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Discrete singular convolution and its application to the analysis of plates with internal supports. Part 2: Applications

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SUMMARY

Part 2 of this series of two papers presents the applications of the discrete singular convolution (DSC) algorithm. The main purpose of this paper is to explore the utility, test the accuracy and examine the convergence of the proposed approach for the vibration analysis of rectangular plates with internal supports. Both partial internal line supports and complex internal supports are considered for 21 square plates of various combinations of edge support conditions. The effects of different size, shape and topology of the internal supports and different boundary conditions on the vibration response of plates are investigated. The partial internal line supports may vary from a central point support to a full range of cross or diagonal line supports. Several closed-loop supports, such as ring, square and rhombus, and their combinations are studied for complex internal supports. Convergence and comparison studies are carried out to establish the correctness and accuracy of the DSC algorithm. The DSC results are compared with those in the available literature obtained by using other methods. Numerical results indicate that the DSC algorithm exhibits controllable accuracy for plate analysis and shows excellent flexibility in handling complex geometries, boundary conditions and support conditions. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: vibration analysis; plates; partial internal line supports; complex internal supports; discrete singular convolution

1. INTRODUCTION

In Part 2 of this series of two papers we explore the usefulness, test the accuracy, and examine the convergence of the discrete singular convolution (DSC) algorithm [1–7] for the vibration analysis of plates with internal supports. Part 2 is a natural extension of the preceding paper

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which formulates the theory and algorithm for the present applications. Particular consideration is given to plates with both partial internal line supports and complex internal supports, which appear frequently in modern engineering structures. For instance, bridge slabs or floor systems may be modelled as plates being not only supported along their edges but also supported by load-bearing walls that can be treated as internal line supports. Other regular internal supports, such as circular internal supports, square internal supports and their combinations, are of practical importance to engineering designs of plates. There is an increasing demand for a reliable theoretical analysis of these plates from engineers. However, it is very challenging to provide a reliable theoretical analysis of plates with complex internal supports due to possible numerical instability. What is required is a numerical method that combines the global methods' accuracy and the local methods' flexibility for the implementation of complex internal support conditions.

The vibration of plates with internal line supports has been investigated by many researchers since the early work by Veletsos and Newmark [8]. By using the Holzer method, Veletsos and Newmark [8] studied the vibration of a rectangular plate with one internal straight line support perpendicular to the edges of the plate. Ungar [9] treated a similar problem by using a semi-graphical approach. Using Bolotin's edge effect approach, Bolotin [10], and later, Moskalenko and Chen [11] analysed plates with two and three internal line supports in one direction. Cheung and Cheung [12] studied multi-span plates with the finite strip method. The modified Bolotin method was employed by Elishakoff and Sternberg [13] to treat multi-span plates in one direction. A similar problem was also studied by Azimi *et al.* [14] by the receptance method. The B-spline functions in association with the Rayleigh–Ritz method were utilized by Mizusawa and Kajita [15] to analyse the free vibration of one-direction continuous plates with arbitrary boundary conditions.

The vibration of rectangular plates with internal line supports which are continuous in two directions has also been studied by many researchers. For example, by using a sine series analysis Takahashi and Chishaki [16] considered rectangular plates with a number of line supports in two directions. In the framework of their finite strip method, Wu and Cheung [17] analysed the free vibration of continuous rectangular plates in one or two directions by using the multi-span vibrating beam functions. In the framework of the Rayleigh–Ritz method, Kim and Dickinson [18] used a set of one-dimensional orthogonal polynomial functions to analyse the free vibration of plates with internal line supports. Liew and Lam [19] analysed eigenmodes and eigenfrequencies of multi-span plates by using a set of two-dimensional orthogonal polynomial functions. Using the pb-2 Ritz method, Liew *et al.* [20] studied the vibration of rectangular Mindlin plates with internal line supports in one or two directions and in the plate diagonal directions. Their method is able to analyse plates with internal straight line supports in an arbitrary direction. Zhou [21–23] modified vibrating beam functions by appropriate polynomials to account for the internal line supports in one or two directions. Cheung and co-workers [24, 25] advanced this approach by combining Zhou's trial functions with the finite layer method to study the vibration of shear-deformable plates with intermediate line supports.

The aforementioned studies are all concentrated on plates with continuous line supports. There is, however, not much work available on the vibration of plates with partial internal line supports. Liew and Wang [26] investigated the vibration of triangular plates with partial internal curved line supports by the pb-2 Ritz method. The point simulation approach was employed to treat the partial curved line supports in the plates which in turn damaged the completeness of the pb-2 Ritz method. Moreover, the analysis of plates with complex internal supports has not received much attention, although there is a great deal of literature on the analysis of plates with internal (full) line and ring supports [8–21, 24, 26]. It is relatively easy for the pb-2 Ritz method [20, 27] to treat plate with complex ring supports and any other internal support topology as long as it can be expressed as a continuous function. However, the efficiency of the Ritz method is dramatically reduced if the internal support topology cannot be analytically expressed [20, 27]. In contrast, local methods, such as finite element approaches, are certainly flexible in imposing the complex internal support conditions. However, the speed of convergence of conventional local methods is relatively low under complex internal support conditions due to the low order approximations used. In this paper, we demonstrate the efficiency and robustness of the DSC algorithm for the treatment of this class of problems.

This paper is organized as follows. In Section 2, we apply the DSC algorithm to the analysis of plates with internal partial line supports, which may vary from a single central point support to partial cross or diagonal line supports. The effectiveness and convenience of the DSC method for treating internal supports are demonstrated. The validity and accuracy of the DSC method for vibration analysis of plates are verified by convergence and comparison studies. Extensive frequency parameters are tabulated for square plates of 21 distinct combinations of edge support conditions with internal supports. Vibration characteristics of square plates with internal supports are examined. Analysis results of plates with complex internal supports are presented in Section 3. A few typical internal support topologies, including ring, square, rhombus and their combinations are considered. Convergence studies are performed to establish the number of DSC grid points required for converged frequencies. The DSC results are compared with available solutions from open literature to validate the correctness of the DSC method. Extensive frequency parameters are presented for square plates of various combination of edge support conditions and having several closed-loop internal supports. These results may serve as useful references for engineers when they design plate structures with complex internal supports. The paper ends with a conclusion.

2. PARTIAL INTERNAL LINE SUPPORT

The application of the DSC algorithm is demonstrated in this section through extensive numerical studies on vibration of square plates with partial internal line supports. Figure 1 depicts 21 distinct boundary conditions which are obtained by combinations of simply supported, elastically supported and clamped edges. For brevity, we shall use the letters S for simply supported edge, C for clamped edge and E for the elastically supported edge and a four-letter designation to represent the edge conditions of the plate. For instance, an ESCS plate will have an elastically supported edge along $X=0$, a simply supported edge along $Y=0$, a clamped edge along $X=1$ and a simply supported edge along $Y=1$, respectively. As shown in Figure 2. the partial internal line supports are arranged to be seven distinct types, i.e. type I: full length diagonal line supports; type II: two-thirds diagonal line supports; type III: one-third diagonal line supports; type IV: a central point support; type V: one-third cross line supports; type VI: two-thirds cross line supports; and type VII: full length cross line supports. Frequency parameters are computed for the combinations of all cases and types. The Poisson

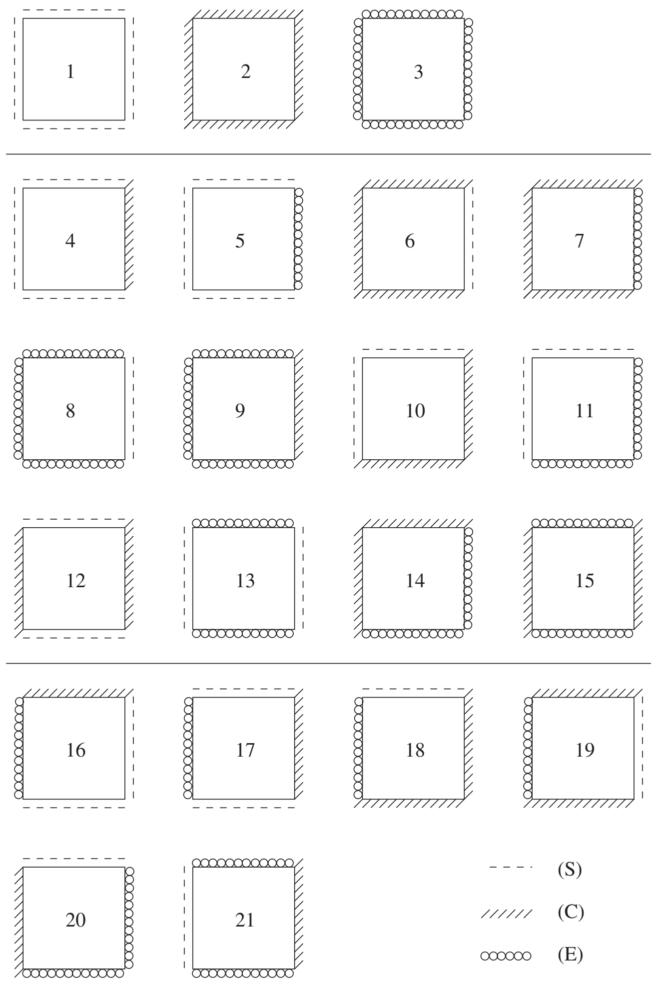


Figure 1. Twenty-one cases of boundary conditions studied in the paper.

ratio v is taken as 0.3 when needed and the vibration frequency is expressed in terms of a non-dimensional frequency parameter given by $(\omega a^2/\pi^2)\sqrt{\rho h/D}$, where $D = Eh^3/[12(1-v^2)]$ and ω, a, ρ, h and E are circular frequency, length, mass density, thickness of the plate and modulus of elasticity, respectively.

