	846	846	846	846	846

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.



TABLE 4  
Logit regressions on the incidence of genocides

	(1)	(2)	(3)	(4)	(5)	(6)
	GEN	GEN	GEN	GEN	GEN	GEN
	(excluding Chile and El Salvador)	(excluding Chile and El Salvador)	(excluding Chile and El Salvador)	(excluding Chile and El Salvador)	(excluding Chile and El Salvador)	(excluding Chile and El Salvador)
Constant	-7.22 (-2.53)	-8.41 (-2.71)	-7.98 (-2.51)	-7.76 (-2.42)	-9.82 (-3.04)	-10.30 (-3.22)
LGDPC	-0.44 (-1.25)	-0.50 (-1.49)	-0.61 (-1.78)	-0.41 (-1.37)	-0.38 (-1.37)	-0.46 (-1.55)
LPOP	0.43 (4.32)	0.46 (4.00)	0.49 (4.68)	0.45 (3.07)	0.48 (3.55)	0.56 (3.90)
PRIMEXP	1.98 (1.19)	1.83 (1.15)	2.10 (1.31)	1.94 (1.07)	1.69 (0.97)	2.02 (1.15)
MOUNTAINS	0.01 (1.12)	0.006 (0.41)	0.003 (0.26)	0.01 (1.18)	0.01 (0.53)	0.005 (0.37)
NONCONT	0.91 (1.13)	1.11 (1.48)	1.19 (1.57)	0.79 (0.74)	1.04 (1.05)	1.18 (1.22)
DEMOCRAC Y	-1.35 (-1.94)	-1.37 (-2.02)	-1.35 (-2.03)	-1.30 (-1.83)	-1.33 (-1.89)	-1.34 (-1.91)
ETHFRAC	0.39 (0.57)		-0.86 (-0.83)	0.33 (0.42)		-1.22 (-1.03)
ETHPOL		2.64 (2.19)	2.99 (2.34)		2.77 (2.28)	3.29 (2.47)
Regional dummies	No	No	No	Yes	Yes	Yes
PseudoR2	0.156	0.181	0.183	0.1575	0.1831	0.1869
N	846	846	846	846	846	846

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.

TABLE 5  
Logit regressions of the incidence of genocides with civil wars

	(1)	(2)	(3)	(4)	(5)	(6)
	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war
Constant	-3.22 (-0.93)	-4.65 (-1.12)	-4.24 (-1.04)	-4.37 (-1.34)	-7.28 (-1.94)	-7.65 (-2.07)
LGDPC	-0.64 (-1.56)	-0.72 (-1.77)	-0.81 (-2.15)	-0.55 (-1.76)	-0.50 (-1.65)	-0.57 (-1.86)
LPOP	0.25 (1.77)	0.29 (1.74)	0.33 (1.89)	0.28 (1.63)	0.33 (1.88)	0.41 (2.19)
PRIMEXP	2.15 (1.39)	1.92 (1.24)	2.11 (1.36)	2.03 (1.23)	1.70 (1.02)	1.95 (1.16)
MOUNTAINS	0.01 (0.97)	0.005 (0.30)	0.00 (0.16)	0.01 (1.13)	0.008 (0.51)	0.004 (0.33)
NONCONT	1.50 (1.79)	1.70 (2.13)	1.76 (2.26)	1.33 (1.26)	1.62 (1.64)	1.75 (1.82)
DEMOCRAC Y	-1.25 (-1.62)	-1.30 (-1.69)	-1.30 (-1.69)	-1.18 (-1.51)	-1.24 (-1.55)	-1.26 (-1.56)
ETHFRAC	0.51 (0.71)		-0.77 (-0.71)	0.42 (0.57)		-1.21 (-1.05)
ETHPOL		2.94 (2.15)	3.22 (2.28)		3.21 (2.41)	3.67 (2.55)
Regional dummies	No	No	No	Yes	Yes	Yes
PseudoR2	0.153	0.182	0.183	0.1560	0.188	0.191
N	846	846	846	846	846	846

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.