2.1. Convergence and comparison studies

To verify the validity and accuracy of the proposed DSC approach, convergence and comparison studies are first carried out for the vibration analysis of plates with internal line supports.

Convergence study is performed on a simply supported square plate (Case 1) with full length cross supports and a simply supported square plate with full length diagonal supports,

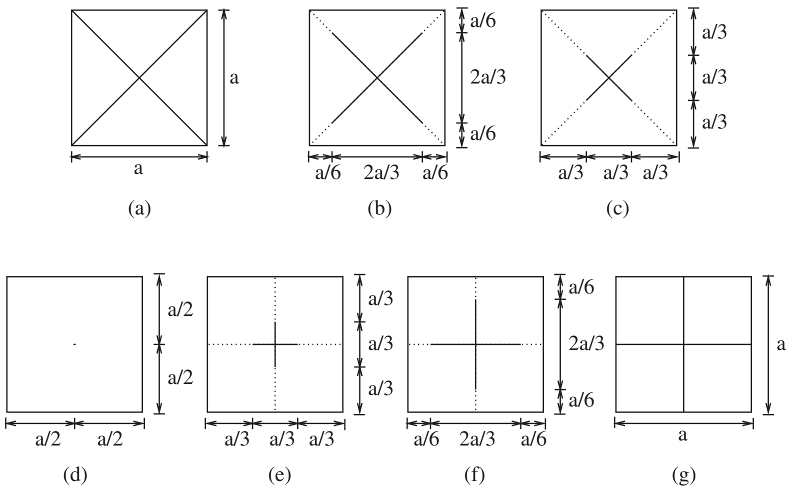
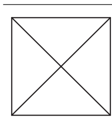
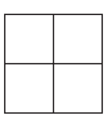


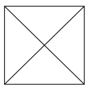
Figure 2. The types and locations of internal point/line supports: (a) Type I; (b) Type II; (c) Type III; (d) Type IV; (e) Type V; (f) Type VI; and (g) Type VII.

Table I. Convergence study of frequency parameters for simply supported square plates (Case 1) with internal line supports (DSC parameters: $M=16, \sigma=2.5; M=32, \sigma=4.0$).

Support type	Mesh size	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
	$N=17$	10.0000	12.2724	12.2724	14.8853	20.0000	23.2292	23.2292	26.0003
	$N=33$	10.0000	12.2681	12.2681	14.8739	20.0000	23.2154	23.2154	26.0000
	$N=65$	10.0000	12.2677	12.2677	14.8729	20.0000	23.2141	23.2141	26.0000
	$N=17$	8.0000	9.5909	9.5909	10.9796	20.0000	20.0000	20.9592	20.9592
	$N=33$	8.0000	9.5842	9.5842	10.9661	20.0000	20.0000	20.9445	20.9445
	$N=65$	8.0000	9.5836	9.5836	10.9648	20.0000	20.0000	20.9430	20.9430

respectively. Frequency parameters for the two plates vibrating in the first eight modes are presented in Table I. The number of DSC grid points used to generate the frequency parameters are chosen as 17^2 (i.e. $N_x=N_y=16$), 33^2 (i.e. $N_x=N_y=32$) and 65^2 (i.e. $N_x=N_y=64$). It is observed that the frequency parameters for the two plates decrease monotonically as the number of grid points increases. It is noted that when the number of DSC grid points is 17^2, the frequency parameters for the first eight modes in the two plates converge to a satisfactory level. To ensure a certain level of reliability, all vibration frequencies presented in the rest of the section are calculated based on the number of DSC grid points N^2 being 37^2. Their DSC parameters are chosen as $M=25$ and $\sigma/\Delta=2.8$. Three sets of candidate DSC parameters $(\sigma/\Delta, M)$ are used in this section, i.e. $(2.5, 16)$, $(2.8, 25)$ and $(4.0, 32)$, respectively [2]. The non-dimensional spring coefficient is set to $K'=100$.

Table II. Comparison studies of frequency parameter for square plate with internal diagonal line supports.

		Mode sequences							
		1	2	3	4	5	6	7	8
Case 1	[20]	9.9999	12.3400	12.3400	15.1034	19.9993	23.4199	23.4200	25.9988
	DSC	10.0000	12.2680	12.2680	14.8737	20.0000	23.2151	23.2151	26.0000
Case 4	[20]	10.4253	12.4259	13.6231	17.0286	20.6182	23.6853	25.1234	26.7506
	DSC	10.4177	12.3434	13.5081	16.7925	20.5954	23.4331	24.8088	26.6964
Case 12	[20]	10.8364	13.3212	15.0208	18.1296	21.2553	24.8655	26.9083	27.4347
	DSC	10.8201	13.2052	14.9380	17.8215	21.2046	24.5201	26.6987	27.3205
Case 6	[20]	11.6383	14.7193	16.0052	18.6497	22.3918	26.6496	28.1759	28.7712
	DSC	11.6096	14.6380	15.8795	18.3899	22.3095	26.4075	27.8812	28.5836
Case 2	[20]	13.3315	16.0837	16.0837	19.3171	24.5341	28.3772	28.3772	31.2966
	DSC	13.3335	15.9898	15.9898	19.0102	24.5411	28.1430	28.1430	31.3060
Case 10	[20]	11.2186	12.5125	15.8942	17.6098	21.6951	23.9464	27.8807	28.1655
	DSC	11.1996	12.4222	15.7776	17.3232	21.6413	23.6518	27.6357	28.0273

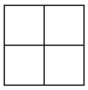
A comprehensive comparison study is carried out for square plates with various edge support conditions and internal line/point supports. Table II presents the frequency parameters of the first eight modes for square plates of various edge support conditions (Cases 1, 4, 12, 6, 2, 10) and with full length diagonal supports. The results are compared with those reported by Liew *et al.* [20] using the pb-2 Ritz method. It is seen from Table II that the DSC results are in good agreement with the Ritz results. It is noted that the values of the DSC results are slightly lower than the Ritz results for most cases.

Table III shows the vibration frequency parameters for square plates of various edge supports and with full length cross supports. For the simply supported square plate, the DSC results have been compared well with results from several other publications [17–21, 25, 28, 29]. For square plates with other edge supporting conditions (see Table III), the current results are compared with those reported by Liew *et al.* [20] using the pb-2 Ritz method and those of Cheung and Zhou [25]. All of these results are in close agreement in general. Particularly, an excellent consistence is found between the present results and those of Cheung and Zhou [25].

Table IV gives the vibration frequency parameters for square plates with a central point support. Many solutions have been found in open literature for a simply supported square plate with a central point support [29–36]. Except for the results reported by Nowacki [30] and Johns and Nataraja [32], the DSC results are in excellent agreement with results from the other researchers [29, 31, 33–36]. For square plates with other edge supporting conditions, the DSC results show close agreement with those by Leissa [29], Kim and Dickinson [33] and Liew *et al.* [36].

The convergence and comparison studies in this section have confirmed the validity and accuracy of the DSC method in dealing with square plates with internal line and point supports.

Table III. Comparison studies of frequency parameter for square plate with central transverse and longitudinal line supports.

		Mode sequences							
		1	2	3	4	5	6	7	8
Case 1	[17]	8.00	9.59	9.60	10.99	20.00	20.16	—	—
	[18]	8.000	9.718	9.718	11.23	20.16	20.16	—	—
	[19]	8.000	9.608	9.608	10.98	20.01	20.01	—	—
	[20]	7.9999	9.6164	9.6164	11.0739	19.9993	19.9993	21.0557	21.0557
	[21]	8.000	9.669	9.669	11.14	20.00	20.00	—	—
	[25]	8.000	9.583	9.583	10.965	20.00	20.00	—	—
	[28]	8.000	9.584	9.584	10.97	20.00	20.00	—	—
	[29]	8.000	9.583	9.583	10.96	20.00	20.00	—	—
	DSC	8.0000	9.5841	9.5841	10.9658	20.0000	20.0000	20.9442	20.9442
Case 4	[20]	8.4391	10.0137	11.0295	12.3498	20.2681	21.1911	21.8974	22.2687
	DSC	8.4354	9.9583	10.9928	12.2290	20.2560	21.1589	21.7779	22.0573
Case 12	[20]	8.8634	11.3867	11.3898	13.5187	21.4671	21.4743	23.0003	23.1798
	DSC	8.8520	11.3246	11.3246	13.3801	21.4022	21.4022	22.8149	22.8924
Case 6	[20]	9.9632	12.1056	12.2849	14.1348	22.1067	23.4675	24.0060	25.4872
	DSC	9.9583	12.0469	12.2290	13.9973	22.0573	23.2478	23.9937	25.3169
Case 2	[20]	10.9644	12.9516	12.9516	14.7497	24.5340	24.6349	25.8056	25.8056
	[25]	10.967	12.902	12.902	14.589	24.548	24.646	—	—
	DSC	10.9658	12.9012	12.9012	14.5882	24.5411	24.6411	25.6751	25.6751
Case 10	[20]	905833	11.0138	11.7868	13.0741	20.9423	22.3488	23.7672	24.7380
	DSC	9.5841	10.9658	11.7353	12.9012	20.9442	22.1905	23.7726	24.5411

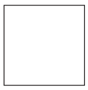
2.2. *Case studies*

Having built our confidence on the DSC approach for the internal line support, we present in this section the case studies of square plates having the 21 prescribed boundary conditions as shown in Figure 1. The elastic support edge is considered as a simply supported edge with rotational spring constraint along the edge.

Tables V and VI present extensive frequency parameters of the first eight modes for the fully simply supported square plates (i.e. Case 1) and the fundamental frequency parameters of the rest 20 cases of square plates. The seven types of internal line/point supports are arranged in the sequence as shown in Figure 2. The results presented in these tables are important for studying the vibration characteristics of square plates with respect to plate boundary conditions and internal line/point supports. These results may also be used by engineers in designing plate structures with complex internal supports.

The vibration responses of a simply supported square plate are studied in more detail to reveal the influence of internal supports on the plate. The frequency parameters and mode contour shapes of the first eight modes are shown in Table V. For plate with a central point support, it is observed that the plate is forced to vibrate in the (antisymmetric–symmetric (AS)

Table IV. Comparison studies of frequency parameter for square plate with central point support.

		Mode sequences							
		1	2	3	4	5	6	7	8
Case 1	[29]	5.0000	5.0000	—	8.0000	—	13.0000	13.0000	—
	[30]	5.3	5.3	—	—	—	—	—	—
	[31]	5.0	5.0	—	—	—	—	—	—
	[32]	5.4	5.4	—	—	—	—	—	—
	[33]	5.0000	5.0000	5.3872	8.0000	10.0000	13.0000	13.0000	15.0159
	[34]	—	—	5.33	—	—	—	—	—
	[35]	5.000	—	5.332	—	10.000	13.00	—	14.86
	[36]	5.00	5.00	5.48	8.00	10.00	13.00	13.00	15.28
	DSC	5.0000	5.0000	5.3376	8.0000	10.0000	13.0000	13.0000	14.8781
Case 4	[29]	5.2357	—	—	8.7272	—	—	—	—
	[33]	5.2357	5.2646	6.7922	8.7273	10.898	13.489	—	—
	DSC	5.1880	5.2357	6.6807	8.7275	10.8800	13.4771	14.2713	15.7634
Case 12	[29]	—	6.134	—	—	11.61	—	—	—
	[33]	5.358	6.134	7.834	9.420	11.61	14.66	—	—
	DSC	5.2609	6.1340	7.5570	9.4179	11.6076	14.6027	14.7718	16.5028
Case 6	[29]	—	7.202	—	10.22	—	—	—	—
	[33]	5.980	7.202	8.028	10.21	12.49	15.39	—	—
	DSC	5.9296	7.2019	7.7919	10.2132	12.4649	15.3916	16.0359	17.5893
Case 2	[29]	7.438	7.438	—	10.97	13.34	—	—	—
	[33]	7.436	7.436	8.136	10.96	13.33	16.72	16.72	19.25
	[36]	7.44	7.44	8.19	10.96	13.33	16.72	16.72	19.37
	DSC	7.4368	7.4368	7.9729	10.9658	13.3335	16.7208	16.7208	18.8954

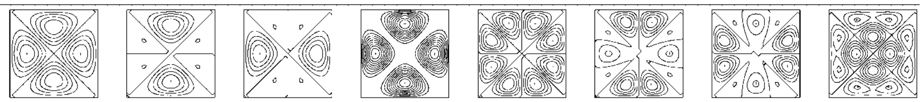
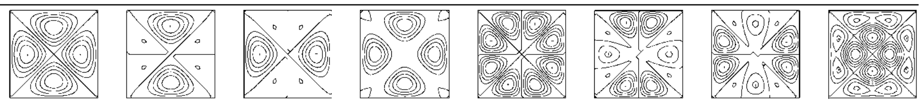
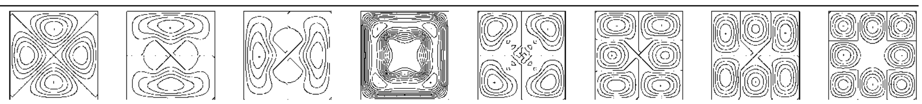
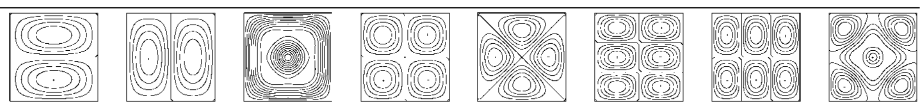
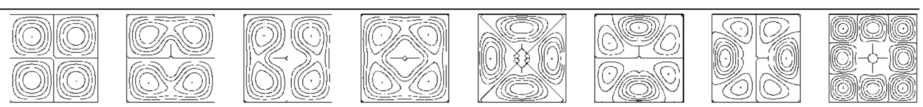
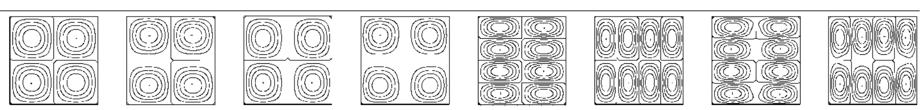
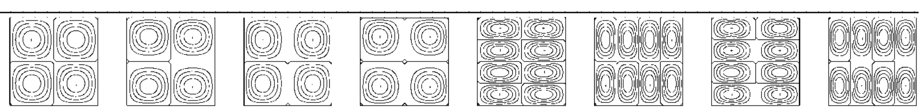
with respect to the x- and y-axis, respectively) and symmetric–antisymmetric (SA) modes as shown by the contour shapes of the first two modes. The third mode is an axisymmetric mode with respect the central point of the plate. There is no nodal line in this mode. A set of cross nodal lines is evident in the fourth mode (AA mode). The fifth mode is an SS mode. However, the nodal lines of the mode is a set of diagonal lines. The sixth and seventh modes are SA and AS modes with three half waves in one direction and two half waves in the other direction. The eighth mode is observed to be an axisymmetric mode with a close loop nodal line.

Table V also shows that when the two-third diagonal line supports are imposed, the plate is forced to vibrate in the same way as a plate with the full length diagonal line supports. The plate behaves quite differently when the one-third diagonal line supports are placed on the plate. Similar trends are also found in the plate with cross line supports.

The fifth frequency parameters for a plate with full length diagonal line supports and full length cross line supports are both 20. However, the mode shapes in the two cases are completely different as evident in Table V.

Table V also shows that the frequency parameters are greater for plates with cross line supports and diagonal line supports than those for plates with a central point support. The

Table V. Frequency parameters and modal shapes for a square plate, Case 1, with internal supports.

Support type	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
								
(I)	10.0000	12.2680	12.2680	14.8737	20.0000	23.2151	23.2151	26.0000
								
(II)	10.0000	12.2679	12.2679	14.8735	20.0000	23.2125	23.2125	26.0000
								
(III)	10.0000	11.2601	11.2601	11.4943	12.8464	14.0712	14.0712	17.5898
								
(IV)	5.0000	5.0000	5.3376	8.0000	10.0000	13.0000	13.0000	14.8781
								
(V)	8.0000	8.5685	8.5685	8.7052	13.8131	15.4903	15.4903	19.1079
								
(VI)	8.0000	9.5644	9.5644	10.9002	20.0000	20.0000	20.7944	20.7944
								
(VII)	8.0000	9.5841	9.5841	10.9658	20.0000	20.0000	20.9442	20.9442

diagonal line supports provide the strongest supports among the three considered internal support types.

Figures 3 and 4 present the variation of frequency parameters versus the support length for a clamped square plate (Case 2) with cross line and diagonal line supports, respectively.