TABLE 6  
Logit regressions for the incidence of genocides and politicides, and genocides with civil wars  
Robustness to regional effects

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	GEN& POL	GEN& POL	GEN& POL	GEN& POL	GEN& POL	GEN& POL	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war	GEN& Civil war
Constant	-8.61 (-3.07)	-8.18 (-2.91)	-7.94 (-2.22)	-7.36 (-1.98)	-7.72 (-2.64)	-6.97 (-2.32)	-6.74 (-1.91)	-6.21 (-1.71)	-4.49 (-1.00)	-4.05 (-0.92)	-3.49 (-0.79)	-2.55 (-0.58)
LGDP	-0.42 (-1.51)	-0.38 (-1.39)	-0.55 (-1.40)	-0.66 (-1.63)	-0.39 (-1.11)	-0.59 (-1.59)	-0.66 (-1.84)	-0.57 (-1.67)	-0.68 (-1.53)	-0.77 (-1.82)	-0.69 (-1.51)	-0.90 (-2.05)
LPOP	0.47 (3.96)	0.42 (3.60)	0.46 (3.53)	0.49 (4.18)	0.37 (3.84)	0.44 (5.34)	0.38 (2.08)	0.30 (1.47)	0.29 (1.64)	0.31 (1.78)	0.19 (1.11)	0.26 (1.56)
PRIMEXP	1.40 (1.03)	1.10 (0.78)	2.09 (1.25)	2.33 (1.39)	1.78 (0.85)	2.41 (1.13)	1.34 (0.84)	0.81 (0.52)	2.02 (1.27)	2.17 (1.36)	1.95 (1.05)	2.51 (1.30)
MOUNTAINS	-0.004 (-0.25)	-0.005 (-0.29)	0.004 (0.30)	0.00 (0.15)	0.01 (0.90)	0.007 (0.58)	-0.03 (-1.22)	-0.03 (-1.23)	0.004 (0.28)	0.002 (0.15)	0.007 (0.45)	0.002 (0.16)
NONCONT	0.16 (0.22)	-0.007 (-0.01)	1.21 (1.52)	1.29 (1.61)	0.33 (0.22)	0.47 (0.32)	1.14 (1.35)	0.82 (1.14)	1.67 (1.93)	1.72 (2.03)	1.31 (1.00)	1.46 (1.12)
DEMOCRACY	-1.05 (-1.61)	-1.07 (-1.64)	-1.406 (-1.67)	-1.39 (-1.69)	-0.79 (-1.21)	-0.74 (-1.18)	-1.41 (-1.61)	-1.14 (-1.66)	-1.22 (-1.41)	-1.22 (-1.42)	-0.86 (-1.02)	-0.81 (-1.01)
ETHPOL	2.70 (2.30)	2.17 (1.29)	2.49 (1.98)	2.78 (2.15)	2.44 (2.26)	3.31 (2.65)	4.65 (3.31)	3.82 (1.95)	2.49 (1.94)	2.7 (2.00)	3.26 (2.34)	4.00 (2.72)
ETHFRAC		0.93 (0.63)		-0.807 (-0.78)		-1.82 (-1.54)		1.71 (0.95)		-0.62 (-0.62)		-1.82 (-1.39)
Eliminated region	SAfrica	SAfrica	Latin Am.	Latin Am.	Asiae	Asiae	SAfrica	SAfrica	Latin Am.	Latin Am.	Asiae	Asiae
PseudoR2	0.167	0.169	0.194	0.197	0.129	0.139	0.204	0.210	0.176	0.177	0.141	0.150
N	580	580	678	678	781	781	580	580	678	678	781	781

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.

TABLE 7  
Logit regressions for the incidence of genocides

	(1)	(2)	(3)	(4)	(5)	(6)
	GEN& POLIT	GEN& POLIT	GEN& POLIT	GEN& POLIT	GEN (excluding El Salvador and Chile)	GEN& Civil war
Constant	-9.35 (-2.92)	-9.56 (-3.08)	-7.75 (-2.57)	-8.57 (-2.74)	-9.68 (-2.93)	-7.44 (-1.96)
LGDP	-0.48 (-1.80)	-0.46 (-1.68)	-0.46 (-1.60)	-0.44 (-1.49)	-0.39 (-1.28)	-0.49 (-1.38)
LPOP	0.53 (3.69)	0.50 (3.61)	0.48 (3.65)	0.47 (3.67)	0.50 (3.70)	0.38 (2.16)
PRIMEXP	2.81 (1.74)	2.28 (1.37)				
MOUNTAINS	0.01 (1.25)	0.01 (0.93)				
NONCONT	1.05 (1.10)	1.05 (1.11)				
DEMOCRACY	-1.12 (-1.77)	-1.13 (-1.81)	-0.90 (-1.74)	-0.90 (-1.76)	-1.06 (-1.86)	-0.83 (-1.28)
ETHDOM	0.93 (1.83)	0.68 (1.20)	0.70 (1.39)	0.36 (0.59)	0.38 (0.58)	0.21 (0.32)
ETHPOL		1.58 (1.33)		1.95 (1.70)	2.29 (1.81)	2.77 (2.06)
Regional dummies	Yes	Yes	Yes	Yes	Yes	Yes
PseudoR2	0.158	0.167	0.132	0.147	0.170	0.158
N	846	846	859	859	859	859