Table VI. Fundamental frequency parameter for square plates with internal line supports (Types I–VII).

Case	Support type						
	I	II	III	IV	V	VI	VII
2	13.3335	13.3335	13.3335	7.4368	10.9658	10.9658	10.9658
3	12.0829	12.0829	12.0829	6.6187	9.7441	9.7441	9.7441
4	10.4177	10.4174	10.2891	5.1880	8.2424	8.4330	8.4354
5	10.3248	10.3246	10.2292	5.1488	8.2071	8.3127	8.3137
6	11.6096	11.6085	11.0560	5.9296	9.9069	9.5883	9.9583
7	12.9059	12.9058	12.8295	7.0143	10.5962	10.6130	10.6130
8	11.2117	11.2114	10.8611	5.6347	9.1409	9.1828	9.1829
9	12.3157	12.3156	12.2844	6.7054	9.9631	9.9934	9.9935
10	10.8201	10.8197	10.5847	5.2609	8.4266	8.8473	8.8520
11	10.6522	10.6521	10.4785	5.2164	8.3850	8.6162	8.6181
12	11.1996	11.1984	10.7365	5.5468	9.5841	9.5841	9.5841
13	10.8489	10.8485	10.5573	5.3255	8.8908	8.8908	8.8908
14	12.5583	12.5583	12.5073	6.8207	10.1773	10.2362	10.2363
15	12.6368	12.6367	12.5569	6.8025	10.3820	10.3820	10.3820
16	10.7378	10.7375	10.5316	5.2395	8.4082	8.7318	8.7349
17	11.0108	11.0101	10.6439	5.4302	9.0641	9.1624	9.1632
18	11.4354	11.4348	10.9735	5.7838	9.3444	9.5493	9.5503
19	11.5538	11.5528	11.0352	5.8779	9.8337	9.8595	9.8596
20	11.3702	11.3697	10.9453	5.7416	9.2963	9.4473	9.4480
21	11.2839	11.2836	10.8918	5.6785	9.2061	9.2870	9.2873

Frequency parameters of several SS, AS and SA modes are depicted in Figures 3 and 4. It is found that the frequency parameters increase with increasing support length in general. The frequency parameters remain the same when the internal support reaches certain length (effective length). The effective length is greater when the plate vibrates in higher modes. There are several modes where the frequency parameters remain constant when the support length increases. This is because the nodal lines of these modes coincide with the internal support lines.

The effect of internal supports on the vibration characteristics of simply supported square plates may also be observed for plates with other edge supporting conditions in Table VI.

The effect of edge support conditions can be studied by examining the frequency parameters in Tables V and VI. It is shown that the frequency parameters increase with the increasing level of boundary constraints from simply supported to elastically supported to clamped. This is because higher level boundary constraints increase the flexural rigidity of the plates, resulting in a higher frequency response. This trend can be found again from the results in Table VI for the other cases.

3. COMPLEX INTERNAL SUPPORT

In this section, the foregoing DSC formulation is used to investigate the influence of complex internal supports and plate boundary conditions on the vibration behaviour of square plates. Again, square plates of 21 combinations of simply supported, clamped and elastically

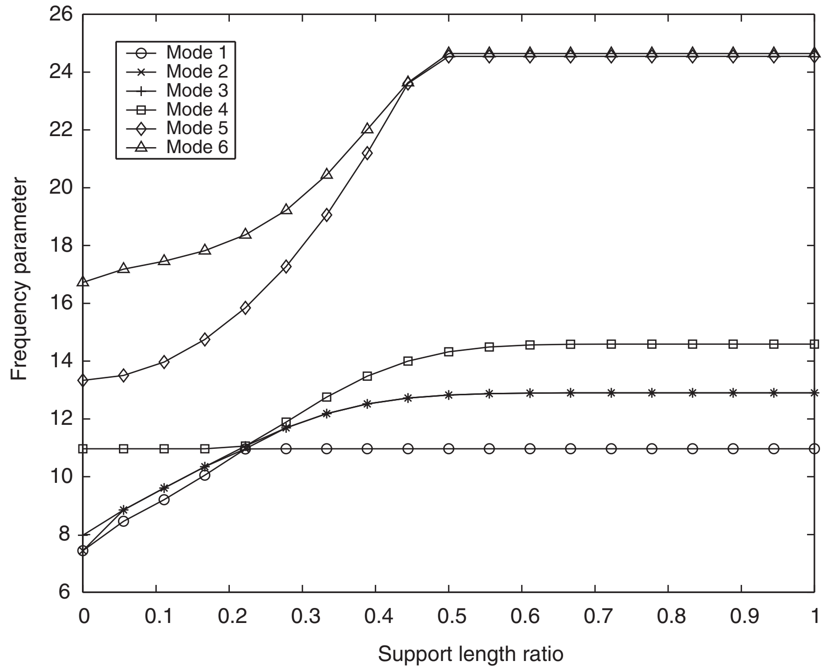


Figure 3. Frequency parameter versus cross support length ratio for a clamped square plate (Case 2).

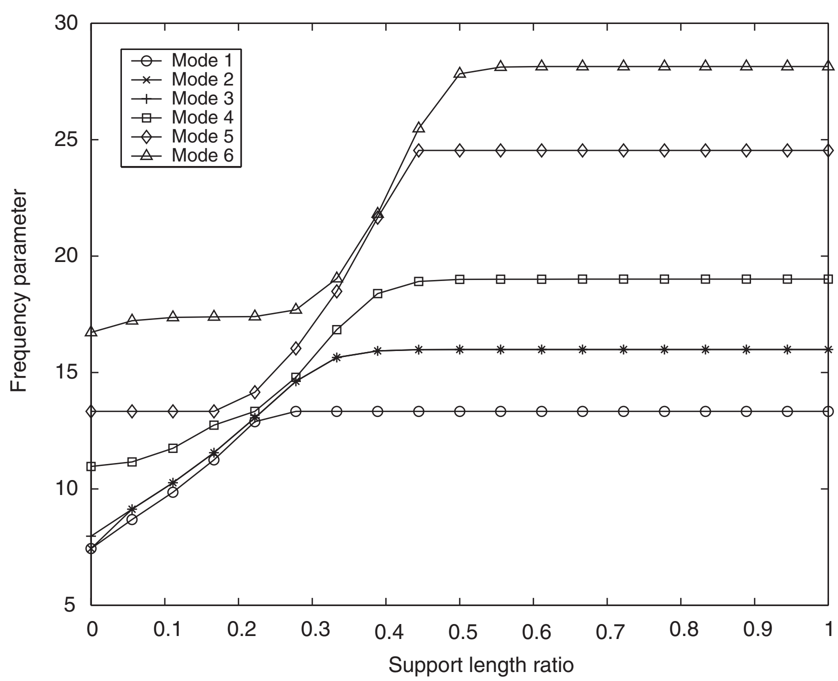


Figure 4. Frequency parameter versus diagonal support length ratio for a clamped square plate (Case 2).

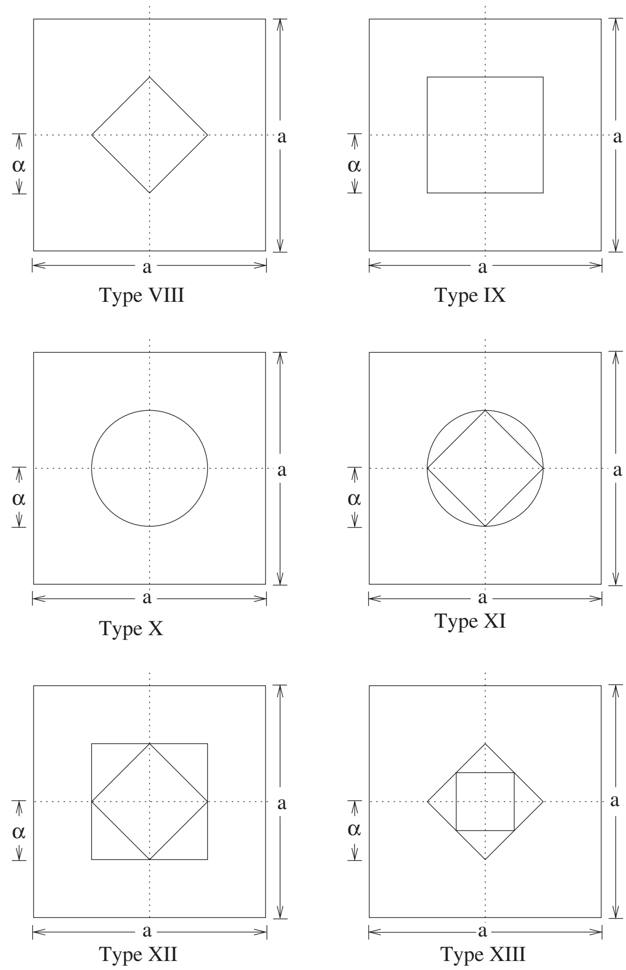


Figure 5. Different types of internal supports.

supported edges are selected in this study as shown in Figure 1. The internal supports considered are square, rhombus and circular loops and three of their combinations (see Figure 5). The size of the loop supports may be determined by a size parameter α (see Figure 5). The DSC parameters and the non-dimensional spring coefficient follow the choices mentioned in Section 2.

3.1. Convergence and comparison studies

As the DSC algorithm for plate analysis is an approximate method, the validity and accuracy of the method need to be examined through convergence and comparison studies. The convergence test is first carried out for three selected square plates, namely, SSSS (Case 1), CCCC (Case 2) and ECCS (Case 18) plates as shown in Figure 1.

Table VII. Convergence study of frequency parameters for SSSS square plates (Case 1) with internal support of Types VIII, IX and XII [DSC parameters: $(N, M, \sigma) \in \{(17, 16, 2.5), (33, 32, 4.0), (65, 32, 4.0)\}$].

Support type	Mesh size	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
VIII ($\alpha = a/4$)	$N=17$	10.2622	10.7291	10.7291	10.8759	18.7813	19.7418	19.7418	21.3115
	$N=33$	10.2831	10.7971	10.7971	10.9747	18.7779	19.7941	19.7941	21.2284
	$N=65$	10.3150	10.8300	10.8300	11.0090	18.8705	19.8852	19.8852	21.2324
IX ($\alpha = a/4$)	$N=17$	10.8087	19.2271	19.2271	20.8514	21.5555	22.2161	23.0031	23.0031
	$N=33$	10.7874	19.1548	19.1548	20.7271	21.3826	22.2158	22.9062	22.9062
	$N=65$	10.7865	19.1676	19.1676	20.7513	21.4126	22.2335	22.9291	22.9291
XII ($\alpha = a/4$)	$N=17$	20.9371	21.3812	21.3812	21.7999	23.8838	23.9522	24.9874	24.9874
	$N=33$	20.8667	21.2677	21.2677	21.6341	23.8532	24.0455	25.1012	25.1012
	$N=65$	20.9182	21.3115	21.3115	21.6652	23.8549	24.1524	25.1952	25.1952

Table VIII. Convergence study of frequency parameters for CCCC square plates (Case 2) with internal supportof Types VIII, IX and XII [DSC parameters: $(N, M, \sigma) \in \{(17, 16, 2.5), (33, 32, 4.0), (65, 32, 4.0)\}$].

Support type	Mesh size	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
VIII ($\alpha = a/4$)	$N=17$	14.4189	15.3680	15.3680	15.5231	22.9564	25.9082	26.4424	26.4424
	$N=33$	14.4056	15.4150	15.4150	15.6081	22.8784	25.7846	26.4433	26.4433
	$N=65$	14.4281	15.4447	15.4447	15.6445	22.8714	25.8768	26.5437	26.5437
IX ($\alpha = a/4$)	$N=17$	11.3174	23.0111	23.0111	30.3086	31.1817	31.1916	32.2591	32.2591
	$N=33$	11.2910	22.9373	22.9373	29.9977	30.7605	31.0316	31.8219	31.8219
	$N=65$	11.2887	22.9326	22.9326	30.0001	30.7667	31.0327	31.8329	31.8329
XII ($\alpha = a/4$)	$N=17$	23.8514	31.7256	31.7893	31.7893	32.1633	34.2092	35.1113	35.1113
	$N=33$	23.7361	31.3267	31.3893	31.3893	31.6816	34.2027	35.0338	35.0338
	$N=65$	23.7334	31.3580	31.4177	31.4177	31.6935	34.3249	35.1402	35.1402

Table IX. Convergence study of frequency parameters for ECCS square plates (Case 18) with internal support of Types VIII, IX and XII [DSC parameters: $(N, M, \sigma) \in \{(17, 16, 2.5), (33, 32, 4.0), (65, 32, 4.0)\}$].

Support type	Mesh size	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
VIII ($\alpha = a/4$)	$N=17$	11.7827	12.8260	14.1675	15.2327	21.0551	22.3651	23.4897	24.9743
	$N=33$	11.7610	12.8890	14.1361	15.2787	21.0010	22.2538	23.4892	24.8660
	$N=65$	11.7837	12.9228	14.1537	15.3089	21.0709	22.2500	23.5594	24.9298
IX ($\alpha = a/4$)	$N=17$	11.1457	20.3178	22.2339	24.1709	25.0421	27.2484	28.2542	30.1451
	$N=33$	11.1172	20.2570	22.0501	23.9405	24.9224	26.9402	27.7768	29.6870
	$N=65$	11.1147	20.2654	22.0346	23.9424	24.9325	26.9203	27.7412	29.6431
XII ($\alpha = a/4$)	$N=17$	22.4598	23.9820	24.4134	27.7642	28.4099	30.8877	31.8720	33.1607
	$N=33$	22.3948	23.8106	24.2413	27.5040	27.9156	30.5392	31.4498	32.9400
	$N=65$	22.4440	23.8101	24.2725	27.5178	27.9128	30.5508	31.4741	32.9647

Table X. Comparison study of frequency parameters for simply supported square plates with an internal ring support (Type X, $\alpha = a/4$).

Case	Method	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
	[38]	9.747	11.95	11.96	13.00	14.56	—	—	—
1	[27]	10.0236	12.8575	12.8575	13.4716	15.5596	19.7088	20.5990	20.5990
	DSC	10.0138	12.7655	12.7655	13.3181	15.3122	19.1912	20.0491	20.0491
	[38]	11.75	17.60	17.60	18.41	18.59	—	—	—
2	[27]	11.8380	18.6134	18.6134	19.4436	20.4630	26.7431	26.7431	27.6433
	DSC	11.7506	18.5325	18.5325	19.2809	20.2246	26.2661	26.2661	26.9000

Table XI. Fundamental frequency parameters for square plates with internal supports (Cases 1–7).

Support type	α	Case 1	2	3	4	5	6	7
VIII	$a/8$	7.4733	11.1446	9.7356	7.6409	7.6261	8.9251	10.4698
	$a/4$	10.2980	14.4161	12.6476	10.5123	10.4900	12.5060	13.7768
	$3a/8$	9.2610	9.8640	9.6516	9.4412	9.3684	9.7509	9.8172
	$a/2$	5.8668	6.0821	5.9817	5.9279	5.8973	6.0359	6.0589
IX	$a/8$	9.1237	13.6253	11.7739	9.3185	9.2991	10.5992	12.6660
	$a/4$	10.7876	11.2901	11.1209	10.9092	10.8690	11.1604	11.2474
	$3a/8$	5.5554	5.7167	5.6375	5.5956	5.5759	5.6762	5.6968
X	$a/8$	7.1195	10.7861	9.3941	7.4244	7.3773	8.7788	10.2495
X	$a/4$	10.0138	11.7506	11.2397	10.4019	10.2873	11.3338	11.6306
	$3a/8$	5.9016	6.0582	5.9874	5.9450	5.9240	6.0235	6.0417
	$a/2$	3.0601	3.8482	3.5815	3.2388	3.1814	3.6290	3.7797
XI	$a/8$	8.0895	12.1113	10.5151	8.2457	8.2329	9.5512	11.2910
	$a/4$	13.6848	20.0515	16.8889	13.7262	13.7255	16.1677	18.3700
	$3a/8$	10.9170	10.9357	10.9259	10.9224	10.9193	10.9317	10.9335
	$a/2$	6.2098	6.2575	6.2302	6.2220	6.2149	6.2459	6.2508
XII	$a/8$	9.4031	14.0580	12.1067	9.5556	9.5432	10.8716	12.9775
	$a/4$	20.9064	23.7348	23.5645	21.0696	21.0591	22.8408	23.7026
	$3a/8$	10.6347	10.6565	10.6447	10.6402	10.6372	10.6511	10.6535
XIII	$a/4$	11.3067	16.1795	13.8932	11.3187	11.3186	13.4902	15.0598
	$a/2$	11.9502	11.9788	11.9644	11.9578	11.9539	11.9720	11.9752

Table VII shows the first eight frequency parameters for an SSSS square plate (Case 1) with internal loop supports of Types VIII, IX and XII, respectively. The size parameter of the internal loop support α is set to be $a/4$. The number of DSC grid points $(N_x + 1) \times (N_y + 1)$ varies from 17^2 to 65^2. Unlike the convergence pattern of the Ritz method [37], where the frequency parameters decrease monotonically as the number of Ritz trial function terms increases, the frequency parameters from the DSC method may increase or decrease

Table XII. Fundamental frequency parameters for square plates with internal supports (Cases 8–14).