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.

TABLE 8

Logit regressions for the incidence of major, intermediate and minor civil wars

	(1)	(2)	(3)	(4)	(5)	(6)
	Major	Major	Intermediate	Intermediate	Minor	Minor
Constant	-4.01 (-1.32)	-4.33 (-1.54)	-7.98 (-2.30)	-7.93 (-2.31)	-0.91 (-0.48)	-1.01 (-0.55)
LGDPC	-0.69 (-3.09)	-0.62 (-2.78)	-0.23 (-0.99)	-0.25 (-1.01)	-0.70 (-4.57)	-0.66 (-3.84)
LPOP	0.33 (2.27)	0.30 (1.94)	0.40 (2.32)	0.40 (2.18)	0.21 (2.11)	0.20 (1.79)
PRIMEXP	0.14 (0.09)	-0.02 (-0.01)	-2.25 (-0.89)	-2.19 (-0.86)	0.55 (0.46)	0.48 (0.38)
MOUNTAINS	0.00 (0.40)	0.005 (0.54)	-0.00 (-0.22)	-0.002 (-0.24)	0.003 (0.59)	0.00 (0.65)
NONCONT	0.34 (0.55)	0.30 (0.50)	0.40 (0.57)	0.40 (0.61)	0.67 (1.51)	0.66 (1.47)
DEMOCRACY	0.04 (0.09)	0.04 (0.09)	0.25 (0.55)	0.25 (0.55)	0.33 (1.12)	0.33 (1.12)
ETHFRAC		0.57 (0.52)		-0.15 (-0.14)		0.23 (0.29)
ETHPOL	2.55 (2.74)	2.33 (2.16)	2.11 (2.75)	2.19 (2.19)	0.61 (1.02)	0.49 (0.63)
PseudoR2	0.128	0.129	0.111	0.112	0.082	0.082
N	846	846	846	846	846	846

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.

TABLE 9  
 Logit regressions for the incidence of genocides  
 Robustness to a reduced set of variables and cross section analysis

	(1)	(2)	(3)	(4)	(5)	(6)
	Five year periods			Cross section		
	GEN& POLIT	GEN (excluding El Salvador and Chile)	GEN& Civil war	GEN& POLIT	GEN (excluding El Salvador and Chile)	GEN& Civil war
Constant	-8.04 (-2.53)	-8.67 (-2.53)	-5.91 (-1.42)	-4.68 (-1.22)	-7.13 (-1.59)	-4.10 (-0.88)
LGDPC	-0.45 (-1.49)	-0.45 (-1.36)	-0.57 (-1.50)	-0.59 (-1.45)	-0.61 (-1.38)	-0.67 (-1.45)
LPOP	0.46 (4.54)	0.49 (4.49)	0.35 (2.14)	0.66 (2.64)	0.81 (2.80)	0.47 (1.89)
DEMOCRACY	-0.89 (-1.70)	-1.09 (-1.89)	-0.86 (-1.31)	-0.69 (-0.97)	-0.92 (-1.16)	-0.25 (-0.32)
ETHFRAC	-0.43 (-0.38)	-1.10 (-0.09)	0.23 (0.17)	-1.14 (-0.64)	-0.12 (-0.06)	0.75 (0.37)
ETHPOL	2.51 (2.32)	2.51 (2.23)	2.53 (2.03)	3.87 (2.16)	4.60 (2.16)	4.23 (1.95)
PseudoR2	0.141	0.160	0.136	0.231	0.302	0.232
N	859	859	859	91	91	91

\* Cluster-corrected standard errors are used to calculate the z-statistics included between parenthesis.