Support type	α	Case 8	9	10	11	12	13	14
	$a/8$	8.4662	9.8554	7.8230	7.7818	8.9144	8.4260	9.9769
VIII	$a/4$	11.6023	12.8590	10.6805	10.6518	12.2664	11.4076	13.0460
	$3a/8$	9.5657	9.7106	9.5890	9.4639	9.6443	9.4840	9.7632
	$a/2$	5.9548	6.0086	5.9813	5.9258	5.9908	5.9282	6.0336
	$a/8$	10.1670	11.9286	9.5223	9.4730	10.5539	10.1002	12.0821
IX	$a/4$	11.0356	11.1628	11.0312	10.9506	11.0379	10.9537	11.2047
	$3a/8$	5.6170	5.6573	5.6355	5.5963	5.6366	5.5966	5.6770
X	$a/8$	8.2972	9.6007	7.7141	7.6221	8.5746	8.0870	9.8039
X	$a/4$	10.9142	11.3724	10.7682	10.5531	11.0190	10.6630	11.4956
	$3a/8$	5.9669	6.0063	5.9839	5.9454	5.9892	5.9466	6.0239
	$a/2$	3.4408	3.6464	3.4171	3.3037	3.4482	3.3166	3.7111
	$a/8$	9.0868	10.6259	8.4232	8.3838	9.5409	9.0675	10.7440
XI	$a/4$	15.0500	16.8965	13.7675	13.7660	16.1676	15.0475	16.8999
	$3a/8$	10.9238	10.9286	10.9270	10.9216	10.9278	10.9217	10.9310
	$a/2$	6.2251	6.2371	6.2339	6.2200	6.2343	6.2200	6.2440
	$a/8$	10.4127	12.2200	9.7306	9.6924	10.8666	10.3974	12.3427
XII	$a/4$	22.2533	23.6160	21.2272	21.2060	22.6805	22.1385	23.6582
	$3a/8$	10.6422	10.6476	10.6456	10.6397	10.6457	10.6397	10.6506
XIII	$a/4$	12.5053	13.9201	11.3293	11.3292	13.4747	12.4837	13.9447
	$a/2$	11.9610	11.9681	11.9649	11.9574	11.9653	11.9576	11.9717

monotonically when the number of DSC grid points increases. There are also cases where the frequency parameter oscillates with increasing DSC grid points. Nevertheless, the frequency parameters show a satisfactory convergence even when the number of DSC grid points is 17^2.

Tables VIII and IX present the frequency parameters against the number of DSC grid points for a CCCC plate (Case 2) and an ECCS plate (Case 18), respectively. The convergence patterns of the frequency parameters in these two cases show the same trends as for the simply supported square plate (Case 1). The frequency parameters converge to an acceptable level with the number of DSC grid points being 17^2. It is observed that even with 17^2 DSC grid points, the frequency parameters for the three selected cases have well converged. To ensure the accuracy and efficiency of the solutions, 41^2 DSC grid points are used for all other calculations in this section.

To verify the correctness of the DSC results, a comparison study is performed against the existing results from the literature. Table X gives the vibration frequencies for a simply supported square plate (SSSS plate) and a clamped square plate (CCCC plate) with a concentric internal ring support. The size parameter of the ring support $\alpha = a/4$. We observed that the frequency parameters generated by the DSC method are in good agreement with those reported by Liew *et al.* [27] and Nagaya [38]. The convergence and comparison

Table XIII. Fundamental frequency parameters for square plates with internal supports (Cases 15–21).

Support type	α	Case 15	16	17	18	19	20	21
VIII	$a/8$	10.3915	7.8015	8.5929	8.6308	8.9251	8.6302	8.4667
	$a/4$	13.5477	10.6669	11.6148	11.7977	12.4829	11.7838	11.6213
	$3a/8$	9.7715	9.5290	9.5624	9.6853	9.7068	9.6349	9.6218
	$a/2$	6.0359	5.9545	5.9593	6.0085	6.0132	5.9838	5.9814
IX	$a/8$	12.5592	9.4965	10.2709	10.3342	10.5988	10.3332	10.1682
	$a/4$	11.2054	10.9909	10.9954	11.1178	11.1199	11.0774	11.0759
	$3a/8$	5.6772	5.6159	5.6165	5.6562	5.6568	5.6368	5.6365
X	$a/8$	10.0424	7.6671	8.2797	8.4988	8.7666	8.4829	8.3149
	$a/4$	11.5194	10.6220	10.8277	11.1637	11.2437	11.0678	11.0146
	$3a/8$	6.0253	5.9652	5.9678	6.0045	6.0072	5.9870	5.9856
	$a/2$	3.7153	3.3606	3.3807	3.5615	3.5729	3.5051	3.4977
XI	$a/8$	11.2373	8.4026	9.2359	9.2538	9.5512	9.2536	9.0870
	$a/4$	18.3676	13.7668	15.1367	15.1414	16.1677	15.1414	15.0500
	$3a/8$	10.9313	10.9244	10.9247	10.9291	10.9295	10.9267	10.9265
	$a/2$	6.2441	6.2270	6.2271	6.2389	6.2392	6.2321	6.2320
XII	$a/8$	12.9346	9.7106	10.5669	10.5812	10.8716	10.5811	10.4128
	$a/4$	23.6714	21.2165	22.3285	22.4504	22.8361	22.4478	22.2556
	$3a/8$	10.6506	10.6426	10.6427	10.6481	10.6481	10.6452	10.6451
XIII	$a/4$	15.0326	11.3293	12.5219	12.5460	13.4901	12.5458	12.5055
	$a/2$	11.9717	11.9612	11.9614	11.9683	11.9685	11.9648	11.9646

studies in this subsection have confirmed the validity and accuracy of the proposed DSC method.

3.2. Case studies

Tables XI–XIII present the fundamental frequency parameter for square plates of 21 combinations of edge support conditions (see Figure 1) with six types of internal complex supports (see Figure 5). The size parameter of the internal supports is chosen to be $\alpha = a/8, 2a/8, 3a/8$ and $4a/8$ for types VIII, X and XI, $\alpha = a/8, 2a/8$ and $3a/8$ for types IX and XII, and $\alpha = 2a/8, 4a/8$ for type XIII, respectively. The effects of the edge support conditions, the types of internal supports and the size parameter of the internal supports on the vibration frequencies may be observed from the results in Tables XI–XIII.

The effect of edge support conditions on the frequency parameters can be examined from results in Tables XI–XIII. It is seen that the frequency parameters for the SSSS plate (Case 1, in Table XI) are smaller than those for the CCCC plate (Case 2, in Table XI). Higher constraint at the edges (in the order from S to E to C) increases the flexural rigidity of the plate, leading to a higher frequency response. Tables XI–XIII also provide information on the effect of internal support type on the frequency parameters. For a given size parameter α, a square plate with Type X internal support (the ring) has the lowest frequency parameters.

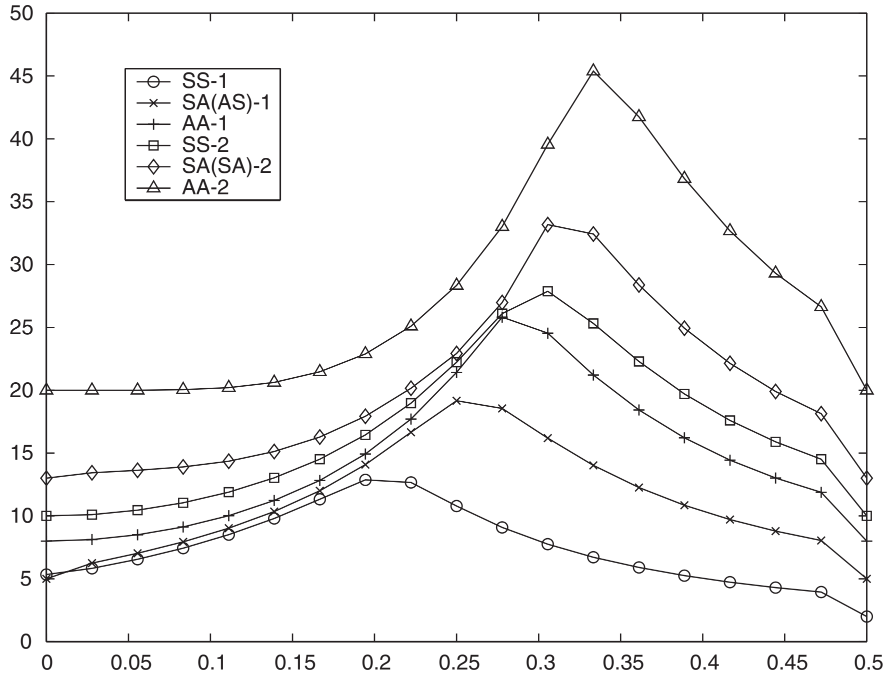


Figure 6. Variation of frequency parameters versus internal support size ratios for SSSS square plates (Case 1).

Higher frequency parameters are found for plates with Type XII support—a combination of a square and a rhombus. In general, for a given edge condition, the larger the unsupported free section of a plate, the lower will be the first vibration mode. As a result, the lowest first frequency parameter (3.0601) is observed for SSSS plate (Case 1) with Type X support and the highest first frequency parameter (23.7348) occurs in the combination of Case 2 and Type XII support.

Figure 6 shows the eight frequency parameters, corresponding to the first two SS (symmetric–symmetric), SA/AS type and AA (antisymmetric–antisymmetric) type modes, respectively, against the support size ratio $2\alpha/a$ for the SSSS square plate (Case 1) with a rhombus internal support (Type VIII support). We can observe that all frequency parameters increase when the value of $2\alpha/a$ varies from 0 to about 0.2. The frequency parameter from the first mode decreases as the value of $2\alpha/a$ is greater than 0.2. The same trend occurs for the other modes with a different key value of $2\alpha/a$. It is evident from Figure 6 that this type of internal support is most effective for the SSSS plate when the support size ratio $2\alpha/a$ is in between 0.2 and 0.35. The variation of frequency parameters under other support types have a similar tendency. In general, large frequency parameters occur at the internal support structure which is about half of the size of the plate.

To have an insight view on the vibration behaviour of plates with complex internal supports, we present selected mode shapes for the SSSS square plate (Case 1). Figures 7–12 show the first six mode shapes of the simply supported SSSS plates (Case 1) having the

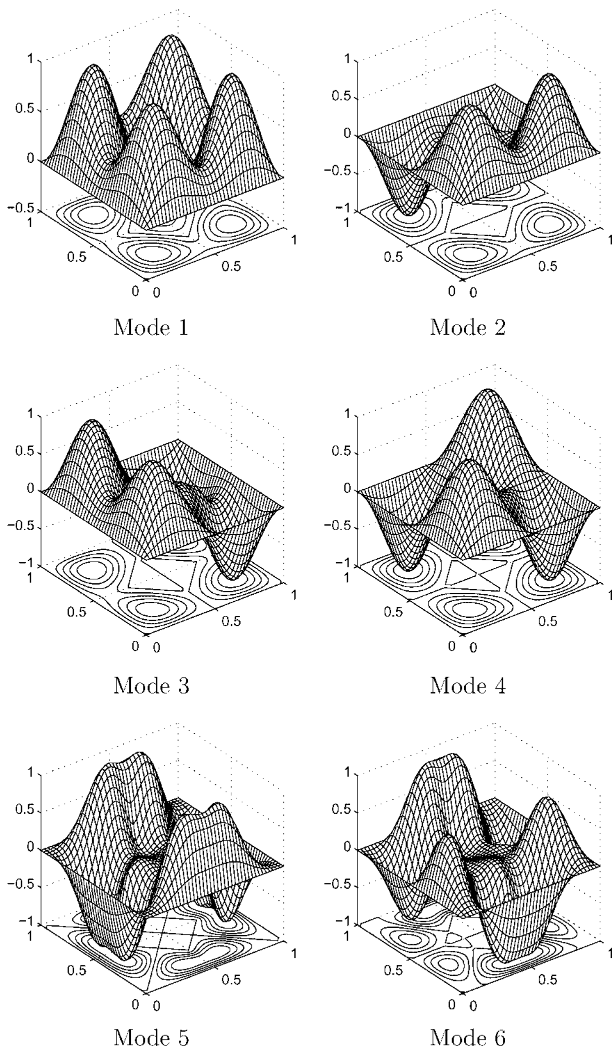


Figure 7. The first six eigenmodes for SSSS square plates (Case 1) with Type VIII internal support ($\alpha = a/4$).

six types of internal supports. The support size ratio $2\alpha/a$ is set to be 0.5 for all cases. Due to the symmetrical characteristics of the edge constraints and the internal supports, the SSSS plate always vibrates in one of its SS, SA, AS or AA modes with respect to the x- and y-axis. Consequently, modes 2 (SA) and 3 (AS) are always degenerated. Two other plates with the CCCC (Case 2) and EEEE (Case 3) edge conditions have very similar mode shapes as those in Figures 7–12. Obviously, since the symmetry is broken fully or partially in Cases 4–21, it is expected that the mode shapes behave differently in these cases.

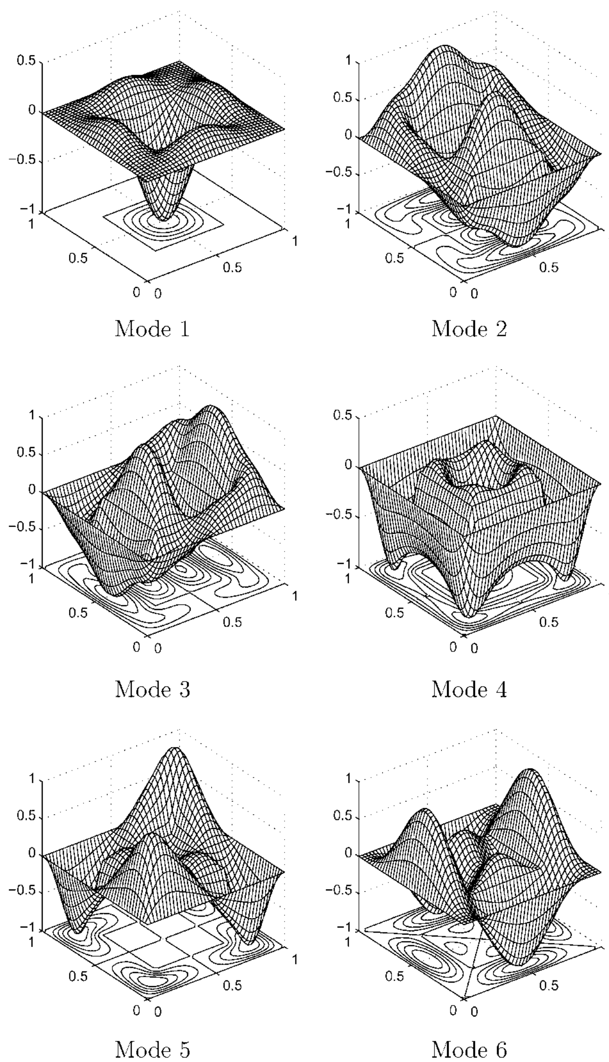


Figure 8. The first six eigenmodes for SSSS square plates (Case 1) with Type IX internal support ($\alpha = a/4$).

4. CONCLUSIONS

This paper explores the utility, tests the accuracy and examines the convergence of the proposed discrete singular convolution (DSC) algorithm for the free vibration analysis of rectangular plates with internal supports. Two classes of internal supports, i.e. partial internal line supports and complex internal supports, are treated in association with 21 combinations of edge support conditions. Particular attention is paid to the effects of different size, shape and topology of the internal supports and different boundary conditions on the vibration response

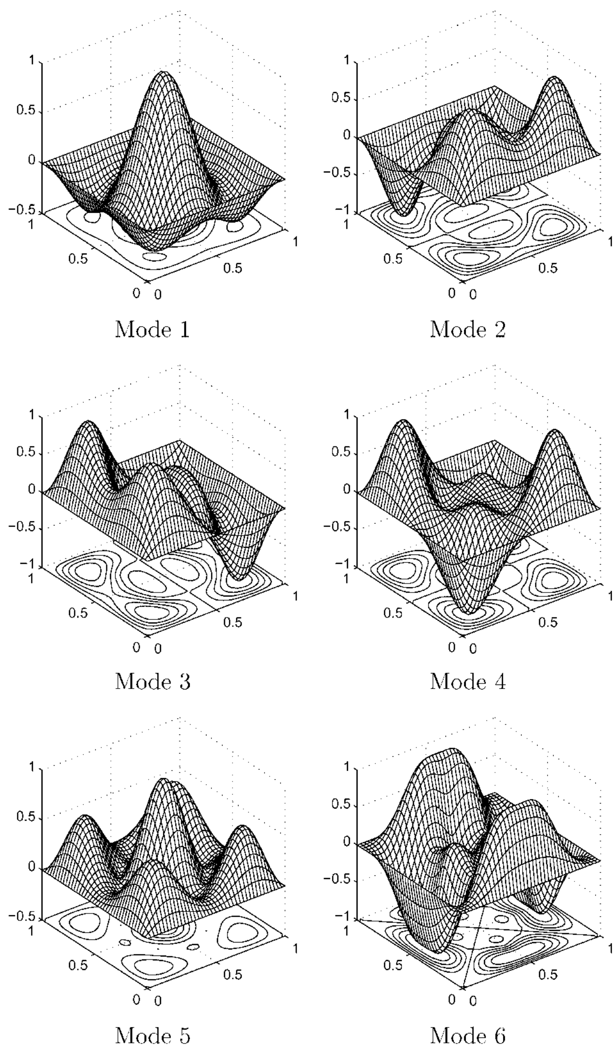


Figure 9. The first six eigenmodes for SSSS square plates (Case 1) with Type X internal support ($\alpha = a/4$).

of plates. For example, the partial internal line supports vary from a central point support to a full range of cross or diagonal line supports. Moreover, several closed-loop supports such as ring, square and rhombus, and their combinations are investigated for complex internal supports. The DSC algorithm is validated by carefully designed convergence studies. Extensive comparison is carried out between the DSC results and those in available literature.

For the partial line supports, the convergence studies have been carried out with two selected square plates with internal cross and diagonal line supports. The correctness of the frequency parameters has been checked against available solutions in the open literature. The

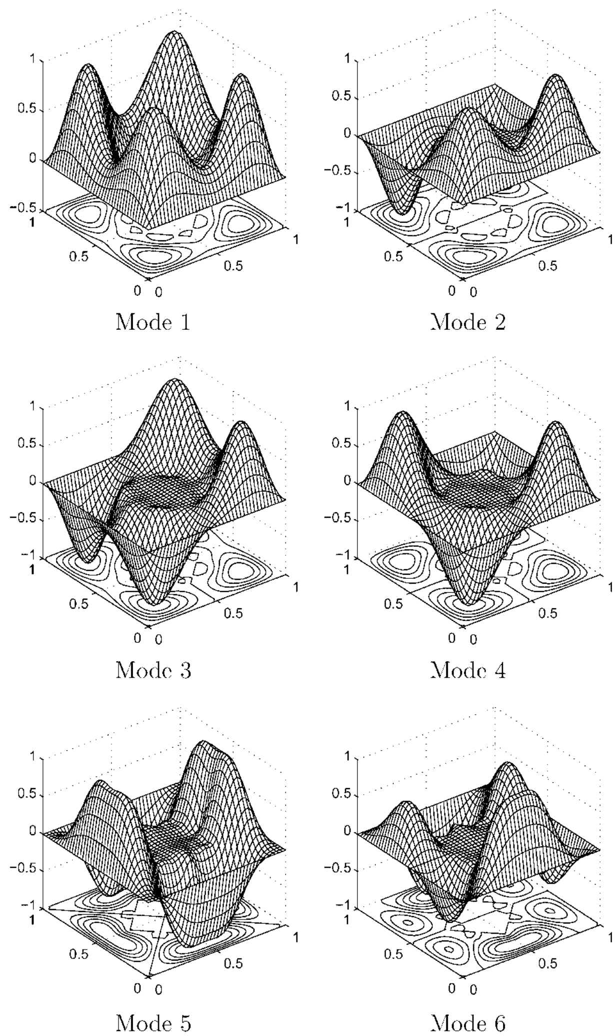


Figure 10. The first six eigenmodes for SSSS square plates (Case 1) with Type XI internal support ($\alpha = a/4$).

DSC results are in good agreement with results produced by finite element methods and Ritz methods. The convergence study showed that the DSC method can produce highly converged frequency parameters when the DSC grid points are doubled and quadrupled. In fact, converged results can be attained with a small mesh size of 17^2 for all tested cases. To ensure the correctness of the present results, a larger mesh size (37^2) is employed in the case study. Extensive frequency parameters have been presented in this paper for square plates with different combinations of edge support conditions and partial internal cross line, diagonal line and central point supports. The general trends of the frequency parameters with respect to the internal supports and boundary conditions are discussed. It is found that partial internal

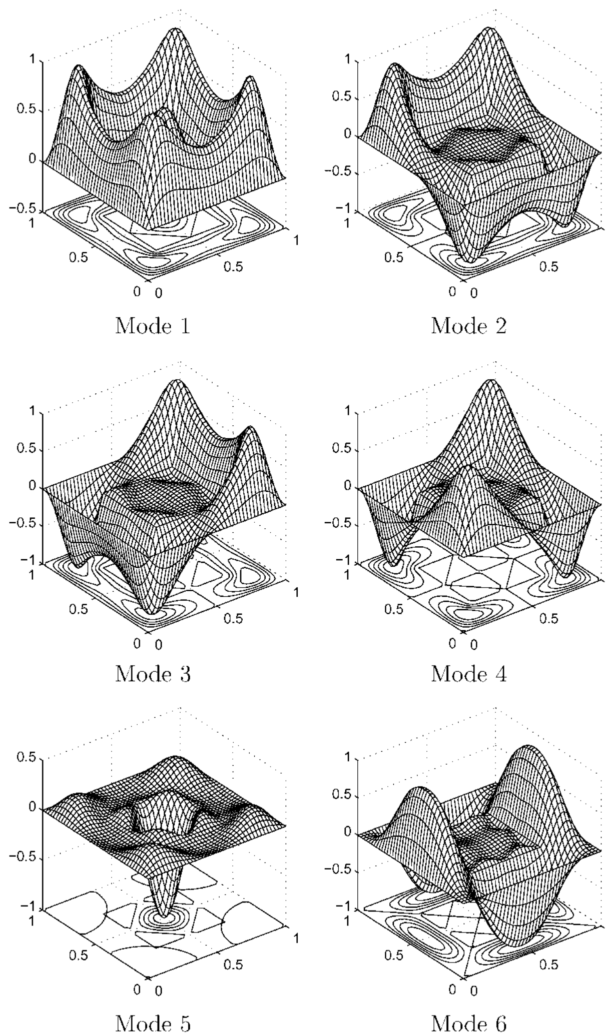


Figure 11. The first six eigenmodes for SSSS square plates (Case 1) with Type XII internal support ($\alpha = a/4$).

cross and diagonal line supports in a plate may produce almost the same effect as for the plate with full length internal cross and diagonal line supports. The diagonal supports are more effective than the cross support in increasing the vibration frequencies of square plates.

The problem of plates with complex internal supports is of great importance in engineering designs and has received little attention in the literature, partially due to the numerical difficulty. The simplicity and flexibility of the DSC method for vibration analysis of rectangular plates with complex internal supports have been demonstrated in this paper. The DSC results are compared with available solutions for the internal ring support from the open

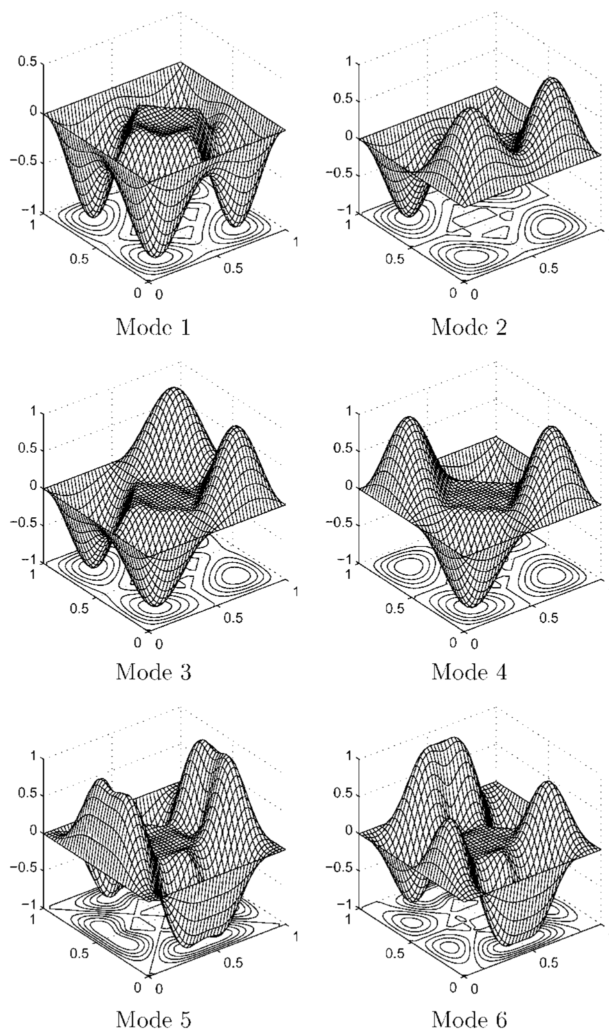


Figure 12. The first six eigenmodes for SSSS square plates (Case 1) with Type XIII internal support ($\alpha = a/4$).

literature. The convergence and comparison studies show that the DSC method can generate accurate vibration frequencies for plates with complex internal supports. Extensive frequency parameters are presented for square plates of 21 combinations of edge support conditions and with six different types of internal supports. The effectiveness of the internal loop supports in increasing vibration frequencies are discussed. It is found that internal supports are most effective when their sizes are about half the size of the plate. The tabulated frequency parameters for square plates with complex internal supports may serve as valuable information for engineers in designing plate structures and as benchmark solutions for researchers to check their numerical methods.

Numerical experiments indicate that the DSC algorithm exhibits controllable accuracy for plate analysis and possesses excellent flexibility in treating complex boundary conditions and support conditions. Although this paper only presents vibration results for square plates with partial line supports and complex internal supports, the DSC method is readily applied to rectangular plates with irregular supports and plates of other shapes.

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